Derivatives

Basic Properties/Formulas/Rules

$\frac{d}{dx}\left(cf(x)\right)=cf'(x)$, c is any constant. $\left(f(x)\pm g(x)\right)'=f'(x)\pm g'(x)$

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$, n is any number. $\frac{d}{dx}(c)=0$, c is any constant.

$(fg)'=f'g+fg'$ – (Product Rule) $\left(\frac{f}{g}\right)'=\frac{f'g-fg'}{g^2}$ – (Quotient Rule)

$\frac{d}{dx}\left(f\left(g(x)\right)\right)=f'\left(g(x)\right)g'(x)$ (Chain Rule)

$\frac{d}{dx}\left(e^{g(x)}\right)=g'(x)e^{g(x)}$ $\frac{d}{dx}\left(\ln g(x)\right)=\frac{g'(x)}{g(x)}$

Common Derivatives

Polynomials

$\frac{d}{dx}(c)=0$ $\frac{d}{dx}(x)=1$ $\frac{d}{dx}(cx)=c$ $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$ $\frac{d}{dx}\left(cx^n\right)=ncx^{n-1}$

Trig Functions

$\frac{d}{dx}(\sin x)=\cos x$ $\frac{d}{dx}(\cos x)=-\sin x$ $\frac{d}{dx}(\tan x)=\sec^2 x$

$\frac{d}{dx}(\sec x)=\sec x\tan x$ $\frac{d}{dx}(\csc x)=-\csc x\cot x$ $\frac{d}{dx}(\cot x)=-\csc^2 x$

Inverse Trig Functions

$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\left(\cos^{-1}x\right)=-\frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$

$\frac{d}{dx}\left(\sec^{-1}x\right)=\frac{1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}\left(\csc^{-1}x\right)=-\frac{1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}\left(\cot^{-1}x\right)=-\frac{1}{1+x^2}$

Exponential/Logarithm Functions

$\frac{d}{dx}\left(a^x\right)=a^x\ln(a)$ $\frac{d}{dx}\left(e^x\right)=e^x$

$\frac{d}{dx}\left(\ln(x)\right)=\frac{1}{x},\ x>0$ $\frac{d}{dx}\left(\ln|x|\right)=\frac{1}{x},\ x\neq 0$ $\frac{d}{dx}\left(\log_a(x)\right)=\frac{1}{x\ln a},\ x>0$

Hyperbolic Trig Functions

$\frac{d}{dx}(\sinh x)=\cosh x$ $\frac{d}{dx}(\cosh x)=\sinh x$ $\frac{d}{dx}(\tanh x)=\operatorname{sech}^2 x$

$\frac{d}{dx}(\operatorname{sech} x)=-\operatorname{sech} x\tanh x$ $\frac{d}{dx}(\operatorname{csch} x)=-\operatorname{csch} x\coth x$ $\frac{d}{dx}(\coth x)=-\operatorname{csch}^2 x$

Common Derivatives and Integrals

Integrals

Basic Properties/Formulas/Rules

$\int cf(x)\,dx=c\int f(x)\,dx$, c is a constant. $\int f(x)\pm g(x)\,dx=\int f(x)\,dx\pm\int g(x)\,dx$

$\int_a^b f(x)\,dx=F(x)\Big|_a^b=F(b)-F(a)$ where $F(x)=\int f(x)\,dx$

$\int_a^b cf(x)\,dx=c\int_a^b f(x)\,dx$, c is a constant. $\int_a^b f(x)\pm g(x)\,dx=\int_a^b f(x)\,dx\pm\int_a^b g(x)\,dx$

$\int_a^a f(x)\,dx=0$ $\int_a^b f(x)\,dx=-\int_b^a f(x)\,dx$

$\int_a^b f(x)\,dx=\int_a^c f(x)\,dx+\int_c^b f(x)\,dx$ $\int_a^b c\,dx=c(b-a)$

If $f(x)\geq 0$ on $a\leq x\leq b$ then $\int_a^b f(x)\,dx\geq 0$

If $f(x)\geq g(x)$ on $a\leq x\leq b$ then $\int_a^b f(x)\,dx\geq\int_a^b g(x)\,dx$

Common Integrals

Polynomials

$\int dx=x+c$ $\int k\,dx=kx+c$ $\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$

$\int\frac{1}{x}dx=\ln|x|+c$ $\int x^{-1}\,dx=\ln|x|+c$ $\int x^{-n}dx=\frac{1}{-n+1}x^{-n+1}+c,\ n\neq 1$

$\int\frac{1}{ax+b}dx=\frac{1}{a}\ln|ax+b|+c$ $\int x^{\frac{p}{q}}dx=\frac{1}{\frac{p}{q}+1}x^{\frac{p}{q}+1}+c=\frac{q}{p+q}x^{\frac{p+q}{q}}+c$

Trig Functions

$\int\cos u\,du=\sin u+c$ $\int\sin u\,du=-\cos u+c$ $\int\sec^2 u\,du=\tan u+c$

$\int\sec u\tan u\,du=\sec u+c$ $\int\csc u\cot u\,du=-\csc u+c$ $\int\csc^2 u\,du=-\cot u+c$

$\int\tan u\,du=\ln|\sec u|+c$ $\int\cot u\,du=\ln|\sin u|+c$

$\int\sec u\,du=\ln|\sec u+\tan u|+c$ $\int\sec^3 u\,du=\frac{1}{2}\left(\sec u\tan u+\ln|\sec u+\tan u|\right)+c$

