# The bigintcalc package

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#### Abstract

This package provides expandable arithmetic operations with big integers that can exceed TeX's number limits.

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<sup>\*</sup>Please report any issues at https://github.com/ho-tex/bigintcalc/issues

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#### 1 Documentation

#### 1.1 Introduction

Package bigintcalc defines arithmetic operations that deal with big integers. Big integers can be given either as explicit integer number or as macro code that expands to an explicit number. *Big* means that there is no limit on the size of the number. Big integers may exceed TEX's range limitation of -2147483647 and 2147483647. Only memory issues will limit the usable range.

In opposite to package intcalc unexpandable command tokens are not supported, even if they are valid TeX numbers like count registers or commands created by \chardef. Nevertheless they may be used, if they are prefixed by \number.

Also  $\varepsilon$ -TEX's \numexpr expressions are not supported directly in the manner of package intcalc. However they can be given if \the\numexpr or \numexpr are used.

The operations have the form of macros that take one or two integers as parameter and return the integer result. The macro name is a three letter operation name prefixed by the package name, e.g. \bigintcalcAdd{10}{43} returns 53.

The macros are fully expandable, exactly two expansion steps generate the result. Therefore the operations may be used nearly everywhere in TEX, even inside \csname, file names, or other expandable contexts.

#### 1.2 Conditions

#### 1.2.1 Preconditions

• Arguments can be anything that expands to a number that consists of optional signs and digits.

• The arguments and return values must be sound. Zero as divisor or factorials of negative numbers will cause errors.

#### 1.2.2 Postconditions

Additional properties of the macros apart from calculating a correct result (of course ©):

- The macros are fully expandable. Thus they can be used inside \edef, \csname, for example.
- Furthermore exactly two expansion steps calculate the result.
- The number consists of one optional minus sign and one or more digits. The first digit is larger than zero for numbers that consists of more than one digit.

In short, the number format is exactly the same as \number generates, but without its range limitation. And the tokens (minus sign, digits) have catcode 12 (other).

Call by value is simulated. First the arguments are converted to numbers.
 Then these numbers are used in the calculations.

Remember that arguments may contain expensive macros or  $\varepsilon$ -TEX expressions. This strategy avoids multiple evaluations of such arguments.

## 1.3 Error handling

Some errors are detected by the macros, example: division by zero. In this cases an undefined control sequence is called and causes a TeX error message, example: \BigIntCalcError:DivisionByZero. The name of the control sequence contains the reason for the error. The TeX error may be ignored. Then the operation returns zero as result. Because the macros are supposed to work in expandible contexts. An traditional error message, however, is not expandable and would break these contexts.

## 1.4 Operations

Some definition equations below use the function Int that converts a real number to an integer. The number is truncated that means rounding to zero:

$$\operatorname{Int}(x) := \begin{cases} \lfloor x \rfloor & \text{if } x \ge 0 \\ \lceil x \rceil & \text{otherwise} \end{cases}$$

#### 1.4.1 Num

\bigintcalcNum  $\{\langle x \rangle\}$ 

Macro \bigintcalcNum converts its argument to a normalized integer number without unnecessary leading zeros or signs. The result matches the regular expression:

## 1.4.2 Inv, Abs, Sgn

## \bigintcalcInv $\{\langle x \rangle\}$

Macro \bigintcalcInv switches the sign.

$$Inv(x) := -x$$

## \bigintcalcAbs $\{\langle x \rangle\}$

Macro \bigintcalcAbs returns the absolute value of integer  $\langle x \rangle$ .

$$Abs(x) := |x|$$

## \bigintcalcSgn $\{\langle x \rangle\}$

Macro \bigintcalcSgn encodes the sign of  $\langle x \rangle$  as number.

$$Sgn(x) := \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

These return values can easily be distinguished by \ifcase:

#### 1.4.3 Min, Max, Cmp

## \bigintcalcMin $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcMin returns the smaller of the two integers.

$$Min(x, y) := \begin{cases} x & \text{if } x < y \\ y & \text{otherwise} \end{cases}$$

## \bigintcalcMax $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcMax returns the larger of the two integers.

4

$$\operatorname{Max}(x,y) := \begin{cases} x & \text{if } x > y \\ y & \text{otherwise} \end{cases}$$

## \bigintcalcCmp $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcCmp encodes the comparison result as number:

$$Cmp(x,y) := \begin{cases} -1 & \text{if } x < y \\ 0 & \text{if } x = y \\ 1 & \text{if } x > y \end{cases}$$

These values can be distinguished by \ifcase:

#### 1.4.4 Odd

## \bigintcalcOdd $\{\langle x angle\}$

$$Odd(x) := \begin{cases} 1 & \text{if } x \text{ is odd} \\ 0 & \text{if } x \text{ is even} \end{cases}$$

#### 1.4.5 Inc, Dec, Add, Sub

## \bigintcalcInc $\{\langle x \rangle\}$

Macro \bigintcalcInc increments  $\langle x \rangle$  by one.

$$Inc(x) := x + 1$$

#### \bigintcalcDec $\{\langle x \rangle\}$

Macro \bigintcalcDec decrements  $\langle x \rangle$  by one.

$$Dec(x) := x - 1$$

## \bigintcalcAdd $\{\langle x angle\}$ $\{\langle y angle\}$

Macro \bigintcalcAdd adds the two numbers.

$$Add(x, y) := x + y$$

## \bigintcalcSub $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcSub calculates the difference.

$$Sub(x, y) := x - y$$

## 1.4.6 Shl, Shr

## \bigintcalcShl $\{\langle x \rangle\}$

Macro \bigintcalcShl implements shifting to the left that means the number is multiplied by two. The sign is preserved.

$$Shl(x) := x * 2$$

## \bigintcalcShr $\{\langle x \rangle\}$

Macro \bigintcalcShr implements shifting to the right. That is equivalent to an integer division by two. The sign is preserved.

$$Shr(x) := Int(x/2)$$

#### 1.4.7 Mul, Sqr, Fac, Pow

## \bigintcalcMul $\{\langle x angle\}$ $\{\langle y angle\}$

Macro \bigintcalcMul calculates the product of  $\langle x \rangle$  and  $\langle y \rangle$ .

$$Mul(x, y) := x * y$$

## \bigintcalcSqr $\{\langle x angle\}$

Macro \bigintcalcSqr returns the square product.

$$Sqr(x) := x^2$$

## \bigintcalcFac $\{\langle x \rangle\}$

Macro \bigintcalcFac returns the factorial of  $\langle x \rangle$ . Negative numbers are not permitted.

$$Fac(x) := x!$$
 for  $x \ge 0$ 

$$(0! = 1)$$

#### \bigintcalcPow Mx My

Macro \bigintcalcPow calculates the value of  $\langle x \rangle$  to the power of  $\langle y \rangle$ . The error "division by zero" is thrown if  $\langle x \rangle$  is zero and  $\langle y \rangle$  is negative. permitted:

$$\operatorname{Pow}(x, y) := \operatorname{Int}(x^y)$$
 for  $x \neq 0$  or  $y \geq 0$ 

$$(0^0 = 1)$$

#### 1.4.8 Div, Mul

## \bigintcalcDiv $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcDiv performs an integer division. Argument  $\langle y \rangle$  must not be zero.

$$\operatorname{Div}(x,y) := \operatorname{Int}(x/y)$$
 for  $y \neq 0$ 

\bigintcalcMod 
$$\{\langle x 
angle\}$$
  $\{\langle y 
angle\}$ 

Macro \bigintcalcMod gets the remainder of the integer division. The sign follows the divisor  $\langle y \rangle$ . Argument  $\langle y \rangle$  must not be zero.

$$Mod(x, y) := x \% y$$
 for  $y \neq 0$ 

The result ranges:

$$-|y| < \operatorname{Mod}(x, y) \le 0 \quad \text{for } y < 0$$
  
 
$$0 \le \operatorname{Mod}(x, y) < y \quad \text{for } y \ge 0$$

## 1.5 Interface for programmers

If the programmer can ensure some more properties about the arguments of the operations, then the following macros are a little more efficient.

In general numbers must obey the following constraints:

- Plain number: digit tokens only, no command tokens.
- Non-negative. Signs are forbidden.
- Delimited by exclamation mark. Curly braces around the number are not allowed and will break the code.

#### \BigIntCalcOdd $\langle number \rangle$ !

1/0 is returned if  $\langle number \rangle$  is odd/even.

\BigIntCalcInc 
$$\langle number \rangle$$
 !

Incrementation.

$$\BigIntCalcDec\ \langle number \rangle$$
 !

Decrementation, positive number without zero.

\BigIntCalcAdd 
$$\langle number A \rangle$$
 !  $\langle number B \rangle$  !

Addition,  $A \geq B$ .

\BigIntCalcSub 
$$\langle number A \rangle$$
 !  $\langle number B \rangle$  !

Subtraction,  $A \geq B$ .

```
\BigIntCalcShl \langle number \rangle !
```

Left shift (multiplication with two).

```
\BigIntCalcShr \langle number \rangle !
```

Right shift (integer division by two).

Multiplication,  $A \geq B$ .

```
\BigIntCalcDiv \langle number A \rangle ! \langle number B \rangle !
```

Division operation.

Modulo operation.

# 2 Implementation

1 (\*package)

## 2.1 Reload check and package identification

Reload check, especially if the package is not used with LATEX.

```
2 \begingroup\catcode61\catcode48\catcode32=10\relax%
   \catcode13=5 % ^^M
   \endlinechar=13 %
4
   \catcode35=6 % #
5
   \catcode39=12 % '
   \colone{1}{catcode44=12 \% }
   \catcode45=12 % -
   \catcode46=12 % .
   \catcode58=12 % :
10
   \catcode64=11 % @
11
   \catcode123=1 % {
12
    \catcode125=2 % }
    \expandafter\let\expandafter\x\csname ver@bigintcalc.sty\endcsname
    \ifx\x\relax % plain-TeX, first loading
15
16
     \def\empty{}%
17
     \ifx\x\empty % LaTeX, first loading,
18
        % variable is initialized, but \ProvidesPackage not yet seen
19
20
21
        \expandafter\ifx\csname PackageInfo\endcsname\relax
22
          \def\x#1#2{%}
23
           \immediate\write-1{Package #1 Info: #2.}%
         }%
24
        \else
25
          26
27
28
        \x{bigintcalc}{The package is already loaded}%
```

```
29
          \aftergroup\endinput
 30
        \fi
      \fi
 31
 32 \endgroup%
Package identification:
 33 \begingroup\catcode61\catcode48\catcode32=10\relax%
      \catcode13=5 % ^^M
      \endlinechar=13 %
 36
      \catcode35=6 % #
     \catcode39=12 % '
     \catcode40=12 % (
 38
      \catcode41=12 % )
 39
      \colone{1} \catcode44=12 % ,
 40
      \colored{catcode45=12 \% -}
 41
      \catcode46=12 % .
 42
      \catcode47=12 % /
 43
      \catcode58=12 % :
 44
      \catcode64=11 % @
 45
      \catcode91=12 % [
 46
      \catcode93=12 % ]
 47
      \catcode123=1 % {
      \catcode125=2 % }
 50
      \expandafter\ifx\csname ProvidesPackage\endcsname\relax
        \def\x#1#2#3[#4]{\endgroup}
 51
          \immediate\write-1{Package: #3 #4}%
 52
          \t 1{#4}%
 53
        }%
 54
 55
      \else
        \def \x#1#2[#3] \endgroup
 56
          #2[{#3}]%
 57
          \ifx#1\@undefined
 58
            \xdef#1{#3}%
 59
          \fi
 60
          \fint 1 relax
 61
 62
            \xdef#1{#3}%
 63
          \fi
        }%
 64
 65
 66 \expandafter\x\csname ver@bigintcalc.sty\endcsname
 67 \ProvidesPackage{bigintcalc}%
      [2019/12/15 v1.5 Expandable calculations on big integers (HO)]%
2.2
      Catcodes
 69 \begingroup\catcode61\catcode48\catcode32=10\relax%
     \catcode13=5 % ^^M
     \endlinechar=13 %
 71
     \catcode123=1 % {
 72
      \catcode125=2 % }
 73
      \catcode64=11 % @
 74
      \def\x{\endgroup
 75
 76
        \expandafter\edef\csname BIC@AtEnd\endcsname{%
```

