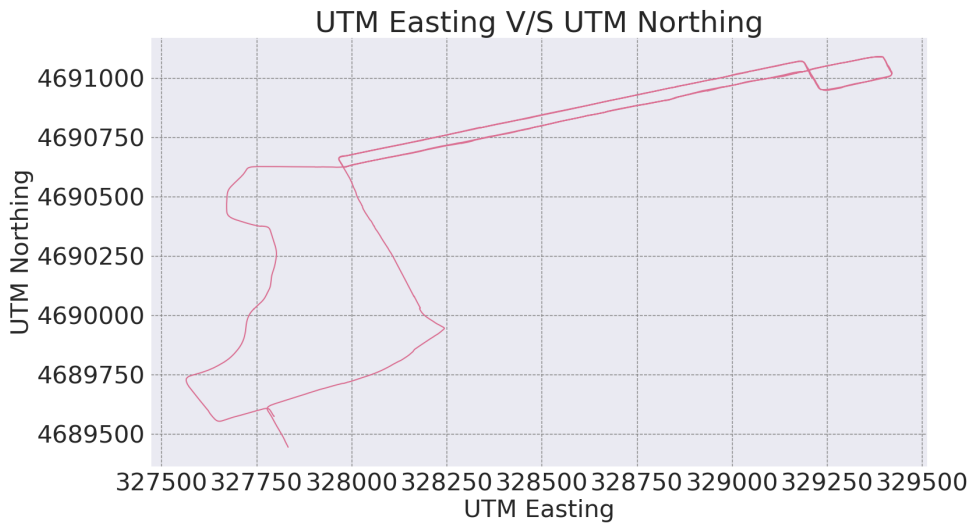
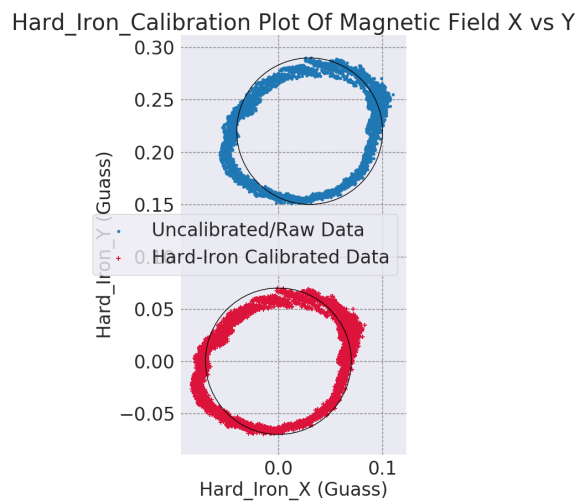
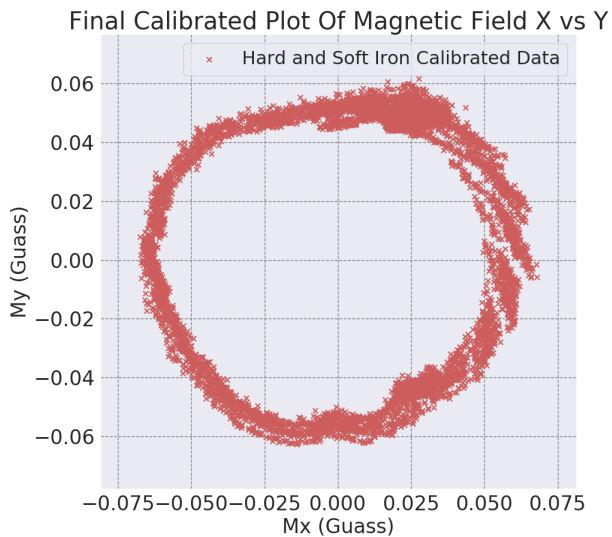


