Homework 3

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Exercise 1

***Introduction to Elementary Particles* (Griffiths, 2e) Exercise 6.8**

The differential cross section is:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{|\mathcal{M}|^2}{(E_a + E_b)^2} \frac{|p_f|}{|p_i|} \tag{1}$$

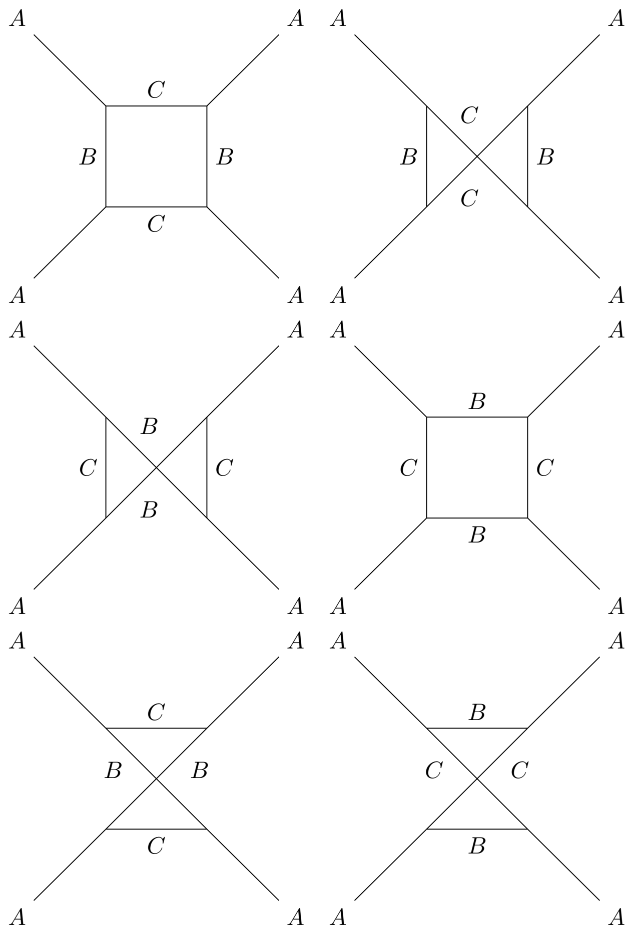
Since the recoil of the target is negligible, $|p_f| = |p_i|$ and since the target is so heavy, $E_b = m_b c^2 \gg E_a$, and $(E_a + E_b)^2 \approx \left(m_b c^2\right)^2 = m_b^2 c^4$.

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S\,|\mathcal{M}|^2}{m_b^2 c^4} \\ &= \left(\frac{\hbar}{8\pi m_b c}\right)^2 |\mathcal{M}|^2 \end{aligned} \tag{2}$$

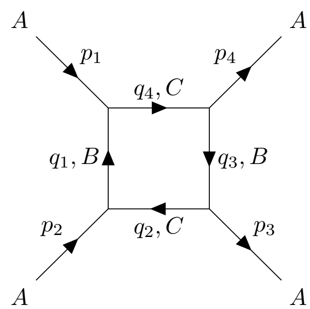
Exercise 2

Introduction to Elementary Particles **(Griffiths, 2e) Exercise 6.12**

Part (a)



Part (b)

