Chapter 22

# Reflection and Refraction of Light

## Quick Quizzes

**1.** (a). In part (a), you can see clear reflections of the headlights and the lights on the top of the truck. The reflection is specular. In part (b), although bright areas appear on the roadway in front of the headlights, the reflection is not as clear, and no separate reflection of the lights from the top of the truck is visible. The reflection in part (b) is mostly diffuse.

**2.** Beams 2 and 4 are reflected; beams 3 and 5 are refracted.

**3.** (b). When light goes from one material into one having a higher index of refraction, it refracts toward the normal line of the boundary between the two materials. If, as the light travels through the new material, the index of refraction continues to increase, the light ray will refract more and more toward the normal line.

**4.** (c). Both the wave speed and the wavelength decrease as the index of refraction increases. The frequency is unchanged.

## Answers to Even Numbered Conceptual Questions

**2.** Ceilings are generally painted a light color so they will reflect more light, making the room brighter. Textured materials are often used on the ceiling to diffuse the reflected light and reduce glare (specular reflections).

**4.** At the altitude of the plane the surface of Earth does not block off the lower half of the rainbow. Thus, the full circle can be seen. You can see such a rainbow by climbing on a stepladder above a garden sprinkler in the middle of a sunny day.

**6.** The spectrum of the light sent back to you from a drop at the top of the rainbow arrives such that the red light (deviated by an angle of 42°) strikes the eye while the violet light (deviated by 40°) passes over your head. Thus, the top of the rainbow looks red. At the bottom of the bow, violet light arrives at your eye and red light is deviated toward the ground. Thus, the bottom part of the bow appears violet.

**8.** A mirage occurs when light changes direction as it moves between batches of air having different indices of refraction. The different indices of refraction occur because the air has different densities at different temperatures. Two images are seen; One from a direct path from the object to you, and the second arriving by rays originally heading toward Earth but refracted to your eye. On a hot day, the Sun makes the surface of blacktop hot, so the air is hot directly above it, becoming cooler as one moves higher into the sky. The "water" we see far in front of us is an image of the blue sky. Adding to the effect is the fact that the image shimmers as the air changes in temperature, giving the appearance of moving water.

**10.** The upright image of the hill is formed by light that has followed a direct path from the hill to the eye of the observer. The second image is a result of refraction in the atmosphere. Some light is reflected from the hill toward the water. As this light passes through warmer layers of air directly above the water, it is refracted back up toward the eye of the observer, resulting in the observation of an inverted image of the hill directly below the upright image.

**12.** The color traveling slowest is bent the most. Thus, X travels more slowly in the glass prism.

**14.** Total internal reflection occurs only when light attempts to move from a medium of high index of refraction to a medium of lower index of refraction. Thus, light moving from air ($n = 1$) to water ($n = 1.33$) cannot undergo total internal reflection.

**16.** Objects beneath the surface of water appear to be raised toward the surface by refraction. Thus, the bottom of the oar appears to be closer to the surface than it really is, and the oar looks to be bent.

**18.** The cross section can be visualized by considering just the two rays of light on the edges of the beam. If the beam of light enters a new medium with a higher index of refraction, the rays bend toward the normal, and the cross section of the refracted beam will be larger than that of the incident beam as suggested by Fig. CQ22.18a. If the new index of refraction is lower, the rays bend away from the normal, and the cross section of the beam is reduced, as shown in Fig. CQ22.18b.

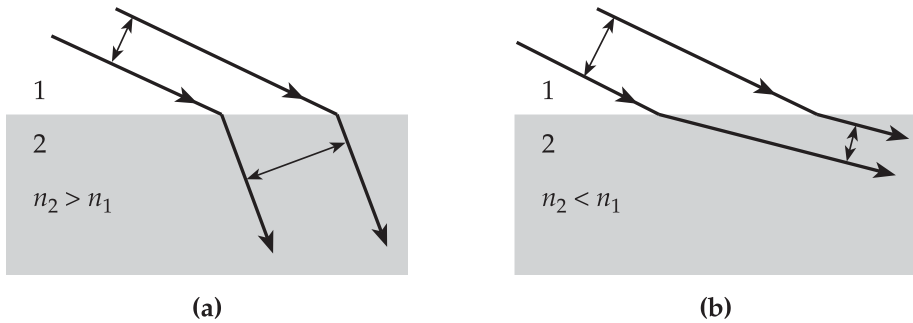



**Figure CQ22.18**

## Answers to Even Numbered Problems

**2.** $2.97 \times 10^8$ m/s

**4.** **(a)** 536 rev/s **(b)** $1.07 \times 10^3$ rev/s

**6.** **(a)** 1.94 m **(b)** 50.0° above horizontal (parallel to incident ray)

**8.** $2.09 \times 10^{-11}$ s

**10.** **(a)** The longer the wavelength, the less it is deviated (or refracted) from the original path.
**(b)** Using data from Figure 22.14, the angles of refraction are:
(400 nm) $\theta_2 = 16.0°$, (500 nm) $\theta_2 = 16.1°$, (650 nm) $\theta_2 = 16.3°$

**12.** **(a)** 327 nm **(b)** 287 nm

**14.** 67.4°

**16.** 53.4°

**18.** First surface: $\theta_i = 30.0°$, $\theta_r = 19.5°$
Second surface: $\theta_i = 19.5°$, $\theta_r = 30.0°$

**20.** $1.06 \times 10^{-10}$ s

**22.** 107 m

**24.** 6.30 cm

**26.** 23.1°

**28.** 2.5 m

**30.** 0.40°

**32.** 4.6°

**34.** **(a)** 24.4° **(b)** 37.0°

**36.** 48.5°

**38.** 67.2°

**40.** 4.54 m

**44.** **(a)** $\theta_1' = 30.0°,\ \theta_2 = 18.8°$ **(b)** $\theta_1' = 30.0°,\ \theta_2 = 50.8°$
**(c)** See solution. **(d)** See solution.

**46.** **(a)** Any angle of incidence $\leq 90°$ **(b)** 30.0°
**(c)** not possible since $n_{polystyrene} < n_{carbon\ disulfide}$

**48.** **(a)** $0.172\ \text{mm/s}$ **(b)** $0.345\ \text{mm/s}$
**(c)** and **(d)** Northward at 50.0° below horizontal.

**50.** 77.5°

**52.** **(a)** $R \geq nd/(n-1)$ **(b)** yes; yes; yes **(c)** 350 $\mu$m

**54.** 7.91°

**56.** 82

**58.** 62.2% of a circle

**60.** The graph is a straight line passing through the origin. From the slope of the graph, $n_{water} = 1.33$.

**62.** **(a)** $n = \left[1 + \left(4t/d^2\right)\right]^{1/2}$ **(b)** 2.10 cm **(c)** violet

## Problem Solutions

**22.1** The total distance the light travels is

$$\Delta d = 2\left( D_{\substack{center\ to\\ center}} - R_{Earth} - R_{Moon} \right)$$

$$= 2\left(3.84\times10^{8} - 6.38\times10^{6} - 1.76\times10^{6}\right)\ \text{m} = 7.52\times10^{8}\ \text{m}$$

Therefore, $v = \dfrac{\Delta d}{\Delta t} = \dfrac{7.52\times10^{8}\ \text{m}}{2.51\ \text{s}} = \boxed{3.00\times10^{8}\ \text{m/s}}$

**22.2** If the wheel has 360 teeth, it turns through an angle of $1/720$ rev in the time it takes the light to make its round trip. From the definition of angular velocity, we see that the time is

$$t = \frac{\theta}{\omega} = \frac{(1/720)\ \text{rev}}{27.5\ \text{rev/s}} = 5.05\times10^{-5}\ \text{s}$$

Hence, the speed of light is $c = \dfrac{2d}{t} = \dfrac{2(7\,500\ \text{m})}{5.05\times10^{-5}\ \text{s}} = \boxed{2.97\times10^{8}\ \text{m/s}}$

**22.3** The experiment is most convincing if the wheel turns fast enough to pass outgoing light through one notch and returning light through the next. Then,

$$\Delta\theta = \left(\frac{1}{720}\ \text{rev}\right)\left(2\pi\ \text{rad/rev}\right) = \frac{2\pi}{720}\ \text{rad}$$

and $\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{\Delta\theta}{2d/c} = \dfrac{c(\Delta\theta)}{2d} = \dfrac{\left(2.998\times10^{8}\ \text{m/s}\right)}{2\left(11.45\times10^{3}\ \text{m}\right)}\left(\dfrac{2\pi}{720}\ \text{rad}\right) = \boxed{114\ \text{rad/s}}$

**22.4** (a) The time for the light to travel to the stationary mirror and back is

$$\Delta t = \frac{2d}{c} = \frac{2\left(35.0\times10^{3}\ \text{m}\right)}{3.00\times10^{8}\ \text{m/s}} = 2.33\times10^{-4}\ \text{s}$$

At the lowest angular speed, the octagonal mirror will have rotated $1/8$ rev in this time, so

$$\omega_{\text{min}} = \frac{\Delta\theta}{\Delta t} = \frac{1/8\ \text{rev}}{2.33\times10^{-4}\ \text{s}} = \boxed{536\ \text{rev/s}}$$

(b) At the next higher angular speed, the mirror will have rotated $2/8$ rev in the elapsed time, or

$$\omega_2 = 2\omega_{\text{min}} = 2\left(536\ \text{rev/s}\right) = \boxed{1.07\times10^{3}\ \text{rev/s}}$$

