Intro to Game theory and Auctions
Assignment 1

Sourabh Mina

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Q 1.

(a) Matching Pennies

(b) Prisoner's Dilemma (Fink, Fink)

(c) A game of rock-paper-scissors

(d) :? (Didn't understand the statement)

Q 2.

(a) For (C,C) to be a SDSE,
For C to be the best possible case for P1, (regardless what P2 has chosen),
$-X > max(-2, -X, -4)$
$\therefore$ It is not possible for any value of X. Similarly, for P2,
$-X > max(-2, -X, -4)$
$\implies$ There is no values of X, for (C,C) is to be SDSE.

(b) (C,C) being WDSE, For C to be best possible case for P1 (in some cases) or equal to the best possible case.
-X $\geq$ max(-2,-X)
$\therefore$ If X<2, max(-2,-X) would be -X,
And -X is $\geq$ -X. OR if X>2, max would be -2. Therefore, making -X $\geq$ -2, i.e. X $\leq$ 2, (Contradictory to the assumption that X>2). AND if X=2, max would be -2. And, -X $\geq$ -2, which would also be true.
$\therefore$ For (C, C) is a WDSE but not an SDSE,
$X \in (-\infty, 2]$.

(c) For (C,C) to be a NE, u_i(C,C) $\geq$ u_i(NC,C) where U_i is a payoff function that represent player i's preferences. $\therefore$ For C to be the best options for P1, (given that P2 has chosen C):-
-X $\geq$ -2.
$\implies$ X $\leq$ 2. Similarly, for P2,
X $\leq$ 2. $\therefore X \in (-\infty, 2]$, which is the same set as WDSE,
$\implies$ There are no values for X, for which (C, C) is an NE but not a WDSE.

(d) For the set of X, for the problem to not have a NE, (as from the previous calculation), it would be negation of ((-∞,2]). $\implies$ $x \in (2, \infty)$.

Q 3. We construct the table, by evaluating each of the payoff for the respective x's and y's.
$\therefore$ Starting with x=1, y=3. U_1(1,3)=|1 − 3|=2.
$\implies U_2 = -2$.
Similarly for other x's and y's, x=2, y=3, U_1(2,3)=|2 − 3|=1.
$\implies U_2(2,3) = -1$.
$\therefore U_1(1,4) = 3$, $U_2(1,4) = -3$; $U_1(2,4) = 2$, $U_2(2,4) = -2$. So the table would look like

P1/P2	3	4
1	2,-2	3,-3
2	1,-1	2,-2

$\therefore$ The Nash Equilibrium for the following problem would be (2,-2) (both when (x,y) are (1,3) and (2,4)).
$\because$ When reached that state, there would not be profitable to deviate from it.

Q 4. To find the Nash Equilibrium, we will be iterating column wise, trying to find the max payoff in each column for P1. (We are doing this in order to find the best response function of P1, given the response of P2 i.e. that particular column) and we would be checking the presence of Nash equilibrium at the best response function point.
$\therefore$ For the particular problem,
For column 1, (if P2 chooses A) The best response of P1 would be choosing D.
$\therefore$ We check the equilibrium conditions at (D,A)or(9.5). (9,5) is not a NE, since for the case when P1 chooses D, P2 is bettor off with choosing D, (rather than A).
Similarly, iterating over the other columns.
For column B,
We get (B,B) or (3,2),
AND (3,2) is a NE, since it is also the best response function of P2 given the choice of P1 (B).
For column C,
We check (B,C) or (2,1),
By checking, it infers out that (2,1) is not a NE. For the last column,
We are at (D,C) or (5,1),
It is also not a NE.
$\therefore$ The game only has one NASH EQUILIBRIUM,
and that is at (B,B) or (3,2).

Q 5. Since 2.3*100=67 (if everyone picks 100), So, thinking from the best response perspective, we should not pick any number greater than 67.
$\because$ There's no chance of it getting becoming 2/3 of the average.
BUT, I am not guessing above 67, it would be natural for every other player to not to guess above 67, since it wont be their best response.
$\therefore$ The new average would come around to be
2/3*67=44.67 which is to be the closest 45,
$\therefore$ We should not guess above 45, since the (2/3) of the average would be ≤ 45.
BUT again, it is true for every other player.
So, no one again guesses above 45,
AND IT CONTINUES...

