§6.1—Antiderivatives & Indefinite Integration

Suppose we have a function *F* whose derivative is given as $f(x) = 4x^3$. From your experience with finding derivatives, you might say that $F(x) = x^4$ since $\frac{d}{dx}\left[x^4\right] = 4x^3$.

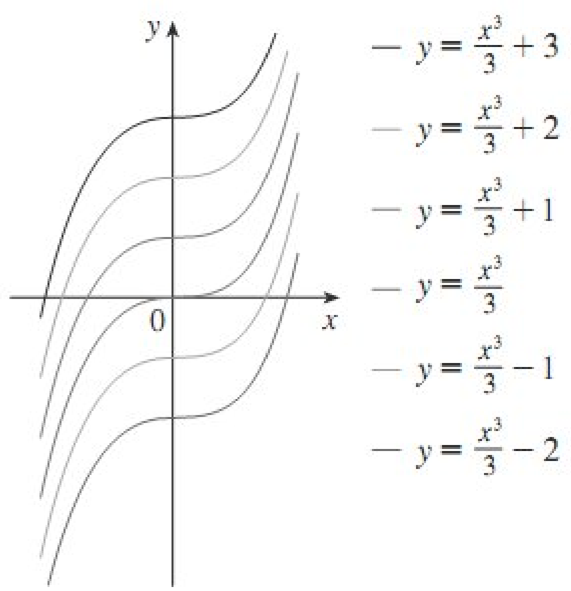
By using your intuition, you have just found *an* **antiderivative**, *F*, of *f*.

Definition

A function F is *an* **antiderivative** of *f* on an interval *I* if $F'(x) = f(x) \quad \forall x \in I$.

Notice that *F* is called AN antiderivative and not THE antiderivative. This is easily understood by looking at the example above.

Some antiderivatives of $f(x) = 4x^3$ are $F(x) = x^4$, $F(x) = x^4 + 2$, $F(x) = x^4 - 52$, and $F(x) = x^4 + \pi$ because in each case, $\frac{d}{dx}\left[F(x)\right] = 4x^3$.



Because of this we can say that the **general antiderivative** of a function $f(x)$ is

$$F(x) + C, \text{ where } C \text{ is an arbitrary constant.}$$

The graph at right show several members of the family of the antiderivatives of x^2.

Example 1:

Find the general antiderivatives of each of the following using you knowledge of how to find derivatives.

a) $f(x) = 2x$ b) $f'(x) = x$ c) $F'(x) = x^2$ d) $g'(x) = \frac{1}{x^2}$ e) $\frac{dy}{dx} = \cos x$

Knowing how to find a derivative of different types of functions will help you find antiderivatives.

Table of Antiderivative Formulas

Function	**Particular antiderivative**		**Function**	**Particular antiderivative**
$cf(x)$	$cF(x)$		$\sin x$	$-\cos x$
$f(x)+g(x)$	$F(x)+G(x)$		$\sec^2 x$	$\tan x$
$x^n,\ n \neq -1$	$\dfrac{x^{n+1}}{n+1}$		$\sec x \tan x$	$\sec x$
$\dfrac{1}{x}$	$\ln\lvert x\rvert$		$\dfrac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
e^x	e^x		$\dfrac{1}{1+x^2}$	$\tan^{-1} x$
$\cos x$	$\sin x$			

Example 2:

Find all functions g such that $g'(x) = 4\sin x + \dfrac{2x^4 - \sqrt{x} + x}{x}$.

Definition

A **differential equation** is an equation explicitly solved for a derivative of a particular equation. **Solving a differential equation** involves finding the original function from which the derivative came. The **general solution** involves $+C$. The **particular solution** uses an initial condition to find the specific value of C.

Example 3:

Solve the differential equation $f'(x) = 3x^2$ if $f(2) = -3$. Find both the general and particular solutions.

The differential equation in Example 3 is called a **separable differential equation** because it is possible to separate all the x and y variables. When given a separable differential equation in Leibniz form, it is **MANDATORY** to show the separation of variables by rewriting the function in **differentiable form**. If $\frac{dy}{dx} = f(x)$, then

$$dy = f(x)\,dx \text{ is the differential form.}$$

The process of finding the antiderivatives of each side of the above equation is called **indefinite integration**. We can denote this operation with an **integral symbol**, $\int$. Taking the integral of both sides of the differential form to find the general solution, we get

$$\int dy = \int f(x)dx$$
$$y = F(x) + C$$

Here's the anatomy of an indefinite integral:

Integral symbol $\longrightarrow \displaystyle\int f(x)dx \longleftarrow$ Variable of integration

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Example 4:

Find the particular solution to the following differential equation if $\frac{dy}{dx} = e^x + 20\left(1+x^2\right)^{-1}$ and $y(0) = -2$.

Example 5:

Find the particular solution to the following differential equation if $\frac{d^2y}{dx^2} = 12x^2 + 6x - 4$ and

a) $y'(1) = 3$ and $y(0) = -6$

b) $y(0) = 4$ and $y(1) = 1$.

Example 6:

a) Evaluate $\int \frac{\sin x}{\cos^2 x} dx$

b) Evaluate $\int \left(\tan^2 p + 4\right) dp$