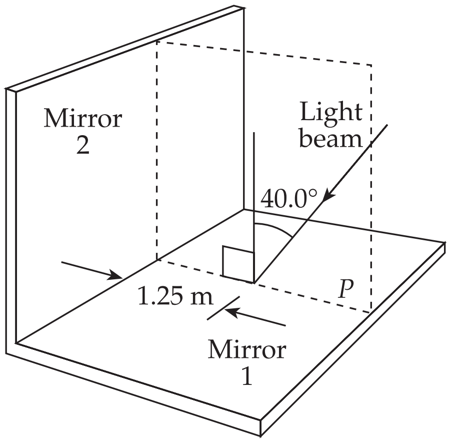
**22.5** (a) For the light beam to make it through both slots, the time for the light to travel distance $d$ must equal the time for the disks to rotate through angle $\theta$. Therefore, if $c$ is the speed of light,

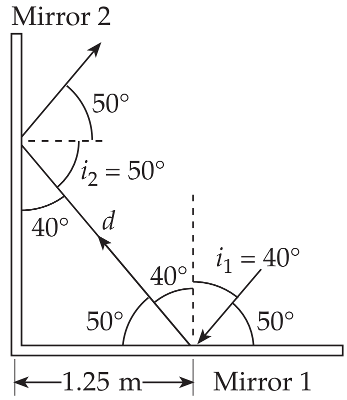
$$t = \frac{d}{c} = \frac{\theta}{\omega}\text{, or}\quad \boxed{c = \frac{\omega d}{\theta}}$$

(b) If $d = 2.500\ \text{m}$, $\theta = \left(\dfrac{1}{60}\ \text{degree}\right)\left(\dfrac{1\ \text{rev}}{360\ \text{degree}}\right) = \dfrac{1\ \text{rev}}{(60)(360)}$, and $\omega = 5\,555\ \text{rev/s}$

$$c = (2.500\ \text{m})\left(5\,555\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{(60)(360)}{1\ \text{rev}}\right) = \boxed{3.000\times10^{8}\ \text{m/s}}$$

**22.6**

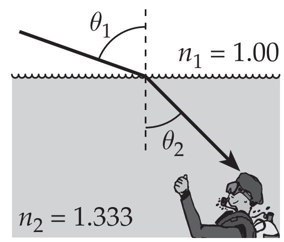



(a) From geometry, $1.25\text{ m} = d\sin 40.0°$, so $d = \boxed{1.94\text{ m}}$

(b) $\boxed{50.0°\text{ above horizontal}}$, or parallel to the incident ray

**22.7** $n_1 \sin\theta_1 = n_2 \sin\theta_2$




$\sin\theta_1 = 1.333 \sin 45.0°$

$\sin\theta_1 = (1.333)(0.707) = 0.943$

$\theta_1 = 70.5° \rightarrow \boxed{19.5°\text{ above the horizontal}}$

**22.8** The speed of light in water is $v_{water} = \dfrac{c}{n_{water}}$, and in Lucite® $v_{Lucite} = \dfrac{c}{n_{Lucite}}$. Thus, the total time required to transverse the double layer is

$$\Delta t_1 = \frac{d_{water}}{v_{water}} + \frac{d_{Lucite}}{v_{Lucite}} = \frac{d_{water}n_{water} + d_{Lucite}n_{Lucite}}{c}$$

The time to travel the same distance in air is $\Delta t_2 = \dfrac{d_{water} + d_{Lucite}}{c}$, so the additional time required for the double layer is

$$\Delta t = \Delta t_1 - \Delta t_2 = \frac{d_{water}\left(n_{water} - 1\right) + d_{Lucite}\left(n_{Lucite} - 1\right)}{c}$$

$$= \frac{\left(1.00\times10^{-2}\text{ m}\right)\left(1.333-1\right) + \left(0.500\times10^{-2}\text{ m}\right)\left(1.59-1\right)}{3.00\times10^{8}\text{ m/s}} = \boxed{2.09\times10^{-11}\text{ s}}$$

**22.9** (a) From Snell's law, $n_2 = \dfrac{n_1 \sin\theta_1}{\sin\theta_2} = \dfrac{(1.00)\sin 30.0°}{\sin 19.24°} = \boxed{1.52}$

(b) $\lambda_2 = \dfrac{\lambda_0}{n_2} = \dfrac{632.8\text{ nm}}{1.52} = \boxed{417\text{ nm}}$

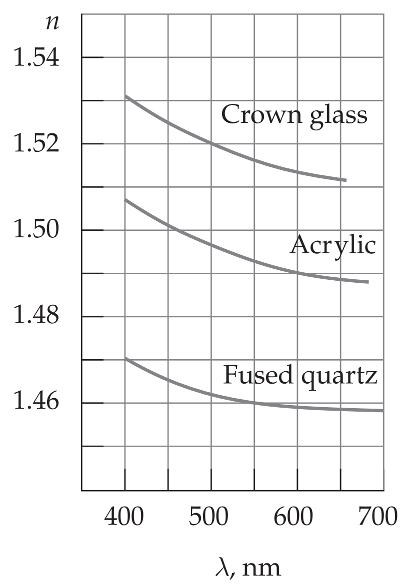
(c) $f = \dfrac{c}{\lambda_0} = \dfrac{3.00\times10^8\text{ m/s}}{632.8\times10^{-9}\text{ m}} = \boxed{4.74\times10^{14}\text{ Hz}}$ in air and in syrup

(d) $v_2 = \dfrac{c}{n_2} = \dfrac{3.00\times10^8\text{ m/s}}{1.52} = \boxed{1.98\times10^8\text{ m/s}}$

**22.10** (a) When light refracts from air $(n_1 = 1.00)$ into the Crown glass, Snell's law gives the angle of refraction as

$$\theta_2 = \sin^{-1}\left(\sin 25.0°/n_{\substack{Crown\\glass}}\right)$$

For first quadrant angles, the sine of the angle increases as the angle increases. Thus, from the above equation, note that $\theta_2$ will increase when the index of refraction of the Crown glass decreases. From Figure 22.14, this means that the longer wavelengths have the largest angles of refraction, and $\boxed{\text{deviate the least}}$ from the original path.




(b) From Figure 22.14, observe that the index of refraction of Crown glass for the given wavelengths is:

$\lambda = 400$ nm: $n_{Crown\ glass} = 1.53$; $\lambda = 500$ nm: $n_{Crown\ glass} = 1.52$;

and $\lambda = 650$ nm: $n_{Crown\ glass} = 1.51$

Thus, Snell's law gives: $\lambda = 400$ nm: $\theta_2 = \sin^{-1}(\sin 25.0°/1.53) = \boxed{16.0°}$

$\lambda = 500$ nm: $\theta_2 = \sin^{-1}(\sin 25.0°/1.52) = \boxed{16.1°}$

$\lambda = 650$ nm: $\theta_2 = \sin^{-1}(\sin 25.0°/1.51) = \boxed{16.3°}$

**22.11** (a) $\lambda_{water} = \dfrac{\lambda_0}{n_{water}}$, so $\lambda_0 = n_{water}\lambda_{water} = (1.333)(438 \text{ nm}) = \boxed{584 \text{ nm}}$

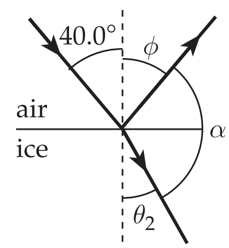
(b) $\lambda_0 = n_{water}\lambda_{water} = n_{benzene}\lambda_{benzene}$

and $$\frac{n_{benzene}}{n_{water}} = \frac{\lambda_{water}}{\lambda_{benzene}} = \frac{438 \text{ nm}}{390 \text{ nm}} = \boxed{1.12}$$

**22.12** (a) $\lambda_{water} = \dfrac{\lambda_0}{n_{water}} = \dfrac{436 \text{ nm}}{1.333} = \boxed{327 \text{ nm}}$

(b) $\lambda_{glass} = \dfrac{\lambda_0}{n_{\substack{crown\\glass}}} = \dfrac{436 \text{ nm}}{1.52} = \boxed{287 \text{ nm}}$

**22.13** From Snell's law,




$$\theta_2 = \sin^{-1}\left[\frac{n_1 \sin\theta_1}{n_2}\right] = \sin^{-1}\left[\frac{(1.00)\sin 40.0°}{1.309}\right] = 29.4°$$

and from the law of reflection, $\phi = \theta_1 = 40.0°$

Hence, the angle between the reflected and refracted rays is

$$\alpha = 180° - \theta_2 - \phi = 180° - 29.4° - 40.0° = \boxed{111°}$$

**22.14** Snell's law may be written as: $\dfrac{\sin\theta_2}{\sin\theta_1} = \dfrac{n_1}{n_2} = \dfrac{c/v_1}{c/v_2} = \dfrac{v_2}{v_1}$

Thus, $\theta_2 = \sin^{-1}\left[\left(\dfrac{v_2}{v_1}\right)\sin\theta_1\right] = \sin^{-1}\left[\left(\dfrac{1\,510 \text{ m/s}}{340 \text{ m/s}}\right)\sin 12.0°\right] = \boxed{67.4°}$

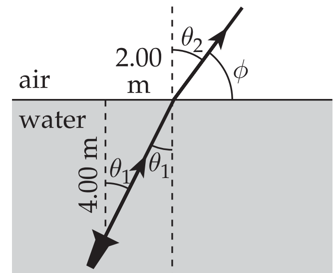
**22.15** The index of refraction of zircon is $n = 1.923$

