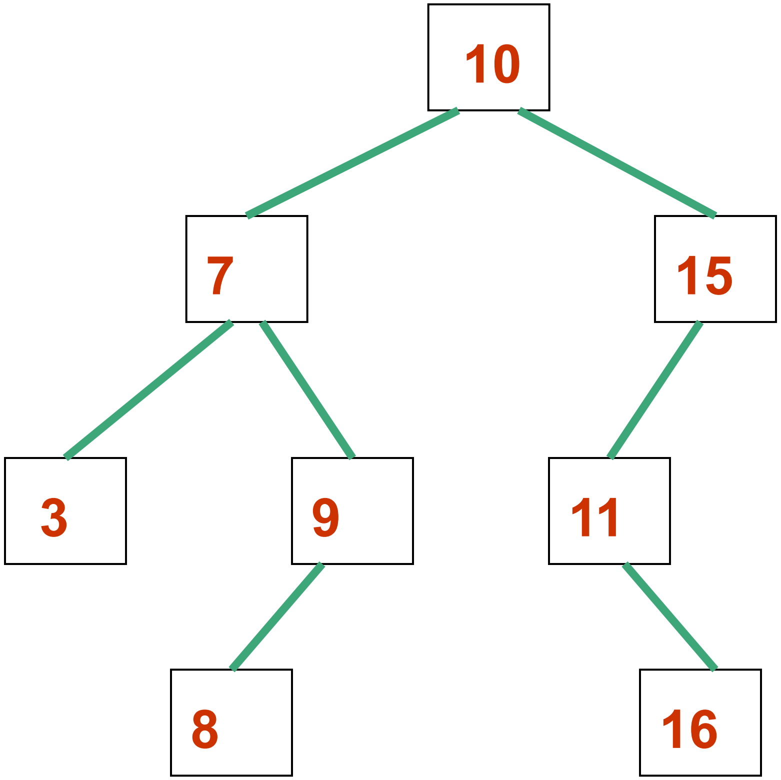
Topic 15

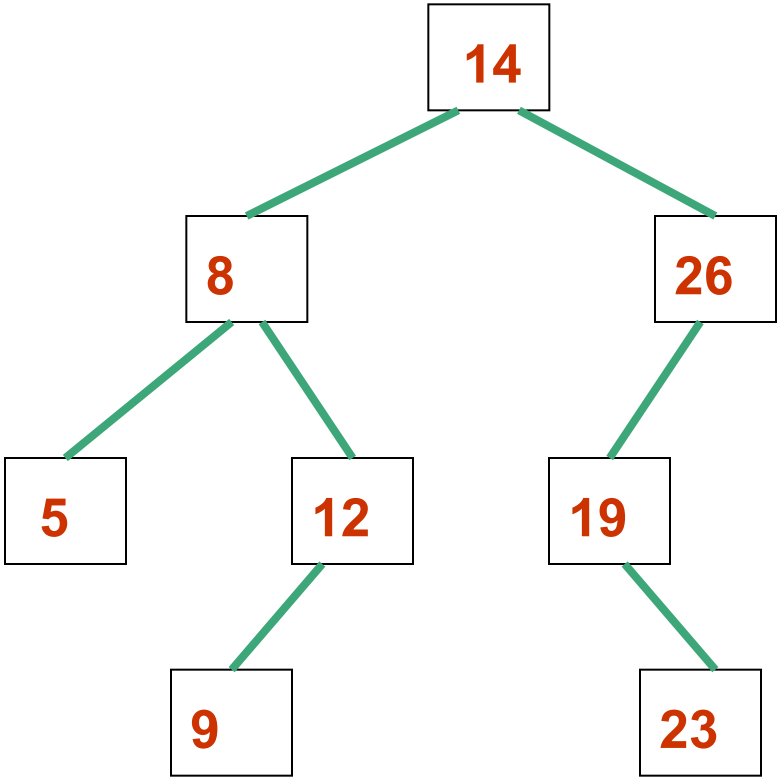
The Binary Search Tree ADT

Binary Search Tree

- A ***binary search tree (BST)*** is a binary tree with an ***ordering*** property of its elements, such that the data in any internal node is
  - ***Greater than*** the data in any node in its left subtree
  - ***Less than*** the data in any node in its right subtree
- ***Note***: this definition does not allow duplicates; some definitions do, in which case we could say "***less than or equal to***"

Examples: are these Binary Search Trees?





Discussion

- Observations:
  - What is in the leftmost node?
  - What is in the rightmost node?

BST Operations

- A binary search tree is a special case of a binary tree
  - So, it has all the operations of a binary tree
- It also has ***operations specific to a BST***:
  - ***add*** an element (requires that the BST property be maintained)
  - ***remove*** an element (requires that the BST property be maintained)
  - ***remove the maximum*** element
  - ***remove the minimum*** element

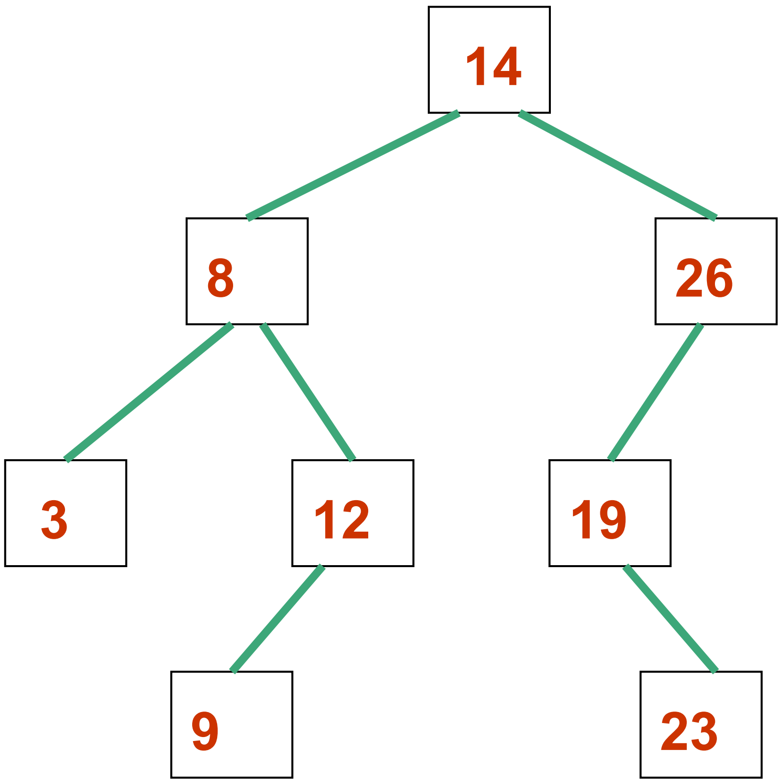
Searching in a BST

- Why is it called a binary ***search*** tree?
  - Data is stored in such a way, that it can be more ***efficiently*** found than in an ordinary binary tree

Searching in a BST

- **Algorithm to *search* for an item in a BST**
  - Compare data item to the root of the (sub)tree
  - If data item = data at root, found
  - If data item < data at root, go to the left; if there is no left child, data item is not in tree
  - If data item > data at root, go to the right; if there is no right child, data item is not in tree

Search Operation – a Recursive Algorithm



To search for a value k; returns true if found or false if not found

If the tree is empty, return false.

