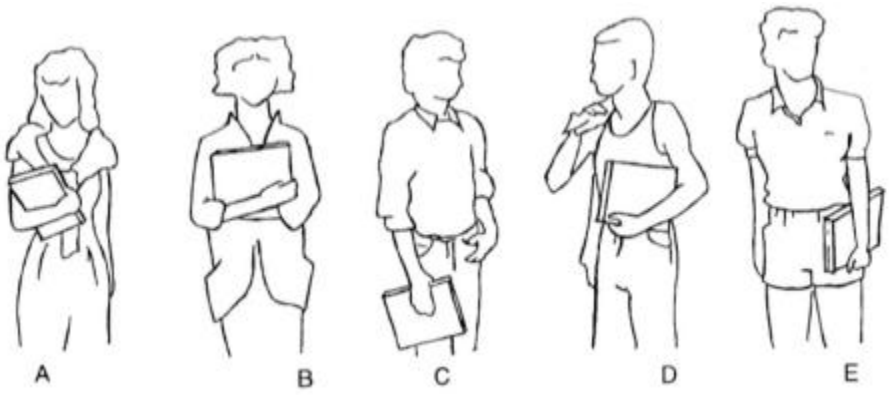
χ^2 Tests for Goodness of Fit and Independence

Jenni (1976) observed book carrying behavior in men and women. She identified five distinct styles of carrying books:



Jenni found that men usually use styles C, D or E to carry their books while women usually use styles A or B to carry their books.

Students in a research methods class observed University of Dayton students and recorded their book carrying style and whether they were male or female. The researcher wanted to know if each of the five book carrying styles is used equally frequently. The researcher also wanted to know if book carrying style is independent of a person's gender.

χ^2 Goodness of Fit Test

To see whether each style of carrying books is equally preferred, we will use the χ^2 goodness of fit test.

1. Step 1: Write the null and alternative hypotheses and specify the probability of making a Type I error:

   H_0: In the general population, there is no preference for any specific style of carrying books.
   H_1: In the general population, one or more of the specific styles of carrying books is preferred over the others.

   $\alpha = .05$

2. Step 2: We will compare the reported *p* value to α. If $p \leq \alpha$, we will reject H_0 and conclude that some of the book carrying styles are preferred more than others.

