RSS

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Home Work : 1

Q. 1

A) Given: 1) rotated about X_A axis by θ
2) translation $[1, 2, 3]^T$

$$ {}^{A}T_B = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \cos\theta & -\sin\theta & 2 \\ 0 & \sin\theta & \cos\theta & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

B) ${}^{B}T_A = ({}^{A}T_B)^{-1}$

$$ = \begin{bmatrix} ({}^{A}R_B)^T & -({}^{B}R_A \cdot t) \\ 0 & 1 \end{bmatrix} $$

1) $$ {}^{B}R_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & S\theta \\ 0 & -S\theta & C\theta \end{bmatrix} $$

$$ -{}^{B}R_A \cdot t = -1 \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & S\theta \\ 0 & -S\theta & C\theta \end{bmatrix}_{3\times3} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times1} $$

$$ = \begin{bmatrix} -1 \\ -2C\theta - 3S\theta \\ 2S\theta - 3C\theta \end{bmatrix} $$

2.

$$ {}^{B}T_A = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & C\theta & S\theta & -2C\theta - 3S\theta \\ 0 & -S\theta & C\theta & 2S\theta - 3C\theta \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

ⓒ Given: $\theta = \pi/4$

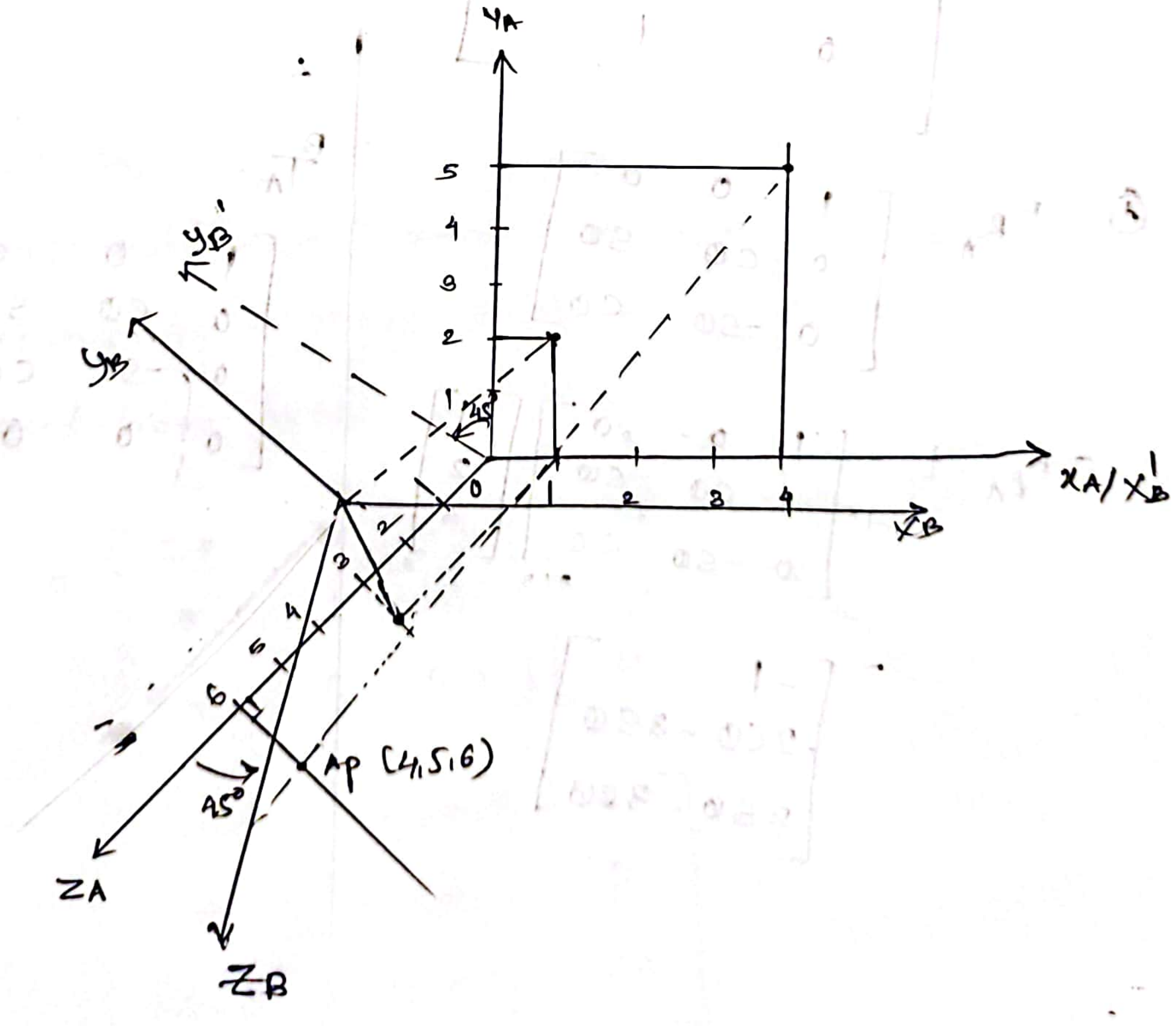
$${}^{A}p = |4, 5, 6|^{T}$$

$${}^{B}p = {}^{B}T_{A} \cdot {}^{A}p$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{5}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ \frac{6}{\sqrt{2}} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ \frac{6}{\sqrt{2}} \\ 0 \\ 1 \end{bmatrix}$$

d)



Q.2

A) Given: 1) rotated about Y_A axis by θ. $\rightarrow \{B\}$

2) rotated about Z_B axis by $\phi \rightarrow \{C\}$

$${}^{A}R_C = {}^{A}R_B \cdot {}^{B}R_C$$

