Solution 1: find the value of $T(2)$ for the recurrence $T(n) = 3(T(n-1)) + 12n$, given that $T(0) = 5$

$$T(n) = 3T(n-1) + 12n \quad \text{--- (I)}$$

put $n=1$

$$T(1) = 3T(0) + 12 \quad \text{--- (II)}$$

put $T(0)=5$ in equation (II)

$$T(1) = 3 \times 5 + 12 = 27$$

put $T=2$ in equation I

$$T(2) = 3T(1) + 12 \times 2 \quad \text{--- (III)}$$

put $T(\cancel{2}1) = 27$ in equation (III)

$$T(2) = 3 \times 27 + 24$$

$$T(2) = 81 + 24$$

$$\boxed{T(2) = 105}$$ Ans

Question ②: Given a Recurrence relation, solve it using the substitution method.

Sol ⓐ: $T(n) = T(n-1) + C$ —— ①

Now put $n = (n-1)$ in equation ①

$T(n-1) = T(n-2) + C$ —— ②

put this $T(n-1)$ in equation ①

$T(n) = T(n-2) + 2C$ —— ③

again put $n = n-1$ in equation ③

$T(n-1) = T(n-3) + 2C$ —— ④

put $T(n-1)$ from equ. ④ to equation ①

$T(n) = T(n-3) + 3C$ —— ⑤

Continue putting this value.

$T(n) = T(n-K) + 3K$ —— ⑥

equation ⑥ is end of recurrence relation.

so $n - K = 0$

$\boxed{K = n}$ & $T(0) = C$

$T(n) = T(0) + 3n$

$\boxed{T(n) = 3n + C}$

So. Time Complexity $T(n) = O(n)$

Sol^n (b) $T(n) = 2T(n/2) + n$ —— (1)

$T(n/2) = 2T(n/4) + n/2$ —— (2)

put value of equ (2) to equ (1)

$$T(n) = 2\left[2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right] + n$$

$$T(n) = 4T\left(\frac{n}{2^2}\right) + 2n \quad \text{—— (3)}$$

again

$$T(n/2) = 4T\left(\frac{n}{2^3}\right) + \frac{2n}{2} \quad \text{—— (4)}$$

put in equation (1)

$$T(n) = 2\left[4T\left(\frac{n}{2^3}\right) + n\right] + n$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 3n \quad \text{—— (5)}$$

repeat ~~K time~~ this

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn \quad \text{—— (6)}$$

Now assum $\frac{n}{2^k} = 1$, and $T(1) = C$

$$n = 2^k \Rightarrow \boxed{k = \log n}$$

$$T(n) = 2^{\log n} T(1) + \log n \cdot n$$

$$\boxed{T(n) = n\log n + nc}$$

$$\boxed{\text{time Complexity} : T(n) = O(n\log n)}$$

Solⁿ (c) $T(n) = 2T(n/2) + c$ —— (1)

$T(n/2) = 2T\left(\frac{n}{2^2}\right) + c$ —— (2)

~~put value~~ from equation (1) & (2) we get

$T(n) = 2\left[2T\left(\frac{n}{2^2}\right) + c\right] + c$

$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + 3c$ —— (3)

again n to n/2 place in equation (3)

$T\left(\frac{n}{2}\right) = 2^2 T\left(\frac{n}{2^3}\right) + 3c$ —— (4)

from equation (4) & (1), we get

$T(n) = 2\left[2^2 T\left(\frac{n}{2^3}\right) + 3c\right] + c$

$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 7c$ —— (5)

repeat this k time, we get

$T(n) = 2^k T\left(\frac{n}{2^k}\right) + (2^k - 1)c$ —— (6)

assume, $\frac{n}{2^k} = 1$, & $T(1) = c_1$ (constant)

$\boxed{k = \log n}$

~~$T(n) = 2^{\log n} T(1) + (2(\log n) + 1)c$~~

~~$T(n) = nc_1 + 2c\log n + c$~~

$T(n) = 2^{\log n} T(1) + (2^{\log n} - 1)c$

$T(n) = nc_1 + nc - c$, assume $c_1 + c = e$

$\boxed{T(n) = en - c}$ time Compl. $\boxed{T(n) = O(n)}$

Sol^n (d) $T(n) = T\left(\frac{n}{2}\right) + c$ —— (1)

put $n = n/2$. in equ (1)

$$T\left(\frac{n}{2}\right) = T\left(\frac{n}{2^2}\right) + c \text{ —— (2)}$$

from equation (1) & (2) we get

$$T(n) = T\left(\frac{n}{2^2}\right) + 2c \text{ —— (3)}$$

again put $n = \frac{n}{2}$ in equation (3)

$$T(n/2) = T\left(\frac{n}{2^3}\right) + 2c \text{ —— (4)}$$

from equation (1) & (4) we get

$$T(n) = T\left(\frac{n}{2^3}\right) + 3c \text{ —— (5)}$$

repeat this k time.

