

Tutorial (Advanced Programming) Worksheet 4:

Assignment 1: Linear Interpolation

Consider an example where you have an unknown function $f(x)$, and you are told that $f(x_a) = y_a$ and $f(x_b) = y_b$. Now you are asked to provide the result of the function evaluation at x_r where $x_a \leq x_r \leq x_b$. Write a program which uses equation (1) to program a function which gets the known values and computes the function value at the point x_r .

$$y_r = y_a + (y_b - y_a) \frac{(x_r - x_a)}{(x_b - x_a)} \quad (1)$$

Assignment 2: Difference Quotient of Arbitrary Functions

Consider an example where you have a known function $f(x)$. You are asked to provide an approximation to the derivative of this function at point x_0 . Write a function `differenceQuotient` with the signature below, which uses equation (2) to calculate and return the difference quotient of function f .

$$f'(x_0) := \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \quad (2)$$

double

`differenceQuotient(double (*f)(double), double x_0, double h = 1e-8);`

Hint: In your tests, try to pass “real” as well as anonymous functions to `differenceQuotient`.

Assignment 3: Greatest Common Divisor

In this task we consider the greatest common divisor algorithm which is defined as follows:

$$\begin{aligned} \gcd(a, 0) &= a \\ \gcd(a, b) &= \gcd(b, a \bmod b) \end{aligned}$$

In the following you will implement this rather simple algorithm in three different ways:

1. Recursively. You are allowed to use temporary variables, but no static or global ones.
2. Iteratively. Same constraints as in the recursive approach.
Hint: Apply your knowledge about “tail-recursion”.

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- Iteratively, but this time without temporary variables.

Hint: You can swap two variables using simple arithmetic (or logical) operations.

However, the function signature should remain the same, with the following properties: The function shall take two integer arguments as input and should return one integer. The latter should contain the greatest common divisor of the provided input arguments.

```
int gcd(int a, int b);
```

Home Assignment 1: Fibonacci

Now, we will cover a very famous sequence, namely the Fibonacci sequence, which is defined by the following recurrence relation:

$$\begin{aligned}F(n) &= F(n-1) + F(n-2) \\F(0) &= 0 \\F(1) &= 1\end{aligned}$$

Again, we want to implement different versions.

- Implement a recursive version directly following the definition. Think about the complexity.
- Implement a second recursive version requiring only one recursive call.
Hint: In addition to the number n , use two formal parameters to cache already computed results. Think about different types of formal parameters you can use.

Questions:

Answer the following questions:

- In Assignment 1, What would happen if $x_a = x_b$? How could you avoid that?
- The second algorithm for the recursive calculation of the Fibonacci numbers requires only one recursive call. Can you reformulate the algorithm such that it uses tail-recursion?