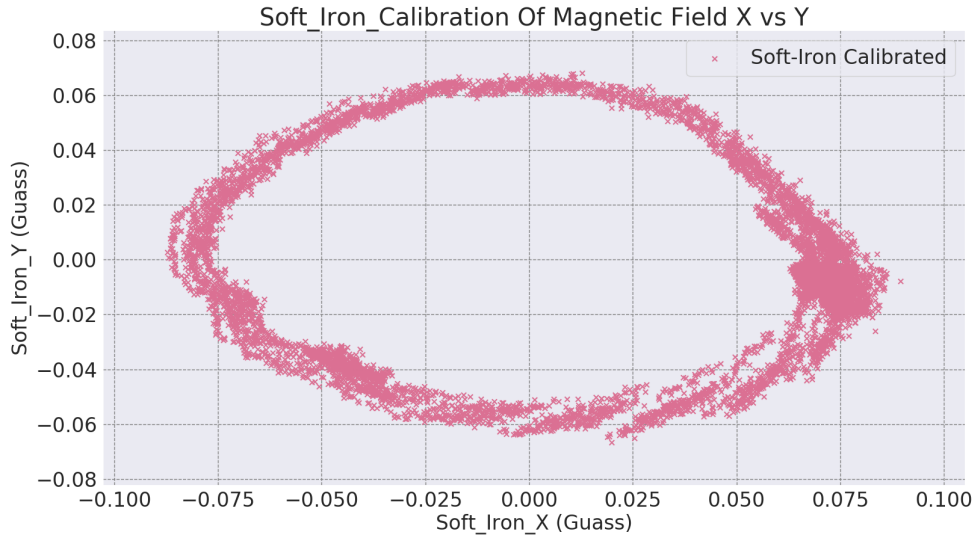
**Robotics Sensing and Navigation**  
**EECE 5554**  
**Navigation with IMU and Magnetometer**  
**LAB-4**

The data was collected in a single rosbag file, and it includes the following message topics /gps, /imu. We collected stationary data for a fixed time at Ruggles and then collected data by going in rounds for magnetometer calibration followed by a stop, then moving data was collected from ruggles to boston commons for dead reckoning and finally a stop back at ruggles. The gps motion can be observed from the below plot.



**Magnetometer Calibration**





**Case 1- No Distortions:** In the event that there are no hard or soft iron distortions present, the measurements should form a circle centered at  $X=0$ ,  $Y=0$ . The radius of the circle equals the magnitude of the magnetic field.

**Case 2 - Hard Iron Distortions:** Hard iron distortions will cause a permanent bias to be present in the magnetic measurements, which leads to a shift in the center of the circle. Suppose  $X=200$  and  $Y=100$ , from this it can be concluded that there is 200 mGauss hard iron bias in the X-axis and 100 mGauss hard iron bias in the Y-axis.

**Case 3 - Hard and Soft Iron Distortions:** Hard iron distortions will only shift the center of the circle away from the origin, they will not distort the shape of the circle in any way. Soft iron distortions, on the other hand, distort and warp the existing magnetic fields. When plotting the magnetic output, soft iron distortions are easy to recognize as they will distort the circular output into an elliptical shape. Every ellipse has a major and minor axis if the major and minor axis are aligned to some degrees from the body frame then it is caused by the soft iron distortions.

#### Hard Iron Calibration -

Compensating for hard-iron distortion is accomplished by determining the x and y offsets and then applying these constants directly to the data.

#### Soft Iron Calibration -

After hard-iron calibration, the origin of the ellipse is at  $(0, 0)$ , and is exhibiting a rotation of  $\theta$  degrees from the X axis.

$$r = \sqrt{X^2 + Y^2} \quad \theta = \arcsin(Y/r) \quad v1 = Rv$$

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad \sigma = q/r$$

$$[-\sin\theta \quad \cos\theta]$$

One method of identifying "r" is to calculate the magnitude of each data point and then identify the maximum of these computed values. The coordinates of this value will correspond with the major axis. Similarly, the minimum value will correspond to the minor axis, "q". Once  $\theta$  has been identified, the rotation matrix "R" given is applied to the vector of magnetometer x and y values. After the rotation, the major axis of the ellipse will be aligned with the reference frame X axis and the minor axis will be aligned with the Y axis. Following the rotation, we can now properly scale the major axis such that the ellipse is converted to

an approximate circle. The scale factor  $\sigma$ , is determined, and is the ratio of the length of the major axis to that of the minor axis. Each magnetometer x value is then divided by this scale factor to produce the desired circle.