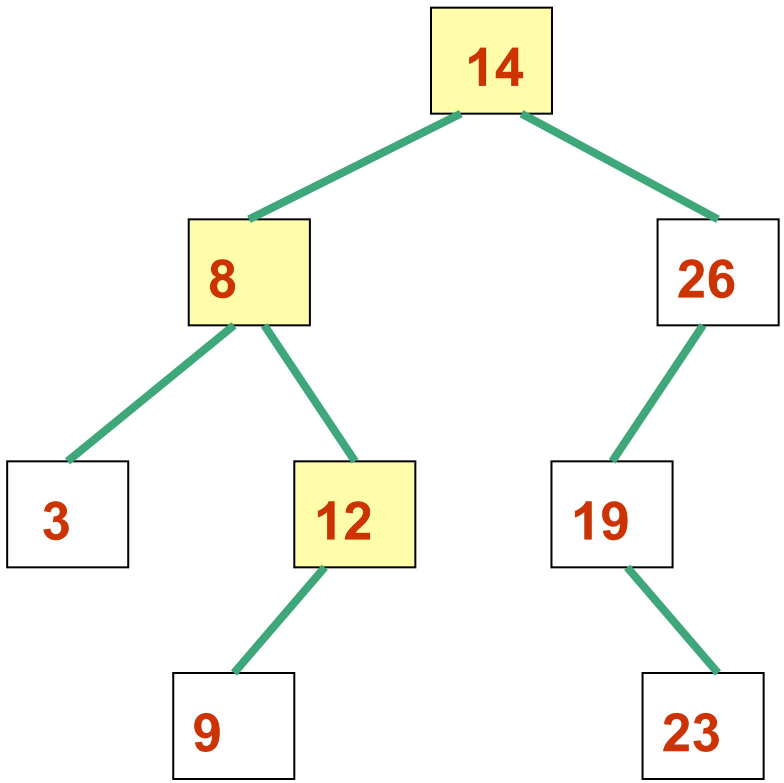
If k == value at root

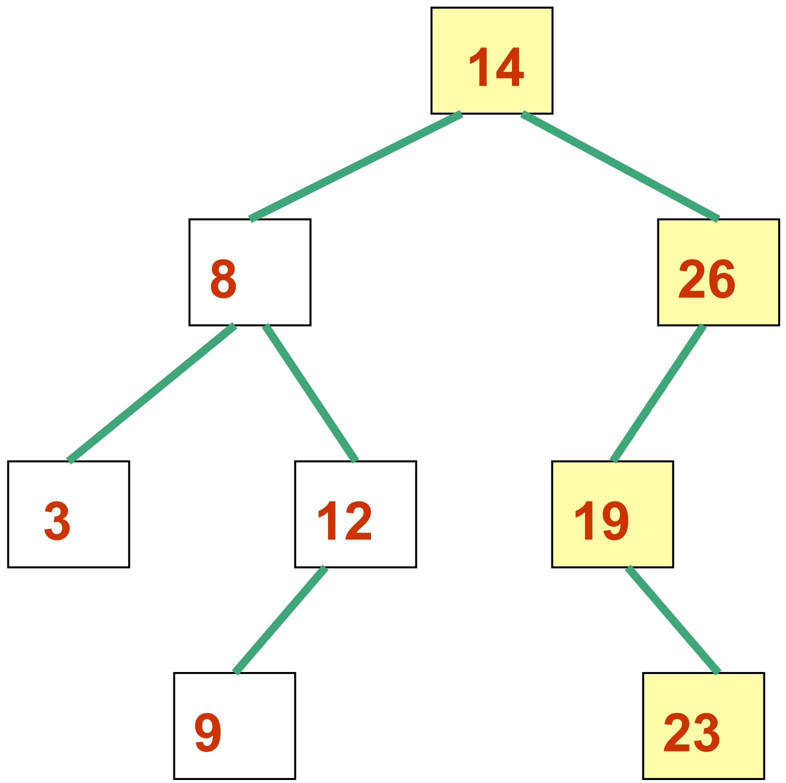
return true: we're done.

If k < value at root
return result from search for k in the left subtree

Else
return result from search for k in the right subtree.

Search Operation





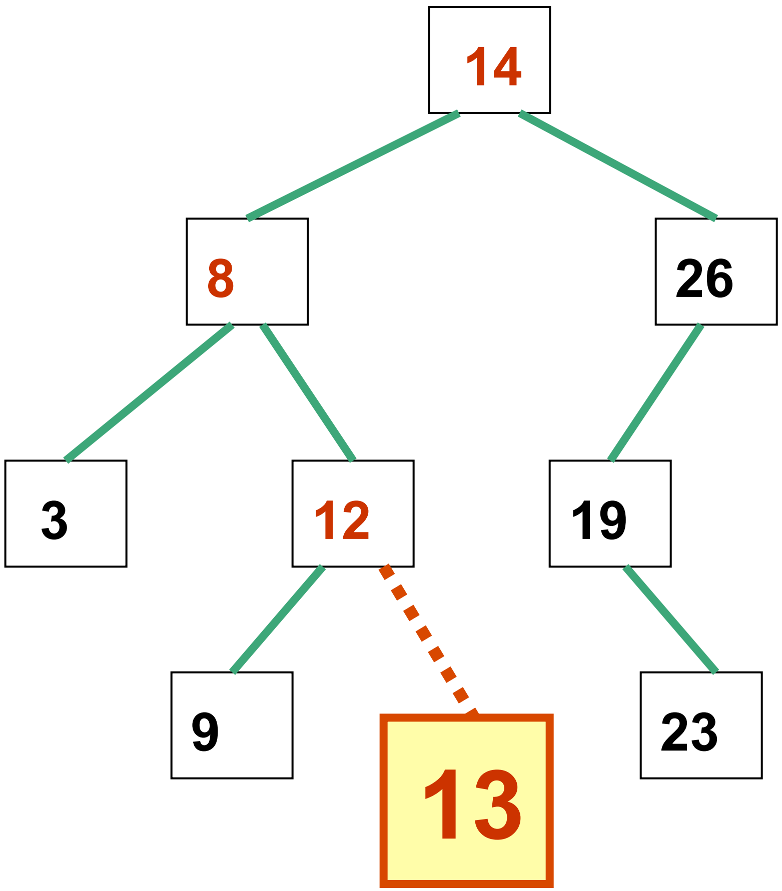
Search for 13: **visited nodes are coloured yellow; return false when node containing 12 has no right child**

Search for 22: **return false when node containing 23 has no left child**

BST Operations: **add**

- To ***add*** an item to a BST:
  - Follow the algorithm for searching, until there is no child
  - Insert at that point
- So, new node will be added as a leaf
- (We are assuming no duplicates allowed)

Add Operation



To insert 13:

Same nodes are visited as when *searching* for 13.

