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Chapter 1

Non-Parametric Test

1. Definition

Non-parametric or distribution free test is a statistical procedure whereby the data does not match a normal distribution. The data used in non-parametric test is frequently of ordinal data type, thus implying it does not depend on arithmetic properties. Consequently, all tests involving the ranking of data are non-parametric and also no statement about the distribution of data is made.

2. Types of Data

Table 1.1 Types of Data

Discrete Data	<p>Discrete data are types of data that take only exact values.</p> <p>Some examples of discrete data are:</p> <ol style="list-style-type: none">1. The number of flight in an hour.2. The number of roses in each plant in a garden.3. The bags sizes of the employees in a company.
Continuous Data	<p>Unlike for discrete data, continuous data can take any numerical value.</p> <p>For instance:</p> <ol style="list-style-type: none">1. The length of a pencil (it can be 7cm, 8.2cm, 9.56cm...).2. The mass of a chocolate cake.3. The time taken by a person to finish his/her homework.
Nominal Data	<p>Nominal data refers to data that is organized in terms of a category or name. Nominal data is non-parametric.</p> <p>Here are some examples:</p> <ol style="list-style-type: none">1. The fat or thin characteristics of individuals.2. The colour of people's hair.3. Gender.4. Religion.

Ordinal Data	<p>Ordinal data deals with the ranking of items in order. It is the comparison of two or more items, i.e., finding the difference between the first, the second and the third and onwards. Ordinal data is also non-parametric.</p> <p>For example:</p> <ol style="list-style-type: none"> 1. Very happy, happy, unhappy, very unhappy. 2. Strongly like, like, neutral, dislike, strongly dislike.
Interval Data	<p>Interval data (also called integer) are types of data for which the measurement can be calculated and the difference in sizes can be compared.</p> <p>For instance:</p> <ol style="list-style-type: none"> 1. Dates. 2. Temperature in Fahrenheit. 3. Years.
Ratio data	<p>Ratio data is similar to interval data, but gives information with respect to an absolute zero. It can be multiplied and divided.</p> <p>Examples of ratio data are:</p> <ol style="list-style-type: none"> 1. Mass. 2. Length. 3. Distance. 4. Speed.

(Types of data, 2012)

1.3 Examples of Non-Parametric statistical tests

Table 1.2 Example of Non-Parametric Test

Non-Parametric Test	Aim of test
Wilcoxon signed rank test.	Comparison of two dependent samples.
Mann-Whitney U-test. Wilcoxon rank sum test.	Comparison of two independent samples.
Friedman test.	Comparison of three or more dependent samples.
Kruskal-Wallis H-test.	Comparison of three or more independent samples.

4. Types of hypotheses

There are two types of hypotheses namely:

1. The null hypothesis,

It is a statement which shows that there is no difference between groups, conditions or variables.

2. The alternative hypothesis, (also known as research hypothesis)

It is a statement which forecasts a difference or relationship between groups, conditions or variables. The alternative hypothesis can be:

- (a)either, **one-tailed hypothesis:** It forecasts a statistically considerable variation in a specific direction.
- (b)or, **two-tailed hypothesis:** It forecasts a statistically considerable change in no specific direction.

5. General steps to carry out a Non-Parametric Test

1. The null hypothesis, and the alternative hypothesis, are stated.
2. The significance level, related with the null hypothesis is set.
is normally set at 5% and therefore the confidence level is 95%.
3. The suitable statistic test is chosen. This is done by taking into account:
 - (a) The number of samples,
 - (b) Whether the samples are dependent or independent,
 - (c) and also, the type of data.
4. The test statistic is then calculated. For small sample, a method particular to a specific statistical test is used. Then, for large samples, the data is approximated to a normal distribution and the z -score is evaluated.
5. The value required to reject the null hypothesis is determined using the suitable table for critical values for the specific statistic.
6. The obtained value is compared with the critical value. This enables us to find the difference based on a specific significance level. Then, we can assert whether the null hypothesis should be rejected or not. For instance, for a two-tailed hypothesis with the null hypothesis is not rejected if
7. The results are explained and a conclusion is drawn out.

(ALejeune, 2010)

Chapter 2

Normality test

Non-parametric Test is carried out with non-normal set of data. In order to know if the samples of data collected are not normally distributed, normality tests are performed. Stating the null and alternative hypothesis:

- : The sampled population is normally distributed.
- : The sampled population is not normally distributed.

2.1 Shapiro-Wilk Test

Testing for normality is not perfect but one of the most robust normality tests is the Shapiro-Wilk test. It tests whether the sample is taken from a population whose distribution is normal.

The null and alternative hypotheses for this test are stated as follows:

- : Data are normally distributed.
- : Data are not normally distributed.

It is acknowledged that for this test to be carried out properly, the sample size must be between 3 and 2,000.

2.1.1 Formula to carry out Shapiro-Wilk Test

Where,

, is the least value in the sample,
, is the mean of the sample,

=

m= expected value
given that
s= is the number of values recorded
Q is called the covariance statics

2.1.2 Interpretation of Shapiro-Wilk Test Values

Accept, when p-value > α, where α is the significance level, given by α=0.05

2.1.3 Shapiro-Wilk Test in Excel

Shapiro-Wilk Normality Test (for 3 ≤ N ≤ 500)						
Index (i)	Data	Sorted	mi	ai		
1	0	0	-1.7394	-0.5150		
2	0	0	-1.2450	-0.3309		
3	0	0	-0.9458	-0.2489		
4	0	0	-0.7137	-0.1879		
5	1	1	-0.5150	-0.1356		
6	1	1	-0.3349	-0.0881		
7	3	3	-0.1651	-0.0435		
8	3	3	0.0000	0.0000		
9	3	3	0.1651	0.0435		
10	7	7	0.3349	0.0881		
11	15	15	0.5150	0.1356		
12	19	19	0.7137	0.1879		
13	22	22	0.9458	0.2489		
14	22	22	1.2450	0.3309		
15	23	23	1.7394	0.5150		

Figure 2.1(a) Data Input for Testing

Descriptive Statistics	
Sample Size (N):	15
Measures of Central Tendency (Location)	
Mean:	7.933
Median:	3.000
Measures of Spread	
Std. Deviation:	9.323
Variance:	86.924
Range:	23.000
IQ Range:	16.500
Measures of Shape	
Skewness:	0.780
Kurtosis:	-1.303
Shapiro-Wilk Test for Normality (Royston's Sign.)	
H0:	Data are normally distributed
HA:	Data are not normally distributed
W Statistics:	0.76931
p-value:	0.00152
Level of significance:	0.05000
Conclusion:	Reject
Data are not normally distributed	

The test should be rejected in order to carry out a non-parametric test

Figure 2.1(b) Testing Data

All the data samples of the different non-parametric tests that will be discussed further in the project have been carried out through the Shapiro Wilk Normality Test. This is to confirm that the data samples are not normally distributed and therefore the data can be used to perform a non-parametric test.

(Formations, études et conseil en statistiques)

Chapter 3

Wilcoxon Signed rank test

3.1 Introduction

The Wilcoxon signed rank sum test is a version of the dependent samples t-Test that can be performed on ordinal (ranked) data. It is an example of a non-parametric test. As for the sign test, the Wilcoxon signed rank sum test is used to test the null hypothesis that is the median of a distribution is equal to some value. It can be used:

- (a) In place of a one-sample t-test,
- (b) In place of a paired t-test or,
- (c) For ordered categorical data where a numerical scale is inappropriate but where it is possible to rank the observations.

2. Steps to perform a Wilcoxon signed rank test

1. The general way

- Step. 1 → State the null hypothesis, H_0 and the alternate hypothesis, H_A .
- Step. 2 → State α .
- Step. 3 → State decision rule.
- Step. 4 → Calculate Z-statistics → _____
- Step. 5 → State Results.
- Step. 6 → State conclusion.

3.2.2 For Paired data

1. State the null hypothesis - in this case it is that the median difference, M , is equal to zero.
2. Calculate each paired difference, $d_i = x_i - y_i$, where x_i , y_i are the pairs of observations.
3. Rank the d_i s, ignoring the signs (i.e. assign rank 1 to the smallest $|d_i|$, rank 2 to the next etc.)
4. Label each rank with its sign, according to the sign of d_i .
5. Calculate W^+ , the sum of the ranks of the positive d_i , and W^- , the sum of the ranks of the negative d_i . (As a check the total, $W^+ + W^-$, should be equal to $n(n+1)/2$, where n is the number of pairs of observations in the sample).

3.2.3 For Single set of data

- 1. State the null hypothesis - the median value is equal to some value M.
- 2. Calculate the difference between each observation and the hypothesized median, $d_i = x_i - M$.
- 3. Rank the d_i s, ignoring the signs (i.e. assign rank 1 to the smallest $|d_i|$, rank 2 to the next etc.)
- 4. Label each rank with its sign, according to the sign of d_i .
- 5. Calculate $W+$, the sum of the ranks of the positive d_i s, and $W-$, the sum of the ranks of the negative dis. (As a check the total, $W+ + W-$, should be equal $ton(n+1)2$, where n is the number of pairs of observations in the sample).

Under the null hypothesis, we would expect the distribution of the differences to be approximately symmetric around zero and the distribution of positives and negatives to be distributed at random among the ranks. Under this assumption, it is possible to work out the exact probability of every possible outcome for W. To carry out the test, we therefore proceed as follows:

- 6. Choose $W = \min (W-, W+)$.
- 7. Use tables of critical values for the Wilcoxon signed rank sum test to find the probability of observing a value of W or more extreme. Most tables give both one-sided and two-sided p-values. If not, double the one-sided p-value to obtain the two-sided p-value. This is an exact test.

3. Example of a Wilcoxon sign test:

- 1. The null hypothesis and alternate hypothesis are defined.

$H_0 =$ There is an increase in the consumer price index.			
$H_A =$ There is no increase in the consumer price index.			

Figure 3.1(a) Null Hypothesis & Alternate Hypothesis

- 2. The significance level is stated.