## 2. Estimation of Yaw Angle

### 2.1 Magnetometer Yaw Estimate before and after calibration

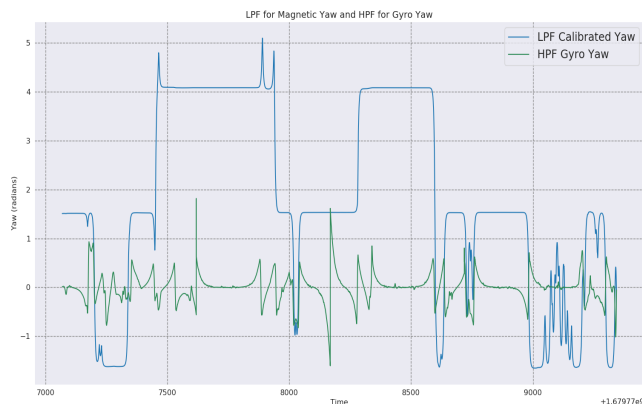
Yaw angle is found after the calibration of hard and soft iron of the magnetometer. It can also be derived from integrating gyroscope data. Both calculated yaw closely matches the form of the yaw angle internally calculated in the IMU. The yaw calculated from the gyroscope data is relatively smoother and less sensitive (fewer peaks) compared to the IMU yaw, while the yaw derived from the magnetometer data is relatively less smooth.

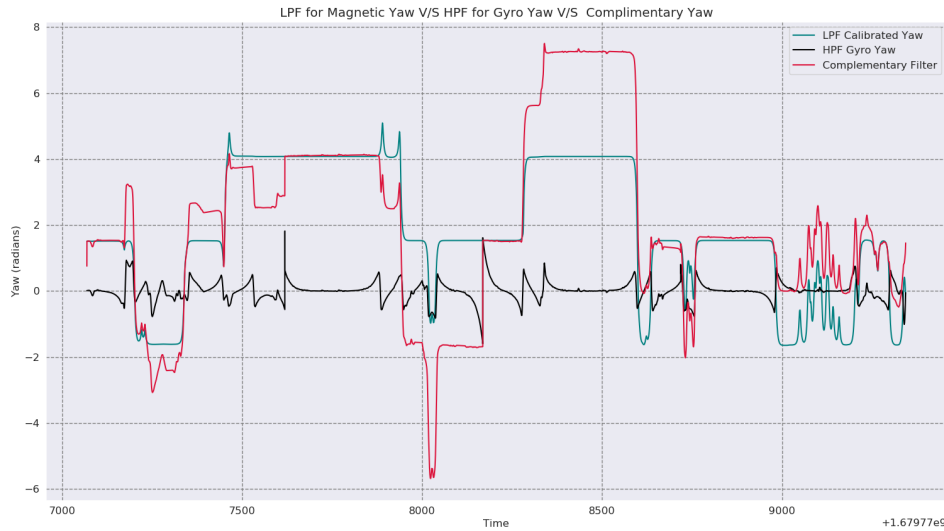
### 2.3 Gyroscope Yaw Estimate and Complementary Filter Yaw Estimate

A LPF is used on the magnetometer data to keep its steady low drift and clip-off any high frequency noise. A HPF is used on the gyro data as the gyroscope data tends to drift over time. So to keep high frequency measurements and clip-off the bias from low frequency drift being integrated a HPF is used. Alpha Value of the complementary filter was set to 0.5 and it gave the closest value and maintained the trend of plot.

