Chapter 7

Rotational Motion and the Law of Gravity

Quick Quizzes

1. (c). For a rotation of more than 180°, the angular displacement must be larger than $\pi = 3.14$ rad. The angular displacements in the three choices are (a) 6 rad − 3 rad = 3 rad, (b) 1 rad − (−1) rad = 2 rad, (c) 5 rad − 1 rad = 4 rad.

2. (b). Because all angular displacements occurred in the same time interval, the displacement with the lowest value will be associated with the lowest average angular speed.

3. (b). From $\alpha = \dfrac{\omega^2 - \omega_0^2}{2\Delta\theta} = \dfrac{\omega^2 - 0}{2\Delta\theta} = \dfrac{\omega^2}{2\Delta\theta}$, it is seen that the case with the smallest angular displacement involves the highest angular acceleration.

4. (b). All points in a rotating rigid body have the same angular speed.

5. (a). Andrea and Chuck have the same angular speed, but Andrea moves in a circle with twice the radius of the circle followed by Chuck. Thus, from $v_t = r\omega$, it is seen that Andrea's tangential speed is twice Chuck's.

6. 1. (e). Since the tangential speed is constant, the tangential acceleration is zero.

   2. (a). The centripetal acceleration, $a_c = v_t^2/r$, is inversely proportional to the radius when the tangential speed is constant.

   3. (b). The angular speed, $\omega = v_t/r$, is inversely proportional to the radius when the tangential speed is constant.

7. (c). Both the velocity and acceleration are changing in direction, so neither of these vector quantities is constant.

8. (b) and (c). According to Newton's law of universal gravitation, the force between the ball and the Earth depends on the product of their masses, so both forces, that of the ball on the Earth, and that of the Earth on the ball, are equal in magnitude. This follows also, of course, from Newton's third law. The ball has large motion compared to the Earth because according to Newton's second law, the force gives a much greater acceleration to the small mass of the ball.

9. (e). From $F = G\,Mm/r^2$, the gravitational force is inversely proportional to the square of the radius of the orbit.

10. (d). The semi-major axis of the asteroid's orbit is 4 times the size of Earth's orbit. Thus, Kepler's third law $(T^2/r^3 = constant)$ indicates that its orbital period is 8 times that of Earth.

Answers to Even Numbered Conceptual Questions

2. If we assume they are separated by about 10 m and their masses are estimated to be 70 kg and 40 kg, then, using the law of universal gravitation, we estimate a gravitational force on the order of 10^{-9} N.

4. The need for a large force toward the center of the circular path on objects near the equator will cause the Earth to bulge at the equator. A force toward the center of the circular path is not needed at the poles, so the radius in this direction will be smaller than at the equator.

6. To a good first approximation, your bathroom scale reading is unaffected because you, Earth, and the scale are all in free fall in the Sun's gravitational field, in orbit around the Sun. To a precise second approximation, you weight slightly less at noon and at midnight than you do at sunrise or sunset. The Sun's gravitational fields is a little weaker at the center of the Earth than at the surface sub-solar point, and a little weaker still on the far side of the planet. When the Sun is high in your sky, its gravity pulls up on you a little more strongly than on the Earth as a whole. At midnight the Sun pulls down on you a little less strongly than it does on the Earth below you. So you can have another doughnut with lunch, and your bedsprings will still last a little longer.

8. The astronaut is accelerating toward the Earth at the same rate as is the spaceship. Thus, if the astronaut drops a wrench, it will float in space next to him. Likewise, he will float in space next to a desk or with reference to the spaceship. Thus, he believes himself to be weightless.

10. Consider one end of a string connected to a spring scale and the other end connected to an object, of true weight w. The tension T in the string will be measured by the scale and construed as the apparent weight. We have $w\text{-}T\text{=}ma_c$. This gives, $T\text{=}w\text{-}ma_c$. Thus, the apparent weight is less than the actual weight by the term ma_c. At the poles the centripetal acceleration is zero. Thus, $T\text{=}w$. However, at the equator the term containing the centripetal acceleration is nonzero, and the apparent weight is less than the true weight.

12. If the acceleration is constant in magnitude and perpendicular to the velocity, the object is moving in a circular path at constant speed. If the acceleration is parallel to the velocity, the object is either speeding up, v and a in same direction, or slowing down, v and a in opposite directions.

14. When an object follows an orbital path, there is a force acting on it that produces the centripetal acceleration. This force and the centripetal acceleration are always directed toward the center of the orbit. In the case of a planet orbiting the Sun, this force is the gravitational force exerted on the planet by the Sun, and it points toward the center of the Sun. Hence, the center of the Sun must coincide with the center of the orbit and lie in the plane of the orbit.

16.   Kepler's second law says that equal areas are swept out in equal times by a line drawn from the Sun to the planet. For this to be so, the planet must move fastest when it is closest to the Sun. This, surprisingly, occurs during the winter.

18.   Yes. The force of friction, a non-conservative force, opposes the motion of the satellite and causes its speed to decrease with time.

Answers to Even Numbered Problems

2. 2.1 m, 1.2×10^2 m , 7.7×10^2 m

4. 4.2×10^{-2} rad/s^2

6. -226 rad/s^2

8. 41 rad/s^2

10. 50 rev

12. 1.02 m

14. 4.9×10^{-2} rad/s

16. 2.36×10^3 m/s^2

18. **(a)** 14.1 m/s   **(b)** 200 m   **(c)** 28.3 s

20. 1.5×10^2 rev/s

22. $0.966\,g$

24. **(b)** 20.1°

26. The required tension in the vine is 1.4×10^3 N . He does not make it.

28. **(a)** 25 kN   **(b)** 12 m/s

30. **(a)** 4.39×10^{20} N toward the Sun   **(b)** 1.99×10^{20} N toward the Earth
      (c) 3.55×10^{22} N toward the Sun

32. 2.59×10^8 m from the center of the Earth

34. 2.00 kg and 3.00 kg

36. **(a)** 5.59×10^3 m/s   **(b)** 3.98 h   **(c)** 1.47×10^3 N

38. **(a)** -4.76×10^9 J   **(b)** 568 N

40. 1.63×10^4 rad/s

42. **(a)** 3.77 m/s^2 **(b)** 3.26 s

44. **(a)** 56.5 rad/s   **(b)** 22.4 rad/s   **(c)** -7.63×10^{-3} rad/s^2

  (d) 1.77×10^5 rad   **(e)** 5.81 km

46. **(a)** $v_{min} = \sqrt{Rg\left(\dfrac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}$, $v_{max} = \sqrt{Rg\left(\dfrac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$

  (b) 8.6 m/s to 17 m/s

48. **(a)** 8.42 N   **(b)** 64.8°   **(c)** 1.67 N

50. **(a)** 0   **(b)** 1.3 kN   **(c)** 2.1 kN

52. **(a)** 2.1 m/s   **(b)** 54°   **(c)** 4.7 m/s

54. **(a)** 1.3×10^{17} N   **(b)** 2.6×10^{12} N/kg

58. **(a)** 15 N, 30 N   **(b)** 0.17

60. **(a)** The upward force of the seat and the downward force of gravity
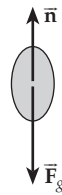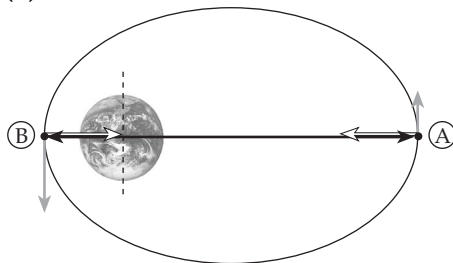

  (b) The upward force of the seat and the downward force of gravity

  (c) The seat exerts the greatest force at the bottom of the track.
  (d) 546 N at the top, 826 N at the bottom. Yes.

62. **(a)**

  (b) Conservation of energy requires the velocity at B be greater than that at A.
  (c) Newton's law of universal gravitation states that the spacecraft experiences a greater gravitational force (and hence has greater acceleration) at B than at A.

64. 2.06×10^3 rev/min

66. **(a)** 2.38×10^5 m/s^2 horizontally inward or 2.43×10^4 g

    (b) 360 N inward perpendicular to the cone

    (c) 4.75×10^4 m/s^2 outward along the wall of the cone

68. **(a)** 2.88 s   **(b)** 12.8 s

70. **(a)** 4.1 m/s   **(b)** 80°   **(c)** 1.7 m   **(d)** no

Problem Solutions

7.1   (a) $\theta = \dfrac{s}{r} = \dfrac{60\,000 \text{ mi}}{1.0 \text{ ft}}\left(\dfrac{5280 \text{ ft}}{1 \text{ mi}}\right) = \boxed{3.2 \times 10^8 \text{ rad}}$

(b) $\theta = 3.2 \times 10^8 \text{ rad}\left(\dfrac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{5.0 \times 10^7 \text{ rev}}$

7.2   The distance traveled is $s = r\theta$, where θ is in radians.

For 30°, $s = r\theta = (4.1 \text{ m})\left[30°\left(\dfrac{\pi \text{ rad}}{180°}\right)\right] = \boxed{2.1 \text{ m}}$

For 30 radians, $s = r\theta = (4.1 \text{ m})(30 \text{ rad}) = \boxed{1.2 \times 10^2 \text{ m}}$

For 30 revolutions, $s = r\theta = (4.1 \text{ m})\left[30 \text{ rev}\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right)\right] = \boxed{7.7 \times 10^2 \text{ m}}$

7.3   The Earth moves through 2π rad in one year $(3.156 \times 10^7 \text{ s})$. Thus,

$$\omega = \frac{2\pi \text{ rad}}{3.156 \times 10^7 \text{ s}} = \boxed{1.99 \times 10^{-7} \text{ rad/s}}$$

Alternatively, the Earth moves through 360° in one year (365.242 days).

