MFDS — Mathematics Assignment-1

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Question 1: Write a code to perform Gauss elimination with and without pivoting for a 2x2 system, taking the number of significant digits (d) to be considered as user input. Using the code, solve the 2x2 system with random coefficients for d = 3, 4, 5 and 6. Display the results in Tabular form.

Solution: Considering the system of linear equations $Ax = b$ for a 2x2 random matrix.

$$\underbrace{\begin{bmatrix} 9.42477796 & 169.64600329 \\ 210.48670779 & 106.81415022 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} \begin{bmatrix} 197.92033718 \\ 169.64600329 \end{bmatrix}$$

Method	REF Matrix	Significant digits 3	4	5	6
without pivoting	$\begin{bmatrix} 9.42477796 & 169.64600329 \\ 0.0 & -3681.94659 \end{bmatrix}$	$\begin{bmatrix} 0.228 \\ 1.154 \end{bmatrix}$	$\begin{bmatrix} 0.2208 \\ 1.1544 \end{bmatrix}$	$\begin{bmatrix} 0.22008 \\ 1.15444 \end{bmatrix}$	$\begin{bmatrix} 0.220134 \\ 1.154437 \end{bmatrix}$
with partial pivoting	$\begin{bmatrix} 210.486708 & 106.814150 \\ 0.04776 & 164.86393 \end{bmatrix}$	$\begin{bmatrix} 0.220 \\ 1.154 \end{bmatrix}$	$\begin{bmatrix} 0.2202 \\ 1.1544 \end{bmatrix}$	$\begin{bmatrix} 0.22013 \\ 1.15444 \end{bmatrix}$	$\begin{bmatrix} 0.220136 \\ 1.154437 \end{bmatrix}$

Matrix-B	
without pivoting	with Partial pivoting
$\begin{bmatrix} 197.92033718 \\ -4280.57486075 \end{bmatrix}$	$\begin{bmatrix} 169.64600329 \\ 190.22426779 \end{bmatrix}$

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Question 2: Write a code to perform

(a) Gauss Jacobi method
(b) Gauss Seidel method

A For a 3X3 system by checking the convergence criteria using a suitable norm. Test the method on a random 3X3 system, which is diagonally dominant and check your results. A comparison between the two methods should be presented in a tabular form. The stopping criteria could be taken as the lowest iteration number when the relative percentage error is less than 1%.

B Generate a random matrix of size 3X3 which cannot be made diagonally dominant and check if the iterates converge. The random entries generated should be of the form n.dddd.

Solution: The random matrix generated to test the convergence

Q2 A: Of a diagonally dominant matrix is as below:-

$$\underbrace{\begin{bmatrix} 9.4248 & 3.1416 & 3.1416 \\ 6.2832 & 15.708 & 0.000 \\ 3.1416 & 12.5664 & 18.8496 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 175.9292 \\ 72.2566 \\ 47.1239 \end{bmatrix}}_{b}$$

$$\text{Normalized Matrix} = \begin{bmatrix} 1.0 & 0.3333 & 0.3333 \\ 0.4 & 1.0 & 0.1 \\ 0.1667 & 0.6667 & 1.0 \end{bmatrix}$$

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Diagonally dominant matrix and Solution Converges

	Gauss-Jacobi			Gauss-Seidel		
No of Iterations	x_1	x_2	x_3	x_1	x_2	x_3
1	18.6666	4.6	2.5	18.6666	-2.8666	1.2995
2	16.2002	-2.8666	-3.6785	19.1889	-3.0756	1.3517
3	20.8481	-1.9201	1.6939	19.2412	-3.0965	1.3569
4	18.742	-3.7392	0.3048	19.2464	-3.0986	1.3574
5	19.8113	-2.8968	1.8686	19.2469	-3.0988	1.3575
6	19.0093	-3.3245	1.1288	19.247	-3.0988	1.3575
7	19.3984	-3.0037	1.5476	19.247	-3.0988	1.3575
8	19.1519	-3.1594	1.2689	19.247	-3.0988	1.3575
9	19.2967	-3.0608	1.4138			
10	19.2155	-3.1187	1.3239	Solution Converges much faster, i.e on 8-th iteration itself. Hence Gauss-Seidel is much faster & Efficient		
11	19.2648	-3.0862	1.376			
12	19.2366	-3.1059	1.3461			
13	19.2531	-3.0946	1.3640			
14	19.2434	-3.1012	1.3537			
15	19.2490	-3.0974	1.3597			
16	19.2458	-3.0996	1.3562			
17	19.2477	-3.0983	1.3582			
18	19.2466	-3.0991	1.3570			
19						

Convergence matrix
$$\begin{bmatrix} 0. & -0.3333 & -0.3333 \\ 0. & 0.13332 & 0.13332 \\ 0. & -0.03332 & -0.03332 \end{bmatrix}$$

Vector 1-norm $= 0.4999 < 1$

Vector 2-norm $= 0.6666 < 1$

Vector-frobenius norm $= 0.5099 < 1$

"Hence matrix satisfies Convergence criteria"

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Non-Diagonally dominant Matrix [Non-converging]

Divergence Table

No of Iterations	Gauss-Jacobi			Gauss-Seidel		
	x_1	x_2	x_3	x_1	x_2	x_3
1	6.1667	35.9999	1.5714	6.1667	11.3331	-7.096
2	-25.4034	5.06975	-10.6209	3.8188	49.1086	-9.8108
3	12.5815	180.0971	29.8837	-24.9447	175.0249	5.0701
4	-173.7919	-133.8609	-38.5439	-144.7491	594.7161	82.0203
5	156.2569	885.3431	219.3269	-571.4305	1993.6408	869.7681
6	-950.9166	-1466.3353	-303.5301	-2024.9023	6656.5366	1364.6131
7	1531.594	5053.7867	1297.9133	-6905.3383	22198.9009	4721.4597
8	-5503.0671	-11282.0293	-2471.0735	-23213.6371	74004.7096	15957.1643
9	11878.5552	31932.5623	7903.2288	-77619.1221	246683.831	53461.3466
10	-34506.4663	-79091.1361	-18137.5925			
11	84050.4029	210612.2351	50741.1551			

Solution is divergin in each iteration.

Random Matrix generated for the above iterative methods:

$$\begin{bmatrix} 18.8496 & 15.708 & 18.8496 \\ 25.1327 & 6.2832 & 25.1327 \\ 25.1327 & 3.1416 & 21.9911 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 116.2389 \\ 226.1947 \\ 34.5575 \end{bmatrix}$$

Matrix after normalization:-

$$\begin{bmatrix} 1.0 & 0.8333 & 1.0 \\ 4.0 & 1.0 & 4.0 \\ 1.4929 & 0.1429 & 1.0 \end{bmatrix}$$

Convergence matrix

$$\begin{bmatrix} 0. & -0.8333 & -1.0 \\ 0. & 3.3332 & 0.0 \\ 0. & 0.47606 & 1.1429 \end{bmatrix}$$

Vector-1-Norm = 4.6426 > 1

Vector 2- Norm = 3.332 > 1

Frobenius -Norm = 3.7865 > 1

As the norm is > 1 the solution is diverging.