$\int\csc u\,du=\ln|\csc u-\cot u|+c$ $\int\csc^3 u\,du=\frac{1}{2}\left(-\csc u\cot u+\ln|\csc u-\cot u|\right)+c$

Exponential/Logarithm Functions

$\int e^u\,du=e^u+c$ $\int a^u\,du=\frac{a^u}{\ln a}+c$ $\int\ln u\,du=u\ln(u)-u+c$

$\int e^{au}\sin(bu)\,du=\frac{e^{au}}{a^2+b^2}\left(a\sin(bu)-b\cos(bu)\right)+c$ $\int ue^u du=(u-1)e^u+c$

$\int e^{au}\cos(bu)\,du=\frac{e^{au}}{a^2+b^2}\left(a\cos(bu)+b\sin(bu)\right)+c$ $\int\frac{1}{u\ln u}du=\ln|\ln u|+c$

Inverse Trig Functions

$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right)+c$ $\qquad \int \sin^{-1}u\,du = u\sin^{-1}u+\sqrt{1-u^2}+c$

$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+c$ $\qquad \int \tan^{-1}u\,du = u\tan^{-1}u-\frac{1}{2}\ln\left(1+u^2\right)+c$

$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right)+c$ $\qquad \int \cos^{-1}u\,du = u\cos^{-1}u-\sqrt{1-u^2}+c$

Hyperbolic Trig Functions

$\int \sinh u\,du = \cosh u + c$ $\qquad \int \cosh u\,du = \sinh u + c$ $\qquad \int \operatorname{sech}^2 u\,du = \tanh u + c$

$\int \operatorname{sech}\tanh u\,du = -\operatorname{sech} u + c$ $\qquad \int \operatorname{csch}\coth u\,du = -\operatorname{csch} u + c$ $\qquad \int \operatorname{csch}^2 u\,du = -\coth u + c$

$\int \tanh u\,du = \ln(\cosh u)+c$ $\qquad \int \operatorname{sech} u\,du = \tan^{-1}\left|\sinh u\right|+c$

Miscellaneous

$\int \frac{1}{a^2-u^2}\,du = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right|+c$ $\qquad \int \frac{1}{u^2-a^2}\,du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right|+c$

$$\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2}+\frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right|+c$$

$$\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2}-\frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right|+c$$

$$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right)+c$$

$$\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2}+\frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right)+c$$

Standard Integration Techniques

Note that all but the first one of these tend to be taught in a Calculus II class.

u Substitution

Given $\int_a^b f(g(x))g'(x)\,dx$ then the substitution $u=g(x)$ will convert this into the integral, $\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$.

Integration by Parts

The standard formulas for integration by parts are,

$$\int u\,dv = uv - \int v\,du \qquad \int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

Choose u and dv and then compute du by differentiating u and compute v by using the fact that $v=\int dv$.

Common Derivatives and Integrals

Trig Substitutions

If the integral contains the following root use the given substitution and formula.

$\sqrt{a^2-b^2x^2} \Rightarrow x=\frac{a}{b}\sin\theta$ and $\cos^2\theta = 1-\sin^2\theta$

$\sqrt{b^2x^2-a^2} \Rightarrow x=\frac{a}{b}\sec\theta$ and $\tan^2\theta = \sec^2\theta - 1$

$\sqrt{a^2+b^2x^2} \Rightarrow x=\frac{a}{b}\tan\theta$ and $\sec^2\theta = 1+\tan^2\theta$

Partial Fractions

If integrating $\int \frac{P(x)}{Q(x)}\,dx$ where the degree (largest exponent) of $P(x)$ is smaller than the degree of $Q(x)$ then factor the denominator as completely as possible and find the partial fraction decomposition of the rational expression. Integrate the partial fraction decomposition (P.F.D.). For each factor in the denominator we get term(s) in the decomposition according to the following table.

Factor in $Q(x)$	Term in P.F.D	Factor in $Q(x)$	Term in P.F.D
$ax+b$	$\frac{A}{ax+b}$	$(ax+b)^k$	$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+\cdots+\frac{A_k}{(ax+b)^k}$
ax^2+bx+c	$\frac{Ax+B}{ax^2+bx+c}$	$(ax^2+bx+c)^k$	$\frac{A_1x+B_1}{ax^2+bx+c}+\cdots+\frac{A_kx+B_k}{(ax^2+bx+c)^k}$

Products and (some) Quotients of Trig Functions

$\int \sin^n x\cos^m x\,dx$

1. **If n is odd.** Strip one sine out and convert the remaining sines to cosines using $\sin^2 x = 1-\cos^2 x$, then use the substitution $u=\cos x$
2. **If m is odd.** Strip one cosine out and convert the remaining cosines to sines using $\cos^2 x = 1-\sin^2 x$, then use the substitution $u=\sin x$
3. **If n and m are both odd.** Use either 1. or 2.
4. **If n and m are both even.** Use double angle formula for sine and/or half angle formulas to reduce the integral into a form that can be integrated.

$\int \tan^n x\sec^m x\,dx$

1. **If n is odd.** Strip one tangent and one secant out and convert the remaining tangents to secants using $\tan^2 x = \sec^2 x - 1$, then use the substitution $u=\sec x$
2. **If m is even.** Strip two secants out and convert the remaining secants to tangents using $\sec^2 x = 1+\tan^2 x$, then use the substitution $u=\tan x$
3. **If n is odd and m is even.** Use either 1. or 2.
4. **If n is even and m is odd.** Each integral will be dealt with differently.

Convert Example : $\cos^6 x = \left(\cos^2 x\right)^3 = \left(1-\sin^2 x\right)^3$