Instead of returning *false* when the node containing 12 has no right child, build the new node, attach it as the right child of the node containing 12, and return *true*.

Add Operation – an Algorithm

To insert a value k into a tree, returning true if successful and false if not

Build a new node for k.

If tree is empty

add new node as root node, return true.

If k == value at root

return false (no duplicates allowed).

If k < value at root

If root has no left child

add new node as left child of root, return true

Else insert k into left subtree of root.

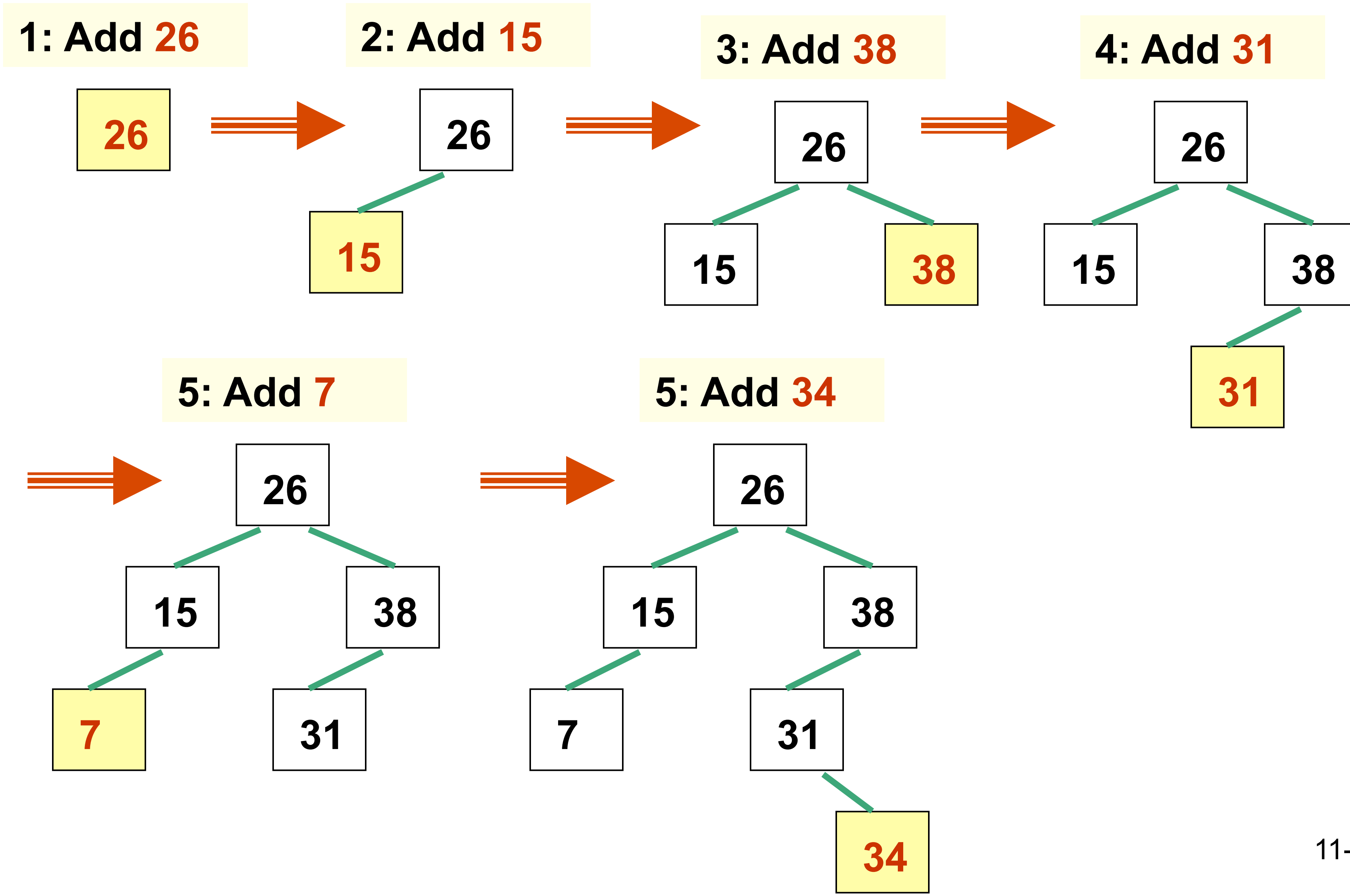
If k > value at root

If root has no right child

add new node as right child of root, return true

Else insert k into the right subtree of root.