\endlinechar=\the\endlinechar\relax
\catcode13=\the\catcode13\relax

\catcode32=\the\catcode32\relax

\catcode35=\the\catcode35\relax

\catcode61=\the\catcode61\relax

\catcode64=\the\catcode64\relax

77

78

79

80

81

82

```
\catcode123=\the\catcode123\relax
                83
                        \catcode125=\the\catcode125\relax
                84
                      }%
                85
                   }%
                86
                87 \x\catcode61\catcode48\catcode32=10\relax%
                88 \catcode13=5 % ^^M
                89 \endlinechar=13 %
                90 \catcode35=6 % #
                91 \catcode64=11 \% 0
                92 \catcode123=1 % {
                93 \catcode125=2 % }
                94 \def\TMP@EnsureCode#1#2{%
                    \edef\BIC@AtEnd{%
                      \BIC@AtEnd
                96
                      \catcode#1=\the\catcode#1\relax
                97
                98
               99
                    \catcode#1=#2\relax
               100 }
               101 \TMP@EnsureCode{33}{12}% !
               102 \TMP@EnsureCode{36}{14}% $ (comment!)
               103 \TMP@EnsureCode{38}{14}% & (comment!)
               104 \TMP@EnsureCode{40}{12}% (
               105 \TMP@EnsureCode{41}{12}% )
               106 \TMP@EnsureCode\{42\}\{12\}\% *
               107 \TMP@EnsureCode{43}{12}% +
               108 \TMP@EnsureCode{45}{12}% -
               109 \TMP@EnsureCode{46}{12}% .
               110 \TMP@EnsureCode{47}{12}% /
               111 \TMP@EnsureCode{58}{11}% : (letter!)
               112 \TMP@EnsureCode{60}{12}% <
               113 \TMP@EnsureCode{62}{12}% >
               114 \TMP@EnsureCode{63}{14}% ? (comment!)
               115 \TMP@EnsureCode{91}{12}% [
               116 \TMP@EnsureCode{93}{12}% ]
               117 \edef\BIC@AtEnd{\BIC@AtEnd\noexpand\endinput}
               119 \expandafter\ifx\csname BIC@TestMode\endcsname\relax
               120 \ensuremath{\setminus} else
               121
                    \catcode63=9 % ? (ignore)
               122 \fi
               123 ? \let\BIC@@TestMode\BIC@TestMode
                   \varepsilon-T<sub>E</sub>X detection
               124 \begingroup\expandafter\expandafter\expandafter\endgroup
               125 \expandafter\ifx\csname numexpr\endcsname\relax
               126 \catcode36=9 % $ (ignore)
               127 \else
               128 \catcode38=9 % & (ignore)
               129 \fi
              2.4 Help macros
    \BIC@Fi
               130 \let\BIC@Fi\fi
\BIC@AfterFi
               131 \def\BIC@AfterFi#1#2\BIC@Fi{\fi#1}%
```

```
\BIC@AfterFiFi
                   132 \def\BIC@AfterFiFi#1#2\BIC@Fi{\fi\fi#1}%
\BIC@AfterFiFiFi
                   133 \def\BIC@AfterFiFiFi#1#2\BIC@Fi{\fi\fi\fi#1}%
     \BIC@Space
                   134 \begingroup
                   135
                        \def\x#1{\endgroup
                   136
                          \let\BIC@Space= #1%
                   137
                       }%
                   138 \x{ }
                  2.5
                         Expand number
                   139 \begingroup\expandafter\expandafter\expandafter\endgroup
                   140 \expandafter\ifx\csname RequirePackage\endcsname\relax
                        \def\TMP@RequirePackage#1[#2]{%
                   142
                           \begingroup\expandafter\expandafter\expandafter\endgroup
                           \expandafter\ifx\csname ver@#1.sty\endcsname\relax
                   143
                            \input #1.sty\relax
                   144
                   145
                          \fi
                   146
                        }%
                        \TMP@RequirePackage{pdftexcmds}[2007/11/11]%
                   147
                   148 \else
                        \RequirePackage{pdftexcmds}[2007/11/11]%
                   149
                   150 \fi
                   151 \begingroup\expandafter\expandafter\expandafter\endgroup
                   152 \expandafter\ifx\csname pdf@escapehex\endcsname\relax
    \BIC@Expand
                        \def\BIC@Expand#1{%
                   153
                          \romannumeral0%
                   154
                          \BIC@@Expand#1!\@nil{}%
                   155
                   156
                        }%
    \BIC@@Expand
                        \def\BIC@@Expand#1#2\@nil#3{%
                   157
                          \expandafter\ifcat\noexpand#1\relax
                   158
                             \expandafter\@firstoftwo
                   159
                   160
                            \expandafter\@secondoftwo
                   161
                   162
                          \fi
                   163
                             \expandafter\BIC@@Expand#1#2\@ni1{#3}%
                   164
                   165
                          }{%
                            \ifx#1!%
                   166
                               \expandafter\@firstoftwo
                   167
                   168
                             \else
                   169
                               \expandafter\@secondoftwo
                   171
                            { #3}{%
                               \BIC@@Expand#2\@ni1{#3#1}%
                   172
                   173
                   174
                          }%
```

