§4.9—Derivatives of Exponential Functions

Example 1:

Sketch the graph of $f(x) = e^x$, then, on the same set of axes, sketch a possible graph of f'(x). What do you notice? Confirm by sketching f'(x) using your calculator's NDERIV capability.

Derivative of e^x

 $\frac{d}{dx} \left[e^x \right] = e^x$. If *u* is a differentiable function of *x*, then $\frac{d}{dx} \left[e^u \right] = e^u \cdot u'$

Example 2:

Find
$$\frac{dy}{dx}$$
 if $y = e^{(x+x^2)}$

Example 3:

Using your calculator, graph $f(x) = 2^x$ and f'(x) using NDERIV. What do you notice? Do the same for $g(x) = 5^x$ and g'(x).

Derivative of b^x

$$\frac{d}{dx} [b^x] = b^x \cdot \ln b$$
. If *u* is a differentiable function of *x*, then $\frac{d}{dx} [b^u] = b^u \cdot \ln b \cdot u'$

Example 4:

At what point on the graph of the function $y = 2^t - 3$ does the tangent line have a slope of 2?

Example 5:

Evaluate the following.

a)
$$\frac{d}{dx} \left[7^{-3/x} \right]$$

b)
$$\frac{d}{dx} \left[e^{\ln x} \right]$$

c)
$$\frac{d}{dt} \left[\left(e^{-t} + e^t \right)^2 \right]$$

a)
$$\frac{d}{dx} \left[7^{-3/x} \right]$$
 b) $\frac{d}{dx} \left[e^{\ln x} \right]$ c) $\frac{d}{dt} \left[\left(e^{-t} + e^{t} \right)^{2} \right]$ d) $\frac{d}{dx} \left[x^{2} \cdot 5^{\sin^{2}(4x)} \right]$ e) $\frac{d}{dx} \left[e^{3x} \right]$

e)
$$\frac{d}{dx} \left[e^3 x \right]$$

Example 6:

Find the coordinates of any points where $f(x) = xe^x$ has a horizontal tangent line.

Example 7:

Find the equation of the tangent line to $y = x^2 e^x - 2xe^x + 2e^x$ at (1,e)

Example 8:

Find
$$\frac{dy}{dx}$$
 for $e^{xy} + x^2 - y^2 = 10$

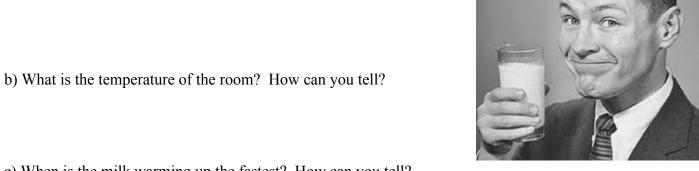
Example 9:

Find
$$\frac{d^2y}{dx^2}$$
 for $y = (3 + 2x)5^{-3x}$

Example 10:

A glass of cold milk from the refrigerator is left on the counter on a hot summer day. Its temperature (in degrees Fahrenheit) after sitting on the counter t minutes is $y = 72 - 30(0.98)^t$. Answer the following by

- analyzing $\frac{dy}{dt}$.
- a) What is the temperature of the refrigerator? How can you tell?



- c) When is the milk warming up the fastest? How can you tell?
- d) Determine algebraically when the temperature of the milk reaches $55^{\circ}F$.
- e) At what rate is the milk warming when its temperature is $55^{\circ}F$? Answer in a complete sentence with appropriate units.