3. Step 3: Calculate the test statistics:

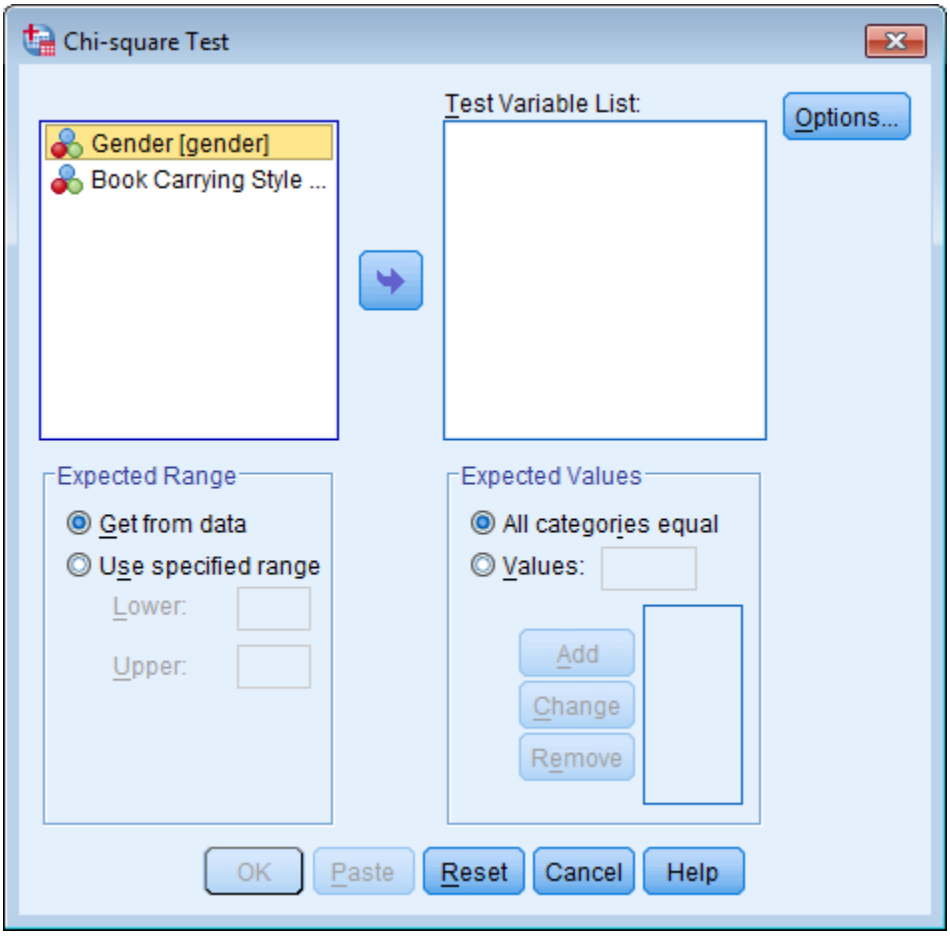
   a. Open SPSS

   b. Either type the data (see the end of the handout for the data) or open a data set. The data used in this sample is available from

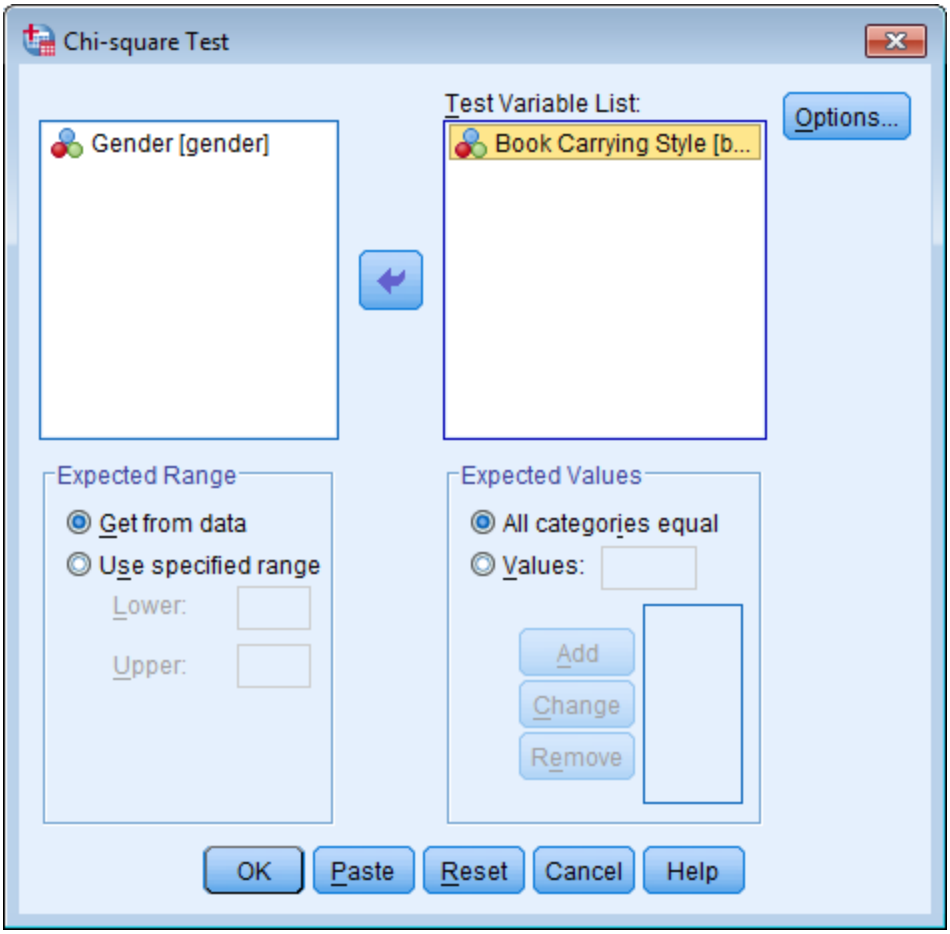
<http://academic.udayton.edu/gregelvers/psy216/spss/chisq.sav>.