$\alpha = 0.05$			
-----------------	--	--	--

Figure 3.1(b) Significance Level

3. The sample data is implemented on an excel sheet as follows:

K17 fx				
	A	B	C	D
1	Input Cost Index for the construction of a single storey house			
2	Wilcoxon Signed test			
3	Table 1.1: Monthly sub-indices by input category, October 2011 to September 2012			
4				
5	Input Categories	2011		2012
6		Oct		Sep
7	LABOUR	\$ 103.30		\$ 75.00
8	HIRE OF PLANT	\$ 101.00		\$ 89.00
9	MATERIALS :	\$ 76.00		\$ 109.70
10	Hardcore (remplissage)	\$ 75.00		\$ 115.40
11	Cement	\$ 94.50		\$ 110.40
12	Sand	\$ 119.40		\$ 119.20
13	Aggregate	\$ 117.30		\$ 117.00
14	Block	\$ 109.80		\$ 114.10
15	Steel bars (armature)	\$ 109.80		\$ 108.40
16	Galvanised corrugated cast iron sheeting	\$ 102.10		\$ 101.50
17	Timber: (a) Carpentry	\$ 108.60		\$ 104.80
18	(b) Joinery	\$ 106.90		\$ 97.60
19	Aluminium openings	\$ 100.00		\$ 100.90
20	Metal openings	\$ 107.70		\$ 107.90
21	Ceramic tiles	\$ 99.10		\$ 102.40
22	Adhesive	\$ 103.60		\$ 104.10
23	Paint	\$ 111.30		\$ 78.00
24	Plumbing	\$ 102.80		\$ 103.40
25	Sanitary installation	\$ 104.20		\$ 100.00
26	Electrical installation	\$ 110.60		\$ 110.10
27	TRANSPORT	\$ 109.10		\$ 109.10
28	Total cost	\$ 2,172.10		\$ 2,178.00
29				

Figure 3.1(c) Implementation of Data in Excel Sheet

4. The difference of prices and the ranks are calculated. Each rank is then assigned to its sign.

Difference	Rank	R+ or R-	hypothesis wi
\$ 28.30	20	R+	
\$ 12.00	19	R+	
\$ (33.70)	2	R-	=IF(F7>0,"R+","R-")
\$ (40.40)	1	R-	
\$ (15.90)	3	R-	=RANK(F7,\$F\$7:\$F\$27,1)
\$ 0.20	11	R+	
\$ 0.30	12	R+	
\$ (4.30)	4	R-	
\$ 1.40	15	R+	
\$ 0.60	14	R+	
\$ 3.80	16	R+	
\$ 9.30	18	R+	
\$ (0.90)	6	R-	
\$ (0.20)	9	R-	
\$ (3.30)	5	R-	
\$ (0.50)	8	R-	
\$ 33.30	21	R+	
\$ (0.60)	7	R-	
\$ 4.20	17	R+	
\$ 0.50	13	R+	
\$ -	10	R-	

Figure 3.1(d) Calculating Difference & Rank

6. The sum of rank of positive differences, $\sum R^+$ and sum of rank of negative differences, $\sum R^-$ are calculated.

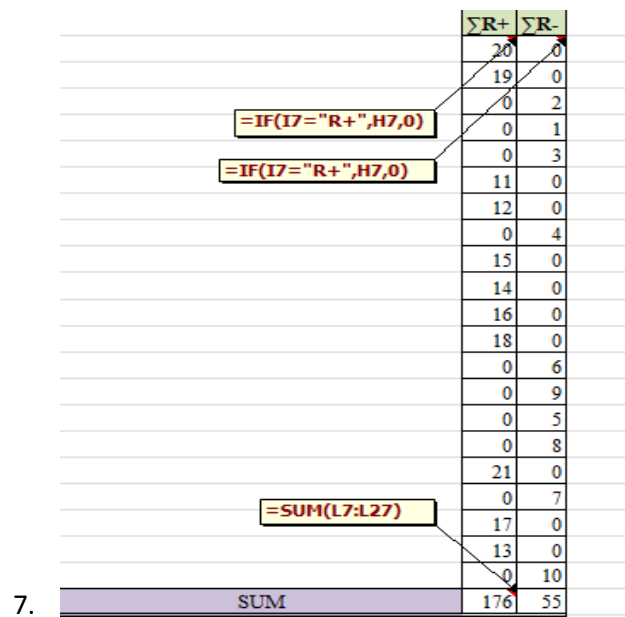


Figure 3.1(e) Rank of Positive & Negative differences Calculated

8. The z-score is then calculated using the following formula:

Where n: the number of items input.
T: the minimum value of both signs rank sum.

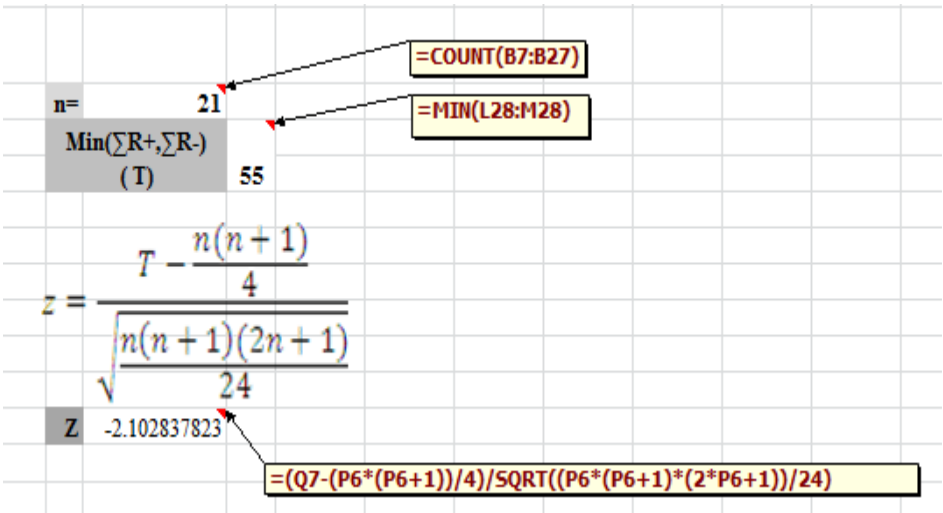


Figure 3.1(f) Z-Score Calculated

9. The results are therefore interpreted and a conclusion is drawn out.

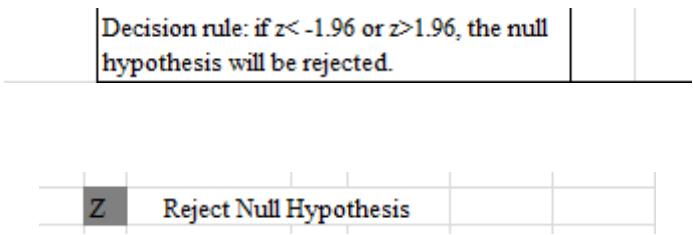


Figure 3.1(g) Interpretation & Conclusion

Hence, if $z < -1.96$ or $z > 1.96$, the null hypothesis will be rejected, i.e. it will imply that there is no increase in the Consumer price index. But, as the value of $z = -0.364955325$, which is greater than -1.96 and less than 1.96 , we therefore accept that there has been an increase in the median difference of the Consumer Price Index.

(The Wilcoxon Rank Sum Test Done in Excel)

(Corder)

Chapter 4

Sign test

1. Introduction

To test the hypothesis, “difference in medians”, the sign test can be used as hypothesis tester, between the constant distributions of two random variables A and B. This non-parametric test makes only some assumptions about the kind of distributions under test. This implies that it has a large probability of being accepted as a suitable test, but may be lacking in statistical power like the Wilcoxon Signed rank test or the T test.

2. Assumptions

Let for .

1. The difference Z_i should be assumed to be independent
2. Each Z_i comes from the same continuous population.
3. The values of A_i and B_i represent are ordered, so the comparisons "greater than", "less than", and "equal to" are meaningful.

3. Characteristics of the sign test

The sign test can be classified into 2 different sets of data, namely:

1. Large sample data
2. Small sample data

4.3.1 Large Sample data

Under sign test, large samples are defined as those samples which contain more than 25 raw data.

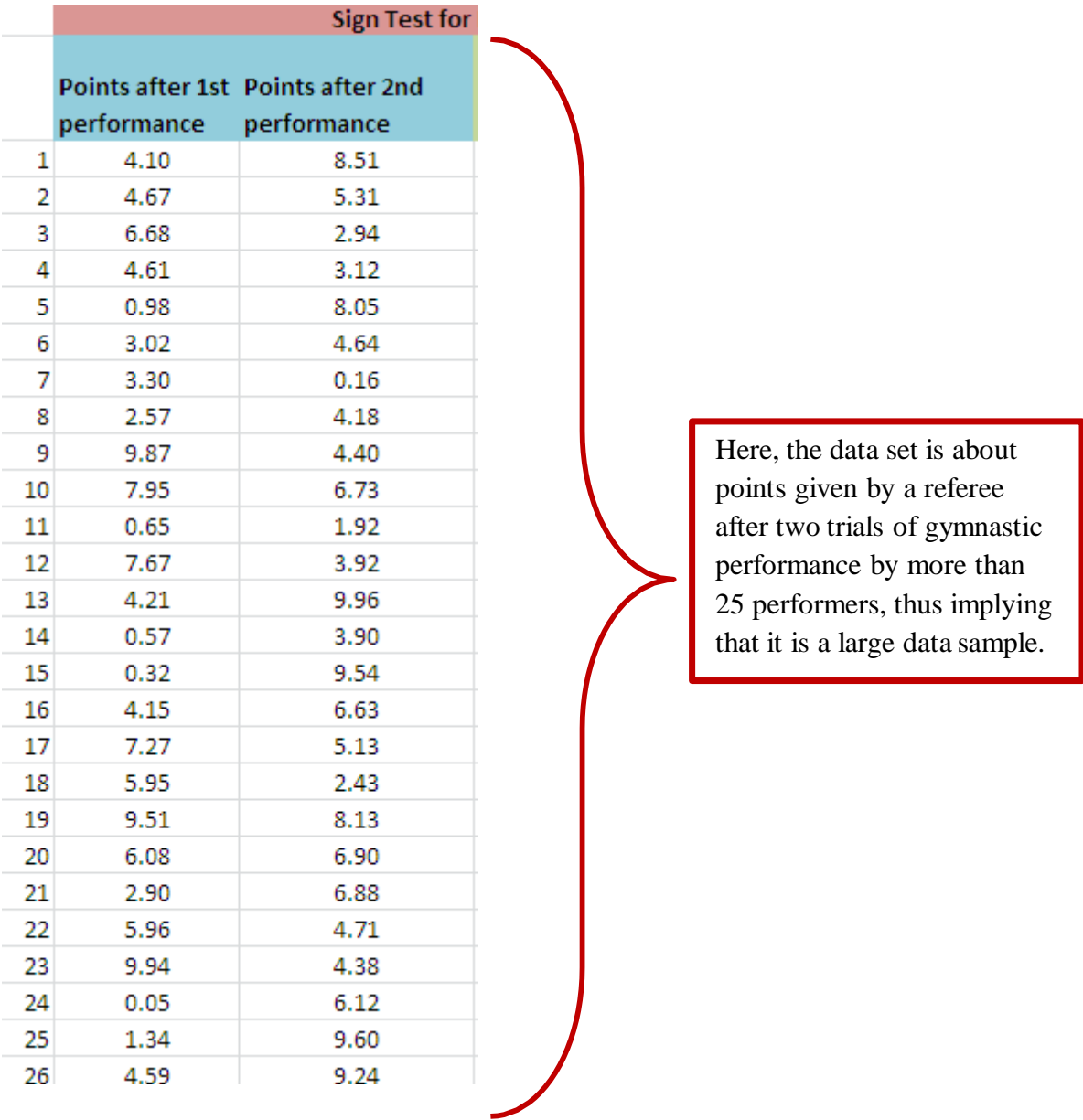


Figure 4.1 Large Sample Data

4.3.2 Small Sample Data

Sign Test for small samp			
		Points after 1st performance	Points after 2nd performance
	1	8.75	4.22
	2	8.13	3.51
	3	5.51	4.74
	4	7.66	6.51
	5	6.78	2.30
	6	4.07	5.30
	7	8.66	0.23
0	8	4.67	1.26
L	9	9.82	3.29
2	10	5.67	6.09
3	11	8.99	3.63
4	12	5.71	2.99
5	13	4.92	2.72
5	14	1.74	4.65
7	15	3.43	5.19
8	16	3.40	9.99
9	17	4.21	1.38
0	18	8.12	7.37
L	19	2.06	9.53
2	20	6.26	4.58

