Q1) The time required for servicing transmissions is normally distributed with mean = 45 minutes and standard deviation = 8 minutes.The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment ?

A) 0.3875

B) 0.2676

C) 0.5

D) 0.6987

Ans) We have a normal distribution with mean=45 and standard deviation=8. Let X be the amount of time it takes to complete the repair on a customer's car.To finish in one hour you must have X <= 50 so the question is to find P(X > 50)

P(X > 50) = 1 - P(X <= 50)

Z = (X-45)/8

Thus the question can be answered by using the normal table to find

P(X <= 50) = P(Z <= (50-45)/8) = P(Z <= 0.625) = 73.4%

Probability that the service manager will not meet his demand will be (100-73.4) = 26.6% or 0.2676

Q2) The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean=38 and standard deviation=6.For each statement below, please specify True or False.If False,explain briefly why;

A) More employees at the processing center are older than 44 than between 38 and 44.

B) A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans) A) We have a normal distribution with mean = 38 and standard deviation = 6. Let X be the number of employees.So according to the question;

Probability of employees greater than age 44 = P(X > 44)

P(X > 44) = 1 - P(X <= 44)

Z = (X - 38)/6

Thus the question can be answered by using the normal table to find

P(X <= 44) = P(Z <= (44-38)/6) = P(Z <= 1) = 84.1345%

Probability that the employees will be greater than the age of 44 = 100 - 84.1345 = 15.86%

So the probability of number of employees between 38 and 44 years of age = P(X < 44) - 0.5

= 84.1345 - 0.5 = 34.1345%

Therefore the statement that "More employees at the processing center are older than 44 than between 38 and 44" is True

B) Probability of employees less than age of 30 = P(X < 30)

Z = (30 - 38)/6

Thus the question can be answered by using the normal table to find

P(X <= 30) = P(Z <= (30-38)/6) = P(Z <= -1.333) = 9.12%

So the number of employees with probability 0.912 of them being under the age 30 = 0.0912*400 = 36.48 (36 employees)

Therefore, the statement B of the question is True.

Q3) If X1 ~ N(mean1,variance1) and X2 ~ N(mean2,variance2) are normal random variables, then what is the difference between 2X1 and X1+X2 ? Discuss both their distributions and parameters.

Ans) As we know that if X ~ N(mean1,variance1) and Y ~ N(mean2,variance2) are two independent random variables then

X + Y ~ N(mean1 + mean2, variance1 + variance2) and X - Y ~ N(mean1 - mean2, variance1 - variance2).

Similarly,if Z = aX + bY, where X and Y are as defined above. Z is linear combination of X and Y, then

Z ~ N(a*mean1 + b*mean2, a^2*variance1 + b^2*variance2)

Therefore in the question

2*X1 ~ N(2*mean, 4*variance) and X1 + X2 ~ N(mean1 + mean2, variance1 + variance2) ~ N(2*mean, 2*variance)

2*X1 - (X1 + X2) = N(4*mean, 6*variance)

Q4) Let X ~ N(100,20^2).Find two values, a and b, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A) 90.5, 105.9

B) 80.2, 119.8

C) 22, 78

D) 48.5, 151.5

E) 90.1, 109.9

Ans) Since we need to find out the values of a and b,which are symmetric about the mean, such that the probability of a random variable taking a value between them is 0.99,we have to work out in a reverse order.

The probability of getting value between a and b should be 0.99

So, the probability of going wrong or the probability outside the a and b area is 0.01 (that is 1 - 0.99)

The probability towards left of a = -0.005 (that is 0.01/2)

The probability towards right from b = +0.005 (that is 0.01/2)

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the standard normal variable Z (Z value),we can calculate the X values.

Z = (X - mean)/standard deviation

For probability 0.005 the Z value is -2.57 (from Z table)

(Z * standard deviation) + mean = X

Z(-0.005)*20 + 100 = -(2.57)*20 + 100 = 48.6

Hence, option D is correct.

Q5) Consider a company that has two different divisions.The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5,3^2) and Profit2 ~ N(7,4^2) respectively. Both the profits are in $ million.

Answer the following questions about the total profit of the company in rupees. Assume that $1 = Rs 45

A) Specify a rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

B) Specify the 5th percentile of the profit (in rupees) for the company.

C) Which of the two divisions has a larger probability of making a loss in a given year ?

Ans) Given that $1 = Rs 45, Profit1 ~ N(5,3^2), Profit2 ~ N(7,4^2)

Thus, company's profit : P ~ N(5+7.3^2+4^2) = N(12,5^2)

A) 95% of the probability lies between 1.96 standard deviation of the mean

This range is;

(12 - 1.96 * 5.12 + 1.96 * 5)

($ 2.2M, $22.8M)

(Rs 99M, Rs 1026M)

B) Fifth percentile is calculated as;

P(Z <= (p-12)/5) = 0.05

From p values of Z score table, we get;

(p-12)/5 = -1.644

p = 12 - 8.22 = 3.78

Thus at $3.78M dollars, or Rs. 170.1M amount, 5th percentile of profit lies or 5th percentile of profit is Rs. 170.1 Million

C) Loss is when profit < 0

Thus: p < 0

The first division of a company, thus has a larger probability of making a loss in a given year.