(a) $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.923} = \boxed{1.56\times10^8 \text{ m/s}}$

(b) The wavelength in the zircon is $\lambda_n = \dfrac{\lambda_0}{n} = \dfrac{632.8 \text{ nm}}{1.923} = \boxed{329.1 \text{ nm}}$

(c) The frequency is $f = \dfrac{v}{\lambda_n} = \dfrac{c}{\lambda_0} = \dfrac{3.00\times10^8 \text{ m/s}}{632.8\times10^{-9} \text{ m}} = \boxed{4.74\times10^{14} \text{ m/s}}$

**22.16** The angle of incidence is




$$\theta_1 = \tan^{-1}\left[\frac{2.00 \text{ m}}{4.00 \text{ m}}\right] = 26.6°$$

Therefore, Snell's law gives

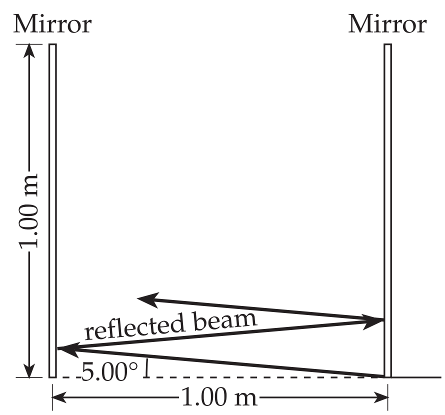
$$\theta_2 = \sin^{-1}\left[\frac{n_1 \sin\theta_1}{n_2}\right]$$

$$= \sin^{-1}\left[\frac{(1.333)\sin 26.6°}{1.00}\right] = 36.6°$$

and the angle the refracted ray makes with the surface is

$$\phi = 90.0° - \theta_2 = 90.0° - 36.6° = \boxed{53.4°}$$

**22.17** The incident light reaches the left-hand mirror at distance




$$(1.00 \text{ m})\tan 5.00° = 0.087\,5 \text{ m}$$

above its bottom edge. The reflected light first reaches the right-hand mirror at height

$$2(0.087\,5 \text{ m}) = 0.175 \text{ m}$$

It bounces between the mirrors with this distance between points of contact with either.

Since $\dfrac{1.00 \text{ m}}{0.175 \text{ m}} = 5.72$, the light reflects

$\boxed{\text{five times from the right-hand mirror and six times from the left}}$

**22.18** At the first surface, the angle of incidence is $\theta_1 = \boxed{30.0°}$, and Snell's law gives

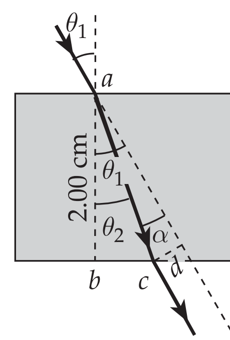
$$\theta_2 = \sin^{-1}\left[\frac{n_{air}\sin\theta_1}{n_{glass}}\right] = \sin^{-1}\left[\frac{(1.00)\sin 30.0°}{1.50}\right] = \boxed{19.5°}$$

Since the second surface is parallel to the first, the angle of incidence at the second surface is $\theta_1 = 19.5°$ and the angle of refraction is

$$\theta_2 = \sin^{-1}\left[\frac{n_{glass}\sin\theta_{glass}}{n_{air}}\right] = \sin^{-1}\left[\frac{(1.50)\sin 19.5°}{1.00}\right] = \boxed{30.0°}$$

Thus, the light emerges traveling parallel to the incident beam.

**22.19** The angle of refraction at the first surface is $\theta_2 = 19.5°$ (see Problem 18). Let $h$ represent the distance from point $a$ to $c$ (that is, the hypotenuse of triangle $abc$). Then,




$$h = \frac{2.00\text{ cm}}{\cos\theta_2} = \frac{2.00\text{ cm}}{\cos 19.5°} = 2.12\text{ cm}$$

Also, $\alpha = \theta_1 - \theta_2 = 30.0° - 19.5° = 10.5°$, so

$$d = h\sin\alpha = (2.12\text{ cm})\sin 10.5° = \boxed{0.388\text{ cm}}$$

**22.20** The distance $h$ the light travels in the glass is

$$h = \frac{2.00\text{ cm}}{\cos 19.5°} = 2.12\text{ cm}$$

The speed of light in the glass is

$$v = \frac{c}{n_{glass}} = \frac{3.00\times10^{8}\text{ m/s}}{1.50} = 2.00\times10^{8}\text{ m/s}$$

Therefore, $t = \dfrac{h}{v} = \dfrac{2.12\times10^{-2}\text{ m}}{2.00\times10^{8}\text{ m/s}} = \boxed{1.06\times10^{-10}\text{ s}}$

**22.21** From Snell's law, the angle of incidence at the air-oil interface is

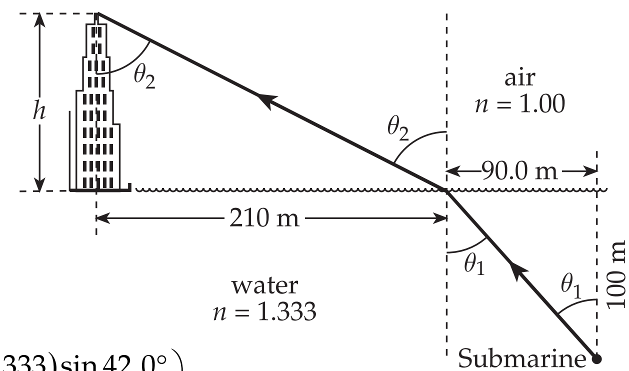
$$\theta = \sin^{-1}\left[\frac{n_{oil}\sin\theta_{oil}}{n_{air}}\right]$$

$$= \sin^{-1}\left[\frac{(1.48)\sin 20.0^\circ}{1.00}\right] = \boxed{30.4^\circ}$$

and the angle of refraction as the light enters the water is

$$\theta' = \sin^{-1}\left[\frac{n_{oil}\sin\theta_{oil}}{n_{water}}\right] = \sin^{-1}\left[\frac{(1.48)\sin 20.0^\circ}{1.333}\right] = \boxed{22.3^\circ}$$

**22.22** The angle of incidence at the water surface is




$$\theta_1 = \tan^{-1}\left(\frac{90.0 \text{ m}}{100 \text{ m}}\right) = 42.0^\circ$$

Then, Snell's law gives the angle of refraction as

$$\theta_2 = \sin^{-1}\left(\frac{n_{water}\sin\theta_1}{n_{air}}\right) = \sin^{-1}\left(\frac{(1.333)\sin 42.0^\circ}{1.00}\right) = 63.1^\circ$$

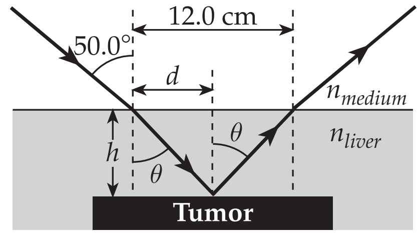
so the height of the building is $h = \dfrac{210 \text{ m}}{\tan\theta_2} = \dfrac{210 \text{ m}}{\tan 63.1^\circ} = \boxed{107 \text{ m}}$

**22.23** $\Delta t = (\text{time to travel 6.20 m in ice}) - (\text{time to travel 6.20 m in air})$

$$\Delta t = \frac{6.20 \text{ m}}{v_{ice}} - \frac{6.20 \text{ m}}{c}$$

Since the speed of light in a medium of refractive index $n$ is $v = \dfrac{c}{n}$

$$\Delta t = (6.20 \text{ m})\left(\frac{1.309}{c} - \frac{1}{c}\right) = \frac{(6.20 \text{ m})(0.309)}{3.00\times10^{8} \text{ m/s}} = 6.39\times10^{-9} \text{ s} = \boxed{6.39 \text{ ns}}$$



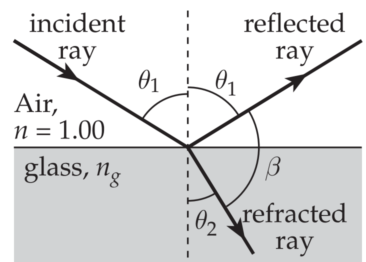

**22.24** From Snell's law, $\sin\theta = \left(\frac{n_{medium}}{n_{liver}}\right)\sin 50.0°$

But, $\frac{n_{medium}}{n_{liver}} = \frac{c/v_{medium}}{c/v_{liver}} = \frac{v_{liver}}{v_{medium}} = 0.900$

so, $\theta = \sin^{-1}\left[(0.900)\sin 50.0°\right] = 43.6°$

From the law of reflection,

$$d = \frac{12.0\text{ cm}}{2} = 6.00\text{ cm}\text{ , and}\qquad h = \frac{d}{\tan\theta} = \frac{6.00\text{ cm}}{\tan(43.6°)} = \boxed{6.30\text{ cm}}$$