Here, unlike the large sample data set, we have only 20 points, which implies that there are only 20 performances. Hence, it is a small data sample.

Figure 4.2 Small Sample Data

4. Step-wise implementation of Sign Test (Large Sample data) in Excel

1. The null hypothesis, the alternate hypothesis and other criteria to be considered during the test are defined. NOTE: These are random numbers generated within the range 0 - 10.

H_o = There is a difference between the points. H_A = There is NO difference between the points. $\alpha = 0.05$ Decision rule= Accept the null hypothesis if the z score is greater than the z critical.
--

Figure 4.3(a) Null & Alternate Hypothesis

2. The sets of data are calculated and ranked.

Sign Test for Large samples				
Points after 1st performance	Points after 2nd performance	Difference	" + numbers "	" - numbers "
8.37	6.16	2.20	1	0
8.42	3.42	5.00	1	0
8.01	3.96	4.05	1	0
4.16	1.09	3.07	1	0
4.38	4.34	0.03	1	0
4.31	8.47	-4.16	0	1
8.68	3.36	5.32	1	0
6.81	5.07	1.74	1	0
4.52	1.16	3.37	1	0
0.15	0.68	-0.54	0	1
7.49	7.97	-0.48	0	1
0.32	2.86	-2.53	0	1
2.15	1.98	-0.17	1	0
0.26	2.95	-2.69	0	1
6.86	4.15	2.71	1	0
6.07	5.55	0.52	1	0
7.78	1.52	6.27	1	0
6.79	1.47	5.33	1	0
8.41	9.47	-1.05	0	1
0.37	0.75	-0.39	0	1
5.58	8.68	-3.10	0	1
4.87	4.66	0.22	1	0

Figure 4.3(b) Sets of data Calculated & Ranked

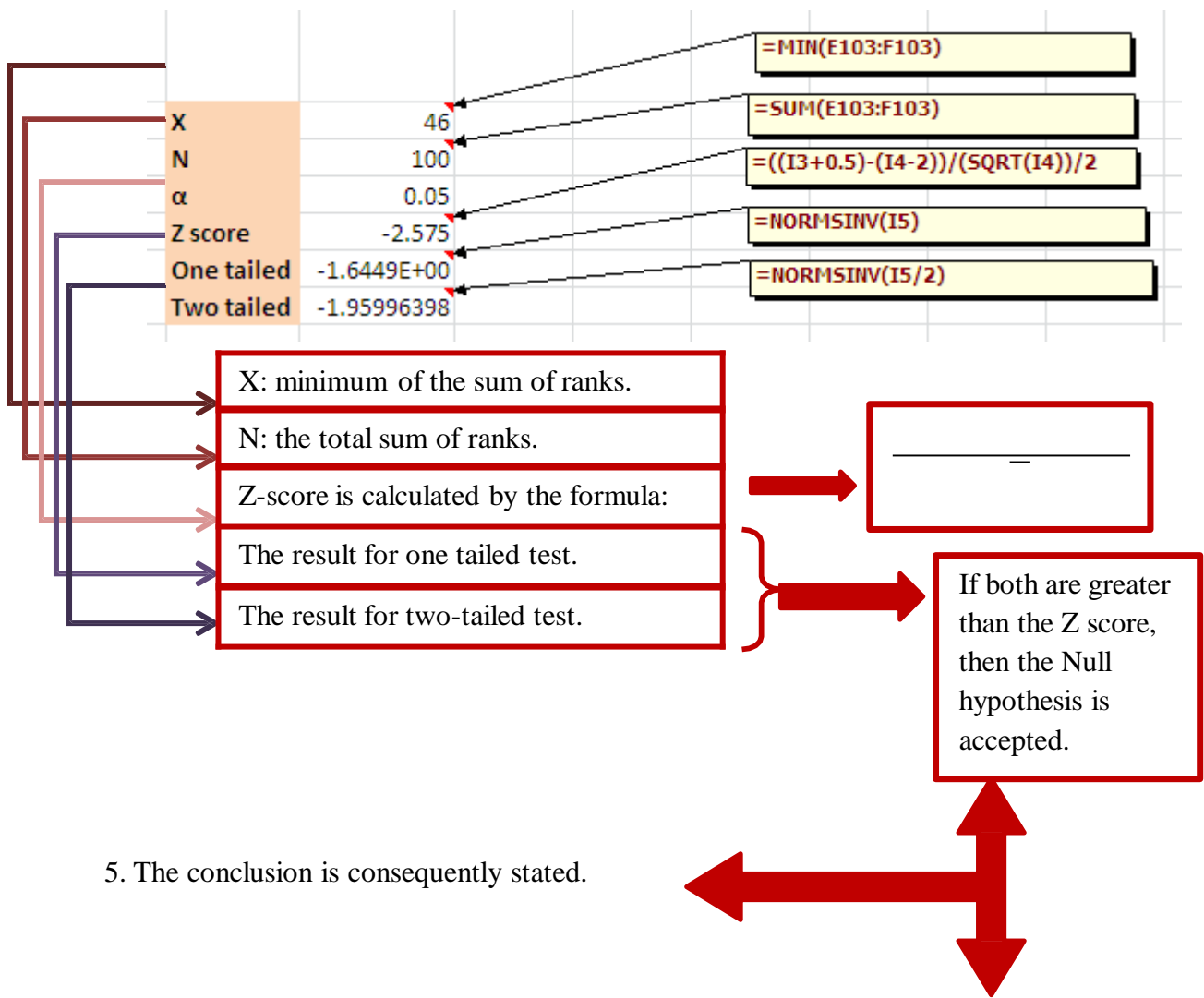
3. The sum of ranks is then evaluated

100	98	4.15	2.77	1.38	1	0
101	99	8.00	2.29	5.71	1	0
102	100	1.59	5.07	-3.49	0	1
103				SUM	46	54
104						
105						
106						

=SUM(E3:E102)

Figure 4.3(c) Calculating Sum of Ranks

4. The data is calculated as follows:



5. The conclusion is consequently stated.

Result	One tailed	Accept Null hypothesis	=IF(I7>I6,"Accept Null hypothesis","Reject Null Hypothesis")
	Two tailed	Accept Null hypothesis	=IF(I8>I6,"Accept Null hypothesis","Reject Null Hypothesis")

Figure 4.3(d) Final Calculations & Conclusion

5. Step-wise implementation of Sign Test (Small Sample data) in Excel

1. The null hypothesis and alternated hypothesis are defined as follows:

$H_0 = \text{There is a difference between the points.}$ $H_A = \text{There is NO difference between the points.}$ $\alpha = 0.05$ Decision rule= if the p-value is greater than α , accept the null hypothesis else reject it.

Figure 4.2(a) Null & Alternative Hypothesis

2. The difference between the two sets of data is evaluated.

	A	B	C	D	E	F
1	Sign Test for small samples					
2		Points after 1st performance	Points after 2nd performance	Difference	" + numbers "	" - numbers "
3	1	5.89	4.36	1.53	1	0
4	2	6.70	2.66	4.04	1	0
5	3	1.68	0.77	0.92	1	0
6	4	4.58	5.51	-0.94	0	1
7	5	8.04	9.95	-1.91	0	1
8	6	6.66	2.45	4.21	1	0
9	7	8.78	7.09	1.70	1	0
10	8	8.20	1.27	=IF(D3<0,0,1)	1	0
11	9	5.56	7.60	-2.04	0	1
12	10	4.99	6.19	-1.21	=IF(E3>0,0,1)	1
13	11	8.22	2.83	5.39	1	0
14	12	4.77	1.25	3.52	1	0
15	13	7.64	5.75	1.88	1	0
16	14	6.34	9.44	-3.10	0	1
17	15	2.54	3.87	-1.33	0	1
18	16	6.09	2.69	3.40	1	0
19	17	4.53	7.66	-3.13	0	1
20	18	0.12	7.35	-7.23	0	1
21	19	0.34	1.15	-0.81	0	1
22	20	8.10	5.76	2.34	1	0

Figure 4.2(b) Sets of data Calculated & Ranked

3. Sum of ranks is calculated to know the value of N.

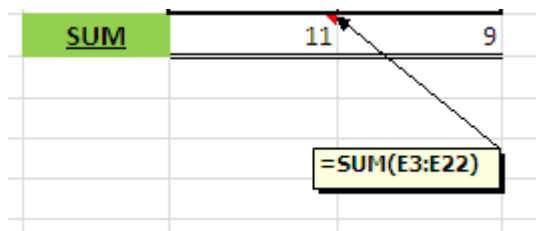


Figure 4.2(c) Calculating Sum of Ranks

4. All criterions required are then displayed.

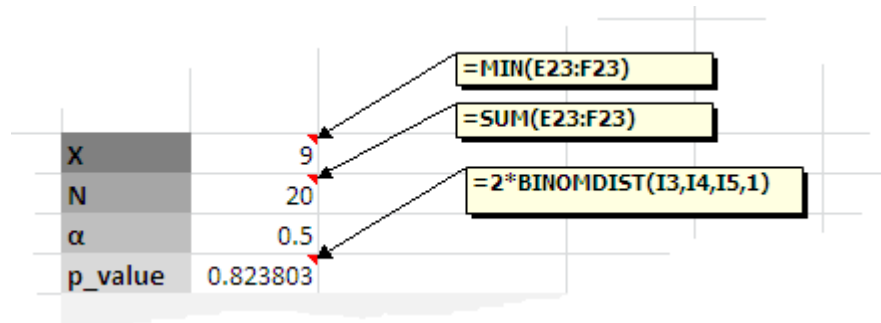


Figure 4.2(d) Displaying Criterions

5. Following the decision rule:

- If the p-value is greater than α , then the null hypothesis is accepted. Else, it is rejected.
-

