Indian Institute of Technology Bombay

Project Report

AE 308
Control Theory

Compensator Design

Ravi Kumar
210010052

Course Instructor: Prof. Arnab Maity

1 Introduction

The given system is a third order type one system with a zero at origin and two poles at -10 and -1. The transfer function of the system is given by:

$$G(s) = \frac{K}{s(0.1s+1)(s+1)}$$

Bode plot and root locus of the system (taking $K = 4$):

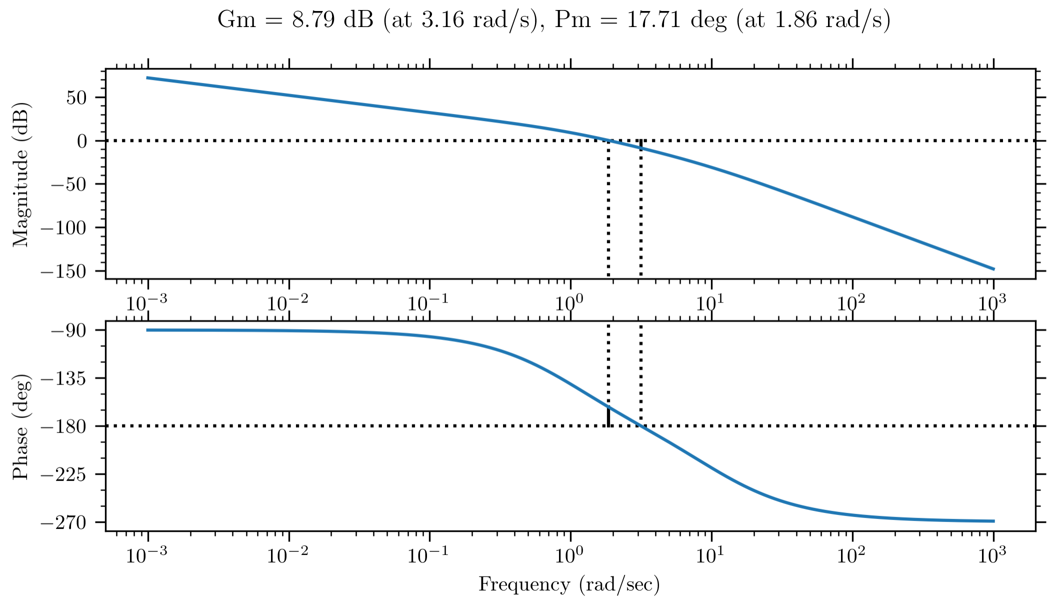

Figure 1: Bode plot of the uncompensated system

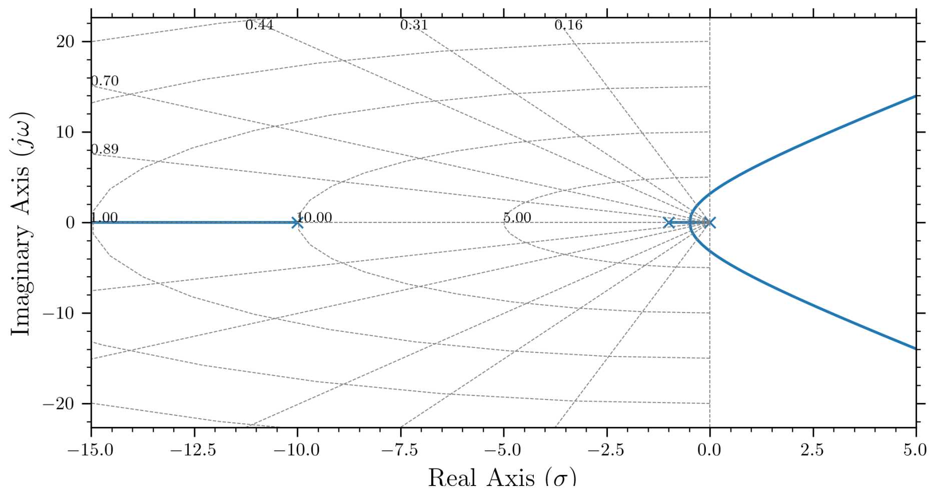

Figure 2: Root locus of the uncompensated system

Step response of the system (taking $K = 4$):

