Adsorber Model Equations Derivation

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\begin{empheq}[box=\tcbhighmath]{equation*}
  \begin{split}
    -\diffty \ddzz{c_i} +  \ddz{(uc_i)}  + \ddt{c_i} + S_i = 0 \\
    -\diffty \ddzz{(Cy_i)} +  \ddzuv{u}{c_i} + \ddt{c_i} + S_i = 0 \\
    -\diffty \ddz{(\ddzuv{C}{y_i})} + \ddzuv{u}{c_i}  + \ddt{c_i} + S_i = 0 \\
    -\diffty \left \{ \ddzudv{y_i}{C} + \ddzudv{C}{y_i} \right \}
        + \ddzuv{u}{c_i}  + \ddt{c_i} + S_i = 0 \\
  \end{split}
\end{empheq}
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$$\boxed{\begin{aligned}
-\mathcal{D}_L \frac{\partial^2 c_i}{\partial z^2} + \frac{\partial (uc_i)}{\partial z} + \frac{\partial c_i}{\partial t} + S_i = 0 \\
-\mathcal{D}_L \frac{\partial^2 (Cy_i)}{\partial z^2} + c_i \frac{\partial u}{\partial z} + u \frac{\partial c_i}{\partial z} + \frac{\partial c_i}{\partial t} + S_i = 0 \\
-\mathcal{D}_L \frac{\partial (y_i \frac{\partial C}{\partial z} + C \frac{\partial y_i}{\partial z})}{\partial z} + c_i \frac{\partial u}{\partial z} + u \frac{\partial c_i}{\partial z} + \frac{\partial c_i}{\partial t} + S_i = 0 \\
-\mathcal{D}_L \left\{ \frac{\partial C}{\partial z}\frac{\partial y_i}{\partial z} + y_i \frac{\partial^2 C}{\partial z^2} + \frac{\partial y_i}{\partial z}\frac{\partial C}{\partial z} + C \frac{\partial^2 y_i}{\partial z^2} \right\} + c_i \frac{\partial u}{\partial z} + u \frac{\partial c_i}{\partial z} + \frac{\partial c_i}{\partial t} + S_i = 0
\end{aligned}}$$

$$\boxed{\begin{aligned}
-\mathcal{D}_L \left\{ C \frac{\partial^2 y_i}{\partial z^2} + 2 \frac{\partial C}{\partial z}\frac{\partial y_i}{\partial z} + y_i \frac{\partial^2 C}{\partial z^2} \right\} + c_i \frac{\partial u}{\partial z} + u \frac{\partial c_i}{\partial z} + \frac{\partial c_i}{\partial t} + S_i = 0 \\
-\mathcal{D}_L \left\{ C \frac{\partial^2 y_i}{\partial z^2} + 2 \frac{\partial C}{\partial z}\frac{\partial y_i}{\partial z} + y_i \frac{\partial^2 C}{\partial z^2} \right\} + C y_i \frac{\partial u}{\partial z} + u \left\{ y_i \frac{\partial C}{\partial z} + C \frac{\partial y_i}{\partial z} \right\} + C \frac{\partial y_i}{\partial t} + y_i \frac{\partial C}{\partial t} + S_i = 0 \\
C \left\{ \frac{\partial y_i}{\partial t} + y_i \frac{\partial u}{\partial z} + u \frac{\partial y_i}{\partial z} \right\} + u y_i \frac{\partial C}{\partial z} + y_i \frac{\partial C}{\partial t} - \mathcal{D}_L \left\{ C \frac{\partial^2 y_i}{\partial z^2} + 2 \frac{\partial C}{\partial z}\frac{\partial y_i}{\partial z} + y_i \frac{\partial^2 C}{\partial z^2} \right\} + S_i = 0 \\
C \left\{ \frac{\partial y_i}{\partial t} + y_i \frac{\partial u}{\partial z} + u \frac{\partial y_i}{\partial z} \right\} + y_i \left\{ u \frac{\partial C}{\partial z} + \frac{\partial C}{\partial t} \right\} - \mathcal{D}_L \left\{ C \frac{\partial^2 y_i}{\partial z^2} + 2 \frac{\partial C}{\partial z}\frac{\partial y_i}{\partial z} + y_i \frac{\partial^2 C}{\partial z^2} \right\} + S_i = 0
\end{aligned}}$$