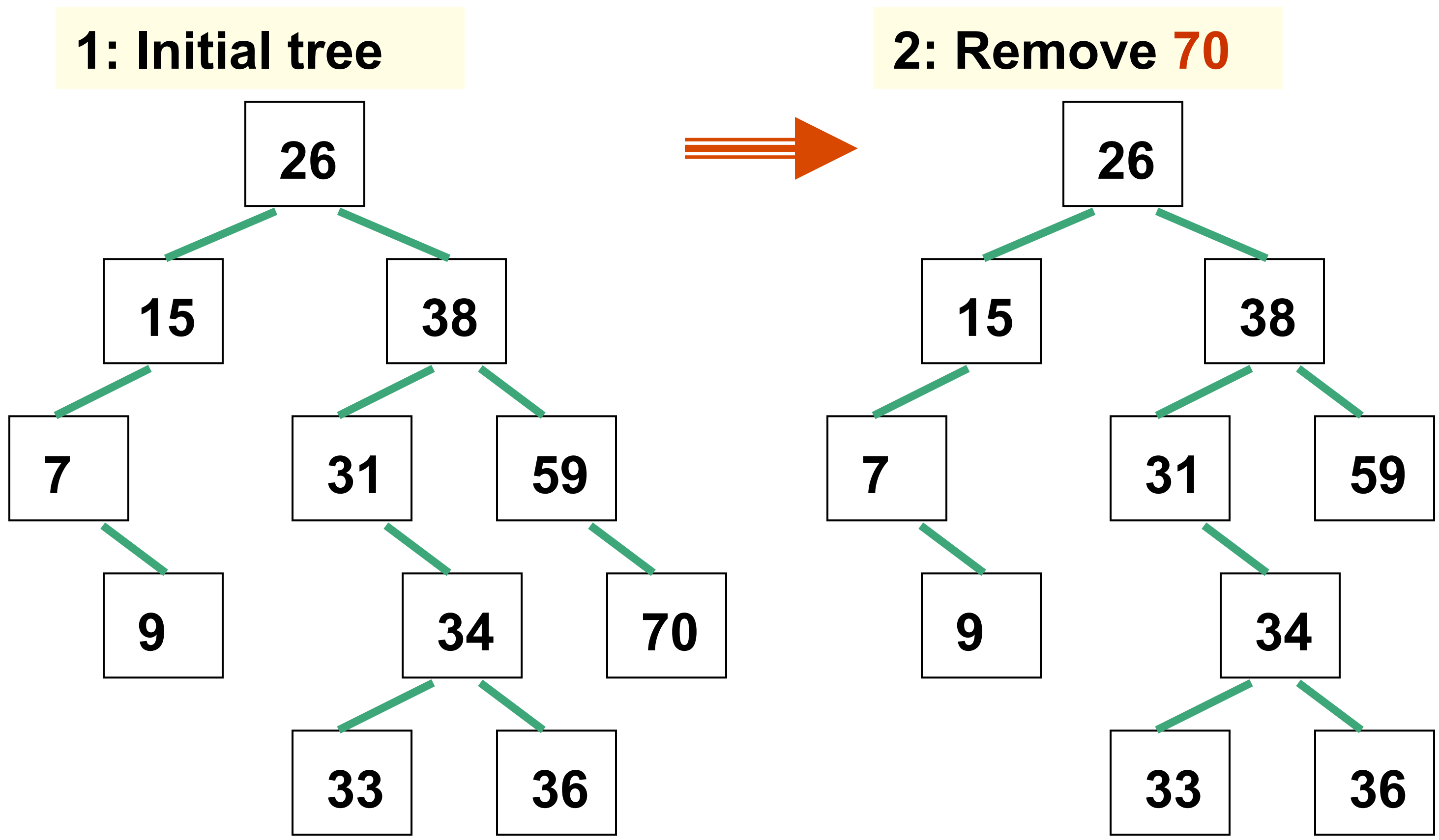
Example: Adding Elements to a BST

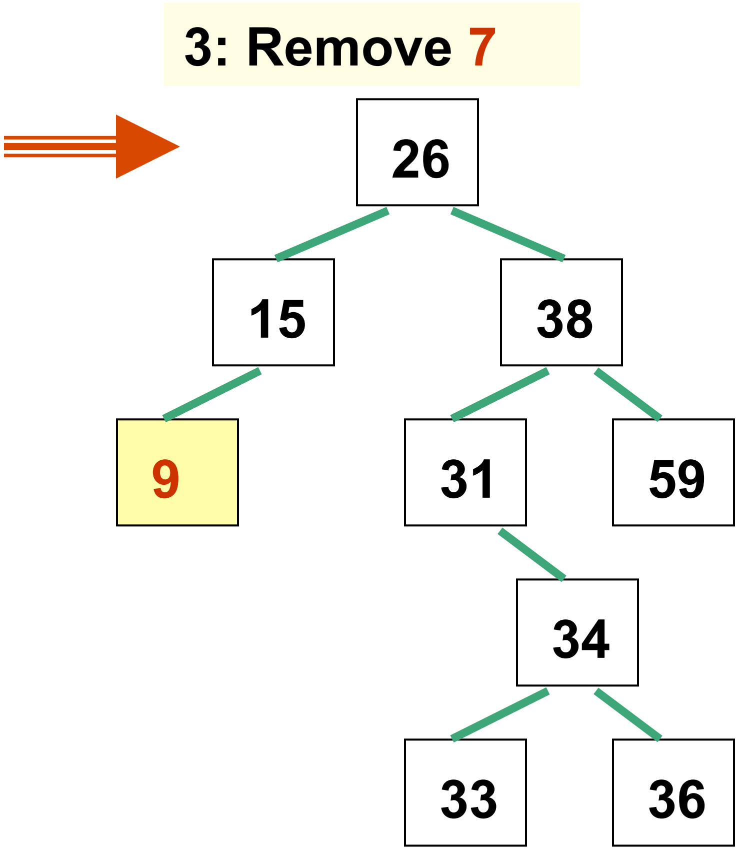


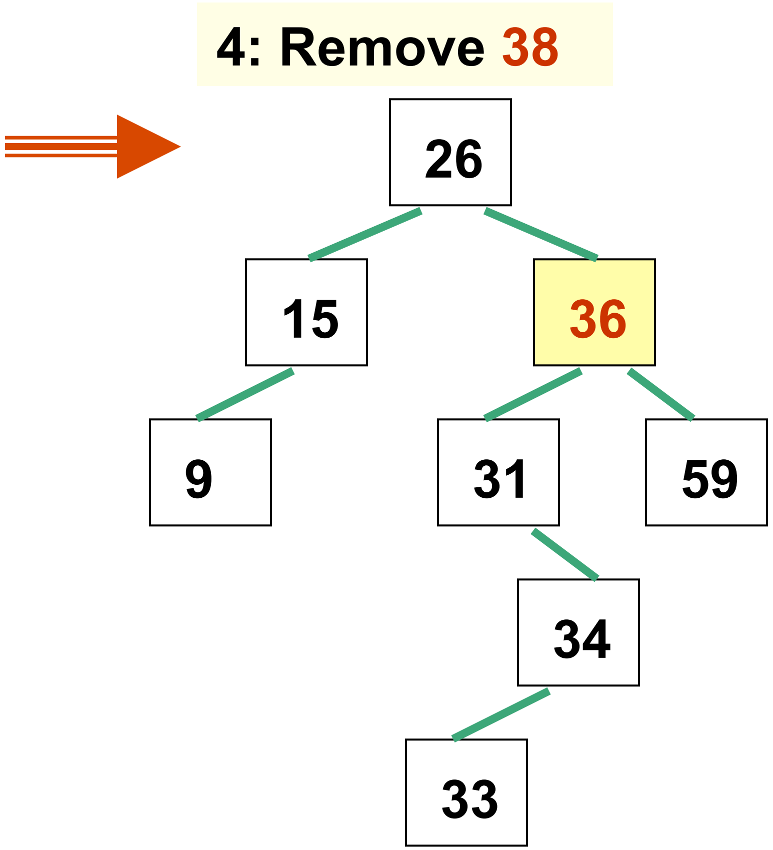
BST Operations: Remove

- ***Case 1**: value to be removed is in a **leaf node***
  - Node can be removed and the tree needs no further rearrangement
- ***Case 2**: value to be removed is in an **interior node***
  - Why can't we just change the link from its parent node to a successor node?
  - We *can* replace the node with its *inorder* predecessor (or successor)
    - Complex, and we will not implement this

