DE: GR20A2069 | GR 20 | SET - 3

II B.Tech I Semester Regular Examinations, February/March 2023

DISCRETE MATHEMATICS

(Information Technology)

3 hours **Max Marks: 70**

uctions:

Question paper comprises of **Part-A** and **Part-B**
Part-A (for 20 marks) must be answered at one place in the answer book.
Part-B (for 50 marks) consists of **five questions with internal choice,** answer all questions.
CO means Course Outcomes. **BL** means Blooms Taxonomy Levels.

PART – A

(Answer ALL questions. All questions carry equal marks)

10 * 2 = 20 Marks

Construct the truth table for $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$.	[2]	CO1	BL3
Show that the formula $q \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$ is a tautology.	[2]	CO1	BL2
What is reflexive Relation? Explain with a suitable example.	[2]	CO2	BL1
Define lattice with a suitable example.	[2]	CO2	BL1
Find the number of arrangements of the letters in TALLAHASSEE which have no adjacent A's.	[2]	CO3	BL1
Prove the following identity: $C(n+1,r) = C(n,r-1) + C(n,r)$	[2]	CO3	BL5
Solve the recurrence relation $a_n = na_{n-1}$ for $n \geq 1$, given that $a_0 = 1$.	[2]	CO4	BL6
Solve the recurrence relation $a_n - 7a_{n-2} + 10a_{n-4} = 0$, $n \geq 4$	[2]	CO4	BL6
Construct two graphs so that they are isomorphic.	[2]	CO5	BL3
Does there exist an Eulerian graph with an odd number of vertices and an even number of edges? Draw such a graph if it exists.	[2]	CO5	BL2

PART – B

(Answer ALL questions. All questions carry equal marks)

5 * 10 = 50 Marks

(a) Prove the validity of the following arguments: [10] CO1 BL5

$(\neg p \vee q) \rightarrow r$

$r \rightarrow (s \vee t)$

$\neg s \wedge \neg u)$

$\neg u \rightarrow \neg t$

$\therefore$ p

(b) Give a proof by contradiction of the following statement: The square of an even integer is an even integer.

OR

3. (a) (i) Obtain the principal conjunctive normal form of the following:
$(\neg p \to q) \wedge (q \leftrightarrow p)$ [1
(ii) Obtain the principal disjunctive normal form of the following:
$\neg(p \vee q) \leftrightarrow (p \wedge q)$

(b) Give a direct proof for the following statement:
For all positive integers m and n, if m and n are perfect squares, mn then is also a perfect square.

4. Define R on $A = \{1,2,3,4,5,6,7,8,9,10,11,12\}$ by $(x, y) \in R$ if $x - y$ is a multiple of 5. (i) Verify that R is an equivalence relation on A, (ii) Describe the distinct equivalence classes of R. [1

OR

5. (a) Consider the set Z of integers. Define aRb by $b = a^r$ for some positive integer r. Show that R is a partial order on Z, that is, show that R is: (i) reflexive; (ii) antisymmetric; (iii) transitive. [1

(b) Consider the set $A = \{ball, bed, dog, let, egg\}$ and define the relation R on A by $R = \{(x, y) : x, y \in A$ and xRy if x and y contain some common letter$\}$. Verify that R is a compatibility relation which is not transitive. Draw the graph of R.

6. (a) Find the number of permutations of the English letters which contain (i) exactly two, (ii) at least two, (iii) exactly three, and (iv) at least three, of the patterns CAR, DOG, PUN, BYTE. [10

(b) How many integers between 1 and 300 (inclusive) are (i) divisible by at least one of 5, 6, 8? (ii) divisible by none of 5, 6, 8?

OR

7. (a) In how many ways 5 number of α's, 4 number of β's and 3 number of γ's can be arranged so that all the identical letters are not in a single block? [10]

(b) A bag contains a large number of red, green, white and black marbles, with at least 24 of each colour. In how many ways can one select 24 of these marbles, so that there are even number of white marbles and at least six black marbles. (Use generating function)

Solve the following recurrence relations by the methods of characteristic roots: $a_n - 6a_{n-1} + 8a_{n-2} = 3^n$, $n \geq 2$, $a_0 = 3$, $a_1 = 7$. [10] CO4 BL3

OR

Find a generating function for the recurrence relation $a_{n+2} - 5a_{n+1} + 6a_n = 2$, $n \geq 0$, and $a_0 = 3$, $a_1 = 7$. Hence solve it. [10] CO4 BL6

(a) What is simple graph? Is it possible to draw a simple graph with 4 vertices and 7 edges? Justify your answer. [10] CO5 BL5

(b) What is binary tree? Give an example. Prove that there are $\frac{1}{2}(n+1)$ pendant vertices in any binary tree with n vertices.

OR

Write down the chromatic number of each of the following graphs. For each graph, devise a suitable colouring and explain. [10] CO5 BL3
(i) a tree, (ii) a graph with exactly one cycle, (iii) the Petersen graph,
(iv) the complete bipartite graph $K_{r,s}$, (v) the complete graph K_n

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