**22.25** As shown at the right, $\theta_1 + \beta + \theta_2 = 180°$

When $\beta = 90°$, this gives $\theta_2 = 90° - \theta_1$

Then, from Snell's law

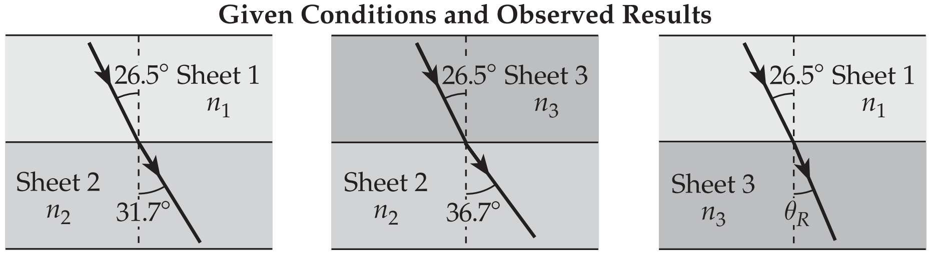
$$\sin\theta_1 = \frac{n_g \sin\theta_2}{n_{air}}$$

$$= n_g \sin(90° - \theta_1) = n_g \cos\theta_1$$

Thus, when $\beta = 90°$, $\frac{\sin\theta_1}{\cos\theta_1} = \tan\theta_1 = n_g$ or $\boxed{\theta_1 = \tan^{-1}(n_g)}$

**22.26**

**Given Conditions and Observed Results**




**Case 1** **Case 2** **Case 3**

For the first placement, Snell's law gives, $n_2 = \frac{n_1 \sin 26.5°}{\sin 31.7°}$

In the second placement, application of Snell's law yields

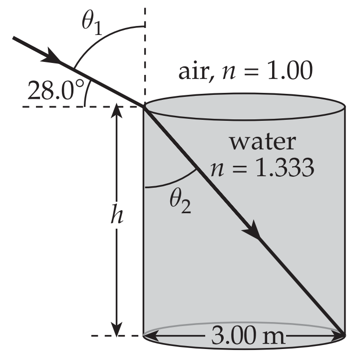
$$n_3 \sin 26.5° = n_2 \sin 36.7° = \left(\frac{n_1 \sin 26.5°}{\sin 31.7°}\right)\sin 36.7°\text{ , or } n_3 = \frac{n_1 \sin 36.7°}{\sin 31.7°}$$

Finally, using Snell's law in the third placement gives

$$\sin\theta_R = \frac{n_1 \sin 26.5°}{n_3} = (n_1 \sin 26.5°)\left(\frac{\sin 31.7°}{n_1 \sin 36.7°}\right) = 0.392$$

and $\theta_R = \boxed{23.1°}$

**22.27** When the Sun is 28.0° above the horizon, the angle of incidence for sunlight at the air-water boundary is




$$\theta_1 = 90.0° - 28.0° = 62.0°$$

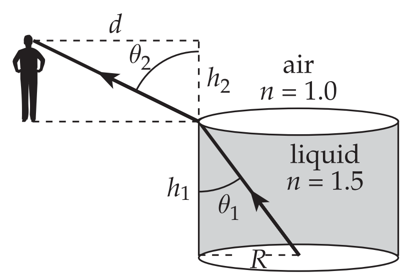
Thus, the angle of refraction is

$$\theta_2 = \sin^{-1}\left[\frac{n_{air} \sin\theta_1}{n_{water}}\right]$$

$$= \sin^{-1}\left[\frac{(1.00)\sin 62.0°}{1.333}\right] = 41.5°$$

The depth of the tank is then $h = \dfrac{3.00\text{ m}}{\tan\theta_2} = \dfrac{3.00\text{ m}}{\tan(41.5°)} = \boxed{3.39\text{ m}}$

**22.28** From the drawing, observe that




$$\theta_1 = \tan^{-1}\left(\frac{R}{h_1}\right) = \tan^{-1}\left(\frac{1.5\text{ m}}{2.0\text{ m}}\right) = 37°$$

Applying Snell's law to the ray shown gives

$$\theta_2 = \sin^{-1}\left(\frac{n_{liquid} \sin\theta_1}{n_{air}}\right) = \sin^{-1}\left(\frac{1.5 \sin 37°}{1.0}\right) = 64°$$

Thus, the distance of the girl from the cistern is

$$d = h_2 \tan\theta_2 = (1.2\text{ m})\tan 64° = \boxed{2.5\text{ m}}$$

**22.29** Using Snell's law gives

$$\theta_{red} = \sin^{-1}\left(\frac{n_{air}\sin\theta_i}{n_{red}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 83.00°}{1.331}\right) = \boxed{48.22°}$$

and $$\theta_{blue} = \sin^{-1}\left(\frac{n_{air}\sin\theta_i}{n_{blue}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 83.00°}{1.340}\right) = \boxed{47.79°}$$

**22.30** The angles of refraction for the two wavelengths are

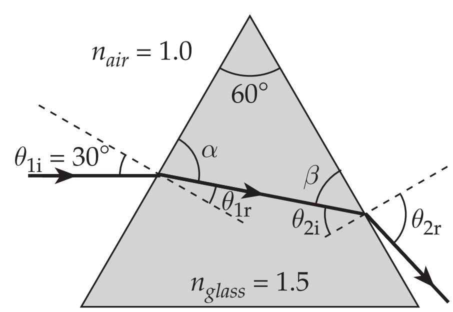
$$\theta_{red} = \sin^{-1}\left(\frac{n_{air}\sin\theta_i}{n_{red}}\right) = \sin^{-1}\left(\frac{(1.00\,0)\sin 30.00°}{1.615}\right) = 18.04°$$

and $$\theta_{blue} = \sin^{-1}\left(\frac{n_{air}\sin\theta_i}{n_{blue}}\right) = \sin^{-1}\left(\frac{(1.00\,0)\sin 30.00°}{1.650}\right) = 17.64°$$

Thus, the angle between the two refracted rays is

$$\Delta\theta = \theta_{red} - \theta_{blue} = 18.04° - 17.64° = \boxed{0.40°}$$

**22.31** (a) The angle of incidence at the first surface is $\theta_{1i} = 30°$, and the angle of refraction is




$$\theta_{1r} = \sin^{-1}\left(\frac{n_{air}\sin\theta_{1i}}{n_{glass}}\right)$$

$$= \sin^{-1}\left(\frac{1.0\sin 30°}{1.5}\right) = \boxed{19°}$$

Also, $\alpha = 90° - \theta_{1r} = 71°$ and $\beta = 180° - 60° - \alpha = 49°$

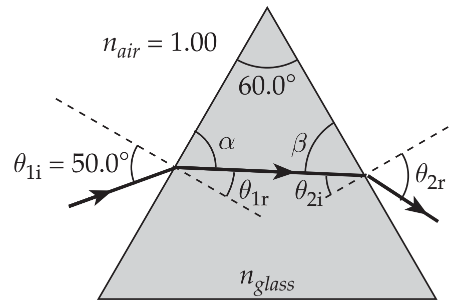
Therefore, the angle of incidence at the second surface is $\theta_{2i} = 90° - \beta = \boxed{41°}$. The angle of refraction at this surface is

$$\theta_{2r} = \sin^{-1}\left(\frac{n_{glass}\sin\theta_{2i}}{n_{air}}\right) = \sin^{-1}\left(\frac{1.5\sin 41°}{1.0}\right) = \boxed{77°}$$

(b) The angle of reflection at each surface equals the angle of incidence at that surface. Thus,

$$(\theta_1)_{reflection} = \theta_{1i} = \boxed{30°}, \text{ and } (\theta_2)_{reflection} = \theta_{2i} = \boxed{41°}$$

**22.32** For the violet light, $n_{glass} = 1.66$, and




$$\theta_{1r} = \sin^{-1}\left(\frac{n_{air}\sin\theta_{1i}}{n_{glass}}\right)$$

$$= \sin^{-1}\left(\frac{1.00\sin 50.0°}{1.66}\right) = 27.5°$$

$$\alpha = 90° - \theta_{1r} = 62.5°,\ \beta = 180.0° - 60.0° - \alpha = 57.5°,$$

and $\theta_{2i} = 90.0° - \beta = 32.5°$. The final angle of refraction of the violet light is

$$\theta_{2r} = \sin^{-1}\left(\frac{n_{glass}\sin\theta_{2i}}{n_{air}}\right) = \sin^{-1}\left(\frac{1.66\sin 32.5°}{1.00}\right) = 63.2°$$

Following the same steps for the red light $(n_{glass} = 1.62)$ gives

$$\theta_{1r} = 28.2°,\ \alpha = 61.8°,\ \beta = 58.2°,\ \theta_{2i} = 31.8°, \text{ and } \theta_{2r} = 58.6°$$

Thus, the angular dispersion of the emerging light is

$$Dispersion = \theta_{2r}\big|_{violet} - \theta_{2r}\big|_{red} = 63.2° - 58.6° = \boxed{4.6°}$$

**22.33** When surrounded by air $(n_2 = 1.00)$, the critical angle of a material is

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1}{n_{material}}\right)$$

(a) For Zircon, $n = 1.923$, and $\theta_c = \sin^{-1}\left(\frac{1}{1.923}\right) = \boxed{31.3°}$

(b) For fluorite, $n = 1.434$, and $\theta_c = \sin^{-1}\left(\frac{1}{1.434}\right) = \boxed{44.2°}$

(c) For ice, $n = 1.309$, and $\theta_c = \sin^{-1}\left(\frac{1}{1.309}\right) = \boxed{49.8°}$

**22.34** When surrounded by air $(n_2 = 1.00)$, the critical angle of a material is

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1}{n_{material}}\right)$$

(a) For diamond, $\theta_c = \sin^{-1}\left(\frac{1}{2.419}\right) = \boxed{24.4°}$

(b) For flint glass, $\theta_c = \sin^{-1}\left(\frac{1}{1.66}\right) = \boxed{37.0°}$

**22.35** When surrounded by water $(n_2 = 1.333)$, the critical angle of a material is

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.333}{n_{material}}\right)$$

(a) For diamond, $\theta_c = \sin^{-1}\left(\frac{1.333}{2.419}\right) = \boxed{33.4°}$

(b) For flint glass, $\theta_c = \sin^{-1}\left(\frac{1.333}{1.66}\right) = \boxed{53.4°}$

**22.36** Using Snell's law, the index of refraction of the liquid is found to be

$$n_{liquid} = \frac{n_{air}\sin\theta_i}{\sin\theta_r} = \frac{(1.00)\sin 30.0°}{\sin 22.0°} = 1.33$$

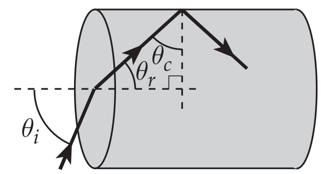
Thus, $\theta_c = \sin^{-1}\left(\frac{n_{air}}{n_{liquid}}\right) = \sin^{-1}\left(\frac{1.00}{1.33}\right) = \boxed{48.5°}$

**22.37** When light attempts to cross a boundary from one medium of refractive index $n_1$ into a new medium of refractive index $n_2 < n_1$, total internal reflection will occur if the angle of incidence exceeds the critical angle given by $\theta_c = \sin^{-1}(n_2/n_1)$.