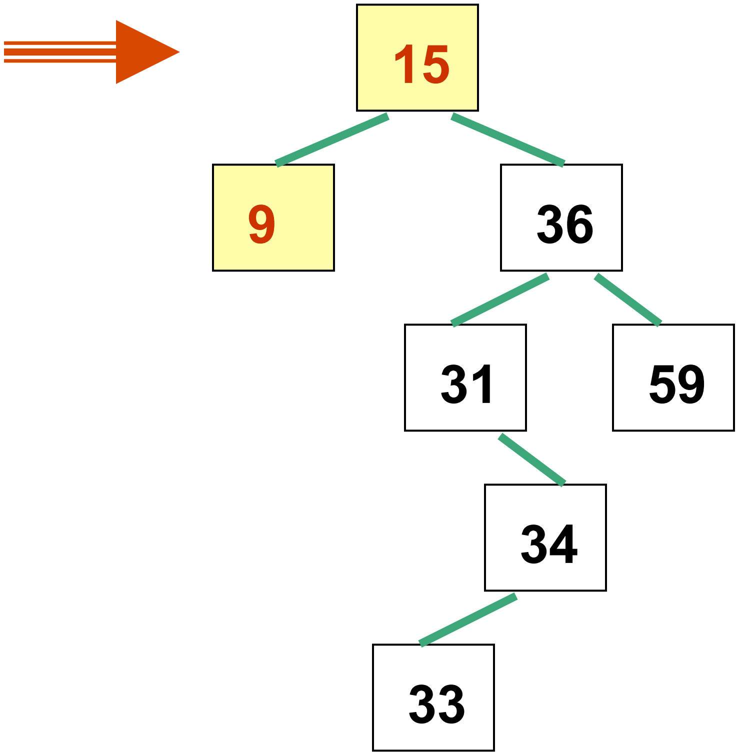
Example: Removing BST Elements



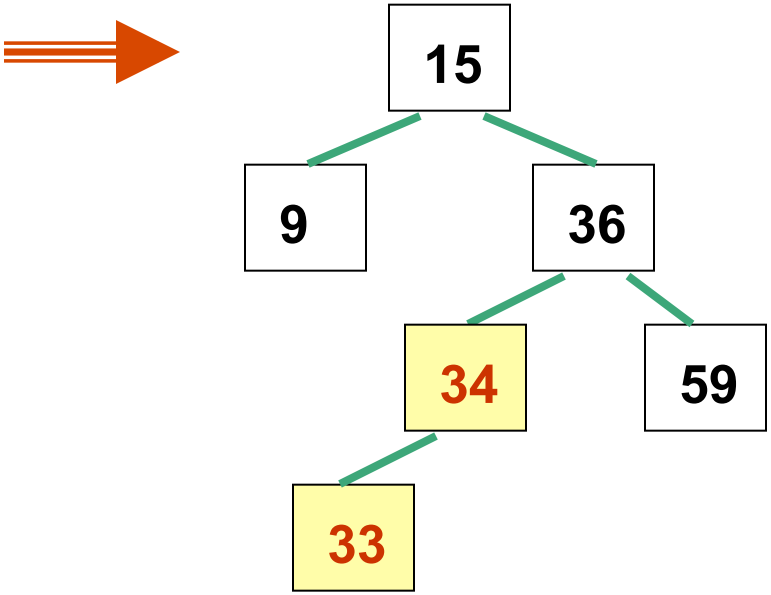

3: Remove 7
26
15
38
9
31
59
34
33
36


4: Remove 38
26
15
36
9
31
59
34
33

5: Remove 26



6: Remove 31



BST Operations: Remove Minimum

- Recall that ***leftmost node*** contains the minimum element
- Three cases:

1)root has no left child (so, root is minimum)