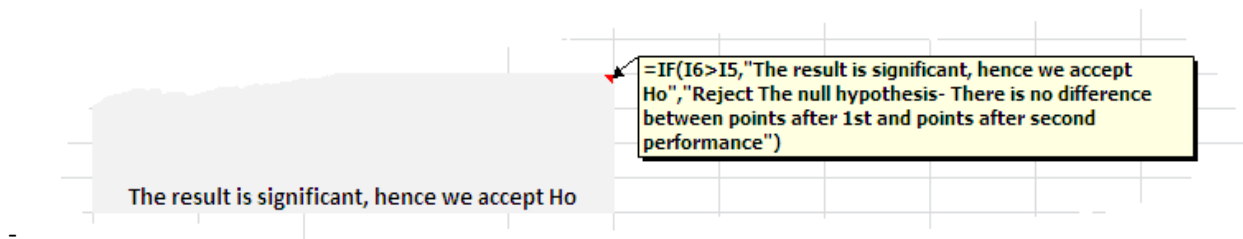


Figure 4.2(e) Conclusion

(The Sign Test (Nonparametric) in Excel)

(Corder)

Chapter 5

The Mann-Whitney U-test

1. Introduction

The Mann-Whitney U-test is a non-parametric statistical method for comparing two groups of sampled data which are independent. Its purpose is to test the null hypothesis that the two samples have similar median or, conversely, whether observations in one sample are likely to have larger values than those in the other sample. The t-test of unrelated samples is a parametric equivalent to the Mann-Whitney U-test.

2. Assumptions

The basic assumptions of a Mann-Whitney U-test are:

- 1. The two samples are random.
- 2. The two samples are independent of each other and so are the observations within each sample.
- 3. The measurement scale is of ordinal type. Therefore, the observations can be arranged in ranks.

Calculations for Mann-Whitney U-test:

Consider a sample of observations where in one group (i.e. from one population) and another sample of observations where in another group (i.e. from another population).

The Mann-Whitney U-test is centered on the difference of each observation in the first sample with each observation in the other sample. The total number of pairwise difference that can be executed is:

If the median is alike for both samples, then each has an even possibility of being smaller or larger than each , implying a probability of ½.

Hence under the (i) null hypothesis –

(ii) Alternate hypothesis –

The number of times an from sample 1 is greater than a from sample 2 is calculated and represented by n_{12} . Likewise, the number of times an from sample 1 is smaller than a from sample 2 is counted and represented by n_{21} . Below the null hypothesis, n_{12} and n_{21} are likely to be almost the same.

3. Steps to perform the Mann-Whitney test

1. The null hypothesis, and the alternative hypothesis, are identified.
2. The significance level, related with the null hypothesis is stated. is normally set at 5% and therefore the confidence level is 95%.
3. All the observations are assembled in terms of magnitude.
4. Below every observation, note down A or B or any other appropriate symbol to show from which sample they come.
5. Below each a , the number of b , that is lesser than it, is recorded. This implies n_{12} . Similarly, below each b , note down the number of a s that is lesser than it. This means n_{21} .
6. The total number of times a is calculated and represented by n_{1+} . Equally, the number of times b is counted and denoted by n_{2+} . The Mann-Whitney U -test statistic for each sample is determined by:

Verify that:

7. Evaluate U . The smaller of the two U statistics is the obtained value.
8. The Mann-Whitney U -test statistical tables are used to evaluate the possibility of finding a value of U or lower. If the test is:
 - a) One-sided: the p-value is the probability calculated itself.
 - b) Two-sided: the probability is doubled to get the p-value.
9. The critical value is compared to the obtained value.
10. The results are then interpreted and a conclusion is drawn out.

Furthermore, if the number of observations is **greater than 20**, a normal approximation is used. For large samples, the z-score is calculated and the normal distribution table is used to get the critical region of the z-scores.

The following formulas are used to evaluate the z-score of the Mann-Whitney U-test for samples of greater quantity:

1. The mean of the samples is calculated.

$$\bar{x} = \frac{\sum x_i}{n}$$

Where, \bar{x} : mean

n_1 : number of values from the first sample

n_2 : number of values from the second sample

2. The standard deviation is also evaluated.

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

where s : standard deviation N

:

3. The z-score for the normal distribution of data can then be calculated.

$$z = \frac{U - \mu_U}{\sigma_U}$$

Where, z : z-score of the sample

U : U-statistic of the sample

5.3.1 Dealing with Ties:

Sometimes, two or more observations can be likely the same. Then, U is calculated by assigning half the tie to the A value and the other half to the B value. Hence, the normal distribution is used with a modification in the standard deviation. Therefore, the formula is:

$$\frac{1}{N} \sum_{j=1}^g \frac{t_j(t_j+1)}{12}$$

Where, σ : standard deviation

N :

g : number of groups of ties

t_j : number of tied ranks in group j

4. Example of a Mann-Whitney U-test:

For instance, the employment rates worldwide between two different years are compared using the Mann-Whitney test.

1. Steps to perform a Mann-Whitney U-test in Microsoft Excel 2010:

- 1. The null hypothesis, and the alternate hypothesis, are defined.

Let data in 1998 be sample 1 and data in 2008 be sample 2.							
Null hypothesis , H_0 : No difference between ranks of employment rates of the two years							
Alternative hypothesis , H_a : There is a difference between the ranks of employment of the two years							

Figure 5.1(a) Null Hypothesis & Alternative Hypothesis

- 2. The significance level, related with the null hypothesis is stated.

Alpha : 0.05			
--------------	--	--	--

Figure 5.1(b) Significance

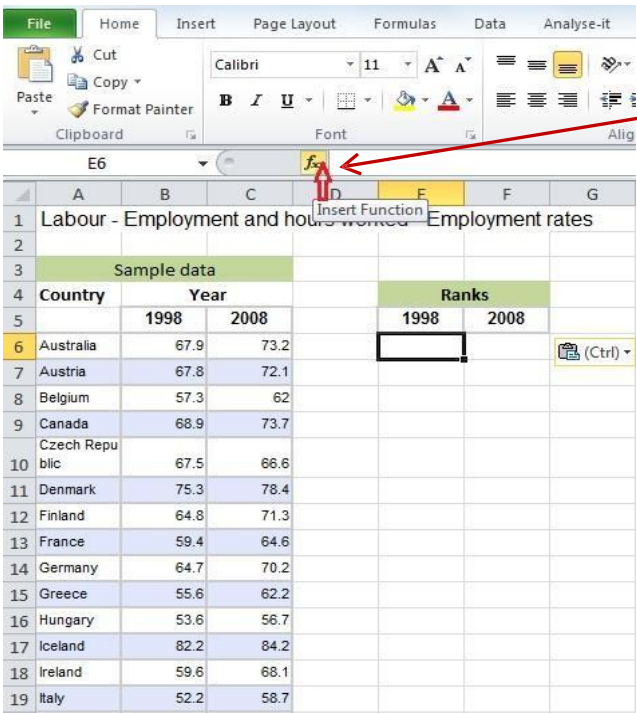
3. The sample data is implemented on an excel sheet as follows:

	A	B	C	D	E	F	G
1	Labour - Employment and hours worked - Employment rates						
2							
3	Sample data						
4	Country	Year					
5		1998	2008				
6	Australia	67.9	73.2				
7	Austria	67.8	72.1				
8	Belgium	57.3	62				
9	Canada	68.9	73.7				
10	Czech Republic	67.5	66.6				
11	Denmark	75.3	78.4				
12	Finland	64.8	71.3				
13	France	59.4	64.6				
14	Germany	64.7	70.2				
15	Greece	55.6	62.2				
16	Hungary	53.6	56.7				
17	Iceland	82.2	84.2				
18	Ireland	59.6	68.1				
19	Italy	52.2	58.7				
20	Japan	69.5	70.7				
21	Korea	59.2	63.8				
22	Luxembourg	60.2	64.4				

Figure 5.1(c) Sample Data

4. The rank of the sample data is calculated as shown below. If there are ties, that is, two or more sample values are similar, then the sample data is given a rank equal to the mean of the ranks that would else be given.

(i)



	A	B	C	D	E	F	G
1	Labour - Employment and hours worked - Employment rates						
2							
3	Sample data						
4	Country	Year					
5		1998	2008				
6	Australia	67.9	73.2				
7	Austria	67.8	72.1				
8	Belgium	57.3	62				
9	Canada	68.9	73.7				
10	Czech Republic	67.5	66.6				
11	Denmark	75.3	78.4				
12	Finland	64.8	71.3				
13	France	59.4	64.6				
14	Germany	64.7	70.2				
15	Greece	55.6	62.2				
16	Hungary	53.6	56.7				
17	Iceland	82.2	84.2				
18	Ireland	59.6	68.1				
19	Italy	52.2	58.7				

Click on Insert
function

(ii)

From the 'or select a category' box, select 'Statistical'

Labour - Employment and hours worked - Employment rates

Sample data		
Country	Year	
	1998	2008
Australia	67.9	73.2
Austria	67.8	72.1
Belgium	57.3	62
Canada	68.9	73.7
Czech Republic	67.5	66.6
Denmark	75.3	78.4
Finland	64.8	71.3
France	59.4	64.6
Germany	64.7	70.2
Greece	55.6	62.2
Hungary	53.6	56.7
Iceland	82.2	84.2
Ireland	59.6	68.1
Italy	52.2	58.7
Japan	69.5	70.7
Korea	59.2	63.8
Luxembourg		

Ranks

1998
=

Insert Function

Search for a function:

Type a brief description of what you want to do and then click Go

Go

Or select a category: Statistical

Select a function:

QUARTILE.EXC
QUARTILE.INC
RANK.AVG
RANK.EQ
RSQ
SKEW
SLOPE

RANK.AVG(number,ref,order)
Returns the rank of a number in a list of numbers: its size relative to other values in the list; if more than one value has the same rank, the average rank is returned.

Help on this function

OK

Cancel

The 'RANK.AVG' function is selected. Then, click OK.

