§4.10—Derivatives of Log Functions & LOG DIFF

<u>Definition</u>: If $b > 0$ and $b \neq 1$, $f(x) = b^x$ is a one-to-one, hence has an inverse, called the logarithm with base *b*. More specifically, if $b^x = y$, then $\log_b y = x$.

Example 1:
Evaluate the following:

a) $\log_2 8$ b) $\log_3 \frac{1}{81}$ c) $\log_{16} 4$

Example 2:
Sketch the graph of $f(x) = \log_4 x$. What is the domain? Range?

<u>Definition</u>: We define the natural logarithm to be $\log_e x$, denoted by $\ln x$. We define the common log to be $\log_{10} x$, denoted by $\log x$.

Example 3:

Find the limit:

a) $\lim\limits_{x\to\infty} \ln(x^2 - x)$ b) $\lim\limits_{x\to 9^+} \log_2 (x-9)$ c) $\lim\limits_{x\to \frac{\pi}{2}^-} \log(\tan x)$ d) $\lim\limits_{x\to 0^+} \log(\cos x)$

Example 4:

Find the domain of $g(x) = \log\left(x^2 - 3x + 2\right)$

Properties of Logarithms:

- $\log_b(MN) = \log_b M + \log_b N$
- $\log_b \dfrac{M}{N} = \log_b M - \log_b N$
- $\log_b M^r = r \log_b M$
- $\log_b b^x = x$
- $b^{\log_b x} = x$
- If $\log_b M = \log_b N$, then $M = N$

Example 5:
Solve the following equations for x.

a) $\log(x+1) = 3$ b) $e^{3-x} = 14$ c) $\ln x - \ln(x-1) = 1$ d) $\log x + \log(x+1) = \log 6$

Example 6:
Find the inverse of the function:

a) $f(x) = \ln(x+2)$ b) $f(x) = \dfrac{10^x}{10^x + 1}$

Example 7:
Find the point where the tangent line to the curve $y = e^{-x}$ is perpendicular to the line $2x - y = 8$.

Change of Base formula:

$\log_b x = \dfrac{\log_a x}{\log_a b}$. Normally, we choose to convert to base *e*: $\log_b x = \dfrac{\ln x}{\ln b}$

Example 8:
Evaluate $\log_2 5$ to 3 decimal places.

Derivatives of Logarithmic Functions

Example 9:
If $y = \ln x$, solve for *x*, then find $\dfrac{dy}{dx}$ using implicit differentiation.

Derivative of $y = \ln x$ and $y = \ln|x|$

$\dfrac{d}{dx}[\ln x] = \dfrac{1}{x}$ $\dfrac{d}{dx}[\ln|x|] = \dfrac{1}{x}$ $\dfrac{d}{dx}[\ln u] = \dfrac{u'}{u}$

Example 10:
Find the derivative of each.

a) $f(x) = \cos(\ln x)$ b) $y = \ln(1 + \ln x)$ c) $f(u) = \ln\sqrt{\dfrac{3u+2}{3u-2}}$

Example 11:

If $y = \log_b x$, use the change of base formula to find $\frac{dy}{dx}$

Example 12:

Find the derivative of $f(x) = \log_3\left(5 - x^4\right)$

Logarithmic Differentiation (LOG DIFF):

This technique involves taking the natural log of BOTH sides of an equation prior to differentiating. We can use this technique in three situations:

i) We are differentiating an "ugly" expression with lots of factors in a product and/or a quotient, thereby making the derivative easier to compute ("Simplify early and often—especially with logs!")

ii) When we are differentiating a function of the form $y = f(x)^{g(x)}$.

iii) When the instructions say "Use Logarithmic Differentiation to . . . "

Example 13:

a) Find the derivative of $y = \dfrac{e^x\left(x^2+2\right)^4}{\sqrt{(x+1)^3}\left(x^2+3\right)^2}$

b) $\frac{d}{dx}\left[(\sin x)^{\cos x}\right] =$

c) Find the derivative using LOG DIFF $y = x^2$

d) $\frac{d}{dx}\left[x^{\ln x}\right] =$