Rearranging

$$\boxed{\begin{aligned}
C \left\{ \frac{\partial y_i}{\partial t} + u \frac{\partial y_i}{\partial z} \right\} + y_i \left\{ C \frac{\partial u}{\partial z} + u \frac{\partial C}{\partial z} + \frac{\partial C}{\partial t} \right\} - \mathcal{D}_L \left\{ C \frac{\partial^2 y_i}{\partial z^2} + 2 \frac{\partial C}{\partial z}\frac{\partial y_i}{\partial z} + y_i \frac{\partial^2 C}{\partial z^2} \right\} + S_i = 0 \\
C \left\{ \frac{\partial y_i}{\partial t} + u \frac{\partial y_i}{\partial z} \right\} + y_i \left\{ \frac{\partial (Cu)}{\partial z} + \frac{\partial C}{\partial t} \right\} - \mathcal{D}_L \left\{ C \frac{\partial^2 y_i}{\partial z^2} + 2 \frac{\partial C}{\partial z}\frac{\partial y_i}{\partial z} + y_i \frac{\partial^2 C}{\partial z^2} \right\} + S_i = 0
\end{aligned}}$$

Total Mass Blance

$$\frac{\partial C}{\partial t} + \frac{\partial (Cu)}{\partial z} + \sum S_i = 0$$

Now Component Balance can be written as

$$C\left\{\frac{\partial y_i}{\partial t} + u\frac{\partial y_i}{\partial z}\right\} - y_i \sum S_i - \mathcal{D}_L\left\{C\frac{\partial^2 y_i}{\partial z^2} + 2\frac{\partial C}{\partial z}\frac{\partial y_i}{\partial z} + y_i\frac{\partial^2 C}{\partial z^2}\right\} + S_i = 0$$

$$C\left\{\frac{\partial y_i}{\partial t} + u\frac{\partial y_i}{\partial z}\right\} - \mathcal{D}_L\left\{C\frac{\partial^2 y_i}{\partial z^2} + 2\frac{\partial C}{\partial z}\frac{\partial y_i}{\partial z} + y_i\frac{\partial^2 C}{\partial z^2}\right\} + S_i - y_i \sum S_i = 0$$

Another way

$$-\mathcal{D}_L\left\{\frac{\partial C}{\partial z}\frac{\partial y_i}{\partial z} + y_i\frac{\partial^2 C}{\partial z^2} + \frac{\partial y_i}{\partial z}\frac{\partial C}{\partial z} + C\frac{\partial^2 y_i}{\partial z^2}\right\} + y_i\frac{\partial (Cu)}{\partial z} + (Cu)\frac{\partial y_i}{\partial z} + \frac{\partial (Cy_i)}{\partial t} + S_i = 0$$

$$-\mathcal{D}_L\left\{\frac{\partial C}{\partial z}\frac{\partial y_i}{\partial z} + y_i\frac{\partial^2 C}{\partial z^2} + \frac{\partial y_i}{\partial z}\frac{\partial C}{\partial z} + C\frac{\partial^2 y_i}{\partial z^2}\right\} + y_i\frac{\partial (Cu)}{\partial z} + Cu\frac{\partial y_i}{\partial z} + y_i\frac{\partial C}{\partial t} + C\frac{\partial y_i}{\partial t} + S_i = 0$$

Total Mass Blance

$$\frac{\partial C}{\partial t} + \frac{\partial (Cu)}{\partial z} + \sum S_i = 0$$

Component Balance

$$C\left\{\frac{\partial y_i}{\partial t} + u\frac{\partial y_i}{\partial z}\right\} - \mathcal{D}_L\left\{C\frac{\partial^2 y_i}{\partial z^2} + 2\frac{\partial C}{\partial z}\frac{\partial y_i}{\partial z} + y_i\frac{\partial^2 C}{\partial z^2}\right\} + S_i - y_i \sum S_i = 0$$

where

$$S_i = \frac{1-\epsilon}{\epsilon}\sum_i \frac{\partial q_i}{\partial t}$$

$$\frac{\partial q_i}{\partial t} = k_i(q_i^* - q_i)$$

Non Dimensional Form

Total Mass Blance in dimensionless form

$$\frac{\partial C^*}{\partial t^*} + \frac{\partial (C^* u^*)}{\partial z^*} + \sum S_i^* = 0$$

Component Balance

$$C^*\left\{\frac{\partial y_i}{\partial t^*} + u^*\frac{\partial y_i}{\partial z^*}\right\} - \frac{1}{P_e}\left\{C^*\frac{\partial^2 y_i}{\partial z^{*2}} + 2\frac{\partial C^*}{\partial z^*}\frac{\partial y_i}{\partial z^*} + y_i\frac{\partial^2 C^*}{\partial z^{*2}}\right\} + S_i^* - y_i \sum S_i{}^* = 0$$

where

$$S_i^* = \frac{1-\epsilon}{\epsilon}k_i\tau(q_i^{**} - q_i^*)$$

$$C^* = C/C^o;\; t^* = t/\tau\;\, z^* = z/L\;\, q_i^* = q_i/C^o;\; P_e = \frac{u^o L}{\mathcal{D}_L};\; \tau = L/u^o$$