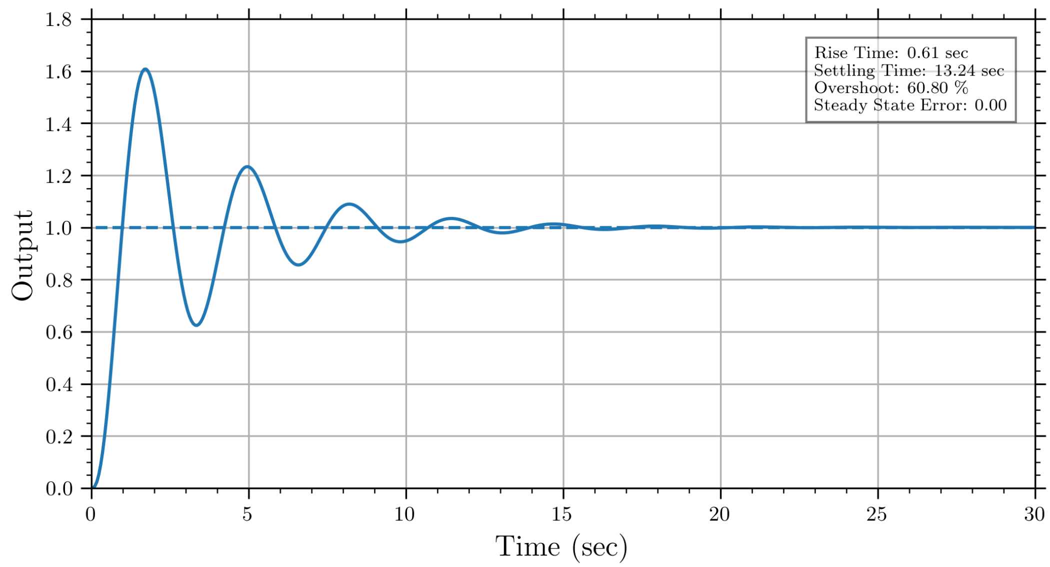

Figure 3: Step response of the uncompensated system

Ramp response of the system (taking $K = 4$):

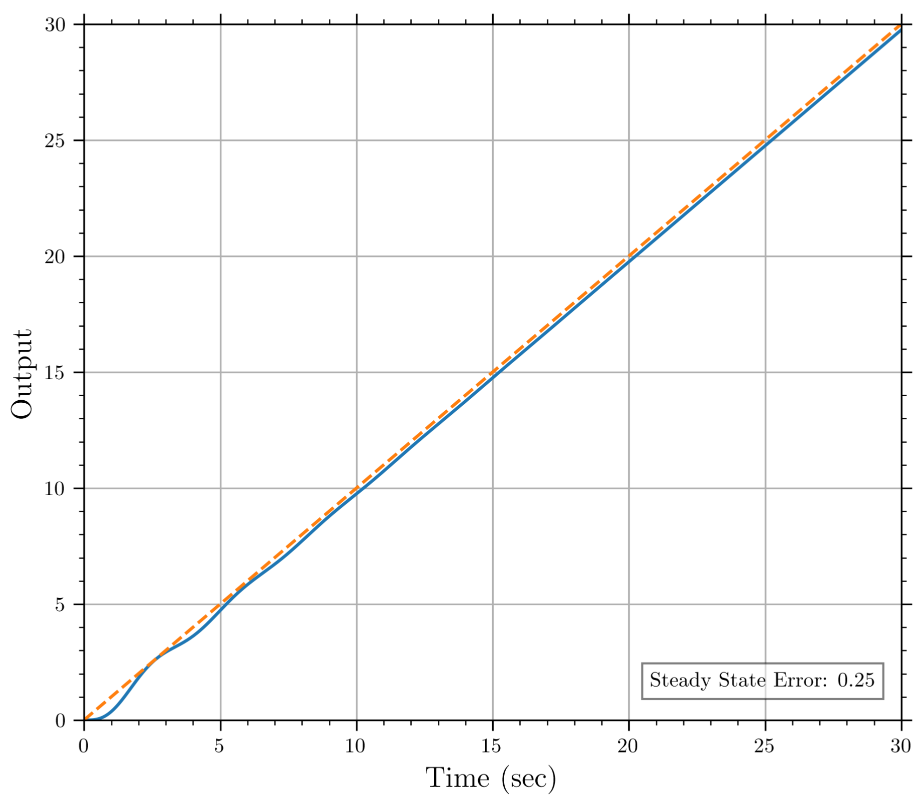

Figure 4: Ramp response of the uncompensated system

2 Control Objectives

The following control objectives are to be achieved:

1. Static velocity error constant (K_v) should be 4.0 sec^{-1}
2. Phase margin should be greater than 45°
3. Gain margin should be greater than or equal to 8 dB

3 Compensator Design

According to the objectives, we will determine the value of K to yield the desired static velocity error constant.

$$K_v = \lim_{s \to 0} sG(s) = K = 4$$

$$\therefore \text{ Our uncompensated system is } G(s) = \frac{4}{s(0.1s+1)(s+1)}$$

Bode Plot Method

The system's initial phase margin is 17.71°. The required phase margin should be greater than 45°. So, we need an additional phase of 27.29°.

For safety, we will take the phase margin to be 50°. The ω for which the phase margin is 50° is 0.723 rad/sec. At this frequency, we get a gain of 13.01 dB.

$$\implies w_{50} = 0.723\text{rad/sec}$$

$$|G(jw_{50})| = 13.01\text{dB}$$

Now, as a general rule, we add the zero a decade before the desired w, i.e. at $w = \frac{w_{50}}{10}$ in a lag compensator. So, location of the zero is -0.0723. We also want gain at this frequency to be lowered by 13.01 dB. Using this information, the location of pole is as follows:

$$20\log_{10}\left[\frac{\text{zero}}{\text{pole}}\right] = |G(jw_{50})|$$

$$\text{pole} = \text{zero} \times 10^{\frac{-|G(jw_{50})|}{20}}$$

$$\therefore \text{pole} = -0.0723 \times 10^{\frac{-13.01}{20}} = -0.0162$$

The transfer function of the lag compensator is given by:

$$G_c(s) = K_c\left[\frac{s+0.0723}{s+0.0162}\right]$$

Using K_c as $\dfrac{\text{pole}}{\text{zero}}$ to get unity gain for compensator, we finally get the following transfer function:

$$G_c(s) = 0.224\left[\frac{s+0.0723}{s+0.0162}\right]$$

Overall transfer function of the compensated system is given by:

$$G_c(s)\cdot G(s) = \frac{4\times 0.224}{s(0.1s+1)(s+1)}\left[\frac{s+0.0723}{s+0.0162}\right]$$

4 Simulation Results

Bode plot and root locus of the compensated system:

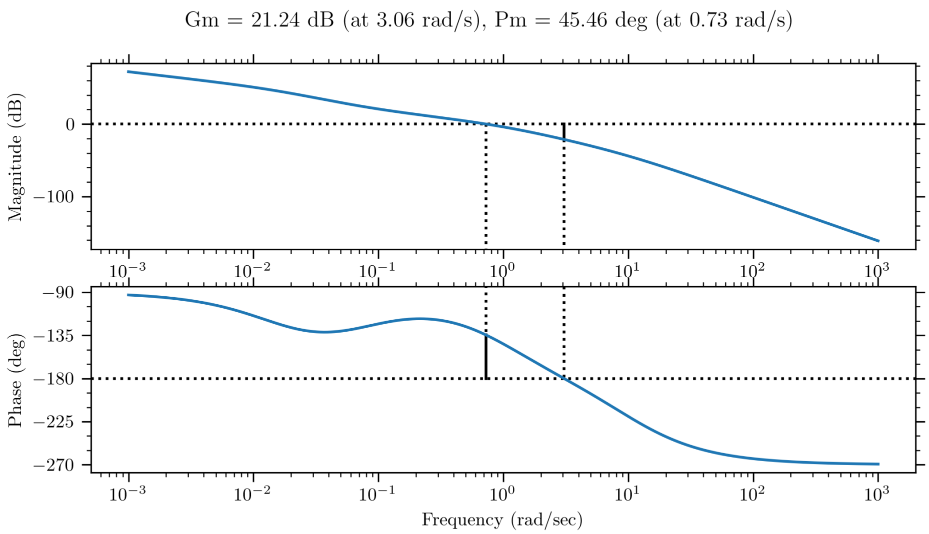

Figure 5: Bode plot of the compensated system

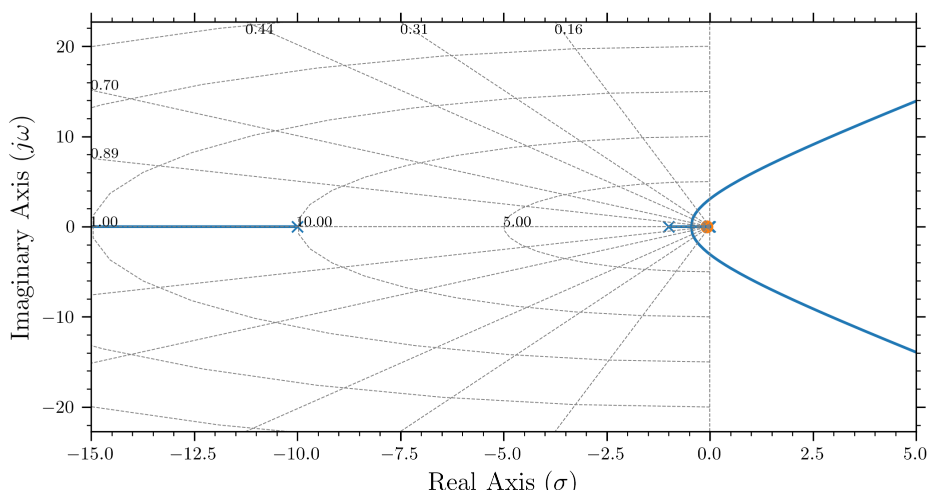

Figure 6: Root locus of the compensated system

Step response of the compensated system:

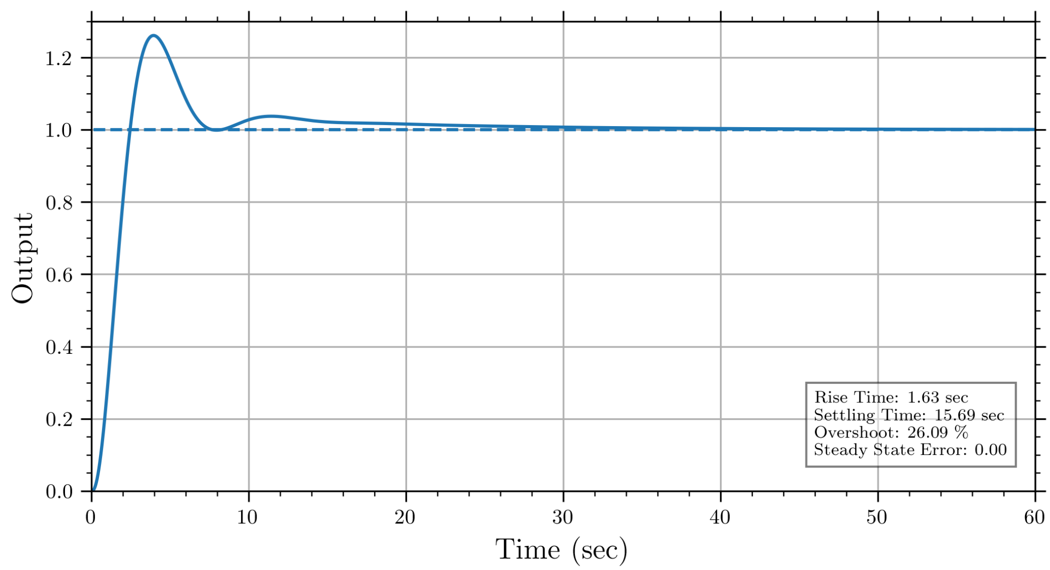

Figure 7: Step response of the compensated system

Ramp response of the compensated system:

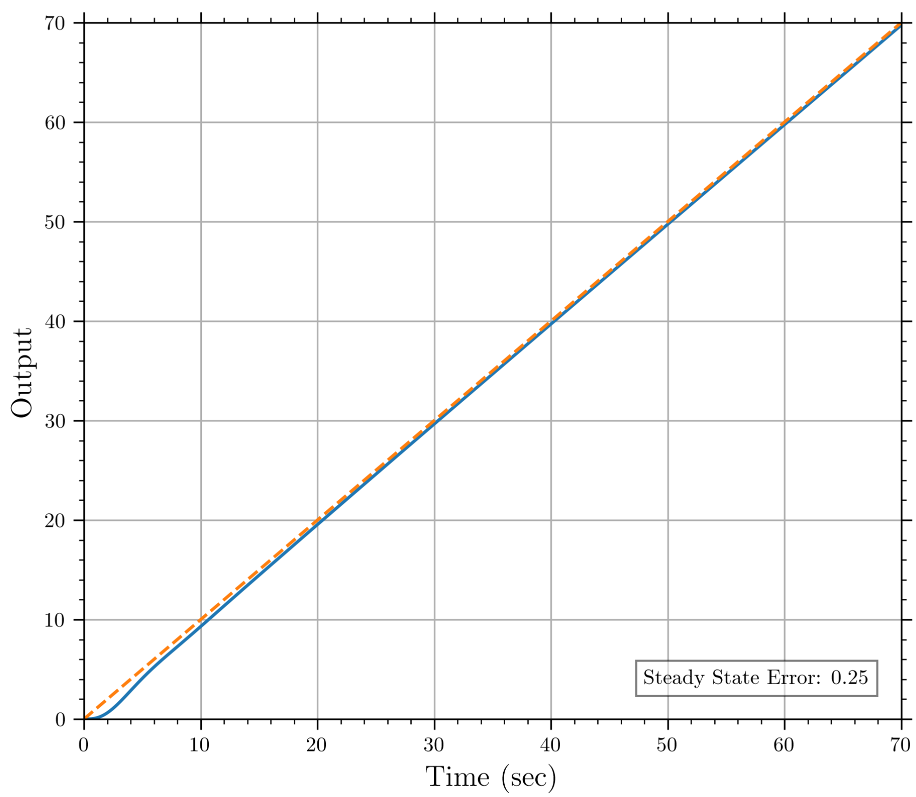

Figure 8: Ramp response of the compensated system

5 Conclusion

We can see that the design requirements have been met, with a gain margin of 21.24 dB $\geq$ 8 dB and a phase margin of $45.46° > 45°$.

By varying the amount of phase margin, we can achieve different compensators that can accomplish the same task. Below is a plot of admissible $\phi_{\max}$ and $\omega_{\max}$ that can be taken to achieve the desired gain margin and phase margins. Where $\phi_{\max} = \sin^{-1}\left(\frac{1-a}{1+a}\right)$ and $\omega_{\max} = \frac{1}{\tau\sqrt{a}}$, for a lag compensator ($a < 1$) of the form $G_c(s) = \frac{a\tau s + 1}{\tau s + 1}$.

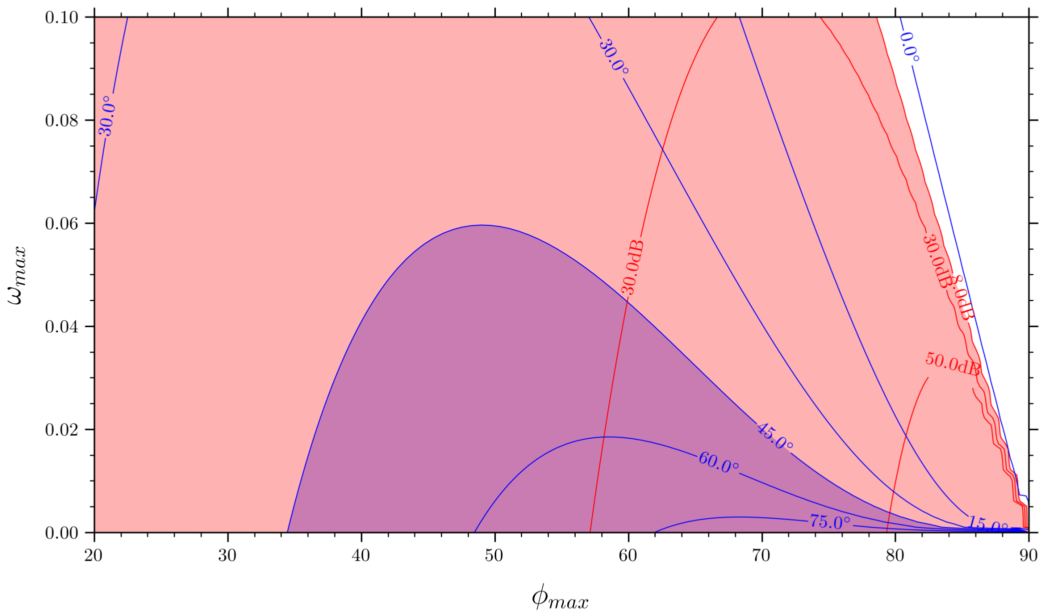

Figure 9: Contours of constant gain margin and phase margins

Any value of $\phi_{\max}$ and $\omega_{\max}$ that lie in the common region of the two contours (red and blue) will satisfy the design requirements.