Q3I would below yaw estimation methods-

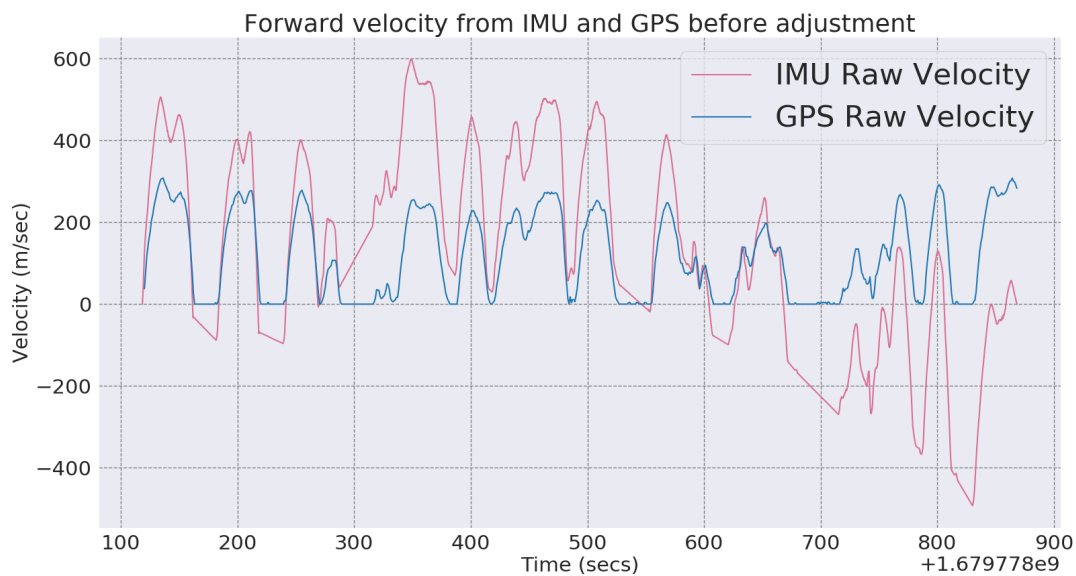
1. Gyroscopes measure angular rate, which can be integrated over time to provide a yaw estimate. For short-term navigation, gyroscopes may be suitable, but for long-term navigation, they should be combined with other sensors.
2. Magnetometers measure the Earth's magnetic field and can provide a yaw estimate based on the magnetic north reference. magnetometers can be trusted for navigation, but in regions with significant magnetic disturbances, their accuracy may be compromised.