175

}%

```
\@firstoftwo
                          \expandafter\ifx\csname @firstoftwo\endcsname\relax
                     176
                     177
                            \long\def\@firstoftwo#1#2{#1}%
                     178
    \@secondoftwo
                           \expandafter\ifx\csname @secondoftwo\endcsname\relax
                     179
                     180
                            \long\def\@secondoftwo#1#2{#2}%
                     181
                     182 \else
       \BIC@Expand
                          \def\BIC@Expand#1{%
                     183
                             \romannumeral0\expandafter\expandafter\BIC@Space
                     184
                             \pdf@unescapehex{%
                     185
                               \expandafter\expandafter\expandafter
                     186
                     187
                               \BIC@StripHexSpace\pdf@escapehex{#1}20\@nil
                            }%
                     188
                          }%
                     189
\BIC@StripHexSpace
                     190
                          \def\BIC@StripHexSpace#120#2\@ni1{%
                            #1%
                     191
                     192
                            \ifx\\#2\\%
                     193
                            \else
                     194
                               \BIC@AfterFi{%
                     195
                                 \BIC@StripHexSpace#2\@nil
                     196
                               }%
                     197
                            \BIC@Fi
                          }%
                     198
                     199 \fi
                           Normalize expanded number
                    2.6
   \BIC@Normalize
                    #1: result sign
                    #2: first token of number
                     200 \def\BIC@Normalize#1#2{%
                          \ifx#2-%
                     202
                            \ifx\\#1\\%
                               \BIC@AfterFiFi{%
                     203
                                 \BIC@Normalize-%
                     204
                               }%
                     205
                            \else
                     206
                               \BIC@AfterFiFi{%
                     207
                                 \BIC@Normalize{}%
                     209
                               }%
                     210
                            \fi
                          \else
                     211
                            \ifx#2+%
                     212
                     213
                               \BIC@AfterFiFi{%
                     214
                                 \BIC@Normalize{#1}%
                               }%
                     215
                            \else
                     216
                              \ifx#20%
                     217
                                \BIC@AfterFiFiFi{%
                     218
                                   \BIC@NormalizeZero{#1}%
                     219
```

```
}%
                        220
                                 \else
                        221
                                   \BIC@AfterFiFiFi{%
                        222
                                      \BIC@NormalizeDigits#1#2%
                        223
                                   }%
                        224
                        225
                                 \fi
                        226
                               \fi
                             \BIC@Fi
                        227
                        228 }
  \BIC@NormalizeZero
                        229 \def\BIC@NormalizeZero#1#2{%
                             \ifx#2!%
                               \BIC@AfterFi{ 0}%
                        231
                        232
                             \else
                               \ifx#20%
                        233
                                 \BIC@AfterFiFi{%
                        234
                                   \BIC@NormalizeZero{#1}%
                        235
                                 }%
                        236
                        237
                               \else
                        238
                                 \BIC@AfterFiFi{%
                                   \BIC@NormalizeDigits#1#2%
                        239
                                 }%
                        240
                               \fi
                        241
                             \BIC@Fi
                        242
                        243 }
\BIC@NormalizeDigits
                        244 \def\BIC@NormalizeDigits#1!{ #1}
                       2.7
                              Num
      \bigintcalcNum
                        245 \def\bigintcalcNum#1{%
                             \romannumeral0%
                             \expandafter\expandafter\BIC@Normalize
                        247
                             \expandafter\expandafter\expandafter{%
                        248
                             \expandafter\expandafter\expandafter}%
                        249
                        250
                             \BIC@Expand{#1}!%
                        251 }
                              Inv, Abs, Sgn
      \bigintcalcInv
                        252 \def\bigintcalcInv#1{%
                             \romannumeralO\expandafter\expandafter\expandafter\BIC@Space
                             \bigintcalcNum{-#1}%
                        255 }
      \bigintcalcAbs
                        256 \ensuremath{\mbox{def\bigintcalcAbs}\#1\%}
                        257
                             \romannumeral0%
                             \verb|\expandafter| expandafter| BIC@Abs|
                        258
                             \bigintcalcNum{#1}%
                        259
                        260 }
```

```
\BIC@Abs
                  261 \ensuremath{\texttt{M}}
                      \ifx#1-%
                         \expandafter\BIC@Space
                  264
                  265
                         \expandafter\BIC@Space
                         \expandafter#1%
                  266
                       \fi
                  267
                  268 }
\bigintcalcSgn
                  269 \def\bigintcalcSgn#1{%
                       \verb|\expandafter| expandafter| BIC@Sgn|
                       \bigintcalcNum{#1}! %
                  272
                  273 }
      \BIC@Sgn
                  274 \def\BIC@Sgn#1#2!{%
                      \ifx#1-%
                  276
                         -1%
                       \else
                  277
                         \ifx#10%
                  278
                           0%
                  279
                         \else
                  280
                  281
                           1%
                         \fi
                  282
                  283
                       \fi
                  284 }
                 2.9
                        Cmp, Min, Max
\bigintcalcCmp
                  285 \def\bigintcalcCmp#1#2{%
                       \number
                  286
                       \expandafter\expandafter\BIC@Cmp
                  287
                       \bigintcalcNum{#2}!{#1}%
                  288
                  289 }
      \BIC@Cmp
                  290 \def\BIC@Cmp#1!#2{%
                      \expandafter\expandafter\expandafter\BIC@@Cmp
                       \bigintcalcNum{#2}!#1!%
                  293 }
     \BIC@@Cmp
                  294 \ensuremath{ \mbox{\mbox{$\sim$}}} 1#2!#3#4!{\%}
                  295
                       \ifx#1-%
                  296
                         \ifx#3-%
                           \BIC@AfterFiFi{%
                  298
                              \BIC@@Cmp#4!#2!%
                  299
                           }%
                         \else
                  300
                           \BIC@AfterFiFi{%
                  301
                             -1 %
                  302
                  303
                           }%
                  304
                         \fi
```

```
305
                      \else
                        \ifx#3-%
                 306
                          \BIC@AfterFiFi{%
                 307
                             1 %
                 308
                          }%
                 309
                        \else
                 310
                 311
                          \BIC@AfterFiFi{%
                             \BIC@CmpLength#1#2!#3#4!#1#2!#3#4!%
                 312
                          }%
                 313
                        \fi
                 314
                      \BIC@Fi
                 315
                 316 }
   \BIC@PosCmp
                 317 \def\BIC@PosCmp#1!#2!{%
                 318 \BIC@CmpLength#1!#2!#1!#2!%
                 319 }
\BIC@CmpLength
                 320 \ensuremath{\texttt{\fmu}}{1}2!#3#4!{\%}
                      \ifx\\#2\\%
                        \ifx\\#4\\%
                 322
                 323
                          \BIC@AfterFiFi\BIC@CmpDiff
                        \else
                 324
                          \BIC@AfterFiFi{%
                 325
                 326
                             \BIC@CmpResult{-1}%
                          }%
                 327
                 328
                        \fi
                 329
                      \else
                 330
                        \ifx\\#4\\%
                          \BIC@AfterFiFi{%
                 331
                             \BIC@CmpResult1%
                 332
                          }%
                 333
                        \else
                 334
                          \BIC@AfterFiFi{%
                 335
                 336
                             \BIC@CmpLength#2!#4!%
                 337
                          }%
                 338
                        \fi
                      \BIC@Fi
                 339
                 340 }
\BIC@CmpResult
                 341 \def\BIC@CmpResult#1#2!#3!{#1 }
  \BIC@CmpDiff
                 \ifnum#1<#3 %
                 344
                        \BIC@AfterFi{%
                 345
                          -1 %
                        }%
                 346
                 347
                      \else
                        \ifnum#1>#3 %
                 348
                          \BIC@AfterFiFi{%
                 349
                 350
                             1 %
                          }%
                 351
                 352
                        \else
                          \ifx\\#2\\%
                 353
                            \BIC@AfterFiFiFi{%
                 354
```

```
0 %
                  355
                              }%
                  356
                  357
                            \else
                              \BIC@AfterFiFiFi{%
                  358
                  359
                                \BIC@CmpDiff#2!#4!%
                              }%
                  360
                  361
                            \fi
                  362
                          \fi
                       \BIC@Fi
                  363
                  364 }
\bigintcalcMin
                  365 \def\bigintcalcMin#1{%
                       \romannumeral0%
                       \verb|\expandafter\expandafter\expandafter\BIC@MinMax| \\
                  367
                  368
                       \bigintcalcNum{#1}!-!%
                  369 }
\bigintcalcMax
                  370 \def\bigintcalcMax#1{%
                       \romannumeral0%
                  371
                  372
                       \expandafter\expandafter\BIC@MinMax
                  373
                       \bigintcalcNum{#1}!!%
                  374 }
   \BIC@MinMax
                 #1: x
                 #2: sign for comparison
                 #3: y
                  375 \def\BIC@MinMax#1!#2!#3{%
                       \expandafter\expandafter\expandafter\BIC@@MinMax
                       \bigintcalcNum{#3}!#1!#2!%
                  377
                  378 }
  \BIC@@MinMax #1: y
                 #3: sign for comparison
                  379 \def\BIC@@MinMax#1!#2!#3!{%
                       \ifnum\BIC@@Cmp#1!#2!=#31 %
                         \BIC@AfterFi{ #1}%
                  381
                  382
                       \else
                         \BIC@AfterFi{ #2}%
                  383
                       \BIC@Fi
                  384
                  385 }
                 2.10
                         Odd
\bigintcalcOdd
                  386 \ensuremath{\mbox{def\bigintcalc0dd}\mbox{$\sharp 1 \mbox{\mbox{$\sharp$}}}}
                       \romannumeral0%
                       \expandafter\expandafter\BIC@Odd
                  388
                       \bigintcalcAbs{#1}!%
                  389
                  390 }
\BigIntCalcOdd
                  391 \def\BigIntCalcOdd#1!{%
                  392 \romannumeral0%
                  393 \BIC@Odd#1!%
                  394 }
```

```
\BIC@Odd #1: x
                                                      395 \ensuremath{\mbox{\sc def}\mbox{\sc BIC@Odd#1#2}\mbox{\sc white}\mbox{\sc def}\mbox{\sc def}\m
                                                                     \ifx#2!%
                                                                             \ifodd#1 %
                                                      398
                                                                                    \BIC@AfterFiFi{ 1}%
                                                                             \else
                                                      399
                                                                                   \BIC@AfterFiFi{ 0}%
                                                      400
                                                                             \fi
                                                      401
                                                      402
                                                                       \else
                                                                             \expandafter\BIC@Odd\expandafter#2%
                                                      403
                                                      404
                                                      405 }
                                                    2.11
                                                                              Inc, Dec
\bigintcalcInc
                                                      406 \def\bigintcalcInc#1{%
                                                                       \romannumeral0%
                                                                       \verb|\expandafter| expandafter| BIC@IncSwitch|
                                                      408
                                                                       \bigintcalcNum{#1}!%
                                                      409
                                                      410 }
\BIC@IncSwitch
                                                      411 \def\BIC@IncSwitch#1#2!{%
                                                                      \ifcase\BIC@@Cmp#1#2!-1!%
                                                                             \BIC@AfterFi{ 0}%
                                                      413
                                                                      \or
                                                      414
                                                                             \BIC@AfterFi{%
                                                      415
                                                                                   \BIC@Inc#1#2!{}%
                                                      416
                                                                            }%
                                                      417
                                                      418
                                                                       \else
                                                      419
                                                                             \BIC@AfterFi{%
                                                                                    \expandafter-\romannumeral0%
                                                      420
                                                      421
                                                                                   \BIC@Dec#2!{}%
                                                      422
                                                                            }%
                                                      423
                                                                      \BIC@Fi
                                                      424 }
\bigintcalcDec
                                                      425 \def\bigintcalcDec#1{%
                                                                     \romannumeral0%
                                                      426
                                                                      \verb|\expandafter| expandafter | BIC@DecSwitch| \\
                                                      427
                                                      428
                                                                      \bigintcalcNum{#1}!%
                                                      429 }
\BIC@DecSwitch
                                                      430 \def\BIC@DecSwitch#1#2!{%
                                                                       \ifcase\BIC@Sgn#1#2! %
                                                                             \BIC@AfterFi{ -1}%
                                                      432
                                                      433
                                                                       \or
                                                      434
                                                                            \BIC@AfterFi{%
                                                                                   \BIC@Dec#1#2!{}%
                                                      435
                                                      436
                                                                            }%
                                                      437
                                                                       \else
                                                      438
                                                                             \BIC@AfterFi{%
                                                                                   \expandafter-\romannumeral0%
                                                      439
                                                                                   \BIC@Inc#2!{}%
                                                      440
```

```
441
                         }%
                       \BIC@Fi
                 442
                 443 }
\BigIntCalcInc
                 444 \def\BigIntCalcInc#1!{%
                      \romannumeral0\BIC@Inc#1!{}%
                 446 }
\BigIntCalcDec
                 447 \def\BigIntCalcDec#1!{%
                 448
                      \romannumeral0\BIC@Dec#1!{}%
                 449 }
      \BIC@Inc
                 450 \def\BIC@Inc#1#2!#3{%
                       \ifx\\#2\\%
                 452
                         \BIC@AfterFi{%
                 453
                           \BIC@@Inc1#1#3!{}%
                         }%
                 454
                       \else
                 455
                         \BIC@AfterFi{%
                 456
                           \BIC@Inc#2!{#1#3}%
                 457
                         }%
                 458
                 459
                       \BIC@Fi
                 460 }
     \BIC@@Inc
                 461 \def\BIC@@Inc#1#2#3!#4{%
                 462
                       \ifcase#1 %
                         \ifx\\#3\\%
                 463
                           \BIC@AfterFiFi{ #2#4}%
                 464
                 465
                         \else
                 466
                           \BIC@AfterFiFi{%
                             \BIC@@IncO#3!{#2#4}%
                 467
                 468
                           }%
                 469
                         \fi
                 470
                       \else
                         \ifnum#2<9 %
                 471
                 472
                           \BIC@AfterFiFi{%
                 473 &
                             \expandafter\BIC@@@Inc\the\numexpr#2+1\relax
                 474 $
                             \expandafter\expandafter\BIC@@@Inc
                 475 $
                             \ifcase#2 \expandafter1%
                 476 $
                             \or\expandafter2%
                 477 $
                             \or\expandafter3%
                 478 $
                             \or\expandafter4%
                 479 $
                             \or\expandafter5%
                 480 $
                             \or\expandafter6%
                 481 $
                             \or\expandafter7%
                 482 $
                             \or\expandafter8%
                 483 $
                             \or\expandafter9%
                 484 $?
                             \else\BigIntCalcError:ThisCannotHappen%
                 485 $
                 486
                             0#3!{#4}%
                           }%
                 487
                 488
                         \else
                           \BIC@AfterFiFi{%
                 489
                             \BIC@@@IncO1#3!{#4}%
                 490
```

```
}%
              491
                     \fi
              492
                   \BIC@Fi
              493
              494 }
\BIC@@@Inc
              495 \def\BIC@@@Inc#1#2#3!#4{%
                   \ifx\\#3\\%
                     \ifnum#2=1 %
              497
                       \BIC@AfterFiFi{ 1#1#4}%
              498
              499
                     \else
                       \BIC@AfterFiFi{ #1#4}%
              500
              501
                     \fi
                   \else
              502
                     \BIC@AfterFi{%
              503
                       \BIC@@Inc#2#3!{#1#4}%
              504
              505
                     }%
                   \BIC@Fi
              506
              507 }
  \BIC@Dec
              508 \def\BIC@Dec#1#2!#3{%
                   \ifx\\#2\\%
              510
                     \BIC@AfterFi{%
                       \BIC@@Dec1#1#3!{}%
              511
                     }%
              512
              513
                   \else
              514
                     \BIC@AfterFi{%
              515
                       \BIC@Dec#2!{#1#3}%
                     }%
              516
                   \BIC@Fi
              517
              518 }
 \BIC@@Dec
              519 \ensuremath{ \mbox{ def\BIC@@Dec#1#2#3!#4{\%} }
                   \ifcase#1 %
              521
                     \ifx\\#3\\%
                       \BIC@AfterFiFi{ #2#4}%
              522
              523
                     \else
                       \BIC@AfterFiFi{%
              524
                          \BIC@@Dec0#3!{#2#4}%
              525
                       }%
              526
              527
                     \fi
              528
                   \else
              529
                     \ifnum#2>0 %
              530
                       \BIC@AfterFiFi{%
              531 &
                          \verb|\expandafter\BIC@@Dec\the\numexpr#2-1\relax| \\
              532 $
                          \expandafter\expandafter\BIC@@@Dec
              533 $
                          \ifcase#2
              534 $?
                            \BigIntCalcError:ThisCannotHappen%
              535 $
                          \or\expandafter0%
              536 $
                          \or\expandafter1%
              537 $
                          \or\expandafter2%
              538 $
                          \or\expandafter3%
              539 $
                          \or\expandafter4%
              540 $
                          \or\expandafter5%
              541 $
                          \or\expandafter6%
              542 $
                          \or\expandafter7%
```

```
543 $
                            \or\expandafter8%
                 544 $?
                            \else\BigIntCalcError:ThisCannotHappen%
                 545 $
                            \fi
                            0#3!{#4}%
                 546
                 547
                          }%
                 548
                        \else
                          \BIC@AfterFiFi{%
                 549
                            \BIC@@@Dec91#3!{#4}%
                 550
                 551
                          }%
                        \fi
                 552
                 553
                      \BIC@Fi
                 554 }
    \BIC@@@Dec
                 555 \def\BIC@@@Dec#1#2#3!#4{%
                     \ifx\\#3\\%
                 556
                        \ifcase#1 %
                 557
                          \ifx\\#4\\%
                 558
                            \BIC@AfterFiFiFi{ 0}%
                 560
                 561
                            \BIC@AfterFiFiFi{ #4}%
                          \fi
                 562
                 563
                        \else
                          \BIC@AfterFiFi{ #1#4}%
                 564
                        \fi
                 565
                 566
                      \else
                        \BIC@AfterFi{%
                 567
                          \BIC@@Dec#2#3!{#1#4}%
                 568
                 569
                        }%
                      \BIC@Fi
                 570
                 571 }
                2.12
                        Add, Sub
\bigintcalcAdd
                 572 \def\bigintcalcAdd#1{%
                 573 \romannumeral0%
                      \expandafter\expandafter\BIC@Add
                      \bigintcalcNum{#1}!%
                 576 }
      \BIC@Add
                 577 \def\BIC@Add#1!#2{%
                      \expandafter\expandafter\expandafter
                      \BIC@AddSwitch\bigintcalcNum{#2}!#1!%
                 579
                 580 }
\bigintcalcSub
                 581 \ensuremath{\mbox{\sc Sub#1#2}}\
                      \romannumeral0%
                 583
                      \expandafter\expandafter\BIC@Add
                 584
                      \bigintcalcNum{-#2}!{#1}%
                 585 }
\BIC@AddSwitch Decision table for \BIC@AddSwitch.
```

