§5.5—Curve Sketching

All of our mathematical studies will now culminate in a single task of sketching the graphs of unfamiliar functions (without a calculator!). Here are some basic guidelines for sketching the graph of $y = f(x)$ by hand:

1. **Find the domain**: for what values of x is $f(x)$ defined? There's no sense talking about a function where it doesn't exist.
2. **Find and name any discontinuities**: Classify any discontinuities as holes, VA's, or Jumps. If there is a VA, plot it now, for asymptotes define the shape of a graph.
3. **Analyze the end behavior**: find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. Identify any horizontal asymptotes. Plot any HA now.
4. **Find easy intercepts**: the y-intercept, $(0, f(0))$ will be easy to find. Finding x-intercepts by setting $f(x) = 0$ might be too difficult. If so, forget about them. We probably don't need them anyway. Plot any intercepts you find now.
5. **Find any symmetry**: If $f(-x) = f(x)$, the function is even and has y-axis symmetry. Obtain the graph in Quadrants I and IV or II and III and you're pretty much done. If $f(-x) = -f(x)$, the function is odd and has origin symmetry. Get half of it done, then take it for a spin 180° to finish up.

Now it's time to B.O.T.C. (Bust Out The Calculus)

6. **Intervals of Increasing/Decreasing**: Find $f'(x)$, then find any critical values. Find the values of f' in between any critical values and any discontinuities.
7. **Find any Relative Extrema**: Using your chart from #6 at the critical values (careful not to call a VA a relative extremum), find the coordinates of any relative extrema. Plot these points now.
8. **Intervals of Concavity**: Find the second derivative, then find any possible inflection values. Find the values of f'' in between any possible inflection values and any discontinuities.
9. **Find any Inflection points**: Using the chart from #8, at the p.i.v.'s (careful not to call a VA an inflection point), find the coordinates of any inflection points. Plot these points now.
10. **Connect the Dots**: sketch the function, making sure the graph rises and falls in accordance with #6 and curves according to #8.

Example 1:

Sketch the graph of $f(x)=\dfrac{2\left(x^2-9\right)}{x^2-4}$.

Example 2:

Sketch the graph of $f(x)=\dfrac{x}{\sqrt{x^2+2}}$

For a rational function whose numerator is one degree larger than the denominator, then a **Slant Asymptote** (SA) might exist. It's equation, of the form $y = mx + b$, is the quotient found using long division as long as the **remainder is non-zero**.

Example 3:

Sketch the graph of $f\left(x\right) = \dfrac{x^3 + 8}{x^2 - 4}$

Example 4:

Sketch the graph of $f\left(x\right) = \dfrac{x^3}{x^2 + 1}$

Example 5:

Sketch the graph of $f(x) = 2x^{5/3} - 5x^{4/3}$

Example 6:

Sketch the graph of $f(x) = xe^{x}$

Example 7:

Sketch the graph of $f\left(x\right)=\dfrac{x^2}{\sqrt{x+1}}$

Example 8:

Sketch the graph of $f\left(x\right)=\ln\left(4-x^2\right)$