Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ID\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_Group\_\_\_\_\_\_\_\_\_\_

10 points each

1. Show that the map  given by  is one-to-one and onto. Is it an isomorphism?
2. Are any two planes through the origin in R3 isomorphic? Explain (find and isomorphism or show that one can never exist.)
3. Stating that a function is “linear” is different than stating that its graph is a line
   1. The function  given by  has a graph that is a line. Show that it is not a linear function.
   2. The function given by does not have a graph that is a line. Show that it is a linear function.
4. Consider the vector space where vector addition and scalar multiplication are not the one inherited from but rather are these *a+b* is the product of a and b, and *r.a* is the *r*-th power of *a* (This was shown to be a vector space in an earlier quiz.) Verify that the natural logarithm map is a homomorphism between these two spaces. Is it an isomorphism?
5. Show that every homomorphism from to  acts via multiplication by a scalar. Conclude that every nontrivial linear transformation of is an isomorphism. Is that true for transformations of ? ?
6. Where is linear, suppose that  for some vectors  from *W*. Answer the following questions with brief explanation.
   1. If the set of *w*’s is independent, must the set of *v*’s also be independent?
   2. If the set of *v*’s is independent, must the set of *w*’s also be independent?
   3. If the set of *w*’s spans W, must the set of *v*’s span *V*?
   4. If the set of *v*’s spans V, must the set of *w*’s span *W*?