- its right child becomes the root

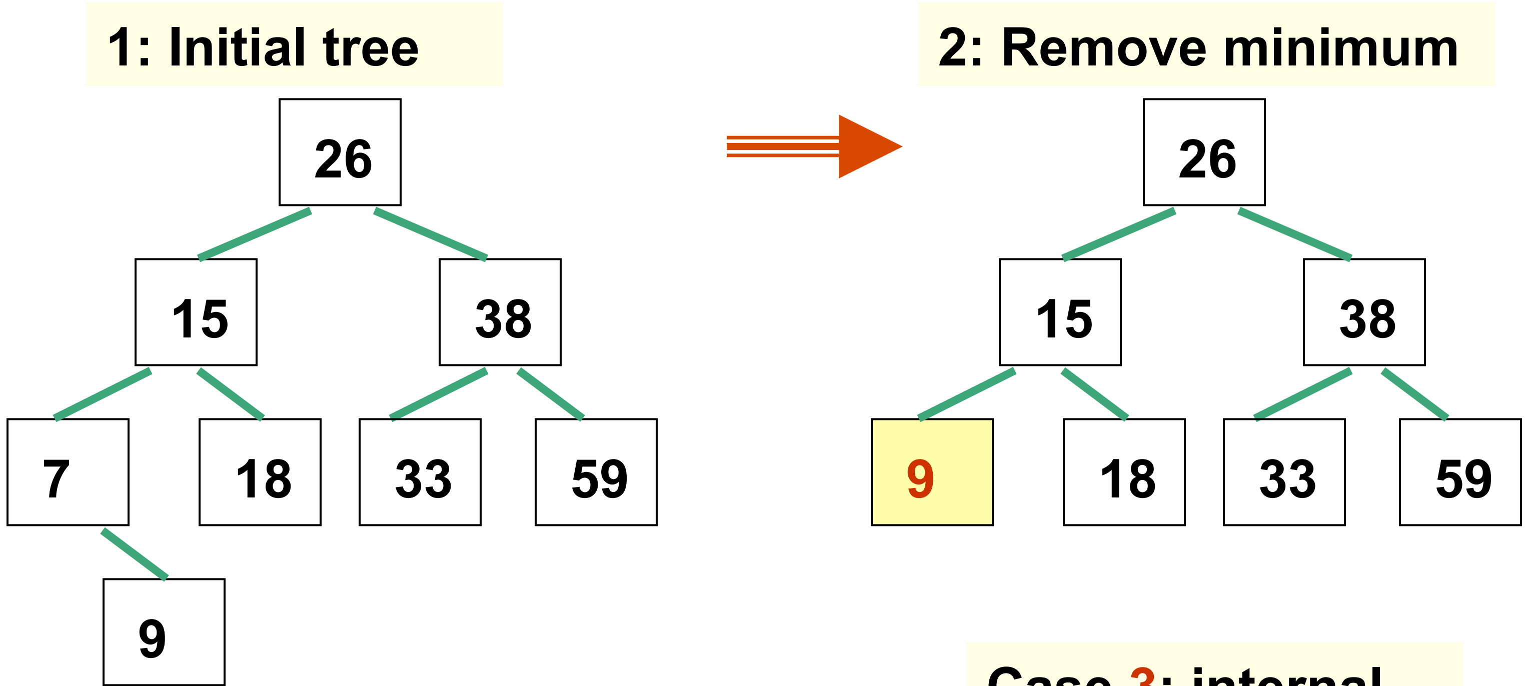
2)leftmost node is a leaf

- set its parent's left child to null

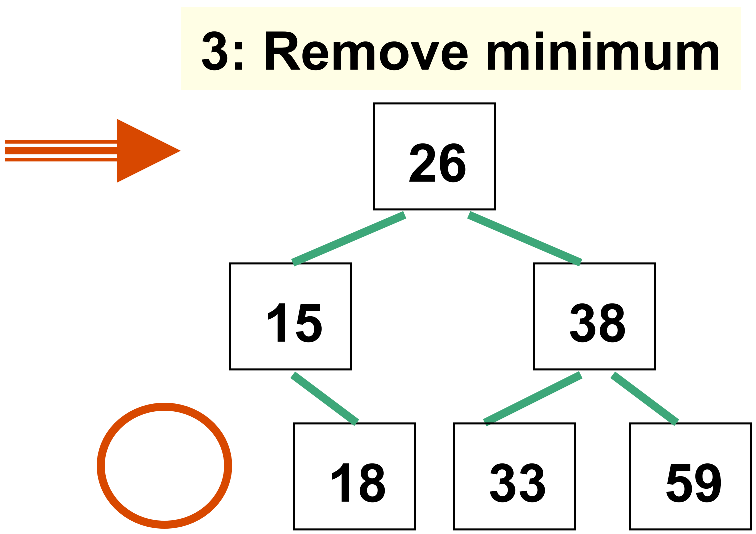
3)leftmost node is internal

- the right child of the node to be removed becomes the parent's left child

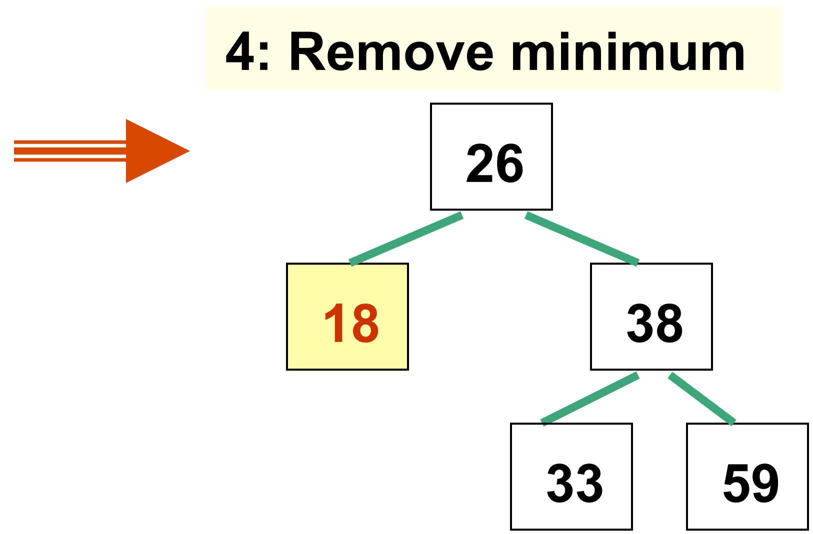
Example: Removing Minimum BST Element



Case 3: internal node removed

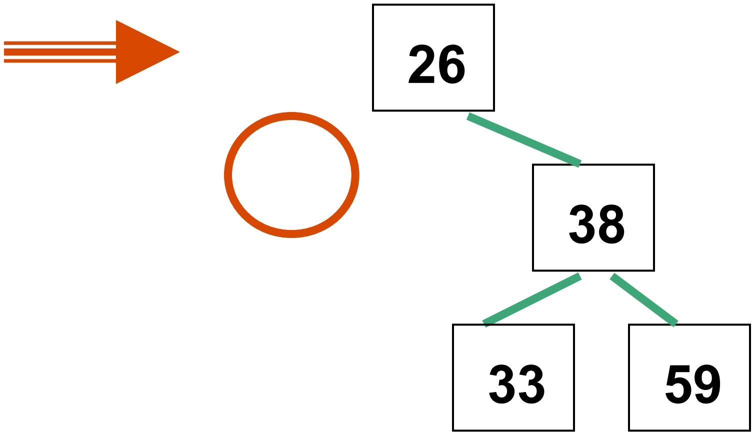


Case 2: leaf node removed

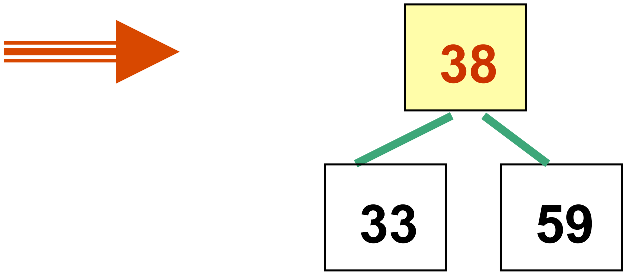


Case 3: internal node removed

5: Remove minimum



6: Remove minimum



Case 2: leaf node removed

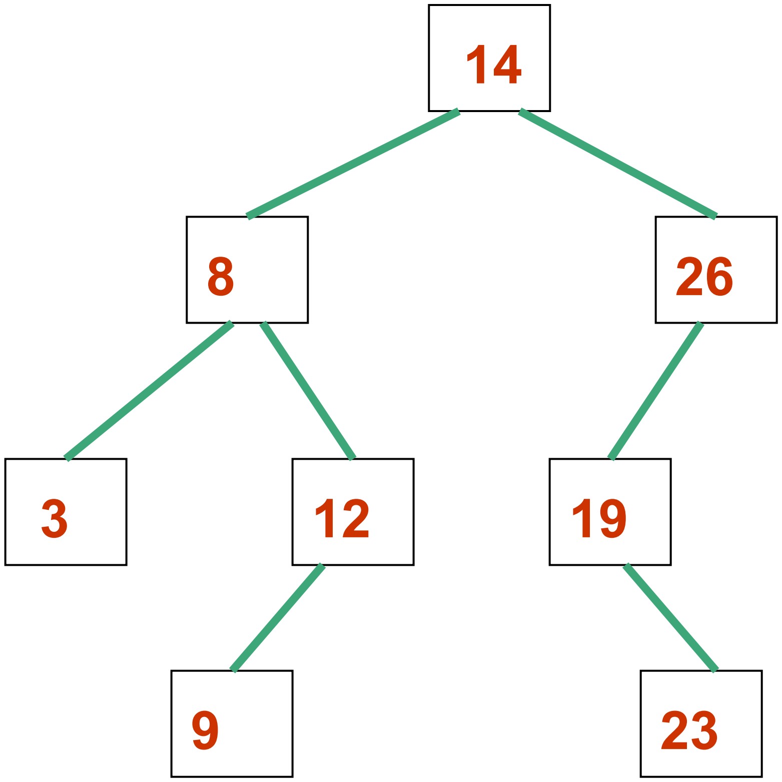
Case 1: root node removed

Binary Search Tree Traversals

- Consider the traversals of a binary search tree: preorder, inorder, postorder, level-order
- Try the traversals on the example on the next page
  - Is there anything special about the ***order of the data*** in the BST, for each traversal?