(a) If $n_1 = 1.53$ and $n_2 = n_{air} = 1.00$, then $\theta_c = \sin^{-1}\left(\frac{1.00}{1.53}\right) = \boxed{40.8°}$

(b) If $n_1 = 1.53$ and $n_2 = n_{water} = 1.333$, then $\theta_c = \sin^{-1}\left(\frac{1.333}{1.53}\right) = \boxed{60.6°}$

**22.38** The critical angle for this material in air is




$$\theta_c = \sin^{-1}\left(\frac{n_{air}}{n_{pipe}}\right) = \sin^{-1}\left(\frac{1.00}{1.36}\right) = 47.3°$$

Thus, $\theta_r = 90.0° - \theta_c = 42.7°$ and from Snell's law,

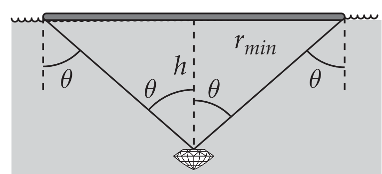
$$\theta_i = \sin^{-1}\left(\frac{n_{pipe}\sin\theta_r}{n_{air}}\right) = \sin^{-1}\left(\frac{(1.36)\sin 42.7°}{1.00}\right) = \boxed{67.2°}$$

**22.39** The light must be totally reflecting from the surface of a hot air layer just above the road surface. The angle of reflection, and hence the critical angle, is $\theta_c = 90.0° - 1.20° = 88.8°$.

Thus, from $\sin\theta_c = \dfrac{n_2}{n_1}$, we find

$$n_2 = n_1 \sin\theta_c = (1.000\ 3)\sin 88.8° = \boxed{1.000\ 08}$$

**22.40** The circular raft must cover the area of the surface through which light from the diamond could emerge. Thus, it must form the base of a cone (with apex at the diamond) whose half angle is $\theta$, where $\theta$ is greater than or equal to the critical angle.




The critical angle at the water-air boundary is

$$\theta_c = \sin^{-1}\left(\frac{n_{air}}{n_{water}}\right) = \sin^{-1}\left(\frac{1.00}{1.333}\right) = 48.6°$$

Thus, the minimum diameter of the raft is

$$2r_{min} = 2h\tan\theta_{min} = 2h\tan\theta_c = 2(2.00\ \text{m})\tan 48.6° = \boxed{4.54\ \text{m}}$$

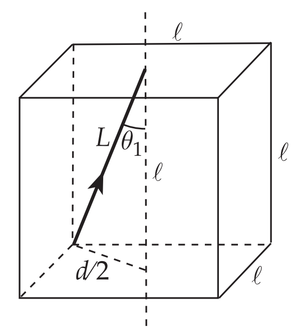
**22.41** (a) Snell's law can be written as $\dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{v_1}{v_2}$. At the critical angle of incidence $(\theta_1 = \theta_c)$, the angle of refraction is 90° and Snell's law becomes $\sin\theta_c = \dfrac{v_1}{v_2}$. At the concrete-air boundary,

$$\theta_c = \sin^{-1}\left(\frac{v_1}{v_2}\right) = \sin^{-1}\left(\frac{343 \text{ m/s}}{1\,850 \text{ m/s}}\right) = \boxed{10.7°}$$

(b) Sound can be totally reflected only if it is initially traveling in the slower medium. Hence, at the concrete-air boundary, the sound must be traveling in $\boxed{\text{air}}$.

(c) $\boxed{\text{Sound in air falling on the wall from most directions is 100\% reflected}}$, so the wall is a good mirror.

**22.42** The sketch at the right shows a light ray entering at the painted corner of the cube and striking the center of one of the three unpainted faces of the cube. The angle of incidence at this face is the angle $\theta_1$ in the triangle shown. Note that one side of this triangle is half the diagonal of a face and is given by




$$\frac{d}{2} = \frac{\sqrt{\ell^2+\ell^2}}{2} = \frac{\ell}{\sqrt{2}}$$

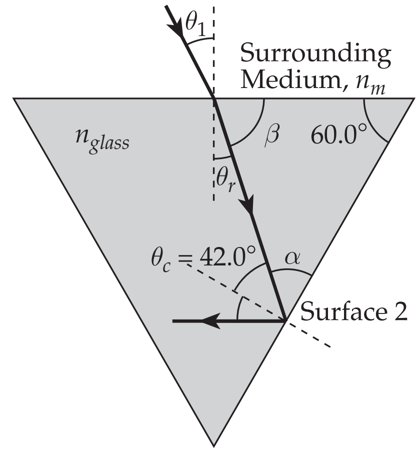
Also, the hypotenuse of this triangle is $L = \sqrt{\ell^2 + \left(\dfrac{d}{2}\right)^2} = \sqrt{\ell^2 + \dfrac{\ell^2}{2}} = \ell\sqrt{\dfrac{3}{2}}$

Thus, $\sin\theta_1 = \dfrac{d/2}{L} = \dfrac{\ell/\sqrt{2}}{\ell\left(\sqrt{3}/\sqrt{2}\right)} = \dfrac{1}{\sqrt{3}}$

For total internal reflection at this face, it is necessary that

$$\sin\theta_1 \geq \sin\theta_c = \frac{n_{air}}{n_{cube}} \quad \text{or} \quad \frac{1}{\sqrt{3}} \geq \frac{1.00}{n} \quad \text{giving} \quad \boxed{n \geq \sqrt{3}}$$

**22.43** If $\theta_c = 42.0°$ at the boundary between the prism glass and the surrounding medium, then $\sin\theta_c = \dfrac{n_2}{n_1}$ gives




$$\frac{n_m}{n_{glass}} = \sin 42.0°$$

or $$\frac{n_{glass}}{n_m} = \frac{1}{\sin 42.0°} = 1.494$$

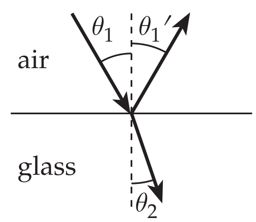
From the geometry shown in the above figure,

$$\alpha = 90.0° - 42.0° = 48.0°,\ \beta = 180° - 60.0° - \alpha = 72.0°$$

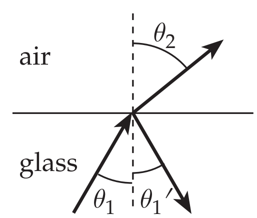
and $\theta_r = 90.0° - \beta = 18.0°$. Thus, applying Snell's law at the first surface gives

$$\theta_1 = \sin^{-1}\left(\frac{n_{glass}\sin\theta_r}{n_m}\right) = \sin^{-1}\left(1.494\sin 18.0°\right) = \boxed{27.5°}$$

**22.44** (a) $\theta_1' = \theta_1 = \boxed{30.0°}$




**Parts (a) and (c)**




**Parts (b) and (d)**

$$\theta_2 = \sin^{-1}\left(\frac{n_1\sin\theta_1}{n_2}\right)$$

$$= \sin^{-1}\left(\frac{(1.00)\sin 30.0°}{1.55}\right)$$

$$= \boxed{18.8°}$$

(b) $\theta_1' = \theta_1 = 30.0°$, $\theta_2 = \sin^{-1}\left(\dfrac{n_1\sin\theta_1}{n_2}\right) = \sin^{-1}\left(\dfrac{(1.55)\sin 30.0°}{1.00}\right) = \boxed{50.8°}$

(c) and (d) The other entries are computed similarly and are shown in the table below.

| (c) air into glass, angles in degrees | | | (d) glass into air, angles in degrees | | |
|---|---|---|---|---|---|
| incidence | reflection | refraction | incidence | reflection | refraction |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 10.0 | 10.0 | 6.43 | 10.0 | 10.0 | 15.6 |
| 20.0 | 20.0 | 12.7 | 20.0 | 20.0 | 32.0 |
| 30.0 | 30.0 | 18.8 | 30.0 | 30.0 | 50.8 |
| 40.0 | 40.0 | 24.5 | 40.0 | 40.0 | 85.1 |
| 50.0 | 50.0 | 29.6 | 50.0 | 50.0 | none* |
| 60.0 | 60.0 | 34.0 | 60.0 | 60.0 | none* |
| 70.0 | 70.0 | 37.3 | 70.0 | 70.0 | none* |
| 80.0 | 80.0 | 39.4 | 80.0 | 80.0 | none* |
| 90.0 | 90.0 | 40.2 | 90.0 | 90.0 | none* |