Thus, $\omega = \dfrac{360°}{365.2 \text{ days}} = \boxed{0.986 \text{ deg/day}}$

7.4   We use $\alpha = \dfrac{\omega_f - \omega_i}{t}$ and find

$$\alpha = \frac{0.20 \text{ rev/s} - 0}{30 \text{ s}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = \boxed{4.2 \times 10^{-2} \text{ rad/s}^2}$$

7.5   (a) $\alpha = \dfrac{\left(2.51 \times 10^4 \text{ rev/min} - 0\right)}{3.20 \text{ s}}\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\dfrac{1 \text{ min}}{60.0 \text{ s}}\right) = \boxed{821 \text{ rad/s}^2}$

(b) $\theta = \omega_i t + \dfrac{1}{2}\alpha t^2 = 0 + \dfrac{1}{2}\left(821 \ \dfrac{\text{rad}}{\text{s}^2}\right)(3.20 \ \text{s})^2 = \boxed{4.21 \times 10^3 \ \text{rad}}$

7.6 $\omega_i = 3\,600 \ \dfrac{\text{rev}}{\text{min}}\left(\dfrac{2\pi \ \text{rad}}{1 \ \text{rev}}\right)\left(\dfrac{1 \ \text{min}}{60.0 \ \text{s}}\right) = 377 \ \text{rad/s}$

$\Delta\theta = 50.0 \ \text{rev}\left(\dfrac{2\pi \ \text{rad}}{1 \ \text{rev}}\right) = 314 \ \text{rad}$

Thus, $\alpha = \dfrac{w^2 - \omega_i^2}{2\,\Delta\theta} = \dfrac{0 - (377 \ \text{rad/s})^2}{2(314 \ \text{rad})} = \boxed{-226 \ \text{rad/s}^2}$

7.7 From $\omega^2 = \omega_i^2 + 2\alpha\,\Delta\theta$, the angular displacement is

$\Delta\theta = \dfrac{\omega^2 - \omega_i^2}{2\alpha} = \dfrac{(2.2 \ \text{rad/s})^2 - (0.60 \ \text{rad/s})^2}{2(0.70 \ \text{rad/s}^2)} = \boxed{3.2 \ \text{rad}}$

7.8 From $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$, the angular acceleration is

$\alpha = \dfrac{2[\Delta\theta - \omega_i t]}{t^2} = \dfrac{2\left[4.7 \ \text{rev}\left(\dfrac{2\pi \ \text{rad}}{1 \ \text{rev}}\right) - 0\right]}{(1.2 \ \text{s})^2} = \boxed{41 \ \text{rad/s}^2}$

7.9 Main Rotor: $v = r\omega = (3.80 \ \text{m})\left(450 \ \dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi \ \text{rad}}{1 \ \text{rev}}\right)\left(\dfrac{1 \ \text{min}}{60 \ \text{s}}\right) = \boxed{179 \ \text{m/s}}$

$v = \left(179 \ \dfrac{\text{m}}{\text{s}}\right)\left(\dfrac{v_{sound}}{343 \ \text{m/s}}\right) = \boxed{0.522\,v_{sound}}$

Tail Rotor: $v = r\omega = (0.510 \ \text{m})\left(4138 \ \dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi \ \text{rad}}{1 \ \text{rev}}\right)\left(\dfrac{1 \ \text{min}}{60 \ \text{s}}\right) = \boxed{221 \ \text{m/s}}$

$v = \left(221 \ \dfrac{\text{m}}{\text{s}}\right)\left(\dfrac{v_{sound}}{343 \ \text{m/s}}\right) = \boxed{0.644\,v_{sound}}$

7.10   We will break the motion into two stages: (1) an acceleration period and (2) a deceleration period.

The angular displacement during the acceleration period is

$$\theta_1 = \omega_{av}t = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left[\frac{(5.0 \text{ rev/s})(2\pi \text{ rad/1 rev}) + 0}{2}\right](8.0 \text{ s}) = 126 \text{ rad}$$

and while decelerating,

$$\theta_2 = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left[\frac{0 + (5.0 \text{ rev/s})(2\pi \text{ rad/1 rev})}{2}\right](12 \text{ s}) = 188 \text{ rad}$$

The total displacement is $\theta = \theta_1 + \theta_2 = \left[(126 + 188) \text{ rad}\right]\left(\dfrac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{50 \text{ rev}}$

7.11   When completely rewound, the tape is a hollow cylinder with a difference between the inner and outer radii of ~1 cm. Let N represent the number of revolutions through which the driving spindle turns in 30 minutes (and hence the number of layers of tape on the spool). We can determine N from:

$$N = \frac{\Delta\theta}{2\pi} = \frac{\omega(\Delta t)}{2\pi} = \frac{(1 \text{ rad/s})\left[(30 \text{ min})(60 \text{ s/1 min})\right]}{2\pi \text{ rad/rev}} = 286 \text{ rev}$$

Then, $thickness \sim \dfrac{1 \text{ cm}}{N}$ $\boxed{\sim 10^{-2} \text{ cm}}$

7.12   The angular displacement of the coin while stopping is

$$\Delta\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{0 - (18.0 \text{ rad/s})^2}{2(-1.90 \text{ rad/s}^2)} = 85.3 \text{ rad}$$

The linear displacement is $\Delta s = r(\Delta\theta)$ with $r = diameter/2 = 1.20 \text{ cm}$, or

$$\Delta s = (1.20 \text{ cm})(85.3 \text{ rad}) = 102 \text{ cm} = \boxed{1.02 \text{ m}}$$

7.13 From $\Delta\theta = \omega_{av}t = \left(\dfrac{\omega + \omega_i}{2}\right)t$, we find the initial angular speed to be

$$\omega_i = \frac{2\,\Delta\theta}{t} - \omega = \frac{2(37.0\ \text{rev})\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right)}{3.00\ \text{s}} - 98.0\ \text{rad/s} = 57.0\ \text{rad/s}$$

The angular acceleration is then

$$\alpha = \frac{\omega - \omega_i}{t} = \frac{98.0\ \text{rad/s} - 57.0\ \text{rad/s}}{3.00\ \text{s}} = \boxed{13.7\ \text{rad/s}^2}$$

7.14 The radius of the cylinder is $r = 2.5\ \text{mi}\left(\dfrac{1609\ \text{m}}{1\ \text{mi}}\right) = 4.0 \times 10^3\ \text{m}$. Thus, from $a_c = r\,\omega^2$, the required angular velocity is

$$\omega = \sqrt{\frac{a_c}{r}} = \sqrt{\frac{9.80\ \text{m/s}^2}{4.0 \times 10^3\ \text{m}}} = \boxed{4.9 \times 10^{-2}\ \text{rad/s}}$$

7.15 The rotational velocity of the Earth is

$$\omega = \left(2\pi\ \frac{\text{rad}}{\text{day}}\right)\left(\frac{1\ \text{day}}{86\,400\ \text{s}}\right) = 7.27 \times 10^{-5}\ \text{rad/s}$$

and the equatorial radius of the Earth is $R_E = 6.38 \times 10^6\ \text{m}$

(a) At the equator, the radius of the circular motion is $r = R_E$.

Thus, $a_c = r\,\omega^2 = \left(6.38 \times 10^6\ \text{m}\right)\left(7.27 \times 10^{-5}\ \text{rad/s}\right)^2 = \boxed{3.37 \times 10^{-2}\ \text{m/s}^2}$

(b) A point at the north pole is on the axis of rotation, so $r = 0$ and $a_c = r\,\omega^2 = \boxed{0}$

7.16 Since the tire rotates at constant speed, the tangential acceleration of the stone is zero. Thus, its only acceleration is the centripetal acceleration given by

$$a_c = \frac{v_t^2}{r} = \frac{\left[\left(60.0\ \dfrac{\text{mi}}{\text{h}}\right)\left(\dfrac{0.447\ \text{m/s}}{1\ \text{mi/h}}\right)\right]^2}{1\ \text{ft}\left(\dfrac{1\ \text{m}}{3.281\ \text{ft}}\right)} = \boxed{2.36 \times 10^3\ \text{m/s}^2}$$

7.17   The final angular velocity is $\omega_f = 78 \dfrac{\text{rev}}{\text{min}}\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 8.17 \text{ rad/s}$

and the radius of the disk is $r = 5.0 \text{ in}\left(\dfrac{2.54 \text{ cm}}{1 \text{ in}}\right) = 12.7 \text{ cm} = 0.127 \text{ m}$

(a)   The tangential acceleration of the bug as the disk speeds up is

$$a_t = r\,\alpha = r\left(\dfrac{\Delta\omega}{\Delta t}\right) = (0.127 \text{ m})\left(\dfrac{8.17 \text{ rad/s}}{3.0 \text{ s}}\right) = \boxed{0.35 \text{ m/s}^2}$$

(b)   The final tangential speed of the bug is

$$v_t = r\,\omega_f = (0.127 \text{ m})(8.17 \text{ rad/s}) = \boxed{1.0 \text{ m/s}}$$

(c)   At $t = 1.0$ s, $\omega = \omega_i + \alpha t = 0 + \left(\dfrac{8.17 \text{ rad/s}}{3.0 \text{ s}}\right)(1.0 \text{ s}) = 2.7 \text{ rad/s}$

Thus, $a_t = r\,\alpha = \boxed{0.35 \text{ m/s}^2}$ as above, while the radial acceleration is

$$a_c = r\,\omega^2 = (0.127 \text{ m})(2.7 \text{ rad/s})^2 = \boxed{0.94 \text{ m/s}^2}$$

The total acceleration is $a = \sqrt{a_c^2 + a_t^2} = \boxed{1.0 \text{ m/s}^2}$, and the angle this acceleration makes with the direction of $\vec{a}_c$ is

$$\theta = \tan^{-1}\left(\dfrac{a_t}{a_c}\right) = \tan^{-1}\left(\dfrac{0.35}{0.94}\right) = \boxed{20^\circ}$$

7.18   (a)   The centripetal acceleration is $a_c = \dfrac{v_t^2}{r}$. Thus, when $a_c = a_t = 0.500 \text{ m/s}^2$, we have

$$v_t = \sqrt{r\,a_c} = \sqrt{(400 \text{ m})(0.500 \text{ m/s}^2)} = \sqrt{200} \text{ m/s} = \boxed{14.1 \text{ m/s}}$$

(b)   At this time, $t = \dfrac{v_t - v_i}{a_t} = \dfrac{\sqrt{200} \text{ m/s} - 0}{0.500 \text{ m/s}^2} = 28.3 \text{ s}$, and the linear displacement is

$$s = (v_t)_{av}\,t = \left(\dfrac{v_t + v_i}{2}\right)t = \left(\dfrac{\sqrt{200} \text{ m/s} + 0}{2}\right)(28.3 \text{ s}) = \boxed{200 \text{ m}}$$

(c)   The time is $t = \boxed{28.3 \text{ s}}$ as found in part (b) above.

7.19 (a) From $\Sigma F_r = ma_c$, we have

$$T = m\left(\frac{v_t^2}{r}\right) = \frac{(55.0 \text{ kg})(4.00 \text{ m/s})^2}{0.800 \text{ m}} = 1.10 \times 10^3 \text{ N} = \boxed{1.10 \text{ kN}}$$

(b) The tension is larger than her weight by a factor of

$$\frac{T}{mg} = \frac{1.10 \times 10^3 \text{ N}}{(55.0 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{2.04 \text{ times}}$$

7.20 Since $F_c = m\frac{v_t^2}{r} = mr\omega^2$, the needed angular velocity is

$$\omega = \sqrt{\frac{F_c}{mr}} = \sqrt{\frac{4.0 \times 10^{-11} \text{ N}}{(3.0 \times 10^{-16} \text{ kg})(0.150 \text{ m})}}$$

$$= (9.4 \times 10^2 \text{ rad/s})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{1.5 \times 10^2 \text{ rev/s}}$$

7.21 Friction between the tires and the roadway is capable of giving the truck a maximum centripetal acceleration of

$$a_{c,\text{max}} = \frac{v_{t,\text{max}}^2}{r} = \frac{(32.0 \text{ m/s})^2}{150 \text{ m}} = 6.83 \text{ m/s}^2$$

If the radius of the curve changes to 75.0 m, the maximum safe speed will be

$$v_{t,\text{max}} = \sqrt{r \, a_{c,\text{max}}} = \sqrt{(75.0 \text{ m})(6.83 \text{ m/s}^2)} = \boxed{22.6 \text{ m/s}}$$

7.22 $a_c = \frac{v_t^2}{r} = \frac{\left[\left(86.5 \frac{\text{km}}{\text{h}}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)\left(\frac{1\,000 \text{ m}}{1 \text{ km}}\right)\right]^2}{61.0 \text{ m}}\left(\frac{1 \, g}{9.80 \text{ m/s}^2}\right) = \boxed{0.966 \, g}$

7.23 (a) $a_c = r\omega^2 = (2.00 \text{ m})(3.00 \text{ rad/s})^2 = \boxed{18.0 \text{ m/s}^2}$

(b) $F_c = ma_c = (50.0 \text{ kg})(18.0 \text{ m/s}^2) = \boxed{900 \text{ N}}$

(c) We know the centripetal acceleration is produced by the force of friction. Therefore, the needed static friction force is $f_s = 900$ N. Also, the normal force is $n = mg = 490$ N. Thus, the minimum coefficient of friction required is

$$\mu_s = \frac{(f_s)_{max}}{n} = \frac{900 \text{ N}}{490 \text{ N}} = \boxed{1.84}$$

So large a coefficient of friction is unreasonable, and she will not be able to stay on the merry-go-round.

