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Concept

Short answer (1 point each)

6. Find the value of c and d that makes the null space of the matrix $\begin{pmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & c & 2 & 2 \\ 0 & 0 & 0 & d & 2 \end{pmatrix}$ 3-dimensional.

   $c = 0 \quad d = 2$.

7. Let A = $[1 \quad 2 \quad 3]$. The dimension of span$(N(A) \cup N(A)^{\perp})$ is 3

   nul(A) ∪ row(A)

8. If the ranks of $n \times n$ matrices A and B are a and b, the minimum rank of AB can be written in terms of a, b, n as 0

   \# pivot

9. A linear function f is on-to but not one-to-one. If the domain is $\mathbb{R}^6$ and codomain is $\mathbb{R}^4$, the nullity of a matrix that represents this function is 2.

   RRE

10. Let B be a matrix of size 5 rows and 2 columns. If AB is invertible, the dimension of $N(A)$ is 3.

    $B = \left[\; \right]_{5\times 2}$ $\quad A = \left[\; \right]_{k\times 5}$

11. Suppose $C(A) = span(\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\4\\6\end{bmatrix}\right\}$ and $N(A) = span(\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}\right\})$. Find A.

    no A

12. Let A be an $n \times m$ matrix such that $N(A) = \{0\}$. The dimension of the column space of $A^T A$ is m.

True or False

13. T Suppose A is an m x n matrix such that $A\boldsymbol{x} = \boldsymbol{b}$ can be solved for any choice of $\boldsymbol{b} \in \mathbb{R}^m$. Then the columns of A form a basis for $\mathbb{R}^m$.
14. T The column space of an m x n matrix is a subspace of $\mathbb{R}^m$.
15. F The rows of $n \times n$ matrix A are linearly independent so the null space of A is an empty set.
16. F Only the columns of A that correspond to the pivot variables can form a basis of the column space of A.

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Show how to solve (3 points each)

17. Let the set { *u*, *v*, *w* } be a basis for subspace *V*. Prove that the set { *u*+*v*, *v*+*w*, *w*+*u* } is also a basis for *V*.

$V = \text{span}(\{u, v, w\})$

$= au + bv + cw \quad \forall a, b, c \in \mathbb{R}$

$\text{span}(\{u+v, v+w, w+u\}) = x(u+v) + y(v+w) + z(w+u) \quad \forall x, y, z \in \mathbb{R}$

$= (x+z)u + (x+y)v + (y+z)w = V$

$\therefore$ $\{u+v, v+w, w+u\}$ เป็น basis ของ V

18. Find matrix A such that $N(A^T)^\perp = N(A) = span(\{\binom{1}{1}\})$ and $A\binom{2}{1} = \binom{3}{3}$.

$A_{2\times2}$

~~$\text{row}(A) = \text{perp}(\text{nul}(A)) = \text{perp}(\text{span}(\{\binom{1}{1}\}))$~~ $= \text{perp}(\{\binom{1}{1}\}) = \text{span}(\{\binom{-1}{1}\})$

$A\binom{2}{1} = \binom{3}{3}$ $\qquad A\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 3 & 0 \end{bmatrix}$

$A\binom{1}{1} = \binom{0}{0}$ $\qquad A\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 3 & 0 \end{bmatrix}$

$A = A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 3 & -3 \end{bmatrix}$

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19. Find the null space of the following $n \times n$ matrix.

$$A = \begin{bmatrix} 1 & 2 & \dots & n \\ 2 & 3 & \dots & n+1 \\ \vdots & \vdots & \ddots & \vdots \\ n & n+1 & \dots & 2n-1 \end{bmatrix}$$

RRE (A)

$$\begin{bmatrix} 1 & 0 & -1 & -2 & \dots & 2-n \\ 0 & 1 & 2 & 3 & \dots & n-1 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & & & & & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$