*total internal reflection

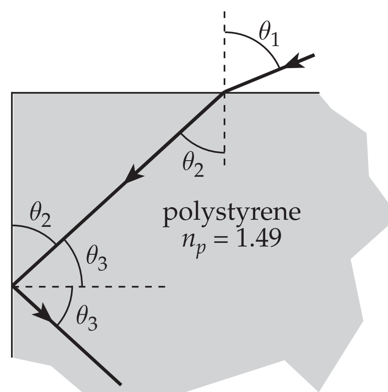
**22.45** At the air-ice boundary, Snell's law gives the angle of refraction in the ice as

$$\theta_{1r} = \sin^{-1}\left(\frac{n_{air}\sin\theta_{1i}}{n_{ice}}\right) = \sin^{-1}\left(\frac{(1.00)\sin 30.0^\circ}{1.309}\right) = 22.5^\circ$$

Since the sides of the ice layer are parallel, the angle of incidence at the ice-water boundary is $\theta_{2i} = \theta_{1r} = 22.5^\circ$. Then, from Snell's law, the angle of refraction in the water is

$$\theta_{2r} = \sin^{-1}\left(\frac{n_{ice}\sin\theta_{2i}}{n_{water}}\right) = \sin^{-1}\left(\frac{(1.309)\sin 22.5^\circ}{1.333}\right) = \boxed{22.0^\circ}$$

**22.46** (a) For polystyrene surrounded by air, total internal reflection at the left vertical face requires that




$$\theta_3 \geq \theta_c = \sin^{-1}\left(\frac{n_{air}}{n_p}\right) = \sin^{-1}\left(\frac{1.00}{1.49}\right) = 42.2^\circ$$

From the geometry shown in the figure at the right,

$$\theta_2 = 90.0^\circ - \theta_3 \leq 90.0^\circ - 42.2^\circ = 47.8^\circ$$

Thus, use of Snell's law at the upper surface gives

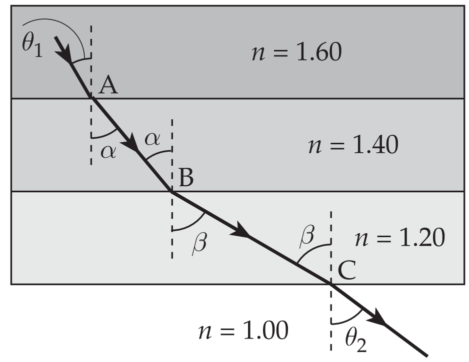
$$\sin\theta_1 = \frac{n_p\sin\theta_2}{n_{air}} \leq \frac{(1.49)\sin 47.8^\circ}{1.00} = 1.10$$

so it is seen that $\boxed{\text{any angle of incidence} \leq 90^\circ}$ at the upper surface will yield total internal reflection at the left vertical face.

(b) Repeating the steps of part (a) with the index of refraction of air replaced by that of water yields $\theta_3 \geq 63.5°$, $\theta_2 \leq 26.5°$, $\sin\theta_1 \leq 0.499$, and $\theta_1 \leq \boxed{30.0°}$.

(c) Total internal reflection is $\boxed{\text{not possible since } n_{polystyrene} < n_{carbon\ disulfide}}$

**22.47** Applying Snell's law at points A, B, and C, gives




$$1.40\sin\alpha = 1.60\sin\theta_1 \qquad (1)$$

$$1.20\sin\beta = 1.40\sin\alpha \qquad (2)$$

and $$1.00\sin\theta_2 = 1.20\sin\beta \qquad (3)$$

Combining equations (1), (2), and (3) yields

$$\sin\theta_2 = 1.60\sin\theta_1 \qquad (4)$$

Note that equation (4) is exactly what Snell's law would yield if the second and third layers of this "sandwich" were ignored. This will always be true if the surfaces of all the layers are parallel to each other.

(a) If $\theta_1 = 30.0°$, then equation (4) gives $\theta_2 = \sin^{-1}(1.60\sin 30.0°) = \boxed{53.1°}$

(b) At the critical angle of incidence on the lowest surface, $\theta_2 = 90.0°$. Then, equation (4) gives

$$\theta_1 = \sin^{-1}\left(\frac{\sin\theta_2}{1.60}\right) = \sin^{-1}\left(\frac{\sin 90.0°}{1.60}\right) = \boxed{38.7°}$$

**22.48** (a) We see the Sun swinging around a circle in the extended plane of our parallel of latitude. Its angular speed is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{86\ 400 \text{ s}} = 7.27\times10^{-5} \text{ rad/s}$$

The direction of sunlight crossing the cell from the window changes at this rate, moving on the opposite wall at speed

$$v = r\omega = (2.37 \text{ m})(7.27\times10^{-5} \text{ rad/s}) = 1.72\times10^{-4} \text{ m/s} = \boxed{0.172 \text{ mm/s}}$$

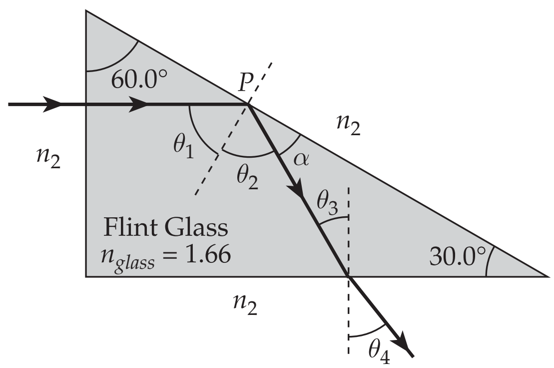
(b) The mirror folds into the cell the motion that would occur in a room twice as wide. Thus,

$$v = (2r)\omega = [2(2.37\text{ m})](7.27\times10^{-5}\text{ rad/s})$$

$$= 3.45\times10^{-4}\text{ m/s} = \boxed{0.345\text{ mm/s}}$$

(c) and (d) As the Sun moves southward and upward at 50.0°, we may regard the corner of the window as fixed, and both patches of light move $\boxed{\text{northward at 50.0° below the horizontal}}$

**22.49** (a) From the geometry of the figure at the right, observe that $\theta_1 = 60.0°$. Also, from the law of reflection, $\theta_2 = \theta_1 = 60.0°$. Therefore, $\alpha = 90.0° - \theta_2 = 30.0°$, and $\theta_3 + 90.0° = 180 - \alpha - 30.0°$ or $\theta_3 = 30.0°$.




Then, since the prism is immersed in water $(n_2 = 1.333)$, Snell's law gives

$$\theta_4 = \sin^{-1}\left(\frac{n_{glass}\sin\theta_3}{n_2}\right) = \sin^{-1}\left(\frac{(1.66)\sin 30.0°}{1.333}\right) = \boxed{38.5°}$$

(b) For refraction to occur at point *P*, it is necessary that $\theta_c > \theta_1$.

Thus, $\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_{glass}}\right) > \theta_1$, which gives

$$n_2 > n_{glass}\sin\theta_1 = (1.66)\sin 60.0° = \boxed{1.44}$$

**22.50** Applying Snell's law to this refraction gives $n_{glass}\sin\theta_2 = n_{air}\sin\theta_1$

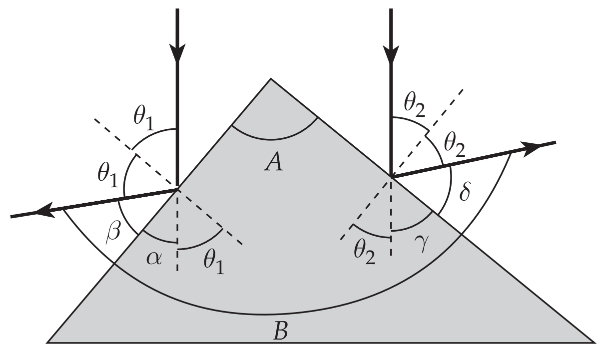
If $\theta_1 = 2\theta_2$, this becomes

$$n_{glass}\sin\theta_2 = \sin(2\theta_2) = 2\sin\theta_2\cos\theta_2 \text{ or } \cos\theta_2 = \frac{n_{glass}}{2}$$

Then, the angle of incidence is

$$\theta_1 = 2\theta_2 = 2\cos^{-1}\left(\frac{n_{glass}}{2}\right) = 2\cos^{-1}\left(\frac{1.56}{2}\right) = \boxed{77.5^\circ}$$

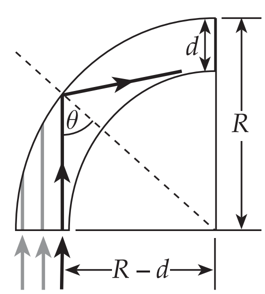
**22.51** In the Figure at the right, observe that $\beta = 90^\circ - \theta_1$ and $\alpha = 90^\circ - \theta_1$. Thus, $\beta = \alpha$.




Similarly, on the right side of the prism, $\delta = 90^\circ - \theta_2$ and $\gamma = 90^\circ - \theta_2$, giving $\delta = \gamma$.