(iii)

Labour - Employment and hours worked - Employment rates

Sample data		
Country	Year	
	1998	2008
Australia	67.9	73.2
Austria	67.8	72.1
Belgium	57.3	62
Canada	68.9	73.7
Czech Republic	67.5	66.6
Denmark	75.3	78.4
Finland	64.8	71.3
France	59.4	64.6
Germany	64.7	70.2
Greece	55.6	62.2
Hungary	53.6	56.7
Iceland	82.2	84.2
Ireland	59.6	68.1
Italy	52.2	58.7
Japan	69.5	70.7
Korea	59.2	63.8
Luxembourg		

Ranks

1998	2008
=RANK.AVG(B6)	

Function Arguments

RANK.AVG

Number B6 = 67.9

Ref = reference

Order = logical

Returns the rank of a number in a list of numbers: its size relative to other values in the list; if more than one value has the same rank, the average rank is returned.

Number is the number for which you want to find the rank.

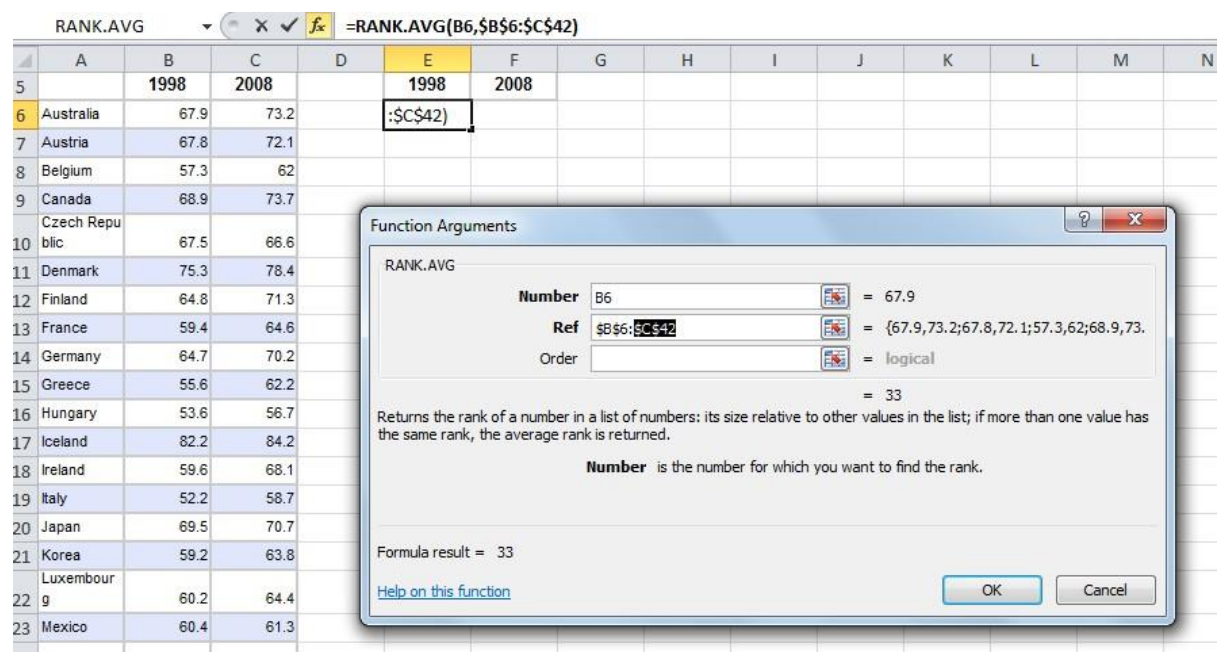
Formula result =

Help on this function

OK

Cancel

(iv)



(v)

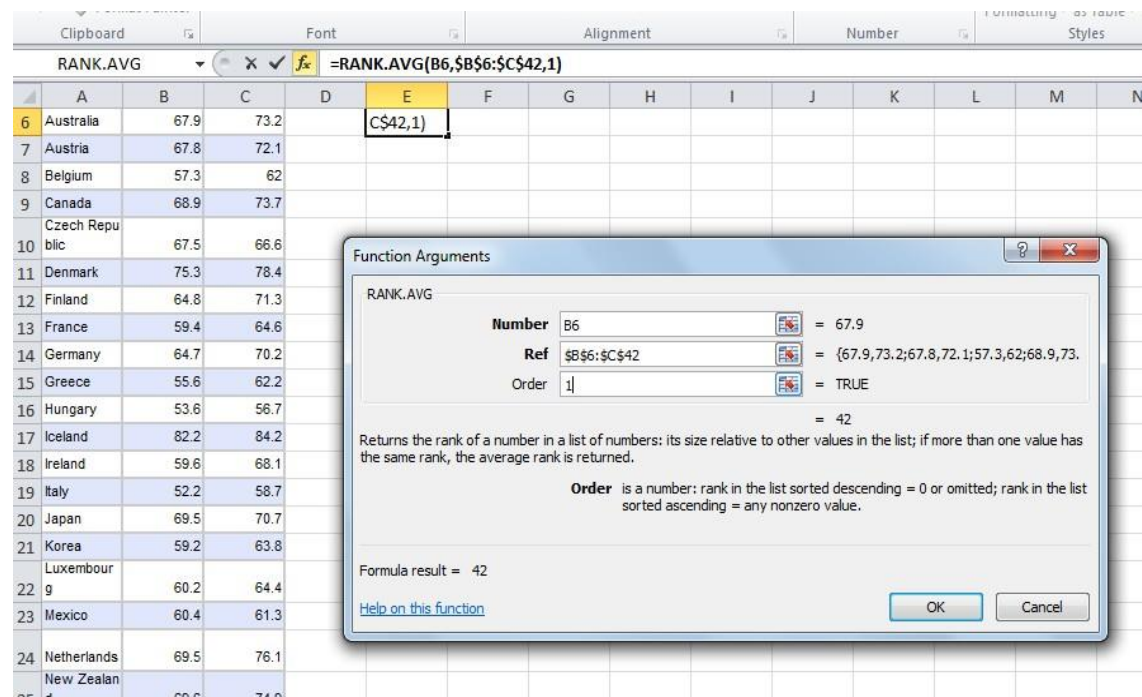


Figure 5.1(d) Ranking Sample Data Using FX Function

(vi)

Clipboard Copy Paste Format Painter Font Aligner

E6 fx =RANK.AVG(B6,\$B\$6:\$C\$42,1)

	A	B	C	D	E	F	G
1	Labour - Employment and hours worked - Employment rates						
2							
3	Sample data						
4	Country	Year			Ranks		
5		1998	2008		1998	2008	
6	Australia	67.9	73.2		46		
7	Austria	67.8	72.1				
8	Belgium	57.3	62				
9	Canada	68.9	73.7				
10	Czech Republic	67.5	66.6				
11	Denmark	75.3	78.4				
12	Finland	64.8	71.3				
13	France	59.4	64.6				
14	Germany	64.7	70.2				
15	Greece	55.6	62.2				
16	Hungary	53.6	56.7				
17	Iceland	82.2	84.2				
18	Ireland	59.6	68.1				
19	Italy	52.2	58.7				
20	Japan	69.5	70.7				
21	Korea	59.2	63.8				
22	Luxembourg	60.2	64.4				
23	Mexico	67.9	61.3				

(vii)

25	New Zealand	67.9	74.9	46	68
26	Norway	67.9	72.1	46	63
27	Poland	57.3	62	10	25.5
28	Portugal	67.9	62	46	25.5
29	Slovak Republic	60.5	62	19	25.5
30	Spain	52.4	65.3	4	37
31	Sweden	71.5	72.1	60	63
32	Switzerland	78	72.1	71	63
33	Turkey	51.4	44.9	2	1
34	United Kingdom	71	72.1	58	63
35	United States	67.9	70.9	46	57
36	OECD total	67.9	62	46	25.5
37	Brazil ¹	67.9	62	46	25.5
38	Chile	57.3	57.8	10	13
39	Estonia	67.9	62	46	25.5
40	Israel ²	57.3	62	10	25.5
41	Russian Federation	57.3	62	10	25.5
42	Slovenia ³	67.9	62	46	25.5
43					

Click and drag to obtain all the

Figure 5.1(e) Click & Drag to Rank

5. The calculations are as follows:

(i)

H	I	J	K	L	M	N	O	P
	Calculations :							
	Let data in 1998 be sample 1 and data in 2008 be sample 2.							
	Null hypothesis , H_0 : No difference between ranks of employment rates of the two years							
	Alternative hypothesis , H_a : There is a difference between the ranks of employment of the two years							
	Alpha : 0.05							
	Critical value : 1.96							

The critical value is calculated from the z table using

(ii)

Year	1998	2008							
Median	67.8	64.6							
	R1	R2							
Sum of ranks	1249	1526							
Counts	N1	N2	N						
	37	37	74						
U1	823								
U2	546								
U	546								
z	-1.4973								
p	0.134316								
Results :									
If $z < -1.96$ or $z > 1.96$, reject H_0									

The Median, Sum of Ranks, Counts, Z-Value & P-Value are calculated using the appropriate formulae.

Figure 5.1(f) Calculations

6. The results are then interpreted and a conclusion is drawn out.

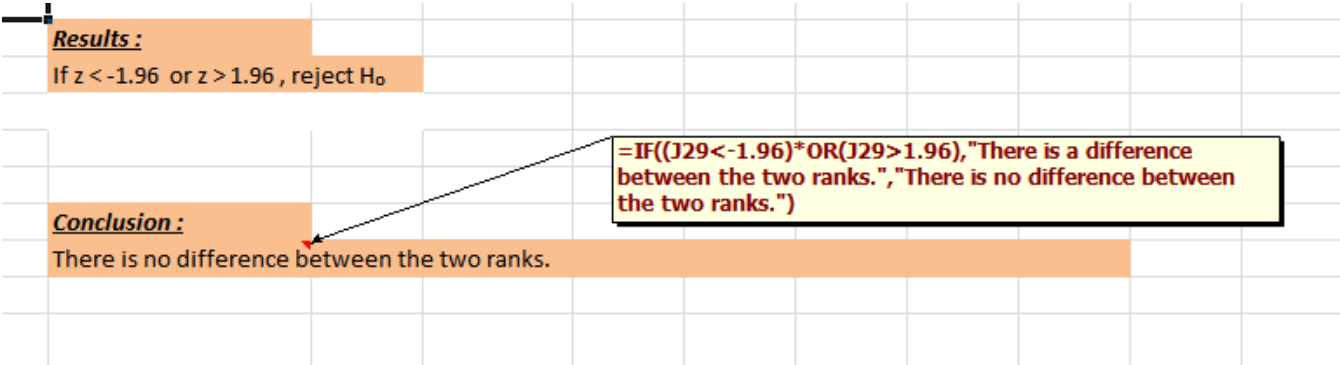


Figure 5.1(g) Result & Conclusion

(Shier)

(Corder)

Chapter 6

The Friedman Test

6.1 Introduction:

The Friedman Test is a non-parametric test which was developed and implemented by **Milton Friedman**. This type of test is used for the comparison of three or more dependent samples. This way of testing involves ranking each row together, then considering the values of ranks by columns. No assumptions are made when implementing this type of test. The Friedman test may be used as an alternative to repeated measures of ANOVA, when the assumption of normality is not met. Just like some of other non-parametric tests, the Friedman Test uses ranks of data rather than their raw data to compute the test statistic. The fact that no assumptions are made in this test, it is not as powerful as the ANOVA. If it is equivalent to the sign test, then this implies that there are only two measures for this test.

