# The intcalc package

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### Abstract

This package provides expandable arithmetic operations with integers.

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<sup>\*</sup>Please report any issues at https://github.com/ho-tex/intcalc/issues

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### 1 Documentation

### 1.1 Introduction

Package intcalc defines arithmetic operations that deal with integers. Integers mean numbers in TeX. The same restrictions apply, the range is limited to [-2147483647, 2147483647].

The operations have the form of macros that take one or two integers as parameter and return the integer result. The macro name is a three letter operation name prefixed by the package name, e.g. \intcalcAdd{10}{43} returns 53.

The macros are fully expandable, exactly two expansion steps generate the result. Therefore the operations may be used nearly everywhere in T<sub>E</sub>X, even inside \number, \csname, file names, or other expandable contexts.

The package contains two implementations of the operations. If  $\varepsilon$ -TEX is detected then the macros are implemented using its features (\numexpr). Otherwise the slower implementation without  $\varepsilon$ -TEX's help is choosen.

### 1.2 Conditions

### 1.2.1 Preconditions

• Arguments can be anything that TEX interprets as "number". Examples: plain numbers, count or length register, macros that expands to a number.

- The arguments are limited to the range -2147483647 until 2147483647. These numbers belong to the range. Note that some operations have additionals restrictions to the range.
- The argument may be expressions that \numexpr understands if ε-T<sub>E</sub>X is available.
- The resulting number must fit in the allowed range.

#### 1.2.2 Postconditions

Additional properties of the macros apart from calculating a correct result (of course ©):

- The macros are fully expandable. Thus they can be used inside \edef, \csname, after \number, for example.
- Furthermore exactly two expansion steps calculate the result.
- The number consists of one optional minus sign and one to ten digits. The first digit is larger than zero for numbers that consists of more than one digit.

In short, the number format is exactly the same as \number generates. And the tokens (minus sign, digits) have catcode 12 (other).

• Call by value is simulated. First the arguments are converted to numbers. Then these numbers are used in the calculations.

Remember that arguments may contain expensive macros or  $\varepsilon$ -TEX expressions. This strategy avoids multiple evaluations of such arguments.

### 1.3 Error handling

There are two kinds of errors if a precondition is violated: Some errors are detected by the macros, example: division by zero. In this cases an undefined control sequence is called and causes a TeX error message, example: \IntCalcError:DivisionByZero. The name of the control sequence contains the reason for the error. The TeX error may be ignored. Then the operation returns zero as result. Because the macros are supposed to work in expandible contexts. An traditional error message, however, is not expandable and would break these contexts.

If a number exceeds the range of -2147483647 until 2147483647, then TeX throws an error "Number too big" and recovers by using biggest allowed value. Example for the negative number -3000000000 is replaced by -2147483647.

### 1.4 Operations

Some definition equations below use the function Int that converts a real number to an integer. The number is truncated that means rounding to zero:

$$\operatorname{Int}(x) := \begin{cases} \lfloor x \rfloor & \text{if } x \ge 0 \\ \lceil x \rceil & \text{otherwise} \end{cases}$$

### 1.4.1 Num

### \intcalcNum $\{\langle x \rangle\}$

Macro \intcalcNum converts its argument to a normalized integer number without unnecessary leading zeros or signs. The result matches the regular expression:

### 1.4.2 Inv, Abs, Sgn

### $\$ intcalcInv $\{\langle x \rangle\}$

Macro \intcalcInv switches the sign.

$$Inv(x) := -x$$

### \intcalcAbs $\{\langle x angle\}$

Macro \intcalcAbs returns the absolute value of integer  $\langle x \rangle$ .

$$Abs(x) := |x|$$

### 

Macro \intcalcSgn encodes the sign of  $\langle x \rangle$  as number.

$$Sgn(x) := \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

These return values can easily be distinguished by \ifcase:

```
\ifcase\intcalcSgn{<x>}
    $x=0$
\or
    $x>0$
\else
    $x<0$
\fi</pre>
```

### 1.4.3 Min, Max, Cmp

### \intcalcMin $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \intcalcMin returns the smaller of the two integers.

$$Min(x, y) := \begin{cases} x & \text{if } x < y \\ y & \text{otherwise} \end{cases}$$

\intcalcMax  $\{\langle x \rangle\}$   $\{\langle y \rangle\}$ 

Macro \intcalcMax returns the larger of the two integers.

$$\operatorname{Max}(x,y) := \begin{cases} x & \text{if } x > y \\ y & \text{otherwise} \end{cases}$$

\intcalcCmp  $\{\langle x \rangle\}$   $\{\langle y \rangle\}$ 

Macro \intcalcCmp encodes the comparison result as number:

$$Cmp(x,y) := \begin{cases} -1 & \text{if } x < y \\ 0 & \text{if } x = y \\ 1 & \text{if } x > y \end{cases}$$

These values can be distinguished by \ifcase:

### 1.4.4 Inc, Dec, Add, Sub

\intcalcInc  $\{\langle x \rangle\}$ 

Macro \intcalcInc increments  $\langle x \rangle$  by one.

$$Inc(x) := x + 1$$

\intcalcDec  $\{\langle x \rangle\}$ 

Macro \intcalcDec decrements  $\langle x \rangle$  by one.

$$Dec(x) := x - 1$$

\intcalcAdd  $\{\langle x \rangle\}$   $\{\langle y \rangle\}$ 

Macro \intcalcAdd adds the two numbers.

$$Add(x, y) := x + y$$

\intcalcSub  $\{\langle x \rangle\}$   $\{\langle y \rangle\}$ 

Macro \intcalcSub calculates the difference.

5

$$Sub(x, y) := x - y$$

### 1.4.5 Shl, Shr

### \intcalcShl $\{\langle x \rangle\}$

Macro \intcalcShl implements shifting to the left that means the number is multiplied by two. Overflow is possible. The sign is preserved.

$$Shl(x) := x * 2$$

### \intcalcShr $\{\langle x \rangle\}$

Macro \intcalcShr implements shifting to the right. That is equivalent to an integer division by two. The sign is preserved.

$$Shr(x) := Int(x/2)$$

### 1.4.6 Mul, Sqr, Fac, Pow

### \intcalcMul $\{\langle x angle\}$ $\{\langle y angle\}$

Macro \intcalcMul calculates the product of  $\langle x \rangle$  and  $\langle y \rangle$ .

$$Mul(x, y) := x * y$$

### \intcalcSqr $\{\langle x \rangle\}$

Macro  $\intcalcSqr$  returns the square product.

$$Sqr(x) := x^2$$

### $\$ intcalcFac $\{\langle x \rangle\}$

Macro \intcalcFac returns the factorial of  $\langle x \rangle$ . Negative numbers are not permitted.

$$Fac(x) := x!$$
 for  $x \ge 0$ 

$$(0! = 1)$$

### \intcalcPow $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \intcalcPow calculates the value of  $\langle x \rangle$  to the power of  $\langle y \rangle$ . The error "division by zero" is thrown if  $\langle x \rangle$  is zero and  $\langle y \rangle$  is negative. permitted:

$$\operatorname{Pow}(x, y) := \operatorname{Int}(x^y)$$
 for  $x \neq 0$  or  $y \geq 0$ 

$$(0^0 = 1)$$

### 1.4.7 Div, Mod

### \intcalcDiv $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \intcalcDiv performs an integer division. Argument  $\langle y \rangle$  must not be zero.

$$\operatorname{Div}(x,y) := \operatorname{Int}(x/y)$$
 for  $y \neq 0$ 

\intcalcMod 
$$\{\langle x \rangle\}$$
  $\{\langle y \rangle\}$ 

Macro \intcalcMod gets the remainder of the integer division. The sign follows the divisor  $\langle y \rangle$ . Argument  $\langle y \rangle$  must not be zero.

$$Mod(x, y) := x \% y$$
 for  $y \neq 0$ 

The result ranges:

$$-|y| < \operatorname{Mod}(x, y) \le 0 \quad \text{for } y < 0$$
  
 
$$0 \le \operatorname{Mod}(x, y) < y \quad \text{for } y \ge 0$$

### 1.5 Interface for programmer

If the programmer can ensure some more properties about the arguments of the operations, then the following macros are a little more efficient.

In general numbers must obey the following constraints:

- Plain number: digit tokens only, no command tokens.
- Non-negative. Signs are forbidden.
- Arguments and the result must fit in range 0..2147483647.
- Delimited by exclamation mark. Curly braces around the number are not allowed and will break the code.

### \IntCalcInc $\langle number \rangle$ !

Incrementation, range: 0..2147483646.

$$\IntCalcDec\ \langle number \rangle$$
 !

Decrementation, range: 1..2147483647.

Addition,  $A \geq B$ .

Subtraction,  $A \geq B$ .

Left shift (multiplication with two), range: 0..1073741823.

Right shift (integer division by two).

Multiplication,  $A \geq B$ .

```
\IntCalcDiv \langle number A \rangle ! \langle number B \rangle !
```

Division operation.

Modulo operation.

# 2 Implementation

1 (\*package)

### 2.1 Reload check and package identification

Reload check, especially if the package is not used with LATEX.

```
2 \begingroup\catcode61\catcode48\catcode32=10\relax%
   \catcode13=5 % ^^M
    \endlinechar=13 %
4
   \catcode35=6 % #
5
   \catcode39=12 % '
   \catcode44=12 % ,
   \catcode45=12 % -
   \catcode46=12 % .
   \catcode58=12 % :
   \catcode64=11 % @
11
   \catcode123=1 % {
12
    \catcode125=2 % }
13
    \expandafter\let\expandafter\x\csname ver@intcalc.sty\endcsname
    \ifx\x\relax % plain-TeX, first loading
    \else
16
      \def\empty{}%
17
     \ifx\x\empty % LaTeX, first loading,
18
        % variable is initialized, but \ProvidesPackage not yet seen
19
20
      \else
21
        \expandafter\ifx\csname PackageInfo\endcsname\relax
22
          \def\x#1#2{%}
           \immediate\write-1{Package #1 Info: #2.}%
23
         }%
24
        \else
25
          26
27
        \x{intcalc}{The package is already loaded}%
28
29
        \aftergroup\endinput
30
      \fi
31
    \fi
32 \endgroup%
```

