Homework 5 Dholakia

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1 Problem 5.3

1.1 Part a

I used Simpson's Rule to find the integral from 0 to 3, with 30 segments. Because the number of segments were so few, Simpson Integration methods would be more accurate.

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Integral from 0 to 3 is: 0.8862073362266246
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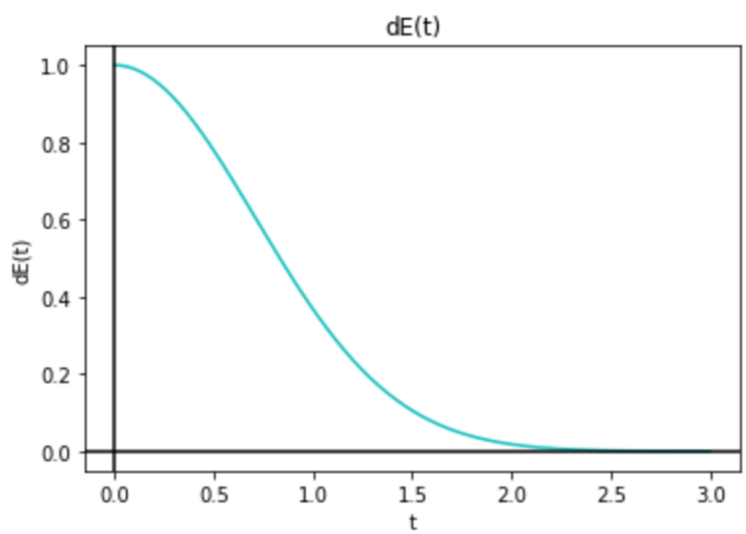

Figure 1:

1.2 Part b

The integral from 0 to 3 is approximately 0.8862, and as x increased, the value of E(x) got closer and closer to that number.

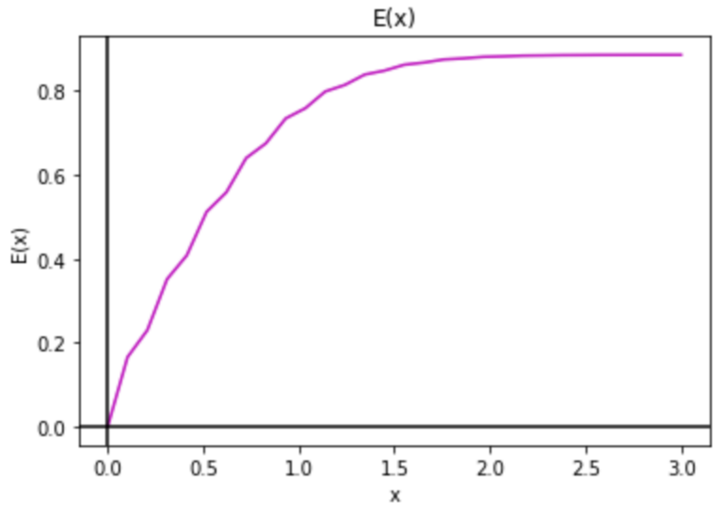

Figure 2:

The error function closely resembles the previous graph:

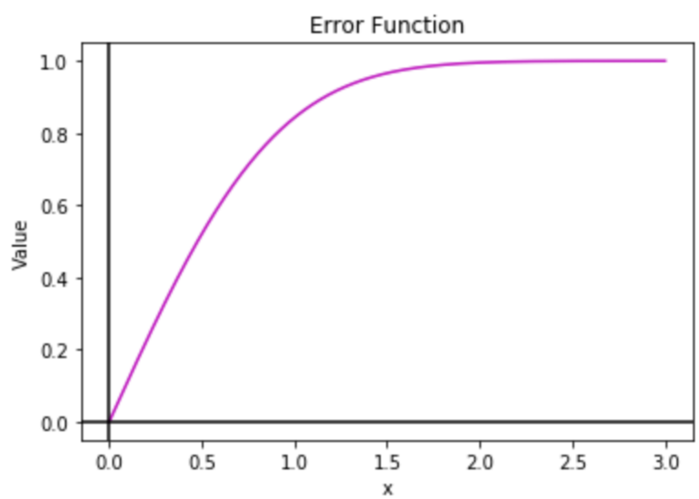

Figure 3: Error Function

2 Problem 5.10

2.1 Part a

Transforming into equation for Period:

$$E = \frac{1}{2}m(\frac{dx}{dt})^2 + V(x)$$

$$E - V(x) = \frac{1}{2}m(\frac{dx}{dt})^2$$

$$\frac{2}{m}(E - V(x)) = (\frac{dx}{dt})^2$$

$$\sqrt{\frac{2}{m}(E - V(x))} = \frac{dx}{dt}$$

$$dt = \frac{dx}{\sqrt{\frac{2}{m}(E-V(x))}}$$

$$\int_0^{T/4} dt = \int_0^a \frac{dx}{\sqrt{\frac{2}{m}(E - V(x))}}$$

$$\frac{T}{4} = \sqrt{\frac{m}{2}} \int_0^a \frac{dx}{\sqrt{\frac{2}{m}(E - V(x))}}$$

$$T = \sqrt{8m} \int_0^a \frac{dx}{\sqrt{\frac{2}{m}(E - V(x))}}$$

2.2 Part c

The period of the anharmonic oscillator approaches ∞ as a approaches 0 because as the "well" flattens out near 0, the time taken for the ball to roll will increase and increase. The smaller the amplitude the larger the time taken (Period).

3 Problem 5.12

3.1 Part a

Stefan Boltzmann Constant

$$I(\omega) = \frac{\hbar}{4\pi^2 c^2} \frac{\omega^3}{(e^{\hbar\omega/k_B T} - 1)}$$

$$W = \frac{\hbar}{4\pi^2 c^2} \int_0^\infty \frac{\omega^3}{(e^{\hbar\omega/k_B T} - 1)} d\omega$$

$x = \frac{\hbar\omega}{k_B T}$, $dx = \frac{\hbar}{k_B T} d\omega$, $d\omega = \frac{k_B T}{\hbar}$

$$W = \frac{\hbar}{4\pi^2 c^2} \int_0^\infty \frac{\omega^3}{(e^x - 1)} d\omega$$

$$W = \frac{\hbar}{4\pi^2 c^2} \frac{k_B^3 T^3}{\hbar^3} \int_0^\infty \frac{x^3}{(e^x - 1)} \frac{k_B T}{\hbar} dx$$

$$W = \frac{k_B^4 T^4}{4\pi^2 c^2 \hbar^3} \int_0^\infty \frac{x^3}{(e^x - 1)} dx$$

3.2 Part b

To find the Integral of the function $\int_0^\infty \frac{x^3}{(e^x-1)} dx$ I used Simpson's Rule and had quadratics fitted to each column (of which there were 10,000) in the area from 0 to 100. This would give me a very close approximation of the integral from 0 to $\infty of the function.$

3.3 Part c

From the integral we obtained in the previous answer, we can use that to multiply the constants in the front part of the equation, $\frac{k_B^4 T^4}{4\pi^2 c^2 \hbar^3}$, by the integral.

This gives us a very accurate approximation of the Stefan Boltzmann Constant.