x < 0	y < 0	-x > -y	_	Add(-x, -y)
		else		Add(-y, -x)
	else	-x > y	_	Sub(-x,y)
		-x=y		0
		else	+	Sub(y, -x)
else	y < 0	x > -y	+	Sub(x, -y)
		x = -y		0
		else	_	Sub(-y,x)
	else	x > y	+	Add(x,y)
		else		Add(y,x)
	else	x > y	+	Add(x,y)

```
586 \def\BIC@AddSwitch#1#2!#3#4!{%
     \int x^{-1} x < 0
588
       \int x#3-\% y < 0
589
          \expandafter-\romannumeral0%
          \ifnum\BIC@PosCmp#2!#4!=1 % -x > -y
590
            \BIC@AfterFiFiFi{%
591
              \BIC@AddXY#2!#4!!!%
592
            }%
593
594
          \else % -x <= -y
595
            \BIC@AfterFiFiFi{%
596
              \BIC@AddXY#4!#2!!!%
            }%
597
          \fi
598
       \leq % y >= 0
599
600
          \label{eq:local_problem} \label{eq:local_problem} $$ \left( \frac{y}{x} - x = y \right) $$
601
            \BIC@AfterFiFiFi{ 0}%
602
          \expandafter-\romannumeral0%
603
            \BIC@AfterFiFiFi{%
604
              \BIC@SubXY#2!#3#4!!!%
605
            }%
606
607
          \else % -x <= y
608
            \BIC@AfterFiFiFi{%
              \BIC@SubXY#3#4!#2!!!%
609
            }%
610
          \fi
611
       \fi
612
     \leq % x >= 0
613
614
       \int x#3-\% y < 0
615
          \ightharpoonup 1#2!#4!\% x = -y
            \BIC@AfterFiFiFi{ 0}%
616
          \or % x > -y
617
            \BIC@AfterFiFiFi{%
618
              \BIC@SubXY#1#2!#4!!!%
619
            }%
620
621
          \else % x <= -y
            \expandafter-\romannumeral0%
622
            \BIC@AfterFiFiFi{%
623
              \BIC@SubXY#4!#1#2!!!%
624
            }%
625
          \fi
626
627
       \else % y >= 0
628
          \int BIC@PosCmp#1#2!#3#4!=1 % x > y
            \BIC@AfterFiFiFi{%
629
              \BIC@AddXY#1#2!#3#4!!!%
630
            }%
631
```

```
\else % x <= y
                 632
                             \BIC@AfterFiFiFi{%
                 633
                               \BIC@AddXY#3#4!#1#2!!!%
                 634
                             }%
                 635
                 636
                           \fi
                         \fi
                 637
                       \BIC@Fi
                 638
                 639 }
\BigIntCalcAdd
                 640 \def\BigIntCalcAdd#1!#2!{%
                      \romannumeral0\BIC@AddXY#1!#2!!!%
                 641
                 642 }
\BigIntCalcSub
                 643 \def\BigIntCalcSub#1!#2!{%
                 644 \romannumeral0\BIC@SubXY#1!#2!!!%
                 645 }
    \BIC@AddXY
                 646 \def\BIC@AddXY#1#2!#3#4!#5!#6!{%
                      \ifx\\#2\\%
                 647
                         \ifx\\#3\\%
                 648
                           \BIC@AfterFiFi{%
                 650
                             \BIC@DoAdd0!#1#5!#60!%
                           }%
                 651
                         \else
                 652
                           \BIC@AfterFiFi{%
                 653
                             \BIC@DoAdd0!#1#5!#3#6!%
                 654
                           }%
                 656
                         \fi
                       \else
                 657
                         \ifx\\#4\\%
                 658
                           \ifx\\#3\\%
                 659
                             \BIC@AfterFiFiFi{%
                 660
                               \BIC@AddXY#2!{}!#1#5!#60!%
                 661
                             }%
                 662
                 663
                           \else
                             \BIC@AfterFiFiFi{%
                 664
                               \BIC@AddXY#2!{}!#1#5!#3#6!%
                 665
                             }%
                 666
                           \fi
                 667
                 668
                         \else
                 669
                           \BIC@AfterFiFi{%
                             \BIC@AddXY#2!#4!#1#5!#3#6!%
                 670
                           }%
                 671
                         \fi
                 672
                       \BIC@Fi
                 673
                 674 }
    \BIC@DoAdd #1: carry
                 #2: reverted result
                 #3#4: reverted x
                 #5#6: reverted y
                 675 \def\BIC@DoAdd#1#2!#3#4!#5#6!{%
                     \ifx\\#4\\%
                 676
                 677
                         \BIC@AfterFi{%
                           \expandafter\BIC@Space
                 678 &
```

```
679 &
                               \theta = 1+#3+#5\
                     680 $
                               \verb|\expandafter| expandafter| BIC@AddResult|
                     681 $
                               \BIC@AddDigit#1#3#5#2%
                             }%
                     682
                     683
                           \else
                             \BIC@AfterFi{%
                     684
                               \expandafter\expandafter\expandafter\BIC@DoAdd
                     685
                               \BIC@AddDigit#1#3#5#2!#4!#6!%
                     686
                             }%
                     687
                           \BIC@Fi
                     688
                     689 }
    \BIC@AddResult
                     690 $ \def\BIC@AddResult#1{%
                           \ifx#10%
                     691 $
                     692 $
                               \expandafter\BIC@Space
                     693 $
                           \else
                     694 $
                               \expandafter\BIC@Space\expandafter#1%
                     695 $ \fi
                     696 $ }%
     \BIC@AddDigit #1: carry
                     #2: digit of x
                     #3: digit of y
                     697 \def\BIC@AddDigit#1#2#3{%}
                     698 \romannumeral0%
                     699 & \expandafter\BIC@@AddDigit\the\numexpr#1+#2+#3!%
                     700 $ \expandafter\BIC@@AddDigit\number%
                     701 $ \csname
                     702 $
                             BIC@AddCarry%
                     703 $
                            \ifcase#1 %
                     704 $
                               #2%
                     705 $
                            \else
                     706 $
                               \footnote{Minimum} $$  \if case #2 1\or 2\or 3\or 4\or 5\or 6\or 7\or 8\or 9\or 10\fi
                     707 $
                            \fi
                     708 $ \endcsname#3!%
                     709 }
    \BIC@@AddDigit
                     710 \def\BIC@@AddDigit#1!{%
                          \ifnum#1<10 %
                     711
                             \BIC@AfterFi{ 0#1}%
                     712
                     713
                          \else
                             \BIC@AfterFi{ #1}%
                     714
                     715 \BIC@Fi
                     716 }
    \BIC@AddCarry0
                     717 $\expandafter\def\csname BIC@AddCarryO\endcsname#1{#1}%
   \BIC@AddCarry10
                     718 \ \expandafter\def\csname BIC@AddCarry10\endcsname#1{1#1}%
\BIC@AddCarry[1-9]
                     719 $ \def\BIC@Temp#1#2{%
                             \expandafter\def\csname BIC@AddCarry#1\endcsname##1{%
                     721 $
                               \ifcase##1 #1\or
```

```
722 $
             723 $?
                       \else\BigIntCalcError:ThisCannotHappen%
             724 $
                       \fi
                   }%
             725 $
             726 $ }%
             727 $ \BIC@Temp 0{1\circ2\circ3\circ4\circ5\circ6\circ7\circ8\circ9}%
             728 $ \BIC@Temp 1{2\or3\or4\or5\or6\or7\or8\or9\or10}%
             729 $ \BIC@Temp 2{3\or4\or5\or6\or7\or8\or9\or10\or11}%
             730 $ \BIC@Temp 3{4\or5\or6\or7\or8\or9\or10\or11\or12}%
             731 $ \BIC@Temp 4{5\or6\or7\or8\or9\or10\or11\or12\or13}%
             732 $ \BIC@Temp 5{6\or7\or8\or9\or10\or11\or12\or13\or14}%
             733 $ \BIC@Temp 6{7\or8\or9\or10\or11\or12\or13\or14\or15}%
             734 $ \BIC@Temp 7{8\or9\or10\or11\or12\or13\or14\or15\or16}%
             735 $ \BIC@Temp 8{9\or10\or11\or12\or13\or14\or15\or16\or17}%
             736 $ \BIC@Temp 9{10\or11\or12\or13\or14\or15\or16\or17\or18}%
\BIC@SubXY Preconditions:
                • x > y, x \ge 0, and y >= 0
                • digits(x) = digits(y)
             737 \def\BIC@SubXY#1#2!#3#4!#5!#6!{%
                  \ifx\\#2\\%
             739
                     \ifx\\#3\\%
             740
                       \BIC@AfterFiFi{%
                         \BIC@DoSub0!#1#5!#60!%
             742
             743
                     \else
                       \BIC@AfterFiFi{%
             744
                         \BIC@DoSub0!#1#5!#3#6!%
             745
                       }%
             746
             747
                     \fi
                   \else
             748
                     \ifx\\#4\\%
             749
                       \ifx\\#3\\%
             750
                         \BIC@AfterFiFiFi{%
             751
             752
                           \BIC@SubXY#2!{}!#1#5!#60!%
                         }%
             753
                       \else
                         \BIC@AfterFiFiFi{%
             755
                           \BIC@SubXY#2!{}!#1#5!#3#6!%
             756
                         }%
             757
                       \fi
             758
             759
                     \else
             760
                       \BIC@AfterFiFi{%
                         \BIC@SubXY#2!#4!#1#5!#3#6!%
             761
             762
                       }%
             763
                     \fi
                   \BIC@Fi
             764
             765 }
\BIC@DoSub #1: carry
            #2: reverted result
            #3#4: reverted x
            #5#6: reverted y
             766 \def\BIC@DoSub#1#2!#3#4!#5#6!{%
                  \ifx\\#4\\%
             767
                     \BIC@AfterFi{%
             768
```

```
769
                                                                                                                                       \expandafter\expandafter\BIC@SubResult
                                                                                              770
                                                                                                                                       \BIC@SubDigit#1#3#5#2%
                                                                                                                             }%
                                                                                              771
                                                                                              772
                                                                                                                     \else
                                                                                              773
                                                                                                                              \BIC@AfterFi{%
                                                                                                                                       \expandafter\expandafter\BIC@DoSub
                                                                                              774
                                                                                                                                       \BIC@SubDigit#1#3#5#2!#4!#6!%
                                                                                              775
                                                                                                                              }%
                                                                                              776
                                                                                                                     \BIC@Fi
                                                                                              777
                                                                                              778 }
                 \BIC@SubResult
                                                                                              779 \def\BIC@SubResult#1{%
                                                                                                                  \ifx#10%
                                                                                              780
                                                                                                                             \expandafter\BIC@SubResult
                                                                                              781
                                                                                              782
                                                                                                                    \else
                                                                                                                              \expandafter\BIC@Space\expandafter#1%
                                                                                              783
                                                                                              784
                                                                                                                   \fi
                                                                                              785 }
                     \BIC@SubDigit
                                                                                         #1: carry
                                                                                          #2: digit of x
                                                                                          #3: digit of y
                                                                                              786 \def\BIC@SubDigit#1#2#3{%
                                                                                              787 \romannumeral0%
                                                                                              788 \ \& \property \prope
                                                                                              789 $ \expandafter\BIC@@AddDigit\number
                                                                                              790 $ \csname
                                                                                                                                       BIC@SubCarry%
                                                                                              791 $
                                                                                              792 $
                                                                                                                                       \ifcase#1 %
                                                                                              793 $
                                                                                                                                                #3%
                                                                                              794 $
                                                                                                                                       \else
                                                                                              795 $
                                                                                                                                               \footnote{Missing the continuous of the contin
                                                                                              796 $
                                                                                                                                       \fi
                                                                                              797 $
                                                                                                                             \endcsname#2!%
                                                                                              798 }
                 \BIC@@SubDigit
                                                                                              799 & \def\BIC@@SubDigit#1!{%
                                                                                              800 &
                                                                                                                         \ifnum#1<0 %
                                                                                              801 &
                                                                                                                                       \BIC@AfterFi{%
                                                                                              802 &
                                                                                                                                                 \expandafter\BIC@Space
                                                                                              803 &
                                                                                                                                                 \expandafter1\the\numexpr#1+10\relax
                                                                                              804 &
                                                                                                                                       }%
                                                                                                                             \else
                                                                                              805 &
                                                                                                                                       \BIC@AfterFi{ 0#1}%
                                                                                              806 &
                                                                                              807 & \BIC@Fi
                                                                                              808 & }%
                 \BIC@SubCarry0
                                                                                              809 $ \expandafter\def\csname BIC@SubCarryO\endcsname#1{#1}%
             \BIC@SubCarry10
                                                                                              810 $\expandafter\def\csname BIC@SubCarry10\endcsname#1{1#1}%
\BIC@SubCarry[1-9]
                                                                                              811 $ \def\BIC@Temp#1#2{%
```

```
812 $
                        \expandafter\def\csname BIC@SubCarry#1\endcsname##1{%
                 813 $
                          \ifcase##1 #2%
                 814 $?
                          \else\BigIntCalcError:ThisCannotHappen%
                 815 $
                          \fi
                        }%
                 816 $
                 817 $ }%
                 \$18  \BIC@Temp 1{19\or0\or1\or2\or3\or4\or5\or6\or7\or8}%
                 819 $ \BIC@Temp 2{18\or19\or0\or1\or2\or3\or4\or5\or6\or7}\%
                 820 \ \DIC@Temp \ 3{17\or18\or19\or0\or1\or2\or3\or4\or5\or6}\%
                 821 $ \BIC@Temp 4{16\or17\or18\or19\or0\or1\or2\or3\or4\or5}%
                 822 $ \BIC@Temp 5{15\or16\or17\or18\or19\or0\or1\or2\or3\or4}%
                 823 $ \BIC@Temp 6{14\or15\or16\or17\or18\or19\or0\or1\or2\or3}%
                 824 $ \BIC@Temp 7{13\or14\or15\or16\or17\or18\or19\or0\or1\or2}%
                 \$25 \BIC@Temp  8{12\or13\or14\or15\or16\or17\or18\or19\or0\or1}%
                 826 $ \BIC@Temp 9{11\or12\or13\or14\or15\or16\or17\or18\or19\or0}%
                2.13
                        ShI, Shr
\bigintcalcShl
                 827 \def\bigintcalcShl#1{%
                 828
                      \romannumeral0%
                      \expandafter\expandafter\BIC@Shl
                 830
                      \bigintcalcNum{#1}!%
                 831 }
      \BIC@Shl
                 832 \def\BIC@Shl#1#2!{%
                 833
                      \ifx#1-%
                 834
                        \BIC@AfterFi{%
                 835
                          \expandafter-\romannumeral0%
                          \BIC@@Sh1#2!!%
                 836 &
                 837 $
                          \BIC@AddXY#2!#2!!!%
                        }%
                 838
                 839
                      \else
                        \BIC@AfterFi{%
                 840
                          \BIC@@Shl#1#2!!%
                 841 &
                 842 $
                          \BIC@AddXY#1#2!#1#2!!!%
                 843
                        }%
                 844
                     \BIC@Fi
                 845 }
\BigIntCalcShl
                 846 \def\BigIntCalcShl#1!{%
                 847 \romannumeral0%
                 848 & \BIC@@Shl#1!!%
                 849 $ \BIC@AddXY#1!#1!!!%
                 850 }
     \BIC@@Shl
                 851 & \def\BIC@@Shl#1#2!{%
                 852 &
                       \ifx\\#2\\%
                 853 &
                          \BIC@AfterFi{%
                            \BIC@@@Sh10!#1%
                 854 &
                 855 &
                          }%
                 856 &
                        \else
                 857 &
                          \BIC@AfterFi{%
                            \BIC@@Sh1#2!#1%
                 858 &
```

```
859 &
                          }%
                 860 & \BIC@Fi
                 861 & }%
    \BIC@@@Shl
                #1: carry
                #2: result
                #3#4: reverted number
                 862 & \def\BIC@@@Shl#1#2!#3#4!{%
                 863 &
                        \ifx\\#4\\%
                 864 &
                          \BIC@AfterFi{%
                            \expandafter\BIC@Space
                 865 &
                            \t \
                 866 &
                 867 &
                          }%
                 868 &
                        \else
                          \BIC@AfterFi{%
                 869 &
                 870 &
                            \verb|\expandafter| BIC0000Shl\the| numexpr#3*2+#1!#2!#4!\% |
                 871 &
                 872 &
                        \BIC@Fi
                 873 & }%
   \BIC@@@@Shl
                 874 & \def\BIC@@@@Shl#1!{%
                 875 & \ifnum#1<10 %
                 876 &
                          \BIC@AfterFi{%
                 877 &
                            \BIC@@@Sh10#1%
                          }%
                 878 &
                 879 &
                       \else
                          \BIC@AfterFi{%
                 880 &
                            \BIC@@@Shl#1%
                 881 &
                          }%
                 882 &
                 883 & \BIC@Fi
                 884 & }%
\bigintcalcShr
                 885 \def\bigintcalcShr#1{%
                      \romannumeral0%
                      \expandafter\expandafter\BIC@Shr
                 887
                      \bigintcalcNum{#1}!%
                 888
                 889 }
      \BIC@Shr
                 890 \def\BIC@Shr#1#2!{%
                 891
                      \ifx#1-%
                 892
                        \expandafter-\romannumeral0%
                 893
                        \BIC@AfterFi{%
                          \BIC@@Shr#2!%
                 894
                        }%
                 895
                      \else
                 896
                        \BIC@AfterFi{%
                 897
                          \BIC@@Shr#1#2!%
                 898
                        }%
                 899
                      \BIC@Fi
                 900
                 901 }
\BigIntCalcShr
                 902 \def\BigIntCalcShr#1!{%
                 903 \romannumeral0%
                 904 \BIC@@Shr#1!%
                 905 }
```

```
\BIC@@Shr
             906 \def\BIC@@Shr#1#2!{%
                   \ifcase#1 %
                     \BIC@AfterFi{ 0}%
             909
                   \or
             910
                     \ifx\\#2\\%
                       \BIC@AfterFiFi{ 0}%
             911
                     \else
             912
                       \BIC@AfterFiFi{%
             913
                         \BIC@@@Shr#1#2!!%
             914
             915
             916
                     \fi
             917
                   \else
                     \BIC@AfterFi{%
             918
                       \BIC@@@Shr0#1#2!!%
             919
                     }%
             920
             921
                   \BIC@Fi
             922 }
\BIC@@@Shr
            #1: carry
             #2#3: number
             #4: result
             923 \def\BIC@@@Shr#1#2#3!#4!{%
             924
                   \ifx\\#3\\%
                     \ifodd#1#2 %
             925
             926
                       \BIC@AfterFiFi{%
             927 &
                          \expandafter\BIC@ShrResult\the\numexpr(#1#2-1)/2\relax
             928 $
                          \expandafter\expandafter\expandafter\BIC@ShrResult
                          \csname BIC@ShrDigit#1#2\endcsname
             929 $
                         #4!%
             930
                       }%
             931
             932
                     \else
                       \BIC@AfterFiFi{%
             933
                         \verb|\expandafter\BIC@ShrResult\the\numexpr#1#2/2\relax|
             934 &
             935 $
                         \expandafter\expandafter\expandafter\BIC@ShrResult
             936 $
                         \csname BIC@ShrDigit#1#2\endcsname
             937
                         #4!%
                       }%
             938
             939
                     \fi
             940
                   \else
                     \ifodd#1#2 %
             941
             942
                       \BIC@AfterFiFi{%
                          \expandafter\BIC@@@@Shr\the\numexpr(#1#2-1)/2\relax1%
             943 &
                          \expandafter\expandafter\BIC@@@Shr
             944 $
             945 $
                         \csname BIC@ShrDigit#1#2\endcsname
                         #3!#4!%
             946
             947
                       }%
                     \else
             948
             949
                       \BIC@AfterFiFi{%
             950 &
                         \ensuremath{\verb| LC00000Shr \rangle the \numexpr#1#2/2\relax0\%}
             951 $
                          \expandafter\expandafter\expandafter\BIC@@@@Shr
             952 $
                         \csname BIC@ShrDigit#1#2\endcsname
             953
                         #3!#4!%
                       }%
             954
             955
                     \fi
                   \BIC@Fi
             956
```