$$T(n) = T\left(\frac{n}{2^k}\right) + kc \text{ —— (6)}$$

assume $\frac{n}{2^k} = 1 \Rightarrow \boxed{k = \log n}$ & $T(1) = d$

constant

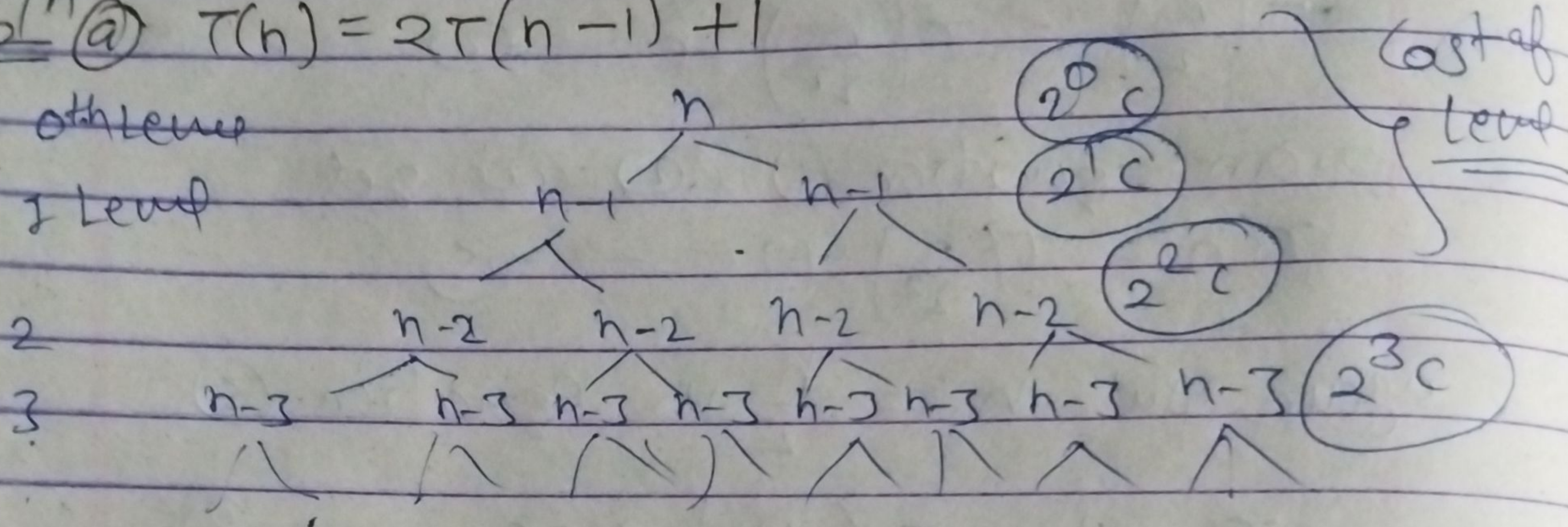
$$T(n) = T(1) + c\log n$$

$$\boxed{T(n) = n\log n + d}$$

Time Complexity : $\boxed{T(n) = O(c\log n)}$

Question (8) Given a recurrence relation. Solve it using recursive tree approach.

Sol^n (a) $T(n) = 2T(n-1) + 1$



k level $n - k = 0$

$$\boxed{k = n}$$

$$2^0 c + 2^1 c + 2^2 c \ldots 2^k c$$

$$\Rightarrow c\,(2^0 + 2^1 + 2^2 + \ldots 2^k)$$

$$\text{Sum of GP Series} = \frac{a(r^n - 1)}{r - 1} = \frac{1(2^n - 1)}{2 - 1}$$

$$= (2^n - 1)$$

$$\boxed{T(n) = O(2^n)}$$

exponential time complexity.

Sol^n (b) $T(n) = 2T(n/2) + n$

~~0 Level~~ n Cost n

~~1st Level~~ $n/2$ $n/2$ $\frac{n}{2} + \frac{n}{2} = n$

2nd level $n/4$ $n/4$ $n/4$ $n/4$ $4 \cdot \left(\frac{n}{4}\right) = n$

$2^k \cdot \left(\frac{n}{2^k}\right) = n$

k^{th} Level $\frac{n}{2^k}$ ...

$$\frac{n}{2^k} = 1, \quad \boxed{k = \log n}$$

sum of the cost of all level.

$T(n)$ $n + n + \ldots$ n (k time)

$T(n) = kn$

$$\boxed{T(n) = n \log n}$$

$$\boxed{\text{Time Complexity: } T(n) = O(n \log n)}$$