Package identification:

```
33 \begingroup\catcode61\catcode48\catcode32=10\relax%
    \catcode13=5 % ^^M
    \endlinechar=13 %
36
    \catcode35=6 % #
    \catcode39=12 % '
37
    \catcode40=12 % (
38
    \colored{1=12 \%}
39
    \colone{1} \catcode44=12 % ,
40
41
    \catcode45=12 % -
42
    \catcode46=12 % .
43
    \catcode47=12 % /
44
    \catcode58=12 % :
    \catcode64=11 % @
45
    \catcode91=12 % [
47
    \catcode93=12 % ]
48
    \catcode123=1 % {
    \catcode125=2 % }
49
    \expandafter\ifx\csname ProvidesPackage\endcsname\relax
50
      \def\x#1#2#3[#4]{\endgroup}
51
        \immediate\write-1{Package: #3 #4}%
52
        \xdef#1{#4}%
53
      }%
54
55
    \else
      \def\x#1#2[#3]{\endgroup}
56
57
        #2[{#3}]%
        \ifx#1\@undefined
58
           \xdef#1{#3}%
59
60
        \fi
        \ifx#1\relax
61
           \xdef#1{#3}%
62
        \fi
63
      ጉ%
64
    \fi
65
66 \expandafter\x\csname ver@intcalc.sty\endcsname
67 \ProvidesPackage{intcalc}%
    [2019/12/15 v1.3 Expandable calculations with integers (HO)]%
```

### 2.2 Catcodes

```
69 \begingroup\catcode61\catcode48\catcode32=10\relax%
    \catcode13=5 % ^^M
71
    \endlinechar=13 %
72
    \catcode123=1 % {
73
    \catcode125=2 % }
74
    \catcode64=11 % @
    \def\x{\endgroup
75
      \expandafter\edef\csname InCa@AtEnd\endcsname{%
76
         \endlinechar=\the\endlinechar\relax
77
         \catcode13=\the\catcode13\relax
78
79
         \catcode32=\the\catcode32\relax
80
         \catcode35=\the\catcode35\relax
81
         \catcode61=\the\catcode61\relax
         \catcode64=\the\catcode64\relax
82
         \catcode123=\the\catcode123\relax
83
         \verb|\catcode| 125 = \verb|\the| catcode| 125 \verb|\relax|
84
85
      }%
86
    }%
```

```
87 \x\catcode61\catcode48\catcode32=10\relax%
            88 \catcode13=5 % ^^M
            89 \endlinechar=13 %
            90 \catcode35=6 \% #
            91 \catcode64=11 % @
            92 \catcode123=1 % {
            93 \catcode125=2 % }
            94 \def\TMP@EnsureCode#1#2{%
                \edef\InCa@AtEnd{%
                  \InCa@AtEnd
                  \catcode#1=\the\catcode#1\relax
            98
            99
                \color= 1=#2\relax
           100 }
           101 \TMP@EnsureCode{33}{12}% !
           102 \TMP@EnsureCode{40}{12}% (
           103 \TMP@EnsureCode{41}{12}% )
           104 \TMP@EnsureCode{42}{12}% *
           105 \TMP@EnsureCode{43}{12}% +
           106 \TMP@EnsureCode{45}{12}% -
           107 \TMP@EnsureCode{47}{12}% /
           108 \TMP@EnsureCode{58}{11}% : (letter!)
           109 \TMP@EnsureCode{60}{12}% <
           110 \TMP@EnsureCode{62}{12}% >
           111 \TMP@EnsureCode{63}{14}% ? (comment!)
           112 \edef\InCa@AtEnd{\InCa@AtEnd\noexpand\endinput}
           114 \verb|\expandafter\ifx\csname InCa@TestMode\endcsname\relax|
           115 \else
           116 \catcode63=9 % ? (ignore)
           117 \fi
           118 ? \let\InCa@@TestMode\InCa@TestMode
                 Macros independent of \varepsilon-TeX
           2.3.1 Abs, Sgn
\InCa@Abs
           119 \def\InCa@Abs#1#2!{%
                \ifx#1-%
           120
           121
                  #2%
                \else
                  #1#2%
           124 \fi
           125 }
\InCa@Sgn
           126 \def\InCa@Sgn#1#2!{%
           127
                \ifx#1-%
           128
                  -1%
                \else
           129
                  \ifx#10%
           130
                    0%
           131
                  \else
           132
                    1%
           133
                  \fi
           134
           135
                \fi
           136 }
```

### 2.3.2 Min, Max, Cmp

```
\InCa@Min
```

```
137 \def\InCa@Min#1!#2!{%
                  \ifnum#1<#2 %
            138
                    #1%
            140
                  \else
            141
                    #2%
            142
                  \fi
            143 }
\InCa@Max
            144 \def\InCa@Max#1!#2!{%
                 \ifnum#1>#2 %
            145
            146
                    #1%
            147
                  \else
            148
                    #2%
            149
                  \fi
            150 }
\InCa@Cmp
            151 \def\InCa@Cmp#1!#2!{%
            152
                  \ifnum#1=#2 %
            153
                    0%
            154
                  \else
                    \ifnum#1<#2 %
            155
                      -%
            156
```

### 2.3.3 Fac

157

158

159 160 } \fi

1%

\fi

\InCa@Fac

It does not make much sense to calculate the faculty by an general algorithm. The allowed range of arguments is too low because of the limited integer domain.

```
161 \def\InCa@Fac#1!{%
    \ifcase#1 1% 0!
162
     \or 1% 1!
163
     \or 2% 2!
     \or 6% 3!
165
     \or 24% 4!
166
     \or 120% 5!
167
     \or 720% 6!
168
     \or 5040% 7!
169
     \or 40320% 8!
170
     \or 362880% 9!
     \or 3628800% 10!
172
     \or 39916800% 11!
173
     \or 479001600% 12!
174
     \else
175
176
       0\IntCalcError:FacNegative%
177
178
         0\IntCalcError:FacOverflow%
179
       \fi
180
     \fi
181
182 }
```

### 2.4 Implementation based on $\varepsilon$ -T<sub>E</sub>X

 $183 \verb|\degingroup\expandafter\expandafte$ 

Only \numexpr is used from  $\varepsilon$ -TEX.

```
184 \expandafter\ifx\csname numexpr\endcsname\relax
              185 \ensuremath{\setminus} else
             2.4.1 Num
\intcalcNum
                    \def\intcalcNum#1{%
              187
                      \the\numexpr#1\relax
              188
             2.4.2 Inv, Abs, Sgn
\intcalcInv
                    \def\intcalcInv#1{%
              189
                      \number-\intcalcNum{#1} %
              190
                   }%
              191
\intcalcAbs
              192
                    \def\intcalcAbs#1{%
              193
                      \number\expandafter\InCa@Abs\the\numexpr#1! %
              194
\intcalcSgn
                    \def\intcalcSgn\#1{\%}
              195
                      \verb|\number| expandafter InCa@Sgn the \\ numexpr#1! %
              196
                   }%
              197
             2.4.3 Min, Max, Cmp
\intcalcMin
                    \def\intcalcMin#1#2{%
              198
                      \number\expandafter\InCa@Min
              200
                      \the\numexpr#1\expandafter!%
                      \the\numexpr#2! %
              201
              202
                   }%
\intcalcMax
              203
                    \def\intcalcMax#1#2{%
              204
                      \number\expandafter\InCa@Max
              205
                      \the\numexpr#1\expandafter!%
                      \the\numexpr#2! %
              207
                   }%
\intcalcCmp
              208
                    \verb|\number| expandafter \\| In Ca@Cmp \\|
              209
              210
                      \the\numexpr#1\expandafter!\the\numexpr#2! %
              211
```

### 2.4.4 Inc, Dec

		,
\intcalcInc		
(111000101110	010	\ dos\:
		\def\intcalcInc#1{%
	$\frac{213}{214}$	<pre>\the\numexpr#1+1\relax }%</pre>
	214	J /6
\intcalcDec		
•	215	\def\intcalcDec#1{%
	216	\the\numexpr#1-1\relax
	217	}%
		370
\IntCalcInc		
	218	\def\IntCalcInc#1!{%
	219	\the\numexpr#1+1\relax
	220	}%
\IntCalcDec		
	221	\def\IntCalcDec#1!{%
	222	\the\numexpr#1-1\relax
	223	}%
	2.4.5	Add, Sub
		,
\intcalcAdd		
	224	\def\intcalcAdd#1#2{%
	225	\the\numexpr#1+(#2)\relax
	226	}%
\intcalcSub		
\IIICCAICDUD		\ 1 C\ :
		\def\intcalcSub#1#2{%
	228	<pre>\the\numexpr#1-(#2)\relax }%</pre>
	229	<b>5%</b>
\IntCalcAdd		
	230	\def\IntCalcAdd#1!#2!{%
	230	\the\numexpr#1+#2\relax
	232	}%
\IntCalcSub		
	233	\def\IntCalcSub#1!#2!{%
	234	\the\numexpr#1-#2\relax
	235	}%
	2.4.6	Shl, Shr
		,
\intcalcShl		
	236	\def\intcalcShl#1{%
	237	\the\numexpr(#1)*2\relax
	238	}%
\intcalcShr		
/Incarconf		2 2 2 44 594
	239	\def\intcalcShr#1{%
	240	<pre>\number\expandafter\InCa@Shr\the\numexpr#1! %</pre>
	241	}%

```
\IntCalcShl
                  \def\IntCalcShl#1!{%
              242
              243
                    \the\numexpr#1*2\relax
              244
\IntCalcShr
                  \def\IntCalcShr#1!{%
              245
              246
                    \theta = \frac{1-1}{else#1 fi/2 relax}
              247
  \InCa@Shr
                  \def\InCa@Shr#1#2!{%
              248
              249
                   \ifx#1-%
                       -\InCa@Shr#2!%
              250
                   \else
              251
                      \ifodd#1#2 %
              252
                        \theta = \frac{1#2-1}{2}
              253
              254
              255
                         \theta = 1#2/2 relax
                       \fi
              256
              257
                   \fi
              258 }%
             2.4.7 Mul, Sqr, Fac
\intcalcMul
              259
                   \def\intcalcMul#1#2{%
              260
                    \t = \m (#1)*(#2)\
              261
                  }%
\IntCalcMul
                  \def\IntCalcMul#1!#2!{%
              262
              263
                   \t \sum_{x=0}^{the\numexpr#1*#2\relax}
              264
\intcalcSqr
                  \def\intcalcSqr#1{%
              265
                    \number\expandafter\InCa@Sqr\the\numexpr#1! %
              266
              267
                  }%
 \InCa@Sqr
                   \def\InCa@Sqr#1!{%
              268
              269
                    \the\numexpr#1*#1\relax
                  }%
              270
\intcalcFac
              271
                   \def\intcalcFac#1{%
              272
                    \number\expandafter\InCa@Fac\the\numexpr#1! %
                  }%
              273
             2.4.8 Pow
\intcalcPow
              274
                  \def\intcalcPow#1#2{%
                    \number\expandafter\InCa@Pow
              275
                    \the\numexpr#1\expandafter!%
              276
              277
                    \the\numexpr#2! %
              278
                 }%
```

```
\InCa@Pow
```

```
\def\InCa@Pow#1#2!#3#4!{%
               279
               280
                       \ightharpoonup \ifcase#3#4 % power = 0
               281
                         1%
               282
                       283
                         #1#2%
                       \or \% power = 2
               284
                         \theta = 1#2*#1#2\
               285
               286
                       \else
                         \footnote{1} \iftit case #1#2 % basis = 0, power <> 0
               287
               288
               289
                           \iny \% power < 0
                             0\IntCalcError:DivisionByZero%
               290
                           \fi
               291
               292
                         \or
                           1% basis = 1
               293
               294
                           \lim 1#2=\m \% basis = -1
               295
                             \ifodd#3#4 %
               296
                               -%
               297
                             \fi
               298
                             1%
               299
                           \else % |basis| > 1
               300
                             ifx#3-% power < 0
               301
               302
                               0%
               303
                             \else % power > 2
                               \InCa@PowRec#1#2!#3#4!1!%
               304
                             \fi
               305
                           \fi
               306
               307
                         \fi
               308
                       \fi
                    }%
               309
\InCa@PowRec
                    Pow(b, p) {
                      PowRec(b, p, 1)
                    PowRec(b, p, r) {
                      if p == 1 then
                        return r*b
                      else
                        ifodd p then
                          return PowRec(b*b, (p-1)/2, r*b) % p div 2 = (p-1)/2
                          return PowRec(b*b, (p-1)/2, r)
                        fi
                      fi
                    }
               310
                     \def\InCa@PowRec#1!#2!#3!{%
                       \int 2=\0
               312
                         \the\numexpr#1*#3\relax
               313
                       \else
                         \ifodd#2 %
               314
                           \expandafter\InCa@PowRec
               315
                           \the\numexpr#1*#1\expandafter!%
               316
               317
                           \the\numexpr(#2-1)/2\expandafter!%
                           \the\numexpr#1*#3\expandafter\expandafter\expandafter!%
               318
               319
               320
                           \expandafter\InCa@PowRec
```

```
321 \the\numexpr#1*#1\expandafter!%
322 \the\numexpr(#2-1)/2\expandafter!%
323 \number#3\expandafter\expandafter\expandafter!%
324 \fi
325 \fi
326 }%
```