957 }

```
\BIC@ShrResult
                       958 & \def\BIC@ShrResult#1#2!{ #2#1}%
                       959 $ \def\BIC@ShrResult#1#2#3!{ #3#1}%
         \BIC@@@Shr #1: new digit
                       #2: carry
                       #3: remaining number
                       #4: result
                       960 \def\BIC@@@Shr#1#2#3!#4!{%
                       961 \BIC@@@Shr#2#3!#4#1!%
                       962 }
\BIC@ShrDigit[00-19]
                       963 $ \def\BIC@Temp#1#2#3#4{%
                       964 $ \expandafter\def\csname BIC@ShrDigit#1#2\endcsname{#3#4}%
                       965 $ }%
                       966 $ \BIC@Temp 0000%
                       967 $ \BIC@Temp 0101%
                       968 $ \BIC@Temp 0210%
                       969 $ \BIC@Temp 0311%
                       970 $ \BIC@Temp 0420%
                       971 $ \BIC@Temp 0521%
                       972 $ \BIC@Temp 0630%
                       973 $ \BIC@Temp 0731%
                       974 $ \BIC@Temp 0840%
                       975 $ \BIC@Temp 0941%
                       976 $ \BIC@Temp 1050%
                       977 $ \BIC@Temp 1151%
                       978 $ \BIC@Temp 1260%
                       979 $ \BIC@Temp 1361%
                       980 $ \BIC@Temp 1470%
                       981 $ \BIC@Temp 1571%
                       982 $ \BIC@Temp 1680%
                       983 $ \BIC@Temp 1781%
                       984 $ \BIC@Temp 1890%
                       985 $ \BIC@Temp 1991%
                       2.14
                               \BIC@Tim
            \BIC@Tim Macro \BIC@Tim implements "Number times digit".
                       #1: plain number without sign
                       #2: digit
           \BIC@@Tim #1#2: number
                       #3: reverted number
                       986 \def\BIC@@Tim#1#2!{%
                       987
                            \ifx\\#2\\%
                               \BIC@AfterFi{%
                       988
                                 \BIC@ProcessTim0!#1%
                       989
                               }%
                       990
                       991
                            \else
                               \BIC@AfterFi{%
                       992
                       993
                                 \BIC@@Tim#2!#1%
                               }%
                       994
                            \BIC@Fi
                       995
                       996 }
```

```
\BIC@ProcessTim #1: carry
                  #2: result
                  #3#4: reverted number
                  #5: digit
                   997 \def\BIC@ProcessTim#1#2!#3#4!#5{%
                        \ifx\\#4\\%
                   998
                           \BIC@AfterFi{%
                   999
                             \expandafter\BIC@Space
                   1000
                   1001 &
                             \theta = 14.4
                  1002 $
                             \romannumeral0\BIC@TimDigit#3#5#1%
                  1003
                             #2%
                           }%
                  1004
                         \else
                  1005
                           \BIC@AfterFi{%
                  1006
                             \expandafter\BIC@@ProcessTim
                  1007
                  1008 &
                             \theta^3*#5+#1%
                  1009 $
                             \romannumeral0\BIC@TimDigit#3#5#1%
                             !#2!#4!#5%
                  1010
                  1011
                           }%
                        \BIC@Fi
                  1012
                  1013 }
\BIC@@ProcessTim #1#2: carry?, new digit
                  #3: new number
                  #4: old number
                  #5: digit
                  1014 \def\BIC@@ProcessTim#1#2!{%
                  1015
                        \ifx\\#2\\%
                           \BIC@AfterFi{%
                  1016
                  1017
                             \BIC@ProcessTimO#1%
                           }%
                  1018
                         \else
                  1019
                           \BIC@AfterFi{%
                  1020
                             \BIC@ProcessTim#1#2%
                  1021
                           }%
                  1022
                        \BIC@Fi
                  1023
                  1024 }
   \BIC@TimDigit #1: digit 0-9
                  #2: digit 3-9
                  #3: carry 0-9
                  1025 $ \def\BIC@TimDigit#1#2#3{%
                          \ifcase#1 % 0
                  1026 $
                             \BIC@AfterFi{ #3}%
                  1027 $
                  1028 $
                           \or % 1
                             \BIC@AfterFi{%
                  1029 $
                  1030 $
                               \expandafter\BIC@Space
                               \number\csname BIC@AddCarry#2\endcsname#3 %
                  1031 $
                  1032 $
                             }%
                           \else
                  1033 $
                             \ifcase#3 %
                  1034 $
                               \BIC@AfterFiFi{%
                  1035 $
                  1036 $
                                 \expandafter\BIC@Space
                  1037 $
                                 \number\csname BIC@MulDigit#2\endcsname#1 %
                               }%
                  1038 $
                             \else
                  1039 $
                               \BIC@AfterFiFi{%
                  1040 $
```

```
1041 $
                                   \expandafter\BIC@Space
                    1042 $
                                   \romannumeral0%
                    1043 $
                                   \expandafter\BIC@AddXY
                                   \number\csname BIC@MulDigit#2\endcsname#1!%
                    1044 $
                    1045 $
                                   #3!!!%
                    1046 $
                                }%
                    1047 $
                              \fi
                           \BIC@Fi
                    1048 $
                    1049 $ }%
\BIC@MulDigit[3-9]
                    1050 $ \def\BIC@Temp#1#2{%
                           \expandafter\def\csname BIC@MulDigit#1\endcsname##1{%
                    1052 $
                              \ifcase##1 0%
                    1053 $
                              \or ##1%
                    1054 $
                              \or #2%
                    1055 $?
                              \else\BigIntCalcError:ThisCannotHappen%
                    1056 $
                              \fi
                    1057 $ }%
                    1058 $ }%
                    1059 $ \BIC@Temp 3{6\or9\or12\or15\or18\or21\or24\or27}%
                    1060 $ \BIC@Temp 4{8\or12\or16\or20\or24\or28\or32\or36}%
                    1061 $ \BIC@Temp 5{10\or15\or20\or25\or30\or35\or40\or45}%
                    1062 $ \BIC@Temp 6{12\or18\or24\or30\or36\or42\or48\or54}%
                    1063 $ \BIC@Temp 7{14\or21\or28\or35\or42\or49\or56\or63}%
                    1064 $ \BIC@Temp 8{16\or24\or32\or40\or48\or56\or64\or72}%
                    1065 $ \BIC@Temp 9{18\or27\or36\or45\or54\or63\or72\or81}%
                    2.15
                            Mul
   \bigintcalcMul
                    1066 \def\bigintcalcMul#1#2{%
                          \romannumeral0%
```

```
\expandafter\expandafter\BIC@Mul
1068
1069 \bigintcalcNum{#1}!{#2}%
1070 }
```

#### \BIC@Mul

```
1071 \def\BIC@Mul#1!#2{%
     \expandafter\expandafter\expandafter\BIC@MulSwitch
      \bigintcalcNum{#2}!#1!%
1074 }
```

\BIC@MulSwitch Decision table for \BIC@MulSwitch.

x = 0				0
x > 0	y = 0			0
	y > 0	x > y	+	Mul(x, y)
		else		Mul(y, x)
	y < 0	x > -y	_	Mul(x, -y)
		else		Mul(-y,x)
x < 0	y = 0			0
	y > 0	-x > y	_	Mul(-x,y)
		else		Mul(y, -x)
	y < 0	-x > -y	+	Mul(-x, -y)
		else		Mul(-y, -x)

```
1075 \def\BIC@MulSwitch#1#2!#3#4!{%
      \ightharpoonup \ \ifcase\BIC@Sgn#1#2! % x = 0
1076
1077
        \BIC@AfterFi{ 0}%
1078
      1079
        \ightharpoonup \ \ifcase\BIC@Sgn#3#4! % y = 0
          \BIC@AfterFiFi{ 0}%
1080
        1081
          \mbox{ifnum}BIC@PosCmp#1#2!#3#4!=1 % x > y
1082
            \BIC@AfterFiFiFi{%
1083
              \BIC@ProcessMul0!#1#2!#3#4!%
1084
1085
            }%
          \else % x <= y
1086
1087
            \BIC@AfterFiFiFi{%
              \BIC@ProcessMul0!#3#4!#1#2!%
1088
            }%
1089
          \fi
1090
        \else % y < 0
1091
1092
          \expandafter-\romannumeral0%
          \label{lower_problem} $$ \prod_BIC@PosCmp#1#2!#4!=1 % x > -y $$
1093
            \BIC@AfterFiFiFi{%
1094
              \BIC@ProcessMul0!#1#2!#4!%
1095
            }%
1096
          \else % x <= -y
1097
1098
            \BIC@AfterFiFiFi{%
              \BIC@ProcessMul0!#4!#1#2!%
1099
1100
            }%
1101
          \fi
        \fi
1102
      \else % x < 0
1103
        \ightharpoonup \ \ifcase\BIC@Sgn#3#4! % y = 0
1104
1105
          \BIC@AfterFiFi{ 0}%
        1106
          \expandafter-\romannumeral0%
1107
          1108
            \BIC@AfterFiFiFi{%
1109
              \BIC@ProcessMul0!#2!#3#4!%
1110
1111
            }%
          \else % -x <= y
1112
1113
            \BIC@AfterFiFiFi{%
1114
              \BIC@ProcessMul0!#3#4!#2!%
            }%
1115
          \fi
1116
        \else % y < 0
1117
          1118
            \BIC@AfterFiFiFi{%
1119
1120
              \BIC@ProcessMul0!#2!#4!%
            }%
1121
1122
          \else % -x <= -y
            \BIC@AfterFiFiFi{%
1123
              \BIC@ProcessMul0!#4!#2!%
1124
            }%
1125
1126
          \fi
1127
        \fi
1128
      \BIC@Fi
1129 }
```

