Chapter 6
Momentum and Collisions

Quick Quizzes

1. (d). We are given no information about the masses of the objects. If the masses are the same, the speeds must be the same (so that they have equal kinetic energies), and then $p_1 = p_2$. If the masses are not the same, the speeds will be different, as will the momenta, and either $p_1 < p_2$, or $p_1 > p_2$, depending on which particle has more mass. Without information about the masses, we cannot choose among these possibilities.

2. (c). Because the momentum of the system (boy + raft) remains constant with zero magnitude, the raft moves towards the shore as the boy walks away from the shore.

3. (c). The total momentum of the car-truck system is conserved. Hence, any change in momentum of the truck must be counterbalanced by an equal magnitude change of opposite sign in the momentum of the car.

4. (a). The total momentum of the two-object system is zero before collision. To conserve momentum, the momentum of the combined object must be zero after the collision. Thus, the combined object must be at rest after the collision.

5. **(a)** Perfectly inelastic. Any collision in which the two objects stick together afterwards is perfectly inelastic.

(b) Inelastic. Both the Frisbee and the skater lose speed (and hence, kinetic energy) in this collision. Thus, the total kinetic energy of the system is not conserved.

(c) Inelastic. The kinetic energy of the Frisbee is conserved. However, the skater loses speed (and hence, kinetic energy) in this collision. Thus, the total kinetic energy of the system is not conserved.

6. (a). If all of the initial kinetic energy is transformed, then nothing is moving after the collision. Consequently, the final momentum of the system is necessarily zero. Because momentum of the system is conserved, the initial momentum of the system must be zero meaning that the two objects must have had equal magnitude momenta in opposite directions before the collision.

Answers to Even Numbered Conceptual Questions

2. The glass, concrete, and steel were part of a rigid structure that shattered upon impact of the airplanes with the towers and upon collapse of the buildings as the steel support structures weakened due to high temperatures of the burning fuel. The sheets of paper floating down were probably not in the vicinity of the direct impact, where they would have burned after being exposed to very high temperatures. The papers were most likely situated on desktops or open file cabinets and were blown out of the buildings as they collapsed.

4. No. Only in a precise head-on collision with equal and opposite momentum can both balls wind up at rest. Yes. In the second case, assuming equal masses for each ball, if Ball 2, originally at rest, is struck squarely by Ball 1, then Ball 2 takes off with the velocity of Ball 1. Then Ball 1 is at rest.

6. The skater gains the most momentum by catching and then throwing the Frisbee.

8. Kinetic energy can be written as $\dfrac{p^2}{2m}$. Thus, even through the particles have the same kinetic energies their momenta may be different due to a difference in mass.

10. The resulting collision is intermediate between an elastic and a completely inelastic collision. Some energy of motion is transformed as the pieces buckle, crumple, and heat up during the collision. Also, a small amount is lost as sound. The most kinetic energy is lost in a head-on collision, so the expectation of damage to the passengers is greatest.

12. The less massive object loses the most kinetic energy in the collision.

14. The superhero is at rest before the toss and the net momentum of the system is zero. When he tosses the piano, say toward the right, something must get an equal amount of momentum to the left to keep the momentum at zero. This something recoiling to the left must be the superhero. He cannot stay at rest.

16. The passenger must undergo a certain momentum change in the collision. This means that a certain impulse must be exerted on the passenger by the steering wheel, the window, an air bag, or something. By increasing the time during which this momentum change occurs, the resulting force on the passenger can be decreased.

18. A certain impulse is required to stop the egg. But, if the time during which the momentum change of the egg occurs is increased, the resulting force on the egg is reduced. The time is increased when the sheet billows out as the egg is brought to a stop. The force is reduced low enough so that the egg will not break.