- ***Question***: what if we wanted to visit the nodes in ***descending*** order?

Binary Search Tree Traversals



Try these traversals:

- **preorder**
- **inorder**
- **postorder**
- **level-order**

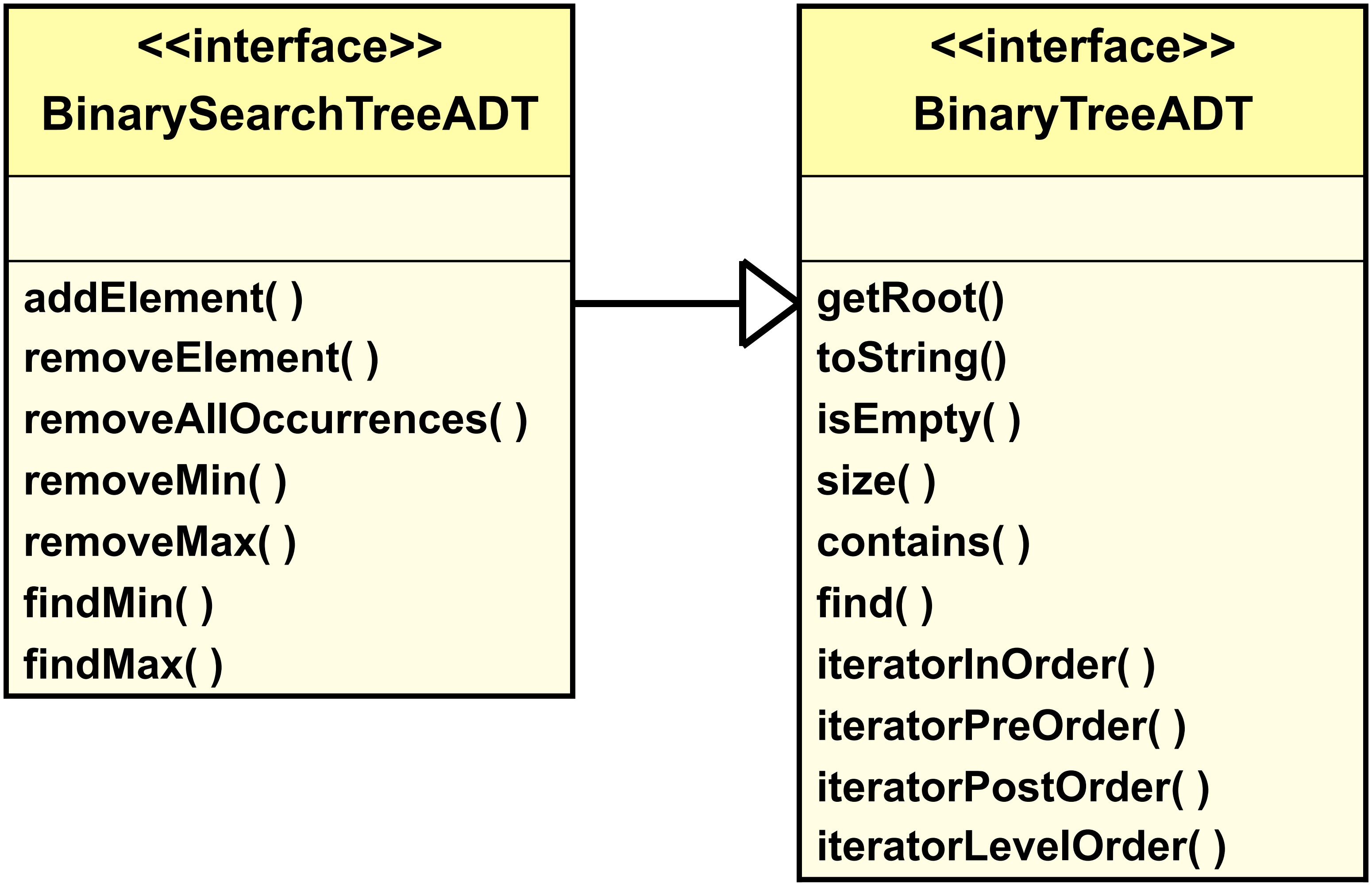
Binary Search Tree ADT

- A BST is just a binary tree with the ordering property imposed on all nodes in the tree
- So, we can define the **BinarySearchTreeADT** interface as an *extension* of the **BinaryTreeADT** interface

```
public interface BinarySearchTreeADT<T> extends
                                        BinaryTreeADT<T>  {
   public void addElement (T element);

   public T removeElement (T targetElement);

   public void removeAllOccurrences (T targetElement);

   public T removeMin( );

   public T removeMax( );

   public T findMin( );

   public T findMax( );
}
```

The BinarySearchTreeADT interface

UML Description of BinarySearchTreeADT



Implementing BSTs using Links

- See ***LinkedBinarySearchTree.java***
  - Constructors: use super()
  - **addElement** method
    - *(does not implement our recursive algorithm of p.12; also, allows duplicates)*
    - note the use of **Comparable**: so that we can use compareTo method to know where to add the new node
  - **removeMin** method
    - essentially implements our algorithm of p. 18

Implementing BSTs using Links

- The special thing about a Binary Search Tree is that finding a specific element is efficient!
  - So, LinkedBinarySearchTree has a **find** method that ***overrides*** the find method of the parent class LinkedBinaryTree
    - It only has to search the appropriate side of the tree
    - It uses a recursive helper method findAgain
  - Note that it does not have a **contains** method that overrides the contains of LinkedBinaryTree – why not?
    - It doesn’t need to, because contains just calls find

Using Binary Search Trees: Implementing Ordered Lists

- A BST can be used to provide *efficient* implementations of other collections!
- We will examine an implementation of an ***Ordered List ADT*** as a ***binary search tree***
- Our implementation is called ***BinarySearchTreeList.java*** (naming convention same as before: this is a BST implementation of a List)

Using BST to Implement Ordered List

- BinarySearchTreeList ***implements*** OrderedListADT
  - Which extends ListADT
  - So it also implements ListADT
  - So, what operations do we need to *implement*?
    - add
    - removeFirst, removeLast, remove, first, last, contains, isEmpty,size, iterator, toString
  - But, for which operations do we actually need to write code? …

Using BST to Implement Ordered List

- BinarySearchTreeList ***extends*** our binary search tree class LinkedBinarySearchTree
  - Which extends LinkedBinaryTree
  - So, what operations have we *inherited* ?
    - addElement, removeElement, removeMin, removeMax, findMin, findMax, find
    - getRoot, isEmpty, size, contains, find, toString, iteratorInOrder, iteratorPreOrder, iteratorPostOrder, iteratorLevelOrder

