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IF2130/II2130 – Organisasi dan Arsitektur Komputer

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Representasi Informasi – Floating Point Operations

Floating Point Operations: Basic Idea

- `x +f y = Round(x + y)` — $x +_f y = \mathrm{Round}(x + y)$
- $x \times_f y = \mathrm{Round}(x \times y)$
- Basic idea
  - First compute exact result
  - Make it fit into desired precision
    - Possibly overflow if exponent too large
    - Possibly round to fit into **`frac`**

Rounding

- Rounding Modes (illustrate with \$ rounding)

	\$1.40	\$1.60	\$1.50	\$2.50	–\$1.50
Towards zero	\$1	\$1	\$1	\$2	–\$1
Round down ($-\infty$)	\$1	\$1	\$1	\$2	–\$2
Round up ($+\infty$)	\$2	\$2	\$2	\$3	–\$1
Nearest Even(default)	\$1	\$2	\$2	\$2	–\$2

Closer Look at Round-To-Even

- Default Rounding Mode
  - Hard to get any other kind without dropping into assembly
  - All others are statistically biased
    - Sum of set of positive numbers will consistently be over- or under-estimated

- Applying to Other Decimal Places / Bit Positions
  - When exactly halfway between two possible values
    - Round so that least significant digit is even
  - E.g., round to nearest hundredth

1.2349999	1.23	(Less than half way)
1.2350001	1.24	(Greater than half way)
1.2350000	1.24	(Half way—round up)
1.2450000	1.24	(Half way—round down)

Rounding Binary Numbers

- Binary Fractional Numbers
  - "Even" when least significant bit is `0`
  - "Half way" when bits to right of rounding position = `100…`$_2$

- Examples
  - Round to nearest 1/4 (2 bits right of binary point)

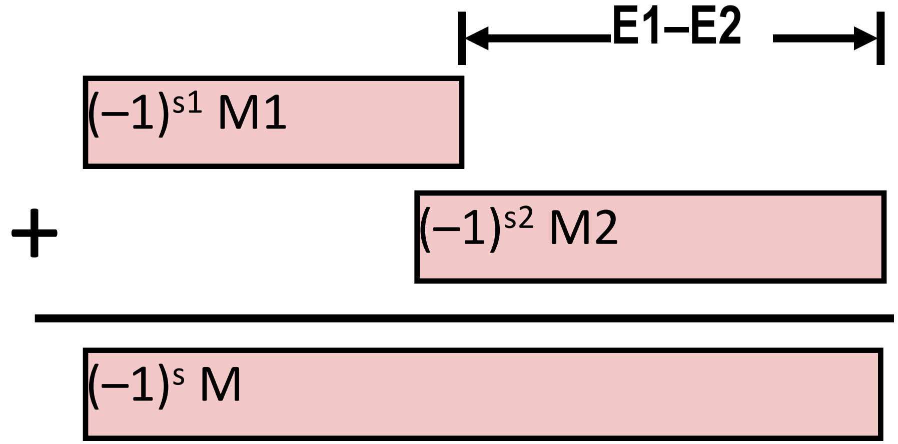
Value	Binary	Rounded	Action	Rounded Value
2 3/32	10.00011_2	10.00_2	(<1/2—down)	2
2 3/16	10.00110_2	10.01_2	(>1/2—up)	2 1/4
2 7/8	10.11100_2	11.00_2	(1/2—up)	3
2 5/8	10.10100_2	10.10_2	(1/2—down)	2 1/2

FP Multiplication

- $(-1)^{s1}\ M1\ \ 2^{E1}\ \ \times\ \ (-1)^{s2}\ M2\ \ 2^{E2}$
- Exact Result: $(-1)^{s}\ M\ \ 2^{E}$
  - Sign s: s1 ^ s2
  - Significand M: M1 x M2
  - Exponent E: E1 + E2
- Fixing
  - If M ≥ 2, shift M right, increment E
  - If E out of range, overflow
  - Round M to fit `frac` precision
- Implementation
  - Biggest chore is multiplying significands

Floating Point Addition

- $(-1)^{s1}\ M1\ \ 2^{E1} \quad + \quad (-1)^{s2}\ M2\ \ 2^{E2}$
  - Assume E1 > E2
- Exact Result: $(-1)^{s}\ M\ \ 2^{E}$
  - Sign s, significand M:
    - Result of signed align & add
  - Exponent E: E1



- Fixing
  - If M ≥ 2, shift M right, increment E
  - if M < 1, shift M left k positions, decrement E by k
  - Overflow if E out of range
  - Round M to fit `frac` precision

Floating Point in C

- C Guarantees Two Levels
  - **`float`** single precision
  - **`double`** double precision

- Conversions/Casting
  - Casting between **`int`**, **`float`**, and **`double`** changes bit representation
  - **`double`**/**`float`** → **`int`**
    - Truncates fractional part
    - Like rounding toward zero
    - Not defined when out of range or NaN: Generally sets to TMin
  - **`int`** → **`double`**
    - Exact conversion, as long as **`int`** has ≤ 53 bit word size
  - **`int`** → **`float`**
    - Will round according to rounding mode

Summary

- IEEE Floating Point has clear mathematical properties
- Represents numbers of form M x 2^E
- One can reason about operations independent of implementation
  - As if computed with perfect precision and then rounded
- Not the same as real arithmetic
  - Violates associativity/distributivity
  - Makes life difficult for compilers & serious numerical applications programmers

End of Segment