Answers to Even Numbered Problems

2. **(a)** 5.40 N·s **(b)** –27.0 J

4. **(a)** 0 **(b)** $1.1\ \text{kg}\cdot\text{m/s}$

6. 1.7 kN

8. 1.91×10^4 N upward

10. **(a)** $7.50\ \text{kg}\cdot\text{m/s}$ westward **(b)** 375 N eastward

12. **(a)** 12.0 N·s **(b)** $6.00\ \text{m/s}$ **(c)** $4.00\ \text{m/s}$

14. 260 N perpendicular to the wall

16. **(a)** $6.3\ \text{kg}\cdot\text{m/s}$ toward the pitcher
(b) 3.2×10^3 N toward the pitcher

18. 62 s

20. **(a)** $0.49\ \text{m/s}$ **(b)** $2.0\times10^{-2}\ \text{m/s}$

22. $v_{thrower} = 2.48\ \text{m/s}$, $v_{catcher} = 2.25\times10^{-2}\ \text{m/s}$

24. **(a)** B exerts a horizontal force on A. **(b)** A exerts a horizontal force on B that is opposite in direction to the force B exerts on A. **(c)** The force on A is equal in magnitude to the force on B, but is oppositely directed. **(d)** Yes. The momentum of the system (the two skaters) is conserved because the net external force on the system is zero (neglecting friction). **(e)** 2.22 m/s

26. $F_{\text{av}} = 3.75\times10^3$ N, no broken bones

28. $5.3\times10^2\ \text{m/s}$

30. $143\ \text{m/s}$

32. **(a)** $20.9\ \text{m/s}$ East **(b)** 8.68×10^3 J into internal energy

34. **(a)** $2.2\ \text{m/s}$ toward the right **(b)** No

36. $-40.0\ \text{cm/s}$ (10.0-g object), $+10.0\ \text{cm/s}$ (15.0-g object)

38. **(a)** $2.50\ \text{m/s}$ **(b)** 3.75×10^4 J

40. **(a)** 0, $1.50\ \text{m/s}$ **(b)** $-1.00\ \text{m/s}$, $1.50\ \text{m/s}$
(c) $1.00\ \text{m/s}$, $1.50\ \text{m/s}$

42. **(a)** $12.4\ \text{m/s}$ at 14.9° N of E **(b)** 7.20 %

44. No, his speed was $41.5\ \text{mi/h}$.

46. 40.5 g

48. 0.556 m

50. $v_{min} = \left(\dfrac{4M}{m}\right)\sqrt{g\ell}$

52. 0.960 m above the level of point B

54. $91\ \text{m/s}$

56. **(a)** $9.90\ \text{m/s}$, $-9.90\ \text{m/s}$ **(b)** $-16.5\ \text{m/s}$, $3.30\ \text{m/s}$
(c) 13.9 m, 0.556 m

58. 0.980 m

60. **(a)** $v_{red} = 0$, $v_{blue} = 3.00\ \text{m/s}$ **(b)** 0.212 m

62. **(a)** $v_m = v_0\sqrt{2}$, $v_{3m} = v_0\sqrt{\dfrac{2}{3}}$ **(b)** 35.3°

64. **(a)** 90.0° **(b)** $3.46\ \text{m/s}$ (cue ball), $2.00\ \text{m/s}$ (target)

66. 0.31 m

68. **(a)** See solution for diagrams.
(b) From Newton's third law, the forces have equal magnitudes and opposite directions.
(c) $\Delta p_A = -2Mv/3$, $\Delta p_B = +2Mv/3$, $\Delta p_C = 0$
(d) Kinetic energy is not conserved in this inelastic collision.

70. $33\ \text{m/s}$, $2.9 \times 10^3\ \text{m/s}^2$

72. **(a)** $1.1\ \text{m/s}$ at 30° from the positive x-axis **(b)** 0.32 or 32%

74. **(a)** $7.1\ \text{m/s}$ **(b)** 2.5 m

Problem Solutions

6.1 The velocity of the ball just before impact is found from $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ as

$$v_1 = -\sqrt{v_{0y}^2 + 2a_y\Delta y} = -\sqrt{0 + 2\left(-9.80 \text{ m/s}^2\right)\left(-1.25 \text{ m}\right)} = -4.95 \text{ m/s}$$

and the rebound velocity with which it leaves the floor is

$$v_2 = +\sqrt{v_f^2 - 2a_y\Delta y} = +\sqrt{0 - 2\left(-9.80 \text{ m/s}^2\right)\left(+0.960 \text{ m}\right)} = +4.34 \text{ m/s}$$

The impulse given the ball by the floor is then

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}\Delta t = \Delta\left(m\vec{\mathbf{v}}\right) = m\left(\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1\right)$$

$$= \left(0.150 \text{ kg}\right)\left[+4.34 \text{ m/s} - \left(-4.95 \text{ m/s}\right)\right] = +1.39 \text{ N}\cdot\text{s} = \boxed{1.39 \text{ N/s upward}}$$

6.2 Assume the initial direction of the ball in the $-x$ direction, away from the net.

(a) $I = \Delta p = m\left(v_f - v_i\right) = \left(0.0600 \text{ kg}\right)\left[40.0 \text{ m/s} - \left(-50.0 \text{ m/s}\right)\right]$ giving
$I = 5.40 \text{ kg}\cdot\text{m/s} = 5.40 \text{ N}\cdot\text{s}$ toward the net.

