

Homework 2

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Exercise 1

An Introduction to Thermal Physics (Schroeder, 1e) Exercise 1.34 (partial)

Part (a)

For each step, we calculate the work done W , change in energy ΔU , and heat added Q . For Step A:

$$\begin{aligned} W_A &= - \int_{V_1}^{V_1} p \, dV = 0 \\ \Delta U_A &= \frac{3}{2} N k_B \Delta T = \frac{3}{2} N k_B \frac{(p_2 - p_1) V_1}{N k_B} = \frac{3 V_1 (p_2 - p_1)}{2} \\ Q_A &= \Delta U - W = \frac{3 V_1 (p_2 - p_1)}{2} \end{aligned} \quad (1)$$

For Step B:

$$\begin{aligned} W_B &= - \int_{V_1}^{V_2} p \, dV = - \int_{V_1}^{V_2} p_2 \, dV = -p_2 (V_2 - V_1) \\ \Delta U_B &= \frac{3}{2} N k_B \Delta T = \frac{3}{2} N k_B \frac{(V_2 - V_1) p_2}{N k_B} = \frac{3 p_2 (V_2 - V_1)}{2} \\ Q_B &= \Delta U - W = \frac{5 p_2 (V_2 - V_1)}{2} \end{aligned} \quad (2)$$

For Step D:

$$\begin{aligned} W_C &= - \int_{V_2}^{V_2} p \, dV = 0 \\ \Delta U_C &= \frac{3}{2} N k_B \Delta T = \frac{3}{2} N k_B \frac{(p_1 - p_2) V_2}{N k_B} = - \frac{3 V_2 (p_2 - p_1)}{2} \\ Q_C &= \Delta U - W = - \frac{3 V_2 (p_2 - p_1)}{2} \end{aligned} \quad (3)$$

For Step C:

$$\begin{aligned}
W_D &= - \int_{V_2}^{V_1} p \, dV = - \int_{V_2}^{V_1} p_1 \, dV = p_1 (V_2 - V_1) \\
\Delta U_D &= \frac{3}{2} N k_B \Delta T = \frac{3}{2} N k_B \frac{(V_1 - V_2) p_1}{N k_B} = - \frac{3 p_1 (V_2 - V_1)}{2} \\
Q_D &= \Delta U - W = - \frac{5 p_1 (V_2 - V_1)}{2}
\end{aligned} \tag{4}$$

Part (b)

The net work W_{net} is

$$\begin{aligned}
W_{\text{net}} &= W_A + W_B + W_D + W_C \\
&= 0 - p_2 (V_2 - V_1) + 0 + p_1 (V_2 - V_1) = - (p_2 - p_1) (V_2 - V_1)
\end{aligned} \tag{5}$$

The net change in energy ΔU_{net} is:

$$\begin{aligned}
\Delta U_{\text{net}} &= \Delta U_A + \Delta U_B + \Delta U_D + \Delta U_C \\
&= \frac{3 V_1 (p_2 - p_1)}{2} + \frac{3 p_2 (V_2 - V_1)}{2} - \frac{3 V_2 (p_2 - p_1)}{2} - \frac{3 p_1 (V_2 - V_1)}{2} \\
&= 0
\end{aligned} \tag{6}$$

The net heat added to the gas Q_{net} is:

$$\begin{aligned}
Q_{\text{net}} &= Q_A + Q_B + Q_D + Q_C \\
&= \frac{3 V_1 (p_2 - p_1)}{2} + \frac{5 p_2 (V_2 - V_1)}{2} - \frac{3 V_2 (p_2 - p_1)}{2} - \frac{5 p_1 (V_2 - V_1)}{2} \\
&= (p_2 - p_1) (V_2 - V_1)
\end{aligned} \tag{7}$$

Exercise 2

Part (a)

Assuming quasistatic compression, the work done on 10 kg of ice by the atmosphere as it melts into water W is:

$$\begin{aligned} W &= - \int_{V_i}^{V_f} p \, dV = (V_i - V_f) p \\ &= \left(\frac{m}{\rho_i} - \frac{m}{\rho_f} \right) p = \left(\frac{1}{\rho_i} - \frac{1}{\rho_f} \right) mp \\ &= \left(\frac{1}{916.23 \, \text{kg/m}^3} - \frac{1}{999.84 \, \text{kg/m}^3} \right) \cdot 10 \, \text{kg} \cdot 101325 \, \text{Pa} \approx 92.478 \, \text{J} \end{aligned} \tag{8}$$

$W = 92.478 \, \text{J}$ of work is done on the water by the atmosphere, so $-92.478 \, \text{J}$ is done on the atmosphere by the water.