7.24   (a) From $\Sigma F_y = 0$, we have

$$n\cos\theta = mg \qquad\qquad (1)$$
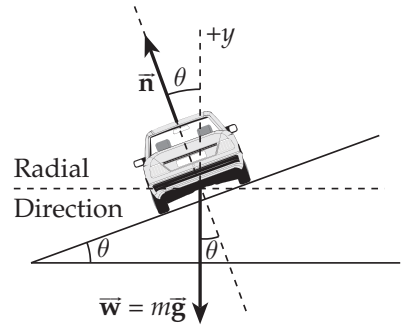
where n is the normal force exerted on the car by the ramp.

Now, require that, $F_{radial\ inward} = ma_c$, or

$$n\sin\theta = m\frac{v_t^2}{r} \qquad\qquad (2)$$

Divide (2) by (1) to obtain $\boxed{\tan\theta = \dfrac{v_t^2}{rg}}$

(b) If $r = 50.0$ m and $v_t = 13.4$ m/s, the needed bank angle is

$$\theta = \tan^{-1}\left[\frac{(13.4 \text{ m/s})^2}{(50.0 \text{ m})(9.80 \text{ m}/^2)}\right] = \boxed{20.1°}$$

7.25   (a) Since the 1.0-kg mass is in equilibrium, the tension in the string is

$$T = mg = (1.0 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{9.8 \text{ N}}$$

(b) The tension in the string must produce the centripetal acceleration of the puck. Hence, $F_c = T = \boxed{9.8 \text{ N}}$.

(c) From $F_c = m_{puck}\left(\dfrac{v_t^2}{r}\right)$, we find $v_t = \sqrt{\dfrac{rF_c}{m_{puck}}} = \sqrt{\dfrac{(1.0 \text{ m})(9.8 \text{ N})}{0.25 \text{ kg}}} = \boxed{6.3 \text{ m/s}}$

7.26 As he passes through the bottom of his swing, the tension in the vine must equal (1) his weight plus (2) the force needed produce the centripetal acceleration. That is,

$$T = mg + m\left(\frac{v_t^2}{r}\right) = (85 \text{ kg})(9.80 \text{ m/s}^2) + (85 \text{ kg})\left[\frac{(8.0 \text{ m/s})^2}{10 \text{ m}}\right] = \boxed{1.4 \times 10^3 \text{ N}}$$

Since $T > 1000$ N, $\boxed{\text{he will not make it}}$.

7.27 (a) The centripetal acceleration is

$$a_c = r\omega^2 = (9.00 \text{ m})\left[\left(4.00 \text{ } \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\right]^2 = \boxed{1.58 \text{ m/s}^2}$$

(b) At the bottom of the circular path, the normal force exerted by the seat must support the weight and also produce the centripetal acceleration.

Thus, $n = m(g + a_c) = (40.0 \text{ kg})\left[(9.80 + 1.58) \text{ m/s}^2\right] = \boxed{455 \text{ N} \text{ upward}}$

(c) At the top of the path, the weight must offset the normal force of the seat plus supply the needed centripetal acceleration. Therefore, $mg = n + ma_c$, or

$$n = m(g - a_c) = (40.0 \text{ kg})\left[(9.80 - 1.58) \text{ m/s}^2\right] = \boxed{329 \text{ N} \text{ upward}}$$

(d) At a point halfway up, the seat exerts an upward vertical component equal to the child's weight (392 N) and a component toward the center having magnitude $F_c = ma_c = (40.0 \text{ kg})(1.58 \text{ m/s}^2) = 63.2 \text{ N}$. The total force exerted by the seat is

$$F_R = \sqrt{(392 \text{ N})^2 + (63.2 \text{ N})^2} = 397 \text{ N} \text{ directed inward and at}$$

$$\theta = \tan^{-1}\left(\frac{392 \text{ N}}{63.2 \text{ N}}\right) = \boxed{80.8° \text{ above the horizontal}}$$

7.28 (a) At A, the track supports the weight and supplies the centripetal acceleration.

Thus, $n = mg + m\frac{v_t^2}{r} = (500 \text{ kg})\left[9.80 \text{ m/s}^2 + \frac{(20.0 \text{ m/s})^2}{10 \text{ m}}\right] = \boxed{25 \text{ kN}}$

(b) At B, the weight must offset the normal force exerted by the track and produce the needed centripetal acceleration, or $mg = n + m\dfrac{v_t^2}{r}$. If the car is on the verge of leaving the track, then $n = 0$ and $mg = m\dfrac{v_t^2}{r}$. Hence,

$$v_t = \sqrt{rg} = \sqrt{(15 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{12 \text{ m/s}}$$

7.29   At the half-way point the spaceship is 1.92×10^8 m from both bodies. The force exerted on the ship by the Earth is directed toward the Earth and has magnitude

$$
\begin{aligned}
F_E &= \frac{Gm_E m_s}{r^2} \\
&= \frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)\left(3.00 \times 10^4 \text{ kg}\right)}{\left(1.92 \times 10^8 \text{ m}\right)^2} = 325 \text{ N}
\end{aligned}
$$

The force exerted on the ship by the Moon is directed toward the Moon and has a magnitude of

$$
\begin{aligned}
F_M &= \frac{Gm_M m_s}{r^2} \\
&= \frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(7.36 \times 10^{22} \text{ kg}\right)\left(3.00 \times 10^4 \text{ kg}\right)}{\left(1.92 \times 10^8 \text{ m}\right)^2} = 4.00 \text{ N}
\end{aligned}
$$

The resultant force is $(325 \text{ N} - 4.00 \text{ N}) = \boxed{321 \text{ N directed toward Earth}}$.

7.30   The Sun-Earth distance is $r_1 = 1.496 \times 10^{11}$ m , the Earth-Moon distance is $r_2 = 3.84 \times 10^8$ m , and the distance from the Sun to the Moon during a solar eclipse is $r_1 - r_2$.

(a) The force exerted on the Moon by the Sun is

$$
\begin{aligned}
F_{MS} &= \frac{Gm_M m_S}{\left(r_1 - r_2\right)^2} \\
&= \frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(7.36 \times 10^{22} \text{ kg}\right)\left(1.991 \times 10^{30} \text{ kg}\right)}{\left(1.496 \times 10^{11} \text{ m} - 3.84 \times 10^8 \text{ m}\right)^2}
\end{aligned}
$$

or $F_{MS} = \boxed{4.39 \times 10^{20} \text{ N toward the Sun}}$

(b) The force exerted on the Moon by the Earth is

$$F_{ME} = \frac{Gm_M m_E}{r_2^2} = \frac{\left(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(7.36 \times 10^{22}\ \text{kg}\right)\left(5.98 \times 10^{24}\ \text{kg}\right)}{\left(3.84 \times 10^8\ \text{m}\right)^2}$$

$$= \boxed{1.99 \times 10^{20}\ \text{N toward the Earth}}$$

(c) The force exerted on the Earth by the Sun is

$$F_{ES} = \frac{Gm_E m_S}{r_1^2} = \frac{\left(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24}\ \text{kg}\right)\left(1.991 \times 10^{30}\ \text{kg}\right)}{\left(1.496 \times 10^{11}\ \text{m}\right)^2}$$
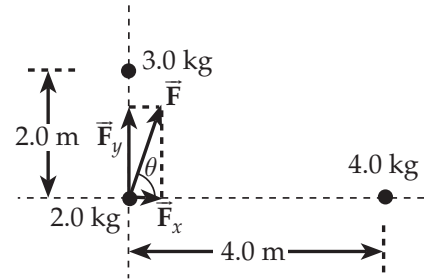
$$= \boxed{3.55 \times 10^{22}\ \text{N toward the Sun}}$$

7.31 The forces exerted on the 2.0-kg by the other bodies are F_x and F_y as shown in the diagram at the right. The magnitudes of these forces are

$$F_x = \frac{\left(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(2.0\ \text{kg}\right)\left(4.0\ \text{kg}\right)}{\left(4.0\ \text{m}\right)^2}$$

$$= 3.3 \times 10^{-11}\ \text{N}$$

and $$F_y = \frac{\left(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left(2.0\ \text{kg}\right)\left(3.0\ \text{kg}\right)}{\left(2.0\ \text{m}\right)^2} = 1.0 \times 10^{-10}\ \text{N}$$

The resultant force exerted on the 2.0-kg is $F = \sqrt{F_x^2 + F_y^2} = \boxed{1.1 \times 10^{-10}\ \text{N}}$

directed at $\theta = \tan^{-1}\left(\dfrac{F_y}{F_x}\right) = \tan^{-1}\left(3.0\right) = \boxed{72° \text{ above the } + x - \text{axis}}$

7.32 The equilibrium position lies between the Earth and the Sun on the line connecting their centers. At this point, the gravitational forces exerted on the object by the Earth and Sun have equal magnitudes and opposite directions. Let this point be located distance r from the center of the Earth. Then, its distance from the Sun is $\left(1.496 \times 10^{11} \text{ m} - r\right)$, and we

may determine the value of r by requiring that $\dfrac{G m_E m}{r^2} = \dfrac{G m_S m}{\left(1.496 \times 10^{11} \text{ m} - r\right)^2}$, where

m_E and m_S are the masses of the Earth and Sun respectively. This reduces to

$\dfrac{\left(1.496 \times 10^{11} \text{ m} - r\right)}{r} = \sqrt{\dfrac{m_S}{m_E}} = 577$, or $1.496 \times 10^{11} \text{ m} = 578 r$, which yields

$r = \boxed{2.59 \times 10^8 \text{ m from center of the Earth}}$

7.33 (a) At the midpoint between the two masses, the forces exerted by the 200-kg and 500-kg masses are oppositely directed, and from $F = \dfrac{G m_1 m_2}{r^2}$ we have

$$\Sigma F = \frac{G\left(50.0 \text{ kg}\right)\left(500 \text{ kg} - 200 \text{ kg}\right)}{\left(0.200 \text{ m}\right)^2} = \boxed{2.50 \times 10^{-5} \text{ N}} \text{ toward the 500-kg}$$

(b) At a point between the two masses and distance d from the 500-kg mass, the net force will be zero when

$$\frac{G\left(50.0 \text{ kg}\right)\left(200 \text{ kg}\right)}{\left(0.400 \text{ m} - d\right)^2} = \frac{G\left(50.0 \text{ kg}\right)\left(500 \text{ kg}\right)}{d^2} \text{ or } d = \boxed{0.245 \text{ m}}$$

Note that the above equation yields a second solution $d = 1.09 \text{ m}$. At that point, the two gravitational forces do have equal magnitudes, but are in the same direction and cannot add to zero.