If you are typing the data, switch to the Variable View and create two variables: gender and bookstyle. If you want to add value labels for gender, the value 1 is female and the value 2 is male. If you want to add value labels for bookstyle, the value 1 is Style A, the value 2 is Style B, the value 3 is Style C, the value 4 is Style D and the value 5 is Style E

c. Analyze | Nonparametric Tests | Legacy Dialogs | Chi Square



d. Move the variable of interest (Book Carrying Style) to the Test Variable List box



e. In this example, the hypothesis is that the expected values are equal. If this was not the case, click the Values: option in the Expected Values group, enter the expected value for the first category and click add. Enter the expected value for the second category and click add. Continue until all of the expected values have been added.

f. Click the OK button

g. The results will appear in the SPSS output viewer. The first section of the results gives the observed and expected frequencies for each book carrying style. For example 115 people were observed using carrying style A while the expected frequency for that style is 51.4:

Book Carrying Style

	Observed N	Expected N	Residual
Style A	115	51.4	63.6
Style B	16	51.4	-35.4
Style C	52	51.4	.6
Style D	15	51.4	-36.4
Style E	59	51.4	7.6
Total	257		

h. The second part of the results gives the value of χ^2 (129.984), its degrees of freedom (4) and the *p* value (.000):

Test Statistics

	Book Carrying Style
Chi-Square	129.984[a]
df	4
Asymp. Sig.	.000

a. 0 cells (.0%) have expected frequencies less than 5. The minimum expected cell frequency is 51.4.

4. Step 4: Make a decision: Because the *p* value (.000) is less than or equal to the α level (.05), we will reject H_0 and conclude that some of the book carrying styles are used more than others.

   We would write:

   The participants preferred some book carrying styles over others, $\chi^2(4, n = 257) = 129.98$, $p = .000$.

χ^2 Test for Independence

The researcher's other question was whether the book carrying style was independent of the person's gender. We will use a χ^2 test for independence to answer this question.

1. Step 1: Write the null and alternative hypotheses and specify the probability of making a Type I error:

   H_0: In the general population, there is no relation between gender and book carrying style
   H_1: In the general population, there is a relation between gender and book carrying style

   $\alpha = .05$

2. Step 2: We will compare the reported *p* value to α. If $p \leq \alpha$, we will reject H_0 and conclude that some of the book carrying styles are preferred more than others.