Part (b)

The heat lost by the 10 kg of ice Q is:

$$Q = ml = 10000 \, \text{g} \cdot 333 \, \text{J/g} = 3330000 \, \text{J} \tag{9}$$

The heat lost far exceeds the work done on the water.

Exercise 3

An Introduction to Thermal Physics (Schroeder, 1e) Exercise 1.41

Part (a)

The heat gained by the water Q is:

$$Q = m_w c_w \Delta T_w = 250 \text{ g} \cdot 4.2 \text{ J/K} \cdot 4 = 4200 \text{ J} \quad (10)$$

The water loses $-Q = -4200 \text{ J}$.

Part (b)

The metal gains as much heat as is lost by the water, and so gains -4200 J .

Part (c)

The heat capacity of the metal C_m is:

$$\begin{aligned} Q &= C_m \Delta T_m \\ C_m &= \frac{Q}{\Delta T_m} = \frac{-4200 \text{ J}}{-76 \text{ K}} \approx 55.263 \text{ J/K} \end{aligned} \quad (11)$$

Part (d)

The specific heat of the chunk c_m is:

$$c_m = \frac{C_m}{m_m} = \frac{Q}{m_m \Delta T_m} = \frac{-4200 \text{ J}}{100 \text{ g} \cdot -76 \text{ K}} \approx 0.553 \text{ J/K} \quad (12)$$

Exercise 4

Part (a)

10000 coins are flipped. Using Stirling's approximation:

$$\begin{aligned}
 \frac{\binom{10000}{5000}}{2^{10000}} &= \frac{1}{2^{10000}} \frac{10000!}{5000!5000!} \\
 &\approx \frac{1}{2^{10000}} \frac{10000^{10000} e^{-10000} \sqrt{2\pi \cdot 10000}}{5000^{5000} e^{-5000} \sqrt{2\pi \cdot 5000} \cdot 5000^{5000} e^{-5000} \sqrt{2\pi \cdot 5000}} \\
 &\approx \frac{1}{\sqrt{2\pi \cdot 50 \cdot 50}} \approx 0.00798
 \end{aligned} \tag{13}$$

There is a 0.564 probability of getting 5000 heads and 5000 tails.

Part (b)

Using Stirling's approximation:

$$\begin{aligned}
 \frac{\binom{10000}{5100}}{2^{10000}} &= \frac{1}{2^{10000}} \frac{10000!}{5100!4900!} \\
 &\approx \frac{1}{2^{10000}} \frac{10000^{10000} e^{-10000} \sqrt{2\pi \cdot 10000}}{5100^{5100} e^{-5100} \sqrt{2\pi \cdot 5100} \cdot 4900^{4900} e^{-4900} \sqrt{2\pi \cdot 4900}} \\
 &\approx \frac{\left(\frac{50}{51}\right)^{5100} \left(\frac{50}{49}\right)^{4900}}{\sqrt{2\pi \cdot 49 \cdot 51}} \approx 0.00107
 \end{aligned} \tag{14}$$

There is a 0.00107 probability of getting 5100 heads and 4900 tails.

Part (c)

Using Stirling's approximation:

$$\begin{aligned}
 \frac{\binom{10000}{5500}}{2^{10000}} &= \frac{1}{2^{10000}} \frac{10000!}{5500!4500!} \\
 &\approx \frac{1}{2^{10000}} \frac{10000^{10000} e^{-10000} \sqrt{2\pi \cdot 10000}}{5500^{5500} e^{-5500} \sqrt{2\pi \cdot 5500} \cdot 4500^{4500} e^{-4500} \sqrt{2\pi \cdot 4500}} \\
 &\approx \frac{\left(\frac{50}{55}\right)^{5500} \left(\frac{50}{45}\right)^{4500}}{\sqrt{2\pi \cdot 45 \cdot 55}} \approx 1.423 \cdot 10^{-24}
 \end{aligned} \tag{15}$$

There is a $1.423 \cdot 10^{-24}$ probability of getting 5500 heads and 4500 tails.