\BigIntCalcMul

1130 \def\BigIntCalcMul#1!#2!{%

```
\romannumeral0%
                1131
                      \BIC@ProcessMul0!#1!#2!%
                1132
                1133 }
\BIC@ProcessMul #1: result
                #2: number x
                #3#4: number y
                1134 \def\BIC@ProcessMul#1!#2!#3#4!{%
                1135
                      \ifx\\#4\\%
                1136
                        \BIC@AfterFi{%
                1137
                          \expandafter\expandafter\BIC@Space
                1138
                          \bigintcalcAdd{\BIC@Tim#2!#3}{#10}%
                1139
                        }%
                1140
                      \else
                1141
                        \BIC@AfterFi{%
                1142
                          \expandafter\expandafter\expandafter\BIC@ProcessMul
                1143
                          \bigintcalcAdd{\BIC@Tim#2!#3}{#10}!#2!#4!%
                        }%
                1144
                      \BIC@Fi
                1145
                1146 }
                2.16
                        Sqr
\bigintcalcSqr
                1147 \def\bigintcalcSqr#1{%
                1148 \romannumeral0%
                      \expandafter\expandafter\BIC@Sqr
                1149
                      \bigintcalcNum{#1}!%
                1150
                1151 }
      \BIC@Sqr
                1152 \def\BIC@Sqr#1{%
                1153
                       \ifx#1-%
                1154
                         \expandafter\BIC@@Sqr
                1155
                        \else
                1156
                         \expandafter\BIC@@Sqr\expandafter#1%
                1157
                       \fi
                1158 }
     \BIC@@Sqr
                1159 \def\BIC@@Sqr#1!{%
                1160 \BIC@ProcessMul0!#1!#1!%
                1161 }
                2.17
                         Fac
\bigintcalcFac
                1162 \def\bigintcalcFac#1{%
                1163 \romannumeral0%
                      \expandafter\expandafter\BIC@Fac
                1165
                      \bigintcalcNum{#1}!%
                1166 }
      \BIC@Fac
                1167 \def\BIC@Fac#1#2!{%
                1168
                      \ifx#1-%
                        \BIC@AfterFi{ 0\BigIntCalcError:FacNegative}%
                1169
```

```
1170
                        \else
                          \ifnum\BIC@PosCmp#1#2!13!<0 %
                  1171
                  1172
                             \ifcase#1#2 %
                                \BIC@AfterFiFiFi{ 1}% 0!
                  1173
                  1174
                             \or\BIC@AfterFiFiFi{ 1}% 1!
                             \or\BIC@AfterFiFiFi{ 2}% 2!
                  1175
                             \or\BIC@AfterFiFiFi{ 6}% 3!
                  1176
                             \or\BIC@AfterFiFiFi{ 24}% 4!
                  1177
                             \or\BIC@AfterFiFiFi{ 120}% 5!
                  1178
                             \or\BIC@AfterFiFiFi{ 720}% 6!
                  1179
                             \or\BIC@AfterFiFiFi{ 5040}% 7!
                  1180
                             \or\BIC@AfterFiFiFi{ 40320}% 8!
                  1181
                  1182
                             \or\BIC@AfterFiFiFi{ 362880}% 9!
                             \or\BIC@AfterFiFiFi{ 3628800}% 10!
                  1183
                             \or\BIC@AfterFiFiFi{ 39916800}% 11!
                  1184
                  1185
                             \or\BIC@AfterFiFiFi{ 479001600}% 12!
                  1186 ?
                             \else\BigIntCalcError:ThisCannotHappen%
                  1187
                             \fi
                          \else
                  1188
                             \BIC@AfterFiFi{%
                  1189
                               \BIC@ProcessFac#1#2!479001600!%
                  1190
                             }%
                  1191
                          \fi
                  1192
                  1193
                        \BIC@Fi
                  1194 }
 \BIC@ProcessFac
                  #1: n
                  #2: result
                  1195 \def\BIC@ProcessFac#1!#2!{%
                        \ifnum\BIC@PosCmp#1!12!=0 %
                          \BIC@AfterFi{ #2}%
                  1197
                  1198
                        \else
                           \BIC@AfterFi{%
                  1199
                             \expandafter\BIC@@ProcessFac
                  1200
                             \romannumeral0\BIC@ProcessMul0!#2!#1!%
                  1201
                  1202
                             !#1!%
                          }%
                  1203
                  1204
                        \BIC@Fi
                  1205 }
\BIC@@ProcessFac
                  #1: result
                  #2: n
                  1206 \def\BIC@@ProcessFac#1!#2!{%
                        \expandafter\BIC@ProcessFac
                  1207
                  1208
                        \romannumeral0\BIC@Dec#2!{}%
                  1209
                        !#1!%
                  1210 }
                  2.18
                          Pow
 \bigintcalcPow
                  #1: basis
                  #2: power
                  1211 \def\bigintcalcPow#1{%
                        \romannumeral0%
                  1212
                  1213
                        \expandafter\expandafter\BIC@Pow
                  1214
                        \bigintcalcNum{#1}!%
                  1215 }
```

```
\BIC@Pow #1: basis
#2: power

1216 \def\BIC@Pow#1!#2{%

1217 \expandafter\expandafter\spandafter\BIC@PowSwitch

1218 \bigintcalcNum{#2}!#1!%

1219 }

\BIC@PowSwitch #1#2: power y
#3#4: basis x

Decision table for \BIC@PowSwitch.
```

y = 0			1
y=1			x
y=2	x < 0		Mul(-x, -x)
	else		Mul(x,x)
y < 0	x = 0		DivisionByZero
	x = 1		1
	x = -1	ifodd(y)	-1
		else	1
	else $( x  > 1)$		0
y > 2	x = 0		0
	x = 1		1
	x = -1	ifodd(y)	-1
		else	1
	$x < -1 \ (x < 0)$	ifodd(y)	$-\operatorname{Pow}(-x,y)$
		else	Pow(-x,y)

 $\overline{\text{Pow}}(x,y)$ 

```
1220 \def\BIC@PowSwitch#1#2!#3#4!{%
1221 \ifcase\ifx\\#2\\%
                \int x = 100 \% y = 0
1222
                \left( x = 111 \% y = 1 \right)
1223
                \left| \frac{y}{2} \right|
1224
1225
                \left( y > 2 \right)
1226
                \fi\fi\fi
1227
              \else
                \int x^{1-3} y < 0
1228
                \else4 % y > 2
1229
1230
                \fi
              \fi
1231
1232
      BIC@AfterFi{ 1}% y = 0
     1233
       \BIC@AfterFi{ #3#4}%
1234
      \ \ y = 2
1235
        \int x^{3}-x < 0
1236
          \BIC@AfterFiFi{%
1237
             \BIC@ProcessMul0!#4!#4!%
1238
1239
          }%
1240
        \else % x >= 0
          \BIC@AfterFiFi{%
1241
            \BIC@ProcessMul0!#3#4!#3#4!%
1242
          }%
1243
        \fi
1244
      \or \% y < 0
1245
        \int \frac{1}{2} 
1246
                  \int ifx#300 \% x = 0
1247
```

else (x > 1)

```
\left( x = 1 \right)
1248
                \else3 % x > 1
1249
                fi\fi
1250
1251
              \else
                \ifcase\BIC@MinusOne#3#4! %
1252
                  3 \% |x| > 1
1253
1254
                \or
                  2 \% x = -1
1255
                \else\BigIntCalcError:ThisCannotHappen%
1256 ?
1257
                \fi
1258
              \fi
         \BIC@AfterFiFi{ 0\BigIntCalcError:DivisionByZero}% x = 0
1259
1260
       1261
         BIC@AfterFiFi{ 1}% x = 1
       1262
         \ifcase\BIC@ModTwo#2! % even(y)
1263
1264
           \BIC@AfterFiFiFi{ 1}%
         \or % odd(y)
1265
           \BIC@AfterFiFiFi{ -1}%
1266
1267 ?
         \else\BigIntCalcError:ThisCannotHappen%
1268
       1269
         \BIC@AfterFiFi{ 0}%
1270
1271 ?
       \else\BigIntCalcError:ThisCannotHappen%
1272
1273
     1274
       \int \frac{1}{x}
                1275
                \left( x = 1 \right)
1276
1277
                \else4 % x > 1
1278
                \fi\fi
              \else
1279
                \int x#3-%
1280
                  \ifcase\BIC@MinusOne#3#4! %
1281
                    3 % x < -1
1282
                  \else
1283
                    2 \% x = -1
                  \fi
1285
1286
                \else
1287
                  4 % x > 1
                \fi
1288
              \fi
1289
         BIC@AfterFiFi{ 0}% x = 0
1290
1291
       \BIC@AfterFiFi{ 1}% x = 1
1292
       1293
         \ifcase\BIC@ModTwo#1#2! % even(y)
1294
           \BIC@AfterFiFiFi{ 1}%
1295
1296
         \or % odd(y)
           \BIC@AfterFiFiFi{ -1}%
1297
1298 ?
         \else\BigIntCalcError:ThisCannotHappen%
1299
         \fi
1300
       1301
         \ifcase\BIC@ModTwo#1#2! % even(y)
           \BIC@AfterFiFiFi{%
1302
1303
             \BIC@PowRec#4!#1#2!1!%
1304
           }%
1305
         \or % odd(y)
```

```
1306
            \expandafter-\romannumeral0%
            \BIC@AfterFiFiFi{%
1307
              \BIC@PowRec#4!#1#2!1!%
1308
            }%
1309
1310 ?
          \else\BigIntCalcError:ThisCannotHappen%
          \fi
1311
        \or % x > 1
1312
          \BIC@AfterFiFi{%
1313
            \BIC@PowRec#3#4!#1#2!1!%
1314
1315
1316 ?
        \else\BigIntCalcError:ThisCannotHappen%
1317
1318 ? \else\BigIntCalcError:ThisCannotHappen%
1319
      \BIC@Fi
1320 }
```

### 2.18.1 Help macros

\BIC@ModTwo

Macro  $\BICQModTwo$  expects a number without sign and returns digit 1 or 0 if the number is odd or even.

```
1321 \def\BIC@ModTwo#1#2!{%
1322
      \ifx\\#2\\%
        \ifodd#1 %
1323
          \BIC@AfterFiFi1%
1324
1325
        \else
1326
          \BIC@AfterFiFi0%
1327
        \fi
1328
      \else
1329
        \BIC@AfterFi{%
1330
          \BIC@ModTwo#2!%
        }%
1331
      \BIC@Fi
1332
1333 }
```

\BIC@MinusOne

Macro \BIC@MinusOne expects a number and returns digit 1 if the number equals minus one and returns 0 otherwise.

```
1334 \def\BIC@MinusOne#1#2!{%

1335 \ifx#1-%

1336 \BIC@@MinusOne#2!%

1337 \else

1338 O%

1339 \fi

1340 }
```

\BIC@@MinusOne

```
1341 \def\BIC@@MinusOne#1#2!{%
1342
      \ifx#11%
1343
        \ifx\\#2\\%
           1%
1344
        \else
1345
           0%
1346
        \fi
1347
      \else
1348
        0%
      \fi
1350
1351 }
```

#### 2.18.2 Recursive calculation

```
Pow(x, y) {
\BIC@PowRec
                     PowRec(x, y, 1)
                   PowRec(x, y, r) {
                     if y == 1 then
                       return r
                     else
                       ifodd y then
                         return PowRec(x*x, y div 2, r*x) % y div 2 = (y-1)/2
                         return PowRec(x*x, y div 2, r)
                       fi
                     fi
                   }
                 #1: x (basis)
              #2#3: y (power)
              #4: r (result)
              1352 \def\BIC@PowRec#1!#2#3!#4!{%
                   \infty BIC@PosCmp#1!#4!=1 % x > r
              1354
                       \BIC@AfterFiFi{%
              1355
                          \BIC@ProcessMul0!#1!#4!%
              1356
                       }%
              1357
              1358
                      \else
                       \BIC@AfterFiFi{%
              1359
              1360
                          \BIC@ProcessMul0!#4!#1!%
                       }%
              1361
                     \fi
              1362
              1363
                   \or
                     \ifcase\BIC@ModTwo#2#3! % even(y)
              1364
              1365
                       \BIC@AfterFiFi{%
              1366
                          \expandafter\BIC@@PowRec\romannumeral0%
                          \BIC@@Shr#2#3!%
              1367
                          !#1!#4!%
              1368
                       }%
              1369
                     \or % odd(y)
              1370
                       \infty BIC@PosCmp#1!#4!=1 % x > r
              1371
              1372
                         \BIC@AfterFiFiFi{%
              1373
                            \expandafter\BIC@@PowRec\romannumeral0%
                            \BIC@ProcessMul0!#1!#4!%
              1374
                           !#1!#2#3!%
              1375
                         }%
              1376
                       \else
              1377
              1378
                          \BIC@AfterFiFiFi{%
              1379
                           \expandafter\BIC@@PowRec\romannumeral0%
                           \BIC@ProcessMul0!#1!#4!%
              1380
                            !#1!#2#3!%
              1381
                         }%
              1382
                       \fi
              1383
                      \else\BigIntCalcError:ThisCannotHappen%
              1384 ?
              1385
              1386 ? \else\BigIntCalcError:ThisCannotHappen%
              1387 \BIC@Fi
             1388 }
\BIC@@PowRec #1: y/2
```

```
#2: x
                                                                               #3: new r (r \text{ or } r * x)
                                                                               1389 \def\BIC@@PowRec#1!#2!#3!{%
                                                                               1390 \expandafter\BIC@PowRec\romannumeral0%
                                                                                                           \BIC@ProcessMul0!#2!#2!%
                                                                               1391
                                                                               1392 !#1!#3!%
                                                                               1393 }
    \BIC@@PowRec #1: r*x #2: x #3: y
                                                                               1394 \def\BIC@@PowRec#1!#2!#3!{%
                                                                                                        \expandafter\BIC@@PowRec\romannumeral0%
                                                                                                        \BIC@@Shr#3!%
                                                                               1396
                                                                               1397 !#2!#1!%
                                                                               1398 }
                                                                               2.19
                                                                                                                     Div
\bigintcalcDiv #1: x
                                                                               #2: y (divisor)
                                                                               1399 \def\bigintcalcDiv#1{%
                                                                               1400 \romannumeral0%
                                                                                                      \expandafter\expandafter\BIC@Div
                                                                               1402 \bigintcalcNum{#1}!%
                                                                              1403 }
                             \BIC@Div #1: x
                                                                              #2: y
                                                                               1404 \def\BIC@Div#1!#2{%
                                                                               1405 \qquad \texttt{\expandafter} \\ \texttt{\
                                                                               1406 \bigintcalcNum{#2}!#1!%
                                                                               1407 }
\BigIntCalcDiv
                                                                               1408 \def\BigIntCalcDiv#1!#2!{%
                                                                               1409 \romannumeral0%
                                                                               1410 \BIC@DivSwitchSign#2!#1!%
                                                                              1411 }
```