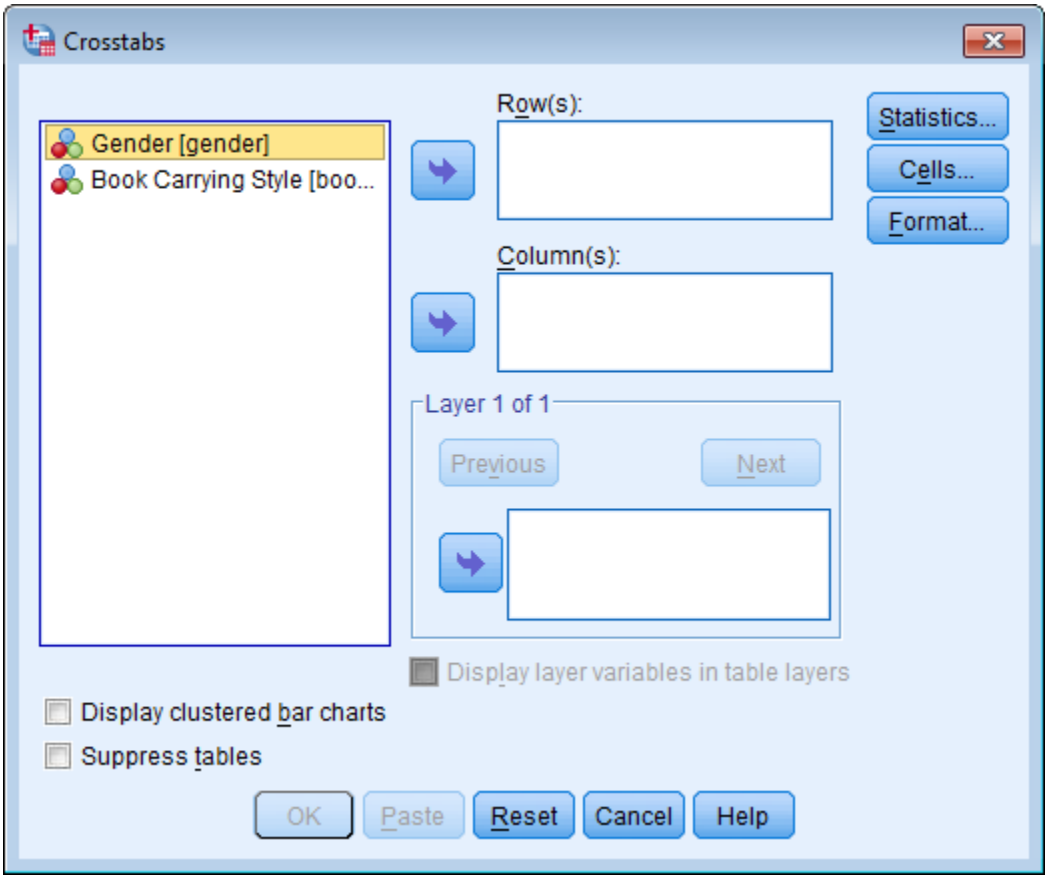
3. Step 3: Calculate the test statistics:

   a. If you have not already done so, open SPSS

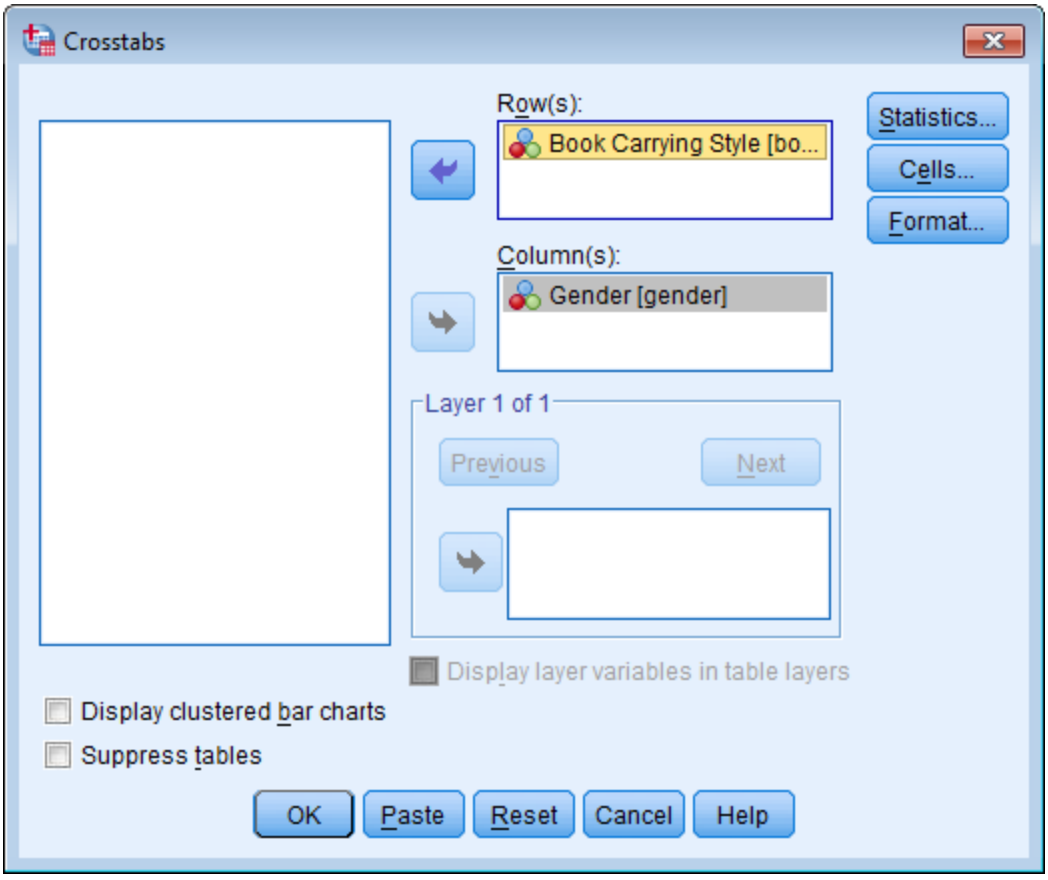
   b. If you have not already done so, either type the data (see the end of the handout for the data) or open a data set. The data used in this sample is available from <http://academic.udayton.edu/gregelvers/psy216/spss/chisq.sav>.