Next, observe that the angle between the reflected rays is $B = (\alpha+\beta)+(\gamma+\delta)$, so $B = 2(\alpha+\gamma)$. Finally, observe that the left side of the prism is sloped at angle $\alpha$ from the vertical, and the right side is sloped at angle $\gamma$. Thus, the angle between the two sides is $A = \alpha+\gamma$, and we obtain the result $B = 2(\alpha+\gamma) = \boxed{2A}$

**22.52** (a) Observe in the sketch at the right that a ray originally traveling along the inner edge will have the smallest angle of incidence when it strikes the outer edge of the fiber in the curve. Thus, if this ray is totally internally reflected, all of the others are also totally reflected.




For this ray to be totally internally reflected it is necessary that

$$\theta \ge \theta_c \qquad \text{or} \qquad \sin\theta \ge \sin\theta_c = \frac{n_{air}}{n_{pipe}} = \frac{1}{n}$$

But, $\sin\theta = \dfrac{R-d}{R}$, so we must have $\dfrac{R-d}{R} \ge \dfrac{1}{n}$

which simplifies to $\boxed{R \ge nd/(n-1)}$

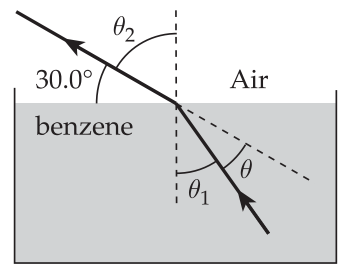
(b) As $d \to 0$, $R \to 0$. This is $\boxed{\text{reasonable}}$ behavior.

As $n$ increases, $R_{\min} = \dfrac{nd}{n-1} = \dfrac{d}{1-1/n}$ decreases. This is $\boxed{\text{reasonable}}$ behavior.

As $n \to 1$, $R_{\min}$ increases. This is $\boxed{\text{reasonable}}$ behavior.

(c) $R_{\min} = \dfrac{nd}{n-1} = \dfrac{(1.40)(100\ \mu\text{m})}{1.40-1} = \boxed{350\ \mu\text{m}}$

**22.53** Consider light from the bottom end of the wire that happens to be headed up along the surface of the wire before and after refraction with angle of incidence $\theta_1$ and angle of refraction $\theta_2 = 60.0°$. Then, from Snell's law, the angle of incidence was

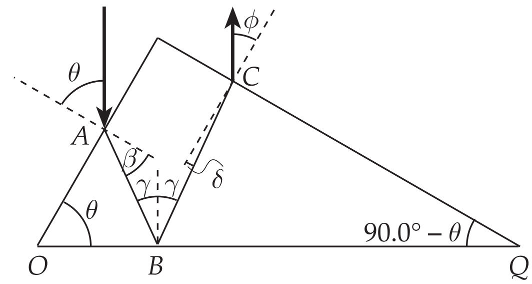



$$\theta_1 = \sin^{-1}\left(\frac{n_{air}\sin\theta_2}{n_{benzene}}\right)$$

$$= \sin^{-1}\left(\frac{(1.00)\sin 60.0°}{1.50}\right) = 35.3°$$

Thus, the wire is bent by angle $\theta = 60.0° - \theta_1 = 60.0° - 35.3° = \boxed{24.7°}$

**22.54** From the sketch at the right, observe that the angle of incidence at $A$ is the same as the prism angle at point $O$. Given that $\theta = 60.0°$, application of Snell's law at point $A$ gives




$$1.50\sin\beta = (1.00)\sin 60.0° \text{ or } \beta = 35.3°$$

From triangle $AOB$, we calculate the angle of incidence and reflection, $\gamma$, at point $B$:

$$\theta + (90.0° - \beta) + (90.0° - \gamma) = 180° \qquad \text{or} \qquad \gamma = \theta - \beta = 60.0° - 35.3° = 24.7°$$

Now, we find the angle of incidence at point $C$ using triangle $BCQ$:

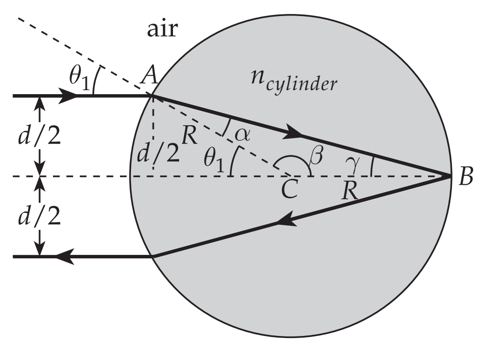
$$(90.0° - \gamma) + (90.0° - \delta) + (90.0° - \theta) = 180°$$

or $\qquad \delta = 90.0° - (\theta + \gamma) = 90.0° - 84.7° = 5.26°$

Finally, application of Snell's law at point $C$ gives $\quad (1.00)\sin\phi = (1.50)\sin(5.26°)$

or $\qquad \phi = \sin^{-1}(1.50\sin 5.26°) = \boxed{7.91°}$

**22.55** The path of a light ray during a reflection and/or refraction process is always reversible. Thus, if the emerging ray is parallel to the incident ray, the path which the light follows through this cylinder must be symmetric about the center line as shown at the right.

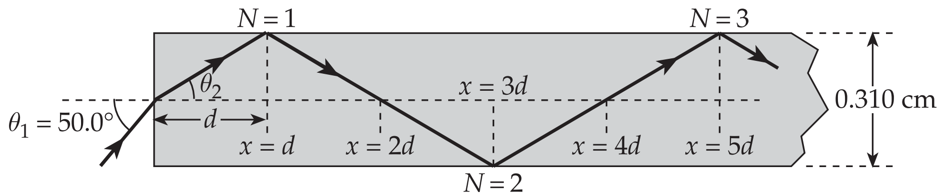



Thus, $\theta_1 = \sin^{-1}\left(\dfrac{d/2}{R}\right) = \sin^{-1}\left(\dfrac{1.00 \text{ m}}{2.00 \text{ m}}\right) = 30.0°$

Triangle $ABC$ is isosceles, so $\gamma = \alpha$ and $\beta = 180° - \alpha - \gamma = 180° - 2\alpha$. Also, $\beta = 180° - \theta_1$ which gives $\alpha = \theta_1/2 = 15.0°$. Then, from applying Snell's law at point $A$,

$$n_{cylinder} = \frac{n_{air}\sin\theta_1}{\sin\alpha} = \frac{(1.00)\sin 30.0°}{\sin 15.0°} = \boxed{1.93}$$

**22.56**




The angle of refraction as the light enters the left end of the slab is

$$\theta_2 = \sin^{-1}\left(\frac{n_{air}\sin\theta_1}{n_{slab}}\right) = \sin^{-1}\left(\frac{(1.00)\sin 50.0^\circ}{1.48}\right) = 31.2^\circ$$

Observe from the figure that the first reflection occurs at $x = d$, the second reflection is at $x = 3d$, the third is at $x = 5d$, and so forth. In general, the $N^{\text{th}}$ reflection occurs at $x = (2N-1)d$ where

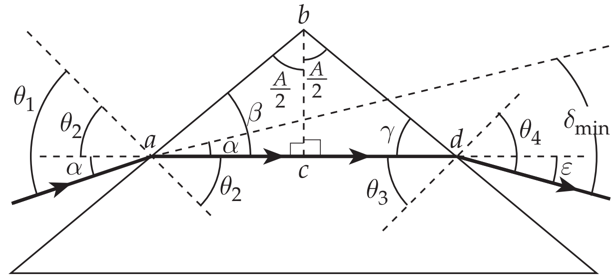
$$d = \frac{(0.310\ \text{cm}/2)}{\tan\theta_2} = \frac{0.310\ \text{cm}}{2\tan 31.2^\circ} = 0.256\ \text{cm}$$

Therefore, the number of reflections made before reaching the other end of the slab at $x = L = 42\ \text{cm}$ is found from $L = (2N-1)d$ to be

$$N = \frac{1}{2}\left(\frac{L}{d}+1\right) = \frac{1}{2}\left(\frac{42\ \text{cm}}{0.256\ \text{cm}}+1\right) = 82.5 \text{ or } \boxed{82 \text{ complete reflections}}$$

**22.57** Refer to the figure given at the right.

$$\delta_{min} = \alpha + \varepsilon = (\theta_1 - \theta_2) + (\theta_4 - \theta_3)$$




Note that triangles $abc$ and $bcd$ are congruent. Therefore, $\beta = \gamma$ and their complementary angles are also equal, or $\theta_2 = \theta_3$.

From Snell's law at point $a$, $\quad \sin\theta_1 = n\sin\theta_2 \quad (1)$

and at point $d$, $\quad \sin\theta_4 = n\sin\theta_3$

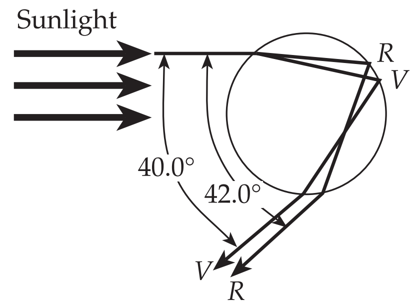
Since $\theta_2 = \theta_3$, comparison of these results shows that $\theta_4 = \theta_1$, so $\delta_{min} = 2(\theta_1 - \theta_2)$.