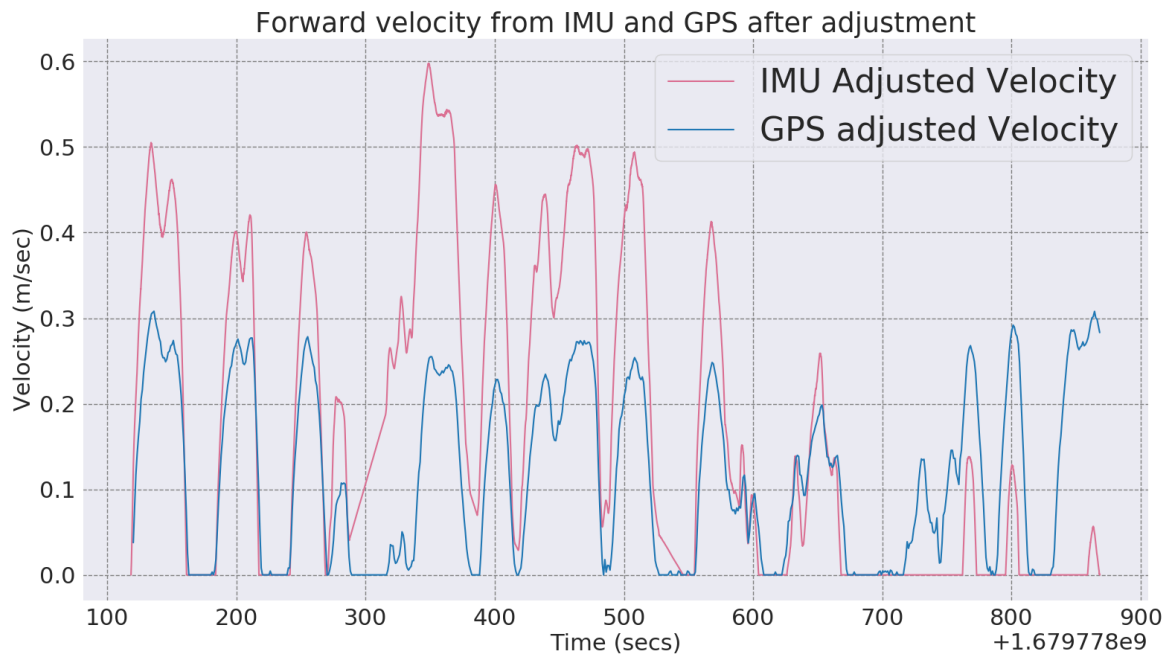




### 3. Estimation of Forward Velocity

Velocity can be found by taking the integral of acceleration and by finding the instantaneous velocity of each point of GPS data. To track the velocity or speed of our vehicle we perform integration on accelerometer data. Here we are using only the X component of the Linear acceleration of the accelerometer, which is mounted as the direction pointing straight out the front of the vehicle. In this experiment we don't consider Y component linear acceleration because the car is not skidding in a sideways direction. Forward velocity can also be calculated by taking the hypotenuse of the X and Y components (easting and northing) of our GPS position data and taking the gradient with respect to time.





### Discrepancies between velocity estimates derived from accelerometer data -

1. To obtain velocity, the acceleration data must be integrated over time. Integration introduces errors that accumulate over time, causing the velocity estimate to drift.
2. GPS, on the other hand, directly measures position, and velocity is obtained by differentiating the position data. While GPS is not immune to errors, it is not subject to the same drift issues as accelerometers.
3. Accelerometers are subject to noise and sensor errors, such as  $\rightarrow$  scale factor errors, and misalignments we see after plotting graphs. These errors can affect the quality of the velocity estimate derived from the accelerometer data.
4. While GPS measurements are also subject to noise and errors, such as atmospheric delays, multipath effects.

### 3.1 Obtain Velocity by Integrating Acceleration

Simplify the equations to get acceleration measured by the inertial sensor (i.e. its acceleration as sensed in the vehicle frame) is:  $\ddot{x}_{obs} = \ddot{X} - \omega \dot{Y} - \omega^2 x_c$ ,  $\ddot{x}_{obs} = \ddot{X}$

Here, all the quantities in these equations are evaluated in the vehicle frame. We can integrate  $\ddot{X}$  to find the velocity. Similarly,  $\ddot{y}_{obs} = \ddot{Y} + \omega \dot{X} + \omega x_c$

can be simplified to  $\ddot{y}_{obs} = \ddot{Y}$  is the velocity calculated. And the  $\omega$  is the z-axis angular velocity from the gyroscope. We assume that the  $\dot{Y} = 0$  (that is, the vehicle is not skidding sideways) and ignore the offset by setting  $x_c = 0$  (meaning that the IMU is on the center of mass of the vehicle, i.e. the point about which the car rotates).