$$= \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta\, c\phi & -c\theta s\phi & s\theta \\ s\phi & c\phi & 0 \\ -s\theta\, c\phi & s\theta s\phi & c\theta \end{bmatrix}$$

B) Let's consider,

① frame in fixed angle is rotated by angle ϕ about Z_A

② rotated by an angle θ about Y_A axis.

Using fixed-frame rotat^n approach

$$= \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} c\phi & -s\phi & 0 \\ s\phi & c\phi & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta\, c\phi & -c\theta s\phi & s\theta \\ s\phi & c\phi & 0 \\ -s\theta\, c\phi & s\theta\, c\phi & c\theta \end{bmatrix}$$

Hence,

first approach is euler's rotation, while second implementation is fixed frame,

comparing the results of 2 conventions, we can see that the results are exactly same

Hence, both are equivalent.

Q.3

frame 1 displaced by $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

A) $${}^{0}T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

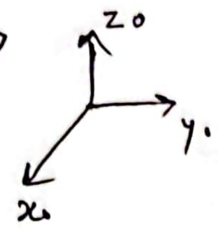
$${}^{0}T_2 = \begin{bmatrix} 0 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}T_3 = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

B). $${}^{3}T_2 = \begin{bmatrix} {}^{3}R_2 & d_3 \\ 0 & 1 \end{bmatrix}$$

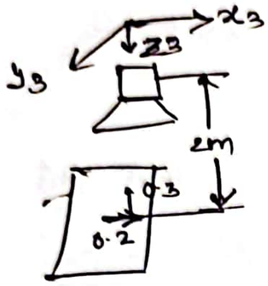
$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c)



Cube is initially at $\begin{pmatrix} -0.5 \\ 1.5 \\ 1 \end{pmatrix}$

& ~~robot~~ cube moves it to → origin $\begin{pmatrix} 0.2 \\ -0.3 \\ 2 \end{pmatrix}$



ie ~~robot~~ cube moves it to $\begin{pmatrix} -0.8 \\ 1.7 \\ 1 \end{pmatrix}$

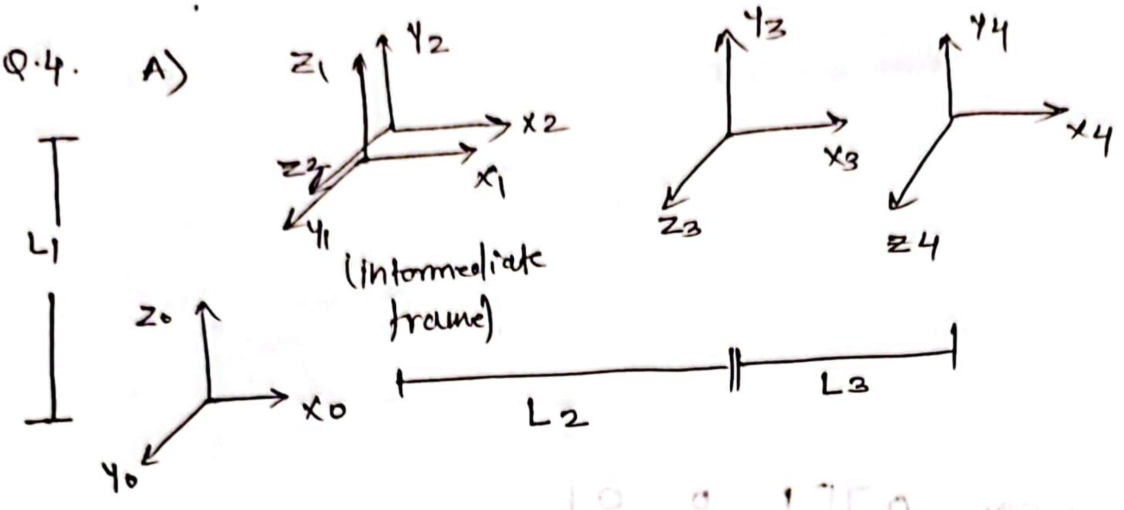
^0P

$$^0p = {}^0T_3 \cdot {}^3p$$

$$= \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.2 \\ -0.3 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.8 \\ 1.7 \\ 1 \\ 1 \end{bmatrix}$$