### 2.4.9 Div, Mod

TEX's \divide truncates,  $\varepsilon$ -TEX's \numexpr rounds the result of a division. The rounding method is called "Symmetric Arithmetic Rounding" or "Round-Half-Up" ("Kaufmännisches Runden" in German):

```
1 = 3 divide 2 = 1.5 = numexpr 3/2 = 2
-1 = -3 divide 2 = -1.5 = numexpr -3/2 = -2
```

Macro \intcalcDiv follows TeX and truncates. The calculation is done by the following formula:

$$Div(X,Y) = (X - (Y-1)/2)/Y$$
 for  $X,Y > 0$  (1)

The operator '/' is \numexpr's division.

\intcalcDiv

```
327 \def\intcalcDiv#1#2{%

328 \number\expandafter\InCa@Div

329 \the\numexpr#1\expandafter!%

330 \the\numexpr#2! %

331 }%
```

\InCa@Div

```
\label{locality} $$ \end{align*} $$ \end{ali
332
333
                                             \ifcase#2 %
334
                                                          0\IntCalcError:DivisionByZero%
                                               \else
335
                                                           \ifcase#1 %
336
                                                                       0%
337
                                                          \else
338
                                                                       \expandafter\InCa@@Div
339
340
                                                                        \romannumeral 0%
                                                                        \int \frac{1}{z}
341
                                                                                   \expandafter-\number-#1%
342
                                                                       \else
343
                                                                                   \expandafter+\number#1%
344
                                                                        \fi
345
346
                                                                       \expandafter!%
347
                                                                       \romannumeral 0%
                                                                       \lim 2<\z0
348
                                                                                   \expandafter-\number-#2%
349
350
                                                                                    \expandafter+\number#2%
351
                                                                       \fi
352
353
                                                                        !%
354
                                                           \fi
                                              \fi
355
356
                              }%
```

 $\IntCalcDiv$ 

```
\def\InCa@Temp#1{%
              357
                      \def\IntCalcDiv##1!##2!{%
              358
              359
                        \number
                        \ifcase##2 %
              360
              361
                          0\IntCalcError:DivisionByZero%
                        \else
              362
                          \ifcase##1 %
              363
                            0%
              364
                          \else
              365
                            \t = \frac{\#1-(\#2-1)/2}{\#2}
              366
              367
              368
                        \fi
              369
                        #1%
                     }%
              370
                    }%
              371
                    \InCa@Temp{ }%
              372
 \InCa@@Div
                    \def\InCa@@Div#1#2!#3#4!{%
              373
                      #1#3%
              374
                      \t \sum_{\#4-1}/2)/\#4\
              375
                   }%
              376
\intcalcMod
                    \def\intcalcMod#1#2{%
              377
                      \number\expandafter\InCa@Mod
              378
                      \the\numexpr#1\expandafter!%
              380
                      \the\numexpr#2! %
                   }%
              381
  \InCa@Mod
                    \def\InCa@Mod#1!#2!{%
                      \ifcase#2 %
              383
              384
                        0\IntCalcError:DivisionByZero%
                      \else
              385
                        \ifcase#1 %
              386
                          0%
              387
              388
                        \else
                          \expandafter\InCa@@Mod
              389
                          \romannumeral 0%
              390
                          \int \frac{1}{z} dx
              391
                            \expandafter-\number-#1%
              392
                          \else
              393
                            \expandafter+\number#1%
              394
              395
                          \fi
                          \expandafter!%
              396
              397
                          \romannumeral 0%
                          \lim 2<\z0
              398
                            \expandafter-\number-#2%
              399
              400
                          \else
                            \expandafter+\number#2%
              401
              402
                          \fi
                          !%
              403
                        \fi
              404
                      \fi
              405
                   }%
              406
```

 $\IntCalcMod$ 

```
407
                   \def\InCa@Temp#1{%
                     \def\IntCalcMod##1!##2!{%
              408
              409
                        \number
                        \ifcase##2 %
              410
              411
                          0\IntCalcError:DivisionByZero%
              412
                        \else
                          \ifcase##1 %
              413
                            0%
              414
                          \else
              415
                            \theta = \frac{\#1-(\#4-1)/2}{\#2*\#2}
              416
                        \fi
              418
              419
                        #1%
                     }%
              420
                   }%
              421
                   \InCa@Temp{ }%
              422
 \InCa@@Mod
                   \def\InCa@@Mod#1#2!#3#4!{%
              423
                     \if#3+%
              425
                        \if#1+%
                          \theta^2-\ln Ca@Div+#2!+#4!*#4\
              426
              427
              428
                          \expandafter\InCa@ModX
                          \the\numexpr-#2+\InCa@@Div+#2!+#4!*#4!#4!%
              429
                        \fi
              430
                      \else
              431
                        -%
              432
                        \if#1+%
              433
                          \expandafter\InCa@ModX
              434
                          \the\numexpr-#2+\InCa@@Div+#2!+#4!*#4!#4!%
              435
              436
                          \theta = \frac{2-\ln Ca@@Div+#2!+#4!*#4\relax}{}
              438
                        \fi
                     \fi
              439
                   }%
              440
 \InCa@ModX
                   \def\InCa@ModX#1!#2!{%
              441
                     \ifcase#1 %
              442
              443
                        0%
              445
                        \theta = 1+#2\
                     \fi
              446
                   }%
              447
                   \expandafter\InCa@AtEnd
              449 \fi%
                    Implementation without \varepsilon-T<sub>E</sub>X
             2.5
             2.5.1
                     Num
\intcalcNum
              450 \def\intcalcNum#1{%
                   \number\expandafter\InCa@FirstOfOne\number#1! %
              452 }
```