(b) $Work = \Delta KE = \dfrac{1}{2}m\left(v_f^2 - v_i^2\right)$

$$= \frac{\left(0.0600 \text{ kg}\right)\left[\left(40.0 \text{ m/s}\right)^2 - \left(50.0 \text{ m/s}\right)^2\right]}{2} = \boxed{-27.0 \text{ J}}$$

6.3 Use $p = mv$:

(a) $p = \left(1.67\times10^{-27} \text{ kg}\right)\left(5.00\times10^{6} \text{ m/s}\right) = \boxed{8.35\times10^{-21} \text{ kg}\cdot\text{m/s}}$

(b) $p = \left(1.50\times10^{-2} \text{ kg}\right)\left(3.00\times10^{2} \text{ m/s}\right) = \boxed{4.50 \text{ kg}\cdot\text{m/s}}$

(c) $p = \left(75.0 \text{ kg}\right)\left(10.0 \text{ m/s}\right) = \boxed{750 \text{ kg}\cdot\text{m/s}}$

(d) $p = \left(5.98\times10^{24} \text{ kg}\right)\left(2.98\times10^{4} \text{ m/s}\right) = \boxed{1.78\times10^{29} \text{ kg}\cdot\text{m/s}}$

6.4 (a) Since the ball was thrown straight upward, it is at rest momentarily ($v = 0$) at its maximum height. Therefore, $p = \boxed{0}$.

(b) The maximum height is found from $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ with $v_y = 0$.

$$0 = v_{0y}^2 + 2(-g)(\Delta y)_{\max} \qquad \text{Thus, } (\Delta y)_{\max} = \frac{v_{0y}^2}{2g}$$

We need the velocity at $\Delta y = \dfrac{(\Delta y)_{\max}}{2} = \dfrac{v_{0y}^2}{4g}$, thus $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ gives

$$v_y^2 = v_{0y}^2 + 2(-g)\left(\frac{v_{0y}^2}{4g}\right) = \frac{v_{0y}^2}{2}, \text{ or } v_y = \frac{v_{0y}}{\sqrt{2}} = \frac{15\ \text{m/s}}{\sqrt{2}}$$

Therefore, $p = mv_y = \dfrac{(0.10\ \text{kg})(15\ \text{m/s})}{\sqrt{2}} = \boxed{1.1\ \text{kg}\cdot\text{m/s}}$ upward.

6.5 (a) If $p_{ball} = p_{bullet}$,

then $v_{ball} = \dfrac{m_{bullet}v_{bullet}}{m_{ball}} = \dfrac{(3.00\times10^{-3}\ \text{kg})(1.50\times10^{3}\ \text{m/s})}{0.145\ \text{kg}} = \boxed{31.0\ \text{m/s}}$

(b) The kinetic energy of the bullet is

$$KE_{bullet} = \frac{1}{2}m_{bullet}v_{bullet}^2 = \frac{(3.00\times10^{-3}\ \text{kg})(1.50\times10^{3}\ \text{m/s})^2}{2} = 3.38\times10^{3}\ \text{J}$$

while that of the baseball is $KE_{ball} = \dfrac{1}{2}m_{ball}v_{ball}^2 = \dfrac{(0.145\ \text{kg})(31.0\ \text{m/s})^2}{2} = 69.7\ \text{J}$

The $\boxed{\text{bullet has the larger kinetic energy}}$ by a factor of 48.4.

