

USING DERIVATIVES TO UNDERSTAND BEHAVIOR OF FUNCTIONS

CRITICAL NUMBERS

- If $f'(c) = 0$ or $f'(c)$ is undefined at $x = c$, then c is a critical number of f .

Note: f must be continuous at $x = c$ and critical numbers refer to the first derivative only.

INCREASING/DECREASING TEST

- If $f'(x) > 0$ on an interval, then f is increasing on that interval.
 - If $f'(x) < 0$ on an interval, then f is decreasing on that interval.
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CONCAVITY TEST

- If $f''(x) > 0$ on an interval, then f is concave up on that interval.
 - If $f''(x) < 0$ on an interval, then f is concave down on that interval.
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FIRST DERIVATIVE TEST for local extreme values

Suppose that c is a critical number of the function f .

- If f' changes from positive to negative at $x = c$, then f has a local maximum at $x = c$.
- If f' changes from negative to positive at $x = c$, then f has a local minimum at $x = c$.

Note: f must be continuous at $x = c$ and not all critical numbers indicate extreme values.

SECOND DERIVATIVE TEST for local extreme values

- If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.
- If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.

Note: f must be continuous at $x = c$

POINT OF INFLECTION TEST

- If f'' changes signs at $x = c$, then f has a point of inflection at $x = c$. Note: f must be continuous at $x = c$.
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FIRST DERIVATIVE TEST for absolute extreme values

Suppose that c is a critical number of the a continuous function defined on an interval.

- If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is an absolute maximum value of f .
 - If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is an absolute minimum value of f .
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CLOSED INTERVAL TEST for absolute extreme values

For a continuous function f on a closed interval $[a, b]$:

- 1) Find the values of f at the critical numbers of f .
 - 2) Find the values of f at the endpoints of the interval.
 - 3) The largest of these values is the absolute maximum value and the smallest of the values is the absolute minimum value.
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GUIDELINES FOR SKETCHING A CURVE

- 1) Domain
 - 2) Intercepts
 - 3) Discontinuities
 - 4) End Behavior
 - 5) Increasing/Decreasing
 - 6) Extreme Values
 - 7) Concavity
 - 8) Points of inflection
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Note: The value of the extreme value is the "y" value. The location of the extreme value is the "x" value.