If you are typing the data, switch to the Variable View and create two variables: gender and bookstyle. If you want to add value labels for gender, the value 1 is female and the value 2 is male. If you want to add value labels for bookstyle, the value 1 is Style A, the value 2 is Style B, the value 3 is Style C, the value 4 is Style D and the value 5 is Style E

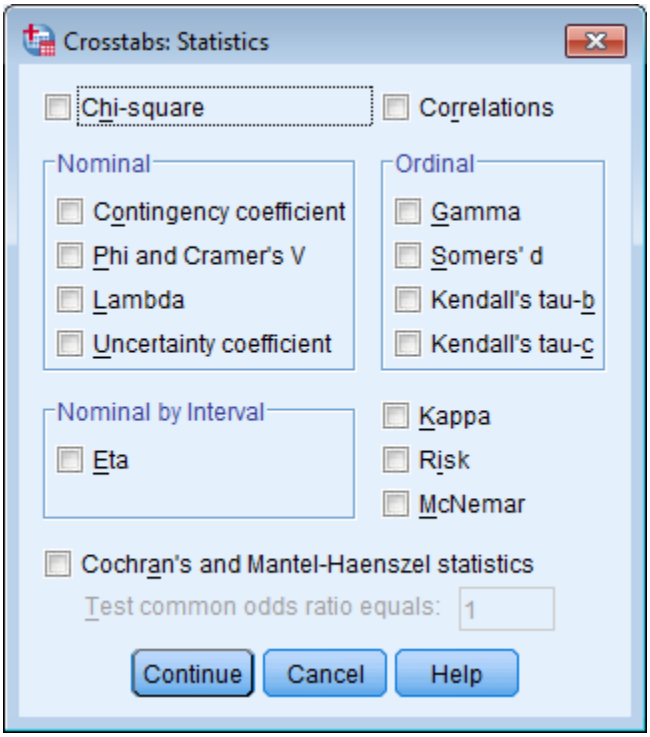
c. Analyze | Descriptive Statistics | Crosstabs



