§4.4—Product & Quotient Rules

- $f(x)$ is the y-value generating "machine."
- $f'(x)$ is the slope value generating "machine."

The *INCORRECT* Product Rule

The derivative of a product of two functions *f* and *g* is the product of the derivatives of *f* and *g*.

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right]=f'(x)\cdot g'(x)$$

The *CORRECT* Product Rule

The derivative of a product of two functions *f* and *g* is .

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right]=f'(x)\cdot g(x)+f(x)\cdot g'(x)$$

Take turns, one derivative at a time per term. The number of factors will equal the number of terms. Multiplication and addition are both commutative, so this one is difficult to mess up, unless you do it incorrectly.

Example 1:
Find the derivative of the following function using the product rule, then verify using the power rule and graphically using NDERIV.

$g(x)=\left(3x-2x^2\right)(4+5x)$

Example 2:
Find the derivative of each of the following

a. $y=5x^3\cos x$

b. $y=2x\cos x-2\sin x$

The *INCORRECT* Quotient Rule

The derivative of a quotient of two functions *f* and *g* is the quotient of the derivatives of *f* and *g*.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{f'(x)}{g'(x)},\ g(x)\neq 0,\ g'(x)\neq 0$$

The *CORRECT* Quotient Rule

The derivative of a quotient of two functions *f* and *g* is

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{\left[g(x)\right]^2},\ g(x)\neq 0$$

Subtraction is not commutative, so this one is more important to get straight. An easier way to remember it is to think of the numerator as the "**HI**" function (it *is* up high, after all), and the denominator as the "**LO**" function (since it's in the bottom.) To make it more alliterative, let's actually call the denominator the "**HO**" function (it *does* rhyme with "LO.") Finally, let "**d**" mean "the derivative of . . ."

The Quotient Rule now becomes

The HOdHI (Quotient) Rule

The derivative of a quotient of two functions HI and HO is

$$\frac{d}{dx}\left[\frac{HI}{HO}\right]=\frac{HOdHI-HIdHO}{HO\cdot HO},\ HO\neq 0$$

Think of the Seven Dwarfs. Think of a Dyslexic Mr. Wilson greeting Tim. Think of Santa Clause in a hurry.

Example 3:

Find the derivative of $y=\dfrac{4x-2}{x^2-1}$. Support your answer graphically by using NDERIV.

Rewriting is still the key . . .

Example 4:

Find an equation of the tangent line to the graph of $f(x)=\dfrac{3-(1/x)}{x+5}$ at $x=-1$

We want to first be effective and second to be efficient.

Example 5:
Differentiate each of the following using the most efficient method.

a. $y=\dfrac{x^2-3x}{6}$ b. $y=\dfrac{5x^4}{8}$ c. $y=2x(x+5)$ d. $y=\dfrac{-3(3x-2x^2)}{7x}$

Example 5:

Find the derivative of $y = \dfrac{\sin x}{\cos x}$ using the quotient rule.

Derivatives of the other Trigonometric Functions

$\dfrac{d}{dx}[\tan x] = \sec^2 x$	$\dfrac{d}{dx}[\cot x] = -\csc^2 x$
$\dfrac{d}{dx}[\sec x] = \sec x \tan x$	$\dfrac{d}{dx}[\csc x] = -\csc x \cot x$

Example 6:

Find the derivatives of each of the following

a. $g(x) = x - \tan x$ b. $y = x \sec x$

When differentiating, it's best to simplify early and often.

Example 7:

Differentiate

a. $f(x) = \dfrac{1 - \cos x}{\sin x}$ b. $g(x) = \csc x - \cot x$

Which was easier? (they're both the same!)

Higher-Order Derivatives

Just as you can obtain a velocity function by differentiating a position function, you can obtain an **acceleration** function by differentiating a velocity function. Alternatively, you can think about obtaining an acceleration function by differentiating a position function ***twice***.

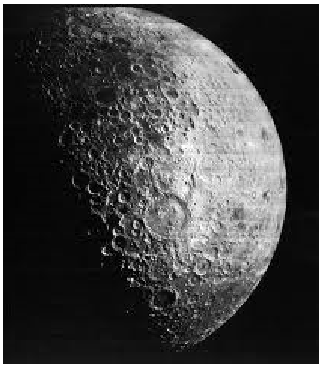
$s(t)$	position function
$v(t)=s'(t)$	velocity function
$a(t)=v'(t)=s''(t)$	acceleration function

The notation $s''(t)$ is called the **second derivative** of $s(t)$ and we can read it as "***s* double prime of *t***."
The second derivative is an example of a higher-order derivative. Why stop at two???

First derivative:	y'	$f'(x)$	$\dfrac{dy}{dx}$	$\dfrac{d}{dx}\left[f(x)\right]$
Second derivative:	y''	$f''(x)$	$\dfrac{d^2y}{dx^2}$	$\dfrac{d^2}{dx^2}\left[f(x)\right]$
Third derivative:	y'''	$f'''(x)$	$\dfrac{d^3y}{dx^3}$	$\dfrac{d^3}{dx^3}\left[f(x)\right]$
Fourth derivative:	$y^{(4)}$	$f^{(4)}(x)$	$\dfrac{d^4y}{dx^4}$	$\dfrac{d^4}{dx^4}\left[f(x)\right]$
$\vdots$				
nth derivative:	$y^{(n)}$	$f^{(n)}(x)$	$\dfrac{d^ny}{dx^n}$	$\dfrac{d^n}{dx^n}\left[f(x)\right]$

Example 8:
Find the first five derivatives of $y=x^3-5x^2+6x-4$

Example 9:



The position function for a falling object on (or near, ≈ 6 feet high) the surface of the moon is given by $s(t) = -0.8t^2 + 2$, where $s(t)$ is the height in meters and t is the time in seconds. What how many times stronger is the acceleration due to gravity on Earth than it is on the moon?

Example 10:

The graph below is the graph of f'. On the same coordinate grid, sketch and label the possible graphs for f'', f''', and f.