Discussion

- First, let’s consider some of the methods of the List ADT that we do ***not*** need to write code for:
  - **contains** method: we can just ***use*** the one from the LinkedBinaryTree class
  - What about the methods
    - **isEmpty**
    - **size**
    - **toString**

Discussion

- To implement the following methods of the OrderedListADT , we can ***call*** the appropriate methods of the LinkedBinarySearchTree class

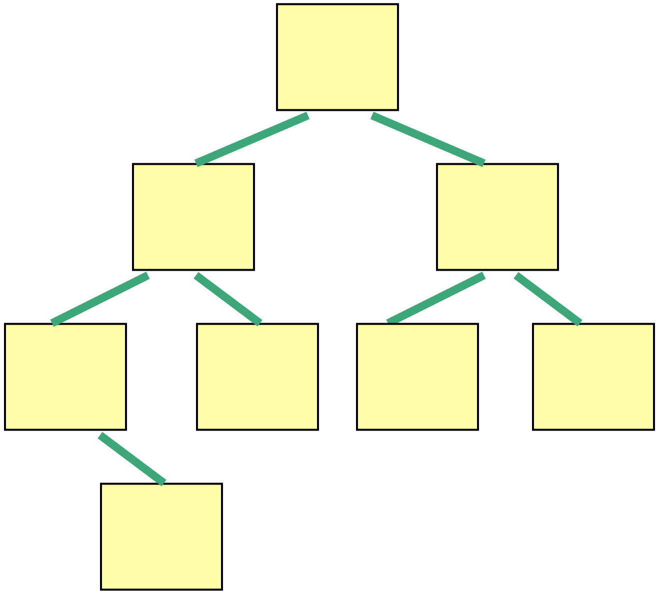
  (fill in the missing ones)

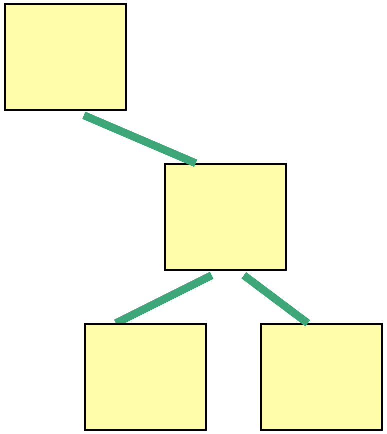
  - **add** call addElement
  - **removeFirst** removeMin
  - **removeLast**
  - **remove**
  - **first**
  - **last**
  - **iterator**

Balanced Trees

- ***Our definition***: a ***balanced tree*** has the property that, for any node in the tree, the height of its left and right subtrees can ***differ by at most 1***
  - Note that conventionally the height of an empty subtree is **-1**

Balanced Trees





Which of these trees is a balanced tree?

Analysis of BST Implementation

- We will now compare the linked list implementation of an ordered list with its BST implementation, making the following important assumptions:
  - The BST is a ***balanced*** tree
  - The maximum level of any node is **$\log_2(n)$**, where **n** is the number of elements stored in the tree

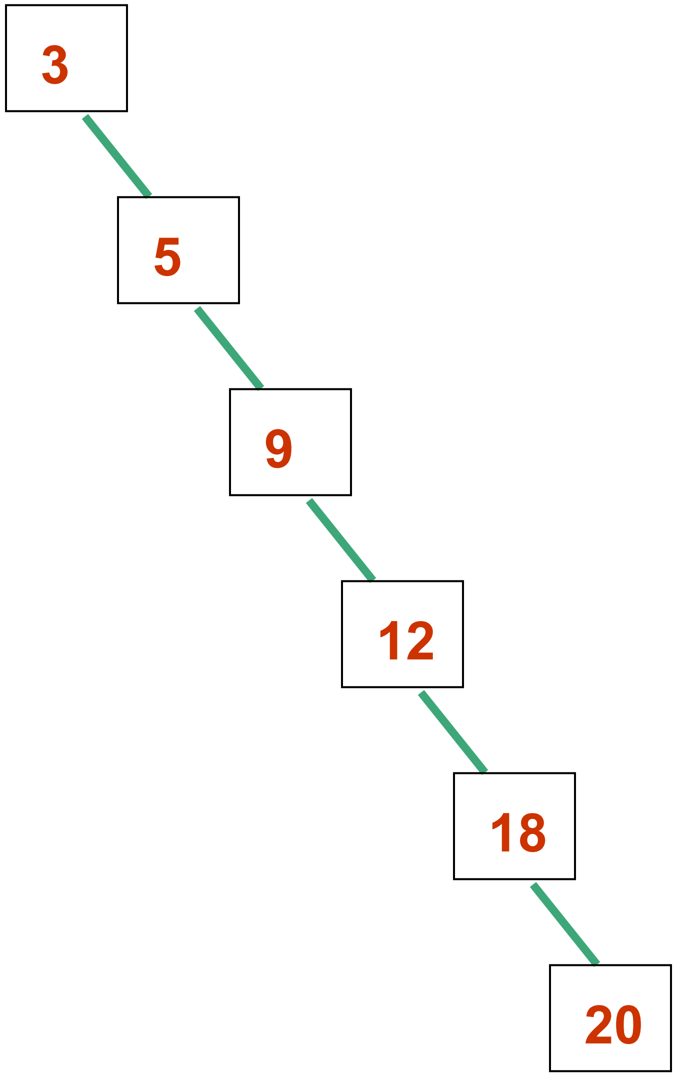
Analysis of Ordered List Implementations: Linked List vs. Balanced BST

Operation	LinkedList	BinarySearchTreeList
removeFirst	O(1)	$O(\log_2 n)$
removeLast	O(n)	$O(\log_2 n)$
remove	O(n)	$O(\log_2 n)$ *but may cause tree to become unbalanced
first	O(1)	$O(\log_2 n)$
last	O(n)	$O(\log_2 n)$
contains	O(n)	$O(\log_2 n)$
isEmpty	O(1)	O(1)
size	O(1)	O(1)
add	O(n)	$O(\log_2 n)$ *

Discussion

- Why is our balance assumption so important?
  - Look at what happens if we insert the following numbers in this order without rebalancing the tree:

3 5 9 12 18 20


3
5
9
12
18
20

Degenerate Binary Trees

- The resulting tree is called a ***degenerate*** binary tree
  - Note that it looks more like a linked list than a tree!
  - But it is actually less efficient than a linked list (***Why?***)

Degenerate Binary Trees

- Degenerate BSTs are far less efficient than balanced BSTs
  - Consider the worst case time complexity for the **`add`** operation:
    - **$O(n)$** for degenerate tree
    - **$O(\log_2 n)$** for balanced tree

Balancing Binary Trees

- There are many approaches to balancing binary trees
  - But they will not be discussed in this course …