7.34 We know that $m_1 + m_2 = 5.00$ kg , or $m_2 = 5.00$ kg $- m_1$

$$F = \frac{Gm_1 m_2}{r^2} \Rightarrow 1.00 \times 10^{-8} \text{ N} = \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \frac{m_1 (5.00 \text{ kg} - m_1)}{(0.200 \text{ m})^2}$$

$$(5.00 \text{ kg}) m_1 - m_1^2 = \frac{(1.00 \times 10^{-8} \text{ N})(0.200 \text{ m})^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} = 6.00 \text{ kg}^2$$

Thus, $m_1^2 - (5.00 \text{ kg}) m_1 + 6.00 \text{ kg}^2 = 0$

or $\quad (m_1 - 3.00 \text{ kg})(m_1 - 2.00 \text{ kg}) = 0$

giving $\boxed{m_1 = 3.00 \text{ kg, so } m_2 = 2.00 \text{ kg}}$

The answer $m_1 = 2.00$ kg and $m_2 = 3.00$ kg is physically equivalent.

7.35 (a) The gravitational force must supply the required centripetal acceleration, so

$$\frac{Gm_E m}{r^2} = m\left(\frac{v_t^2}{r}\right). \text{ This reduces to } r = \frac{Gm_E}{v_t^2}, \text{ which gives}$$

$$r = \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \frac{(5.98 \times 10^{24} \text{ kg})}{(5\,000 \text{ m/s})^2} = 1.595 \times 10^7 \text{ m}$$

The altitude above the surface of the Earth is then

$$h = r - R_E = 1.595 \times 10^7 \text{ m} - 6.38 \times 10^6 \text{ m} = \boxed{9.57 \times 10^6 \text{ m}}$$

(b) The time required to complete one orbit is

$$T = \frac{\text{circumference of orbit}}{\text{orbital speed}} = \frac{2\pi (1.595 \times 10^7 \text{ m})}{5\,000 \text{ m/s}} = 2.00 \times 10^4 \text{ s} = \boxed{5.57 \text{ h}}$$

7.36 (a) The satellite moves in an orbit of radius $r = 2R_E$ and the gravitational force supplies the required centripetal acceleration. Hence, $m(v_t^2/2R_E) = Gm_E m/(2R_E)^2$, or

$$v_t = \sqrt{\frac{Gm_E}{2R_E}} = \sqrt{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \frac{(5.98 \times 10^{24} \text{ kg})}{2(6.38 \times 10^6 \text{ m})}} = \boxed{5.59 \times 10^3 \text{ m/s}}$$

(b) The period of the satellite's motion is

$$T = \frac{2\pi r}{v_t} = \frac{2\pi\left[2\left(6.38 \times 10^6 \text{ m}\right)\right]}{5.59 \times 10^3 \text{ m/s}} = 1.43 \times 10^4 \text{ s} = \boxed{3.98 \text{ h}}$$

(c) The gravitational force acting on the satellite is $F = G m_E m / r^2$, or

$$F = \left(6.67 \times 10^{-11} \; \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \frac{\left(5.98 \times 10^{24} \text{ kg}\right)\left(600 \text{ kg}\right)}{\left[2\left(6.38 \times 10^6 \text{ m}\right)\right]^2} = \boxed{1.47 \times 10^3 \text{ N}}$$

7.37 The gravitational force exerted on Io by Jupiter provides the centripetal acceleration, so

$$m\left(\frac{v_t^2}{r}\right) = \frac{GMm}{r^2}, \text{ or } M = \frac{r v_t^2}{G}$$

The orbital speed of Io is

$$v_t = \frac{2\pi r}{T} = \frac{2\pi\left(4.22 \times 10^8 \text{ m}\right)}{\left(1.77 \text{ days}\right)\left(86\,400 \text{ s/day}\right)} = 1.73 \times 10^4 \text{ m/s}$$

Thus, $\quad M = \dfrac{\left(4.22 \times 10^8 \text{ m}\right)\left(1.73 \times 10^4 \text{ m/s}\right)^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} = \boxed{1.90 \times 10^{27} \text{ kg}}$

7.38 The radius of the satellite's orbit is

$$r = R_E + h = 6.38 \times 10^6 \text{ m} + 2.00 \times 10^6 \text{ m} = 8.38 \times 10^6 \text{ m}$$

(a) $\quad PE_g = -\dfrac{GM_E m}{r}$

$$= -\left(6.67 \times 10^{-11} \; \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \frac{\left(5.98 \times 10^{24} \text{ kg}\right)\left(100 \text{ kg}\right)}{8.38 \times 10^6 \text{ m}} = \boxed{-4.76 \times 10^9 \text{ J}}$$

(b) $\quad F = \dfrac{GM_E m}{r^2} = \left(6.67 \times 10^{-11} \; \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) \frac{\left(5.98 \times 10^{24} \text{ kg}\right)\left(100 \text{ kg}\right)}{\left(8.38 \times 10^6 \text{ m}\right)^2} = \boxed{568 \text{ N}}$

7.39　The radius of the satellite's orbit is

$$r = R_E + h = 6.38 \times 10^6 \text{ m} + 200 \times 10^3 \text{ m} = 6.58 \times 10^6 \text{ m}$$

(a)　Since the gravitational force provides the centripetal acceleration,

$$m\left(\frac{v_t^2}{r}\right) = \frac{Gm_E m}{r^2}, \text{ or}$$

$$v_t = \sqrt{\frac{Gm_E}{r}} = \sqrt{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98 \times 10^{24} \text{ kg}\right)}{\left(6.58 \times 10^6 \text{ m}\right)}} = 7.79 \times 10^3 \text{ m/s}$$

Hence, the period of the orbital motion is

$$T = \frac{2\pi r}{v_t} = \frac{2\pi\left(6.58 \times 10^6 \text{ m}\right)}{7.79 \times 10^3 \text{ m/s}} = 5.31 \times 10^3 \text{ s} = \boxed{1.48 \text{ h}}$$

(b)　The orbital speed is $v_t = \boxed{7.79 \times 10^3 \text{ m/s}}$ as computed above.

(c)　Assuming the satellite is launched from a point on the equator of the Earth, its initial speed is the rotational speed of the launch point, or

$$v_i = \frac{2\pi R_E}{1 \text{ day}} = \frac{2\pi\left(6.38 \times 10^6 \text{ m}\right)}{86\,400 \text{ s}} = 464 \text{ m/s}$$

The work-kinetic energy theorem gives the energy input required to place the satellite in orbit as $W_{nc} = \left(KE + PE_g\right)_f - \left(KE + PE_g\right)_i$, or

$$W_{nc} = \left(\frac{1}{2}mv_t^2 - \frac{GM_E m}{r}\right) - \left(\frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E}\right) = m\left[\frac{v_t^2 - v_i^2}{2} + GM_E\left(\frac{1}{R_E} - \frac{1}{r}\right)\right]$$

Substitution of appropriate numeric values into this result gives the minimum energy input as $W_{nc} = \boxed{6.43 \times 10^9 \text{ J}}$

7.40   The gravitational force on a small parcel of material at the star's equator supplies the centripetal acceleration, or $\dfrac{GM_s\,m}{R_s^2} = m\left(\dfrac{v_t^2}{R_s}\right) = m\left(R_s\,\omega^2\right)$

Hence, $\omega = \sqrt{GM_s/R_s^3}$

$$= \sqrt{\dfrac{\left(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2\right)\left[2\left(1.99 \times 10^{30}\ \text{kg}\right)\right]}{\left(10.0 \times 10^3\ \text{m}\right)^3}} = \boxed{1.63 \times 10^4\ \text{rad/s}}$$

7.41   (a) $\omega = \dfrac{v_t}{r} = \dfrac{\left(98.0\ \text{mi/h}\right)\left(\dfrac{0.447\ \text{m/s}}{1\ \text{mi/h}}\right)}{0.742\ \text{m}} = 59.0\ \dfrac{\text{rad}}{\text{s}}\left(\dfrac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = \boxed{9.40\ \text{rev/s}}$

(b) $\alpha = \dfrac{\omega^2 - \omega_i^2}{2\,\Delta\theta} = \dfrac{\left(9.40\ \text{rev/s}\right)^2 - 0}{2\left(1\ \text{rev}\right)} = \boxed{44.1\ \text{rev/s}^2}$

$a_c = \dfrac{v_t^2}{r} = \dfrac{\left[\left(98.0\ \text{mi/h}\right)\left(\dfrac{0.447\ \text{m/s}}{1\ \text{mi/h}}\right)\right]^2}{0.742\ \text{m}} = \boxed{2.59 \times 10^3\ \text{m/s}^2}$

$a_t = r\alpha = \left(0.742\ \text{m}\right)\left[44.1\ \dfrac{\text{rev}}{\text{s}^2}\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\right] = \boxed{206\ \text{m/s}^2}$

(c)   In the radial direction at the release point, the hand supports the weight of the ball and also supplies the centripetal acceleration. Thus, $F_r = mg + ma_r = m\left(g + a_r\right)$ or

$$F_r = \left(0.198\ \text{kg}\right)\left(9.80\ \text{m/s}^2 + 2.59 \times 10^3\ \text{m/s}^2\right) = \boxed{514\ \text{N}}$$

In the tangential direction, the hand supplies only the tangential acceleration, so

$$F_t = ma_t = \left(0.198\ \text{kg}\right)\left(206\ \text{m/s}^2\right) = \boxed{40.7\ \text{N}}$$

7.42 (a) The gravitational force exerted on an object of mass m near the surface (i.e., at distance R from the center) of a planet of mass M is

$$F_g = \frac{GMm}{R^2} . \text{ Equating this to } F_g = mg \text{ gives } mg = \frac{GMm}{R^2} , \text{ or}$$

$$g = \frac{GM}{R^2} \text{ as the acceleration of gravity at the planet's surface.}$$

For Mars,

$$g = \left(6.67 \times 10^{-11} \ \frac{N \cdot m^2}{kg^2} \right) \frac{\left[0.107\ 4\left(5.98 \times 10^{24} \ kg\right) \right]}{\left[0.528\ 2\left(6.38 \times 10^6 \ m\right) \right]^2} = \boxed{3.77 \ m/s^2}$$

(b) From $\Delta y = v_i t + \dfrac{1}{2} a_y t^2$ with $v_i = 0$ and $a_y = -g$, we find

$$t = \sqrt{\frac{2(\Delta y)}{-g}} = \sqrt{\frac{2(-20.0 \ m)}{-3.77 \ m/s^2}} = \boxed{3.26 \ s}$$

7.43 The angular velocity of the ball is $\omega = 0.500 \ \text{rev/s} = \pi \ \text{rad/s}$

(a) $v_t = r\omega = (0.800 \ m)(\pi \ \text{rad/s}) = \boxed{2.51 \ m/s}$

(b) $a_c = \dfrac{v_t^2}{r} = r\omega^2 = (0.800 \ m)(\pi \ \text{rad/s})^2 = \boxed{7.90 \ m/s^2}$

(c) We imagine that the weight of the ball is supported by a frictionless platform. Then, the rope tension need only produce the centripetal acceleration. The force required to produce the needed centripetal acceleration is $F = m\left(v_t^2/r\right)$. Thus, if the maximum force the rope can exert is 100 N, the maximum tangential speed of the ball is

$$\left(v_t\right)_{max} = \sqrt{\frac{r \ F_{max}}{m}} = \sqrt{\frac{(0.800 \ m)(100 \ N)}{5.00 \ kg}} = \boxed{4.00 \ m/s}$$

7.44 (a) $\omega_i = \dfrac{v_t}{r_i} = \dfrac{1.30 \ m/s}{2.30 \times 10^{-2} \ m} = \boxed{56.5 \ \text{rad/s}}$

(b) $\omega_f = \dfrac{v_t}{r_f} = \dfrac{1.30 \ m/s}{5.80 \times 10^{-2} \ m} = \boxed{22.4 \ \text{rad/s}}$

