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Home Work #3

Ques ①

$$f(x) = \begin{cases} \frac{3}{2}(1-x^2) & 0 \le x \le 1 \end{cases}$$

ⓐ Check that this is a valid PDF.

For a PDF to be valid, it should integrate 1 and should be non-negative.

$$\int_0^1 f(x)\,dx = \frac{3}{2}\int_0^1 (1-x^2)\,dx = \frac{3}{2}\left[x - \frac{x^3}{3}\right]_0^1$$

$$= \frac{3}{2}\left(1 - \frac{1}{3}\right) = \frac{3}{2} \times \frac{2}{3} = 1$$

$\therefore$ $f(x)$ is a valid PDF

ⓑ Calculate expected value

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\,dx$$

$$= \frac{3}{2}\int_0^1 x(1-x^2)\,dx = \frac{3}{2}\int_0^1 (x - x^3)\,dx$$

$$= \frac{3}{2}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{3}{2}\left[\frac{1}{2} - \frac{1}{4}\right] = \frac{3}{2} \times \frac{1}{4}$$

$$= \frac{3}{8}$$ Ans

ⓒ Calculate standard deviation of X

$$\text{St. Dev.} = \sqrt{\text{Variance}}$$

$$\text{Variance (Var)} = \int_{-\infty}^{\infty} x^2 f(x) - (E(x))^2$$

$$Var = \frac{3}{2}\int_0^1 x^2(1-x^2)\,dx - \left(\frac{3}{8}\right)^2$$

$$= \frac{3}{2}\int_0^1 x^2 - x^4\,dx - \frac{9}{64}$$

$$= \frac{3}{2}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 - \frac{9}{64} = \frac{3}{2}\left[\frac{1}{3} - \frac{1}{5}\right] - \frac{9}{64}$$

$$= \frac{3}{2}\left(\frac{2}{15}\right) - \frac{9}{64} = \frac{1}{5} - \frac{9}{64}$$

$$= \frac{64-45}{320} = \frac{19}{320} \text{ Ans}$$

$$\text{stdev.} = \sqrt{var} = \sqrt{\frac{19}{320}} = 0.24367 \text{ Ans}$$

Ques ② Average last time for a part = 10 years

ⓐ prob. that a computer part lasts more than 6 years.

Average time $(\mu) = 10$

$$\mu = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{\mu} = \frac{1}{10} = 0.1$$

$$P(\text{last more than 6 years}) = 1 - P(x \le 6)$$

$$= 1 - [1 - e^{-\lambda x}]$$

$$= 1 - [1 - e^{-0.6}]$$

$$= 1 - 0.4511$$

$$= 0.5489 \text{ Ans}$$

(b) Since, one part lasts for 10 years on an average.
Three parts if used one after another will last for $3 \times 10 = 30$ years. on an average.

(c) Probability that a computer part will last between 9 and 11 years

Ans = $p(x < 11) - p(X < 9)$

$= (1 - e^{-0.1 \times 11}) - (1 - e^{-0.1 \times 9})$

$= 1 - e^{-1.1} - 1 + e^{-0.9}$

$= 0.0736$ Ans

Ques (3) $Y \sim U(100, 300)$

$$f(y) = \begin{cases} \frac{1}{200} & \text{for } 100 \leq y \leq 300 \\ 0 & \text{for } y < 100 \text{ or } y > 300 \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

(a) $p(Y > 174) = (300 - 174) \times \frac{1}{200}$

$= 0.63$ Ans

(b) $p(100 < Y < 226) = \frac{226 - 100}{200}$

$= 0.63$ Ans

(c) Expected value $(E(x)) = \frac{a+b}{2} = \frac{300+100}{2} = \frac{400}{2}$

$= 200$ Ans

Variance (Var) $= \frac{(b-a)^2}{12} = \frac{(300-100)^2}{12} = \frac{(200)^2}{12}$

$= 3333.33$ Ans

Standard Deviation $(\sigma) = \sqrt{Var}$

$= 57.7349$ Ans

Ques 4. Average customer arrival per hour $= 30$

(a) On average, how many minutes elapse b/w two successive arrivals?

per hour 30 customer mean, on an average, each customer takes 2 minutes

$\therefore$ 2 minutes elapse b/w two successive arrivals.

(b) Since for 1 customer, it takes 2 minutes to arrive.
3 customers would take $2 \times 3 \rightarrow 6$ minutes to arrive.

(c) P(less than 1 minutes for next customer) i.e. event

$\mu = 2$

$\lambda = \frac{1}{\mu} = \frac{1}{2} = 0.5$

$P(\text{event}) = 1 - e^{0.5 \times 1}$

$= 0.3934$ Ans

(d) P(takes more than 5 minutes for the next customer to arrive) i.e. event

$$P(\text{event}) = P(y > 5) = 1 - P(y \leq 5)$$

$$= 1 - (1 - e^{-\lambda x})$$

$$= 1 - (1 - e^{0.5 \times 5})$$

$$= 0.082 \text{ Ans}$$