6.6 From the impulse-momentum theorem, $F_{av}(\Delta t) = \Delta p = mv_f - mv_i$

Thus, $F_{av} = \dfrac{m(v_f - v_i)}{(\Delta t)} = \dfrac{(55\times10^{-3}\ \text{kg})(2.0\times10^{2}\ \text{ft/s} - 0)}{0.0020\ \text{s} - 0}\left(\dfrac{1\ \text{m/s}}{3.281\ \text{ft/s}}\right) = \boxed{1.7\ \text{kN}}$

6.7 If the diver starts from rest and drops vertically into the water, the velocity just before impact is found from

$$\left(KE + PE_g\right)_f = \left(KE + PE_g\right)_i$$

$$\frac{1}{2}mv_{impact}^2 + 0 = 0 + mgh \quad \Rightarrow \quad v_{impact} = \sqrt{2gh}$$

With the diver at rest after an impact time of Δt, the average force during impact is given by $F_{av} = \dfrac{m\left(0 - v_{impact}\right)}{\Delta t} = \dfrac{-m\sqrt{2gh}}{\Delta t}$ or $F_{av} = \dfrac{m\sqrt{2gh}}{\Delta t}$ (directed upward)

Assuming a mass of 55 kg and an impact time of ~1.0 s, the magnitude of this average force is

$$F_{av} = \frac{(55\text{ kg})\sqrt{2\left(9.80\text{ m/s}^2\right)(10\text{ m})}}{1.0\text{ s}} = 770\text{ N, or } \boxed{\sim 10^3\text{ N}}$$

6.8 The speed just before impact is given by $\left(KE + PE_g\right)_f = \left(KE + PE_g\right)_i$ as

$$\frac{1}{2}mv_{impact}^2 + 0 = 0 + mgh\text{, or } v_{impact} = \sqrt{2gh}$$

The time required for the stuntman to travel distance d as the mattresses bring him to rest is

$$\Delta t = \frac{d}{v_{av}} = \frac{d}{\left(0 + v_{impact}\right)/2} = \frac{2d}{v_{impact}} = \frac{2d}{\sqrt{2gh}}$$

Taking upward as positive, the impulse-momentum theorem gives the average net force exerted on the stuntman as he comes to rest as

$$\left(\vec{\mathbf{F}}_{av}\right)_{net} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{0 - m\left(-v_{impact}\right)}{\Delta t} = \frac{+m\sqrt{2gh}}{2d/\sqrt{2gh}} = \frac{mgh}{d} = \frac{(75.0\text{ kg})\left(9.80\text{ m/s}^2\right)(25.0\text{ m})}{1.00\text{ m}} = +1.84 \times 10^4\text{ N}$$

or $\left(\vec{\mathbf{F}}_{av}\right)_{net} = 1.84 \times 10^4\text{ N upward}$. But, this net upward force is the sum of an upward force exerted by the mattresses and the downward gravitational force, $F_{net} = +F_{mattress} - mg$. Thus, the average upward force exerted by the mattresses is

$$F_{mattress} = F_{net} + mg = 1.84 \times 10^4\text{ N} + (75.0\text{ kg})\left(9.80\text{ m/s}^2\right) = \boxed{1.91 \times 10^4\text{ N}}$$

6.9 $I = F_{av}(\Delta t) = \Delta p = m(\Delta v)$

Thus, $|I| = m|\Delta v| = (70.0 \text{ kg})(5.20 \text{ m/s} - 0) = \boxed{364 \text{ kg}\cdot\text{m/s}}$, and

$$F_{av} = \frac{I}{\Delta t} = \frac{364 \text{ kg}\cdot\text{m/s}}{0.832 \text{ s}} = 438 \text{ kg}\cdot\text{m/s}^2$$

or $\vec{\mathbf{F}}_{av} = \boxed{438 \text{ N directed forward}}$

6.10 Choose toward the east as the positive direction.

(a) The impulse delivered to the ball as it is caught is

$$\vec{\mathbf{I}} = \Delta\vec{\mathbf{p}} = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i = 0 - (0.500 \text{ kg})(+15.0 \text{ m/s}) = -7.50 \text{ kg}\cdot\text{m/s}$$

or $\vec{\mathbf{I}} = \boxed{7.50 \text{ kg}\cdot\text{m/s westward}}$

(b) The average force exerted by the ball on the receiver is the negative of the average force exerted by the receiver on the ball, or

$$\left(\vec{\mathbf{F}}_{av}\right)_{receiver} = -\left(\vec{\mathbf{F}}_{av}\right)_{ball} = -\frac{\vec{\mathbf{I}}}{\Delta t} = -\left(\frac{-7.50 \text{ kg}\cdot\text{m/s}}{0.0200 \text{ s}}\right) = +375 \text{ N}$$

$$\left(\vec{\mathbf{F}}_{av}\right)_{receiver} = \boxed{375 \text{ N eastward}}$$