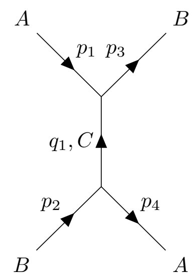


$$
\begin{aligned}
&\iiiint (-ig)^4 \frac{i}{q_1^2 - m_B^2 c^2} \frac{i}{q_2^2 - m_C^2 c^2} \frac{i}{q_3^2 - m_C^2 c^2} \frac{i}{q_4^2 - m_B^2 c^2} \\
&\cdot (2\pi)^4 \delta^4 (p_1 + q_1 - q_4) (2\pi)^4 \delta^4 (p_2 + q_2 - q_1) \\
&\cdot (2\pi)^4 \delta^4 (p_3 - q_3 + q_2) (2\pi)^4 \delta^4 (p_4 - q_4 + q_3) \\
&\cdot \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \frac{d^4 q_3}{(2\pi)^4} \frac{d^4 q_4}{(2\pi)^4} \\
&= g^4 \iiiint d^4 q_1 d^4 q_2 d^4 q_3 d^4 q_4 \frac{1}{q_1^2 q_2^2 q_3^2 q_4^2} \\
&\cdot \delta^4 (p_1 + q_1 - q_4) \delta^4 (p_2 + q_2 - q_1) \delta^4 (p_3 - q_3 + q_2) \delta^4 (p_4 - q_4 + q_3) \\
&= g^4 \iiint d^4 q_1 d^4 q_2 d^4 q_3 \frac{1}{q_1^2 q_2^2 q_3^2 (p_1 + q_1)^2} \\
&\cdot \delta^4 (p_2 + q_2 - q_1) \delta^4 (p_3 - q_3 + q_2) \delta^4 (p_4 - p_1 - q_1 + q_3) \\
&= g^4 \iint d^4 q_1 d^4 q_2 \frac{1}{q_1^2 q_2^2 (p_3 + q_2)^2 (p_1 + q_1)^2} \\
&\cdot \delta^4 (p_2 + q_2 - q_1) \delta^4 (p_4 - p_1 - q_1 + p_3 + q_2) \\
&= g^4 \iint \frac{\delta^4 (p_4 - p_1 + p_3 - p_2)}{q_1^2 (q_1 - p_2)^2 (p_3 + q_1 - p_1)^2 (p_1 + q_1)^2} d^4 q_1 \\
\mathcal{M} &= i \left(\frac{g}{2\pi} \right)^4 \iint \frac{1}{q^2 (q - p_2)^2 (p_3 + q - p_1)^2 (p_1 + q)^2} d^4 q
\end{aligned}
\tag{3}
$$

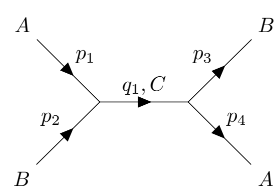
Exercise 3

Introduction to Elementary Particles **(Griffiths, 2e) Exercise 6.15**

Part (a)



$$
\begin{aligned}
&\int (-ig)^2 \frac{i}{q_1^2 - m_C^2 c^2} \\
&\cdot (2\pi)^4 \delta^4 (p_1 + q_1 - p_3) (2\pi)^4 \delta^4 (p_2 - q_1 - p_4) \frac{d^4 q_1}{(2\pi)^4} \\
&= -ig^2 (2\pi)^4 \int \frac{\delta^4 (p_1 + q_1 - p_3) \delta^4 (p_2 - q_1 - p_4)}{q_1^2 - m_C^2 c^2} d^4 q_1 \\
&= -ig^2 (2\pi)^4 \frac{\delta^4 (p_1 + p_2 - p_3 - p_4)}{(p_1 - p_3)^2 - m_C^2 c^2} \\
\mathcal{M}_1 &= \frac{g^2}{(p_1 - p_3)^2 - m_C^2 c^2}
\end{aligned} \tag{4}
$$



$$
\begin{aligned}
&\int (-ig)^2 \, \frac{i}{q_1^2 - m_C^2 c^2} \\
&\cdot (2\pi)^4 \, \delta^4 \left(p_1 + p_2 - q_1\right) (2\pi)^4 \, \delta^4 \left(p_3 + p_4 - q_1\right) \frac{d^4 q_1}{(2\pi)^4} \\
&= -ig^2 \, (2\pi)^4 \int \frac{\delta^4 \left(p_1 + p_2 - q_1\right) \delta^4 \left(p_3 + p_4 - q_1\right)}{q_1^2 - m_C^2 c^2} d^4 q_1 \\
&= -ig^2 \, (2\pi)^4 \, \frac{\delta^4 \left(p_3 + p_4 - p_1 - p_2\right)}{\left(p_1 + p_2\right)^2 - m_C^2 c^2} \qquad (5) \\
\mathcal{M}_2 &= \frac{g^2}{\left(p_1 + p_2\right)^2 - m_C^2 c^2} \\
\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 &= g^2 \left(\frac{1}{\left(p_1 - p_3\right)^2 - m_C^2 c^2} + \frac{1}{\left(p_1 + p_2\right)^2 - m_C^2 c^2} \right)
\end{aligned}
$$