(c) The duration of the recording is

$$\Delta t = (74 \text{ min})(60 \text{ s/min}) + 33 \text{ s} = 4\,473 \text{ s}$$

Thus, $\alpha_{av} = \dfrac{\omega_f - \omega_i}{\Delta t} = \dfrac{(22.4 - 56.8) \text{ rad/s}}{4\,473 \text{ s}} = \boxed{-7.63 \times 10^{-3} \text{ rad/s}^2}$

(d) $\Delta\theta = \dfrac{\omega_f^2 - \omega_i^2}{2\alpha} = \dfrac{(22.4 \text{ rad/s})^2 - (56.5 \text{ rad/s})^2}{2(-7.63 \times 10^{-3} \text{ rad/s}^2)} = \boxed{1.77 \times 10^5 \text{ rad}}$

(e) The track moves past the lens at a constant speed of $v_t = 1.30 \text{ m/s}$ for 4 473 seconds. Therefore, the length of the spiral track is

$$\Delta s = v_t (\Delta t) = (1.30 \text{ m/s})(4\,473 \text{ s}) = 5.81 \times 10^3 \text{ m} = \boxed{5.81 \text{ km}}$$

7.45   The radius of the satellite's orbit is

$$r = R_E + h = 6.38 \times 10^6 \text{ m} + (1.50 \times 10^2 \text{ mi})(1\,609 \text{ m/1 mi}) = 6.62 \times 10^6 \text{ m}$$

(a) The required centripetal acceleration is produced by the gravitational force, so $m\left(\dfrac{v_t^2}{r}\right) = \dfrac{GM_E m}{r^2}$, which gives $v_t = \sqrt{\dfrac{GM_E}{r}}$

$$v_t = \sqrt{\left(6.67 \times 10^{-11} \dfrac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\dfrac{(5.98 \times 10^{24} \text{ kg})}{6.62 \times 10^6 \text{ m}}} = \boxed{7.76 \times 10^3 \text{ m/s}}$$

(b) The time for one complete revolution is

$$T = \dfrac{2\pi r}{v_t} = \dfrac{2\pi(6.62 \times 10^6 \text{ m})}{7.76 \times 10^3 \text{ m/s}} = 5.36 \times 10^3 \text{ s} = \boxed{89.3 \text{ min}}$$

7.46   (a) When the car is about to slip down the incline, the friction force, $\vec{f}$, is directed up the incline as shown and has the magnitude $f = \mu n$. Thus,

$$\Sigma F_y = n\cos\theta + \mu n\sin\theta - mg = 0$$

or $n = \dfrac{mg}{\cos\theta + \mu\sin\theta}$ (1)

Also, $\Sigma F_x = n\sin\theta - \mu n\cos\theta = m\left(\dfrac{v_{min}^2}{R}\right)$
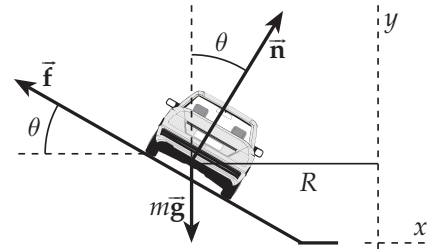
or $v_{min} = \sqrt{\dfrac{nR}{m}(\sin\theta - \mu\cos\theta)}$ (2)

Substituting equation (1) into (2) gives

$$v_{min} = \sqrt{Rg\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)} = \boxed{\sqrt{Rg\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}}$$

If the car is about to slip up the incline, $f = \mu n$ is directed down the slope (opposite to what is shown in the sketch). Then,

$$\Sigma F_y = n\cos\theta - \mu n\sin\theta - mg = 0 \text{ , or } n = \frac{mg}{\cos\theta - \mu\sin\theta}$$ (3)

Also, $\Sigma F_x = n\sin\theta + \mu n\cos\theta = m\left(\dfrac{v_{max}^2}{R}\right)$, or

$$v_{max} = \sqrt{\frac{nR}{m}(\sin\theta + \mu\cos\theta)}$$ (4)

Combining equations (3) and (4) gives

$$v_{max} = \sqrt{Rg\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)} = \boxed{\sqrt{Rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}}$$

(b) If $R = 100$ m, $\theta = 10°$, and $\mu = 0.10$, the lower and upper limits of safe speeds are:

$$v_{min} = \sqrt{(100 \ m)(9.8 \ m/s^2)\left(\frac{\tan 10° - 0.10}{1 + 0.10\tan 10°}\right)} = \boxed{8.6 \ m/s}$$

and $\qquad v_{max} = \sqrt{(100 \ m)(9.8 \ m/s^2)\left(\dfrac{\tan 10° + 0.10}{1 - 0.10\tan 10°}\right)} = \boxed{17 \ m/s}$

7.47   (a)   When the car is at the top of the arc, the normal force is upward and the weight downward. The net force directed downward, toward the center of the circular path and hence supplying the centripetal acceleration, is $\Sigma F_{down} = mg - n = m\left(v_t^2/r\right)$.

Thus, the normal force is $\boxed{n = m\left(g - v_t^2/r\right)}$

(b)   If $r = 30.0$ m and $n \to 0$, then $g - \dfrac{v_t^2}{r} \to 0$ or the speed of the car must be

$$v_t = \sqrt{rg} = \sqrt{(30.0 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{17.1 \text{ m/s}}$$

7.48   (a)   At the lowest point on the path, the net upward force (i.e., the force directed toward the center of the path and supplying the centripetal acceleration) is

$$\Sigma F_{up} = T - mg = m\left(\frac{v_t^2}{r}\right), \text{ so the tension in the cable is}$$

$$T = m\left(g + \frac{v_t^2}{r}\right) = (0.400 \text{ kg})\left(9.80 \text{ m/s}^2 + \frac{(3.00 \text{ m/s})^2}{0.800 \text{ m}}\right) = \boxed{8.42 \text{ N}}$$

(b)   Using conservation of mechanical energy, $\left(KE + PE_g\right)_f = \left(KE + PE_g\right)_i$, as the bob goes from the lowest to the highest point on the path gives

$$0 + mg\left[L(1 - \cos\theta_{max})\right] = \frac{1}{2}mv_i^2 + 0 \text{ , or } \cos\theta_{max} = 1 - \frac{v_i^2}{2gL}$$

$$\theta_{max} = \cos^{-1}\left(1 - \frac{v_i^2}{2gL}\right) = \cos^{-1}\left(1 - \frac{(3.00 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(0.800 \text{ m})}\right) = \boxed{64.8°}$$

(c)   At the highest point on the path, the bob is at rest and the net radial force is

$$\Sigma F_r = T - mg\cos\theta_{max} = m\left(\frac{v_t^2}{r}\right) = 0$$

Therefore,

$$T = mg\cos\theta_{max} = (0.400 \text{ kg})(9.80 \text{ m/s}^2)\cos(64.8°) = \boxed{1.67 \text{ N}}$$

7.49 The speed the person has due to the rotation of the Earth is $v_t = r\omega$ where r is the distance from the rotation axis and ω is the angular velocity of rotation.

The person's apparent weight, $\left(F_g\right)_{apparent}$, equals the magnitude of the upward normal force exerted on him by the scales. The true weight, $\left(F_g\right)_{true} = mg$, is directed downward. The net downward force produces the needed centripetal acceleration, or

$$\Sigma F_{down} = -n + \left(F_g\right)_{true} = -\left(F_g\right)_{apparent} + \left(F_g\right)_{true} = m\left(\frac{v_t^2}{r}\right) = mr\omega^2$$

(a) At the equator, $r = R_E$, so $\boxed{\left(F_g\right)_{true} = \left(F_g\right)_{apparent} + mR_E\,\omega^2} > \left(F_g\right)_{apparent}$

(b) At the equator, it is given that $r\omega^2 = 0.0340$ m/s^2 , so the apparent weight is

$$\left(F_g\right)_{apparent} = \left(F_g\right)_{true} - mr\omega^2 = (75.0 \text{ kg})\left[(9.80 - 0.0340) \text{ m/s}^2\right] = \boxed{732 \text{ N}}$$

At either pole, $r = 0$ (the person is on the rotation axis) and

$$\left(F_g\right)_{apparent} = \left(F_g\right)_{true} = mg = (75.0 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{735 \text{ N}}$$

7.50 When the rope makes angle θ with the vertical, the net force directed toward the center of the circular path is $\Sigma F_r = T - mg\cos\theta$ as shown in the sketch. This force supplies the needed centripetal acceleration, so

$$T - mg\cos\theta = m\left(\frac{v_t^2}{r}\right), \text{ or } T = m\left(g\cos\theta + \frac{v_t^2}{r}\right)$$
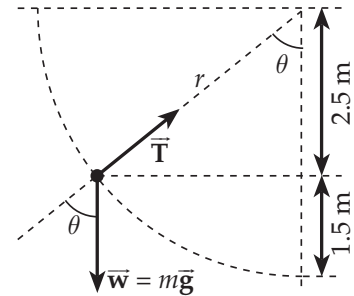
Using conservation of mechanical energy, with $KE = 0$ at $\theta = 90°$ and $PE_g = 0$ at the bottom of the arc, the speed when the rope is at angle θ from the vertical is given by $\frac{1}{2}mv_t^2 + mg(r - r\cos\theta) = 0 + mgr$, or $v_t^2 = 2gr\cos\theta$. The expression for the tension in the rope at angle θ then reduces to $T = 3mg\cos\theta$.

(a) At the beginning of the motion, $\theta = 90°$ and $T = \boxed{0}$

(b)   At 1.5 m from the bottom of the arc, $\cos\theta = \dfrac{2.5\ \text{m}}{r} = \dfrac{2.5\ \text{m}}{4.0\ \text{m}} = 0.63$ and the tension is

$$T = 3(70\ \text{kg})(9.8\ \text{m/s}^2)(0.63) = 1.3 \times 10^3\ \text{N} = \boxed{1.3\ \text{kN}}$$

(c)   At the bottom of the arc, $\theta = 0°$ and $\cos\theta = 1.0$, so the tension is

$$T = 3(70\ \text{kg})(9.8\ \text{m/s}^2)(1.0) = 2.1 \times 10^3\ \text{N} = \boxed{2.1\ \text{kN}}$$

7.51   The normal force exerted on the person by the cylindrical wall must provide the centripetal acceleration, so $n = m(r\omega^2)$.