\BIC@DivSwitchSign Decision table for \BIC@DivSwitchSign.

y = 0		DivisionByZero
y > 0	x = 0	0
	x > 0	DivSwitch(+, x, y)
	x < 0	DivSwitch(-, -x, y)
y < 0	x = 0	0
	x > 0	DivSwitch(-, x, -y)
	x < 0	DivSwitch(+, -x, -y)

```
#1: y (divisor)
#2: x

1412 \def\BIC@DivSwitchSign#1#2!#3#4!{%

1413 \ifcase\BIC@Sgn#1#2! % y = 0

1414 \BIC@AfterFi{ O\BigIntCalcError:DivisionByZero}%

1415 \or % y > 0

1416 \ifcase\BIC@Sgn#3#4! % x = 0

1417 \BIC@AfterFif{ 0}%
```

```
1418
          \BIC@AfterFiFi{%
1419
            \BIC@DivSwitch{}#3#4!#1#2!%
1420
          }%
1421
        \else % x < 0
1422
          \BIC@AfterFiFi{%
1423
            \BIC@DivSwitch-#4!#1#2!%
1424
          }%
1425
1426
        \fi
      \else % y < 0
1427
        \ightharpoonup \ \ifcase\BIC@Sgn#3#4! % x = 0
1428
1429
          \BIC@AfterFiFi{ 0}%
1430
        \BIC@AfterFiFi{%
1431
            \BIC@DivSwitch-#3#4!#2!%
1432
          }%
1433
        \else % x < 0
1434
1435
          \BIC@AfterFiFi{%
            \BIC@DivSwitch{}#4!#2!%
1436
          }%
1437
1438
        \fi
      \BIC@Fi
1439
1440 }
```

\BIC@DivSwitch Decision table for \BIC@DivSwitch.

y = x	sign 1	
y > x	0	
y < x	y = 1	sign x
	y=2	sign Shr(x)
	y=4	sign Shr(Shr(x))
	else	sign ProcessDiv $(x, y)$

```
#1: sign
#2: x
#3#4: y (y \neq 0)
1441 \def\BIC@DivSwitch#1#2!#3#4!{%
                            \infty = \infty = x
1442
1443
                                          \BIC@AfterFi{ #11}%
1444
                                \or % y > x
                                         \BIC@AfterFi{ 0}%
1445
                                \ensuremath{\mbox{\sc lse}}\ \ensuremath{\mbox{\sc y}}\ \ensuremath{\mbox{\sc y}}\ \ensuremath{\mbox{\sc x}}\ \ensuremath{\mbox{\sc x}}\ \ensuremath{\mbox{\sc y}}\ \ensuremath{\mbox{\sc y}}\ \ensuremath{\mbox{\sc x}}\ \ensuremath{\mbox{\sc y}}\ \ensuremath{\mbox
1446
                                          \ifx\\#1\\%
1447
                                           \else
1448
                                                      \expandafter-\romannumeral0%
1449
                                           \fi
1450
1451
                                           \ifcase ifx \#4\%
                                                                                            \int x#310 \% y = 1
1452
1453
                                                                                            \left| \frac{321 \% y = 2}{} \right|
                                                                                            \ensuremath{$\ $$ \ensuremath{$\ $$}$} = 4
1454
                                                                                            \ensuremath{\verb|\ensuremath{|}} 4 > 2
1455
                                                                                            fi\fi\fi
1456
1457
                                                                                  \else
                                                                                           3 % y > 2
1458
                                                                                \fi
1459
                                                      \BIC@AfterFiFi{ #2}% y = 1
1460
                                          1461
                                                      \BIC@AfterFiFi{%
1462
```

```
\BIC@@Shr#2!%
                                                     1463
                                                                                   }%
                                                     1464
                                                                             1465
                                                                                   \BIC@AfterFiFi{%
                                                     1466
                                                                                         \expandafter\BIC@@Shr\romannumeral0%
                                                     1467
                                                                                               \BIC@@Shr#2!!%
                                                     1468
                                                                                   }%
                                                     1469
                                                                             \or % y > 2
                                                     1470
                                                                                   \BIC@AfterFiFi{%
                                                     1471
                                                                                         \BIC@DivStartX#2!#3#4!!!%
                                                     1472
                                                     1473
                                                     1474 ?
                                                                             \else\BigIntCalcError:ThisCannotHappen%
                                                     1475
                                                                      \BIC@Fi
                                                     1476
                                                     1477 }
   \BIC@ProcessDiv #1#2: x
                                                     #3#4: y
                                                     #5: collect first digits of x
                                                     #6: corresponding digits of y
                                                     1478 \def\BIC@DivStartX#1#2!#3#4!#5!#6!{%
                                                                     \ifx\\#4\\%
                                                     1479
                                                                             \BIC@AfterFi{%
                                                     1480
                                                     1481
                                                                                   \BIC@DivStartYii#6#3#4!{#5#1}#2=!%
                                                                             }%
                                                     1482
                                                                       \else
                                                     1483
                                                                            \BIC@AfterFi{%
                                                     1484
                                                                                   \BIC@DivStartX#2!#4!#5#1!#6#3!%
                                                     1485
                                                                            }%
                                                     1486
                                                                     \BIC@Fi
                                                     1487
                                                     1488 }
\BIC@DivStartYii #1: y
                                                     #2: x, =
                                                     1489 \def\BIC@DivStartYii#1!{%
                                                     1490 \expandafter\BIC@DivStartYiv\romannumeral0%
                                                     1491 \BIC@Shl#1!%
                                                     1492
                                                                     !#1!%
                                                     1493 }
\BIC@DivStartYiv #1: 2y
                                                     #2: y
                                                     #3: x, =
                                                     1494 \ensuremath{ \mbox{\sc 1494} \mbox{\sc 1494}} \ensuremath{ \m
                                                     1495 \expandafter\BIC@DivStartYvi\romannumeral0%
                                                     1496 \BIC@Shl#1!%
                                                     1497 !#1!%
                                                     1498 }
\BIC@DivStartYvi #1: 4y
                                                     #2: 2y
                                                     #3: y
                                                     #4: x, =
                                                     1499 \def\BIC@DivStartYvi#1!#2!{%
                                                     1500 \expandafter\BIC@DivStartYviii\romannumeral0%
                                                                   \BIC@AddXY#1!#2!!!%
                                                     1501
                                                     1502 !#1!#2!%
                                                     1503 }
```

```
\BIC@DivStartYviii #1: 6y
                    #2: 4y
                    #3: 2y
                    #4: y
                    #5: x, =
                    1504 \def\BIC@DivStartYviii#1!#2!{%
                    1505 \expandafter\BIC@DivStart\romannumeral0%
                    1506 \BIC@Sh1#2!%
                    1507 !#1!#2!%
                    1508 }
    \BIC@DivStart #1: 8y
                    #2: 6y
                    #3: 4y
                    #4: 2y
                    #5: y
                    #6: x, =
                    1509 \def\BIC@DivStart#1!#2!#3!#4!#5!#6!{%
                    1510 \BIC@ProcessDiv#6!!#5!#4!#3!#2!#1!=%
                    1511 }
  \BIC@ProcessDiv #1#2#3: x, =
                    #4: result
                    #5: y
                    #6: 2y
                    #7: 4y
                    #8: 6y
                    #9: 8y
                    1512 \def\BIC@ProcessDiv#1#2#3!#4!#5!{%
                    1513 \ifcase\BIC@PosCmp#5!#1!% y = #1
                            \ifx#2=%
                    1514
                              \BIC@AfterFiFi{\BIC@DivCleanup{#41}}%
                    1515
                    1516
                               \BIC@AfterFiFi{%
                    1517
                                 \BIC@ProcessDiv#2#3!#41!#5!%
                    1518
                              }%
                    1519
                           \fi
                    1520
                          \or % y > #1
                    1521
                    1522
                            \ifx#2=%
                               \BIC@AfterFiFi{\BIC@DivCleanup{#40}}%
                    1523
                    1524
                            \else
                              \ifx\\#4\\%
                    1525
                                 \BIC@AfterFiFiFi{%
                    1526
                                   \BIC@ProcessDiv{#1#2}#3!!#5!%
                    1527
                                }%
                    1528
                    1529
                               \else
                                 \BIC@AfterFiFiFi{%
                    1530
                                   \BIC@ProcessDiv{#1#2}#3!#40!#5!%
                    1531
                    1532
                                }%
                              \fi
                    1533
                            \fi
                    1534
                          \else % y < #1
                    1535
                            \BIC@AfterFi{%
                    1536
                               \BIC@@ProcessDiv{#1}#2#3!#4!#5!%
                    1537
                            }%
                    1538
                          \BIC@Fi
                    1539
                    1540 }
```

```
\BIC@DivCleanup
                   #1: result
                    #2: garbage
                    1541 \def\BIC@DivCleanup#1#2={ #1}%
 \BIC@@ProcessDiv
                    1542 \def\BIC@@ProcessDiv#1#2#3!#4!#5!#6!#7!{%
                         \ifcase\BIC@PosCmp#7!#1!% 4y = #1
                            \ifx#2=%
                    1544
                              \BIC@AfterFiFi{\BIC@DivCleanup{#44}}%
                    1545
                    1546
                            \else
                             \BIC@AfterFiFi{%
                    1547
                               \BIC@ProcessDiv#2#3!#44!#5!#6!#7!%
                    1548
                             }%
                    1549
                           \fi
                    1550
                          \or % 4y > #1
                    1551
                            \ifcase\BIC@PosCmp#6!#1!% 2y = #1
                    1552
                    1553
                              \ifx#2=%
                    1554
                                \BIC@AfterFiFiFi{\BIC@DivCleanup{#42}}%
                    1555
                                \BIC@AfterFiFiFi{%
                    1556
                                  \BIC@ProcessDiv#2#3!#42!#5!#6!#7!%
                    1557
                                }%
                    1558
                              \fi
                    1559
                            \or % 2y > #1
                    1560
                    1561
                              \ifx#2=%
                                \BIC@AfterFiFiFi{\BIC@DivCleanup{#41}}%
                    1562
                    1563
                                \BIC@AfterFiFiFi{%
                    1564
                                  \BIC@DivSub#1!#5!#2#3!#41!#5!#6!#7!%
                    1565
                                }%
                    1566
                    1567
                              \fi
                    1568
                            \else % 2y < #1
                              \BIC@AfterFiFi{%
                    1569
                                \expandafter\BIC@ProcessDivII\romannumeral0%
                    1570
                                \BIC@SubXY#1!#6!!!%
                    1571
                                !#2#3!#4!#5!23%
                    1572
                                #6!#7!%
                    1573
                    1574
                              }%
                            \fi
                    1575
                    1576
                          \else % 4y < #1
                            \BIC@AfterFi{%
                    1577
                              \BIC@@@ProcessDiv{#1}#2#3!#4!#5!#6!#7!%
                    1578
                            }%
                    1579
                    1580
                          \BIC@Fi
                    1581 }
      \BIC@DivSub Next token group: #1-#2 and next digit #3.
                    1582 \def\BIC@DivSub#1!#2!#3{%
                          \expandafter\BIC@ProcessDiv\expandafter{%
                    1584
                            \romannumeral0%
                            \BIC@SubXY#1!#2!!!%
                    1585
                            #3%
                    1586
                    1587 }%
                    1588 }
\BIC@ProcessDivII #1: x' - 2y
                   #2#3: remaining x, =
                    #4: result
```

```
#5: y
                    #6: first possible result digit
                    #7: second possible result digit
                    1589 \def\BIC@ProcessDivII#1!#2#3!#4!#5!#6#7{%
                          \ifcase\BIC@PosCmp#5!#1!% y = #1
                    1591
                            \ifx#2=%
                              \BIC@AfterFiFi{\BIC@DivCleanup{#4#7}}%
                    1592
                            \else
                    1593
                              \BIC@AfterFiFi{%
                    1594
                                \BIC@ProcessDiv#2#3!#4#7!#5!%
                    1595
                              }%
                    1596
                    1597
                            \fi
                          \or % y > #1
                    1598
                            \ifx#2=%
                    1599
                              \BIC@AfterFiFi{\BIC@DivCleanup{#4#6}}%
                    1600
                            \else
                    1601
                              \BIC@AfterFiFi{%
                    1602
                    1603
                                 \BIC@ProcessDiv{#1#2}#3!#4#6!#5!%
                    1604
                    1605
                            \fi
                          \else % y < #1
                    1606
                            \ifx#2=%
                    1607
                              \BIC@AfterFiFi{\BIC@DivCleanup{#4#7}}%
                    1608
                    1609
                            \else
                    1610
                              \BIC@AfterFiFi{%
                                 \BIC@DivSub#1!#5!#2#3!#4#7!#5!%
                    1611
                    1612
                              }%
                            \fi
                    1613
                          \BIC@Fi
                    1614
                    1615 }
\BIC@ProcessDivIV #1#2#3: x, =, x > 4y
                    #4: result
                    #5: y
                    #6: 2y
                    #7: 4y
                    #8: 6y
                    #9: 8y
                    1616 \def\BIC@@@ProcessDiv#1#2#3!#4!#5!#6!#7!#8!#9!{%
                         \ifcase\BIC@PosCmp#8!#1!% 6y = #1
                    1617
                            \ifx#2=%
                    1618
                              \BIC@AfterFiFi{\BIC@DivCleanup{#46}}%
                    1619
                            \else
                    1620
                    1621
                              \BIC@AfterFiFi{%
                    1622
                                \BIC@ProcessDiv#2#3!#46!#5!#6!#7!#8!#9!%
                    1623
                              }%
                            \fi
                    1624
                          \or % 6y > #1
                    1625
                    1626
                            \BIC@AfterFi{%
                              \expandafter\BIC@ProcessDivII\romannumeral0%
                    1627
                    1628
                              \BIC@SubXY#1!#7!!!%
                    1629
                              !#2#3!#4!#5!45%
                              #6!#7!#8!#9!%
                    1630
                    1631
                            }%
                          \else % 6y < #1
                    1632
                            \ifcase\BIC@PosCmp#9!#1!% 8y = #1
                    1633
                    1634
                              \ifx#2=%
```

```
\else
                 1636
                 1637
                             \BIC@AfterFiFiFi{%
                               \BIC@ProcessDiv#2#3!#48!#5!#6!#7!#8!#9!%
                 1638
                             }%
                 1639
                           \fi
                 1640
                         \or % 8y > #1
                 1641
                           \verb|\BIC@AfterFiFi{||}|
                 1642
                             \expandafter\BIC@ProcessDivII\romannumeral0%
                 1643
                             \BIC@SubXY#1!#8!!!%
                 1644
                 1645
                             !#2#3!#4!#5!67%
                             #6!#7!#8!#9!%
                 1646
                 1647
                           }%
                 1648
                         \else % 8y < #1
                           \BIC@AfterFiFi{%
                 1649
                 1650
                             \expandafter\BIC@ProcessDivII\romannumeral0%
                 1651
                             \BIC@SubXY#1!#9!!!%
                 1652
                             !#2#3!#4!#5!89%
                             #6!#7!#8!#9!%
                 1653
                           }%
                 1654
                         \fi
                 1655
                       \BIC@Fi
                 1656
                 1657 }
                 2.20
                         Mod
\bigintcalcMod #1: x
                 #2: y
                 1658 \def\bigintcalcMod#1{%
                       \romannumeral0%
                 1659
                       \verb|\expandafter| expandafter| BIC@Mod|
                 1660
                       \bigintcalcNum{#1}!%
                 1661
                 1662 }
      \BIC@Mod #1: x
                 #2: y
                 1663 \def\BIC@Mod#1!#2{%
                       \expandafter\expandafter\expandafter\BIC@ModSwitchSign
                 1665
                       \bigintcalcNum{#2}!#1!%
                 1666 }
\BigIntCalcMod
                 1667 \def\BigIntCalcMod#1!#2!{%
                 1668
                      \romannumeral0%
                       \BIC@ModSwitchSign#2!#1!%
                 1669
                 1670 }
```