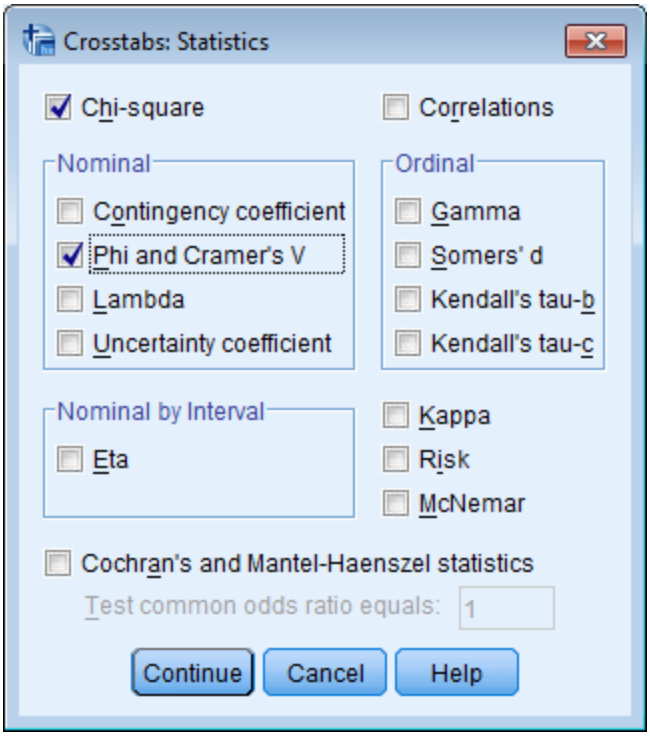
d. Move one of the variables in the relation into the Row(s): box and the other variable in the relation into the Column(s): box. In this example, move Book Carrying Style to the Row(s): box and move Gender to the Column(s): box:



e. Click the Statistics… button

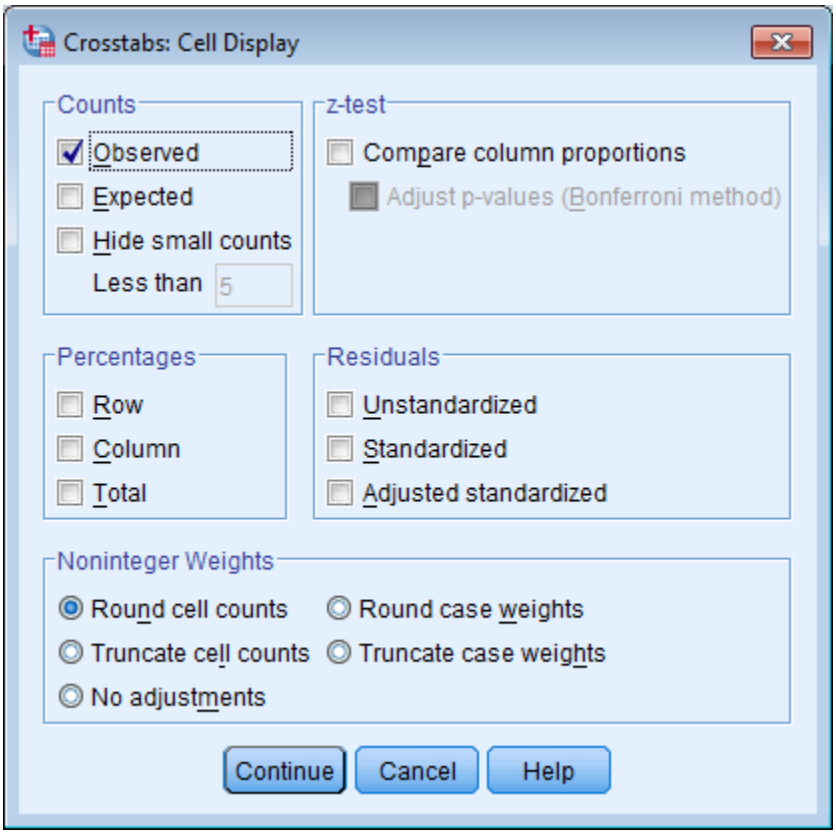


f. Check Chi-square and Phi and Cramer's V

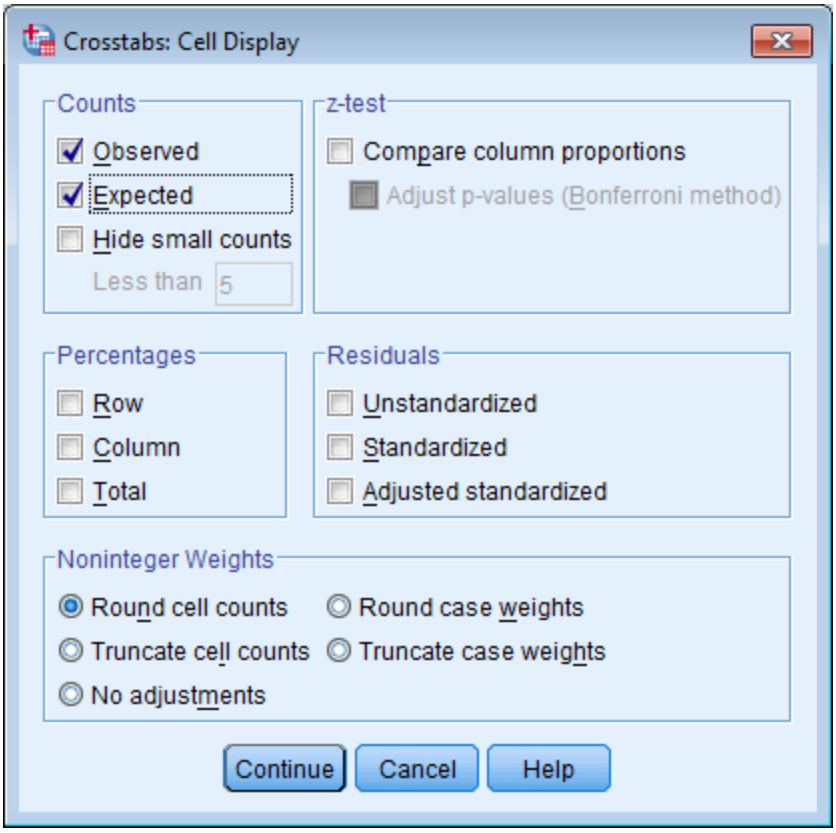


g. Click Continue

h. Click the Cells… button



i. Check Expected



j. Click Continue

k. Click OK

l. The results will appear in the output viewer. The Book Carrying Style * Gender Crosstabulation output gives the observed (Count) and expected (Expected Count) frequencies for males and females for each book carrying style. It also gives the row and column totals needed if you are calculating the expected frequencies by hand.

Book Carrying Style * Gender Crosstabulation

			Gender		
			Female	Male	Total
Book Carrying Style	Style A	Count	107	8	115
		Expected Count	68.5	46.5	115.0
	Style B	Count	15	1	16
		Expected Count	9.5	6.5	16.0
	Style C	Count	17	35	52
		Expected Count	31.0	21.0	52.0
	Style D	Count	5	10	15
		Expected Count	8.9	6.1	15.0
	Style E	Count	9	50	59
		Expected Count	35.1	23.9	59.0
Total		Count	153	104	257
		Expected Count	153.0	104.0	257.0

m. The Chi-Squares Tests output gives the value of χ^2 (129.220, from the Pearson Chi-Square row), the degrees of freedom (4, also from the Pearson Chi-Square row) and the *p* value (.000, from the Pearson Chi-Square row and the Asymp. Sig. (2-sided) column):

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	129.220[a]	4	.000
Likelihood Ratio	146.100	4	.000
Linear-by-Linear Association	117.485	1	.000
N of Valid Cases	257		

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 6.07.

n. The Symmetric Measures output gives ϕ (.709) and Cramer's V (also .709)

Symmetric Measures

		Value	Approx. Sig.
Nominal by Nominal	Phi	.709	.000
	Cramer's V	.709	.000
N of Valid Cases		257	

4. Step 4: Make a decision: Because the *p* value associated with χ2 (.000) is less than or equal to α (.05), we reject H0 and conclude that book carrying style depends on the person's gender.