### 2.5.2 Inv, Abs, Sgn

```
\intcalcInv
                    453 \ensuremath{\mbox{def}\mbox{intcalcInv#1}}\xspace \ensuremath{\mbox{\%}}
                    454 \number\expandafter\InCa@FirstOfOne\number-#1! \%
                    455 }
\InCa@FirstOfOne
                    456 \def\InCa@FirstOfOne#1!{#1}
     \intcalcAbs
                    457 \ensuremath{\mbox{def}\mbox{intcalcAbs#1}}
                    458 \number\expandafter\InCa@Abs\number#1! \%
                    459 }
     \intcalcSgn
                    460 \def\intcalcSgn#1{%
                    461 \number\expandafter\InCa@Sgn\number#1! %
                    462 }
                   2.5.3 Min, Max, Cmp
     \intcalcMin
                    463 \def\intcalcMin#1#2{%
                         \number\expandafter\InCa@Min
                    465
                         \number\number#1\expandafter!\number#2! %
                    466 }
     \intcalcMax
                    467 \def\intcalcMax#1#2{%
                        \number\expandafter\InCa@Max
                    469 \number#1\expandafter!\number#2! %
                    470 }
     \intcalcCmp
                    471 \def\intcalcCmp#1#2{%
                    472 \number\expandafter\InCa@Cmp
                    473 \number\number#1\expandafter!\number#2! %
                    474 }%
                   2.5.4 Inc, Dec
     \intcalcInc
                    475 \def\intcalcInc#1{%
                         \number\expandafter\InCa@IncSwitch\number#1! %
                    477 }
 \InCa@IncSwitch
                    478 \def\InCa@IncSwitch#1#2!{%
                    479
                        \ifx#1-%
                           -%
                    480
                            \csname InCa@Empty%
                    481
                           \InCa@Dec#2!%
                    482
                    483 \else
                           \csname InCa@Empty%
                    484
                          \InCa@Inc#1#2!\%
                    485
                    486 \fi
                    487 }
```

```
\intcalcDec
                          488 \ensuremath{\mbox{def}\mbox{intcalcDec#1}}\
                                \number\expandafter\InCa@DecSwitch\number#1! %
                          490 }
     \InCa@DecSwitch
                          491 \ensuremath{\mbox{\sc def}\mbox{\sc lnCa@DecSwitch#1#2!}\mbox{\sc witch#1#2!}\mbox{\sc witch#1#2!}\mbox{\sc witch#1#2!}\label{fig:eq:491}
                          492 \ifx#1-%
                                   -%
                          493
                                   \csname InCa@Empty%
                          494
                                   \expandafter\InCa@Inc#2!%
                          495
                          496
                          497
                                   \ifx#10%
                                     -1%
                          498
                                   \else
                          499
                                      \csname InCa@Empty%
                          500
                                      \InCa@Dec#1#2!%
                          501
                          502
                                   \fi
                          503
                                \fi
                          504 }
         \IntCalcInc
                          505 \def\IntCalcInc#1!{%
                          506 \number\csname InCa@Empty\InCa@Inc#1! %
                          507 }
         \IntCalcDec
                          508 \def\IntCalcDec#1!{%
                          509 \number\csname InCa@Empty\InCa@Dec#1! %
                          510 }
            \InCa@Inc
                          \ifx#2!%
                          512
                                   \csname InCa@IncDigit#1\endcsname1%
                          513
                          514
                                 \else
                          515
                                   \csname InCa@IncDigit#1%
                          516
                                   \expandafter\InCa@Inc\expandafter#2%
                          517
                               \fi
                          518 }
\InCa@IncDigit[0-8]
                          519 \ensuremath{\mbox{\sc S}}19\ensuremath{\mbox{\sc S}}19\ensuremath{\mbox{\sc S}}19\%
                                \expandafter\def\csname InCa@IncDigit#1\endcsname##1{%
                                   \endcsname
                          522
                                   0%
                                   \ifcase##1 %
                          523
                                     #1%
                          524
                                   \else
                          525
                                      #2%
                          526
                                   \fi
                          527
                          528 }%
                          529 }
                          530 \InCa@Temp 01
                          531 \InCa@Temp 12
                          532 \InCa@Temp 23
                          533 \InCa@Temp 34
```

```
534 \InCa@Temp 45
                                                                                                                            535 \InCa@Temp 56
                                                                                                                            536 \InCa@Temp 67
                                                                                                                            537 \InCa@Temp 78
                                                                                                                            538 \InCa@Temp 89
                      \InCa@IncDigit9
                                                                                                                            539 \expandafter\def\csname InCa@IncDigit9\endcsname#1{%
                                                                                                                                                         \expandafter\endcsname
                                                                                                                                                         \ifcase#1 %
                                                                                                                            541
                                                                                                                                                                    09%
                                                                                                                            542
                                                                                                                                                         \else
                                                                                                                            543
                                                                                                                            544
                                                                                                                                                                    10%
                                                                                                                            545
                                                                                                                                                        \fi
                                                                                                                            546 }
                                                        \InCa@Dec
                                                                                                                            547 \ensuremath{\mbox{\sc 1}}\ensuremath{\mbox{\sc 1}}\ensuremath{\mb
                                                                                                                            548
                                                                                                                                                        \ifx#2!%
                                                                                                                                                                    \csname InCa@DecDigit#1\endcsname1%
                                                                                                                            549
                                                                                                                            550
                                                                                                                            551
                                                                                                                                                                    \csname InCa@DecDigit#1%
                                                                                                                            552
                                                                                                                                                                    \expandafter\InCa@Dec\expandafter#2%
                                                                                                                            553
                                                                                                                                                        \fi
                                                                                                                            554 }
\InCa@DecDigit[1-9]
                                                                                                                            555 \ensuremath{\mbox{\sc 1nCa@Temp}\#1\#2}\ensuremath{\mbox{\sc 1}}\xspace \ensuremath{\mbox{\sc 1
                                                                                                                                                         \expandafter\def\csname InCa@DecDigit#1\endcsname##1{%
                                                                                                                            557
                                                                                                                                                                    \endcsname
                                                                                                                            558
                                                                                                                                                                    0%
                                                                                                                            559
                                                                                                                                                                    \ifcase##1 %
                                                                                                                                                                                #1%
                                                                                                                            560
                                                                                                                                                                    \else
                                                                                                                            561
                                                                                                                            562
                                                                                                                                                                                #2%
                                                                                                                            563
                                                                                                                                                                    \fi
                                                                                                                                                     }%
                                                                                                                            564
                                                                                                                            565 }
                                                                                                                            566 \setminus InCa@Temp 98
                                                                                                                            567 \InCa@Temp 87
                                                                                                                            568 \InCa@Temp 76
                                                                                                                            569 \InCa@Temp 65
                                                                                                                            570 \InCa@Temp 54
                                                                                                                            571 \InCa@Temp 43
                                                                                                                            572 \InCa@Temp 32
                                                                                                                            573 \InCa@Temp 21
                                                                                                                            574 \InCa@Temp 10
                     \InCa@DecDigit0
                                                                                                                            575 \expandafter\def\csname InCa@DecDigitO\endcsname#1{%
                                                                                                                                                         \expandafter\endcsname
                                                                                                                                                         \ifcase#1 %
                                                                                                                            577
                                                                                                                            578
                                                                                                                                                                    00%
                                                                                                                                                        \else
                                                                                                                            579
                                                                                                                            580
                                                                                                                                                                    19%
                                                                                                                            581
                                                                                                                                                        \fi
                                                                                                                            582 }
```

### 2.5.5 Add, Sub

#### \intcalcAdd

```
583 \ensuremath{\mbox{\sc Mdd}}\fi 1#2{%
                     \number
                       \expandafter\InCa@AddSwitch
                       \number\number#1\expandafter!%
               586
                       \number#2! %
               587
               588 }
\intcalcSub
               589 \def\intcalcSub#1#2{%
               590
                     \number
                       \expandafter\InCa@AddSwitch
                       \number\number#1\expandafter!%
                       \number-\number#2! %
               593
               594 }
```

\InCa@AddSwitch Decision table for \InCa@AddSwitch. The sign of negative numbers can be removed by a simple \@gobble instead of the more expensive \number-.

x < 0	y < 0	x < y	_	Add(-x, -y)
		else		Add(-y, -x)
	else	-x > y	_	Sub(-x,y)
		else	+	Sub(y, -x)
else	y < 0	x > -y	+	Sub(x, -y)
		else	_	Sub(-y,x)
	else	x > y	+	Add(x,y)
		else		Add(y,x)

```
595 \ensuremath{\mbox{\sc System}}\xspace 195 \ensuremath{\mbox{\sc Sy
                          \lim 1<\z0
597
                                     \lim 2<\z0
598
                                               \ifnum#1<#2 %
599
                                                          \expandafter\InCa@Add\number-#1\expandafter!%
600
                                                          \@gobble#2!%
601
602
                                                           \expandafter\InCa@Add\number-#2\expandafter!%
603
604
                                                           \@gobble#1!%
605
                                     \else
606
                                               \ifnum-#1>#2 %
607
                                                          -%
608
609
                                                          \expandafter\InCa@Sub\@gobble#1!#2!%
610
                                                           \expandafter\InCa@Sub\number#2\expandafter!%
611
                                                          \@gobble#1!%
612
613
                                               \fi
                                    \fi
614
615
                           \else
                                    \lim 2<\z0
616
617
                                               \ifnum#1>-#2 %
                                                           \expandafter\InCa@Sub\number#1\expandafter!%
618
                                                           \@gobble#2!%
619
                                                \else
620
                                                          -%
621
```

```
\expandafter\InCa@Sub\@gobble#2!#1!%
              622
                       \fi
              623
              624
                     \else
                       \ifnum#1>#2 %
              625
                         \InCa@Add#1!#2!%
              626
                       \else
              627
                         \InCa@Add#2!#1!%
              628
                       \fi
              629
                     \fi
              630
                   \fi
              631
              632 }
\IntCalcAdd
              633 \def\IntCalcAdd#1!#2!{%
              634 \number\InCa@Add#1!#2! %
              635 }
\IntCalcSub
              636 \def\IntCalcSub#1!#2!{%
              637 \number\InCa@Sub#1!#2! %
              638 }
\InCa@Space
              639 \begingroup
              640 \def\x#1{\endgroup
              641
                   \let\InCa@Space= #1%
              642 }%
              643 \x{ }
  \InCa@Add
              644 \def\InCa@Add#1!#2!{%
              645 \ifcase#2 %
                     #1%
              646
              647
                   \else
                    \InCa@@Add#1!#2!00000000\InCa@Space
              648
              649 \fi
              650 }
  \InCa@Sub
              651 \def\InCa@Sub#1!#2!{%
              652
                  \ifnum#1=#2 %
                     0%
              653
                   \else
              654
                     \InCa@@Sub#1!#2!00000000\InCa@Space
              655
                   \fi
              656
              657 }
\InCa@@Add
              658 \def\InCa@@Add#1!#2#3!{%
                   \ifx\InCa@Empty#3\InCa@Empty
              660
                     \@ReturnAfterElseFi{%
                       \InCa@@@Add!!#1!#2%
              661
                     }%
              662
                   \else
              663
              664
                     \@ReturnAfterFi{%
                       \InCa@@Add#1!#3!#2%
              665
                     }%
              666
              667
                   \fi
              668 }
```

```
\InCa@@Sub
                   669 \def\InCa@@Sub#1!#2#3!{%
                        \ifx\InCa@Empty#3\InCa@Empty
                           \@ReturnAfterElseFi{%
                   671
                   672
                             \InCa@@@Sub!!#1!#2%
                           }%
                   673
                         \else
                   674
                           \@ReturnAfterFi{%
                   675
                             \InCa@@Sub#1!#3!#2%
                   676
                          }%
                   677
                   678
                         \fi
                   679 }
     \InCa@@@Add
                   680 \def\InCa@@@Add#1!#2!#3#4!#5{%
                         \ifx\InCa@Empty#4\InCa@Empty
                   682
                           \csname InCa@Empty%
                           \@ReturnAfterElseFi{%
                   683
                             \InCa@ProcessAdd#1#3!#5#2%
                   684
                          }%
                   685
                   686
                         \else
                   687
                           \@ReturnAfterFi{%
                             \InCa@@@Add#1#3!#5#2!#4!%
                          ጉ%
                   689
                        \fi
                   690
                   691 }
     \InCa@@Sub
                   692 \def\InCa@@@Sub#1!#2!#3#4!#5{%
                         \ifx\InCa@Empty#4\InCa@Empty
                   694
                           \csname @gobble%
                           \@ReturnAfterElseFi{%
                   695
                   696
                             \InCa@ProcessSub#1#3!#5#2%
                   697
                           }%
                         \else
                   698
                           \@ReturnAfterFi{%
                   699
                             \InCa@@@Sub#1#3!#5#2!#4!%
                   700
                          }%
                   701
                   702
                         \fi
                   703 }
\InCa@ProcessAdd
                   704 \def\InCa@ProcessAdd#1#2!#3#4{%
                         \ifx\InCa@Empty#2\InCa@Empty
                   706
                           \csname InCa@AddDigit#1\endcsname#3%
                   707
                           \romannumeral0#4%
                   708
                         \else
                           \csname InCa@AddDigit#1\csname InCa@DigitCarry#3%
                   709
                   710
                           \@ReturnAfterFi{%
                   711
                             \InCa@ProcessAdd#2!#4%
                   712
                          }%
                   713
                        \fi
                   714 }
\InCa@ProcessSub
                   715 \def\InCa@ProcessSub#1#2!#3#4{%
                        \ifx\InCa@Empty#2\InCa@Empty
                           \csname InCa@SubDigit#1\endcsname#3%
                   717
```

```
718
                                 \romannumeral0#4%
                         719
                              \else
                                 \csname InCa@SubDigit#1\csname InCa@DigitCarry#3%
                         720
                                 \@ReturnAfterFi{%
                         721
                         722
                                   \InCa@ProcessSub#2!#4%
                                 }%
                         723
                         724
                              \fi
                         725 }
\InCa@DigitCarry[0-9]
                         726 \def\InCa@Temp#1#2{%
                              \expandafter\def\csname InCa@DigitCarry#1\endcsname##1{%
                         727
                         728
                                 \ifcase##1 %
                                   \endcsname#1%
                         729
                                 \else
                         730
                                   \endcsname#2%
                         731
                                 \fi
                         732
                             }%
                         733
                         734 }
                         735 \InCa@Temp 01
                         736 \InCa@Temp 12
                         737 \InCa@Temp 23
                         738 \InCa@Temp 34
                         739 \InCa@Temp 45
                         740 \InCa@Temp 56
                         741 \InCa@Temp 67
                         742 \InCa@Temp 78
                         743 \InCa@Temp 89
                         744 \setminus InCa@Temp 9{\{10\}}
      \InCa@AddDigit0
                         745 \expandafter\def\csname InCa@AddDigit0\endcsname#1{%  
                         746
                              \ifnum#1>9 %
                                 \endcsname10%
                         747
                         748
                              \else
                                \endcsname0#1%
                         749
                         750
                              \fi
                         751 }
  \InCa@AddDigit[1-9]
                         752 \def\InCa@Temp#1#2#3{%
                              \expandafter\def\csname InCa@AddDigit#1\endcsname##1{%
                         754
                                 \ifnum##1>#2 %
                                   \endcsname 1%
                         755
                         756
                                 \else
                         757
                                   \endcsname 0%
                                 \fi
                         758
                                 \ifcase##1 #1% 0
                         759
                         760
                                 #3%
                                 \else #1% 10
                         761
                         762
                                 \fi
                         763
                              }%
                         764 }
                         765 \InCa@Temp 18{%
                              \or 2% 1
                         766
                              \or 3% 2
                         767
                              \or 4% 3
                         768
                         769
                              \or 5% 4
```

```
770 \or 6% 5
    \or 7% 6
771
    \or 8% 7
772
    \or 9% 8
773
774 \or 0% 9
775 }%
776 \InCa@Temp 27{%
    \or 3% 1
777
778
    \or 4% 2
779
    \or 5% 3
    \or 6% 4
781
    \or 7% 5
782
    \or 8% 6
    \or 9% 7
783
784 \or 0% 8
785 \or 1% 9
786 }%
787 \InCa@Temp 36{%
788
    \or 4% 1
789
    \or 5% 2
    \or 6% 3
790
791
    \or 7% 4
792
    \or 8% 5
793
    \or 9% 6
    \or 0% 7
795 \or 1% 8
796 \or 2% 9
797 }%
798 \InCa@Temp 45{%
799 \or 5% 1
    \or 6% 2
800
    \or 7% 3
801
802
    \or 8% 4
    \or 9% 5
803
804
    \or 0% 6
    \or 1% 7
805
    \or 2% 8
806
    \or 3% 9
808 }%
809 \InCa@Temp 54{%
810
    \or 6% 1
    \or 7% 2
811
    \or 8% 3
812
    \or 9% 4
813
814
    \or 0% 5
815
    \or 1% 6
816
    \or 2% 7
817
    \or 3% 8
818
    \or 4% 9
819 }%
820 \InCa@Temp 63{%
821
   \or 7% 1
822
    \or 8% 2
    \or 9% 3
823
824
    \or 0% 4
825
    \or 1% 5
    \or 2% 6
826
```

