

1. Find an SVD of the matrix $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$.
Clearly describe the steps used for the computation.

2. An SVD of matrix A is given by

$$\begin{bmatrix} 0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8 & -0.6 \\ 0 & 0.6 & 0.8 \end{bmatrix}^T$$

(a) Matrix A has _____ rows and _____ columns.

Rank of A = _____ Nullity of A = _____

(b) From the SVD, write down a basis of each of the following subspaces:
 $r(A)$, $N(A)$, $c(A)$ and $N(A^T)$.

(c) How many different SVDs does matrix A have? Explain.

(d) Let A^+ be the pseudoinverse of A . Find $A^+A \begin{pmatrix} 1 \\ 0.2 \\ 1.4 \end{pmatrix}$.

3. Let A be a 3×3 symmetric matrix with $\det(A)=240$. Suppose that two eigenvalues of A are 20 and 6 with corresponding eigenvectors $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. Find A and its SVD.