Some examples of its use are:

- n wine judges each rate k different wines. Are any wines ranked consistently higher or lower than the others?
- n wines are each rated by k different judges. Are the judges' ratings consistent with each other?
- n welders each use k welding torches, and the ensuing welds were rated on quality. Do any of the torches produce consistently better or worse welds?

We use the Friedman test for one way repeated measure analysis of variance by ranks. This way of testing resembles to that of **Kruskal Wallis one way analysis of variance by ranks**.

2. Step-wise implementation of Friedman Test on Excel

1. The null and alternative hypotheses are defined.

$H_0 =$ There is no difference between 3 conditions
$H_A =$ There is a difference between these 3 conditions.

Figure 6.1(a) Null & Alternative Hypothesis

2. The value of $\alpha= 0.05$ is stated.

15	α	0.05
----	----------	------

Figure 6.1(b) Significance

3. The samples are then ranked and the summation of ranks is carried out.

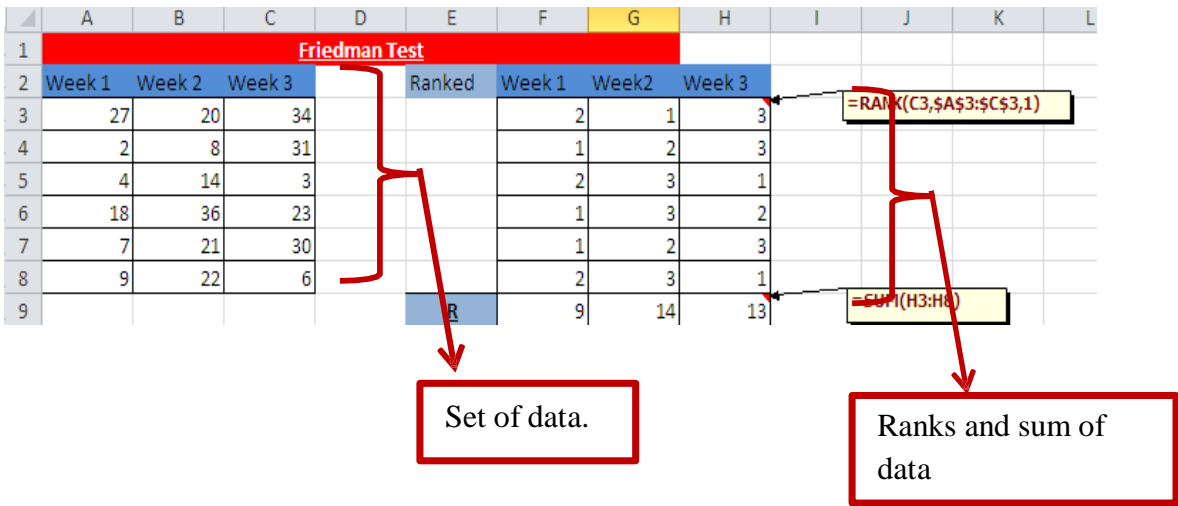


Figure 6.1(c) Ranking & Summation

4. The value of k and n are calculated, where

- n: the number of test attempts (rows) and,
- k: the number of columns.

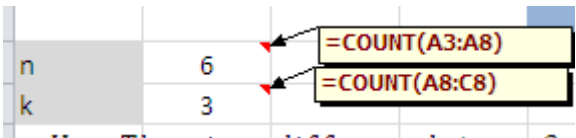


Figure 6.1(d) Finding Value of k & n

5. The Degree of Freedom, df, is then evaluated using the formula:

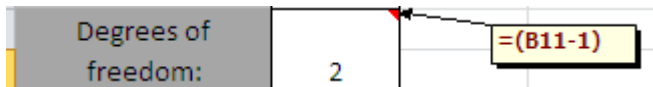


Figure 6.1(e) Degree of Freedom

6. The decision rule is stated.

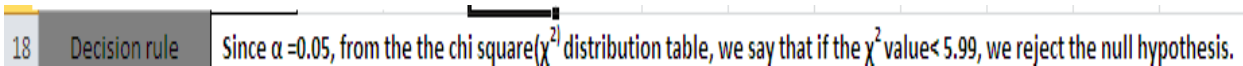


Figure 6.1(f) Decision Rule

7. Consequently, the Friedman test statistic, is calculated as follows:

(a) if there is no ties from the ranks, then is evaluated using the following formula

$$\chi^2 = \frac{12}{n(n+1)} \sum_{i=1}^k \frac{R_i^2}{k} - \frac{3n(n+1)}{4}$$

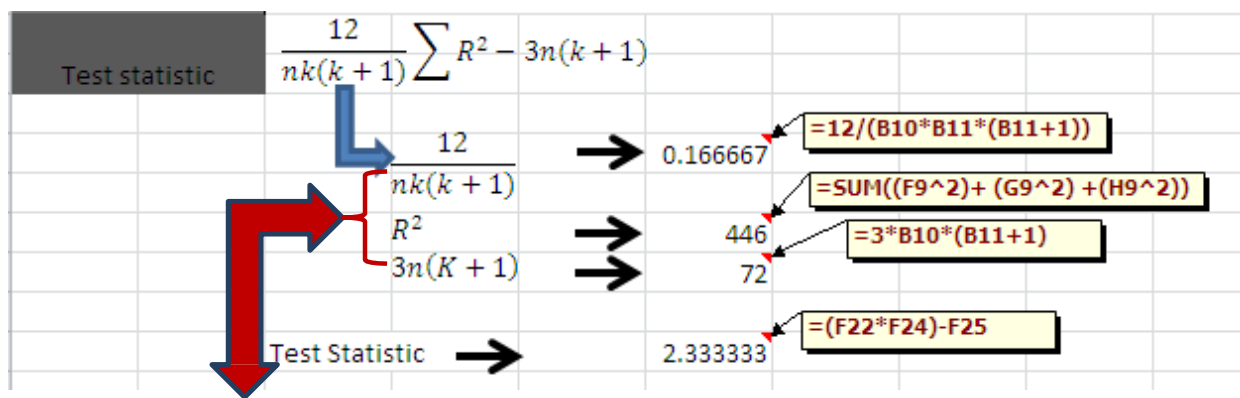
where n: number of rows
k: number of columns
sum of ranks from column, i

(b) if there is ties in the ranking of values, then is obtained by:

$$\chi^2 = \frac{12}{n(n+1)} \sum_{i=1}^k \frac{R_i^2}{k} - \frac{3n(n+1)}{4} - \frac{1}{n} \sum_{j=1}^n \frac{t_j^3 - t_j}{t_j(t_j+1)}$$

where rank corresponding to subject j in column i.
ties correction which is calculated by

$$t_j = \text{number of ties for subject } j$$



The Test statistic Equation is broken into parts in order to facilitate the computation of the result.

8. A conclusion is then drawn out.

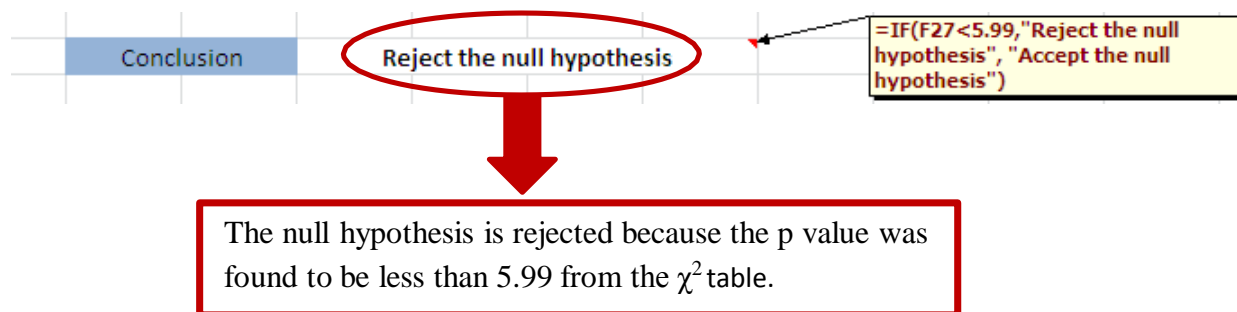


Figure 6.1(g) Calculation & Conclusion

(Friedman's two-way ANOVA for ranks)

(Corder)

Chapter 7

The Kruskal-Wallis H-test

1. Introduction

The Kruskal-Wallis H-test is a non-parametric statistical procedure for comparing more than two samples that are independent. The parametric equivalent to this test is the one-way analysis of variance (ANOVA).

Yet ANOVA is used for normally distributed data, but Kruskal Wallis can be performed without the data being normally distributed

The H-test is a generalization of the Mann-Whitney test, a test for knowing whether the two samples chosen are taken from the same population. The p value for both the Kruskal Wallis and the Mann-Whitney test are equal.

However Kruskal Wallis unlike Mann-Whitney test is used in samples to evaluate their degree of association.

2. Description of samples

Kruskal Wallis tests liable to be performed when the different samples of data meet the following criteria:

- similar distributions
- data in each sample should be >5
- same shape of distribution as the population
- data can and must be ranked
- independent

3. Description of the Kruskal-Wallis H-test

The Kruskal-Wallis test has the following characteristics:

- test static is nearly chi square distributed
- test equality of >2 population median
- uses K samples of data
- used when there is one nominal variable and one measurement variable
- used in case of one nominal and one ranked variable
- apply Kruskal Wallis when data is not normally distributed
- is the significance level
- Data need to be ranked
- to calculate degrees of freedom= no of sample-1
- the rank of each sample is calculated
- use average rank whenever there is a tie

4. Calculations and steps for Kruskal-Wallis H-test

The hypothesis is stated in terms of populations.