To formulate it into a formula, if my best response for the problem is x, (which would be in turn the best response for every other player) due to symmetry.
Also,
x should be 2/3 of the average.
$\therefore x = 2/3 * x$
$\implies$ x=0.
Therefore, the best response for everyone would be choosing 0.
$\implies$ (0,0,0....) would be the NASH EQUILIBRIUM for the problem.

Q 6. A strategic game would consist of :-

- **A set of players** : In this case, would be the N people.
- **A set of actions** : It could be finalised as 0 or 1. (As per the participation, 1 for contributing).
- **Preference over the set of action profile** : Which as per question :
  (0,good arrives) > (1,good arrives) > (0, good does not arrive) > (1, good does not arrive)

And, to find the NASH EQUILIBRIUM for this strategic game, We iterate over the possibilities on the basis on number of player participating.
I) If more than k people contribute:-
There would be no possibility of a NE,
$\because$ any contributing player would be better off, if they deviate from the social norm of contributing, and the good will still be providing, (since preference of (0,good arrives) > (1,good arrives)),
$\therefore$ There is no possibility of NE.

II) If Less than k people contribute
If there are some people contributing, there won't be any NE since, the contributor could choose to not to contribute and be better off. ($\because$ (0, good does not arrive) > (1, good does not arrive)).
BUT, if there is no one contributing, i.e. zero contributors,
$\therefore$ Anyone that would switch from 0 to 1, will be picking a worse choice for himself (cause the good will still not be provided, since k cannot be 1, since k $\geq$2).
$\implies$ (0,0,0...) is a NE for the strategic problem.

III) IF exactly k people contribute
It can be proven that any combination of k players contributing would be a NASH EQUILIBRIUM.
$\because$ If a contributor decides to deviate, the public good will not be provided, and it would worse case for him.
AND, for a non contributing person, switching to 1, makes no sense since the good is coming either way.
$\therefore$ It would be better of him to not switch.
$\implies$ A Nash equilibrium found, when exactly K person contribute.
$\therefore$ The strategic game has total number of $\binom{n}{k}$+1 number of Nash Equilbria.

Q 7. $\because$ There is a unique equilibrium, it is tempting to check the equilibrium at the symmetry of the problem, i.e. at (5,5).
As for current state, each player gets, a fair split of 5-5.
If one player tries to deviate from it by increasing their choice of number,
i.e. > 5.
It would make the sum, greater than 10, and would receive the split as (10-choice of the lower bidder), i.e. (10-5)=5;
Thus, it is not profitable to increase their choice from the (5,5).
AND, if someone, deviates from it by decreasing the value,
It would make the sum <10.
$\therefore$ That player would be getting the split as the number of their choice, which would in fact, be lower than 5.
$\implies$ Its not profitable for the player to deviate from (5,5). Thus, (5,5) is the only unique Nash equilibrium, which is in fact a Weakly dominated Nash Equilibrium.

Q 8. :/

Q 9. A set of players : P1 and P2
Set of actions : Placing the bid from [0 to k] lets assume b1 and b2
Preference : It would be the profit from bidding, (assuming no sentimental backgrounds for bidding).
Again,
Dividing the cases on the basis of bids to check for Nash Equilibrium for each case,
I) If b1 $\neq$ b2
If they have different bids, there's no possibility for a Nash equilibrium, since the smaller bidder is always better of at bidding 0.

II) If $(b1 = b2) \leq k/2$
It would be profitable for the player to increase their bids to get the K worth item, rather than getting k/2.
$\therefore$ Again no chance of a Nash Equilibrium.

III) If $k \geq (b1 = b2) \geq k/2$
Again, the current state is not profitable for both the players, as they could increase the bid, and be more profitable. Again,
No possibility of a NE.

IV) If $(b1 = b2) = k$
In this situation it would be better for any bidder to bid 0 and deviate from the equilibrium to get more profit.
($\because$ Staying at k, would result both of them to get the object worth k/2, thus making it a loss.)
$\therefore$ This is not a Nash Equilibrium condition.

We have covered all the cases, and found no state, where deviation from it would result in lower payoff than it.
$\implies$ There is no Nash Equilibrium for this strategical problem.