If the minimum acceptable coefficient of friction is present, the person is on the verge of slipping and the maximum static friction force equals the person's weight, or $(f_s)_{max} = (\mu_s)_{min}\, n = mg$

Thus, $(\mu_s)_{min} = \dfrac{mg}{n} = \dfrac{g}{r\omega^2} = \dfrac{9.80\ \text{m/s}^2}{(3.00\ \text{m})(5.00\ \text{rad/s})^2} = \boxed{0.131}$

7.52   The horizontal component of the tension in the cord is the only force directed toward the center of the circular path, so it must supply the centripetal acceleration. Thus,

$$T\sin\theta = m\left(\frac{v_t^2}{r}\right) = m\left(\frac{v_t^2}{L\sin\theta}\right), \text{ or } T\sin^2\theta = \frac{mv_t^2}{L} \quad (1)$$
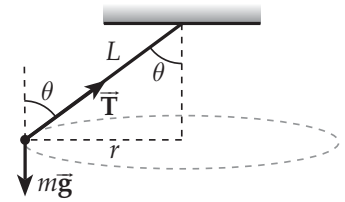
Also, the vertical component of the tension must support the weight of the ball, or

$$T\cos\theta = mg \tag{2}$$

(a)   Dividing equation (1) by (2) gives

$$\frac{\sin^2\theta}{\cos\theta} = \frac{v_t^2}{Lg}, \text{ or } v_t = \sin\theta\sqrt{\frac{Lg}{\cos\theta}} \tag{3}$$

With $L = 1.5$ m/s and $\theta = 30°$,

$$v_t = \sin 30°\sqrt{\frac{(1.5\ \text{m})(9.8\ \text{m/s}^2)}{\cos 30°}} = \boxed{2.1\ \text{m/s}}$$

(b) From equation (3), with $\sin^2\theta = 1 - \cos^2\theta$, we find

$$\frac{1-\cos^2\theta}{\cos\theta} = \frac{v_t^2}{Lg} \quad \text{or} \quad \cos^2\theta + \left(\frac{v_t^2}{Lg}\right)\cos\theta - 1 = 0$$

Solving this quadratic equation for $\cos\theta$ gives

$$\cos\theta = -\left(\frac{v_t^2}{2Lg}\right) \pm \sqrt{\left(\frac{v_t^2}{2Lg}\right)^2 + 1}$$

If $L = 1.5$ m and $v_t = 4.0$ m/s, this yields solutions: $\cos\theta = -1.7$ (which is impossible), and $\cos\theta = +0.59$ (which is possible).

Thus, $\theta = \cos^{-1}(0.59) = \boxed{54°}$

(c) From equation (2), when $T = 9.8$ N and the cord is about to break, the angle is

$$\theta = \cos^{-1}\left(\frac{mg}{T}\right) = \cos^{-1}\left(\frac{(0.50\ \text{kg})(9.8\ \text{m/s}^2)}{9.8\ \text{N}}\right) = 60°$$

Then equation (3) gives

$$v_t = \sin\theta\sqrt{\frac{Lg}{\cos\theta}} = \sin 60°\sqrt{\frac{(1.5\ \text{m})(9.8\ \text{m/s}^2)}{\cos 60°}} = \boxed{4.7\ \text{m/s}}$$

7.53   Choosing $PE_g = 0$ at the top of the hill, the speed of the skier after dropping distance h is found using conservation of mechanical energy as

$$\frac{1}{2}mv_t^2 - mgh = 0 + 0, \quad \text{or} \quad v_t^2 = 2gh$$

The net force directed toward the center of the circular path, and providing the centripetal acceleration, is

$$\Sigma F_r = mg\cos\theta - n = m\left(\frac{v_t^2}{R}\right)$$
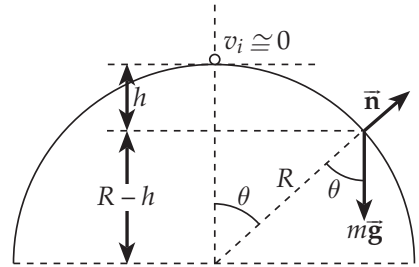
Solving for the normal force, after making the substitutions $v_t^2 = 2gh$ and $\cos\theta = \dfrac{R-h}{R} = 1 - \dfrac{h}{R}$,

gives $n = mg\left(1 - \dfrac{h}{R}\right) - m\left(\dfrac{2gh}{R}\right) = mg\left(1 - \dfrac{3h}{R}\right)$

The skier leaves the hill when $n \to 0$. This occurs when

$$1 - \dfrac{3h}{R} = 0 \text{ or } \boxed{h = \dfrac{R}{3}}$$

7.54   (a) $F = \dfrac{GMm}{r^2}$

$$= \left(6.67 \times 10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\dfrac{\left[100\left(1.99 \times 10^{30}\ \text{kg}\right)\right]\left(1000\ \text{kg}\right)}{\left[10 \times 10^3\ \text{m} + 50\ \text{m}\right]^2} = \boxed{1.3 \times 10^{17}\ \text{N}}$$

(b) $\Delta F = \dfrac{GM(1.0\ \text{kg})}{r_{front}^2} - \dfrac{GM(1.0\ \text{kg})}{r_{back}^2}$, or $\dfrac{\Delta F}{1.0\ \text{kg}} = GM\left[\dfrac{1}{r_{front}^2} - \dfrac{1}{r_{back}^2}\right]$

$$\dfrac{\Delta F}{1.0\ \text{kg}} = \left(6.67 \times 10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(1.99 \times 10^{32}\ \text{kg}\right)\left[\dfrac{1}{\left(10^4\ \text{m}\right)^2} - \dfrac{1}{\left(10^4\ \text{m}+100\ \text{m}\right)^2}\right]$$

$$= \boxed{2.6 \times 10^{12}\ \text{N/kg}}$$

7.55   Define the following symbols: M_m = mass of moon, M_e = mass of the Earth, R_m = radius of moon, R_e = radius of the Earth, and r = radius of the Moon's orbit around the Earth.

We interpret "lunar escape speed" to be the escape speed from the surface of a stationary moon alone in the universe. Then,

$$v_{launch} = 2v_{escape} = 2\sqrt{\dfrac{2GM_m}{R_m}}\ , \text{ or } v_{launch}^2 = \dfrac{8GM_m}{R_m}$$

Applying conservation of mechanical energy from launch to impact gives

$$\frac{1}{2}mv_{impact}^2 + \left(PE_g\right)_f = \frac{1}{2}mv_{launch}^2 + \left(PE_g\right)_i, \text{ or}$$

$$v_{impact} = \sqrt{v_{launch}^2 + \frac{2}{m}\left[\left(PE_g\right)_i - \left(PE_g\right)_f\right]}$$

The needed potential energies are

$$\left(PE_g\right)_i = -\frac{G\,M_m\,m}{R_m} - \frac{G\,M_e\,m}{r} \text{ and } \left(PE_g\right)_f = -\frac{G\,M_e\,m}{R_e} - \frac{G\,M_m\,m}{r}$$

Using these potential energies and the expression for v_{launch}^2 from above, the equation for the impact speed reduces to

$$v_{impact} = \sqrt{2G\left(\frac{3M_m}{R_m} + \frac{M_e}{R_e} - \frac{\left(M_e - M_m\right)}{r}\right)}$$

With numeric values of $G = 6.67 \times 10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$, $M_m = 7.36 \times 10^{22}\ \text{kg}$, $R_m = 1.74 \times 10^6\ \text{m}$, $R_e = 6.38 \times 10^6\ \text{m}$, and $r = 3.84 \times 10^8\ \text{m}$, we find

$$v_{impact} = 1.18 \times 10^4\ \text{m/s} = \boxed{11.8\ \text{km/s}}$$

7.56 The escape speed from the surface of a planet of radius R and mass M is given by

$$v_e = \sqrt{\frac{2GM}{R}}$$

If the planet has uniform density, ρ, the mass is given by

$$M = \rho(volume) = \rho\left(4\pi R^3/3\right) = 4\pi\rho R^3/3$$

The expression for the escape speed then becomes

$$v_e = \sqrt{\frac{2G}{R}\left(\frac{4\pi\rho R^3}{3}\right)} = \left(\sqrt{\frac{8\pi\rho G}{3}}\right)R = \boxed{(constant)R}$$

or the escape speed is directly proportional to the radius of the planet.

7.57 If the block will just make it through the top of the loop, the force required to produce the centripetal acceleration at point C must equal the block's weight, or $m\left(v_c^2/R\right)=mg$.

This gives $v_c = \sqrt{Rg}$ as the required speed of the block at point C.

We apply the work-energy theorem in the form

$$W_{nc} = \left(KE + PE_g + PE_s\right)_f - \left(KE + PE_g + PE_s\right)_i$$

from when the block is first released until it reaches point C to obtain

$$f_k\left(\overline{AB}\right)\cos 180° = \frac{1}{2}mv_c^2 + mg\left(2R\right)+0-0-0-\frac{1}{2}kd^2$$

The friction force is $f_k = u_k\left(mg\right)$, and for minimum initial compression of the spring, $v_c^2 = Rg$ as found above. Thus, the work-energy equation reduces to

$$d_{min} = \sqrt{\frac{2\mu_k mg\left(\overline{AB}\right)+mRg+2mg\left(2R\right)}{k}} = \sqrt{\frac{mg\left(2\mu_k\,\overline{AB}+5R\right)}{k}}$$

$$d_{min} = \sqrt{\frac{\left(0.50\text{ kg}\right)\left(9.8\text{ m/s}^2\right)\left[2\left(0.30\right)\left(2.5\text{ m}\right)+5\left(1.5\text{ m}\right)\right]}{78.4\text{ N/m}}} = \boxed{0.75\text{ m}}$$

7.58 Choosing $y = 0$ and $PE_g = 0$ at the level of point B, applying the work-energy theorem to the block's motion gives

$$W_{nc} = \frac{1}{2}mv^2 + mgy - \frac{1}{2}mv_0^2 - mg\left(2R\right), \text{ or } v^2 = v_0^2 + \frac{2W_{nc}}{m} + 2g\left(2R - y\right) \qquad (1)$$

(a) At point A, $y = R$ and $W_{nc} = 0$ (no non-conservative force has done work on the block yet). Thus, $v_A^2 = v_0^2 + 2gR$. The normal force exerted on the block by the track must supply the centripetal acceleration at point A, so

$$n_A = m\left(\frac{v_A^2}{R}\right) = m\left(\frac{v_0^2}{R} + 2g\right)$$

$$= \left(0.50\text{ kg}\right)\left(\frac{\left(4.0\text{ m/s}\right)^2}{1.5\text{ m}} + 2\left(9.8\text{ m/s}^2\right)\right) = \boxed{15\text{ N}}$$

At point B, $y = 0$ and W_{nc} is still zero. Thus, $v_B^2 = v_0^2 + 4gR$. Here, the normal force must supply the centripetal acceleration *and* support the weight of the block. Therefore,

$$n_B = m\left(\frac{v_B^2}{R}\right) + mg = m\left(\frac{v_0^2}{R} + 5g\right)$$

$$= (0.50 \text{ kg})\left[\frac{(4.0 \text{ m/s})^2}{1.5 \text{ m}} + 5(9.8 \text{ m/s}^2)\right] = \boxed{30 \text{ N}}$$

(b) When the block reaches point C, $y = 2R$ and $W_{nc} = -f_k L = -\mu_k (mg) L$. At this point, the normal force is to be zero, so the weight alone must supply the centripetal acceleration. Thus, $m(v_c^2 / R) = mg$, or the required speed at point C is $v_c^2 = Rg$. Substituting this into equation (1) yields $Rg = v_0^2 - 2\mu_k gL + 0$, or

$$\mu_k = \frac{v_0^2 - Rg}{2gL} = \frac{(4.0 \text{ m/s})^2 - (1.5 \text{ m})(9.8 \text{ m/s}^2)}{2(9.8 \text{ m/s}^2)(0.40 \text{ m})} = \boxed{0.17}$$

7.59　(a) At point A, the weight of the coaster must be just large enough to supply the centripetal acceleration. Thus, $m\left(\dfrac{v_A^2}{R}\right) = mg$, or $v_A^2 = Rg$

Applying conservation of mechanical energy from the start to point A,

$$\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv_A^2 + mg\left(\frac{2h}{3}\right), \text{ or } v_0^2 = v_A^2 - \frac{2gh}{3}$$

Using the value of $v_A^2 = Rg$, so the car barely stays on track at A, gives

$$v_0 = \sqrt{Rg - \frac{2gh}{3}} = \boxed{\sqrt{g\left(R - \frac{2h}{3}\right)}}$$

(b) If the speed of the coaster is to be zero at point B, conservation of mechanical energy from the start to point B gives

$$0 + mgh' = \frac{1}{2}mv_0^2 + mgh = \frac{1}{2}m\left[g\left(R - \frac{2h}{3}\right)\right] + mgh$$

or $h' = \boxed{\dfrac{R}{2} + \dfrac{2h}{3}}$

7.60   (a)   When the passenger is at the top, the radial forces producing the centripetal acceleration are the upward force of the seat and the downward force of gravity. The downward force must exceed the upward force to yield a net force toward the center of the circular path.
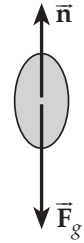
(b)   At the lowest point on the path, the radial forces contributing to the centripetal acceleration are again the upward force of the seat and the downward force of gravity. However, the upward force must now exceed the downward force to yield a net force directed toward the center of the circular path.
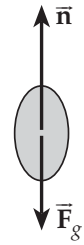
(c)   The seat must exert the greatest force on the passenger at the lowest point on the circular path.