Also worth noted is the observation of the median of the population while doing the Kruskal-Wallis test.

- In order to perform the Kruskal-Wallis H-test, the first step is to combine all the samples and perform a rank ordering on all the values.

Calculate the Kruskal Wallis by using the following formula:

.

Where,

N = Number of values obtained from every grouped samples,
= Summation of ranks taken from a particular sample and
Number of values from the equivalent sum of rank.

- While performing the H-test, the degree of freedom which is written as df is determined by means of the formula:

Where,

df stands for degrees of freedom

k for the number of groups or samples

- Then p-value is calculated

If P-Value is less than 5% or greater than 10%, reject null hypothesis.

If p-Value lie between 5% and 10% accept null hypothesis.

7.5 Implementing Kruskal-Wallis H-test in excel 2010

Example:

An investigation is carried out using 3 different groups of person to see how many glasses of alcoholic drinks they consume per day although having gone through the alcoholism therapy.

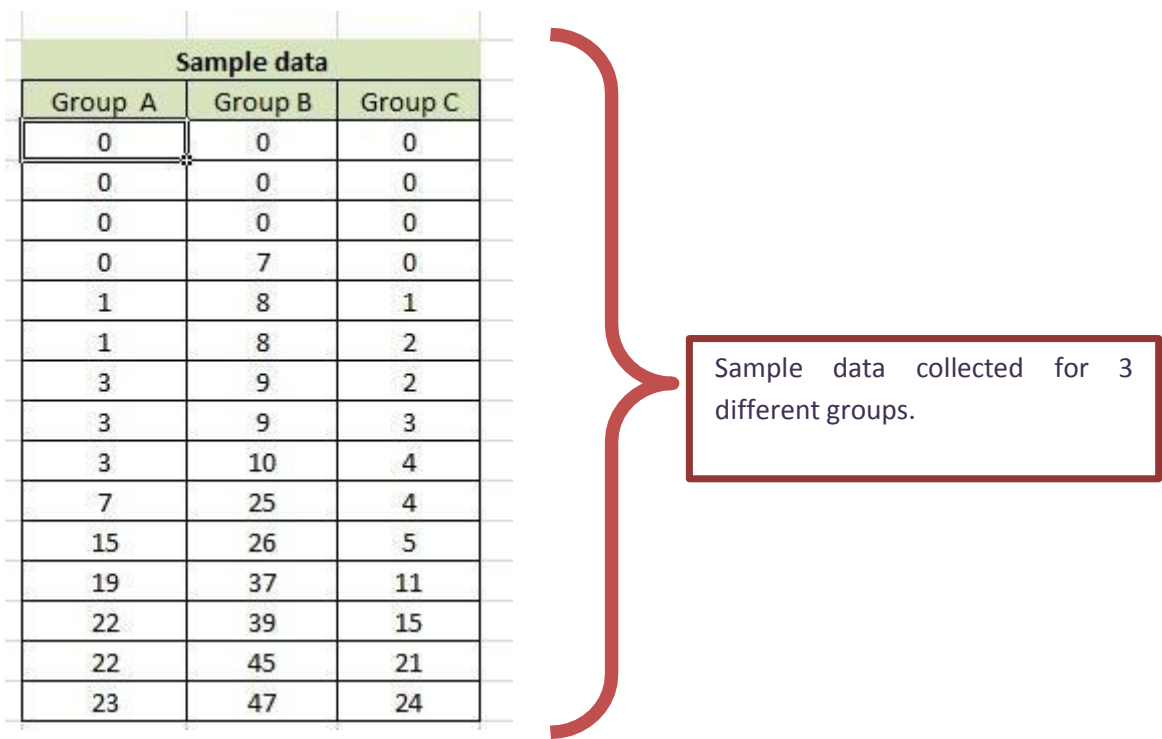


Figure 7.1(a) Sample Data

Step 1:

All the sample data are combined into one group and ranked.

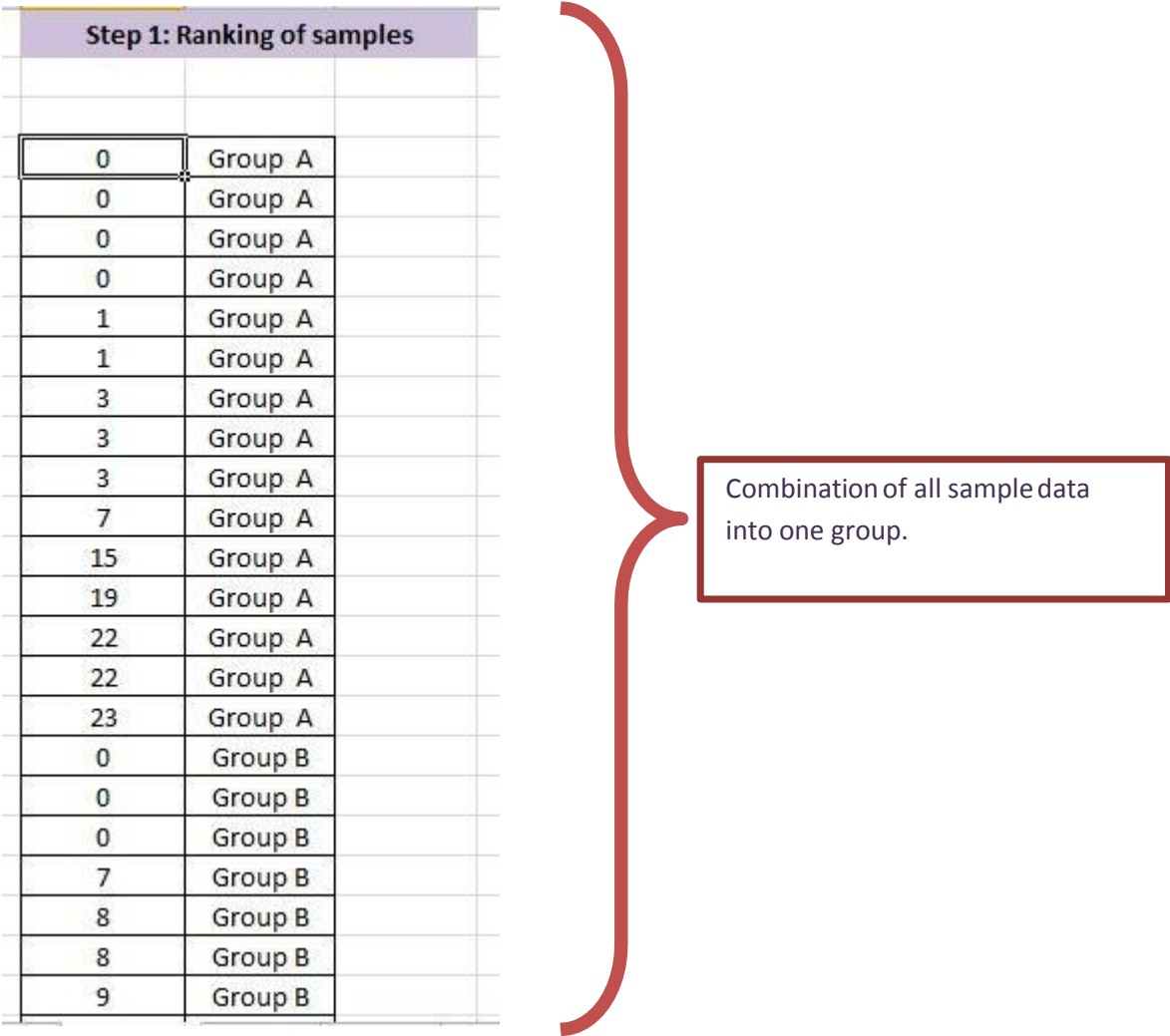


Figure 7.1(b) Combining all Sample Data

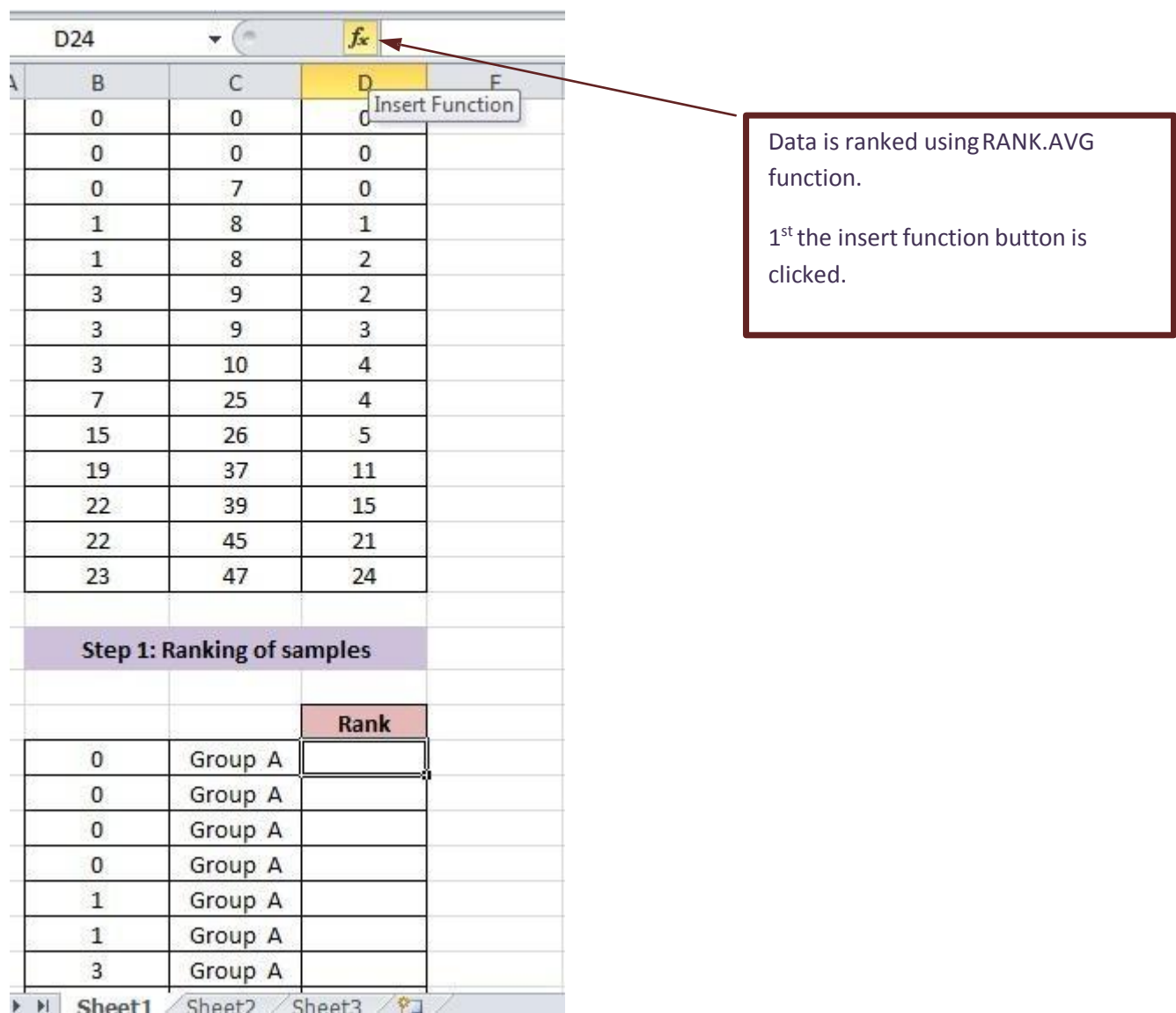
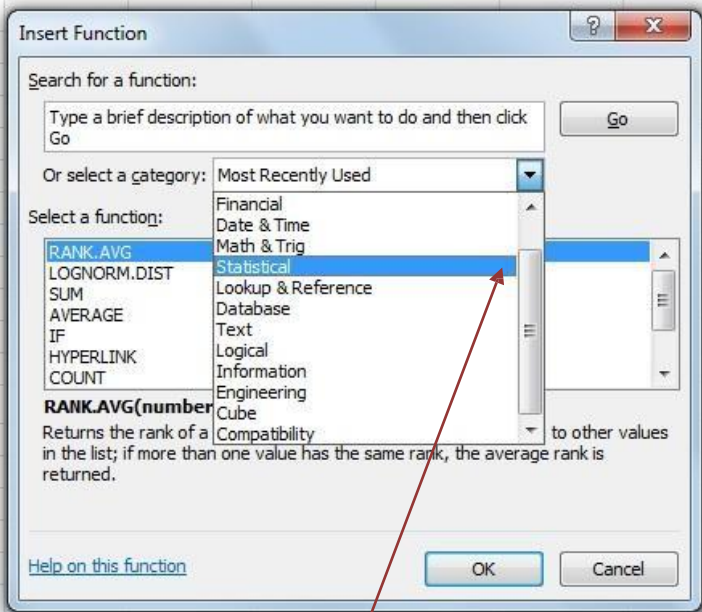


