Chapter 8
Rotational Equilibrium and Rotational Dynamics

Quick Quizzes

1. (d). A larger torque is needed to turn the screw. Increasing the radius of the screwdriver handle provides a greater lever arm and hence an increased torque.

2. (b). Since the object has a constant net torque acting on it, it will experience a constant angular acceleration. Thus, the angular velocity will change at a constant rate.

3. (b). The hollow cylinder has the larger moment of inertia, so it will be given the smaller angular acceleration and take longer to stop.

4. (a). The hollow sphere has the larger moment of inertia, so it will have the higher rotational kinetic energy.

5. (c). The box. All objects have the same potential energy associated with them before they are released. As the objects move down the inclines, this potential energy is transformed to kinetic energy. For the ball and cylinder, the transformation is into both rotational and translational kinetic energy. The box has only translational kinetic energy. Because the kinetic energies of the ball and cylinder are split into two types, their translational kinetic energy is necessarily less than that of the box. Consequently, their translational speeds are less than that of the box, so the ball and cylinder will lag behind.

6. (c). Apply conservation of angular momentum to the system (the two disks) before and after the second disk is added to get the result: $I_1\omega_1 = \left(I_1 + I_2\right)\omega$.

7. (a). Earth already bulges slightly at the Equator, and is slightly flat at the poles. If more mass moved towards the Equator, it would essentially move the mass to a greater distance from the axis of rotation, and increase the moment of inertia. Because conservation of angular momentum requires that $\omega_z I_z = \text{const}$, an increase in the moment of inertia would decrease the angular velocity, and slow down the spinning of Earth. Thus, the length of each day would increase.

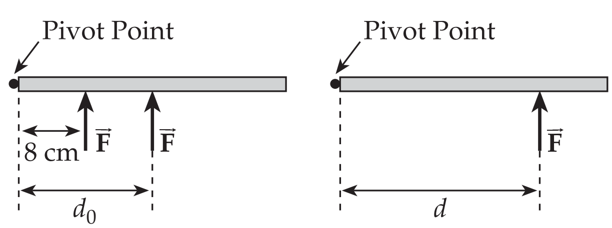
Answers to Even Numbered Conceptual Questions

2. If the bar is, say, seven feet above the ground, a high jumper has to lift his center of gravity approximately to a height of seven feet in order to clear the bar. A tall person already has his center of gravity higher than that of a short person. Thus, the taller athlete has to raise his center of gravity through a smaller distance.

4. The lever arm of a particular force is found with respect to some reference point. Thus, an origin for calculating torques must be specified. However, for an object in equilibrium, the calculation of the total torque is independent of the location of the origin.

6. We assume that the melt-water would form a thin shell of mass $m = 2.3 \times 10^{19}$ kg and radius $R_E = 6.38 \times 10^6$ m around the Earth. This shell would increase Earth's moment of inertia by an amount $\Delta I = \frac{2}{3} m R_E^2$. If we treat Earth as a uniform solid sphere, this would represent a fractional increase of $\frac{\Delta I}{I_0} = \frac{\frac{2}{3} m R_E^2}{\frac{2}{5} M_E R_E^2} = \frac{5}{3}\left(\frac{m}{M_E}\right)$. Thus, the fractional increase in the moment of inertial would be on the order of $\frac{10^{19}\ \text{kg}}{10^{24}\ \text{kg}} = 10^{-6}$. In this process, angular momentum would be conserved, so the quantity $I\omega = I\left(\frac{2\pi}{T}\right)$, where T is the rotation period or length of a day, must remain constant. Therefore, the length of the day must also experience a fractional increase on the order of 10^{-6}. This would give an increase in the length of the day of $\Delta T \sim 10^{-6} T_0 = 10^{-6}(86\,400\ \text{s}) = 0.0864\ \text{s}$, or $\Delta T \sim 10^{-1}$ s.

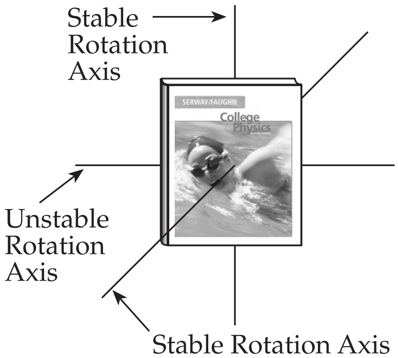
8. The critical factor is the total torque being exerted about the line of the hinges. For simplicity, we assume that the paleontologist and the botanist exert equal magnitude forces. The free body diagram of the original situation is shown on the left and that for the modified situation is shown on the right in the sketches below:



Original Situation Modified Situation

In order for the torque exerted on the door in the modified situation to equal that of the original situation, it is necessary that $Fd = Fd_0 + F(8\ \text{cm})$ or $d = d_0 + 8\ \text{cm}$. Thus, the paleontologist would need to relocate about 8 cm farther from the hinge.

10.



12. After the head crosses the bar, the jumper should arch his back so the head and legs are lower than the midsection of the body. In this position, the center of gravity may pass under the bar while the midsection of the body is still above the bar. As the feet approach the bar, the legs should be straightened to avoid hitting the bar.

14. **(a)** Consider two people, at the ends of a long table, pushing with equal magnitude forces directed in opposite directions perpendicular to the length of the table. The net force will be zero, yet the net torque is not zero.

(b) Consider a falling body. The net force acting on it is its weight, yet the net torque about the center of gravity is zero.

16. As the cat falls, angular momentum must be conserved. Thus, if the upper half of the body twists in one direction, something must get an equal angular momentum in the opposite direction. Rotating the lower half of the body in the opposite direction satisfies the law of conservation of angular momentum.

18. All solid spheres reach the bottom of the hill at the same time. The speed at the bottom does not depend on the sphere's mass or radius, but only on how its mass is distributed and the height of the hill.

Answers to Even Numbered Problems

2. 0.642 N·m counterclockwise

4. $F_y + R_y - F_g = 0,\ \ F_x - R_x = 0,\ \ F_y \ell \cos\theta - F_g\left(\frac{\ell}{2}\right)\cos\theta - F_x \ell \sin\theta = 0$

6. –168 N·m

8. $x_{cg} = 6.69 \times 10^{-3}$ nm, $y_{cg} = 0$

10. 139 grams

12. (–1.5 m, –1.5 m)

14. $x_{\text{cg}} = 0.459$ m, $y_{\text{cg}} = 0.103$ m

16. $T = 1.68 \times 10^3$ N, $R = 2.34 \times 10^3$ N, $\theta = 21.2°$

18. 567 N (left end), 333 N (right end)

20. **(b)** T = 343 N, H = 171 N, V = 683 N **(c)** 5.14 m

22. **(a)** 392 N **(b)** H = 339 N (to right), V = 0

24. **(a)** 267 N (to right), 1.30 kN (upward) **(b)** $\mu_s = 0.324$

26. T = 1.47 kN, H = 1.33 kN (to right), V = 2.58 kN (upward)

28. 2.8 m

30. 149 N·m, 66.0 N·m, 215 N·m

32. 0.30

34. **(a)** 872 N **(b)** 1.40 kN

36. **(a)** $5.35\ \text{m/s}^2$ downward **(b)** 42.8 m **(c)** $8.91\ \text{rad/s}^2$

38. $30.3\ \text{rev/s}$

40. $10.9\ \text{rad/s}$

42. **(a)** 1.37×10^8 J **(b)** 5.10 h

44. $36\ \text{rad/s}$

46. $0.91\ \text{km/s}$

48. $1.17\ \text{rad/s}$

50. **(a)** $1.9\ \text{rad/s}$ **(b)** $KE_i = 2.5\ \text{J}, KE_f = 6.4\ \text{J}$

52. **(a)** $3.6\ \text{rad/s}$ **(b)** $5.4 \times 10^2\ \text{J}$, work done by the man as he walks inward

54. $12.3\ \text{m/s}^2$

56. The weight must be 508 N or more. The person could be a child. We assume the stove is a uniform box with feet at its corners. We ignore the masses of the backsplash and the oven door. If the oven door is heavy, the minimum weight for the person would be somewhat less than 508 N.

58. **(a)** 1.04 kN **(b)** 973 N at 67.7° above the horizontal to the right

60. **(a)** 46.8 N **(b)** $0.234\ \text{kg} \cdot \text{m}^2$ **(c)** $40.0\ \text{rad/s}$

62. $T_1 = 11.2\ \text{N}, T_2 = 1.39\ \text{N}, F = 7.23\ \text{N}$

66. **(a)** 61.2 J **(b)** 50.8 J

68. **(a)** $3.75 \times 10^3\ \text{kg} \cdot \text{m}^2/\text{s}$ **(b)** 1.88 kJ **(c)** $3.75 \times 10^3\ \text{kg} \cdot \text{m}^2/\text{s}$
(d) $10.0\ \text{m/s}$ **(e)** 7.50 kJ **(f)** 5.62 kJ

70. $T_{right} = 1.59\ \text{kN}, T_{left} = 1.01\ \text{kN}$

72. 24 m

74. $\frac{3}{8}w$

76. 9.00 ft

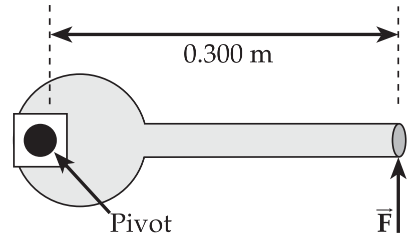
78. **(a)** A smooth (frictionless) surface cannot exert a force parallel to itself. Thus, a smooth vertical wall can exert only horizontal forces, normal to its surface.
(b) $L\sin\theta$ **(c)** $(L/2)\cos\theta$ **(d)** 2.5 m

82. Strut AB: 7 200 N compression; Strut AC: 6 200 N tension; Strut BC: 7 200 N tension; Strut BD: 12 000 N compression; Strut CD: 7 200 N tension; Strut CE: 6 200 N tension; Strut DE: 7 200 N compression;

84. $5.7\ \text{rad/s}$

Problem Solutions

8.1 To exert a given torque using minimum force, the lever arm should be as large as possible. In this case, the maximum lever arm is used when the force is applied at the end of the wrench and perpendicular to the handle.

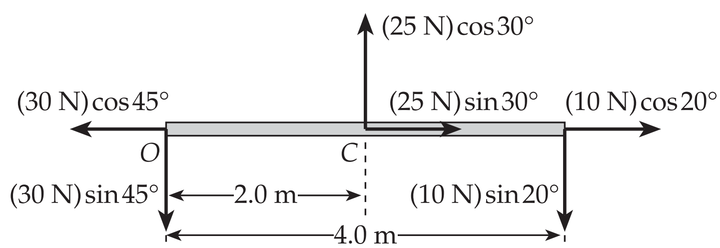


Then, $F_{\min} = \dfrac{\tau}{d_{\max}} = \dfrac{40.0\ \text{N}\cdot\text{m}}{0.300\ \text{m}} = \boxed{133\ \text{N}}$

8.2 The lever arm is $d = \left(1.20\times10^{-2}\ \text{m}\right)\cos 48.0° = 8.03\times10^{-3}\ \text{m}$, and the torque is

$$\tau = Fd = (80.0\ \text{N})\left(8.03\times10^{-3}\ \text{m}\right) = \boxed{0.642\ \text{N}\cdot\text{m}\ \ \text{counterclockwise}}$$

8.3 First resolve all of the forces shown in Figure P8.3 into components parallel to and perpendicular to the beam as shown in the sketch below.



(a) $\tau_O = +\left[(25\ \text{N})\cos 30°\right](2.0\ \text{m}) - \left[(10\ \text{N})\sin 20°\right](4.0\ \text{m}) = \boxed{+30\ \text{N}\cdot\text{m}}$

or $\tau_O = \boxed{30\ \text{N}\cdot\text{m counterclockwise}}$

(b) $\tau_C = +\left[(30\ \text{N})\sin 45°\right](2.0\ \text{m}) - \left[(10\ \text{N})\sin 20°\right](2.0\ \text{m}) = \boxed{+36\ \text{N}\cdot\text{m}}$

or $\tau_C = \boxed{36\ \text{N}\cdot\text{m counterclockwise}}$

8.4 The object is in both translational and rotational equilibrium. Thus, we may write:

$$\Sigma F_x = 0 \;\Rightarrow\; \boxed{F_x - R_x = 0}$$

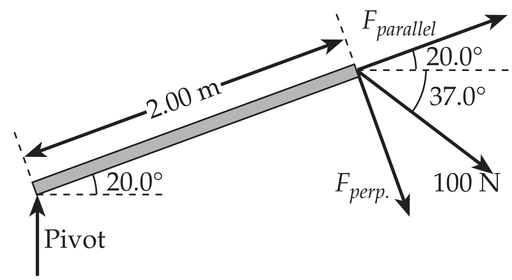
$$\Sigma F_y = 0 \;\Rightarrow\; \boxed{F_y + R_y - F_g = 0}$$

and $$\Sigma \tau_O = 0 \;\Rightarrow\; \boxed{F_y\left(\ell\cos\theta\right) - F_x\left(\ell\sin\theta\right) - F_g\left(\frac{\ell}{2}\cos\theta\right) = 0}$$

8.5 $$|\tau| = F\cdot(\textit{lever arm}) = (mg)\cdot\left[L\sin\theta\right]$$

$$= (3.0\text{ kg})(9.8\text{ m/s}^2)\cdot\left[(2.0\text{ m})\sin 5.0°\right] = \boxed{5.1\text{ N}\cdot\text{m}}$$

8.6 Resolve the 100-N force into components parallel to and perpendicular to the rod, as



$$F_{parallel} = (100\text{ N})\cos(20.0° + 37.0°) = 54.5\text{ N}$$

and

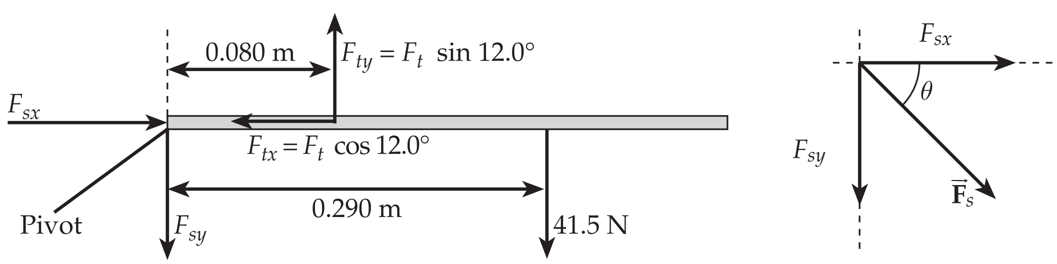
$$F_{perp.} = (100\text{ N})\sin(20.0° + 37.0°) = 83.9\text{ N}$$

The torque due to the 100-N force is equal to the sum of the torques of its components.

Thus,

$$\tau = (54.5\text{ N})(0) - (83.9\text{ N})(2.00\text{ m}) = \boxed{-168\text{ N}\cdot\text{m}}$$

8.7



Requiring that $\Sigma\tau = 0$, using the shoulder joint at point O as a pivot, gives

$$\Sigma\tau = (F_t \sin 12.0°)(0.080\text{ m}) - (41.5\text{ N})(0.290\text{ m}) = 0\text{, or } F_t = \boxed{724\text{ N}}$$

Then $\Sigma F_y = 0 \;\Rightarrow\qquad -F_{sy} + (724\text{ N})\sin 12.0° - 41.5\text{ N}=0$

yielding $\qquad F_{sy} = 109\text{ N}$

$\Sigma F_x = 0$ gives $\quad F_{sx} - (724\text{ N})\cos 12.0°=0$, or $F_{sx} = 708\text{ N}$

Therefore, $\qquad F_s = \sqrt{F_{sx}^2 + F_{sy}^2} = \sqrt{(708\text{ N})^2 + (109\text{ N})^2} = \boxed{716\text{ N}}$

8.8 If the mass of a hydrogen atom is 1.00 u (that is, 1 unit), then the mass of the oxygen atom is 16.0 u.

$$x_{cg} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{(16.0\text{ u})(0) + 2(1.00\text{ u})\left[(0.100\text{ nm})\cos 53.0°\right]}{(16.0 + 1.00 + 1.00)\text{ u}} = \boxed{6.69\times 10^{-3}\text{ nm}}$$

$$y_{cg} = \frac{\Sigma m_i y_i}{\Sigma m_i}$$

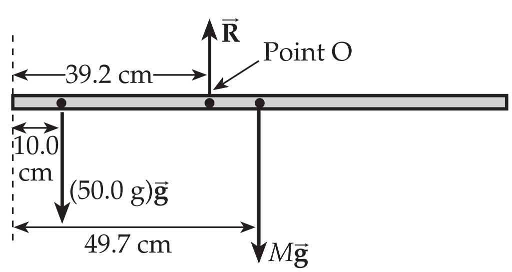
$$= \frac{(16.0)(0) + (1.00)\left[(0.100)\sin 53.0°\right] + (1.00)\left[-(0.100)\sin 53.0°\right]\text{ u}\cdot\text{nm}}{(16.0 + 1.00 + 1.00)\text{ u}} = \boxed{0}$$

8.9 Require that $\Sigma\tau = 0$ about an axis through the elbow and perpendicular to the page. This gives

$$\Sigma\tau = +\left[(2.00\text{ kg})\left(9.80\text{ m/s}^2\right)\right](25.0\text{ cm}+8.00\text{ cm}) - \left(F_B\cos 75.0°\right)(8.00\text{ cm}) = 0$$

or $\qquad F_B = \dfrac{(19.6\text{ N})(33.0\text{ cm})}{(8.00\text{ cm})\cos 75.0°} = \boxed{312\text{ N}}$

8.10 Since the bare meter stick balances at the 49.7 cm mark when placed on the fulcrum, the center of gravity of the meter stick is located 49.7 cm from the zero end. Thus, the entire weight of the meter stick may be considered to be concentrated at this point. The free-body diagram of the stick when it is balanced with the 50.0-g mass attached at the 10.0 cm mark is as given at the right.