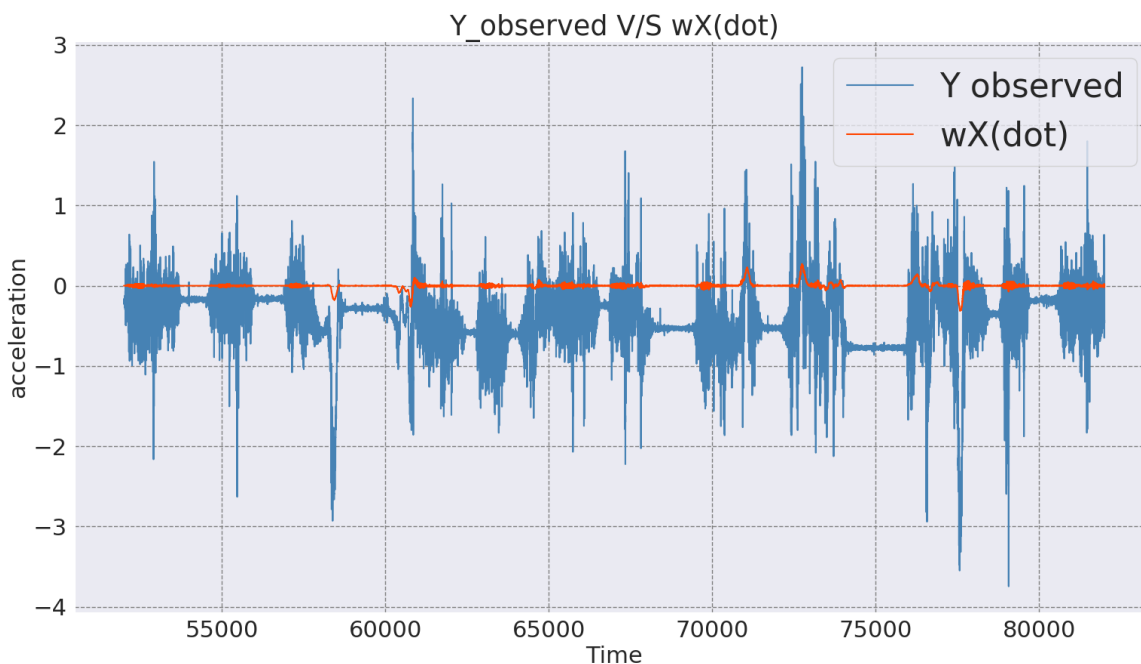
Q6) This equation represents the relationship between the observed linear acceleration ( $\ddot{y}_{obs}$ ) of a point on a rigid body and the components of acceleration that contribute to it. The equation takes into account the linear acceleration of the center of mass ( $\ddot{Y}$ ), the centripetal acceleration due to rotation ( $\omega\dot{X}$ ), and the tangential acceleration due to the change in angular velocity ( $\omega\dot{X}_c$ ).

It is possible to relate the centripetal acceleration to the linear acceleration, and comparing the two can help determine the motion characteristics of the body. But there might be difference due to -

- 1.If the rigid body is not only rotating but also translating through space with a linear acceleration, there will be a difference.
2. If the angular velocity ( $\omega$ ) of the rotating body is changing over time,tangential acceleration ( $\omega\dot{X}_c$ ) will also contribute to the difference between  $\ddot{y}_{obs}$  and  $\omega\dot{X}$ .
3. Errors in the sensors could contribute to the difference between the two values.

#### 4. Dead Reckoning

Dead Reckoning is the process of calculating the current position of a moving object from knowledge of its previously determined position. Dead reckoning is the only navigation choice, especially in the absence of landmarks of known position. Dead reckoning using IMU can be done by multiplying our estimated forward velocity by yaw angle.



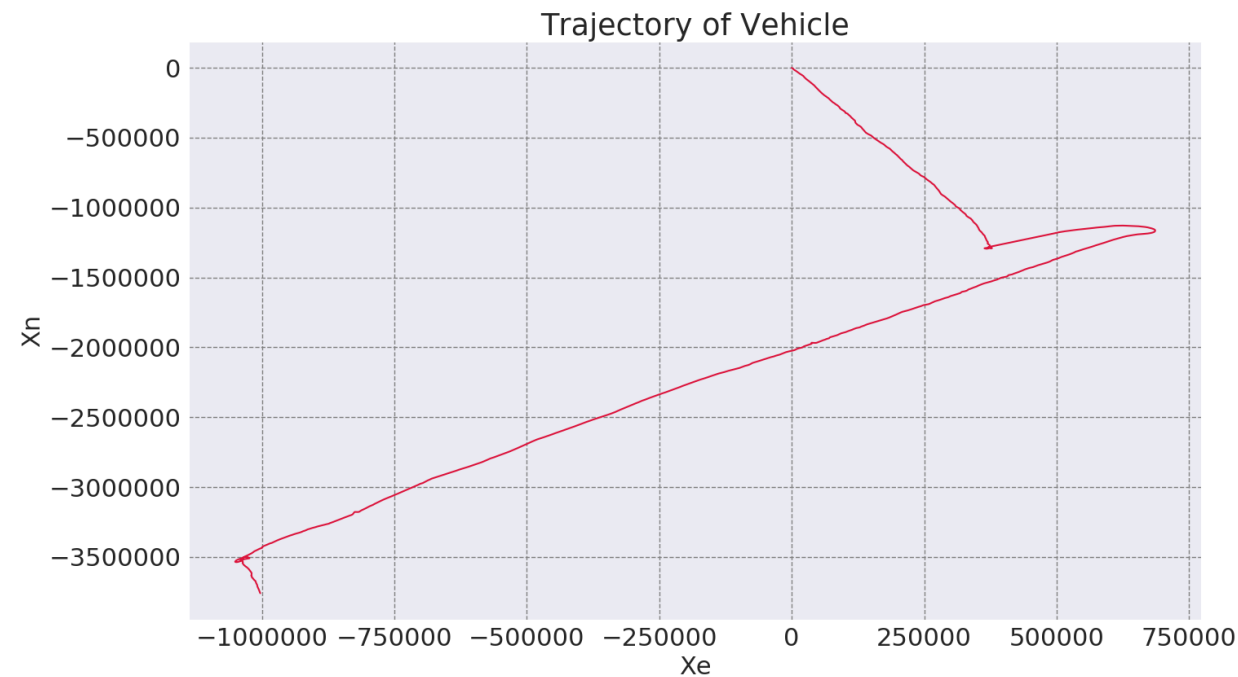
Q8)