Q.4. A)



DH Table

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	4	θ_1
2	$\pi/2$	0	0	θ_2
3	0	L_2	0	θ_3
4	0	L_3	0	0

We have,

$$^{i-1}T_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -d_n s\alpha_{i-1} \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & d_n c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

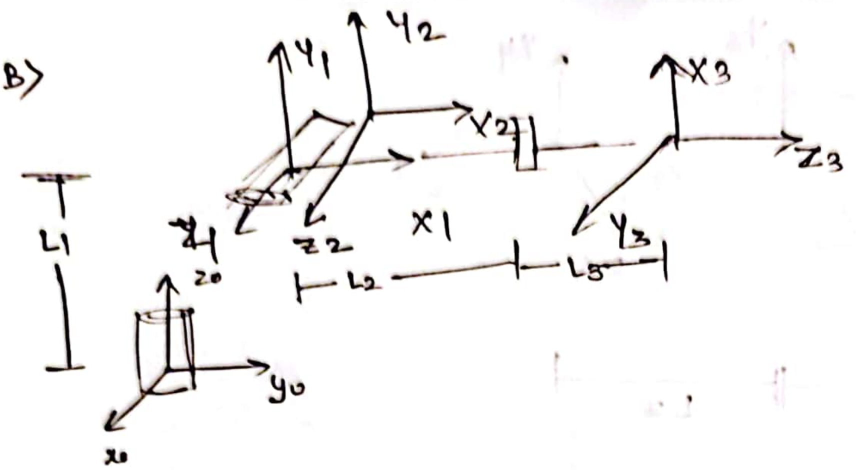
$$^1T_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^3T_4 = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_4 = {}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4$$

B)



$$^0R_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$^0R_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 \\ S\theta_1 & 0 & -C\theta_1 \\ 0 & 1 & 0 \end{bmatrix} \qquad {}^0d_1 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix} \quad — (1)$$

$$^0T_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0T_3 = {}^0T_1 \cdot {}^1T_2 \cdot {}^2T_3$$

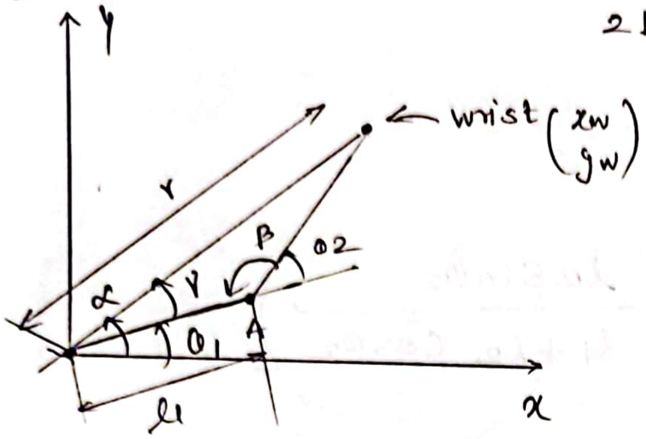
$$^1T_2 = \begin{bmatrix} -S\theta_2 & 0 & C\theta_2 & 0 \\ C\theta_2 & 0 & S\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2+L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q.5.

2 DOF Planar Arm.

A)



A) $r^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos\beta$

$$\cos\beta = \frac{l_1^2 + l_2^2 - r^2}{2 l_1 l_2}$$

B) $\theta_2 = \pi - \beta$

taking cosine on bothsides

$$C\theta_2 = -C\beta$$

$$= -\left(\frac{l_1^2 + l_2^2 - r^2}{2 l_1 l_2}\right)$$

$$\cos\theta_2 = \frac{r^2 - l_1^2 - l_2^2}{2 l_1 l_2}$$

$$\theta_2 = a\cos\left(\frac{r^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

c)