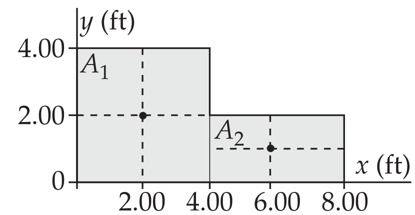


Requiring that the sum of the torques about point O be zero yields

$$+\left[(50.0\text{ g})\cancel{g}\right](39.2\text{ cm}-10.0\text{ cm})-M\cancel{g}(49.7\text{ cm}-39.2\text{ cm})=0$$

or $$M=(50.0\text{ g})\left(\frac{39.2\text{ cm}-10.0\text{ cm}}{49.7\text{ cm}-39.2\text{ cm}}\right)=\boxed{139\text{ g}}$$

8.11 Consider the remaining plywood to consist of two parts: A_1 is a 4.00-ft by 4.00-ft section with center of gravity located at $(2.00\text{ ft, }2.00\text{ ft})$, while A_2 is a 2.00-ft by 4.00-ft section with center of gravity at $(6.00\text{ ft, }1.00\text{ ft})$. Since the plywood is uniform, its mass per area σ is constant and the mass of a section having area A is $m=\sigma A$. The center of gravity of the remaining plywood has coordinates given by:



$$x_{cg}=\frac{\Sigma m_i x_i}{\Sigma m_i}=\frac{\cancel{\sigma}A_1x_1+\cancel{\sigma}A_2x_2}{\cancel{\sigma}A_1+\cancel{\sigma}A_2}=\frac{(16.0\text{ ft}^2)(2.00\text{ ft})+(8.00\text{ ft}^2)(6.00\text{ ft})}{(16.0\text{ ft}^2)+(8.00\text{ ft}^2)}=\boxed{3.33\text{ ft}}$$

and $$y_{cg}=\frac{\Sigma m_i y_i}{\Sigma m_i}=\frac{\cancel{\sigma}A_1y_1+\cancel{\sigma}A_2y_2}{\cancel{\sigma}A_1+\cancel{\sigma}A_2}=\frac{(16.0\text{ ft}^2)(2.00\text{ ft})+(8.00\text{ ft}^2)(1.00\text{ ft})}{(16.0\text{ ft}^2)+(8.00\text{ ft}^2)}=\boxed{1.67\text{ ft}}$$

8.12 Requiring that $x_{cg}=\frac{\Sigma m_i x_i}{\Sigma m_i}=0$ gives

$$\frac{(5.0\text{ kg})(0)+(3.0\text{ kg})(0)+(4.0\text{ kg})(3.0\text{ m})+(8.0\text{ kg})x}{(5.0+3.0+4.0+8.0)\text{ kg}}=0$$

or $8.0x+12\text{ m}=0$ which yields $x=-1.5\text{ m}$

Also, requiring that $y_{cg}=\frac{\Sigma m_i y_i}{\Sigma m_i}=0$ gives

$$\frac{(5.0\text{ kg})(0)+(3.0\text{ kg})(4.0\text{ m})+(4.0\text{ kg})(0)+(8.0\text{ kg})y}{(5.0+3.0+4.0+8.0)\text{ kg}}=0$$

or $8.0y+12\text{ m}=0$ yielding $y=-1.5\text{ m}$

Thus, the 8.0-kg object should be placed at coordinates $\boxed{(-1.5\text{ m},-1.5\text{ m})}$

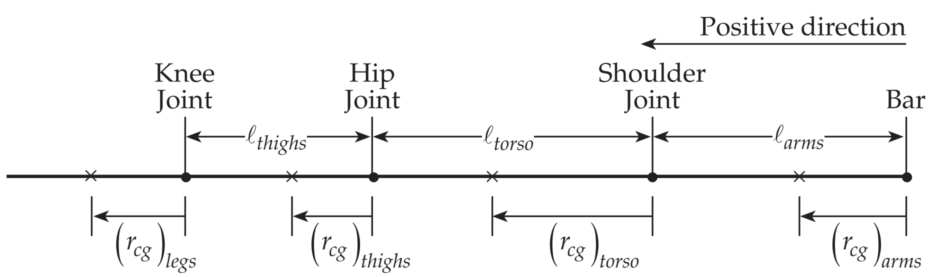
8.13 In each case, the distance from the bar to the center of mass of the body is given by

$$x_{cg} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{m_{arms} x_{arms} + m_{torso} x_{torso} + m_{thighs} x_{thighs} + m_{legs} x_{legs}}{m_{arms} + m_{torso} + m_{thighs} + m_{legs}}$$

where the distance x for any body part is the distance from the bar to the center of gravity of that body part. <u>In each case, we shall take the positive direction for distances to run from the bar toward the location of the head</u>.

Note that: $\Sigma m_i = (6.87 + 33.57 + 14.07 + 7.54)\ \text{kg} = 62.05\ \text{kg}$

With the body positioned as shown in Figure P8.13b, the distances x for each body part is computed using the sketch given below:



$$x_{arms} = +(r_{cg})_{arms} = +0.239\ \text{m}$$

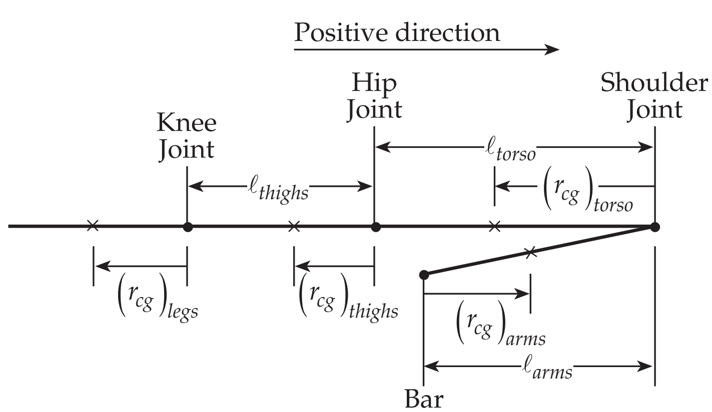
$$x_{torso} = \ell_{arms} + (r_{cg})_{torso} = +0.548\ \text{m} + 0.337\ \text{m} = 0.885\ \text{m}$$

$$x_{thighs} = \ell_{arms} + \ell_{torso} + (r_{cg})_{thighs} = (+0.548 + 0.601 + 0.151)\ \text{m} = 1.30\ \text{m}$$

$$x_{legs} = \ell_{arms} + \ell_{torso} + \ell_{thighs} + (r_{cg})_{legs} = (+0.548 + 0.601 + 0.374 + 0.227)\ \text{m} = 1.75\ \text{m}$$

With these distances and the given masses we find: $x_{cg} = \dfrac{+62.8\ \text{kg}\cdot\text{m}}{62.05\ \text{kg}} = \boxed{+1.01\ \text{m}}$

With the body positioned as shown in Figure P8.13c, we use the following sketch to determine the distance x for each body part:



$$x_{arms} = +\left(r_{cg}\right)_{arms} = +0.239 \text{ m}$$

$$x_{torso} = \ell_{arms} - \left(r_{cg}\right)_{torso} = +0.548 \text{ m} - 0.337 \text{ m} = +0.211 \text{ m}$$

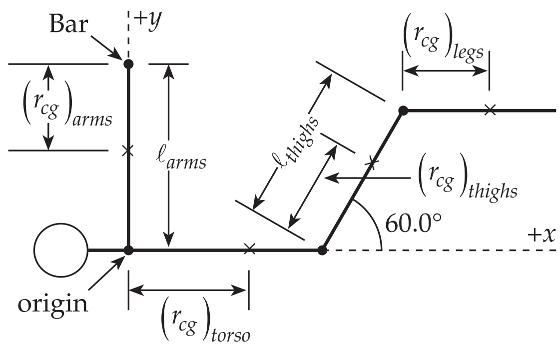
$$x_{thighs} = \ell_{arms} - \ell_{torso} - \left(r_{cg}\right)_{thighs} = \left(+0.548 - 0.601 - 0.151\right) \text{ m} = -0.204 \text{ m}$$

$$x_{legs} = \ell_{arms} - \ell_{torso} - \ell_{thighs} - \left(r_{cg}\right)_{legs} = \left(+0.548 - 0.601 - 0.374 - 0.227\right) \text{ m} = -0.654 \text{ m}$$

With these distances, the location (relative to the bar) of the center of gravity of the body is:

$$x_{cg} = \frac{+0.924 \text{ kg}\cdot\text{m}}{62.05 \text{ kg}} = \boxed{+0.015 \text{ m}} = \boxed{0.015 \text{ m towards the head}}$$

8.14 With the coordinate system shown below, the coordinates of the center of gravity of each body part may be computed:



$$x_{cg,\,arms} = 0 \qquad y_{cg,\,arms} = \ell_{arms} - \left(r_{cg}\right)_{arms} = 0.309 \text{ m}$$

$$x_{cg,\,torso} = \left(r_{cg}\right)_{torso} = 0.337 \text{ m} \qquad y_{cg,\,torso} = 0$$

$$x_{cg,\,thighs} = \ell_{torso} + \left(r_{cg}\right)_{thighs} \cos 60.0° = 0.676 \text{ m} \qquad y_{cg,\,thighs} = \left(r_{cg}\right)_{thighs} \sin 60.0° = 0.131 \text{ m}$$

$$x_{cg,\,legs} = \ell_{torso} + \ell_{thighs} \cos 60.0° + \left(r_{cg}\right)_{legs} = 1.02 \text{ m} \qquad y_{cg,\,legs} = \ell_{thighs} \sin 60.0° = 0.324 \text{ m}$$

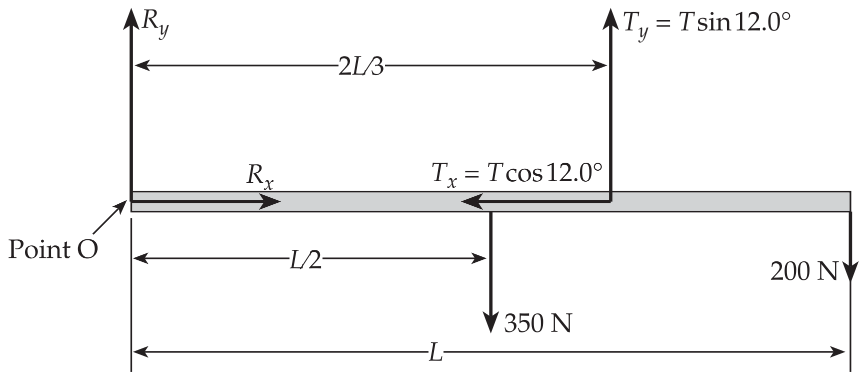
With these coordinates for individual body parts and the masses given in Problem 8.13, the coordinates of the center of mass for the entire body are found to be:

$$x_{cg} = \frac{m_{arms}x_{cg,\,arms} + m_{torso}x_{cg,\,torso} + m_{thighs}x_{cg,\,thighs} + m_{legs}x_{cg,\,legs}}{m_{arms} + m_{torso} + m_{thighs} + m_{legs}} = \frac{28.5 \text{ kg}\cdot\text{m}}{62.05 \text{ kg}} = \boxed{0.459 \text{ m}}$$

and

$$y_{cg} = \frac{m_{arms}y_{cg,\,arms} + m_{torso}y_{cg,\,torso} + m_{thighs}y_{cg,\,thighs} + m_{legs}y_{cg,\,legs}}{m_{arms} + m_{torso} + m_{thighs} + m_{legs}} = \frac{6.41 \text{ kg}\cdot\text{m}}{62.05 \text{ kg}} = \boxed{0.103 \text{ m}}$$

8.15 The free-body diagram for the spine is shown below.



When the spine is in rotational equilibrium, the sum of the torques about the left end (point O) must be zero. Thus,

$$+T_y\left(\frac{2L}{3}\right)-(350\text{ N})\left(\frac{L}{2}\right)-(200\text{ N})(L)=0$$

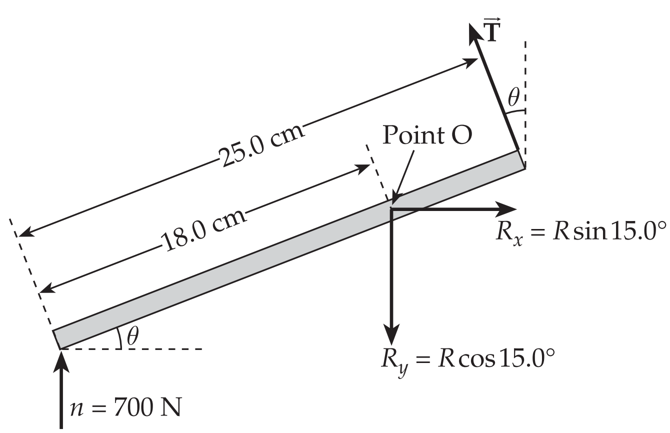
yielding $T_y = T\sin 12.0° = 562\text{ N}$

The tension in the back muscle is then $T=\dfrac{562\text{ N}}{\sin 12.0°}=2.71\times 10^3\text{ N}=\boxed{2.71\text{ kN}}$

The spine is also in translational equilibrium, so $\Sigma F_x = 0 \Rightarrow R_x - T_x = 0$ and the compressional force in the spine is

$$R_x = T_x = T\cos 12.0° = (2.71\text{ kN})\cos 12.0° = \boxed{2.65\text{ kN}}$$

8.16 In the free-body diagram of the foot given at the right, note that the force $\vec{\mathbf{R}}$ (exerted on the foot by the tibia) has been replaced by its horizontal and vertical components. Employing both conditions of equilibrium (using point O as the pivot point) gives the following three equations:



$$\Sigma F_x = 0 \Rightarrow R\sin 15.0° - T\sin\theta = 0$$

or $$R=\frac{T\sin\theta}{\sin 15.0°} \qquad (1)$$

$$\Sigma F_y = 0 \Rightarrow 700 \text{ N} - R\cos 15.0° + T\cos\theta = 0 \qquad (2)$$

$$\Sigma \tau_O = 0 \Rightarrow -(700 \text{ N})\left[(18.0 \text{ cm})\cos\theta\right] + T(25.0 \text{ cm} - 18.0 \text{ cm}) = 0$$

or $$T = (1\,800 \text{ N})\cos\theta \qquad (3)$$

Substituting Equation (3) into Equation (1) gives: $R = \left(\dfrac{1\,800 \text{ N}}{\sin 15.0°}\right)\sin\theta\cos\theta$ (4)

Substituting Equations (3) and (4) into Equation (2) yields

$$\left(\frac{1\,800 \text{ N}}{\tan 15.0°}\right)\sin\theta\cos\theta - (1\,800 \text{ N})\cos^2\theta = 700 \text{ N}$$

which reduces to: $\sin\theta\cos\theta = (\tan 15.0°)\cos^2\theta + 0.104\,2$

Squaring this result and using the identity $\sin^2\theta = 1 - \cos^2\theta$ gives

$$\left[\tan^2(15.0°) + 1\right]\cos^4\theta + \left[(2\tan 15.0°)(0.104\,2) - 1\right]\cos^2 + (0.104\,2)^2 = 0$$

In this last result, let $u = \cos^2\theta$ and evaluate the constants to obtain the quadratic equation

$$(1.071\,8)u^2 - (0.944\,2)u + (0.010\,9) = 0$$

The quadratic formula yields the solutions $u = 0.869\,3$ and $u = 0.011\,7$.