Figure 7.1(c) Ranking data using fx function part 1

0	0	0
0	7	0
1	8	1
1	8	2
3	9	2
3	9	3
3	10	4
7	25	4
15	26	5
19	37	11
22	39	15
22	45	21
23	47	24

Step 1: Ranking of samples

		Rank
0	Group A	=
0	Group A	
0	Group A	
0	Group A	
1	Group A	
1	Group A	
3	Group A	



2nd statistical is selected in the insert function window.

Figure 7.1(d) Ranking data using fx function part 2

Insert Function

Search for a function:

Type a brief description of what you want to do and then click Go

Or select a category: Most Recently Used

Select a function:

RANK.AVG
LOGNORM.DIST
SUM
AVERAGE
IF
HYPERLINK
COUNT

RANK.AVG(number,ref,order)

Returns the rank of a number in a list of numbers: its size relative to other values in the list; if more than one value has the same rank, the average rank is returned.

[Help on this function](#)

OK Cancel

Figure 7.1(e) Ranking data using fx function part 3

The following cells are selected.

Click and drag to rank all data.

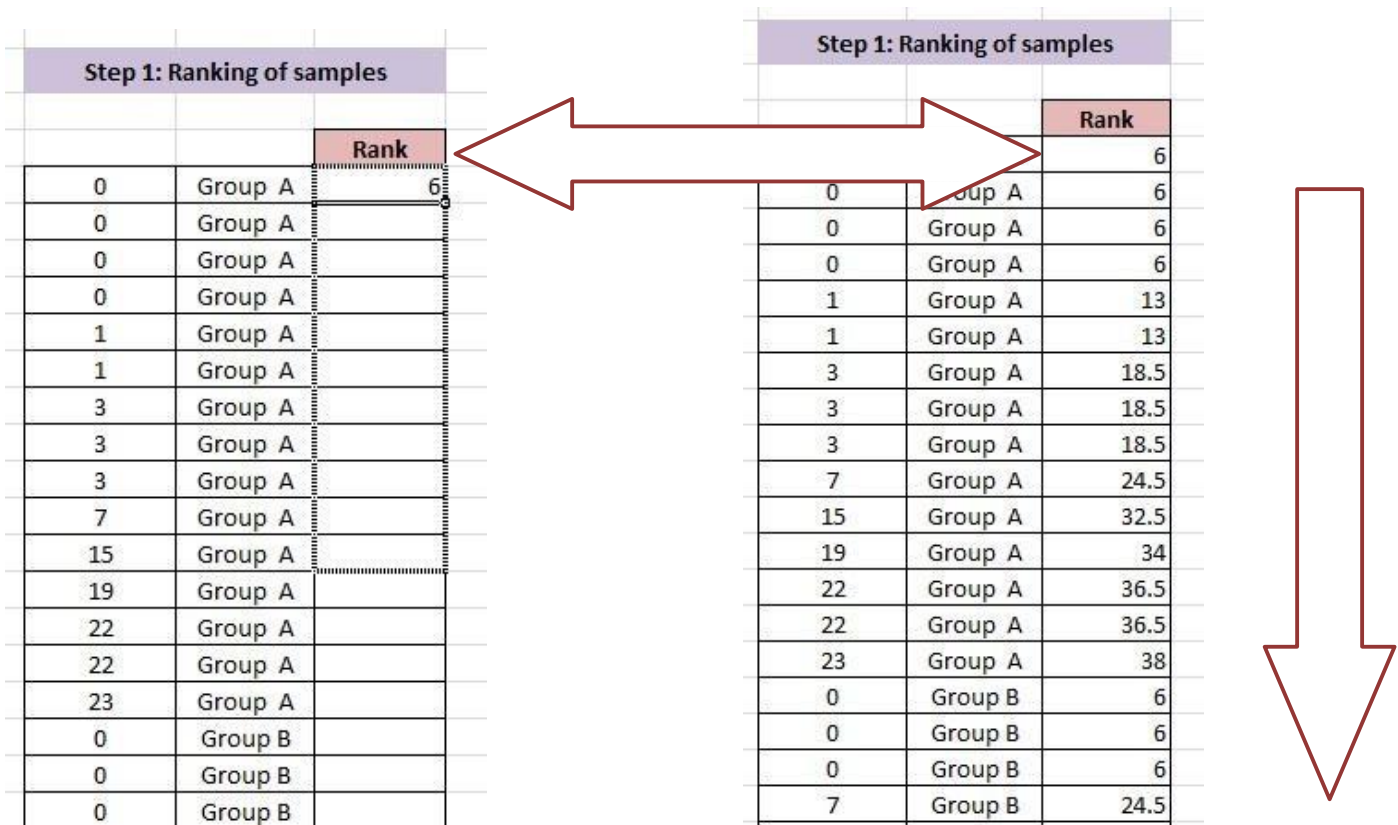


Figure 7.1(g) Click & Drag

Step 2:

Rank data values are copied back into the original group and sum of ranks are calculated for each group.

Step 2: Putting back sample into original positions				
	Group A	Group B	Group C	
	6	6	6	
	6	6	6	
	6	6	6	
	6	24.5	6	
	13	26.5	13	
	13	26.5	15.5	
	18.5	28.5	15.5	
	18.5	28.5	18.5	
	18.5	30	21.5	
	24.5	40	21.5	
	32.5	41	23	
	34	42	31	
	36.5	43	32.5	
	36.5	44	35	
	38	45	39	
Sum of ranks	R_1 307.5	R_2 437.5	R_3 290.0	
	=SUM(I6:I20)	=SUM(K6:K20)	=SUM(M6:M20)	

Figure 7.1(h) Recopying Rank to original group & Calculating Sum

Step 3:

Null hypothesis , H_0 : There is no difference between ranks of the three different groups.		
Alternative hypothesis , H_a : There is a difference between the ranks of the three different groups.		
N1	Count of Samples in Group A	15
N2	Count of Samples in Group B	15
N3	Count of Samples in Group C	15
n	Total no of Samples	45
k	NO of Groups	3
df	Degree of Freedom (k-1)	2
Alpha	Significance	0.05

H Statistics	5.01932
P-Value	8%

Step 4 Conclusion :

P-Value
< 5% or >10% reject Null Hypothesis
>5% and <10% accept Null Hypothesis

Conclusion : There is no difference between ranks of the three different groups.

Figure 7.1(i) Result & Conclusion

(The Kruskal-Wallis Test Done in Excel)

(Corder)

Summary and Condition

Non-Parametric Test plays an important role in the field of statistics. In this project, we have seen the different types of non-parametric test and its importance. We have shown the mathematical procedures of the non-parametric tests namely the Wilcoxon Signed Ranked test, Sign test, Mann Whitney test, Kruskal Wallis and Friedman test. Next, using our skills in mathematics, a stepwise implementation of these non-parametric tests has also been done on MS Excel 2010.

Throughout the project, we have realized that non parametric tests have allowed us to test for not- normally distributed samples of data more easily.

Division of Work

	Krisen Pareanen	Vianee Bastalingum	Lovena Nithoo	Jagoo Kushal Girish	Mardiapoullé Annielle
Report					
Cover, Bibliography & References	Done				
Acknowledgement		Done			
Abstract		Done			
Table of Figures	Done				
Table of Contents	Done				
Non-Parametric Test & General Steps			Done		
Types of Data			Edited		Done
Wilcoxon Ranked Sign Test		Partly Done & Edited	Partly Done & Prt Scrn	Partly Done	
Sign Test		Edited	Edited	Done & Prt Scrn	
Mann-Whitney		Prt Scrn	Done		
Friedman Test			Edited	Done	
Kruskal Wallis	Done & Prt Scrn	Edited			
Normality test	Edited	Edited & Prt Scrn			Done
Summary & Conclusion	Edited	Done			

	Krisen Pareanen	Vianee Bastalingum	Lovena Nithoo	Jagoo Kushal Girish	Mardiapoullé Annielle
Excel					
Wilcoxon Ranked Sign Test		Edited		Done	
Sign Test		Edited		Done	
Mann-Whitney		Done			
Kruskal Wallis	Done	Edited			
Friedman		Edited		Done	
Built-up of Project	Done	Done			

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