$$\gamma = a\tan\left(\frac{l_2 \sin\theta_2}{l_1 + l_2 \cos\theta_2}\right)$$

d) $$\alpha = \operatorname{atan}\left(\frac{y_w}{x_w}\right)$$

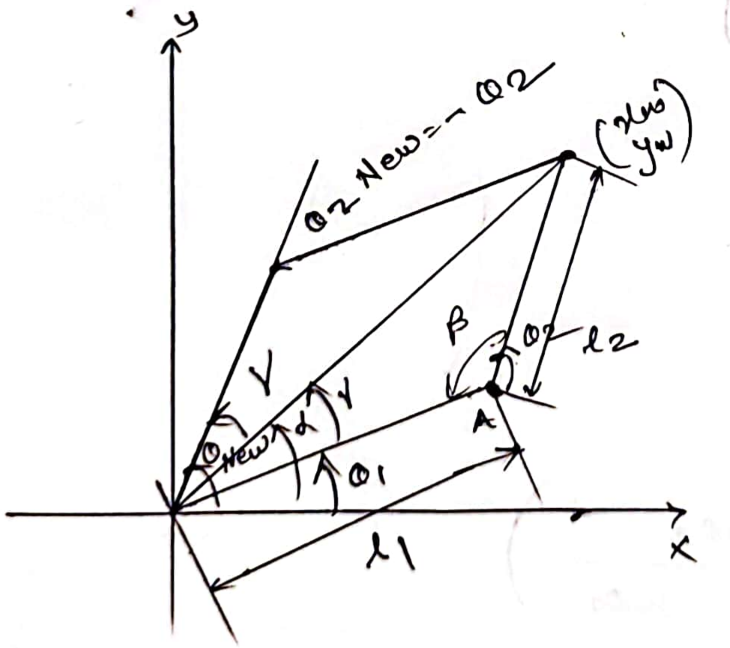
$$\theta_1 = \operatorname{atan}\left(\frac{y_w}{x_w}\right) - \operatorname{atan}\left(\frac{l_2 \sin\theta_2}{l_1 + l_2 \cos\theta_2}\right)$$

e) depending on $[x_w, y_w]^T$ there may be exactly 0, 1, 2 solutions.

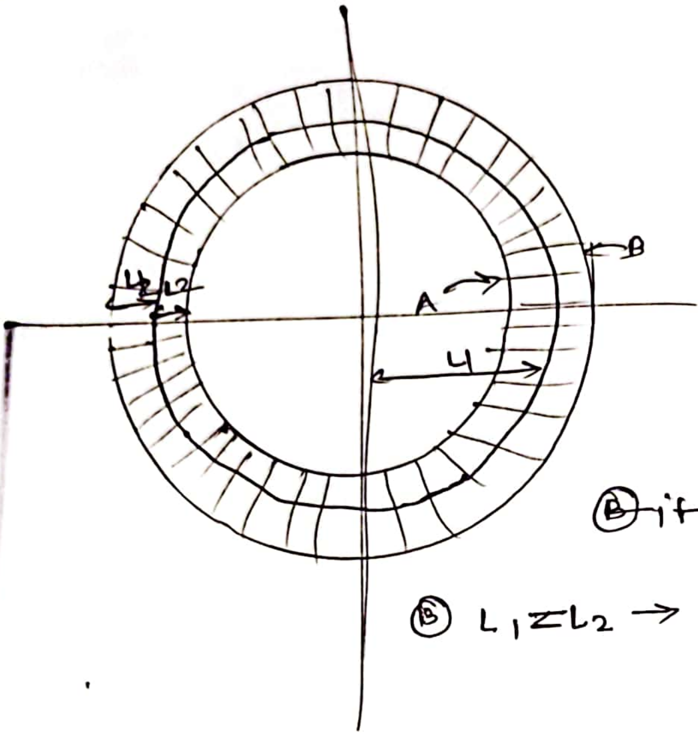
$$\theta_1 = \operatorname{atan}\left(\frac{y_w}{x_w}\right) \pm \operatorname{atan}\left(\frac{l_2 \sin\theta_2}{l_1 + l_2 \cos\theta_2}\right)$$

$$\theta_2 = \pm \operatorname{acos}\left(\frac{r^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

Here, $\left(r^2 = x_w^2 + y_w^2\right)$



$$\theta_1 \text{ New} = \alpha + \gamma$$
$$\theta_2 \text{ New} = -\theta_2$$



0 solutions:

Ⓐ $L_1 > L_2$ & $L_2 \ngtr L_1$ → the workspace is defined between $|L_1 - L_2|$ to outer radius $L_1 + L_2$.

If $\begin{pmatrix} x_w \\ y_w \end{pmatrix}$ lies outside → shaded circle then → No solution.

~~Ⓑ if $L_1 = L_2$~~

Ⓑ $L_1 = L_2$ → inside circle disappears.
whole circle will be workspace.
All points inside circle will be reachable

1 solution – occurs when $\begin{pmatrix} x_w \\ y_w \end{pmatrix}$ lies on the peripheri of the 2 circles – A & B.

occurs everywhere else in the shaded region.

2 solutions – occurs everywhere else in the shaded region.