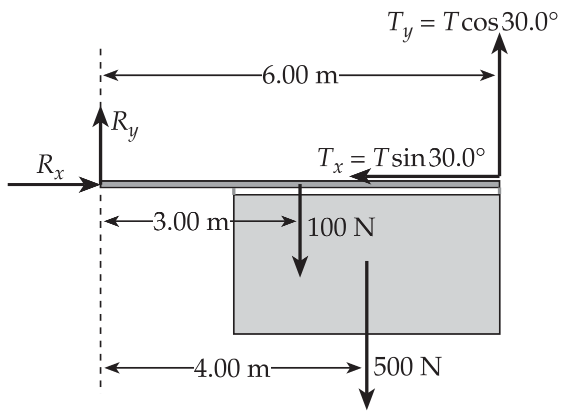
Thus, $\theta = \cos^{-1}\left(\sqrt{0.869\,3}\right) = 21.2°$ or $\theta = \cos^{-1}\left(\sqrt{0.011\,7}\right) = 83.8°$

We ignore the second solution since it is physically impossible for the human foot to stand with the sole inclined at 83.8° to the floor. We are the left with: $\theta = \boxed{21.2°}$

Equation (3) then yields: $T = (1\,800 \text{ N})\cos 21.2° = \boxed{1.68 \times 10^3 \text{ N}}$

and Equation (1) gives: $R = \dfrac{(1.68 \times 10^3 \text{ N})\sin 21.2°}{\sin 15.0°} = \boxed{2.34 \times 10^3 \text{ N}}$

8.17 Consider the torques about an axis perpendicular to the page through the left end of the rod.



$$\Sigma\tau = 0 \Rightarrow T = \frac{(100 \text{ N})(3.00 \text{ m}) + (500 \text{ N})(4.00 \text{ m})}{(6.00 \text{ m})\cos 30.0°}$$

$$T = \boxed{443 \text{ N}}$$

$$\Sigma F_x = 0 \Rightarrow R_x = T\sin 30.0° = (443 \text{ N})\sin 30.0°$$

$$R_x = \boxed{221 \text{ N toward the right}}$$

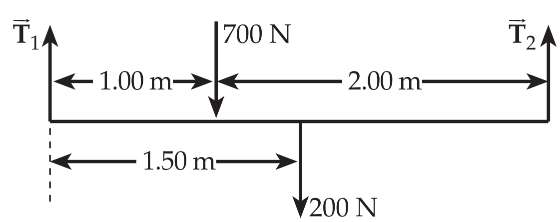
$$\Sigma F_y = 0 \Rightarrow R_y + T\cos 30.0° - 100 \text{ N} - 500 \text{ N} = 0$$

$$R_y = 600 \text{ N} - (443 \text{ N})\cos 30.0° = \boxed{217 \text{ N upward}}$$

8.18 Consider the torques about an axis perpendicular to the page through the left end of the scaffold.

$$\Sigma\tau = 0 \Rightarrow T_1(0) - (700 \text{ N})(1.00 \text{ m}) - (200 \text{ N})(1.50 \text{ m}) + T_2(3.00 \text{ m}) = 0$$

From which, $T_2 = \boxed{333 \text{ N}}$



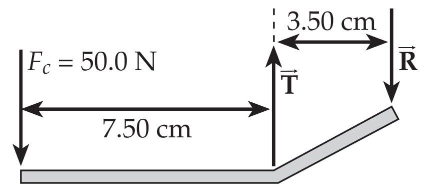
Then, from $\Sigma F_y = 0$, we have

$$T_1 + T_2 - 700 \text{ N} - 200 \text{ N} = 0$$

or

$$T_1 = 900 \text{ N} - T_2 = 900 \text{ N} - 333 \text{ N} = \boxed{567 \text{ N}}$$

8.19 Consider the torques about an axis perpendicular to the page and through the point where the force $\vec{\mathbf{T}}$ acts on the jawbone.

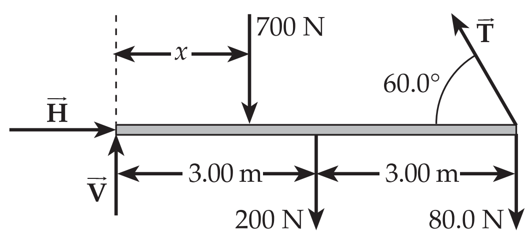


$$\Sigma\tau = 0 \Rightarrow (50.0\text{ N})(7.50\text{ cm}) - R(3.50\text{ cm}) = 0$$

which yields $R = \boxed{107\text{ N}}$

Then, $\Sigma F_y = 0 \Rightarrow -(50.0\text{ N}) + T - 107\text{ N} = 0$, or $T = \boxed{157\text{ N}}$

8.20 (a) See the diagram below:



(b) If $x = 1.00$ m, then

$$\Sigma\tau)_{left\ end} = 0 \Rightarrow -(700\text{ N})(1.00\text{ m}) - (200\text{ N})(3.00\text{ m}) - (80.0\text{ N})(6.00\text{ m}) + (T\sin 60.0°)(6.00\text{ m}) = 0$$

giving $T = \boxed{343\text{ N}}$

Then, $\Sigma F_x = 0 \Rightarrow H - T\cos 60.0° = 0$, or $H = (343\text{ N})\cos 60.0° = \boxed{171\text{ N}}$

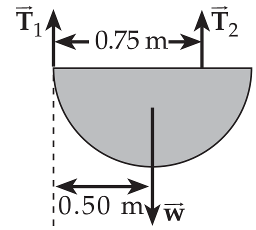
and $\Sigma F_y = 0 \Rightarrow V - 980\text{ N} + (343\text{ N})\sin 60.0° = 0$, or $V = \boxed{683\text{ N}}$

(c) When the wire is on the verge of breaking, $T = 900$ N and

$$\Sigma\tau)_{left\ end} = -(700\text{ N})x_{\max} - (200\text{ N})(3.00\text{ m}) - (80.0\text{ N})(6.00\text{ m}) + [(900\text{ N})\sin 60.0°](6.00\text{ m}) = 0$$

which gives $x_{max} = \boxed{5.14\text{ m}}$

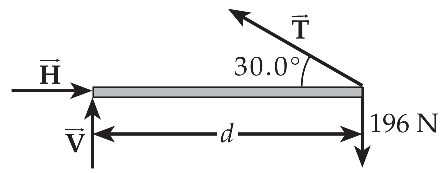
8.21 We call the tension in the cord at the left end of the sign, T_1 and the tension in the cord near the right end T_2. Consider the torques about an axis perpendicular to the page and through the left end of the sign.



$$\Sigma\tau = -w(0.50 \text{ m}) + T_2(0.75 \text{ m}) = 0 \text{, so } T_2 = \boxed{\frac{2}{3}w}$$

From $\Sigma F_y = 0$, $T_1 + T_2 - w = 0$, or $T_1 = w - T_2 = w - \frac{2}{3}w = \boxed{\frac{1}{3}w}$

8.22 (a) Consider the torques about an axis perpendicular to the page and through the left end of the horizontal beam.



$$\Sigma\tau = +(T\sin 30.0°)d - (196 \text{ N})d = 0,$$

giving $T = \boxed{392 \text{ N}}$

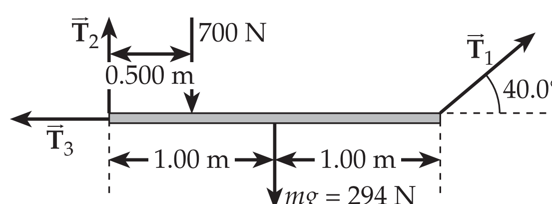
(b) From $\Sigma F_x = 0$, $H - T\cos 30.0° = 0$, or

$$H = (392 \text{ N})\cos 30.0° = \boxed{339 \text{ N to the right}}$$

From $\Sigma F_y = 0$, $V + T\sin 30.0° - 196 \text{ N} = 0$

or $V = 196 \text{ N} - (392 \text{ N})\sin 30.0° = \boxed{0}$

8.23 Consider the torques about an axis perpendicular to the page and through the left end of the plank.



$\Sigma\tau = 0$ gives

$$-(700 \text{ N})(0.500 \text{ m}) - (294 \text{ N})(1.00 \text{ m}) + (T_1\sin 40.0°)(2.00 \text{ m}) = 0$$

or $T_1 = \boxed{501 \text{ N}}$

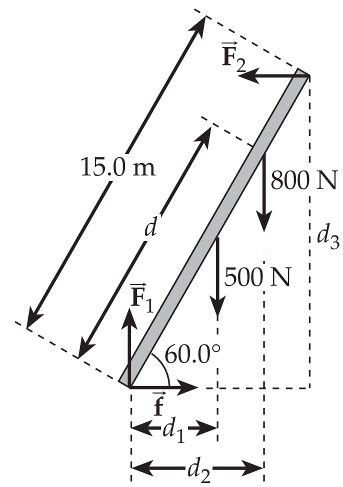
Then, $\Sigma F_x = 0$ gives $-T_3 + T_1 \cos 40.0° = 0$, or

$$T_3 = (501\text{ N})\cos 40.0° = \boxed{384\text{ N}}$$

From $\Sigma F_y = 0$, $T_2 - 994\text{ N} + T_1 \sin 40.0° = 0$,

or $$T_2 = 994\text{ N} - (501\text{ N})\sin 40.0° = \boxed{672\text{ N}}$$

8.24 First, we compute some needed dimensions:



$$d_1 = (7.50\text{ m})\cos 60.0° = 3.75\text{ m}$$

$$d_2 = d\cos 60.0° = (0.500)\ d$$

$$d_3 = (15.0\text{ m})\sin 60.0° = 13.0\text{ m}$$

Using an axis perpendicular to the page and through the lower end of the ladder, $\Sigma\tau = 0$ gives

$$-(500\text{ N})d_1 - (800\text{ N})d_2 + F_2 d_3 = 0$$

or $$F_2 = \frac{1875\text{ N}\cdot\text{m} + (800\text{ N})[(0.500)d]}{13.0\text{ m}} \qquad (1)$$

(a) When d = 4.00 m, equation (1) gives $F_2 = 267\text{ N to the left}$.

Then, $\Sigma F_x = 0$ gives $f - 267\text{ N} = 0$, or $f = \boxed{267\text{ N to the right}}$

and $\Sigma F_y = 0$ yields $F_1 - 500\text{ N} - 800\text{ N} = 0$, or $F_1 = \boxed{1.30\text{ kN upward}}$

(b) When d = 9.00 m, equation (1) gives $F_2 = 421\text{ N to the left}$.

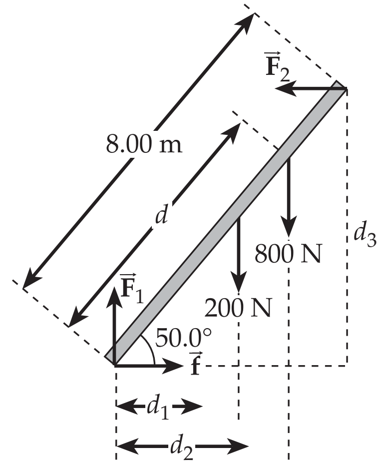
Then, $\Sigma F_x = 0$ gives $f = 421\text{ N}$ to the right, while

$\Sigma F_y = 0$ yields $F_1 = 1.30 \times 10^3\text{ N} = 1.30\text{ kN}$ as before.

If the ladder is ready to slip under these conditions, then $f = (f_s)_{\max}$,

and $$\mu_s = \frac{(f_s)_{\max}}{n} = \frac{(f_s)_{\max}}{F_1} = \frac{421\text{ N}}{1.30\times 10^3\text{ N}} = \boxed{0.324}$$

8.25 The required dimensions are:



$$d_1 = (4.00\ \text{m})\cos 50.0° = 2.57\ \text{m}$$

$$d_2 = d\cos 50.0° = (0.643)\ d$$

$$d_3 = (8.00\ \text{m})\sin 50.0° = 6.13\ \text{m}$$

$$\Sigma F_y = 0 \ \text{yields}\ F_1 - 200\ \text{N} - 800\ \text{N} = 0$$

or $$F_1 = 1.00 \times 10^3\ \text{N}$$

When the ladder is on the verge of slipping,

$$f = (f_s)_{\max} = \mu_s n = \mu_s F_1, \qquad \text{or} \qquad f = (0.600)(1.00\times 10^3\ \text{N}) = 600\ \text{N}$$

Then, $\Sigma F_x = 0$ gives $F_2 = 600\ \text{N to the left}$.

Finally, using an axis perpendicular to the page and through the lower end of the ladder, $\Sigma\tau = 0$ gives

$$-(200\ \text{N})(2.57\ \text{m}) - (800\ \text{N})(0.643)d + (600\ \text{N})(6.13\ \text{m}) = 0$$

or $$d = \frac{(3.68\times 10^3 - 550)\ \text{N}\cdot\text{m}}{0.643(800\ \text{N})} = \boxed{6.15\ \text{m}}$$ when the ladder is ready to slip.

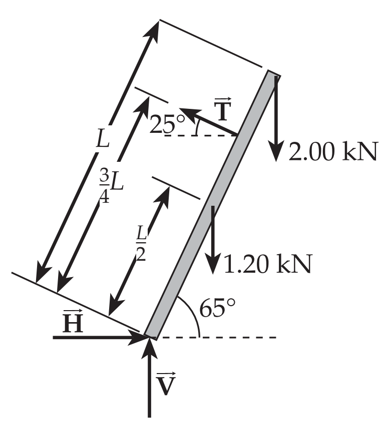
8.26 Observe that the cable is perpendicular to the boom. Then, using $\Sigma\tau = 0$ for an axis perpendicular to the page and through the lower end of the boom gives

$$-(1.20\ \text{kN})\left(\frac{L}{2}\cos 65°\right) + T\left(\frac{3}{4}L\right) - (2.00\ \text{kN})(L\cos 65°) = 0 \text{ or } T = \boxed{1.47\ \text{kN}}$$

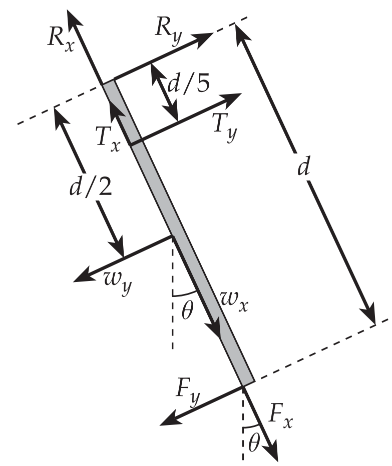
From $\Sigma F_x = 0$, $H = T\cos 25° = \boxed{1.33\ \text{kN to the right}}$

and $\Sigma F_y = 0$ gives,

$$V = 3.20\ \text{kN} - T\sin 25° = \boxed{2.58\ \text{kN upward}}$$



8.27 First, we resolve all forces into components parallel to and perpendicular to the tibia, as shown. Note that $\theta = 40.0°$ and



$$w_y = (30.0\text{ N})\sin 40.0° = 19.3\text{ N}$$

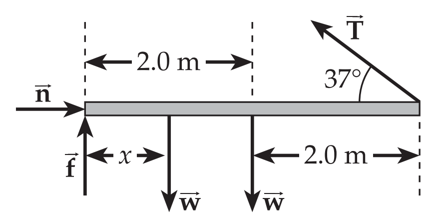
$$F_y = (12.5\text{ N})\sin 40.0° = 8.03\text{ N}$$

and $\quad T_y = T\sin 25.0°$

Using $\Sigma\tau = 0$ for an axis perpendicular to the page and through the upper end of the tibia gives

$$\left(T\sin 25.0°\right)\frac{d}{5} - (19.3\text{ N})\frac{d}{2} - (8.03\text{ N})d = 0, \quad \text{or} \quad T = \boxed{209\text{ N}}$$

8.28 When $x = x_{\text{min}}$, the rod is on the verge of slipping, so $f = (f_s)_{\text{max}} = \mu_s n = 0.50n$



From $\Sigma F_x = 0$,

$$n - T\cos 37° = 0, \quad \text{or} \quad n = 0.80T$$

Thus, $\quad f = 0.50(0.80T) = 0.40T$

From $\Sigma F_y = 0$, $f + T\sin 37° - 2w = 0$, or $\quad 0.40T + 0.60T - 2w = 0$

giving $\; T = 2w$

Using $\Sigma\tau = 0$ for an axis perpendicular to the page and through the left end of the beam gives $-w\cdot x_{\text{min}} - w(2.0\text{ m}) + \left[(2w)\sin 37°\right](4.0\text{ m}) = 0$

which reduces to $\qquad x_{\text{min}} = \boxed{2.8\text{ m}}$

8.29 The moment of inertia for rotations about an axis is $I = \Sigma m_i r_i^2$, where r_i is the distance mass m_i is from that axis.