Exercise 5

An Introduction to Thermal Physics (Schroeder, 1e) Exercise 2.7

Oscillator	#1	#2	#3	#4	#1	#2	#3	#4
Energy	2	0	0	0	1	0	0	0
	0	2	0	0	0	1	0	0
	0	0	2	0	0	0	1	0
	0	0	0	2	0	0	0	1
	1	1	0	0	0	0	0	0
	1	0	1	0				
	1	0	0	1				
	0	1	1	0				
	0	1	0	1				
	0	0	1	1				

There are three macrostates with the following multiplicities:

$$\Omega(0) = 1, \Omega(1) = 4, \Omega(2) = 10 \quad (16)$$

Exercise 6

An Introduction to Thermal Physics (Schroeder, 1e) Exercise 2.25
(partial, extended)

Part (a)

At the end of a long random walk, you are most likely to end up at your starting position, as you are expected to get equal numbers of heads and tails, and consequently equal number of steps to the left and right.

Part (b)

At the end of the walk, you are y steps to the right of your starting position. Let N_L be the number of steps taken to the left, and N_R to the right. Then:

$$\begin{aligned} N_R + N_L &= N \\ N_R - N_L &= y \end{aligned} \tag{17}$$

Solving for N_L, N_R in terms of N and y :

$$\begin{aligned} N_R &= \frac{N+y}{2} \\ N_L &= \frac{N-y}{2} \end{aligned} \tag{18}$$

The multiplicity Ω is:

$$\begin{aligned} \Omega &= \binom{N}{N_R} = \binom{N}{\frac{N+y}{2}} \\ &= \frac{N!}{(\frac{N+y}{2})!(\frac{N-y}{2})!} \end{aligned} \tag{19}$$

Part (c)

In the limit where $N \gg 1$ and $N \gg y$, using Stirling's approximation:

$$\begin{aligned}
\ln \Omega &= \ln \frac{N!}{\left(\frac{N+y}{2}\right)! \left(\frac{N-y}{2}\right)!} = \ln N! - \ln \left(\frac{N+y}{2}\right)! - \ln \left(\frac{N-y}{2}\right)! \\
&\approx \left(N \ln N - N + \frac{1}{2} \ln 2\pi N \right) \\
&\quad - \left(\frac{N+y}{2} \ln \frac{N+y}{2} - \frac{N+y}{2} + \frac{1}{2} \ln 2\pi \frac{N+y}{2} \right) \\
&\quad - \left(\frac{N-y}{2} \ln \frac{N-y}{2} - \frac{N-y}{2} + \frac{1}{2} \ln 2\pi \frac{N-y}{2} \right) \\
&\approx N \ln N - \frac{N+y}{2} \ln \frac{N+y}{2} - \frac{N-y}{2} \ln \frac{N-y}{2} + \frac{1}{2} \ln \frac{N}{2\pi^{\frac{N+y}{2} \frac{N-y}{2}}} \\
&\approx N \ln N + \frac{1}{2} \ln \frac{N}{2\pi^{\frac{N+y}{2} \frac{N-y}{2}}} - \frac{N+y}{2} \left(\ln \frac{N}{2} + \ln \left(1 + \frac{y}{N}\right) \right) \\
&\quad - \frac{N-y}{2} \left(\ln \frac{N}{2} + \ln \left(1 - \frac{y}{N}\right) \right) \\
&\approx N \ln 2 + \frac{1}{2} \ln \frac{N}{2\pi^{\frac{N+y}{2} \frac{N-y}{2}}} - \frac{N+y}{2} \ln \left(1 + \frac{y}{N}\right) - \frac{N-y}{2} \ln \left(1 - \frac{y}{N}\right) \\
&\approx N \ln 2 + \frac{1}{2} \ln \frac{2}{N\pi} - \frac{N+y}{2} \frac{y}{N} - \frac{N-y}{2} \cdot -\frac{y}{N} \\
&\approx N \ln 2 + \frac{1}{2} \ln \frac{2}{N\pi} - \frac{y^2}{N} \\
\Omega &\approx e^{N \ln 2 + \frac{1}{2} \ln \frac{2}{N\pi} - \frac{y^2}{N}} \approx 2^N \sqrt{\frac{2}{N\pi}} e^{-\frac{y^2}{N}}
\end{aligned} \tag{20}$$