Part (b)

$$
\begin{aligned}
p_1 = \left(\frac{E}{c}, \vec{p}_1\right), p_2 &= \left(\frac{E}{c}, \vec{p}_2\right) = \left(\frac{E}{c}, -\vec{p}_1\right) \\
p_3 = \left(\frac{E}{c}, \vec{p}_3\right), p_4 &= \left(\frac{E}{c}, \vec{p}_4\right) = \left(\frac{E}{c}, -\vec{p}_3\right) \\
\left(p_1 + p_2\right)^2 &= \left(2\frac{E}{c}, \mathbf{0}\right)^2 = 4\frac{E^2}{c^2} \\
\left(p_1 - p_3\right)^2 &= \left(0, \vec{p}_1 + \vec{p}_3\right)^2 \\
&= -\left(\vec{p}_1 + \vec{p}_3\right)^2 = -\vec{p}_1^2 - \vec{p}_3^2 - 2|\vec{p}_1||\vec{p}_2| \cos\theta \\
&= -2p^2 \left(1 + \cos\theta\right) = -4p^2 \cos^2 \frac{\theta}{2} \\
\frac{d\sigma}{d\Omega} &= \left(\frac{\hbar c}{8\pi}\right)^2 \frac{|\mathcal{M}|^2}{(E+E)^2} \frac{|p_f|}{|p_i|} \qquad (6) \\
&= \left(\frac{\hbar c}{8\pi}\right)^2 \frac{g^4}{4E^2} \left(-\frac{1}{4\left(\frac{E^2}{c^2} - m^2 c^2\right) \cos^2 \frac{\theta}{2}} + \frac{c^2}{4E^2} \right)^2 \\
&= \left(\frac{\hbar c}{8\pi}\right)^2 \frac{g^4 c^4}{64E^2} \left(-\frac{1}{(E^2 - m^2 c^4) \cos^2 \frac{\theta}{2}} + \frac{1}{E^2} \right)^2 \\
&= -\left(\frac{\hbar c}{8\pi}\right)^2 \frac{g^4 c^4}{64E^2} \left(\frac{E^2 \tan^2 \frac{\theta}{2} + m^2 c^4}{E^2 (E^2 - m^2 c^4)} \right)^2 \\
&= -\left(\frac{\hbar g^2 c^3}{64\pi} \frac{E^2 \tan^2 \frac{\theta}{2} + m^2 c^4}{E^3 (E^2 - m^2 c^4)} \right)^2
\end{aligned}
$$

Part (c)

In the large m_B limit:

$$p_1 = \left(\frac{E}{c}, \vec{p}_1\right), p_2 = (m_B c, \mathbf{0})$$
$$p_3 = (m_B c, \mathbf{0}), p_4 = \left(\frac{E}{c}, \vec{p}_4\right)$$

$$\begin{aligned}
s &= (p_1 + p_2)^2 c^2 = \left(\frac{E}{c} + m_B c, \vec{p}_1\right)^2 c^2 \\
&= \left(\left(\frac{E}{c} + m_B c\right)^2 - |\vec{p}_1|^2\right) c^2 \\
&= \left(\left(\frac{E}{c} + m_B c\right)^2 - \left(\frac{E^2}{c^2} - m_A^2 c^2\right)\right) c^2 \\
&= \left(2Em_B + m_B^2 c^2 + m_A^2 c^2\right) c^2 \approx m_B^2 c^4 \\
t &= (p_1 - p_3)^2 c^2 = \left(\frac{E}{c} - m_B c, \vec{p}_1\right)^2 c^2 \\
&= \left(\left(\frac{E}{c} - m_B c\right)^2 - |\vec{p}_1|^2\right) c^2 \\
&= \left(\left(\frac{E}{c} - m_B c\right)^2 - \left(\frac{E^2}{c^2} - m_A^2 c^2\right)\right) c^2 \\
&= \left(-2Em_B + m_B^2 c^2 + m_A^2 c^2\right) c^2 \approx m_B^2 c^4 \\
\frac{d\sigma}{d\Omega} &= \left(\frac{\hbar}{8\pi m_B c}\right)^2 g^4 \left(\frac{2}{m_B^2 c^2 - m_C^2 c^2}\right)^2 \\
&\approx \left(\frac{\hbar}{8\pi m_B c}\right)^2 g^4 \left(\frac{2}{m_B^2 c^2}\right)^2 \\
&\approx \left(\frac{\hbar g^2}{4\pi m_B^3 c^3}\right)^2
\end{aligned} \tag{7}$$

Part (d)

$$\begin{aligned}
\sigma &= \int \frac{d\sigma}{d\Omega}\, d\Omega \\
&= 4\pi \frac{d\sigma}{d\Omega} \\
&= 4\pi \left(\frac{\hbar g^2}{4\pi m_B^3 c^3}\right)^2 = \frac{1}{\pi}\left(\frac{\hbar g^2}{2m_B^3 c^3}\right)^2
\end{aligned} \tag{8}$$