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Homework #5

Ques ① Suppose that we have a sample of 100 people.....
Soln:-

Total number of people $(n) = 100$
Average height $E(X_i) = \mu = 150$
Standard Deviation $(\sigma) = 25$

Let Y is the total height

$$Y = X_1 + X_2 + X_3 + \dots + X_n \text{ , where } n = 100$$

$$EY = n\mu = 100 \times 150 = 15000$$

$$P(\text{height} < 17000) = P\left(\frac{Y - EY}{\sigma(\sqrt{n})} < \frac{17000 - EY}{\sigma(\sqrt{n})}\right)$$

$$= P\left(\frac{Y - 15000}{25 * 10} < \frac{17000 - 15000}{25 * 10}\right)$$

$$= P\left(\frac{Y - 15000}{250} < \frac{2000}{250}\right)$$

$$= \phi(8) \quad \text{Ans}$$

$\because \phi(8)$ is a large value and closer to 1 z-score.
therefore, prob. that total height is < 17000 is 1

Ques ② Assuming that random variable Y belongs to......
Soln:- $Y \sim \text{Binomial}(n = 49, p = 2/3)$

$$\mu = p = 2/3$$

$$\text{variance} = p(1-p) = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$

$$\text{Standard Deviation} = \sqrt{\text{Var}} = \frac{\sqrt{2}}{3}$$

$$P(Y>15) = 1 - P(Y \le 15)$$

$$= 1 - P\left(\frac{Y - \mu n}{\sigma(\sqrt{n})} \le \frac{15 - \mu n}{\sigma(\sqrt{n})}\right)$$

$$= 1 - P\left(\frac{Y - (\frac{2}{3} * 49)}{\frac{\sqrt{2}}{3} \times \sqrt{49}} \le \frac{15 - 49(\frac{2}{3})}{\frac{\sqrt{2}}{3} \times \sqrt{49}}\right)$$

$$= 1 - \phi(-5.35)$$

as $\phi(-5.35) \approx 0$, $P(Y>15) \approx 1$ Ans

Ques 3) The random variable X_i is the waiting time ......

Soln:-

$E(X_i) = \mu = 20$

std. dev. $(\sigma) = 8$

$n = 50$

Let Y be the total waiting time, then

$$Y = X_1 + X_2 + \ldots\ldots + X_{50}$$

~~Prob. of Y~~ $P(Y \text{ b/w } 900 \ \& \ 1100) = P\left(\frac{900 - n\mu}{\sigma\sqrt{n}} < \frac{Y - n\mu}{\sigma\sqrt{n}} < \frac{1100 - n\mu}{\sigma\sqrt{n}}\right)$

$$P(\text{event}) \approx \phi\left(\frac{1100 - n\mu}{\sigma\sqrt{n}}\right) - \phi\left(\frac{900 - n\mu}{\sigma\sqrt{n}}\right)$$

$$= \phi\left(\frac{1100 - 1000}{8 \times \sqrt{50}}\right) - \phi\left(\frac{900 - 1000}{8 \times \sqrt{50}}\right)$$

$$= \phi(1.768) - \phi(-1.768)$$

$$= 0.9612 - 0.0368$$

$$= 0.9224$$ Ans

Ques ④ Suppose that the weight of people in a specific ....

Soln:

$\mu = 150$

standard deviation $(\sigma) = 20$

(i) $P(\text{weight} < 140 \text{ pounds}) = P\left(\frac{X-\mu}{\sigma} < \frac{140-\mu}{\sigma}\right)$

$= P\left(Z < \frac{140-150}{20}\right)$

$= P(Z < -1/2)$

$= \phi(-0.5)$

$= \underline{0.3085}$ Ans

Hence, population that weighs < 140 pounds is $\underline{30.85\%}$.

(ii) $P(\text{weight} > 170 \text{ pounds}) = P\left(\frac{X-\mu}{\sigma} > \frac{170-\mu}{\sigma}\right)$

$= P\left(Z > \frac{170-150}{20}\right)$

$= 1 - \phi(1.0)$

$= 1 - 0.8413$

$= \underline{0.1587}$ Ans

Hence, population that weighs > 170 pounds is $\underline{15.87\%}$