CS 765 Assignment 1

21000060, 210050112, 210050162

February 2024

1 Question 2

The choice of the exponential distribution stems from its ability to represent the inter-arrival time of a system where, at any given moment, there exists a probability of an event occurring—such as a transaction being generated by any peer. Let β represent a parameter proportional to the likelihood of a transaction being generated instantly. Hence, within a very small time interval δ, the likelihood of an event can be approximated as $\beta\delta$. Consequently, the probability of no block occurring for a duration of $n\delta$, or equivalently, the probability of the inter-arrival time exceeding $n\delta$, is expressed as $(1-\beta\delta)^n$. Letting $n\delta$ be denoted as x, and considering the inter-arrival time as a random variable I, we arrive at the expression

$$P(I > x) = \left(1 - \frac{\beta x}{n}\right)^n.$$

As δ approaches zero to imply events can happen at any moment, and letting n approach infinity to ensure a non-zero time interval, we obtain

$$P(I > x) = \lim_{n\to\infty} \left(1 - \frac{\beta x}{n}\right)^n = e^{-\beta x}.$$

This expression represents the Cumulative Distribution Function of the exponential distribution.

2 Question 5

The exponential distribution is characterized by its probability distribution function:

$$P(I = x) = \beta e^{-\beta x}$$

where a larger β implies a higher likelihood of obtaining a smaller value when sampling the random variable. The mean of this distribution is given by $1/\beta$. In our scenario, this suggests:

$$\frac{1}{\beta} = \frac{96 \text{ kbits}}{c_{ij}}$$

This indicates that if c_{ij} is inversely related to the mean, it is directly related to the β parameter of the exponential distribution, from which d_{ij} is sampled. This adjustment is necessary because as the link speed c_{ij} increases, the probability of receiving a smaller value for the inter-arrival time between queues (modeled as a Poisson process) – which corresponds to the queuing delay d_{ij} – also increases. Higher link speeds facilitate faster packet transfer across the link, resulting in reduced waiting times at the queue.

3 Question 7

Our goal is to find Tk, ensuring that the average time between any two blocks generated by different nodes is I. Additionally, we aim to have nodes with higher hashing power produce blocks more frequently. To achieve this, we assign a mean time of I/hi for each peer, ensuring that nodes with greater hashing power experience shorter inter-arrival times.

To calculate the average time between blocks from any two peers, we consider the probability that none of the peers generates a block within a given time frame. Mathematically, this is expressed as:

$$P(I_1, I_2, ..I_n > x) = P(I_1 > x) \cdot P(I_2 > x) \cdot ...$$

This computation relies on the independence of the random variables. By assigning a mean time of I/hi for each peer, the expression simplifies to:

$$\prod_{i=1}^{n} e^{-h_i x/I} = e^{-\sum h_i x/I} = e^{-x/I}$$

Here, the mean of this distribution equals the desired value of I, justifying the choice of I/hi for the mean time.