827

\or 3% 7

```
\or 4% 8
                        828
                              \or 5% 9
                        829
                        830 }%
                        831 \InCa@Temp 72{%
                              \or 8% 1
                        832
                        833
                              \or 9% 2
                              \or 0% 3
                        834
                              \or 1% 4
                        835
                              \or 2% 5
                        836
                              \or 3% 6
                        837
                              \or 4% 7
                        839
                              \or 5% 8
                        840
                              \or 6% 9
                        841 }%
                        842 \InCa@Temp 81{%
                        843
                              \or 9% 1
                              \or 0% 2
                        844
                              \or 1% 3
                              \or 2% 4
                        846
                              \or 3% 5
                        847
                              \or 4% 6
                        848
                              \or 5% 7
                        849
                              \or 6% 8
                        850
                              \or 7% 9
                        852 }%
                        853 \label{linealTemp} 90\%
                              \or 0% 1
                        854
                              \or 1% 2
                        855
                              \or 2% 3
                        856
                              \or 3% 4
                        857
                              \or 4% 5
                              \or 5% 6
                        859
                              \or 6% 7
                        860
                              \or 7% 8
                        861
                              \or 8% 9
                        862
                        863 }%
\InCa@SubDigit[0-9]
                        864 \left\lceil \frac{1nCa@Temp#1#2}{%} \right\rceil
                        865
                              \expandafter\def\csname InCa@SubDigit#1\endcsname##1{%
                                \ifnum##1>#1 %
                        866
                                  \endcsname 1%
                        867
                                \else
                        868
                                  \endcsname 0%
                        869
                                \fi
                        870
                        871
                                \ifcase##1 #1% 0
                        872
                                #2%
                                \else #1% 10
                        873
                                \fi
                        874
                              }%
                        875
                        876 }
                        877 \InCa@Temp 0{%
                              \or 9% 1
                              \or 8% 2
                        879
                              \or 7% 3
                        880
                              \or 6% 4
                        881
                              \or 5% 5
                        882
                              \or 4% 6
                        883
```

```
\or 3% 7
884
    \or 2% 8
885
886 \or 1% 9
887 }
888 \ \ \ 1nCa@Temp \ 1\%
889
     \or 0% 1
     \or 9% 2
890
     \or 8% 3
891
     \or 7% 4
892
     \or 6% 5
893
    \or 5% 6
894
895
    \or 4% 7
896
    \or 3% 8
897 \or 2% 9
898 }
899 \InCa@Temp 2{%
900
    \or 1% 1
     \or 0% 2
901
     \or 9% 3
902
903
     \or 8% 4
     \or 7% 5
904
905
    \or 6% 6
    \or 5% 7
906
    \or 4% 8
907
908 \or 3% 9
909 }
910 \InCa@Temp 3\%
    \or 2% 1
911
     \or 1% 2
912
     \or 0% 3
913
     \or 9% 4
914
     \or 8% 5
915
916
     \or 7% 6
     \or 6% 7
917
918
    \or 5% 8
919
    \or 4% 9
920 }
921 \InCa@Temp 4{%
922
    \or 3% 1
    \or 2% 2
923
924
    \or 1% 3
     \or 0% 4
925
     \or 9% 5
926
     \or 8% 6
927
928
     \or 7% 7
929
     \or 6% 8
930
     \or 5% 9
931 }
932 \InCa@Temp 5{%
933
    \or 4% 1
    \or 3% 2
    \or 2% 3
936
    \or 1% 4
     \or 0% 5
937
     \or 9% 6
938
     \or 8% 7
939
     \or 7% 8
940
941
     \or 6% 9
```

```
942 }
943 \InCa@Temp 6{%
944 \or 5% 1
      \or 4% 2
945
      \or 3% 3
946
      \or 2% 4
947
     \or 1% 5
948
     \or 0% 6
949
     \or 9% 7
950
     \or 8% 8
951
     \or 7% 9
952
953 }
954 \InCa@Temp 7{%
     \or 6% 1
955
     \or 5% 2
956
      \or 4% 3
957
     \or 3% 4
958
     \or 2% 5
959
     \or 1% 6
960
     \or 0% 7
961
     \or 9% 8
962
963
     \or 8% 9
964 }
965 \InCa@Temp 8{%
     \or 7% 1
     \or 6% 2
967
     \or 5% 3
968
     \or 4% 4
969
     \or 3% 5
970
      \or 2% 6
971
      \or 1% 7
972
      \or 0% 8
974
     \or 9% 9
975 }
976 \InCa@Temp 9{%
977
     \or 8% 1
     \or 7% 2
978
     \or 6% 3
980
     \or 5% 4
     \or 4% 5
981
982
     \or 3% 6
     \or 2% 7
983
     \or 1% 8
984
     \or 0% 9
985
986 }
2.5.6 Shl, Shr
987 \def\intcalcShl#1{%
988 \number\expandafter\InCa@ShlSwitch\number#1! %
989 }
990 \def\InCa@ShlSwitch#1#2!{%
991 \ifx#1-%
992
       -\csname InCa@Empty%
        \InCa@Shl#2!%
```

\intcalcShl

\InCa@ShlSwitch

```
994
                          \else
                            \csname InCa@Empty%
                     995
                            \InCa@Shl#1#2!%
                     996
                          \fi
                     997
                     998 }
       \IntCalcShl
                     999 \def\IntCalcShl#1!{%
                     1000
                          \number
                          \csname InCa@Empty%
                     1001
                     1002 \InCa@Shl#1! %
                     1003 }
  \IntCal@ShlDigit
                     1004 \def\InCa@Shl#1#2{%
                          \ifx#2!%
                            \csname InCa@ShlDigit#1\endcsname0%
                     1006
                          \else
                     1007
                            \csname InCa@ShlDigit#1%
                     1008
                            \@ReturnAfterFi{%
                     1009
                     1010
                              \InCa@Sh1#2%
                     1011
                            }%
                     1012 \fi
                     1013 }
   \InCa@ShlDigit0
                     1015 \endcsname0%
                     1016 }
\InCa@ShlDigit[1-9]
                     1017 \def\InCa@Temp#1#2#3#4#5{%
                          \expandafter\def\csname InCa@ShlDigit#1\endcsname##1{%
                            \expandafter\endcsname
                     1019
                            \ifcase##1 %
                     1020
                              #2#3%
                     1021
                     1022
                             \else
                     1023
                              #4#5%
                     1024
                            \fi
                     1025
                         }%
                     1026 }
                     1027 \InCa@Temp 10203
                     1028 \InCa@Temp 20405
                     1029 \InCa@Temp 30607
                     1030 \InCa@Temp 40809
                     1031 \InCa@Temp 51011
                     1032 \InCa@Temp 61213
                     1033 \InCa@Temp 71415
                     1034 \setminus InCa@Temp 81617
                     1035 \InCa@Temp 91819
       \intcalcShr
                     1036 \def\intcalcShr#1{%
                          \number\expandafter\InCa@ShrSwitch\number#1! %
                     1038 }
```

