§4.3—Differentiation Rules

- $\dfrac{dy}{dx}$ is a **noun**. It means "the derivative of y with respect to x."
- $\dfrac{d}{dx}$ is a **verb**. It means "take the derivative with respect to x" of the expression that follows.

The Constant Rule

The derivative of a constant function is 0. That is, if c is a real number, then $\dfrac{d}{dx}[c]=0$. The derivative of $y=c$ is $\dfrac{dy}{dx}=0$.

Example 1:
Find the derivative of the following functions:

a. $y=8$ b. $f(x)=0$ c. $s(t)=3$ d. $y=k\pi^2$, k is a constant

The Power(ful) Rule

If n is a real number and a is some constant in the function $f(x)=ax^n$, then $\dfrac{d}{dx}\left[ax^n\right]=anx^{n-1}$. Equivalently, $f'(x)=anx^{n-1}$.

Example 2:
Find the derivative of the following functions:

a. $f(x)=2x^3$ b. $g(x)=\dfrac{\sqrt[3]{x}}{3}$ c. $y=\dfrac{5}{3x^{\pi}}$ d. $y=\dfrac{6}{\sqrt[5]{x^3}}$

Example 3:

Find the slope of the graph of $f(x)=\frac{x^4}{2}$ when

a. $x=-1$ b. $x=0$ c. $x=1$

Example 4:

Find an equation of the a) tangent line and the b) normal line to the graph of $f(x)=3x^2$ when $x=-2$. Then c) find the other point where the normal line intersects $f(x)$.

Example 5:

Find the coordinates where the function $f(x)=-5x^2$ has tangents lines with the following slopes.

a. $m=-3$ b. $m=0$ c. $m=\frac{2}{3}$

Rewriting is very important when using the Power Rule. This is worth repeating. Rewriting is very important when using the Power Rule. An expression MUST be in the form ax^n and n MUST be a real number.

Example 6:
Rewrite, evaluate (differentiate), and then simplify the following:

a. $\frac{d}{dx}\left[\frac{5}{2x^{\sqrt{2}}}\right]$ b. $\frac{d}{dx}\left[\frac{4}{(2x)^3}\right]$ c. $\frac{d}{dt}\left[\frac{7t}{3\sqrt{t}}\right]$ d. $\frac{d}{dm}\left[\frac{6}{(3m)^{-2}}\right]$

The Sum and Difference & Konstant Rules

The derivative of the sum of two functions *f* and *g* is the sum of the derivatives of *f* and *g*. Similarly, the derivative of the difference of two functions *f* and *g* is the difference of the derivatives of *f* and *g*.

$\frac{d}{dx}\left[f(x)+g(x)\right]=f'(x)+g'(x)$ Sum Rule

$\frac{d}{dx}\left[f(x)-g(x)\right]=f'(x)-g'(x)$ Difference Rule

$\frac{d}{dx}\left[kf(x)\right]=k\frac{d}{dx}\left[f(x)\right]$ Konstant Rule

Example 7:
Find the derivative of the following functions:

a. $f(x)=x^3-4x+5$ b. $g(x)=-\frac{x^4}{2}+2x^3-5x$ c. $y=\frac{2x^3-3x^2+7x+5}{2\sqrt{x}}$ d. $y=x(3x+2)^2$

Example 8:
Find the coordinates and equations of any horizontal tangents to the curve $y = x^4 - 2x^2 + 2$.

The Sine and Cosine Rules

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

Example 9:
Find the derivatives of the following functions:

a. $y = 2\sin x$ b. $f(x) = \frac{\sin x}{2}$ c. $f(x) = x + \cos x$

Example 10:

Find the slope of the graph of $f(x) = \sin x$ at a) the origin, and b) at $x = \frac{4\pi}{3}$

The derivative of a function at a point gives

- The slope of the tangent line at that point
- The instantaneous rate of change at that point

Applications of the Derivative: Motion

Let $s(t)$ be a position function as a function of time t and $v(t)$ be a velocity function as a function of time. Then,

The average rate of change of position on a time interval from $t = a$ to $t = b$ is called **average velocity.**

- Average velocity $= \dfrac{\text{change in distance}}{\text{change in time}} = \dfrac{\Delta s}{\Delta t} = \dfrac{s(b) - s(a)}{b - a}$ = slope of secant line

The instantaneous rate of change of position at time t is called **instantaneous velocity**.

- Instantaneous velocity $= v(t) = s'(t)$ =slope of tangent line

Example 11:
A calculus textbook is dropped from a height of 100 feet. Its height s in feet at time t seconds is given by the position function $s(t) = -16t^2 + 100$. Find the following:

1. The average velocity over each of the following intervals

   a. $[1,2]$ b. $[1,1.5]$ c. $[1,1.1]$ d. $[1,1.01]$

2. The instantaneous velocity at $t = 1$ second.

3. How fast is the book traveling when it hits the ground?

Example 12:

If $f(x)=\begin{cases} 3-x, & x<1 \\ ax^2+bx, & x\ge 1 \end{cases}$, find the values of a and b such that $f(x)$ differentiable everywhere.