4 Question 8

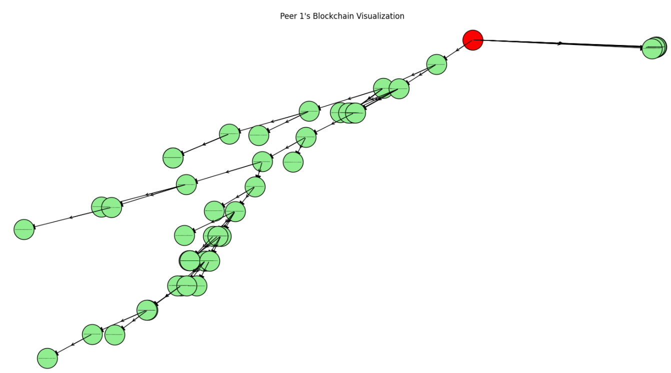

Figure 1: peers=20, meanTx = 1000, z0 = 0.9, z1=0.9

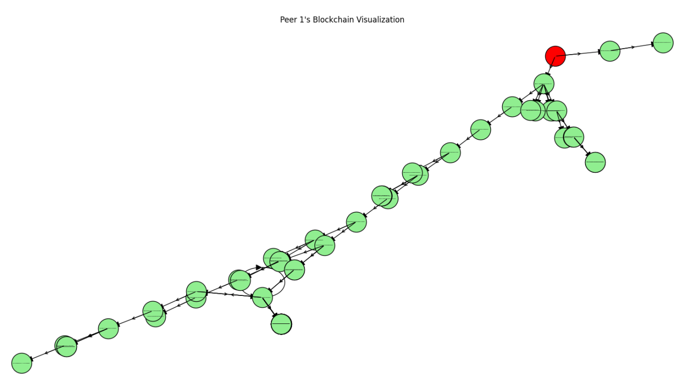

Figure 2: peers=20, meanTx = 1000, z0 = 0.5, z1=0.9

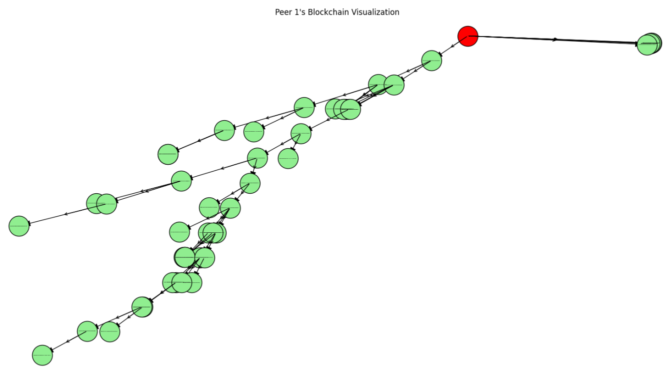

Figure 3: peers=20, meanTx = 1000, z0 = 0.9, z1=0.5

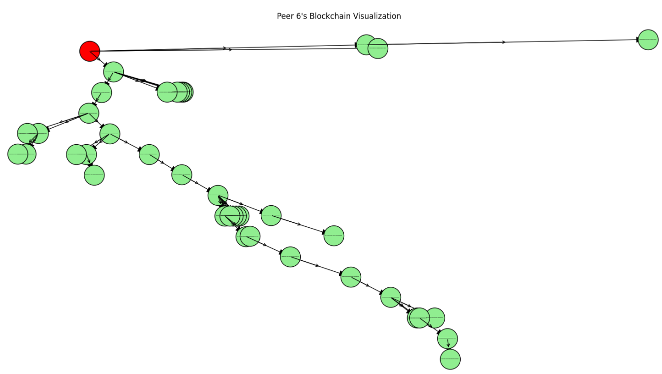

Figure 4: peers=20, meanTx = 1000, z0 = 0.1, z1=0.1

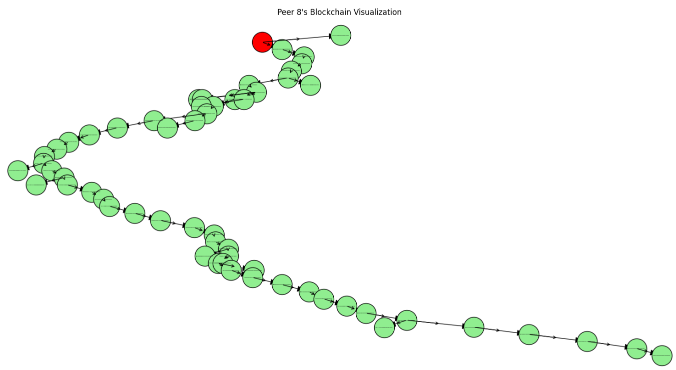

Figure 5: peers=20, meanTx = 10000, z0 = 0.1, z1=0.1

5 Visualization and Experimentation

5.1 Peers

Increasing the number of peers generally leads to increased branching, as each block now needs to propagate through a larger network, which takes more time. Consequently, the probability of another block being generated during this time interval increases.

For Slow: 50.0, LowCPU: 50.0, Mtransactiontime: 100.0, with 5 peers, the ratio of blocks in the longest chain to the total number of blocks is 0.20408163265306123 (total blocks: 478), and for 20 peers, the ratio remains the same at 0.11764705882352941 (total blocks: 138).

With Slow: 50.0, LowCPU: 50.0, Mtransactiontime: 1000.0, for 5 peers, the ratio of blocks in the longest chain to the total number of blocks is 0.5306122448979592 (total blocks: 477), and for 20 peers, the ratio changes to 0.41933483870967744 (total blocks: 138).

When Slow: 50.0, LowCPU: 50.0, Mtransactiontime: 10000.0, with 5 peers, the ratio of blocks in the longest chain to the total number of blocks is 0.8205128205128205 (total blocks: 478), and for 20 peers, the ratio becomes 0.7368421052631579 (total blocks: 137).

5.2 CPU

On increasing the number of low hashing power nodes, each block takes more time to generate and hence, gives more time to resolve forks and hence branching should be less on increasing the number of low CPU nodes. Taking Peers: 20 Slow: 50.0 Mtransactiontime: 100.0, for LowCPU= 90, we get the Ratio of Blocks in Longest Chain to Total Number of Blocks : 0.07407407407407407

(Total blocks : 162), for LowCPU = 70, we get the Ratio of Blocks in Longest Chain to Total Number of Blocks : 0.09259259259259259, for LowCPU = 50, we get the Ratio of Blocks in Longest Chain to Total Number of Blocks : 0.11764764704882616 (Total blocks : 187), for LowCPU = 30, we get the same ratio as Ratio of Blocks in Longest Chain to Total Number of Blocks : 0.2780748631016043 (Total blocks : 187) and for LowCPU = 10, we get the same ratio as Ratio of Blocks in Longest Chain to Total Number of Blocks : 0.3592392389754819 (Total blocks : 186)

5.3 Speed

Increasing the number of slow peers generally leads to increased branching, as each block now takes longer to propagate through the same network. Consequently, the probability of another block being generated during this time interval increases.

With 20 peers, LowCPU at 90.0, and Mtransactiontime at 10000.0, for LowCPU = 10, the ratio of blocks in the longest chain to the total number of blocks is 0.8060453400503779 (total blocks: 218). For LowCPU = 50, the ratio is 0.7810945273631841 (total blocks: 217), and for LowCPU = 90, the ratio remains the same at 0.7663316582914573 (total blocks: 221).