\InCa@ShrSwitch

30

```
1039 \def\InCa@ShrSwitch#1#2!{%
                   \ifx#1-%
             1040
                      -\InCa@Shr#2!%
             1041
                    \else
             1042
                     \InCa@Shr#1#2!%
             1043
                   \fi
             1044
             1045 }
\IntCalcShr
             1046 \def\IntCalcShr#1!{%
             1047 \number\InCa@Shr#1! %
             1048 }
  \InCa@Shr
             1049 \def\InCa@Shr#1#2{%
                   \InCa@ShrDigit#1!%
                    \ifx#2!%
             1051
                    \else
             1052
                      \@ReturnAfterFi{%
             1053
                        \ifodd#1 %
             1054
             1055
                          \@ReturnAfterElseFi{%
             1056
                            \InCa@Shr{1#2}%
             1057
                          }%
             1058
                        \else
                          \expandafter\InCa@Shr\expandafter#2%
             1059
             1060
             1061
                      }%
             1062
                    \fi
             1063 }
             1064 \def\InCa@ShrDigit#1!{%
                   \ifcase#1 0% 0
                   \or 0% 1
             1067
                   \or 1% 2
                   \or 1% 3
             1068
                    \or 2% 4
             1069
                    \or 2% 5
             1070
                    \or 3% 6
             1071
                    \or 3% 7
             1072
                    \or 4% 8
             1073
             1074
                    \or 4% 9
                   \or 5% 10
             1075
                   \or 5% 11
             1076
                   \or 6% 12
             1077
                   \or 6% 13
             1078
             1079
                   \or 7% 14
             1080
                   \or 7% 15
                   \or 8% 16
             1081
                   \or 8% 17
             1082
                    \or 9% 18
             1083
                    \or 9% 19
             1084
             1085
                    \fi
             1086 }
             2.5.7 \InCa@Tim
```

\InCa@Tim Macro \InCa@Tim implements "Number times digit".

1087 \def\InCa@Temp#1{%

```
\def\InCa@Tim##1##2{%}
                   1088
                           \number
                   1089
                             \ifcase##2 % 0
                   1090
                   1091
                               0%
                   1092
                             \or % 1
                                ##1%
                   1093
                             \else % 2-9
                   1094
                                \csname InCa@Empty%
                   1095
                                \InCa@ProcessTim##2##1!%
                   1096
                             \fi
                   1097
                   1098
                           #1%
                   1099
                        }%
                   1100 }
                   1101 \InCa@Temp{ }
\InCa@ProcessTim
                   1102 \def\InCa@ProcessTim#1#2#3{%
                   1103
                        \ifx#3!%
                           \csname InCa@TimDigit#2\endcsname#10%
                   1104
                   1105
                   1106
                           \csname InCa@TimDigit#2\csname InCa@Param#1%
                   1107
                           \@ReturnAfterFi{%
                             \InCa@ProcessTim#1#3%
                   1108
                           }%
                   1109
                        \fi
                   1110
                   1111 }
\InCa@Param[0-9]
                   1112 \def\InCa@Temp#1{%
                         \expandafter\def\csname InCa@Param#1\endcsname{%
                           \endcsname#1%
                   1114
                         }%
                   1115
                   1116 }
                   1117 \InCa@Temp 0%
                   1118 \InCa@Temp 1%
                   1119 \InCa@Temp 2%
                   1120 \InCa@Temp 3%
                   1121 \InCa@Temp 4%
                   1122 \InCa@Temp 5%
                   1123 \InCa@Temp 6%
                   1124 \InCa@Temp 7%
                   1125 \InCa@Temp 8%
                   1126 \InCa@Temp 9%
 \InCa@TimDigit0
                   1127 \expandafter\def\csname InCa@TimDigitO\endcsname#1#2{%
                   1128
                         \endcsname
                   1129
                         0#2%
                   1130 }
 \InCa@TimDigit1
                   1131 \expandafter\def\csname InCa@TimDigit1\endcsname#1#2{\%}
                   1132
                         \ifcase#2 %
                           \endcsname 0#1%
                   1133
                         \else
                   1134
                   1135
                           \csname InCa@AddDigit#1\endcsname #2%
                         \fi
                   1136
                   1137 }
```

### \InCa@TimDigit[2-9]

```
1138 \def\InCa@Temp#1#2{%}
     \expandafter\def\csname InCa@TimDigit#1\endcsname##1{%
1140
        \expandafter\InCa@TimDigitCarry
1141
        \number
          \ifcase##1 0% 0
1142
          #2%
1143
1144
          \fi
        !%
1145
     }%
1146
1147 }
1148 \InCa@Temp 2{%
1149 \or 2% 1
     \or 4% 2
1150
     \or 6% 3
1151
     \or 8% 4
1152
     \or 10% 5
1153
1154
     \or 12% 6
     \or 14% 7
1155
     \or 16% 8
1156
1157 \or 18% 9
1158 }
1159 \InCa@Temp 3{%
1160 \or 3% 1
1161
      \or 6% 2
     \or 9% 3
1162
1163
     \or 12% 4
     \or 15% 5
1164
     \or 18% 6
1165
     \or 21% 7
1166
1167
     \or 24% 8
1168 \or 27% 9
1169 }
1170 \InCa@Temp 4{%
1171 \or 4% 1
     \or 8% 2
1172
1173
     \or 12% 3
1174
     \or 16% 4
     \or 20% 5
1175
     \or 24% 6
1176
     \or 28% 7
1177
     \or 32% 8
1178
1179 \or 36% 9
1180 }
1181 \InCa@Temp 5{%
1182 \or 5% 1
     \or 10% 2
1183
     \or 15% 3
1184
      \or 20% 4
1185
1186
      \or 25% 5
1187
      \or 30% 6
      \or 35% 7
1188
     \or 40% 8
1189
     \or 45% 9
1190
1191 }
1192 \InCa@Temp 6{%
1193 \or 6% 1
1194
     \or 12% 2
```

```
\or 18% 3
                      1195
                           \or 24% 4
                      1196
                            \or 30% 5
                      1197
                            \or 36% 6
                      1198
                            \or 42% 7
                      1199
                      1200
                            \or 48% 8
                      1201 \or 54% 9
                      1202 }
                      1203 \InCa@Temp 7{%
                           \or 7% 1
                      1204
                           \or 14% 2
                      1205
                      1206
                           \or 21% 3
                      1207
                           \or 28% 4
                           \or 35% 5
                      1208
                           \or 42% 6
                      1209
                           \or 49% 7
                      1210
                           \or 56% 8
                      1211
                           \or 63% 9
                      1212
                      1213 }
                      1214 \InCa@Temp 8{%
                      1215 \or 8% 1
                           \or 16% 2
                      1216
                           \or 24% 3
                      1217
                      1218 \or 32% 4
                      1219 \or 40% 5
                      1220 \or 48% 6
                           \or 56% 7
                      1221
                           \or 64% 8
                      1222
                      1223 \or 72% 9
                      1224 }
                      1225 \InCa@Temp 9{%
                      1226 \or 9% 1
                            \or 18% 2
                      1227
                           \or 27% 3
                      1228
                           \or 36% 4
                      1229
                           \or 45% 5
                      1230
                           \or 54% 6
                      1231
                      1232 \or 63% 7
                      1233 \or 72% 8
                      1234 \or 81% 9
                      1235 }
 \InCa@TimDigitCarry
                      1236 \def\InCa@TimDigitCarry#1!{%
                      1237 \ifnum#1<10 %
                      1238
                              \csname InCa@AddDigit#1\expandafter\endcsname
                      1239
                           \else
                              \@ReturnAfterFi{%
                      1240
                                \InCa@@TimDigitCarry#1!%
                      1241
                              }%
                      1242
                      1243 \fi
                      1244 }
\InCa@@TimDigitCarry
                      1245 \def\InCa@@TimDigitCarry#1#2!#3{%
                      1246 \csname InCa@DigitCarry#1%
                      1247 \csname InCa@AddDigit#2\endcsname #3%
                      1248 }
```

#### 2.5.8 Mul

#### \intcalcMul

```
1249 \def\intcalcMul#1#2{%

1250 \number

1251 \expandafter\InCa@MulSwitch

1252 \number\number#1\expandafter!%

1253 \number#2! %

1254 }
```

\InCa@MulSwitch Decision table for \InCa@MulSwitch.