(d)   At the top of the loop, $\Sigma F_r = m\dfrac{v^2}{r} = F_g - n$

or $n = F_g - m\dfrac{v^2}{r} = m\left(g - \dfrac{v^2}{r}\right) = (70.0 \text{ kg})\left(9.80 \text{ m/s}^2 - \dfrac{(4.00 \text{ m/s})^2}{8.00 \text{ m}}\right) = \boxed{546 \text{ N}}$

At the bottom of the loop, $\Sigma F_r = m\dfrac{v^2}{r} = n - F_g$

or $n = F_g + m\dfrac{v^2}{r} = m\left(g + \dfrac{v^2}{r}\right) = (70.0 \text{ kg})\left(9.80 \text{ m/s}^2 + \dfrac{(4.00 \text{ m/s})^2}{8.00 \text{ m}}\right) = \boxed{826 \text{ N}}$

7.61   (a)   In order to launch yourself into orbit by running, your running speed must be such that the gravitational force acting on you exactly equals the force needed to produce the centripetal acceleration. That is, $GMm/r^2 = mv_t^2/r$, where M is the mass of the asteroid and r is its radius. Since $M = density \times volume = \rho\left(\dfrac{4}{3}\pi r^3\right)$, this requirement

becomes $G\rho\left(\dfrac{4}{3}\pi r^3\right)\dfrac{m}{r^2} = \dfrac{m v_t^2}{r}$ or $r = \sqrt{\dfrac{3v_t^2}{4\pi G\rho}}$.

The radius of the asteroid would then be

$$r = \sqrt{\frac{3(8.50 \ \text{m/s})^2}{4\pi(6.673 \times 10^{-11} \ \text{N} \cdot \text{m}^2/\text{kg}^2)(1.10 \times 10^3 \ \text{kg/m}^3)}} = 1.53 \times 10^4 \ \text{m}$$
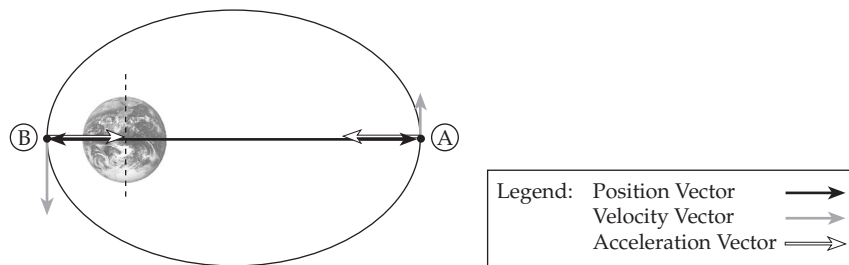
or $r = \boxed{15.3 \ \text{km}}$

(b) The mass of the asteroid is given by

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = (1.10 \times 10^3 \ \text{kg/m}^3)\frac{4}{3}\pi(1.53 \times 10^4 \ \text{m})^3 = \boxed{1.66 \times 10^{16} \ \text{kg}}$$

(c) Your period will be

$$T = \frac{2\pi}{\omega} = \frac{2\pi r}{v_t} = \frac{2\pi(1.53 \times 10^4 \ \text{m})}{8.50 \ \text{m/s}} = \boxed{1.13 \times 10^4 \ \text{s}}$$

7.62 (a)

Legend: Position Vector
          Velocity Vector
          Acceleration Vector

(b) The velocity vector at A is shorter than that at B. The gravitational force acting on the spacecraft is a conservative force, so the total mechanical energy of the craft is constant. The gravitational potential energy at A is larger than at B. Hence, the kinetic energy (and therefore the velocity) at A must be less than at B.

(c) The acceleration vector at A is shorter than that at B. From Newton's second law, the acceleration of the spacecraft is directly proportional to the force acting on it. Since the gravitational force at A is weaker than that at B, the acceleration at A must be less than the acceleration at B.

7.63 From the sketch at the right, observe that the radial component of the gravitational force acting on the piece of laundry has a magnitude of $mg \cos \phi = mg \cos(90.0° - \theta)$. The total radial force acting on the laundry produces the centripetal acceleration, so
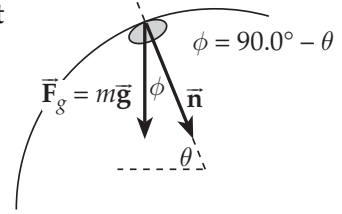
$$\Sigma F_r = n + mg \cos(90.0° - \theta) = m\frac{v_t^2}{r}$$

At $\theta = 68.0°$, the laundry loses contact with the tub and the normal force goes to zero. At this point, the tangential speed of the laundry is

$$v_t = \sqrt{rg \cos(90.0° - 68.0°)} = \sqrt{(0.330 \text{ m})(9.80 \text{ m/s}^2)\cos(22.0°)} = 1.73 \text{ m/s}$$

and the angular speed (rate of revolution) is given by

$$\omega = \frac{v_t}{r} = \frac{1.73 \text{ m/s}}{0.330 \text{ m}} = 5.25 \frac{\text{rad}}{\text{s}}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{0.835 \frac{\text{rev}}{\text{s}} = 50.1 \frac{\text{rev}}{\text{min}}}$$

7.64 The centripetal acceleration of a particle at distance r from the axis is $a_c = \frac{v_t^2}{r} = r\omega^2$. If we are to have $a_c = 100g$, then it is necessary that

$$r\omega^2 = 100g \quad \text{or} \quad \omega = \sqrt{\frac{100g}{r}}$$

The required rotation rate increases as r decreases. In order to maintain the required acceleration for all particles in the casting, we use the minimum value of r and find

$$\omega = \sqrt{\frac{100g}{r_{min}}} = \sqrt{\frac{100(9.80 \text{ m/s}^2)}{2.10 \times 10^{-2} \text{ m}}} = 216 \frac{\text{rad}}{\text{s}}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{60.0 \text{ s}}{1 \text{ min}}\right) = \boxed{2.06 \times 10^3 \frac{\text{rev}}{\text{min}}}$$

7.65 The sketch at the right shows the car as it passes the highest point on the bump. Taking upward as positive, we have

$$\Sigma F_y = ma_y \implies n - mg = m\left(-\frac{v^2}{r}\right)$$

or $\qquad n = m\left(g - \frac{v^2}{r}\right)$
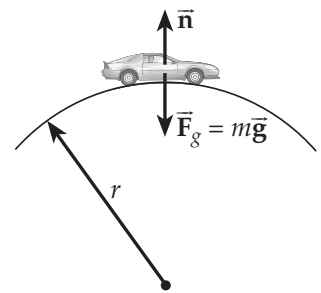
(a) If $v = 8.94$ m/s , the normal force exerted by the road is

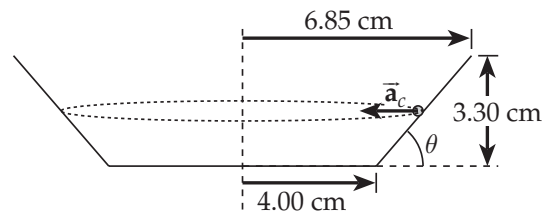$$n = \left(1\,800\ \text{kg}\right)\left[9.80\ \frac{\text{m}}{\text{s}^2} - \frac{\left(8.94\ \text{m/s}\right)^2}{20.4\ \text{m}}\right] = 1.06 \times 10^4\ \text{N} = \boxed{10.6\ \text{kN}}$$

(b) When the car is on the verge of losing contact with the road, $n = 0$. This gives $g = v^2/r$ and the speed must be

$$v = \sqrt{rg} = \sqrt{\left(20.4\ \text{m}\right)\left(9.80\ \text{m/s}^2\right)} = \boxed{14.1\ \text{m/s}}$$

7.66 (a) At a point midway between the top and bottom of the basket, the piece of fruit moves in a horizontal circle of radius

$$r = \frac{6.85\ \text{cm} + 4.00\ \text{cm}}{2} = 5.42\ \text{cm}$$

and its centripetal acceleration is

$$a_c = \frac{v_t^2}{r} = r\omega^2 = \left(5.42 \times 10^{-2}\ \text{m}\right)\left[20\,000\ \frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60.0\ \text{s}}\right)\right]^2$$

or $a_c = \boxed{2.38 \times 10^5\ \text{m/s}^2\ \text{horizontally inward}}$

$$a_c = 2.38 \times 10^5\ \text{m/s}^2\left(\frac{1\ g}{9.80\ \text{m/s}^2}\right) = \boxed{2.43 \times 10^4\ g}$$

(b) In a reference frame rotating with the basket, the piece of fruit behaves as if it had an horizontally outward force of magnitude ma_c acting on it. Other forces acting on the piece are a normal force perpendicular to the cone and a kinetic friction force parallel to the cone as shown at the right. In this reference frame, the piece accelerates upward along the cone at
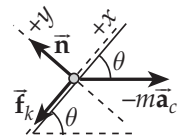
$$\theta = \tan^{-1}\left(\frac{3.30\ \text{cm}}{6.85\ \text{cm} - 4.00\ \text{cm}}\right) = 49.2°$$

$$\Sigma F_y = ma_y \Rightarrow n - ma_c \sin 49.2° = 0$$

or $n = \left(2.00 \times 10^{-3}\ \text{kg}\right)\left(2.38 \times 10^5\ \text{m/s}^2\right)\sin 49.2° = \boxed{360\text{N}}$

(c)   The kinetic friction force is $f_k = \mu_k n = (0.600)(360 \text{ N}) = 216 \text{ N}$. Then, referring to the sketch above, we find

$$\Sigma F_x = ma_x \Rightarrow ma_c \cos 49.2° - f_k = ma_x$$

or $a_x = a_c \cos 49.2° - \dfrac{f_k}{m} = (2.38 \times 10^5 \text{ m/s}^2)\cos 49.2° - \dfrac{216 \text{ N}}{2.00 \times 10^{-3} \text{ kg}}$

giving $a_x = \boxed{4.75 \times 10^4 \text{ m/s}^2 \text{ outward along the wall of the cone}}$

7.67   The angular speed of the luggage is $\omega = 2\pi/T$ where T is the time for one complete rotation of the carousel. The resultant force acting on the luggage must be directed toward the center of the horizontal circular path (that is, in the $+x$ direction). The magnitude of this resultant force must be

$$ma_c = m\left(\frac{v_t^2}{r}\right) = mr\omega^2$$

Thus, $\Sigma F_x = ma_x \Rightarrow f_s \cos\theta - n\sin\theta = ma_c$ (1)