But, $\theta_2 = 90° - \beta = 90° - (90° - A/2)$, or $\theta_2 = A/2$

Thus, $\delta_{min} = 2\left(\theta_1 - \dfrac{A}{2}\right)$, or $\theta_1 = \dfrac{A + \delta_{min}}{2}$ and equation (1) then yields

$$n = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin\left[\frac{1}{2}(A + \delta_{min})\right]}{\sin\left(\frac{1}{2}A\right)}$$

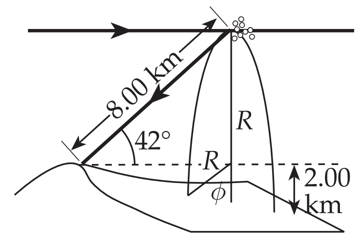
**22.58** Horizontal light rays from the setting Sun pass above the hiker. Those light rays that encounter raindrops at 40.0° to 42.0° from the hiker's shadow are twice refracted and once reflected and reach the hiker as the rainbow.




The hiker sees a greater percentage of the violet inner edge, so we consider the red outer edge. The radius $R$ of the circle of droplets is

$$R = (8.00\text{ km})\sin 42.0° = 5.35\text{ km}$$

Then the angle $\phi$, between the vertical and the radius where the bow touches the ground, is given by



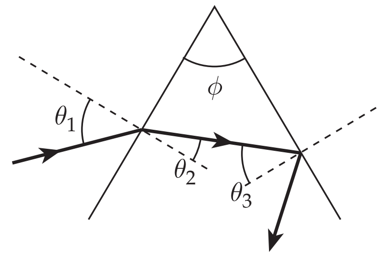

$$\cos\phi = \frac{2.00\text{ km}}{R} = \frac{2.00\text{ km}}{5.35\text{ km}} = 0.374$$

or $\phi = 68.1°$

The angle filled by the visible bow is $360° - (2 \times 68.1°) = 224°$, so the visible bow is

$$\frac{224°}{360°} = \boxed{62.2\%\text{ of a circle}}$$

**22.59** Applying Snell's law at the first surface in the figure at the right gives the angle of incidence as




$$\theta_1 = \sin^{-1}\left(\frac{n\sin\theta_2}{n_{air}}\right) = \sin^{-1}\left(n\sin\theta_2\right) \qquad (1)$$

Since the sum of the interior angles of a triangle equals 180°, observe that $\phi + \left(90° - \theta_2\right) + \left(90° - \theta_3\right) = 180°$, which reduces to $\theta_2 = \phi - \theta_3$. Thus, equation (1) becomes

$$\theta_1 = \sin^{-1}\left[n\sin\left(\phi - \theta_3\right)\right] = \sin^{-1}\left[n\left(\sin\phi\cos\theta_3 - \cos\phi\sin\theta_3\right)\right]$$

At the smallest allowed value for $\theta_1$, $\theta_3$ is equal to the critical angle at the second surface, or $\sin\theta_3 = \sin\theta_c = \frac{n_{air}}{n} = \frac{1}{n}$. Then,

$$\cos\theta_3 = \sqrt{1 - \sin^2\theta_3} = \sqrt{1 - \frac{1}{n^2}} = \frac{\sqrt{n^2 - 1}}{n},$$

and $$\theta_1 = \sin^{-1}\left[n\left(\sin\phi\,\frac{\sqrt{n^2-1}}{n} - \frac{1}{n}\cos\phi\right)\right] = \boxed{\sin^{-1}\left(\sqrt{n^2-1}\,\sin\phi - \cos\phi\right)}$$

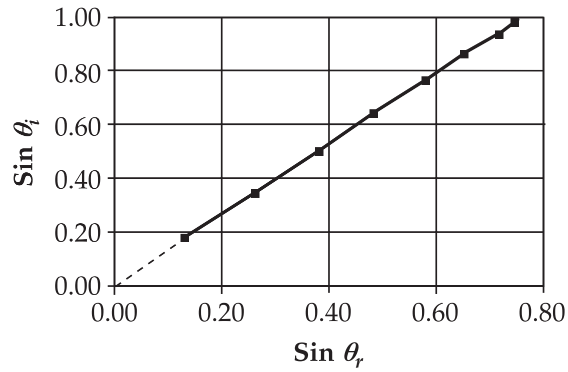
Note that when $\tan\phi < 1/\sqrt{n^2 - 1}$, the value of $\theta_1$ given by this result is negative. This simply means that, when $\tan\phi < 1/\sqrt{n^2 - 1}$ and the refracted beam is striking the second surface of the prism at the critical angle, the incident beam at the first surface will be on the opposite side of the normal line from what is shown in the figure.

**22.60** Snell's law would predict that $n_{air} \sin\theta_i = n_{water} \sin\theta_r$, or since $n_{air} = 1.00$,

$$\sin\theta_i = n_{water} \sin\theta_r$$

Comparing this equation to the equation of a straight line, $y = mx + b$, shows that if Snell's law is valid, a graph of $\sin\theta_i$ versus $\sin\theta_r$ should yield a straight line that would pass through the origin if extended and would have a slope equal to $n_{water}$ .

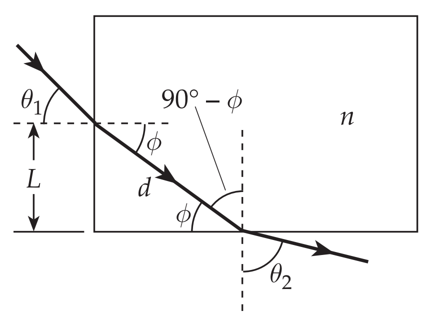
| $\theta_i$ (deg) | $\theta_r$ (deg) | $\sin\theta_i$ | $\sin\theta_r$ |
|---|---|---|---|
| 10.0 | 7.50 | 0.174 | 0.131 |
| 20.0 | 15.1 | 0.342 | 0.261 |
| 30.0 | 22.3 | 0.500 | 0.379 |
| 40.0 | 28.7 | 0.643 | 0.480 |
| 50.0 | 35.2 | 0.766 | 0.576 |
| 60.0 | 40.3 | 0.866 | 0.647 |
| 70.0 | 45.3 | 0.940 | 0.711 |
| 80.0 | 47.7 | 0.985 | 0.740 |




The straightness of the graph line and the fact that its extension passes through the origin demonstrates the validity of Snell's law. Using the end points of the graph line to calculate its slope gives the value of the index of refraction of water as

$$n_{water} = slope = \frac{0.985 - 0.174}{0.740 - 0.131} = 1.33$$

**22.61** (a) If $\theta_1 = 45.0°$, application of Snell's law at the point where the beam enters the plastic block gives




$$(1.00)\sin 45.0° = n\sin\phi \qquad [1]$$

Application of Snell's law at the point where the beam emerges from the plastic, with $\theta_2 = 76.0°$ gives

$$n\sin(90° - \phi) = (1.00)\sin 76° \qquad \text{or} \qquad (1.00)\sin 76° = n\cos\phi \qquad [2]$$

Dividing Equation [1] by Equation [2], we obtain

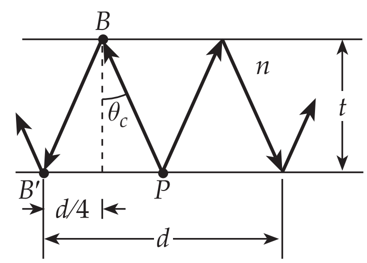
$$\tan\phi = \frac{\sin 45.0°}{\sin 76°} = 0.729 \qquad \text{and} \qquad \phi = 36.1°$$

Thus, from Equation [1],
$$n = \frac{\sin 45.0°}{\sin\phi} = \frac{\sin 45.0°}{\sin 36.1°} = \boxed{1.20}$$

(b) Observe from the figure above that $\sin\phi = L/d$. Thus, the distance the light travels inside the plastic is $d = L/\sin\phi$, and if $L = 50.0\text{ cm} = 0.500\text{ m}$, the time required is

$$\Delta t = \frac{d}{v} = \frac{L/\sin\phi}{c/n} = \frac{nL}{c\sin\phi} = \frac{1.20(0.500\text{ m})}{(3.00\times10^{8}\text{ m/s})\sin 36.1^\circ} = 3.40\times10^{-9}\text{ s} = \boxed{3.40\text{ ns}}$$

**22.62** (a) At the upper surface of the glass, the critical angle is given by




$$\sin\theta_c = \frac{n_{air}}{n_{glass}} = \frac{1}{n}$$

Consider the critical ray $PBB'$: $\tan\theta_c = \dfrac{d/4}{t} = \dfrac{d}{4t}$

But, also, $\tan^2\theta_c = \dfrac{\sin^2\theta_c}{\cos^2\theta_c} = \dfrac{\sin^2\theta_c}{1-\sin^2\theta_c} = \dfrac{1/n^2}{1-1/n^2} = \dfrac{1}{n^2-1}$

Thus, $\left(\dfrac{d}{4t}\right)^2 = \dfrac{1}{n^2-1}$ or $n^2-1 = \left(\dfrac{4t}{d}\right)^2$ giving $\boxed{n = \sqrt{1+\left(\dfrac{4t}{d}\right)^2}}$

(b) Using the result from Part (a) and solving for *d* gives $d = \dfrac{4t}{\sqrt{n^2-1}}$

Thus, if $n = 1.52$ and $t = 0.600\text{ cm}$, then $d = \dfrac{4(0.600\text{ cm})}{\sqrt{(1.52)^2-1}} = \boxed{2.10\text{ cm}}$

(c) The color at the inner edge of the white halo is associated with the wavelength forming the smallest diameter dark spot. Considering the result from above, $d = 4t/\sqrt{n^2-1}$, we see that this is the wavelength for which the glass has the highest refractive index. From a dispersion graph, such as Figure 22.14 in the textbook, this is seen to be the shorter wavelength. Thus, the inner edge will appear $\boxed{\text{violet}}$.