\BIC@AfterFiFiFi{\BIC@DivCleanup{#48}}%

\BIC@ModSwitchSign Decision table for \BIC@ModSwitchSign.

1635

y = 0	DivisionByZero	
y > 0	x = 0	0
	else	ModSwitch(+, x, y)
y < 0		ModSwitch(-, -x, -y)

```
#1#2: y
#3#4: x
```

 $1671 \ensuremath{\mbox{\mbox{$1$}}} 1671 \ensuremath{\mbox{\mbox{$4$}}} 182!#3#4!{\%}$ 

```
\ifcase\ifx\\#2\%
1672
                                                           \int x#100 \% y = 0
1673
                                                            \else1 % y > 0
1674
1675
                                                            \fi
1676
                                                        \else
                                                               \int x^{1-2} y < 0
1677
                                                               \else1 % y > 0
1678
                                                               \fi
1679
                                                       \fi
1680
                               \BIC@AfterFi{ O\BigIntCalcError:DivisionByZero}%
1681
1682
                               \left(\frac{4}{\pi}300 \right) = 1 \left(\frac{\pi}{\pi} x = 0\right)
1683
1684
                                       \BIC@AfterFiFi{ 0}%
1685
                               \else
                                       \BIC@AfterFiFi{%
1686
                                               \BIC@ModSwitch{}#3#4!#1#2!%
1687
                                       }%
1688
1689
                               \fi
                       \else % y < 0
1690
                               1691
                                                                   \int x#300 % x = 0
1692
                                                                   \ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath}\ensuremath{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ens
1693
                                                                   \fi
1694
1695
                                                            \else
                                                                   ifx#3-2 % x < 0
1696
1697
                                                                   \else1 % x > 0
1698
                                                                   \fi
                                                           \fi
1699
                                       \BIC@AfterFiFi{ 0}%
1700
                               1701
1702
                                       \BIC@AfterFiFi{%
1703
                                               \BIC@ModSwitch--#3#4!#2!%
1704
                                       }%
                               \else % x < 0
1705
                                       \BIC@AfterFiFi{%
1706
                                               \BIC@ModSwitch-#4!#2!%
1707
1708
                                       }%
1709
                               \fi
1710
                       \BIC@Fi
1711 }
```

\BIC@ModSwitch Decision table for \BIC@ModSwitch.

y=1		0	
y=2	ifodd(x)	sign 1	
	else	0	
y > 2	x < 0	$z \leftarrow x - (x/y) * y;$	(z < 0) ? z + y : z
	x > 0	x - (x/y) * y	

```
#1: sign
#2#3: x
#4#5: y

1712 \def\BIC@ModSwitch#1#2#3!#4#5!{%
1713 \ifcase\ifx\\#5\\%
1714 \ifx#410 % y = 1
1715 \else\ifx#421 % y = 2
1716 \else2 % y > 2
1717 \fi\fi
```

```
\else2 % y > 2
                1718
                1719
                             \fi
                        BIC@AfterFi{ 0}% y = 1
                1720
                1721
                      \ \ y = 2
                1722
                        \ifcase\BIC@ModTwo#2#3! % even(x)
                          \BIC@AfterFiFi{ 0}%
                1723
                        \or % odd(x)
                1724
                          \BIC@AfterFiFi{ #11}%
                1725
                        \else\BigIntCalcError:ThisCannotHappen%
                1726 ?
                1727
                        \fi
                1728
                      \ifx\\#1\\%
                1729
                1730
                        \else
                1731
                          \expandafter\BIC@Space\romannumeral0%
                          \expandafter\BIC@ModMinus\romannumeral0%
                1732
                1733
                        \fi
                        \int x^2-x < 0
                1734
                1735
                          \BIC@AfterFiFi{%
                            \expandafter\expandafter\BIC@ModX
                1736
                            \bigintcalcSub{#2#3}{%
                1737
                               \label{limits} $$ \left( \frac{4445}{\sigma^2} \right) = \frac{243}{445} %
                1738
                            }!#4#5!%
                1739
                          }%
                1740
                1741
                        \else % x > 0
                          \BIC@AfterFiFi{%
                1742
                1743
                            \expandafter\expandafter\BIC@Space
                1744
                            \bigintcalcSub{#2#3}{%
                1745
                               \bigintcalcMul{#4#5}{\bigintcalcDiv{#2#3}{#4#5}}%
                1746
                            }%
                          }%
                1747
                        \fi
                1749 ? \else\BigIntCalcError:ThisCannotHappen%
                1750
                     \BIC@Fi
                1751 }
\BIC@ModMinus
                1752 \def\BIC@ModMinus#1{\%
                1753
                      \ifx#10%
                        \BIC@AfterFi{ 0}%
                1754
                1755
                      \else
                        \BIC@AfterFi{ -#1}%
                1756
                1757
                     \BIC@Fi
                1758 }
    \BIC@ModX #1#2: z
                #3: x
                1759 \def\BIC@ModX#1#2!#3!{%
                      \int x^{1-} z < 0
                1760
                        \BIC@AfterFi{%
                1761
                          \expandafter\BIC@Space\romannumeral0%
                1762
                          \BIC@SubXY#3!#2!!!%
                1763
                        }%
                1764
                      \else % z >= 0
                1765
                        \BIC@AfterFi{ #1#2}%
                1766
                1767
                      \BIC@Fi
                1768 }
                1769 \BIC@AtEnd%
                1770 \langle /package \rangle
```

### 3 Installation

#### 3.1 Download

**Package.** This package is available on CTAN<sup>1</sup>:

CTAN:macros/latex/contrib/bigintcalc/bigintcalc.dtx The source file.

CTAN:macros/latex/contrib/bigintcalc/bigintcalc.pdf Documentation.

**Bundle.** All the packages of the bundle 'bigintcalc' are also available in a TDS compliant ZIP archive. There the packages are already unpacked and the documentation files are generated. The files and directories obey the TDS standard.

CTAN:install/macros/latex/contrib/bigintcalc.tds.zip

TDS refers to the standard "A Directory Structure for TEX Files" (CTAN:pkg/tds). Directories with texmf in their name are usually organized this way.

#### 3.2 Bundle installation

Unpacking. Unpack the bigintcalc.tds.zip in the TDS tree (also known as texmf tree) of your choice. Example (linux):

```
unzip bigintcalc.tds.zip -d ~/texmf
```

### 3.3 Package installation

**Unpacking.** The .dtx file is a self-extracting docstrip archive. The files are extracted by running the .dtx through plain T<sub>F</sub>X:

```
tex bigintcalc.dtx
```

**TDS.** Now the different files must be moved into the different directories in your installation TDS tree (also known as texmf tree):

```
\label{eq:bigintcalc.sty} \begin{array}{l} \texttt{bigintcalc.sty} \to \texttt{tex/generic/bigintcalc/bigintcalc.sty} \\ \texttt{bigintcalc.pdf} \to \texttt{doc/latex/bigintcalc/bigintcalc.pdf} \\ \texttt{bigintcalc.dtx} \to \texttt{source/latex/bigintcalc/bigintcalc.dtx} \end{array}
```

If you have a docstrip.cfg that configures and enables docstrip's TDS installing feature, then some files can already be in the right place, see the documentation of docstrip.

### 3.4 Refresh file name databases

If your TEX distribution (TEX Live, MiKTEX, ...) relies on file name databases, you must refresh these. For example, TEX Live users run texhash or mktexlsr.

### 3.5 Some details for the interested

Unpacking with LATEX. The .dtx chooses its action depending on the format:

plain T<sub>E</sub>X: Run docstrip and extract the files.

LATEX: Generate the documentation.

<sup>1</sup>CTAN:pkg/bigintcalc

If you insist on using LATEX for docstrip (really, docstrip does not need LATEX), then inform the autodetect routine about your intention:

```
latex \let\install=y\input{bigintcalc.dtx}
```

Do not forget to quote the argument according to the demands of your shell.

Generating the documentation. You can use both the .dtx or the .drv to generate the documentation. The process can be configured by the configuration file ltxdoc.cfg. For instance, put this line into this file, if you want to have A4 as paper format:

```
\PassOptionsToClass{a4paper}{article}
```

An example follows how to generate the documentation with pdfLATEX:

```
pdflatex bigintcalc.dtx
makeindex -s gind.ist bigintcalc.idx
pdflatex bigintcalc.dtx
makeindex -s gind.ist bigintcalc.idx
pdflatex bigintcalc.dtx
```

## 4 History

## [2007/09/27 v1.0]

• First version.

## [2007/11/11 v1.1]

• Use of package pdftexcmds for LuaTeX support.

# [2011/01/30 v1.2]

• Already loaded package files are not input in plain TEX.

# [2012/04/08 v1.3]

• Fix: pdftexcmds wasn't loaded in case of LATEX.

# [2016/05/16 v1.4]

• Documentation updates.

# [2019/12/15 v1.5]

• Documentation updates.

### 5 Index

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