x < 0	y < 0	x < y	+	Mul(-x, -y)
		else		Mul(-y, -x)
	else	-x > y	_	Mul(-x,y)
		else		Mul(y, -x)
else	y < 0	x > -y	_	Mul(x, -y)
		else		Mul(-y,x)
	else	x > y	+	Mul(x, y)
		else		Mul(y, x)

```
1255 \def\InCa@MulSwitch#1!#2!{%
      \lim 1<\z0
1256
        \ifnum#2<\z@
1257
          \ifnum#1<#2 %
1258
1259
            \expandafter\InCa@Mul\number-#1\expandafter!%
1260
            \@gobble#2!%
          \else
1261
            \expandafter\InCa@Mul\number-#2\expandafter!%
1262
            \@gobble#1!%
1263
          \fi
1264
        \else
1265
1266
          -%
          \ifnum-#1>#2 %
1267
            \expandafter\InCa@Mul\@gobble#1!#2!%
1268
1269
            \expandafter\InCa@Mul\number#2\expandafter!%
1270
1271
            \@gobble#1!%
          \fi
1272
        \fi
1273
1274
      \else
        \lim 2<\z0
1275
          -%
1276
          \ifnum#1>-#2 %
1277
            \expandafter\InCa@Mul\number#1\expandafter!%
1278
            \@gobble#2!%
1279
1280
          \else
1281
             \expandafter\InCa@Mul\@gobble#2!#1!%
1282
          \fi
        \else
1283
          \ifnum#1>#2 %
1284
            \InCa@Mul#1!#2!%
1285
1286
          \else
            \InCa@Mul#2!#1!%
1287
1288
        \fi
1289
1290
      \fi
1291 }
```

```
\IntCalcMul
             1292 \def\IntCalcMul#1!#2!{%
                   \number\InCa@Mul#1!#2! %
             1294 }
  \InCa@Mul
             1295 \def\InCa@Mul#1!#2!{%
                   \ifcase#2 %
             1296
                      0%
             1297
                    \or
             1298
              1299
                      #1%
              1300
                      \csname InCa@Empty%
             1301
             1302
                      \expandafter\InCa@Shl#1!%
                    \else
             1303
                      \ifnum#2<10 %
             1304
                        \InCa@Tim{#1}#2%
             1305
             1306
                      \else
             1307
                        \InCa@ProcessMul!#2!#1!%
             1308
             1309
                    \fi
             1310 }
  \InCa@Mul
             1311 \def\InCa@ProcessMul#1!#2#3!#4!{%
                   \ifx\InCa@Empty#3\InCa@Empty
             1313
                      \expandafter\InCa@Add\number
                      #10\expandafter\expandafter\expandafter!%
             1314
                      \InCa@Tim{#4}#2!%
             1315
             1316
                    \else
                      \ifx\InCa@Empty#1\InCa@Empty
             1317
                        \expandafter\expandafter\InCa@ProcessMul
             1318
                        \label{localim} $$\prod_{a\in\mathbb{Z}_{+}^{2}}\
             1319
             1320
                        #3!#4!%
             1321
                      \else
             1322
                        \expandafter\InCa@ProcessMul\number
                        \expandafter\InCa@Add\number%
             1323
             1324
                        #10\expandafter\expandafter!%
             1325
                        \InCa@Tim{#4}#2!!%
             1326
                        #3!#4!%
             1327
                      \fi
                    \fi
             1328
             1329 }
             2.5.9 Sqr, Fac
\intcalcSqr
             1330 \def\intcalcSqr#1{%
             1331
                    \number\expandafter\InCa@Sqr\number#1! %
             1332 }
  \InCa@Sqr
             1333 \def\InCa@Sqr#1#2!{%
             1334
                    \ifx#1-%
             1335
                      \InCa@Mul#2!#2!%
             1336
                    \else
                      \InCa@Mul#1#2!#1#2!%
             1337
```

```
1338 \fi
             1339 }
\intcalcFac
             1340 \ensuremath{\mbox{\sc H1}\%}
                   \number\expandafter\InCa@Fac\number#1! %
             1342 }
             2.5.10 Pow
\intcalcPow
             1343 \def\intcalcPow#1#2{%
                    \number\expandafter\InCa@Pow
                    \number\number#1\expandafter!%
             1345
                    \number#2! %
             1346
             1347 }
  \InCa@Pow
             1348 \def\InCa@Pow#1#2!#3#4!{%
                    1350
                    \or \% power = 1
             1351
                      #1#2%
             1352
                    1353
                      \ifx#1-%
             1354
                        \InCa@Mul#2!#2!%
             1355
             1356
                      \else
                        \InCa@Mul#1#2!#1#2!%
             1357
             1358
                      \fi
                    \else
             1359
                      \footnote{1} \iftit{ifcase#1#2 % basis = 0, power <> 0
             1360
             1361
                        \iny 3-\% power < 0
             1362
                          0\IntCalcError:DivisionByZero%
             1363
                        \fi
             1364
                      \or
             1365
                        1% basis = 1
             1366
                      \else
             1367
                        \lim 1#2=\m \% basis = -1
             1368
             1369
                          \ifodd#3#4 %
                            -%
             1370
                          \fi
             1371
                          1%
             1372
                        \else % |basis| > 1
             1373
                          \iny 3-\% power < 0
             1374
                            0%
             1375
                          \else % power > 2
             1376
                            \int x#1-\% basis < 0
             1377
                              \ifodd#3#4 %
             1378
                                -%
             1379
                              \fi
             1380
                              \InCa@PowRec#2!#3#4!1!%
             1381
             1382
                              \InCa@PowRec#1#2!#3#4!1!%
             1383
                            \fi
             1384
                          \fi
             1385
                        \fi
             1386
                      \fi
             1387
```

```
1388 \fi
              1389 }
\InCa@PowRec
                    Pow(b, p) {
                      PowRec(b, p, 1)
                    PowRec(b, p, r) {
                      if p == 1 then
                        return r
                      else
                        ifodd p then
                          return PowRec(b*b, p div 2, r*b) % p div 2 = (p-1)/2
                        else
                          return PowRec(b*b, p div 2, r)
                        fi
                      fi
                    }
              1390 \def\InCa@PowRec#1!#2!#3!{%
                    \lim 2=\0
                      \ifnum#1>#3 %
              1392
              1393
                         \InCa@Mul#1!#3!%
                       \else
              1394
                         \InCa@Mul#3!#1!%
              1395
              1396
                      \fi
              1397
                     \else
              1398
                      \expandafter\InCa@PowRec
                      \number\InCa@Mul#1!#1!\expandafter!%
              1399
                      \number\intcalcShr{#2}\expandafter!%
              1400
                      \number
              1401
                      \ifodd#2 %
              1402
                         \ifnum#1>#3 %
              1403
                           \InCa@Mul#1!#3!%
              1404
              1405
                         \else
              1406
                           \InCa@Mul#3!#1!%
                         \fi
              1407
                      \else
              1408
                         #3%
              1409
              1410
                      \fi
              1411
                       \expandafter!%
                    \fi
              1412
              1413 }
              2.5.11 Div
 \intcalcDiv
              1414 \def\intcalcDiv#1#2{%
              1415
                    \number\expandafter\InCa@Div
                    \number\number#1\expandafter!%
              1416
              1417
                    \number#2! %
              1418 }
   \InCa@Div
              1419 \def\InCa@Div#1!#2!{%
              1420
                    \ifcase#2 %
                      0\IntCalcError:DivisionByZero%
              1421
              1422
                    \else
              1423
                      \ifcase#1 %
                         0%
              1424
```

```
1425
                                                                                                                                               \else
                                                                                          1426
                                                                                                                                                              \expandafter\InCa@DivSwitch
                                                                                          1427
                                                                                                                                                              \number#1\expandafter!%
                                                                                                                                                               \number#2!%
                                                                                          1428
                                                                                          1429
                                                                                                                                                 \fi
                                                                                                                                 \fi
                                                                                          1430
                                                                                          1431 }
\IntCalcDiv
                                                                                          1432 \left( \frac{1}{432} \right)
                                                                                                                                  \label{lintCalcDiv} $$ \end{tabular} $
                                                                                                                                               \number
                                                                                         1434
                                                                                                                                               \ifcase##2 %
                                                                                          1435
                                                                                                                                                              0\IntCalcError:DivisionByZero%
                                                                                          1436
                                                                                          1437
                                                                                                                                               \else
                                                                                                                                                              \ifcase##1 %
                                                                                          1438
                                                                                                                                                                           0%
                                                                                          1439
                                                                                                                                                              \else
                                                                                          1440
                                                                                                                                                                            \InCa@@Div##1!##2!%
                                                                                          1441
                                                                                          1442
                                                                                                                                                              \fi
                                                                                          1443
                                                                                                                                               \fi
                                                                                                                                               #1%
                                                                                          1444
                                                                                          1445 }%
                                                                                          1446 }
                                                                                         1447 \InCa@Temp{ }%
```

\InCa@DivSwitch Decision table for \InCa@DivSwitch.

x < 0	y < 0	+	$\mathrm{Div}(-x,-y)$
	else	_	$\mathrm{Div}(-x,y)$
else	y < 0	_	$\mathrm{Div}(x, -y)$
	else	+	$\mathrm{Div}(x,y)$

```
1448 \ensuremath{\mbox{\mbox{$1$}}} 1448 \ensuremath{\mbox{\mbox{$1$}}} 182! \ensuremath{\mbox{$1$}} 
                                                                1449
                                                                                            \int \frac{1}{z} dx
                                                                                                         \lim 2<\z0
                                                                1450
                                                                                                                    \expandafter\InCa@@Div\number-#1\expandafter!%
                                                                1451
                                                                                                                    \@gobble#2!%
                                                                1452
                                                                                                         \else
                                                                1453
                                                                                                                    -%
                                                                1454
                                                                                                                    \expandafter\InCa@@Div\@gobble#1!#2!%
                                                                1455
                                                                                                         \fi
                                                               1456
                                                                1457
                                                                                                \else
                                                                                                         \lim 2<\z0
                                                                1458
                                                                1459
                                                                1460
                                                                                                                    \expandafter\InCa@@Div\number#1\expandafter!%
                                                                                                                    \@gobble#2!%
                                                                1461
                                                                1462
                                                                                                          \else
                                                                                                                    \InCa@@Div#1!#2!%
                                                                1463
                                                                                                         \fi
                                                                1464
                                                                1465
                                                                                              \fi
                                                                1466 }
\InCa@@Div
                                                                1467 \def\InCa@@Div#1!#2!{%
                                                                1468 \ifnum#1>#2 %
                                                                                                        \ifcase#2 % 0 already catched
                                                                1469
                                                                                                                    \IntCalcError:ThisCannotHappen%
                                                                1470 ?
```

```
\or % 1
                1471
                          #1%
                1472
                        \or % 2
                1473
                          \InCa@Shr#1!%
                1474
                1475
                         \else
                          \InCa@DivStart!#1!#2!#2!%
                1476
                        \fi
                1477
                      \else
                1478
                        \ifnum#1=#2 %
                1479
                          1%
                1480
                1481
                        \else
                          0%
                1482
                1483
                        \fi
                1484
                     \fi
                1485 }
\InCa@DivStart
                1486 \def\InCa@DivStart#1!#2#3!#4#5{%
                     \ifx#5!%
                1487
                        \@ReturnAfterElseFi{%
                1488
                          \InCa@DivStartI{#1#2}#3=!%
                1489
                1490
                        }%
                1491
                      \else
                1492
                        \@ReturnAfterFi{%
                1493
                          \InCa@DivStart{#1#2}!#3!#5%
                        }%
                1494
                1495 \fi
                1496 }
  \InCa@StartI
                1497 \def\InCa@DivStartI#1!#2!{%
                      \expandafter\InCa@DivStartII
                1498
                      \number#2\expandafter\expandafter\expandafter!%
                1499
                1500
                      1501
                      #1!%
                1502 }
\InCa@StartII
                1503 \def\InCa@DivStartII#1!#2!{%
                      \expandafter\InCa@DivStartIII
                1504
                      \number#1\expandafter!%
                1505
                      \number#2\expandafter\expandafter\expandafter!%
                1506
                      \intcalcShl{#2}!%
                1507
                1508 }
\InCa@StartIII
                1509 \def\InCa@DivStartIII#1!#2!#3!{%
                      \expandafter\InCa@DivStartIV
                1510
                      \number#1\expandafter!%
                1511
                1512
                      \number#2\expandafter!%
                      \number#3\expandafter!%
                1513
                      \number\InCa@Add#3!#2!\expandafter\expandafter\expandafter!%
                1514
                1515
                      \intcalcShl{#3}!%
                1516 }
 \InCa@StartIV
                1517 \def\InCa@DivStartIV#1!#2!#3!#4!#5!#6!{%
                1518 \InCa@ProcessDiv#6!#1!#2!#3!#4!#5!/%
                1519 }
```

```
1520 \def\InCa@ProcessDiv#1#2#3!#4!#5!#6!#7!#8!#9/{%
1521
     #9%
      \ifnum#1<#4 % 0
1522
1523
        0%
        \ifx#2=%
1524
        \else
1525
          \InCa@ProcessDiv{#1#2}#3!#4!#5!#6!#7!#8!%
1526
1527
        \fi
      \else % 1-9
1528
1529
        \ifnum#1<#5 % 1
1530
          1%
          \ifx#2=%
1531
1532
          \else
1533
            \expandafter\InCa@ProcessDiv\expandafter{%
1534
               \number\InCa@Sub#1!#4!%
1535
               #2%
            }#3!#4!#5!#6!#7!#8!%
1536
          \fi
1537
        \else % 2-9
1538
          \ifnum#1<#7 % 2 3 4 5
1539
            \ifnum#1<#6 % 2 3
1540
               \@ReturnAfterElseFi{%
1541
1542
                 \expandafter\InCa@@ProcessDiv
1543
                 \number\InCa@Sub#1!#5!!%
                 23%
1544
              }%
1545
            \else % 4 5
1546
               \@ReturnAfterFi{%
1547
1548
                 \expandafter\InCa@@ProcessDiv
                 \number\InCa@Sub#1!#6!!%
1549
1550
              }%
1551
            \fi
1552
            #2#3!#4!#5!#6!#7!#8!%
1553
          \else % 6 7 8 9
1554
            \ifnum#1<#8 % 6 7
1555
1556
               \@ReturnAfterElseFi{%
                 \expandafter\InCa@@ProcessDiv
1557
                 \number\InCa@Sub#1!#7!!%
1558
                 67%
1559
              }%
1560
            \else % 8 9
1561
               \@ReturnAfterFi{%
1562
                 \expandafter\InCa@@ProcessDiv
1563
                 \number\InCa@Sub#1!#8!!%
1564
                 89%
1565
              }%
1566
            \fi
1567
1568
            #2#3!#4!#5!#6!#7!#8!%
1569
        \fi
1570
      \fi
1571
      \ifx#2=%
1572
        \expandafter\@gobble
1573
      \fi
1574
1575
     /%
1576 }
```

```
1577 \def\InCa@@ProcessDiv#1!#2#3#4#5!#6!{%
                        \ifnum#1<#6 %
                  1578
                  1579
                          #2%
                  1580
                          \@ReturnAfterElseFi{%
                            \int x#4=%
                  1581
                               \expandafter\InCa@CleanupIV
                  1582
                            \else
                  1583
                  1584
                               \@ReturnAfterFi{%
                                 \InCa@ProcessDiv{#1#4}#5!#6!%
                  1585
                  1586
                  1587
                            \fi
                          }%
                  1588
                  1589
                        \else
                          #3%
                  1590
                          \@ReturnAfterFi{%
                  1591
                  1592
                            \ifx#4=%
                               \expandafter\InCa@CleanupIV
                  1593
                  1594
                               \@ReturnAfterFi{%
                  1595
                                 \expandafter\InCa@ProcessDiv\expandafter{%
                  1596
                                   \number\InCa@Sub#1!#6! %
                  1597
                                   #4%
                  1598
                  1599
                                }#5!#6!%
                  1600
                              }%
                  1601
                            \fi
                          }%
                  1602
                        \fi
                  1603
                  1604 }
\InCa@CleanupIV
                  1605 \def\InCa@CleanupIV#1!#2!#3!#4!{}
                  2.5.12 Mod
    \intcalcMod
                  1606 \def\intcalcMod#1#2{%
                        \number\expandafter\InCa@Mod
                  1607
                  1608
                        \number\number#1\expandafter!%
                        \number#2! %
                  1610 }
```