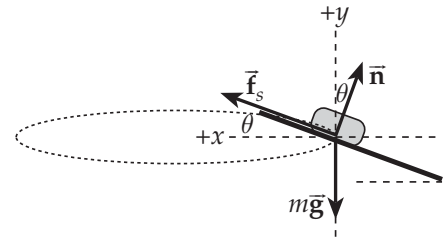and $\Sigma F_y = ma_y \Rightarrow f_s \sin\theta + n\cos\theta - mg = 0$

or $n = \dfrac{mg - f_s \sin\theta}{\cos\theta}$ (2)

Substituting Equation (2) into Equation (1) gives

$$f_s \cos\theta - mg\tan\theta + f_s\left(\frac{\sin^2\theta}{\cos\theta}\right) = ma_c$$

or $f_s = \dfrac{ma_c + mg\tan\theta}{\cos\theta + \sin^2\theta/\cos\theta}$ (3)

(a) With $T = 38.0$ s and $r = 7.46$ m , we find that

$\omega = 0.165$ rad/s and $ma_c = mr\omega^2 = (30.0 \text{ kg})(7.46 \text{ m})(0.165 \text{ rad/s})^2 = 6.12$ N

Equation (3) then gives the friction force as

$$f_s = \frac{6.12 \text{ N} + (30.0 \text{ kg})(9.80 \text{ m/s}^2)\tan 20.0°}{\cos 20.0° + \dfrac{\sin^2 20.0°}{\cos 20.0°}} = \frac{113 \text{ N}}{1.06} = \boxed{106 \text{ N}}$$

(b) If $T = 34.0$s and $r = 7.94$ m , then $\omega = 0.185$ rad/s and

$$ma_c = mr\omega^2 = (30.0 \text{ kg})(7.94 \text{ m/s}^2)(0.185 \text{ rad/s})^2 = 8.13 \text{ N}$$

From Equation (1),

$$f_s = \frac{8.13 \text{ N} + (30.0 \text{ kg})(9.80 \text{ m/s}^2)\tan 20.0°}{\cos 20.0° + \dfrac{\sin^2 20.0°}{\cos 20.0°}} = \frac{115 \text{ N}}{1.06} = 108 \text{ N}$$

while Equation (2) yields

$$n = \frac{(30.0 \text{ kg})(9.80 \text{ m/s}^2) - (108 \text{ N})\sin 20.0°}{\cos 20.0°} = 273 \text{ N}$$

Since the luggage is on the verge of slipping, $f_s = (f_s)_{max} = \mu_s n$ and the coefficient of static friction must be

$$\mu_s = \frac{f_s}{n} = \frac{108 \text{ N}}{273 \text{ N}} = \boxed{0.396}$$

7.68   Choose $t = 0$ when the dog first sees the bone and $\theta = 0$ at the angular position of the dog at this instant. The angular positions of the dog and the bone at any time $t > 0$ are given by $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$.

For the dog: $\theta_0 = 0$, $\omega_0 = 0.750$ rad/s , and $\alpha = 0$ so $\theta_{dog} = (0.750 \text{ rad/s})t$

For the bone: $\theta_0 = \dfrac{2\pi \text{ rad}}{3}$, $\omega_0 = 0$, and $\alpha = 0.0150$ rad/s^2 giving

$$\theta_{bone} = \frac{2\pi \text{ rad}}{3} + (7.50 \times 10^{-3} \text{ rad/s}^2)t^2$$

(a) The dog overtakes the bone when $\theta_{dog} = \theta_{bone}$. At this time,

$$(0.750 \ \text{rad/s})t = \frac{2\pi \ \text{rad}}{3} + (7.50 \times 10^{-3} \ \text{rad/s}^2)t^2$$

or $t^2 - (100 \ \text{s})t + 279 \ \text{s}^2 = 0$ with solutions of and $t = 2.88$ s and $t = 97.1$ s.

The dog first overtakes the bone at $t = \boxed{2.88 \ \text{s}}$. At $t = 97.1$ s , the angular positions are once again equal after the dog and bone have passed each several times.

(b) If the dog overruns the bone, it will next pass the bone when $\theta_{dog} = \theta_{bone} + 2\pi$ rad . At this time, we have

$$(0.750 \ \text{rad/s})t = \frac{2\pi \ \text{rad}}{3} + (7.50 \times 10^{-3} \ \text{rad/s}^2)t^2 + 2\pi \ \text{rad}$$

This reduces to $t^2 - (100 \ \text{s})t + 1.12 \times 10^3 \ \text{s}^2 = 0$ and has solutions $t = 12.8$ s and $t = 87.2$ s . Thus, the first occurrence of this scenario is at $t = \boxed{12.8 \ \text{s}}$

7.69 (a) The chain does not slip on the front sprocket wheel, so the linear speed of a link in the chain must equal the tangential speed of a tooth on this sprocket wheel. The angular speed of this wheel is the same as the rate the bicyclist is pedaling. Thus,

$$v_{link} = (v_t)_{\substack{front \\ sprocket}} = r\omega = \left(\frac{0.152 \ \text{m}}{2}\right)\left(76.0 \ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \ \text{rad}}{1 \ \text{rev}}\right)\left(\frac{1 \ \text{min}}{60.0 \ \text{s}}\right) = \boxed{0.605 \ \text{m/s}}$$

(b) Also, the chain does not slip on the rear sprocket wheel. Hence, the tangential speed of a tooth on this sprocket must equal the linear speed of a link in the chain. This gives

$$\omega_{\substack{rear \\ sprocket}} = \frac{(v_t)_{\substack{rear \\ sprocket}}}{r_{\substack{rear \\ sprocket}}} = \frac{v_{link}}{r_{\substack{rear \\ sprocket}}} = \frac{0.605 \ \text{m/s}}{\frac{1}{2}(7.00 \times 10^{-2} \ \text{m})} = 17.3 \ \text{rad/s}$$

Since the rear sprocket wheel is rigidly attached to the rear tire of the bicycle, the angular speed of the tires must be the same as that of this sprocket wheel. Thus,

$$\omega_{tire} = \boxed{17.3 \ \text{rad/s}}$$

(c) If the drive wheel of the bicycle does not slip on the roadway, the translational speed of the axle (and hence, that of the bicycle as a whole) is the same as the tangential speed of a point on the rim of the drive wheel. Therefore,

$$v_{bicycle} = \left(v_t\right)_{\substack{rear \\ tire}} = r_{\substack{rear \\ tire}}\omega_{\substack{rear \\ tire}} = \tfrac{1}{2}(0.673 \text{ m})(17.3 \text{ rad/s}) = \boxed{5.82 \text{ m/s}}$$

(d) The $\boxed{\text{length of the pedal cranks}}$ was not used in this solution.

7.70 The maximum lift force is $\left(F_L\right)_{max} = Cv^2$, where $C = 0.018 \text{ N}\cdot\text{s}^2/\text{m}^2$ and v is the flying speed. For the bat to stay aloft, the vertical component of the lift force must equal the weight, or $F_L\cos\theta = mg$ where θ is the banking angle. The horizontal component of this force supplies the centripetal acceleration needed to make a turn, or $F_L\sin\theta = m\dfrac{v^2}{r}$ where r is the radius of the turn.

(a) To stay aloft while flying at minimum speed, the bat must have $\theta = 0$ (to give $\cos\theta = (\cos\theta)_{max} = 1$) and also use the maximum lift force possible at that speed. That is, we need

$$\left(F_L\right)_{max}\left(\cos\theta\right)_{max} = mg \text{ ,or} \qquad Cv_{min}^2(1) = mg$$

Thus, we see that minimum flying speed is

$$v_{min} = \sqrt{\frac{mg}{C}} = \sqrt{\frac{(0.031 \text{ kg})(9.8 \text{ m/s}^2)}{0.018 \text{ N}\cdot\text{s}^2/\text{m}^2}} = \boxed{4.1 \text{ m/s}}$$

(b) To maintain horizontal flight while banking at the maximum possible angle, we must have $\left(F_L\right)_{max}\cos\theta_{max} = mg$, or $Cv^2\cos\theta_{max} = mg$. For $v = 10 \text{ m/s}$, this yields

$$\cos\theta_{max} = \frac{mg}{Cv^2} = \frac{(0.031 \text{ kg})(9.8 \text{ m/s}^2)}{(0.018 \text{ N}\cdot\text{s}^2/\text{m}^2)(10 \text{ m/s})^2} = 0.17 \quad \text{or} \quad \theta_{max} = \boxed{80°}$$

(c) The horizontal component of the lift force supplies the centripetal acceleration in a turn, $F_L \sin\theta = mv^2/r$. Thus, the minimum radius turn possible is given by

$$r_{min} = \frac{mv^2}{(F_L)_{max}(\sin\theta)_{max}} = \frac{m\cancel{v^2}}{C\cancel{v^2}\sin\theta_{max}} = \frac{m}{C\sin\theta_{max}}$$

where we have recognized that $\sin\theta$ has its maximum value at the largest allowable value of θ. For a flying speed of $v = 10$ m/s, the maximum allowable bank angle is $\theta_{max} = 80°$ as found in part (b). The minimum radius turn possible at this flying speed is then

$$r_{min} = \frac{0.031 \text{ kg}}{(0.018 \text{ N}\cdot\text{s}^2/\text{m}^2)\sin 80.0°} = \boxed{1.7 \text{ m}}$$

(d) $\boxed{\text{No}}$. Flying slower actually increases the minimum radius of the achievable turns. As found in part (c), $r_{min} = m/C\sin\theta_{max}$. To see how this depends on the flying speed, recall that the vertical component of the lift force must equal the weight or $F_L \cos\theta = mg$. At the maximum allowable bank angle, $\cos\theta$ will be a minimum. This occurs when $F_L = (F_L)_{max} = Cv^2$. Thus, $\cos\theta_{max} = mg/Cv^2$ and

$$\sin\theta_{max} = \sqrt{1 - \cos^2\theta_{max}} = \sqrt{1 - \left(\frac{mg}{Cv^2}\right)^2}$$

This gives the minimum radius turn possible at flying speed v as

$$r_{min} = \frac{m}{C\sqrt{1 - \left(\frac{mg}{Cv^2}\right)^2}}$$

Decreasing the flying speed v will decrease the denominator of this expression, yielding a larger value for the minimum radius of achievable turns.

7.71 (a) The gravitational force exerted on an object of mass m located on the surface of a spherical body of mass M and radius R may be written as

$$F_g = G\frac{Mm}{R^2} = mg$$

Thus, the acceleration gravity at the surface of this spherical body is

$$g = \frac{F_g}{m} = G\frac{M}{R^2}$$

If $M = 1.5M_{Sun} = 1.5\left(1.991 \times 10^{30}\ \text{kg}\right) = 3.0 \times 10^{30}\ \text{kg}$ and $R = 10.0\ \text{km} = 1.00 \times 10^4\ \text{m}$,

$$g = \left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(3.0 \times 10^{30}\ \text{kg}\right)}{\left(1.00 \times 10^4\ \text{m}\right)^2} = \boxed{2.0 \times 10^{12}\ \text{m/s}^2}$$

(b) $F_g = mg = \left(0.120\ \text{kg}\right)\left(2.0 \times 10^{12}\ \text{m/s}^2\right) = \boxed{2.4 \times 10^{11}\ \text{N}}$

(c) $PE_g = mgh = \left(70.0\ \text{kg}\right)\left(2.0 \times 10^{12}\ \text{m/s}^2\right)\left(1.00 \times 10^{-2}\ \text{m}\right) = \boxed{1.4 \times 10^{12}\ \text{J}}$