Let us assume two convex function $f(x)$ and $g(x)$

To prove that $f(x) - g(x) = h(x) \rightarrow$ not convex f^n

We know that,

A f^n $f: R \rightarrow R$ is convex, if for all $x_1, x_2 \in \mathbb{R}$ and for all $\lambda \in [0, 1]$, we have

$$f(\lambda x_1 + (1-\lambda) x_2) \leq \lambda f(x_1) + (1-\lambda) f(x_2)$$

Similarly, a f^n $g(x)$ is convex, if

$$g(\lambda x_1 + (1-\lambda) x_2) \leq \lambda g(x_1) + (1-\lambda) g(x_2)$$

take ~~example~~ two convex example

$f(x) = x^2 + x$, & $g(x) = x^2$

to check convexity 2nd derivative, $\cancel{f''(x) \& g''(x)} \geq 0$
$f''(x) \geq 0$ & $g''(x) \geq 0$

$f'(x) = 2x$	$g'(x) = 2x$
$f''(x) = 2 \geq 0$	$g''(x) = 2 \geq 0$

So both f^n are convex.

Now, $h(x) = f(x) - g(x)$

$= (x^2 + x) - x^2 = x$

$h(x) = x$

$\Rightarrow h'(x) = 1 \Rightarrow h''(x) = 0$

means $h(x)$ is not strictly convex, but is at most affine (linear), we know a f^n is convex, if and only if its second derivative is non-negative. While linear f^n are convex, they are not necessarily convex in the strict sense.

~~Next~~ Next example

$f(x) = e^x$, $g(x) = 2e^x$

$f''(x) = e^x \geq 0$, $g''(x) = 2e^x \geq 0$

$\downarrow$ convex $\qquad \downarrow$ convex

$h(x) = f(x) - g(x)$

$= -e^x$

$h''(x) = -e^x \leq 0$, $\forall x$, so that is strictly concave & thus not convex.

So,

The difference of two convex f^n is not necessarily convex