\intcalc@Mod Pseudocode/decision table for \intcalc@Mod.

```
if
               DivisionByZero
       y = 0
               -\operatorname{Mod}(-x,-y)
elsif y < 0
elsif x = 0 0
elsif y=1
elsif y = 2 ifodd(x)? 1:0
               z \leftarrow x - (x/y) * y; \quad (z < 0) ? z + y : z
elsif x < 0
                x - (x/y) * y
_{\mathrm{else}}
```

```
1611 \def\InCa@Mod#1!#2!{%
1612 \ifcase#2 %
1613
       0\IntCalcError:DivisionByZero%
1614
     \else
       \lim 2<\z0
1615
```

```
-%
             1616
             1617
                       \expandafter\InCa@Mod
             1618
                       \number-#1\expandafter!%
                        \number-#2!%
             1619
             1620
                      \else
                        \ifcase#1 %
             1621
                          0%
             1622
                        \else
             1623
                          \ifcase#2 % 0 already catched
             1624
                            \IntCalcError:ThisCannotHappen%
             1625 ?
             1626
                          \or % 1
             1627
                            0%
             1628
                          \or % 2
                            \ifodd#1 1\else 0\fi
             1629
                          \else
             1630
                            1631
             1632
                              \expandafter\InCa@ModShift
             1633
                              \number-%
                                \expandafter\InCa@Sub
             1634
                                \number\@gobble#1\expandafter!%
             1635
                                1636
                                  \verb|\expandafter\InCa@Div\@gobble#1!#2!%|
             1637
                               }!%
             1638
             1639
                              !#2!%
             1640
                            \else
             1641
                              \expandafter\InCa@Sub\number#1\expandafter!%
                              \number\intcalcMul{#2}{\InCa@Div#1!#2!}!%
             1642
             1643
                            \fi
                          \fi
             1644
                       \fi
             1645
                     \fi
             1646
             1647
                   \fi
             1648 }
\IntCalcMod
             1649 \def\InCa@Temp#1{%
                   \def\IntCalcMod##1!##2!{%
             1650
             1651
                     \number
             1652
                     \ifcase##2 %
                       0\IntCalcError:DivisionByZero%
             1653
                     \else
             1654
                       \ifcase##1 %
             1655
                          0%
             1656
                       \else
             1657
                          \ifcase##2 % 0 already catched
             1658
             1659 ?
                            \IntCalcError:ThisCannotHappen
             1660
                          \or % 1
                            0%
             1661
                          \or % 2
             1662
                            1663
                          \else
             1664
                            \expandafter\InCa@Sub\number##1\expandafter!%
             1665
             1666
                            \number\intcalcMul{##2}{\InCa@Div##1!##2!}!%
                          \fi
             1667
                        \fi
             1668
                     \fi
             1669
                     #1%
             1670
             1671
                   }%
```

```
1672 }
                      1673 \InCa@Temp{ }%
     \InCa@ModShift
                      1674 \def\InCa@ModShift#1!#2!{%
                      1675
                             \int \frac{1}{z^0}
                               \expandafter\InCa@Sub\number#2\expandafter!%
                      1676
                               \@gobble#1!%
                      1677
                             \else
                      1678
                      1679
                               #1%
                            \fi
                      1680
                      1681 }
                      2.5.13 Help macros
        \InCa@Empty
                      1682 \def\InCa@Empty{}
           \@gobble
                      1683 \expandafter\ifx\csname @gobble\endcsname\relax
                      1684 \leq \lceil 0 \rceil 
                      1685 \fi
    \@ReturnAfterFi
                      1686 \long\def\@ReturnAfterFi#1\fi{\fi#1}%
\@ReturnAfterElseFi
                      1687 \end{center} $1687 \end{center} IclseFi#1\else#2\fi{fi#1}%
                      1688 \InCa@AtEnd%
                      1689 (/package)
```

## 3 Installation

### 3.1 Download

**Package.** This package is available on CTAN<sup>1</sup>:

CTAN:macros/latex/contrib/intcalc/intcalc.dtx The source file.

CTAN:macros/latex/contrib/intcalc/intcalc.pdf Documentation.

**Bundle.** All the packages of the bundle 'intcalc' are also available in a TDS compliant ZIP archive. There the packages are already unpacked and the documentation files are generated. The files and directories obey the TDS standard.

CTAN:install/macros/latex/contrib/intcalc.tds.zip

TDS refers to the standard "A Directory Structure for TEX Files" (CTAN:pkg/tds). Directories with texmf in their name are usually organized this way.

### 3.2 Bundle installation

Unpacking. Unpack the intcalc.tds.zip in the TDS tree (also known as texmf tree) of your choice. Example (linux):

```
unzip intcalc.tds.zip -d ~/texmf
```

<sup>1</sup>CTAN:pkg/intcalc

### 3.3 Package installation

**Unpacking.** The .dtx file is a self-extracting docstrip archive. The files are extracted by running the .dtx through plain T<sub>F</sub>X:

```
tex intcalc.dtx
```

**TDS.** Now the different files must be moved into the different directories in your installation TDS tree (also known as texmf tree):

```
\label{eq:control_interaction} intcalc.sty \rightarrow tex/generic/intcalc/intcalc.sty \\ intcalc.pdf \rightarrow doc/latex/intcalc/intcalc.pdf \\ intcalc.dtx \rightarrow source/latex/intcalc/intcalc.dtx
```

If you have a docstrip.cfg that configures and enables docstrip's TDS installing feature, then some files can already be in the right place, see the documentation of docstrip.

### 3.4 Refresh file name databases

If your T<sub>E</sub>X distribution (T<sub>E</sub>X Live, MiKT<sub>E</sub>X, ...) relies on file name databases, you must refresh these. For example, T<sub>E</sub>X Live users run texhash or mktexlsr.

#### 3.5 Some details for the interested

Unpacking with LATEX. The .dtx chooses its action depending on the format:

plain T<sub>F</sub>X: Run docstrip and extract the files.

LATEX: Generate the documentation.

If you insist on using LATEX for docstrip (really, docstrip does not need LATEX), then inform the autodetect routine about your intention:

```
latex \let\install=y\input{intcalc.dtx}
```

Do not forget to quote the argument according to the demands of your shell.

Generating the documentation. You can use both the .dtx or the .drv to generate the documentation. The process can be configured by the configuration file ltxdoc.cfg. For instance, put this line into this file, if you want to have A4 as paper format:

```
\PassOptionsToClass{a4paper}{article}
```

An example follows how to generate the documentation with pdfIAT<sub>F</sub>X:

```
pdflatex intcalc.dtx
makeindex -s gind.ist intcalc.idx
pdflatex intcalc.dtx
makeindex -s gind.ist intcalc.idx
pdflatex intcalc.dtx
```

# 4 History

### [2007/09/09 v1.0]

• First version.

# [2007/09/27 v1.1]

- \intcalcNum added.
- $\bullet$  \intcalcSh1 and \intcalcShr allow negative numbers. The sign is preserved.
- Reuse \@gobble instead of own macro \IntCalc@Gobble.
- Small fixes.
- Shorter internal prefix.
- $\bullet\,$  Some programmer's interface.

# [2016/05/16 v1.2]

 $\bullet$  Documentation updates.

# [2019/12/15 v1.3]

• Documentation updates.

## 5 Index

Numbers written in italic refer to the page where the corresponding entry is described; numbers underlined refer to the code line of the definition; plain numbers refer to the code lines where the entry is used.

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