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2110201 Quiz3

Basic Skill

1. (2 points) A non-empty subset of $S \subset \mathbb{R}^n$ is called a subspace if it is

   (a)____

   (b)____

2. (4 points) Find the null space of the following matrix

$$\begin{bmatrix} 0 & 1 & 7 & 0 & 8 & 2 & 4 \\ 0 & 0 & 0 & 0 & 5 & 1 & 1 \\ 0 & 0 & 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 \end{bmatrix}$$

3. (2 points) Let $S = \text{span}(\{v_1, v_2\})$ where $v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$. Find $S^{\perp}$.

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4. (8 points) Let $A = \begin{bmatrix} 2 & 2 & 3 & 3 \\ -3 & -3 & 2 & 2 \\ 2 & 2 & 1 & 1 \end{bmatrix}$.

   Given that $\mathrm{RRE}(A) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $\mathrm{RRE}(A^T) = \begin{bmatrix} 1 & 0 & \frac{7}{13} \\ 0 & 1 & \frac{-4}{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

   Find a basis for the null space, column space, row space and left null space of A.
   What is the rank and nullity of A?

5. (4 points) Which of the following sets in $\mathbb{R}^3$ is dependent? Which is independent?

   a. $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}, \begin{pmatrix} 5 \\ 6 \\ 7 \end{pmatrix} \right\}$ b. $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \right\}$

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Concept

Short answer (1 point each)

6. Find the value of c and d that makes the null space of the matrix $\begin{pmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & c & 2 & 2 \\ 0 & 0 & 0 & d & 2 \end{pmatrix}$ 3-dimensional.
___________________________________.

7. Let $A = [1 \quad 2 \quad 3]$. The dimension of span$(N(A) \cup N(A)^{\perp})$ is _____________

8. If the ranks of $n \times n$ matrices A and B are a and b, the minimum rank of AB can be written in terms of a, b, n as _____________________________

9. A linear function f is on-to but not one-to-one. If the domain is $\mathbb{R}^6$ and codomain is $\mathbb{R}^4$, the nullity of a matrix that represents this function is _______.

10. Let B be a matrix of size 5 rows and 2 columns. If AB is invertible, the dimension of $N(A)$ is___.

11. Suppose $C(A) = span(\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\4\\6\end{bmatrix}\right\})$ and $N(A) = span(\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}\right\})$. Find A.

12. Let A be an $n \times m$ matrix such that $N(A) = \{0\}$. The dimension of the column space of $A^T A$ is _____.

True or False

13. _____ Suppose A is an m x n matrix such that $A\boldsymbol{x} = \boldsymbol{b}$ can be solved for any choice of $\boldsymbol{b} \in \mathbb{R}^m$. Then the columns of A form a basis for $\mathbb{R}^m$.
14. _____ The column space of an m x n matrix is a subspace of $\mathbb{R}^m$.
15. _____The rows of $n \times n$ matrix A are linearly independent so the null space of A is an empty set.
16. _____Only the columns of A that correspond to the pivot variables can form a basis of the column space of A.

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<u>Show how to solve (3 points each)</u>

17. Let the set { *u*, *v*, *w* } be a basis for subspace *V*. Prove that the set { *u*+*v*, *v*+*w*, *w*+*u* } is also a basis for *V*.

18. Find matrix A such that $N(A^T)^{\perp} = N(A) = span(\left\{\begin{pmatrix}1\\1\end{pmatrix}\right\})$ and $A\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}3\\3\end{pmatrix}$.

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19. Find the null space of the following $n \times n$ matrix.

$$\begin{bmatrix} 1 & 2 & \dots & n \\ 2 & 3 & \dots & n+1 \\ \vdots & \vdots & \ddots & \vdots \\ n & n+1 & \dots & 2n-1 \end{bmatrix}$$