Introduction to Game Theory and Auctions

Week 1-3

ASSIGNMENT - 1

Deadline: 11:59 PM, 20th May (Friday) Max mark: 100

Shorthand Notations: NE is Nash Equilibrium, WDSE is Weakly Dominant Strategy Equilibrium, SDSE is Strongly Dominant Strategy Equilibrium , S is the Strategy set, C is the Cost Function

1. Give examples of two player games:
   a) Having no NE
   a) unique NE which is not a WDSE
   b) a unique WDSE which is not an SDSE
   c) a unique DSE and another profile which is only an NE. **(3+3+3+3)**

2. Consider the following variant of prisoners' dilemma problem

P1/P2	NC	C
NC	-4,-4	-2,-X
C	-X,-2	-X,-X

   Find the values of x for which:
   a) (C, C) is an SDSE
   b) (C, C) is a WDSE but not an SDSE
   c) (C, C) is an NE but not a WDSE
   d) (C, C) is not even an NE **(12)**

3. Compute NE for two player game with S1 = {1, 2}, S2 = {3, 4}, such that

$$u_1(x, y) + u_2(x, y) = 0 \quad (1)$$

$$u_1(x, y) = |x - y| \quad (2)$$

   for all x, y lying in the set S1 and S2 respectively. **(6)**

4. Find all NE's of the following game.

P1/P2	A	B	C	D
A	5,2	2,6	1,4	0,4
B	0,0	3,2	2,1	1,1
C	7,0	2,2	1,5	5,1
D	9,5	1,3	0,2	4,8

(8)

5. Consider 10 players simultaneously announce an integer between 0 and 100. A prize of \$100 is split equally between all players whose number is closest to 2/3 of the average. Other players get nothing. Find the NE of this game. **(15)**

6. Each of n people choose whether or not to contribute a fixed amount toward the provision of a public good. The good is provided if and only if at least k people contribute, where $2 \leq k \leq n$; if it is not provided, contributions are not refunded. Each person ranks outcomes from best to worst as follows: (i) any outcome in which the good is provided and he does not contribute, (ii) any outcome in which the good is provided and he contributes, (iii) any outcome in which the good is not provided and he does not contribute, (iv) any outcome in which the good is not provided and he contributes. Formulate this situation as a strategic game and find all the Nash equilibria. **(15)**

7. Two players simultaneously report a number in the interval [0, 10]. They divide \$10 between them as follows. If the sum of their reported numbers is at most 10, they just take the split and the remaining money is burnt. If their sum is greater than 10, the smaller bidder gets the money he asked for, and the remaining money is given to the larger bidder. If their sum is greater than 10 and they bid equally, then they get \$5 each. There is a unique NE of this game, find it. **(10)**

8. Consider Bertrand's model of duopoly. Let $C_i(q) = c_i \times q$, with $c_1 < c_2$.
Consider the setting where the market is equally split between both the firms when prices are equal. Find the Nash Equilibria, if one exists. **(12)**

9. Two people submit sealed bids for an object worth \$K to each of them. The winner is the person whose bid is higher; in the event of a tie each person receives half of the object, which he values at \$K/2. Each person pays his bid, whether or not he wins. Find the Nash Equilibria, if one exists. **(10)**