The data was processed to find the vehicle's position and direction using heading and IMU measurements. The IMU path was adjusted to match the GPS path by scaling it down by 0.75 times. To check the accuracy, GPS and IMU paths were plotted together. The IMU path didn't perfectly match the

GPS path but followed a similar figure-eight pattern. The position error could be due to issues with the IMU, calibration errors, or changes in vehicle motion. In our test, GPS and IMU positions matched within 2 meters for about 15 minutes before they started to drift apart because of IMU errors. Sometimes, IMU can give better position estimates than GPS. In conclusion, the dead reckoning analysis with IMU data can estimate the vehicle's position with decent accuracy.

Taking into account the raw IMU measurements, we can rely on the IMU for navigation for about 4-5 seconds. Using Allan deviation, we can extend this time by an additional 12-13 seconds to improve the IMU-based position estimation. From the observations, it appears that the IMU and GPS data align well until the first turn, as shown in the provided graph. To maintain an accurate trajectory, it's necessary to apply a position fix and adjust the data, such as scaling and modifying the headings.

Unfortunately, I was not able to manipulate and twist data to align it properly with the GPS data.



#### 4.2 Estimation of $x_c$

The position and velocity of the center-of-mass (CM) of the vehicle is shown by  $R$  and  $V$ , respectively. The inertial sensor is displaced from the CM by  $r = (x_c, 0, 0)$  note that this vector is constant in the vehicle frame and assumes that the displacement of the IMU sensor is only along the x-axis. The velocity of the inertial sensor is:

$v = V + \omega \times r$        $\omega$  is the is the z-axis angular velocity from the gyroscope

$r = (x_c, 0, 0)$

$V$  is actual linear velocity

$v$  stands for measured linear velocity

$$\ddot{x} = \dot{v} + \omega \times v = \ddot{X} + \dot{\omega} \times r + \omega \times \dot{X} + \omega \times (\omega \times r)$$

Then the range of possible  $x_c$  values were found using the equation:  $x_c = (V - v)/\omega$

Using the above equation we come to know that  $x_c$  and  $\omega$  are inversely proportional, which means, If we take small values of  $\omega$  it will give large value of  $x_c$  and vice versa. This method may isolate the radius of rotation of the car making the turn. And, when the car was driving in a straight line at a constant speed  $V$  was set as the minimum velocity, which would return a range of more reasonable estimates.

$X_c = (y_{\text{observed}} - \dot{x} * \omega) / \dot{\omega}$

Offset is determined by taking the mean of  $x_c \rightarrow 0.47808478112468217$  meters

It can be said that the sensor was 47.8 cm away from the center of gravity of the car. The sensor was placed near the dashboard of the car.

Note:

I was unable to submit my assignment on time due to unforeseen circumstances. I had an eye injury that prevented me from completing the assignment as planned. I had already reached out to TA Skanda and explained my situation, and they were kind enough to grant me an extension. However, due to the severity of my injury, I was unable to complete the assignment even within the extended deadline. I apologize for any inconvenience caused by my delay in submitting the assignment.