   We would write:

   Book carrying style depends on a person's gender, $\chi^2(4, n = 257) = 129.22$, $p = .000$, $\phi = .709$.

The frequency data:

	Female	Male	Total
Style A	107	8	115
Style B	15	1	16
Style C	17	35	52
Style D	5	10	15
Style E	9	50	59
Total	153	104	257

χ^2 Goodness of Fit:

1. Step 1 is the same as Step 1 using SPSS

2. Step 2: The χ^2 has 4 degrees of freedom (df = number of categories – 1 = 5 – 1 = 4). The critical value of χ^2 for 4 degrees of freedom with $\alpha = .05$ is 9.488 (from a table of critical values of χ^2). If the calculated value of χ^2 is greater than or equal to the critical χ^2, we will reject H_0.

3. Step 3: Calculate the test statistic:

    a. The hypothesis is that all of the book carrying styles are used equally. Given that there are 257 observations and 5 book carrying styles, the expected frequency, if equal, would be 257 / 5 = 51.4

    b. Calculate (Observed – Expected)^2 / Expected for each of the five book carrying styles and sum:

	Style A	**Style B**	**Style C**	**Style D**	**Style E**
Observed	115	16	52	15	59
Expected	51.4	51.4	51.4	51.4	51.4
(O – E)	63.6	-35.4	0.6	-36.4	7.6
$(O - E)^2$	4044.96	1253.16	0.36	1324.96	57.76
$(O - E)^2 / E$	78.696	24.381	0.007	25.777	1.124

    $\chi^2 = 78.696 + 24.381 + 0.007 + 25.777 + 1.124 = 129.985$

4. Step 4: Make a decision: The observed value of χ^2 (129.985) is larger than the critical value of χ^2 (9.488) so we reject H_0 and conclude that it is likely that some book carrying styles are used more than others.

χ^2 Test for Independence

1. Step 1 is the same as Step 1 using SPSS

2. Step 2: The χ^2 has 4 degrees of freedom (df = (number of categories for one variable – 1) · (number of categories for the other variable – 1) = (5 – 1) · (2 – 1) = 4). The critical value of χ^2 for 4 degrees of freedom with $\alpha = .05$ is 9.488 (from a table of critical values of χ^2). If the calculated value of χ^2 is greater than or equal to the critical χ^2, we will reject H_0.

3. Step 3: Calculate the test statistic:

   a. Calculate the expected frequencies for each cell based on the row and column totals:

   $f_e = f_c \cdot f_r / n$

   b.

		Female	**Male**	Total
Style A	Obs.	107	8	115
	Exp.	153·115/257=68.46	104·115/257=46.54	
Style B	Obs.	15	1	16
	Exp.	153·16/257=9.53	104·16/257=6.47	
Style C	Obs.	17	35	52
	Exp.	153·52/257=30.96	104·52/257=21.04	
Style D	Obs.	5	10	15
	Exp.	153·15/257=8.93	104·15/257=6.07	
Style E	Obs.	9	50	59
	Exp.	153·59/257=35.12	104·59/257=23.88	
Total		153	104	257

   b. Calculate (Observed – Expected)^2 / Expected for each cell and sum:

   $\chi^2 = (107 - 68.46)^2 / 68.46 + (15 - 9.53)^2 / 9.53 + (17 - 30.96)^2 / 30.96 + (5 - 8.93)^2 / 8.93 + (9 - 35.12)^2 / 35.12 + (8 - 46.54)^2 / 46.54 + (1 - 6.47)^2 / 6.47 + (35 - 21.04)^2 / 21.04 + (10 - 6.07)^2 / 6.07 + (50 - 23.88)^2 / 23.88 = 129.20$

   c. Calculate $\phi = \sqrt{(\chi^2/n)} = \sqrt{(129.20 / 257)} = .709$

4. Step 4: Make a decision: The observed value of χ^2 (129.20) is larger than the critical value of χ^2 (9.488) so we reject H_0 and conclude that it is likely that book carrying style depends on the person's gender.