(a) For rotation about the x-axis,

$$I_x = (3.00\text{ kg})(3.00\text{ m})^2 + (2.00\text{ kg})(3.00\text{ m})^2 +$$

$$(2.00\text{ kg})(3.00\text{ m})^2 + (4.00\text{ kg})(3.00\text{ m})^2 = \boxed{99.0\text{ kg}\cdot\text{m}^2}$$

(b) When rotating about the y-axis,

$$I_y = (3.00\text{ kg})(2.00\text{ m})^2 + (2.00\text{ kg})(2.00\text{ m})^2 +$$

$$(2.00\text{ kg})(2.00\text{ m})^2 + (4.00\text{ kg})(2.00\text{ m})^2 = \boxed{44.0\text{ kg}\cdot\text{m}^2}$$

(c) For rotations about an axis perpendicular to the page through point O, the distance r_i for each mass is $r_i = \sqrt{(2.00\text{ m})^2 + (3.00\text{ m})^2} = \sqrt{13.0}\text{ m}$

Thus, $I_O = \left[(3.00 + 2.00 + 2.00 + 4.00)\text{ kg}\right](13.0\text{ m}^2) = \boxed{143\text{ kg}\cdot\text{m}^2}$

8.30 The required torque in each case is $\tau = I\alpha$.

Thus, $\tau_x = I_x\,\alpha = (99.0\text{ kg}\cdot\text{m}^2)(1.50\text{ rad/s}^2) = \boxed{149\text{ N}\cdot\text{m}}$

$$\tau_y = I_y\,\alpha = (44.0\text{ kg}\cdot\text{m}^2)(1.50\text{ rad/s}^2) = \boxed{66.0\text{ N}\cdot\text{m}}$$

and $\tau_O = I_O\,\alpha = (143\text{ kg}\cdot\text{m}^2)(1.50\text{ rad/s}^2) = \boxed{215\text{ N}\cdot\text{m}}$

8.31 (a) $\tau = F\cdot r = (0.800\text{ N})(30.0\text{ m}) = \boxed{24.0\text{ N}\cdot\text{m}}$

(b) $\alpha = \dfrac{\tau}{I} = \dfrac{\tau}{mr^2} = \dfrac{24.0\text{ N}\cdot\text{m}}{(0.750\text{ kg})(30.0\text{ m})^2} = \boxed{0.0356\text{ rad/s}^2}$

(c) $a_t = r\alpha = (30.0\text{ m})(0.0356\text{ rad/s}^2) = \boxed{1.07\text{ m/s}^2}$

8.32 The angular acceleration is $\alpha = \dfrac{\omega_f - \omega_i}{\Delta t} = -\left(\dfrac{\omega_i}{\Delta t}\right)$ since $\omega_f = 0$.

Thus, the torque is $\tau = I\,\alpha = -\left(\dfrac{I\,\omega_i}{\Delta t}\right)$. But, the torque is also $\tau = -f\,r$, so the magnitude of the required friction force is

$$f = \frac{I\,\omega_i}{r(\Delta t)} = \frac{\left(12\ \text{kg}\cdot\text{m}^2\right)\left(50\ \text{rev/min}\right)}{(0.50\ \text{m})(6.0\ \text{s})}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 21\ \text{N}$$

Therefore, the coefficient of friction is $\mu_k = \dfrac{f}{n} = \dfrac{21\ \text{N}}{70\ \text{N}} = \boxed{0.30}$

8.33 (a) $\tau_{net} = I\,\alpha = \left(6.8\times10^{-4}\ \text{kg}\cdot\text{m}^2\right)\left(66\ \text{rad/s}^2\right) = 4.5\times10^{-2}\ \text{N}\cdot\text{m}$

The torque exerted by the fish is $\tau_{fish} = F\cdot r$, and this also equals

$$\tau_{fish} = \tau_{net} + \tau_{friction} = \left(4.5\times10^{-2} + 1.3\right)\ \text{N}\cdot\text{m}$$

Thus, $F = \dfrac{\tau_{fish}}{r} = \dfrac{\left(4.5\times10^{-2}+1.3\right)\ \text{N}\cdot\text{m}}{4.0\times10^{-2}\ \text{m}} = \boxed{34\ \text{N}}$

(b) $\theta = \omega_i\,t + \dfrac{1}{2}\alpha t^2 = 0 + \dfrac{1}{2}\left(66\ \text{rad/s}^2\right)(0.50\ \text{s})^2 = \left(\dfrac{33}{4}\right)\ \text{rad}$

so $s = r\,\theta = \left(4.0\times10^{-2}\ \text{m}\right)\left(\dfrac{33}{4}\right)\ \text{rad} = 0.33\ \text{m} = \boxed{33\ \text{cm}}$

8.34 $I = MR^2 = (1.80\ \text{kg})(0.320\ \text{m})^2 = 0.184\ \text{kg}\cdot\text{m}^2$

$\tau_{net} = \tau_{applied} - \tau_{resistive} = I\,\alpha$, or $F\cdot r - f\cdot R = I\,\alpha$

yielding $F = \dfrac{I\,\alpha + f\cdot R}{r}$

(a) $F = \dfrac{\left(0.184\ \text{kg}\cdot\text{m}^2\right)\left(4.50\ \text{rad/s}^2\right) + (120\ \text{N})(0.320\ \text{m})}{4.50\times10^{-2}\ \text{m}} = \boxed{872\ \text{N}}$

(b) $F = \dfrac{\left(0.184\ \text{kg}\cdot\text{m}^2\right)\left(4.50\ \text{rad/s}^2\right) + (120\ \text{N})(0.320\ \text{m})}{2.80\times10^{-2}\ \text{m}} = \boxed{1.40\ \text{kN}}$

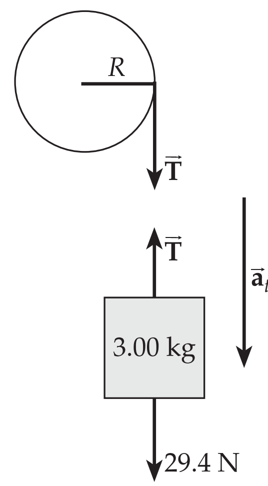
8.35 $I = \frac{1}{2}MR^2 = \frac{1}{2}(150\text{ kg})(1.50\text{ m})^2 = 169\text{ kg}\cdot\text{m}^2$

and $\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(0.500\text{ rev/s} - 0)}{2.00\text{ s}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \frac{\pi}{2}\text{ rad/s}^2$

Thus, $\tau = F\cdot r = I\alpha$ gives

$$F = \frac{I\alpha}{r} = \frac{(169\text{ kg}\cdot\text{m}^2)\left(\frac{\pi}{2}\text{ rad/s}^2\right)}{1.50\text{ m}} = \boxed{177\text{ N}}$$

8.36 The moment of inertia of the reel is



$$I = \frac{1}{2}MR^2 = \frac{1}{2}(5.00\text{ kg})(0.600\text{ m})^2 = 0.900\text{ kg}\cdot\text{m}^2$$

Applying Newton's second law to the falling bucket gives

$$29.4\text{ N} - T = (3.00\text{ kg})a_t \qquad (1)$$

Then, Newton's second law for the reel gives

$$\tau = TR = I\alpha = I\left(\frac{a_t}{R}\right)$$

or $T = \frac{Ia_t}{R^2} = \frac{(0.900\text{ kg}\cdot\text{m}^2)}{(0.600\text{ m})^2}a_t = (2.50\text{ kg})a_t \qquad (2)$

(a) Solving equations (1) and (2) simultaneously gives

$$a_t = \boxed{5.35\text{ m/s}^2\text{ downward}}$$

(b) $\Delta y = v_0 t + \frac{1}{2}a_t t^2 = 0 + \frac{1}{2}(5.35\text{ m/s}^2)(4.00\text{ s})^2 = \boxed{42.8\text{ m}}$

(c) $\alpha = \frac{a_t}{R} = \frac{5.35\text{ m/s}^2}{0.600\text{ m}} = \boxed{8.91\text{ rad/s}^2}$

8.37 The initial angular velocity of the wheel is zero, and the final angular velocity is

$$\omega_f = \frac{v}{r} = \frac{50.0\ \text{m/s}}{1.25\ \text{m}} = 40.0\ \text{rad/s}$$

Hence, the angular acceleration is

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{40.0\ \text{rad/s} - 0}{0.480\ \text{s}} = 83.3\ \text{rad/s}^2$$

The torque acting on the wheel is $\tau = f_k \cdot r$, so $\tau = I\,\alpha$ gives

$$f_k = \frac{I\,\alpha}{r} = \frac{\left(110\ \text{kg}\cdot\text{m}^2\right)\left(83.3\ \text{rad/s}^2\right)}{1.25\ \text{m}} = 7.33\times 10^3\ \text{N}$$

Thus, the coefficient of friction is $\mu_k = \dfrac{f_k}{n} = \dfrac{7.33\times 10^3\ \text{N}}{1.40\times 10^4\ \text{N}} = \boxed{0.524}$

8.38 The work done on the grindstone is $W_{net} = F\cdot s = F\cdot\left(r\,\theta\right) = \left(F\cdot r\right)\theta = \tau\cdot\theta$

Thus, $W_{net} = \tau\cdot\theta = \frac{1}{2}I\,\omega_f^2 - \frac{1}{2}I\,\omega_i^2$

or $$(25.0\ \text{N}\cdot\text{m})(15.0\ \text{rev})\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = \frac{1}{2}\left(0.130\ \text{kg}\cdot\text{m}^2\right)\omega_f^2 - 0$$

This yields $\omega_f = \left(190\ \dfrac{\text{rad}}{\text{s}}\right)\left(\dfrac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = \boxed{30.3\ \text{rev/s}}$

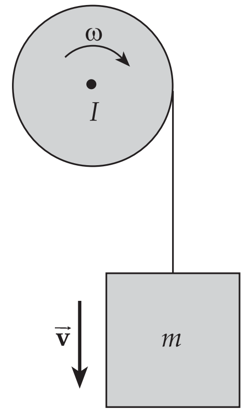
8.39 (a) $KE_{trans} = \frac{1}{2}mv_t^2 = \frac{1}{2}\left(10.0\ \text{kg}\right)\left(10.0\ \text{m/s}\right)^2 = \boxed{500\ \text{J}}$

(b) $$KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v_t^2}{R^2}\right)$$

$$= \frac{1}{4}mv_t^2 = \frac{1}{4}\left(10.0\ \text{kg}\right)\left(10.0\ \text{m/s}\right)^2 = \boxed{250\ \text{J}}$$

(c) $KE_{total} = KE_{trans} + KE_{rot} = \boxed{750\ \text{J}}$

8.40 As the bucket drops, it loses gravitational potential energy. The spool gains rotational kinetic energy and the bucket gains translational kinetic energy. Since the string does not slip on the spool, $v = r\omega$ where r is the radius of the spool. The moment of inertia of the spool is $I = \frac{1}{2}Mr^2$, where M is the mass of the spool. Conservation of energy gives



$$\left(KE_t + KE_r + PE_g\right)_f = \left(KE_t + KE_r + PE_g\right)_i$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgy_f = 0 + 0 + mgy_i$$

or $$\frac{1}{2}m(r\omega)^2 + \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega^2 = mg\left(y_i - y_f\right)$$

This gives $$\omega = \sqrt{\frac{2mg\left(y_i - y_f\right)}{\left(m + \frac{1}{2}M\right)r^2}} = \sqrt{\frac{2(3.00\text{ kg})(9.80\text{ m/s}^2)(4.00\text{ m})}{\left[3.00\text{ kg} + \frac{1}{2}(5.00\text{ kg})\right](0.600\text{ m})^2}} = \boxed{10.9\text{ rad/s}}$$

8.41 The moment of inertia of the cylinder is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\left(\frac{w}{g}\right)R^2 = \frac{1}{2}\left(\frac{800\text{ N}}{9.80\text{ m/s}^2}\right)(1.50\text{ m})^2 = 91.8\text{ kg}\cdot\text{m}^2$$

The angular acceleration is given by

$$\alpha = \frac{\tau}{I} = \frac{F\cdot R}{I} = \frac{(50.0\text{ N})(1.50\text{ m})}{91.8\text{ kg}\cdot\text{m}^2} = 0.817\text{ rad/s}^2$$

At $t = 3.00$ s, the angular velocity is

$$\omega = \omega_i + \alpha t = 0 + \left(0.817\text{ rad/s}^2\right)(3.00\text{ s}) = 2.45\text{ rad/s}$$

and the kinetic energy is

$$KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(91.8\text{ kg}\cdot\text{m}^2\right)(2.45\text{ rad/s})^2 = \boxed{276\text{ J}}$$

8.42 (a) The moment of inertial of the flywheel is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(500\ \text{kg})(2.00\ \text{m})^2 = 1.00\times10^3\ \text{kg}\cdot\text{m}^2$$

and the angular velocity is

$$\omega = \left(5000\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 524\ \text{rad/s}$$

Therefore, the stored kinetic energy is

$$KE_{stored} = \frac{1}{2}I\omega^2 = \frac{1}{2}(1.00\times10^3\ \text{kg}\cdot\text{m}^2)(524\ \text{rad/s})^2 = \boxed{1.37\times10^8\ \text{J}}$$

(b) A 10.0-hp motor supplies energy at the rate of

$$\mathcal{P} = (10.0\ \text{hp})\left(\frac{746\ \text{W}}{1\ \text{hp}}\right) = 7.46\times10^3\ \text{J/s}$$

The time the flywheel could supply energy at this rate is

$$t = \frac{KE_{stored}}{\mathcal{P}} = \frac{1.37\times10^8\ \text{J}}{7.46\times10^3\ \text{J/s}} = 1.84\times10^4\ \text{s} = \boxed{5.10\ \text{h}}$$

8.43 Using $W_{net} = KE_f - KE_i = \frac{1}{2}I\omega_f^2 - 0$, we have

$$\omega_f = \sqrt{\frac{2W_{net}}{I}} = \sqrt{\frac{2F\cdot s}{I}} = \sqrt{\frac{2(5.57\ \text{N})(0.800\ \text{m})}{4.00\times10^{-4}\ \text{kg}\cdot\text{m}^2}} = \boxed{149\ \text{rad/s}}$$

8.44 Using conservation of mechanical energy,

$$\left(KE_{trans} + KE_{rot} + PE_g\right)_f = \left(KE_{trans} + KE_{rot} + PE_g\right)_i$$

or $$\frac{1}{2}Mv_t^2 + \frac{1}{2}I\omega^2 + 0 = 0 + 0 + Mg\left(L\sin\theta\right)$$

Since $I = \frac{2}{5}MR^2$ for a solid sphere and $v_t = R\omega$ when rolling without slipping, this becomes $\frac{1}{2}MR^2\omega^2 + \frac{1}{5}MR^2\omega^2 = Mg\left(L\sin\theta\right)$ and reduces to

$$\omega = \sqrt{\frac{10gL\sin\theta}{7R^2}} = \sqrt{\frac{10\left(9.8\text{ m/s}^2\right)\left(6.0\text{ m}\right)\sin 37^\circ}{7\left(0.20\text{ m}\right)^2}} = \boxed{36\text{ rad/s}}$$

8.45 Each mass moves in a circular path of radius $r = 0.500\text{ m/s}$ about the center of the connecting rod. Their angular speed is

$$\omega = \frac{v}{r} = \frac{5.00\text{ m/s}}{0.500\text{ m}} = 10.0\text{ m/s}$$

Neglecting the moment of inertia of the light connecting rod, the angular momentum of this rotating system is

$$L = I\omega = \left[m_1r^2 + m_2r^2\right]\omega = \left(4.00\text{ kg} + 3.00\text{ kg}\right)\left(0.500\text{ m}\right)^2\left(10.0\text{ rad/s}\right) = \boxed{17.5\text{ J}\cdot\text{s}}$$

8.46 Using conservation of angular momentum, $L_{aphelion} = L_{perihelion}$.

Thus, $\left(mr_a^2\right)\omega_a = \left(mr_p^2\right)\omega_p$. Since $\omega = \frac{v_t}{r}$ at both aphelion and perihelion, this is equivalent to $\left(mr_a^2\right)\frac{v_a}{r_a} = \left(mr_p^2\right)\frac{v_p}{r_p}$, giving

$$v_a = \left(\frac{r_p}{r_a}\right)v_p = \left(\frac{0.59\text{ A.U.}}{35\text{ A.U.}}\right)\left(54\text{ km/s}\right) = \boxed{0.91\text{ km/s}}$$

8.47 The initial moment of inertia of the system is

$$I_i = \Sigma m_i r_i^2 = 4\left[M(1.0\text{ m})^2\right] = M\left(4.0\text{ m}^2\right)$$

The moment of inertia of the system after the spokes are shortened is

$$I_f = \Sigma m_f r_f^2 = 4\left[M(0.50\text{ m})^2\right] = M\left(1.0\text{ m}^2\right)$$

From conservation of angular momentum, $I_f\,\omega_f = I_i\,\omega_i$,

or $$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = (4)(2.0\text{ rev/s}) = \boxed{8.0\text{ rev/s}}$$

8.48 From conservation of angular momentum: $\left(I_{child} + I_{m\text{-}g\text{-}r}\right)_f \omega_f = \left(I_{child} + I_{m\text{-}g\text{-}r}\right)_i \omega_i$

where $I_{m\text{-}g\text{-}r} = 275\text{ kg}\cdot\text{m}^2$ is the constant moment of inertia of the merry-go-round.

Treating the child as a point object, $I_{child} = mr^2$ where r is the distance the child is from the rotation axis. Conservation of angular momentum then gives

$$\omega_f = \left(\frac{mr_i^2 + I_{m\text{-}g\text{-}r}}{mr_f^2 + I_{m\text{-}g\text{-}r}}\right)\omega_i = \left[\frac{(25.0\text{ kg})(1.00\text{ m})^2 + 275\text{ kg}\cdot\text{m}^2}{(25.0\text{ kg})(2.00\text{ m})^2 + 275\text{ kg}\cdot\text{m}^2}\right](14.0\text{ rev/min})$$

or $$\omega_f = 11.2\ \frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = \boxed{1.17\text{ rad/s}}$$

8.49 The moment of inertia of the cylinder before the putty arrives is

$$I_i = \frac{1}{2}MR^2 = \frac{1}{2}(10.0\text{ kg})(1.00\text{ m})^2 = 5.00\text{ kg}\cdot\text{m}^2$$

After the putty sticks to the cylinder, the moment of inertia is

$$I_f = I_i + mr^2 = 5.00\text{ kg}\cdot\text{m}^2 + (0.250\text{ kg})(0.900\text{ m})^2 = 5.20\text{ kg}\cdot\text{m}^2$$

Conservation of angular momentum gives $I_f\,\omega_f = I_i\,\omega_i$,

or $$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{5.00\text{ kg}\cdot\text{m}^2}{5.20\text{ kg}\cdot\text{m}^2}\right)(7.00\text{ rad/s}) = \boxed{6.73\text{ rad/s}}$$

8.50 The total angular momentum of the system is

$$I_{total} = I_{masses} + I_{student} = 2\left(mr^2\right) + 3.0\ \text{kg}\cdot\text{m}^2$$

Initially, $r = 1.0\ \text{m}$, and $I_i = 2\left[(3.0\ \text{kg})(1.0\ \text{m})^2\right] + 3.0\ \text{kg}\cdot\text{m}^2 = 9.0\ \text{kg}\cdot\text{m}^2$

Afterward, $r = 0.30\ \text{m}$, so

$$I_f = 2\left[(3.0\ \text{kg})(0.30\ \text{m})^2\right] + 3.0\ \text{kg}\cdot\text{m}^2 = 3.5\ \text{kg}\cdot\text{m}^2$$

(a) From conservation of angular momentum, $I_f\,\omega_f = I_i\,\omega_i$, or

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{9.0\ \text{kg}\cdot\text{m}^2}{3.5\ \text{kg}\cdot\text{m}^2}\right)(0.75\ \text{rad/s}) = \boxed{1.9\ \text{rad/s}}$$

(b) $KE_i = \dfrac{1}{2}I_i\,\omega_i^2 = \dfrac{1}{2}\left(9.0\ \text{kg}\cdot\text{m}^2\right)(0.75\ \text{rad/s})^2 = \boxed{2.5\ \text{J}}$

$$KE_f = \frac{1}{2}I_f\,\omega_f^2 = \frac{1}{2}\left(3.5\ \text{kg}\cdot\text{m}^2\right)(1.9\ \text{rad/s})^2 = \boxed{6.4\ \text{J}}$$

8.51 The initial angular velocity of the puck is $\omega_i = \dfrac{(v_t)_i}{r_i} = \dfrac{0.800\ \text{m/s}}{0.400\ \text{m}} = 2.00\ \dfrac{\text{rad}}{\text{s}}$

Since the tension in the string does not exert a torque about the axis of revolution, the angular momentum of the puck is conserved, or $I_f\,\omega_f = I_i\,\omega_i$.

Thus, $\omega_f = \left(\dfrac{I_i}{I_f}\right)\omega_i = \left(\dfrac{mr_i^2}{mr_f^2}\right)\omega_i = \left(\dfrac{0.400\ \text{m}}{0.250\ \text{m}}\right)^2(2.00\ \text{rad/s}) = 5.12\ \text{rad/s}$

The net work done on the puck is

$$W_{net} = KE_f - KE_i = \frac{1}{2}I_f\,\omega_f^2 - \frac{1}{2}I_i\,\omega_i^2 = \frac{1}{2}\left[\left(mr_f^2\right)\omega_f^2 - \left(mr_i^2\right)\omega_i^2\right] = \frac{m}{2}\left[r_f^2\omega_f^2 - r_i^2\omega_i^2\right]$$

or $\quad W_{net} = \dfrac{(0.120\ \text{kg})}{2}\left[(0.250\ \text{m})^2(5.12\ \text{rad/s})^2 - (0.400\ \text{m})^2(2.00\ \text{rad/s})^2\right]$

This yields $W_{net} = \boxed{5.99\times10^{-2}\ \text{J}}$

8.52 The initial angular velocity of the system is

$$\omega_i = \left(0.20\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 0.40\pi\ \text{rad/s}$$

The total moment of inertia is given by

$$I = I_{man} + I_{cylinder} = mr^2 + \frac{1}{2}MR^2 = (80\ \text{kg})r^2 + \frac{1}{2}(25\ \text{kg})(2.0\ \text{m})^2$$

Initially, the man is at $r = 2.0\ \text{m}$ from the axis, and this gives $I_i = 3.7\times10^2\ \text{kg}\cdot\text{m}^2$. At the end, when $r = 1.0\ \text{m}$, the moment of inertia is $I_f = 1.3\times10^2\ \text{kg}\cdot\text{m}^2$.

(a) From conservation of angular momentum, $I_f\,\omega_f = I_i\,\omega_i$, or

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{3.7\times10^2\ \text{kg}\cdot\text{m}^2}{1.3\times10^2\ \text{kg}\cdot\text{m}^2}\right)(0.40\pi\ \text{rad/s}) = 1.14\pi\ \text{rad/s} = \boxed{3.6\ \text{rad/s}}$$

(b) The change in kinetic energy is $\Delta KE = \frac{1}{2}I_f\,\omega_f^2 - \frac{1}{2}I_f\,\omega_i^2$, or

$$\Delta KE = \frac{1}{2}(1.3\times10^2\ \text{kg}\cdot\text{m}^2)\left(1.14\pi\ \frac{\text{rad}}{\text{s}}\right)^2 - \frac{1}{2}(3.7\times10^2\ \text{kg}\cdot\text{m}^2)\left(0.40\pi\ \frac{\text{rad}}{\text{s}}\right)^2$$

or $\Delta KE = \boxed{5.4\times10^2\ \text{J}}$. The difference is the $\boxed{\text{work done by the man}}$ as he walks inward.

8.53 (a) The table turns counterclockwise, opposite to the way the woman walks. Its angular momentum cancels that of the woman so the total angular momentum maintains a constant value of $L_{total} = L_{woman} + L_{table} = 0$.

Since the final angular momentum is $L_{total} = I_w\,\omega_w + I_t\,\omega_t = 0$, we have

$$\omega_t = -\left(\frac{I_w}{I_t}\right)\omega_w = -\left(\frac{m_w r^2}{I_t}\right)\left(\frac{v_w}{r}\right) = -\left(\frac{m_w r}{I_t}\right)v_w$$

or $$\omega_t = -\left[\frac{(60.0\ \text{kg})(2.00\ \text{m})}{500\ \text{kg}\cdot\text{m}^2}\right](-1.50\ \text{m/s}) = 0.360\ \text{rad/s}$$

Hence $\omega_{table} = \boxed{0.360\ \text{rad/s counterclockwise}}$

(b) $W_{net} = \Delta KE = KE_f - 0 = \frac{1}{2}mv_w^2 + \frac{1}{2}I_t\omega_t^2$

$$W_{net} = \frac{1}{2}(60.0\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}(500\text{ kg}\cdot\text{m}^2)(0.360\text{ rad/s})^2 = \boxed{99.9\text{ J}}$$

8.54 For one of the crew, $\Sigma F_c = ma_c$ becomes $n = m\left(\frac{v_t^2}{r}\right) = mr\omega_i^2$

We require $n = mg$, so the initial angular velocity must be $\omega_i = \sqrt{\frac{g}{r}}$

From conservation of angular momentum, $I_f\omega_f = I_i\omega_i$, or $\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i$

Thus, the angular velocity of the station during the union meeting is

$$\omega_f = \left(\frac{I_i}{I_f}\right)\sqrt{\frac{g}{r}} = \left[\frac{5.00\times10^8\text{ kg}\cdot\text{m}^2 + 150(65.0\text{ kg})(100\text{ m})^2}{5.00\times10^8\text{ kg}\cdot\text{m}^2 + 50(65.0\text{ kg})(100\text{ m})^2}\right]\sqrt{\frac{g}{r}} = 1.12\sqrt{\frac{g}{r}}$$

The centripetal acceleration experienced by the managers still on the rim is

$$a_c = r\omega_f^2 = r12.3\text{ m/s}^2\,(1.12)^2\frac{g}{r} = (1.12)^2\left(9.80\text{ m/s}^2\right) = \boxed{12.3\text{ m/s}^2}$$

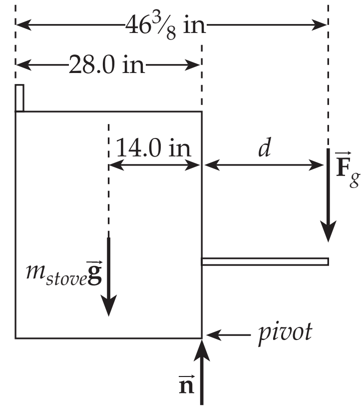
8.55 (a) From conservation of angular momentum, $I_f\omega_f = I_i\omega_i$,

so $\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \boxed{\left(\frac{I_1}{I_1+I_2}\right)\omega_o}$

(b) $KE_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(I_1+I_2)\left(\frac{I_1}{I_1+I_2}\right)^2\omega_o^2 = \left(\frac{I_1}{I_1+I_2}\right)\left[\frac{1}{2}I_1\omega_o^2\right] = \left(\frac{I_1}{I_1+I_2}\right)KE_i$

or $\frac{KE_f}{KE_i} = \boxed{\frac{I_1}{I_1+I_2}}$. Since this is less than 1.0, kinetic energy was lost.

8.56. For simplicity, we consider the stove to be a uniform box with feet at its corners, ignoring the mass of the backsplash and oven door. The free-body diagram at the right shows our simplified stove on the verge of tipping forward about the front feet when a person of weight F_g is located on the front edge of the door. Since the stove is still in equilibrium,



$$\Sigma\tau = 0 \;\Rightarrow\; (m_{stove}g)(14.0\text{ in}) - F_g d = 0$$

or $$F_g = \frac{(m_{stove}g)(14.0\text{ in})}{d} = \frac{(68.0\text{ kg})(9.80\text{ m/s}^2)(14.0\text{ in})}{46\;3/8\text{ in} - 28.0\text{ in}} = 508\text{ N}\left(\frac{0.2248\text{ lb}}{1\text{ N}}\right) = 114\text{ lb}$$

Thus, the minimum weight required to tip the stove over is $\boxed{\text{508 N or about 114 lbs}}$. This could possibly be a child. If the oven door is heavy, the required weight of the person would be somewhat less than that calculated here.

8.57 (a) Since no horizontal force acts on the child-boat system, the center of gravity of this system will remain stationary, or

$$x_{cg} = \frac{m_{child}x_{child} + m_{boat}x_{boat}}{m_{child} + m_{boat}} = \text{ constant}$$

The masses do not change, so this result becomes $m_{child}x_{child} + m_{boat}x_{boat} = \text{ constant}$

Thus, $\boxed{\text{as the child walks to the right, the boat will move to the left.}}$

(b) Measuring distances from the stationary pier, with away from the pier being positive, the child is initially at $x = 3.00\text{ m}$ and the center of gravity of the boat is at $x = 5.00\text{ m}$. At the end, the child is at the right end of the boat, so $(x_{child})_f = (x_{boat})_f + 2.00\text{ m}$. Since the center of gravity of the system does not move, we have $(m_{child}x_{child} + m_{boat}x_{boat})_f = (m_{child}x_{child} + m_{boat}x_{boat})_i$

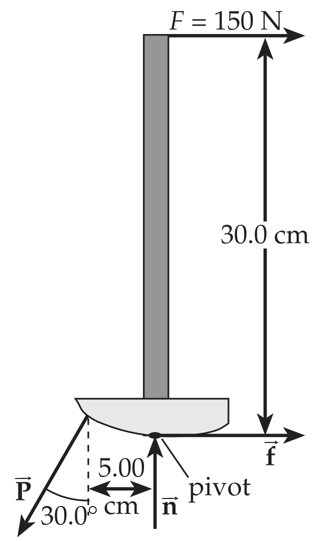
or $m_{child}(x_{child})_f + m_{boat}\left[(x_{child})_f - 2.00\text{ m}\right] = m_{child}(3.00\text{ m}) + m_{boat}(5.00\text{ m})$

and $$(x_{child})_f = \frac{m_{child}(3.00\text{ m}) + m_{boat}(5.00\text{ m} + 2.00\text{ m})}{m_{child} + m_{boat}}$$

$$(x_{child})_f = \frac{(40.0\text{ kg})(3.00\text{ m}) + (70.0\text{ kg})(5.00\text{ m} + 2.00\text{ m})}{40.0\text{ kg} + 70.0\text{ kg}} = \boxed{5.55\text{ m}}$$

(c) When the child arrives at the right end of the boat, the greatest distance from the pier that he can reach is $x_{\max} = (x_{child})_f + 1.00 \text{ m} = 5.55 \text{ m} + 1.00 \text{ m} = 6.55 \text{ m}$. This leaves him $\boxed{\text{0.45 m short of reaching the turtle}}$

8.58 (a) Choose an axis perpendicular to the page and passing through the indicated pivot. Then, $\Sigma\tau = 0$ gives



$$+(P \cdot \cos 30.0°)(5.00 \text{ cm}) - (150 \text{ N})(30.0 \text{ cm}) = 0$$

so $P = \dfrac{(150 \text{ N})(30.0 \text{ cm})}{(5.00 \text{ cm})\cos 30.0°} = \boxed{1.04 \text{ kN}}$

(b) $\Sigma F_y = 0 \Rightarrow n - P\cos 30.0° = 0$, giving

$$n = P\cos 30.0° = \left(1.04 \times 10^3 \text{ N}\right)\cos 30.0° = 900 \text{ N}$$

$\Sigma F_x = 0 \Rightarrow f + F - P\sin 30.0° = 0$, or

$$f = P\sin 30.0° - F = \left(1.04 \times 10^3 \text{ N}\right)\sin 30.0° - 150 \text{ N} = 370 \text{ N}$$

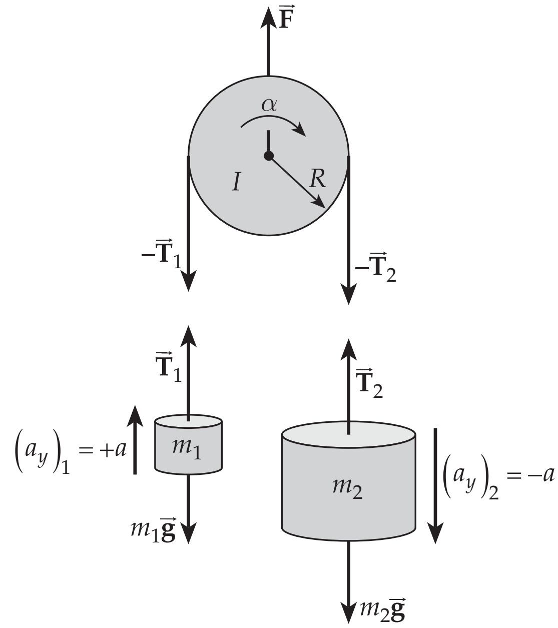
The resultant force exerted on the hammer at the pivot is

$$R = \sqrt{f^2 + n^2} = \sqrt{(370 \text{ N})^2 + (900 \text{ N})^2} = 973 \text{ N}$$

at $\theta = \tan^{-1}\left(\dfrac{n}{f}\right) = \tan^{-1}\left(\dfrac{900 \text{ N}}{370 \text{ N}}\right) = 67.7°$

or $\vec{\mathbf{R}} = \boxed{\text{973 N at 67.7° above the horizontal to the right}}$

8.59 We draw separate free-body diagrams of the pulley and each of the masses. Since the cord does not slip on the pulley, the angular acceleration of the pulley is related to the magnitude of the linear accelerations of the masses by $\alpha = a/r$.



Taking counterclockwise as positive, and applying the rotational form of Newton's second law to the pulley gives:

$$\Sigma\tau = 0 \;\Rightarrow\; T_1R - T_2R = I(-\alpha) = I\left(-\frac{a}{R}\right)$$

or $$T_2 - T_1 = \left(\frac{I}{R^2}\right)a \qquad (1)$$

Applying the translational form of Newton's second law, $\Sigma F_y = ma_y$, to each of the masses gives:

$$T_1 - m_1g = m_1(+a) \qquad \text{or} \qquad T_1 = m_1(g+a) \qquad (2)$$

and

$$T_2 - m_2g = m_2(-a) \qquad \text{or} \qquad T_2 = m_2(g-a) \qquad (3)$$

(a) Substituting Equations (2) and (3) into Equation (1) gives

$$m_2g - m_2a - m_1g - m_1a = \left(\frac{I}{R^2}\right)a \qquad \text{or} \qquad a = \frac{(m_2-m_1)g}{m_2+m_1+I/R^2}$$

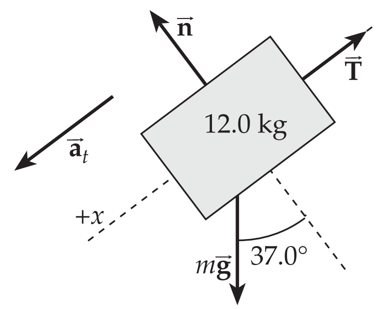
$$a = \frac{(5.0\text{ kg} - 2.0\text{ kg})(9.8\text{ m/s}^2)}{5.0\text{ kg} + 2.0\text{ kg} + (5.0\text{ kg}\cdot\text{m}^2)/(0.50\text{ m})^2} = \boxed{1.1\text{ m/s}^2}$$

(b) Equations (2) and (3) then yield

$$T_1 = m_1(g+a) = (2.0\text{ kg})(9.8\text{ m/s} + 1.1\text{ m/s}\;) = \boxed{22\text{ N}}$$

and $$T_2 = m_2(g-a) = (5.0\text{ kg})(9.8\text{ m/s} - 1.1\text{ m/s}\;) = \boxed{44\text{ N}}$$

8.60 (a) Consider the free-body diagram of the block given at the right. If the $+x$-axis is directed down the incline, $\Sigma F_x = ma_x$ gives

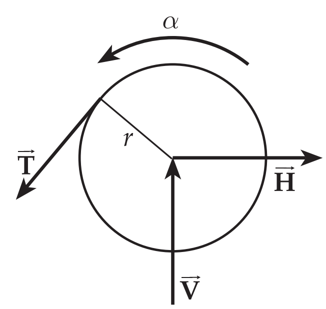


$$mg\sin 37.0° - T = ma_t\text{, or } T = m\left(g\sin 37.0° - a_t\right)$$

$$T = (12.0\text{ kg})\left[\left(9.80\text{ m/s}^2\right)\sin 37.0° - 2.00\text{ m/s}^2\right]$$

$$= \boxed{46.8\text{ N}}$$

(b) Now, consider the free-body diagram of the pulley. Choose an axis perpendicular to the page and passing through the center of the pulley,

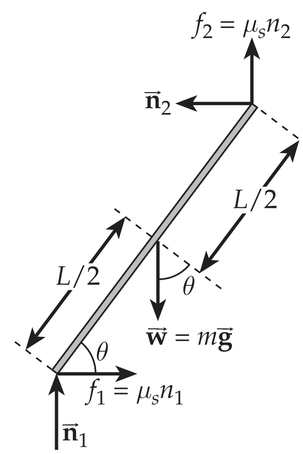


$$\Sigma\tau = I\alpha \text{ gives } T\cdot r = I\left(\frac{a_t}{r}\right)\text{, or}$$

$$I = \frac{T\cdot r^2}{a_t} = \frac{(46.8\text{ N})(0.100\text{ m})^2}{2.00\text{ m/s}^2} = \boxed{0.234\text{ kg}\cdot\text{m}^2}$$

(c) $$\omega = \omega_i + \alpha t = 0 + \left(\frac{a_t}{r}\right)t = \left(\frac{2.00\text{ m/s}^2}{0.100\text{ m}}\right)(2.00\text{ s}) = \boxed{40.0\text{ rad/s}}$$

8.61 If the ladder is on the verge of slipping, $f = (f_s)_{\max} = \mu_s n$ at both the floor and the wall.



From $\Sigma F_x = 0$, we find $f_1 - n_2 = 0$

or $\quad n_2 = \mu_s n_1$ (1)

Also, $\Sigma F_y = 0$ gives $n_1 - w + \mu_s n_2 = 0$

Using equation (1), this becomes

$$n_1 - w + \mu_s\left(\mu_s n_1\right) = 0$$

or $\quad n_1 = \dfrac{w}{1+\mu_s^2} = \dfrac{w}{1.25} = 0.800\,w$ (2)

Thus, equation (1) gives $n_2 = 0.500(0.800\,w) = 0.400\,w$ (3)

Choose an axis perpendicular to the page and passing through the lower end of the ladder. Then, $\Sigma\tau = 0$ yields

$$-w\left(\frac{L}{2}\cos\theta\right) + n_2\left(L\sin\theta\right) + f_2\left(L\cos\theta\right) = 0$$

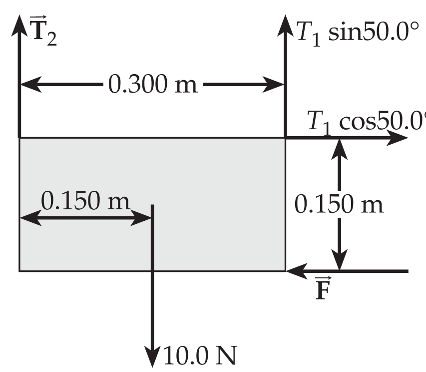
Making the substitutions $n_2 = 0.400\,w$ and $f_2 = \mu_s n_2 = 0.200\,w$, this becomes

$$-w\left(\frac{L}{2}\cos\theta\right) + \left(0.400\,w\right)\left(L\sin\theta\right) + \left(0.200\,w\right)\left(L\cos\theta\right) = 0$$

and reduces to $\sin\theta = \left(\dfrac{0.500 - 0.200}{0.400}\right)\cos\theta$

Hence, $\tan\theta = 0.750$ and $\theta = \boxed{36.9°}$

8.62 Use an axis perpendicular to the page and passing through the lower left corner of the frame. Then, $\Sigma\tau = 0$ gives



$$-(10.0\text{ N})(0.150\text{ m}) - \left(T_1\cos 50.0°\right)(0.150\text{ m}) + \left(T_1\sin 50.0°\right)(0.300\text{ m}) = 0$$

or $\quad T_1 = \boxed{11.2\text{ N}}$

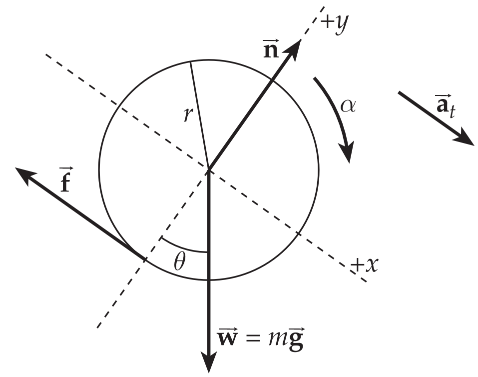
Then, using $\Sigma F_y = 0$, obtain

$$T_2 + (11.2\text{ N})\sin 50.0° - 10.0\text{ N}=0$$

or $\quad T_2 = \boxed{1.39\text{ N}}$

Finally, $\Sigma F_x = 0$ gives $F - T_1\cos 50.0° = 0$, or $F = (11.2\text{ N})\cos 50.0° = \boxed{7.23\text{ N}}$

8.63 Consider the free-body diagram of an object rolling down the incline. If it rolls without slipping, $a_t = r\alpha$.



From $\Sigma F_x = ma_x$, we obtain

$$mg\sin\theta - f = ma_t \qquad (1)$$

Now, consider an axis perpendicular to the page and passing through the center of the object.

$\Sigma\tau = I\alpha$ becomes $f\cdot r = I\alpha = I\left(\frac{a_t}{r}\right)$, or $f = \left(\frac{I}{r^2}\right)a_t$

Substitute this result into equation (1) and simplify to obtain

$$a_t = \frac{g\sin\theta}{\left(1+\frac{I}{mr^2}\right)}$$

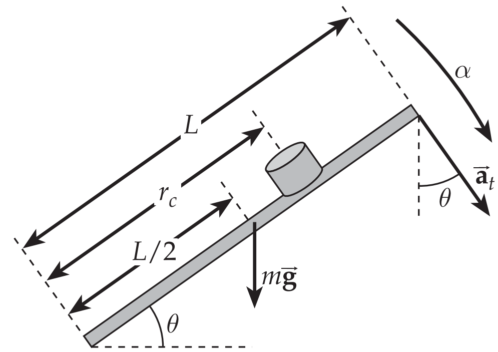
as the linear acceleration of the center of gravity of the object.

For a sphere, $I = \frac{2}{5}mr^2$, so $\boxed{a_{sphere} = \frac{g\sin\theta}{1.4}}$. For a disk, $I = \frac{1}{2}mr^2$, and

$\boxed{a_{disk} = \frac{g\sin\theta}{1.5}}$. Finally, for a ring, $I = mr^2$, so $\boxed{a_{ring} = \frac{g\sin\theta}{2.0}}$

Thus, we find $a_{sphere} > a_{disk} > a_{ring}$, so the sphere wins and the ring comes in last.

8.64 Choose an axis perpendicular to the page and passing through the lower end of the board. Then, when the support rod is removed, $\Sigma\tau = I\alpha$ gives the initial angular acceleration of the board as



$$\Sigma\tau = I\alpha \;\Rightarrow\; (mg)\left(\frac{L}{2}\cos\theta\right) = \left(\frac{1}{3}mL^2\right)\alpha$$

or $$\alpha = \frac{3g\cos\theta}{2L}$$

The tangential acceleration of the upper end of the board is $a_t = L\alpha = \dfrac{3g\cos\theta}{2}$ and the vertical component of this is

$$a_y = a_t\cos\theta = \frac{3g\cos^2\theta}{2}$$

(a) If $a_y > g$ (the vertical acceleration of the freely-falling ball), the board will get ahead of the ball. Thus, the criterion is

$$\frac{3g\cos^2\theta}{2} > g \text{ or } \cos^2\theta > \frac{2}{3}\text{, which yields } \theta < \cos^{-1}\left(\sqrt{\frac{2}{3}}\right) = \boxed{35.3°}$$

(b) For the ball to land in the cup, the cup must strike the table directly below the initial position of the ball. Thus, when starting from the limiting angle, we must have

$$r_c = L\cos\theta = \frac{L\cos^2\theta}{\cos\theta} = \boxed{\frac{2}{3}\left(\frac{L}{\cos\theta}\right)}$$

8.65 Let m_p be the mass of the pulley, m_1 be the mass of the sliding block, and m_2 be the mass of the counterweight.

(a) The moment of inertia of the pulley is $I = \frac{1}{2}m_pR_p^2$ and its angular velocity at any time is $\omega = \dfrac{v}{R_p}$, where v is the linear speed of the other objects. The friction force retarding the sliding block is $f_k = \mu_k n = \mu_k(m_1 g)$

Choose $PE_g = 0$ at the level of the counterweight when the sliding object reaches the second photogate. Then, from the work-energy theorem,

$$W_{nc} = \left(KE_{trans} + KE_{rot} + PE_g\right)_f - \left(KE_{trans} + KE_{rot} + PE_g\right)_i$$

$$-f_k \cdot s = \frac{1}{2}(m_1 + m_2)v_f^2 + \frac{1}{2}\left(\frac{1}{2}m_p R_p^2\right)\left(\frac{v_f^2}{R_p^2}\right) + 0$$
$$-\frac{1}{2}(m_1 + m_2)v_i^2 \; -\frac{1}{2}\left(\frac{1}{2}m_p R_p^2\right)\left(\frac{v_i^2}{R_p^2}\right) - m_2 g s$$

or $\frac{1}{2}\left(m_1 + m_2 + \frac{1}{2}m_p\right)v_f^2 = \frac{1}{2}\left(m_1 + m_2 + \frac{1}{2}m_p\right)v_i^2 + m_2 g s - \mu_k\left(m_1 g\right)\cdot s$

This reduces to $v_f = \sqrt{v_i^2 + \dfrac{2(m_2 - \mu_k m_1)gs}{m_1 + m_2 + \frac{1}{2}m_p}}$

and yields

$$v_f = \sqrt{\left(0.820\ \frac{\text{m}}{\text{s}}\right)^2 + \frac{2(0.208\text{ kg})(9.80\text{ m/s}^2)(0.700\text{ m})}{1.45\text{ kg}}} = \boxed{1.63\text{ m/s}}$$

(b) $\omega_f = \dfrac{v_f}{R_p} = \dfrac{1.63\text{ m/s}}{0.030\,0\text{ m}} = \boxed{54.2\text{ rad/s}}$

8.66 (a) The frame and the center of each wheel moves forward at $v = 3.35$ m/s and each wheel also turns at angular speed $\omega = v/R$. The total kinetic energy of the bicycle is $KE = KE_t + KE_r$, or

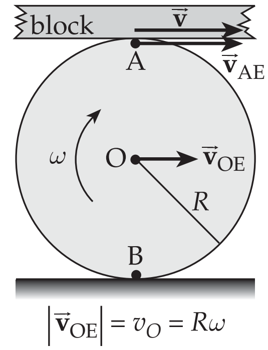
$$KE = \frac{1}{2}\left(m_{frame} + 2m_{wheel}\right)v^2 + 2\left(\frac{1}{2}I_{wheel}\,\omega^2\right)$$
$$= \frac{1}{2}\left(m_{frame} + 2m_{wheel}\right)v^2 + \frac{1}{2}\left(m_{wheel}R^2\right)\left(\frac{v^2}{R^2}\right)$$

This yields

$$KE = \frac{1}{2}\left(m_{frame} + 3m_{wheel}\right)v^2$$

$$= \frac{1}{2}\left[8.44\text{ kg} + 3(0.820\text{ kg})\right](3.35\,\text{m/s})^2 = \boxed{61.2\text{ J}}$$

(b) Since the block does not slip on the roller, its forward speed must equal that of point A, the uppermost point on the rim of the roller. That is, $v = |\vec{\mathbf{v}}_{AE}|$ where $\vec{\mathbf{v}}_{AE}$ is the velocity of A relative to Earth.



Since the roller does not slip on the ground, the velocity of point O (the roller center) must have the same magnitude as the tangential speed of point B (the point on the roller rim in contact with the ground). That is, $|\vec{\mathbf{v}}_{OE}| = R\omega = v_O$. Also, note that the velocity of point A relative to the roller center has a magnitude equal to the tangential speed $R\omega$, or $|\vec{\mathbf{v}}_{AO}| = R\omega = v_O$.

From the discussion of relative velocities in Chapter 3, we know that $\vec{\mathbf{v}}_{AE} = \vec{\mathbf{v}}_{AO} + \vec{\mathbf{v}}_{OE}$. Since all of these velocities are in the same direction, we may add their magnitudes getting $|\vec{\mathbf{v}}_{AE}| = |\vec{\mathbf{v}}_{AO}| + |\vec{\mathbf{v}}_{OE}|$, or $v = v_O + v_O = 2v_O = 2R\omega$.

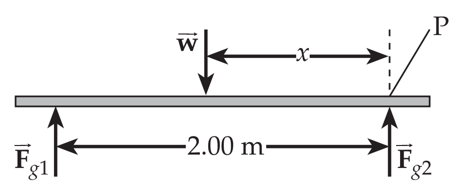
The total kinetic energy is $KE = KE_t + KE_r$, or

$$KE = \frac{1}{2}m_{stone}v^2 + 2\left[\frac{1}{2}m_{tree}\left(\frac{v}{2}\right)^2\right] + 2\left(\frac{1}{2}I_{tree}\,\omega^2\right)$$
$$= \left(\frac{1}{2}m_{stone} + \frac{1}{4}m_{tree}\right)v^2 + \frac{1}{2}m_{tree}R^2\left(\frac{v^2}{4R^2}\right)$$

This gives $KE = \frac{1}{2}\left(m_{stone} + \frac{3}{4}m_{tree}\right)v^2$, or

$$KE = \frac{1}{2}\left[844\text{ kg} + \frac{3}{4}(82.0\text{ kg})\right](0.335\,\text{m/s})^2 = \boxed{50.8\text{ J}}$$

8.67 We neglect the weight of the board and assume that the woman's feet are directly above the point of support by the rightmost scale. Then, the free-body diagram for the situation is as shown at the right.



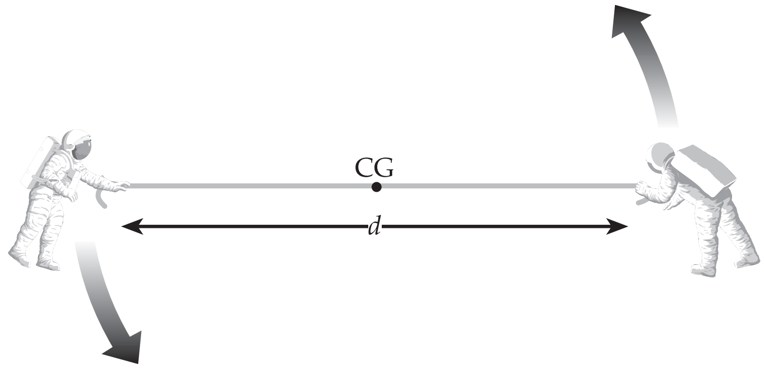
From $\Sigma F_y = 0$, we have $F_{g1} + F_{g2} - w = 0$, or $w = 380\text{ N} + 320\text{ N}=700\text{ N}$

Choose an axis perpendicular to the page and passing through point P.

Then $\Sigma\tau = 0$ gives $w\cdot x - F_{g1}(2.00\text{ m}) = 0$, or

$$x = \frac{F_{g1}(2.00\text{ m})}{w} = \frac{(380\text{ N})(2.00\text{ m})}{700\text{ N}} = \boxed{1.09\text{ m}}$$

8.68



We treat each astronaut as a point object, m, moving at speed v in a circle of radius $r = d/2$. Then the total angular momentum is

$$L = I_1\omega + I_2\omega = 2\left[\left(mr^2\right)\left(\frac{v}{r}\right)\right] = 2mvr$$

(a) $L_i = 2mv_ir_i = 2(75.0\text{ kg})(5.00\text{ m/s})(5.00\text{ m})$

$L_i = \boxed{3.75\times10^3\text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $KE_i = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = 2\left(\frac{1}{2}mv_i^2\right)$

$KE_i = (75.0\text{ kg})(5.00\text{ m/s})^2 = 1.88\times10^3\text{ J} = \boxed{1.88\text{ kJ}}$

(c) Angular momentum is conserved: $L_f = L_i = \boxed{3.75\times10^3\text{ kg}\cdot\text{m}^2/\text{s}}$

(d) $v_f = \frac{L_f}{2(mr_f)} = \frac{3.75\times10^3\text{ kg}\cdot\text{m}^2/\text{s}}{2(75.0\text{ kg})(2.50\text{ m})} = \boxed{10.0\text{ m/s}}$

(e) $KE_f = 2\left(\frac{1}{2}mv_f^2\right) = (75.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{7.50\text{ kJ}}$

(f) $W_{net} = KE_f - KE_i = \boxed{5.62\text{ kJ}}$

8.69 (a) $L_i = 2\left[Mv\left(\frac{d}{2}\right)\right] = \boxed{Mvd}$

(b) $KE_i = 2\left(\frac{1}{2}Mv_i^2\right) = \boxed{Mv^2}$

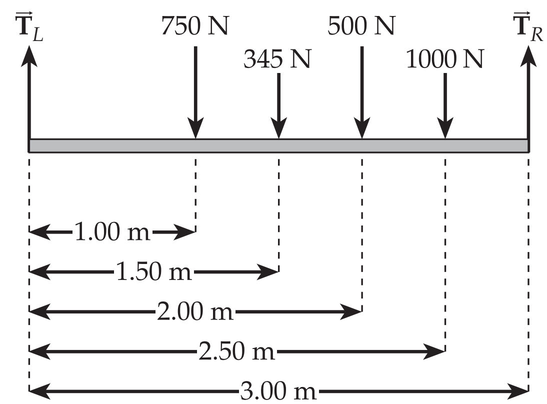
(c) $L_f = L_i = \boxed{Mvd}$

(d) $v_f = \dfrac{L_f}{2\left(Mr_f\right)} = \dfrac{Mvd}{2M\left(d/4\right)} = \boxed{2v}$

(e) $KE_f = 2\left(\dfrac{1}{2}Mv_f^2\right) = M\left(2v\right)^2 = \boxed{4Mv^2}$

(f) $W_{net} = KE_f - KE_i = \boxed{3Mv^2}$

8.70 Choose an axis that is perpendicular to the page and passing through the left end of the scaffold. Then $\Sigma\tau = 0$ gives



$$-(750\text{ N})(1.00\text{ m}) - (345\text{ N})(1.50\text{ m}) - (500\text{ N})(2.00\text{ m}) - (1000\text{ N})(2.50\text{ m}) + T_R(3.00\text{ m}) = 0$$

or $\quad T_R = 1.59\times10^3\text{ N} = \boxed{1.59\text{ kN}}$

Then,

$$\Sigma F_y = 0 \Rightarrow T_L = (750+345+500+1000)\text{ N} - 1.59\times10^3\text{ N} = \boxed{1.01\text{ kN}}$$

8.71 First, we define the following symbols:

I_P = moment of inertia due to mass of people on the equator
I_E = moment of inertia of Earth alone (without people)
ω = angular velocity of Earth (due to rotation on its axis)
$T = \dfrac{2\pi}{\omega}$ = rotational period of Earth (length of the day)
R = radius of Earth

The initial angular momentum of the system (before people start running) is

$$L_i = I_P\omega_i + I_E\omega_i = \left(I_P + I_E\right)\omega_i$$

When Earth has angular speed ω, the tangential speed of a point on the equator is $v_t = R\omega$. Thus, when the people run eastward along the equator at speed v relative to the surface of Earth, their tangential speed is $v_p = v_t + v = R\omega + v$ and their angular speed is $\omega_P = \dfrac{v_p}{R} = \omega + \dfrac{v}{R}$

The angular momentum of the system after the people begin to run is

$$L_f = I_P\,\omega_p + I_E\,\omega = I_P\left(\omega + \frac{v}{R}\right) + I_E\,\omega = \left(I_P + I_E\right)\omega + \frac{I_P v}{R}$$

Since no external torques have acted on the system, angular momentum is conserved $\left(L_f = L_i\right)$, giving $\left(I_P + I_E\right)\omega + \dfrac{I_P v}{R} = \left(I_P + I_E\right)\omega_i$. Thus, the final angular velocity of Earth is $\omega = \omega_i - \dfrac{I_P v}{\left(I_P + I_E\right)R} = \omega_i\left(1 - x\right)$, where $x \equiv \dfrac{I_P v}{\left(I_P + I_E\right)R\,\omega_i}$

The new length of the day is $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\omega_i\left(1-x\right)} = \dfrac{T_i}{1-x} \approx T_i\left(1+x\right)$, so the increase in the length of the day is $\Delta T = T - T_i \approx T_i\,x = T_i\left[\dfrac{I_P v}{\left(I_P + I_E\right)R\,\omega_i}\right]$. Since $\omega_i = \dfrac{2\pi}{T_i}$, this may be written as $\Delta T \approx \dfrac{T_i^2 I_P\,v}{2\pi\left(I_P + I_E\right)R}$

To obtain a numeric answer, we compute

$$I_P = m_p R^2 = \left[\left(5.5\times10^9\right)\left(70\ \text{kg}\right)\right]\left(6.38\times10^6\ \text{m}\right)^2 = 1.57\times10^{25}\ \text{kg}\cdot\text{m}^2$$

and

$$I_E = \frac{2}{5}m_E R^2 = \frac{2}{5}\left(5.98\times10^{24}\ \text{kg}\right)\left(6.38\times10^6\ \text{m}\right)^2 = 9.74\times10^{37}\ \text{kg}\cdot\text{m}^2$$

Thus,

$$\Delta T \approx \frac{\left(8.64\times10^4\ \text{s}\right)^2\left(1.57\times10^{25}\ \text{kg}\cdot\text{m}^2\right)\left(2.5\ \text{m/s}\right)}{2\pi\left[\left(1.57\times10^{25} + 9.74\times10^{37}\right)\ \text{kg}\cdot\text{m}^2\right]\left(6.38\times10^6\ \text{m}\right)} = \boxed{7.5\times10^{-11}\ \text{s}}$$

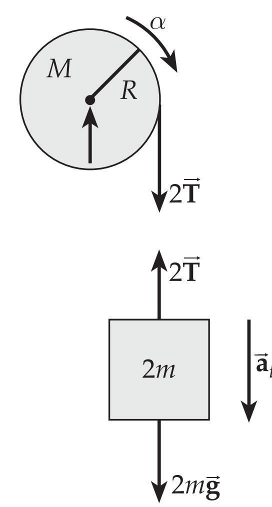
8.72 Choose $PE_g = 0$ at the level of the base of the ramp. Then, conservation of mechanical energy gives

$$\left(KE_{trans} + KE_{rot} + PE_g\right)_f = \left(KE_{trans} + KE_{rot} + PE_g\right)_i$$

$$0 + 0 + (mg)(s\sin\theta) = \frac{1}{2}mv_i^2 + \frac{1}{2}\left(mR^2\right)\left(\frac{v_i}{R}\right)^2 + 0$$

or $s = \dfrac{v_i^2}{g\sin\theta} = \dfrac{R^2\omega_i^2}{g\sin\theta} = \dfrac{(3.0\text{ m})^2(3.0\text{ rad/s})^2}{\left(9.80\text{ m/s}^2\right)\sin 20°} = \boxed{24\text{ m}}$

8.73 Choose an axis perpendicular to the page and passing through the center of the cylinder. Then, applying $\Sigma\tau = I\alpha$ to the cylinder gives



$$(2T)\cdot R = \left(\frac{1}{2}MR^2\right)\alpha = \left(\frac{1}{2}MR^2\right)\left(\frac{a_t}{R}\right), \text{ or } T = \frac{1}{4}Ma_t \qquad (1)$$

Now apply $\Sigma F_y = ma_y$ to the falling objects to obtain

$$(2m)g - 2T = (2m)a_t, \text{ or } a_t = g - \frac{T}{m} \qquad (2)$$

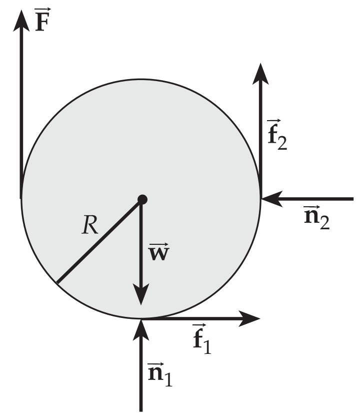
(a) Substituting equation (2) into (1) yields

$$T = \frac{Mg}{4} - \left(\frac{M}{4m}\right)T, \text{ which reduces to } T = \boxed{\frac{Mmg}{M+4m}}$$

(b) From equation (2) above,

$$a_t = g - \frac{1}{m}\left(\frac{Mmg}{M+4m}\right) = g - \frac{Mg}{M+4m} = \boxed{\frac{4mg}{M+4m}}$$

8.74 Slipping occurs simultaneously at both the bottom and side contact points. Just before slipping occurs, both static friction forces must have their maximum values. When the cylinder is about to slip, $f_1 = \mu_s n_1 = 0.5n_1$ and $f_2 = \mu_s n_2 = 0.5n_2$. Choose an axis perpendicular to the page and passing through the center of the cylinder.



Then, $\Sigma\tau = 0 \Rightarrow f_1 \cdot R + f_2 \cdot R - F \cdot R = 0$

or $\qquad F = f_1 + f_2 \qquad (1)$

From $\Sigma F_x = 0$, $f_1 = n_2 = \dfrac{f_2}{\mu_s} = 2f_2 \qquad (2)$

Combining equation (2) with equation (1) gives

$F = 2f_2 + f_2 = 3f_2$, or $f_2 = \dfrac{F}{3}$. Then equation (2) yields $f_1 = \dfrac{2F}{3}$

From $\Sigma F_y = 0$, $w = F + f_2 + n_1 = F + f_2 + \dfrac{f_1}{\mu_s} = F + f_2 + 2f_1 = F + \dfrac{F}{3} + 2\left(\dfrac{2F}{3}\right)$

or $\qquad w = \dfrac{8F}{3}$. Solving for the applied force, $F = \boxed{\dfrac{3w}{8}}$

8.75 (a) The magnitude of angular acceleration may be written as

$$|\alpha| = \frac{|\omega_f - \omega_i|}{\Delta t} = \frac{\left|\dfrac{2\pi}{T_f} - \dfrac{2\pi}{T_i}\right|}{\Delta t} = \frac{2\pi|T_i - T_f|}{T_i T_f(\Delta t)}$$

where T is the period of rotation. In this case, $\Delta t = 100$ yr, $|T_i - T_f| \sim 10^{-3}$ s, $T_i = 1 \text{ day} = 8.64 \times 10^4$ s, and $T_f = T_i - 10^{-3} \text{ s} \approx T_i$. Thus,

$$|\alpha| \sim \frac{2\pi(10^{-3}\ \text{s})}{(8.64 \times 10^4\ \text{s})^2(100\ \text{yr})}\left(\frac{1\ \text{yr}}{3.156 \times 10^7\ \text{s}}\right) \text{ giving } \boxed{|\alpha| \sim 10^{-22}\ \text{rad/s}^2}$$

(b) If we consider Earth to be a uniform solid sphere, then

$$I = \frac{2}{5}M_E R_E^2 = \frac{2}{5}\left(5.98\times10^{24}\text{ kg}\right)\left(6.38\times10^{6}\text{ m}\right)^2 = 9.74\times10^{37}\text{ kg}\cdot\text{m}^2$$

or $I \sim 10^{38}\text{ kg}\cdot\text{m}^2$

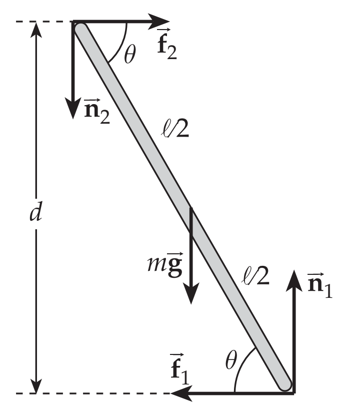
The torque required to produce the acceleration found in part (a) is therefore

$$\tau = I\alpha \sim \left(10^{38}\text{ kg}\cdot\text{m}^2\right)\left(10^{-22}\text{ m/s}^2\right) \quad \text{or} \quad \boxed{\tau \sim 10^{16}\text{ N}\cdot\text{m}}$$

(c) The average person might be able to exert a maximum force on the order of 10^3 N (about 220 lbs). The lever arm then needed to produce the torque found in part (b) is

$$d = \frac{\tau}{F} \sim \frac{10^{16}\text{ N}\cdot\text{m}}{10^{3}\text{ N}} \quad \text{or} \quad \boxed{d \sim 10^{13}\text{ m}}$$

8.76 The free-body diagram at the right shows the pole when it still in equilibrium, but on the verge of slipping.



$$\Sigma F_y = 0 \Rightarrow n_1 = n_2 + mg > n_2$$

and it is known that $(\mu_s)_1 > (\mu_s)_2$. Thus, we realize that the pole will slip at the ceiling before slipping at the floor.

Considering the torques about an axis perpendicular to the page and through the lower end of the pole gives:

$$\Sigma\tau = 0 \Rightarrow +n_2(\ell\cos\theta) + mg\left(\frac{\ell}{2}\cos\theta\right) - f_2(\ell\sin\theta) = 0$$

When the pole is on the verge of slipping, $f_2 = \mu_2 n_2$. Hence, at the critical angle of tilt,

$$n_2(\mu_2\sin\theta - \cos\theta) = \frac{mg}{2}\cos\theta \text{ or } \qquad n_2(\mu_2\tan\theta - 1) = \frac{mg}{2} > 0$$

Since $n_2 > 0$, it is necessary that $\mu_2\tan\theta - 1 > 0$ or $\tan\theta > \dfrac{1}{\mu_2}$

Thus, for the pole to remain in equilibrium, it is necessary that

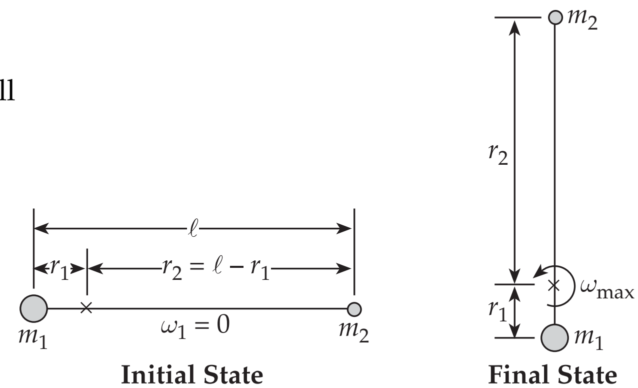
$$\theta > \tan^{-1}\left(\frac{1}{\mu_2}\right) = \tan^{-1}\left(\frac{1}{0.576}\right) = 60.1°$$

But $d = \ell \sin\theta$, so the restriction on the length of the pole is

$$\ell = \frac{d}{\sin\theta} < \frac{d}{\sin 60.1°} = \frac{7.80 \text{ ft}}{\sin 60.1°} \qquad \text{or} \qquad \ell < \boxed{9.00 \text{ ft}}$$

8.77 The large mass ($m_1 = 60.0$ kg) moves in a circular path of radius $r_1 = 0.140$ m, while the radius of the path for the small mass ($m_2 = 0.120$ kg) is $r_2 = \ell - r_1 = 3.00 \text{ m} - 0.140 \text{ m} = 2.86 \text{ m}$.

The system has maximum angular speed when the rod is in the vertical position as shown at the right.



We take $PE_g = 0$ at the level of the horizontal rotation axis and use conservation of energy to find:

$$KE_f + \left(PE_g\right)_f = KE_i + \left(PE_g\right)_i \quad \Rightarrow \quad \left(\tfrac{1}{2}I_1\omega_{\text{max}}^2 + \tfrac{1}{2}I_2\omega_{\text{max}}^2\right) + \left(m_2 g r_2 - m_1 g r_1\right) = 0 + 0$$

Approximating the two objects as point masses, we have $I_1 = m_1 r_1^2$ and $I_2 = m_2 r_2^2$. The energy conservation equation then becomes $\frac{1}{2}\left(m_1 r_1^2 + m_2 r_2^2\right)\omega_{\text{max}}^2 = \left(m_1 r_1 - m_2 r_2\right)g$ and yields

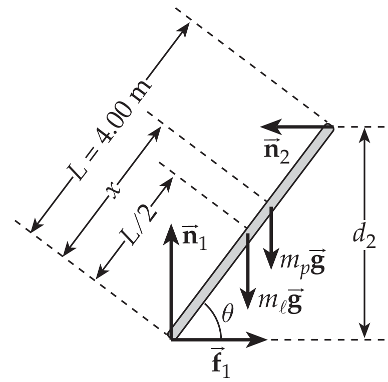
$$\omega_{\text{max}} = \sqrt{\frac{2\left(m_1 r_1 - m_2 r_2\right)g}{m_1 r_1^2 + m_2 r_2^2}} = \sqrt{\frac{2\left[(60.0 \text{ kg})(0.140 \text{ m}) - (0.120 \text{ kg})(2.86 \text{ m})\right]\left(9.80 \text{ m/s}^2\right)}{(60.0 \text{ kg})(0.140 \text{ m})^2 + (0.120 \text{ kg})(2.86 \text{ m})^2}}$$

or $\omega_{\text{max}} = 8.56$ rad/s. The maximum linear speed of the small mass object is then

$$\left(v_2\right)_{\text{max}} = r_2\omega_{\text{max}} = (2.86 \text{ m})(8.56 \text{ rad/s}) = \boxed{24.5 \text{ m/s}}$$

8.78 (a) A smooth (that is, frictionless) wall cannot exert a force parallel to its surface. Thus, the only force the vertical wall can exert on the upper end of the ladder is a horizontal normal force.

(b) Consider the free-body diagram of the ladder given at the right. If the rotation axis is perpendicular to the page and passing through the lower end of the ladder, the lever arm of the normal force $\vec{\mathbf{n}}_2$ that the wall exerts on the upper end of the ladder is



$$d_2 = \boxed{L\sin\theta}$$

(c) The lever arm of the force of gravity, $m_\ell \vec{\mathbf{g}}$, acting on the ladder is

$$d_\ell = (L/2)\cos\theta = \boxed{(L\cos\theta)/2}$$

(d) Refer to the free-body diagram given in part (b) of this solution and make use of the fact that the ladder is in both translational and rotational equilibrium.

$$\Sigma F_y = 0 \;\Rightarrow\; n_1 - m_\ell g - m_p g = 0 \qquad \text{or} \qquad n_1 = \left(m_\ell + m_p\right)g$$

When the ladder is on the verge of slipping, $f_1 = \left(f_1\right)_{\text{max}} = \mu_s n_1 = \mu_s\left(m_\ell + m_p\right)g$

Then $\Sigma F_x = 0 \;\Rightarrow\; n_2 = f_1$ or $\qquad n_2 = \mu_s\left(m_\ell + m_p\right)g$

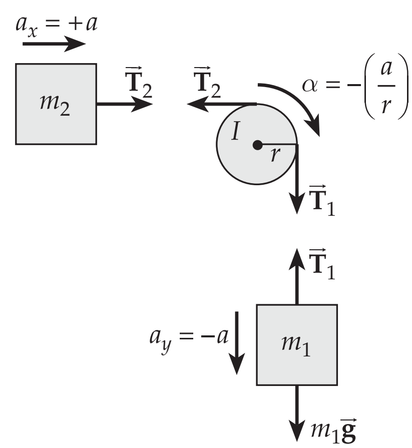
Finally, $\Sigma\tau = 0 \;\Rightarrow\; n_2\left(L\sin\theta\right) - m_\ell g\left(\dfrac{L}{2}\cos\theta\right) - m_p g x\cos\theta = 0$ where x is the maximum distance the painter can go up the ladder before it will start to slip. Solving for x gives

$$x = \frac{n_2\left(L\sin\theta\right) - m_\ell g\left(\dfrac{L}{2}\cos\theta\right)}{m_p g\cos\theta} = \mu_s\left(\frac{m_\ell}{m_p} + 1\right)L\tan\theta - \left(\frac{m_\ell}{2m_p}\right)L$$

and using the given numerical data, we find

$$x = (0.45)\left(\frac{30\text{ kg}}{80\text{ kg}} + 1\right)(4.0\text{ m})\tan 53^\circ - \left[\frac{30\text{ kg}}{2(80\text{ kg})}\right](4.0\text{ m}) = \boxed{2.5\text{ m}}$$

8.79 (a) Free-body diagrams for each block and the pulley are given at the right. Observe that the angular acceleration of the pulley will be clockwise in direction and has been given a negative sign. Since $\Sigma\tau = I\alpha$, the positive sense for torques and angular acceleration must be the same (counterclockwise).



For m_1: $\Sigma F_y = ma_y \;\Rightarrow\; T_1 - m_1 g = m_1(-a)$

or $\quad T_1 = m_1(g - a) \qquad (1)$

For m_2: $\Sigma F_x = ma_x \;\Rightarrow\; T_2 = m_2 a \qquad (2)$

For the pulley: $\Sigma\tau = I\alpha \;\Rightarrow\; T_2 r - T_1 r = I(-a/r)$

or $\quad T_1 - T_2 = \left(\dfrac{I}{r^2}\right)a \qquad (3)$

Substitute Equations (1) and (2) into Equation (3) and solve for a to obtain

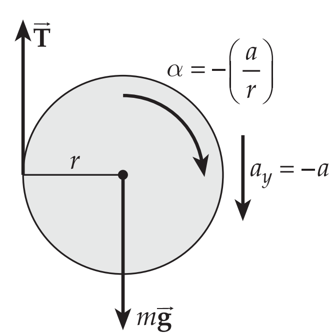
$$a = \frac{m_1 g}{(I/r^2) + m_1 + m_2}$$

or $a = \dfrac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{(0.500\text{ kg}\cdot\text{m}^2)/(0.300\text{ m})^2 + 4.00\text{ kg} + 3.00\text{ kg}} = \boxed{3.12\text{ m/s}^2}$

(b) Equation (1) above gives: $T_1 = (4.00\text{ kg})(9.80\text{ m/s}^2 - 3.12\text{ m/s}^2) = \boxed{26.7\text{ N}}$

and Equation (2) yields: $T_2 = (3.00\text{ kg})(3.12\text{ m/s}^2) = \boxed{9.37\text{ N}}$

8.80 (a) Note that the cylinder has both translational and rotational motion. The center of gravity accelerates downward while the cylinder rotates around the center of gravity. Thus, we apply both the translational and the rotational forms of Newton's second law to the cylinder:



$\Sigma F_y = ma_y \;\Rightarrow\; T - mg = m(-a)$

or $\quad T = m(g - a) \qquad (1)$

$\Sigma\tau = I\alpha \;\Rightarrow\; -Tr = I(-a/r)$

For a uniform, solid cylinder, $I = \frac{1}{2}mr^2$ so our last result becomes

$$Tr = \left(\frac{mr^2}{2}\right)\left(\frac{a}{r}\right) \qquad \text{or} \qquad a = \frac{2T}{m} \tag{2}$$

Substituting Equation (2) into Equation (1) gives

$T = mg - 2T$ and solving for T yields $T = \boxed{mg/3}$

(b) From Equation (2) above, $a = \dfrac{2T}{m} = \dfrac{2}{m}\left(\dfrac{mg}{3}\right) = \boxed{2g/3}$

(c) Considering the translational motion of the center of gravity, $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ gives

$$v_y = \sqrt{0 + 2\left(-\frac{2g}{3}\right)(-h)} = \boxed{\sqrt{4gh/3}}$$

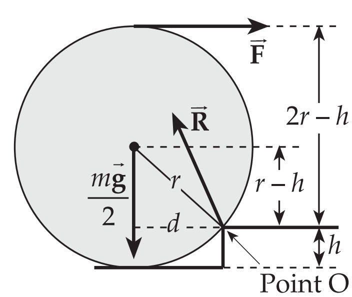
Using conservation of energy with $PE_g = 0$ at the final level of the cylinder gives

$$\left(KE_t + KE_r + PE_g\right)_f = \left(KE_t + KE_r + PE_g\right)_i \text{ or } \tfrac{1}{2}mv_y^2 + \tfrac{1}{2}I\omega^2 + 0 = 0 + 0 + mgh$$

Since $\omega = v_y/r$ and $I = \frac{1}{2}mr^2$, this becomes $\dfrac{1}{2}mv_y^2 + \dfrac{1}{2}\left(\dfrac{1}{2}m\cancel{r^2}\right)\left(\dfrac{v_y^2}{\cancel{r^2}}\right) = mgh$

or $\dfrac{3}{4}mv_y^2 = mgh$ yielding $v_y = \boxed{\sqrt{4gh/3}}$

8.81. (a) The free-body diagram at the right shows one of the main wheels at the instant when it is still in equilibrium but on the verge of going over the curb. Note that at this time, the ground exerts no force on the bottom of the wheel. $\vec{\mathbf{R}}$ is the reaction force exerted on the wheel by the curb at point O. We shall apply the second condition of equilibrium, with a pivot chosen at point O.



The Pythagorean theorem gives the lever arm of the gravitational force about point O as

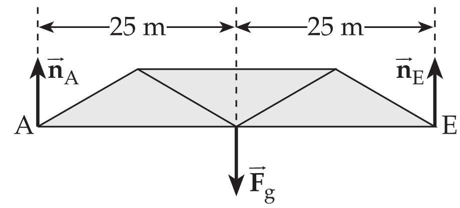
$$d = \sqrt{r^2 - (r-h)^2} = \sqrt{2rh - h^2}$$

Thus, $\Sigma\tau = 0 \Rightarrow -F(2r-h) + \left(\dfrac{mg}{2}\right)\sqrt{2rh - h^2} = 0$ or $F = \boxed{\dfrac{mg\sqrt{2rh-h^2}}{2(2r-h)}}$

(b) Using the given numeric data and retaining only one significant figure in our estimate, we find

$$F = \frac{(1\,400\text{ N})\sqrt{2(0.30\text{ m})(0.10\text{ m}) - (0.10\text{ m})^2}}{2\left[2(0.30\text{ m}) - 0.10\text{ m}\right]} = \boxed{3\times10^2\text{ N}}$$

8.82. Since the structure is free to slide horizontally at each end, the ground exerts only vertical forces on points A and E. With this realization, consider the structure as a whole as shown in the free-body diagram at the right:

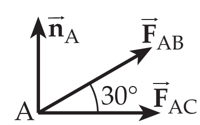


Considering point A as a pivot and requiring that $\Sigma\tau = 0$ gives

$$n_E(50\text{ m}) - (7\,200\text{ N})(25\text{ m}) = 0 \qquad \text{or} \qquad n_E = 3\,600\text{ N}$$

Then $\Sigma F_y = 0 \Rightarrow n_A = 7\,200\text{ N} - n_E = 7\,200\text{ N} - 3\,600\text{ N}$ or $n_A = 3\,600\text{ N}$

Now consider the free-body diagram of joint A shown at the right. Note that it has been assumed that members AB and AC are under tension (that is, these members *pull* on joint A). If either of these assumptions is incorrect, the computed force will have a negative sign, but the magnitude will be correct.



$$\Sigma F_y = 0 \Rightarrow F_{AB}\sin 30° + n_A = 0$$

or $$F_{AB} = \frac{-n_A}{\sin 30°} = \frac{-3\,600\text{ N}}{\sin 30°} = -7\,200\text{ N}$$

Thus, the force in strut AB is really a $\boxed{\text{compression force of magnitude 7 200 N}}$

Also, $\Sigma F_x = 0 \Rightarrow F_{AC} = -F_{AB}\cos 30° = -(-7\,200\text{ N})\cos 30° = +6\,200\text{ N}$

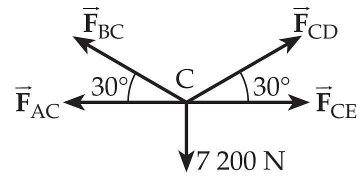
or the force in strut AC is a $\boxed{\text{tension force of magnitude 6 200 N}}$

In a manner identical to the analysis of joint A given above, one can analyze the free-body diagram of joint E and find that:

the force in strut DE is a $\boxed{\text{compression force of magnitude 7 200 N}}$

and, the force in strut CE is a $\boxed{\text{tension force of magnitude 6 200 N}}$

Now, consider the free-body diagram of joint C:



$$\Sigma F_x = 0 \;\Rightarrow\; \left(F_{CE} - F_{AC}\right) + \left(F_{CD} - F_{BC}\right)\cos 30° = 0$$

or $\quad \left(6\,200\text{ N} - 6\,200\text{ N}\right) + \left(F_{CD} - F_{BC}\right)\cos 30° = 0$

which yields: $F_{CD} = F_{BC}$

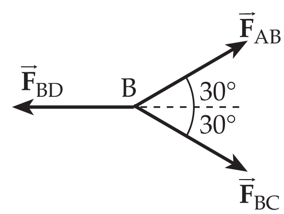
Then, $\Sigma F_y = 0 \;\Rightarrow\; \left(F_{BC} + F_{CD}\right)\sin 30° = 7\,200\text{ N}$

Since $F_{CD} = F_{BC}$, this reduces to $\quad 2F_{BC}\sin 30° = 7\,200\text{ N}$ or $F_{BC} = \dfrac{7\,200\text{ N}}{2\sin 30°} = 7\,200\text{ N}$

Thus: the force in strut BC is a $\boxed{\text{tension force of magnitude 7 200 N}}$

and the force in strut CD is a $\boxed{\text{tension force of magnitude 7 200 N}}$

Finally, consider the free-body diagram of joint B. Recall that the force in strut AB is a compression force while that in strut BC is a tension force. Note that the diagram assumes that the force in strut BD is a compression force.

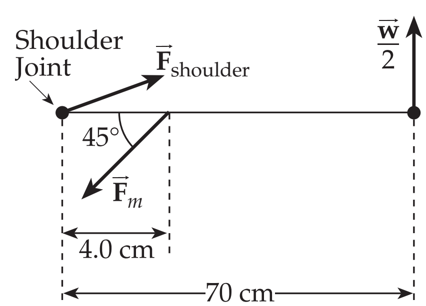


$$\Sigma F_x = 0 \;\Rightarrow\; F_{BD} = \left(F_{AB} + F_{BC}\right)\cos 30°$$

or $\quad F_{BD} = 2\left(7\,200\text{N}\right)\cos 30° = 12\,000\text{ N}$

The force in strut BD is a $\boxed{\text{compression force of magnitude 12 000 N}}$

8.83. Considering the shoulder joint as the pivot, the second condition of equilibrium gives:



$$\Sigma\tau = 0 \;\Rightarrow\; \frac{w}{2}(70\text{ cm}) - \left(F_m \sin 45°\right)(4.0\text{ cm}) = 0$$

or $\quad F_m = \dfrac{w(70\text{ cm})}{2(4.0\text{ cm})\sin 45°} = 12.4w$

Recall that this is the total force exerted on the arm by a set of two muscles. If we approximate that the two muscles of this pair exert equal magnitude forces, the force exerted by each muscle is

$$F_{\substack{each\\muscle}} = \frac{F_m}{2} = \frac{12.4w}{2} = \boxed{6.2w} = 6.2(750\text{ N}) = 4.6\times 10^3\text{ N} = \boxed{4.6\text{ kN}}$$

8.84. Observe that since the torque opposing the rotational motion of the gymnast is constant, the work done by non-conservative forces as the gymnast goes from position 1 to position 2 (an angular displacement of $\pi/2$ rad) will be the same as that done while the gymnast goes from position 2 to position 3 (another angular displacement of $\pi/2$ rad).

Choose $PE_g = 0$ at the level of the bar, and let the distance from the bar to the center of gravity of the outstretched body be r_{cg}. Applying the work-energy theorem, $W_{nc} = \left(KE + PE_g\right)_f - \left(KE + PE_g\right)_i$, to the rotation from position 1 to position 2 gives

$$\left(W_{nc}\right)_{12} = \left(\tfrac{1}{2}I\omega_2^2 + 0\right) - \left(0 + mgr_{\text{cg}}\right) \text{ or } \left(W_{nc}\right)_{12} = \tfrac{1}{2}I\omega_2^2 - mgr_{\text{cg}} \tag{1}$$

Now, apply the work-energy theorem to the rotation from position 2 to position 3 to obtain:

$$\left(W_{nc}\right)_{23} = \left[\tfrac{1}{2}I\omega_3^2 + mg\left(-r_{\text{cg}}\right)\right] - \left(\tfrac{1}{2}I\omega_2^2 + 0\right) \text{ or } \left(W_{nc}\right)_{23} = \tfrac{1}{2}I\omega_3^2 - \tfrac{1}{2}I\omega_2^2 - mgr_{\text{cg}} \tag{2}$$

Since the frictional torque is constant and these two segments of the motion involve equal angular displacements, $\left(W_{nc}\right)_{23} = \left(W_{nc}\right)_{12}$. Thus, equating Equation (2) to Equation (1) gives:

$$\tfrac{1}{2}I\omega_3^2 - \tfrac{1}{2}I\omega_2^2 - \cancel{mgr_{\text{cg}}} = \tfrac{1}{2}I\omega_2^2 - \cancel{mgr_{\text{cg}}}$$

which yields $\omega_3^2 = 2\omega_2^2$ or $\omega_3 = \sqrt{2}\omega_2 = \sqrt{2}\left(4.0 \text{ rad/s}\right) = \boxed{5.7 \text{ rad/s}}$