6.11 (a) The impulse equals the area under the F versus t graph. This area is the sum of the area of the rectangle plus the area of the triangle. Thus,

$$I = (2.0 \text{ N})(3.0 \text{ s}) + \frac{1}{2}(2.0 \text{ N})(2.0 \text{ s}) = \boxed{8.0 \text{ N}\cdot\text{s}}$$

(b) $I = F_{av}(\Delta t) = \Delta p = m(v_f - v_i)$

$$8.0 \text{ N}\cdot\text{s} = (1.5 \text{ kg})v_f - 0, \text{ giving } v_f = \boxed{5.3 \text{ m/s}}$$

(c) $I = F_{\text{av}}(\Delta t) = \Delta p = m(v_f - v_i)$, so $v_f = v_i + \dfrac{I}{m}$

$$v_f = -2.0 \text{ m/s} + \frac{8.0 \text{ N}\cdot\text{s}}{1.5 \text{ kg}} = \boxed{3.3 \text{ m/s}}$$

6.12 (a) Impulse = area under curve = (two triangular areas of altitude 4.00 N and base 2.00 s) + (one rectangular area of width 1.00 s and height of 4.00 N.)

Thus, $$I = 2\left[\frac{(4.00 \text{ N})(2.00 \text{ s})}{2}\right] + (4.00 \text{ N})(1.00 \text{ s}) = \boxed{12.0 \text{ N}\cdot\text{s}}$$

(b) $I = F_{\text{av}}(\Delta t) = \Delta p = m(v_f - v_i)$, so $v_f = v_i + \dfrac{I}{m}$

$$v_f = 0 + \frac{12.0 \text{ N}\cdot\text{s}}{2.00 \text{ kg}} = \boxed{6.00 \text{ m/s}}$$

(c) $$v_f = v_i + \frac{I}{m} = -2.00 \text{ m/s} + \frac{12.0 \text{ N}\cdot\text{s}}{2.00 \text{ kg}} = \boxed{4.00 \text{ m/s}}$$

6.13 (a) The impulse is the area under the curve between 0 and 3.0 s.

This is: $I = (4.0 \text{ N})(3.0 \text{ s}) = \boxed{12 \text{ N}\cdot\text{s}}$

(b) The area under the curve between 0 and 5.0 s is:

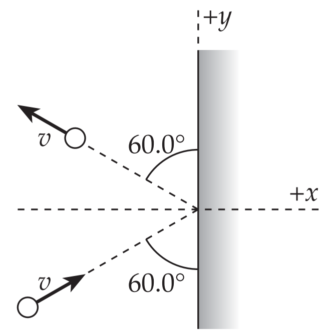
$$I = (4.0 \text{ N})(3.0 \text{ s}) + (-2.0 \text{ N})(2.0 \text{ s}) = \boxed{8.0 \text{ N}\cdot\text{s}}$$

(c) $I = F_{\text{av}}(\Delta t) = \Delta p = m(v_f - v_i)$, so $v_f = v_i + \dfrac{I}{m}$

at 3.0 s: $$v_f = v_i + \frac{I}{m} = 0 + \frac{12 \text{ N}\cdot\text{s}}{1.50 \text{ kg}} = \boxed{8.0 \text{ m/s}}$$

at 5.0 s: $$v_f = v_i + \frac{I}{m} = 0 + \frac{8.0 \text{ N}\cdot\text{s}}{1.50 \text{ kg}} = \boxed{5.3 \text{ m/s}}$$

6.14 $\vec{\mathbf{F}}_{\text{av}} = \dfrac{\Delta\vec{\mathbf{p}}}{\Delta t}$ so $\left(F_{\text{av}}\right)_x = \dfrac{\Delta p_x}{\Delta t}$ and $\left(F_{\text{av}}\right)_y = \dfrac{\Delta p_y}{\Delta t}$



$$\left(F_{\text{av}}\right)_y = \frac{m\left[\left(v_y\right)_f - \left(v_y\right)_i\right]}{\Delta t} = \frac{m\left[v\cos 60.0° - v\cos 60.0°\right]}{\Delta t} = 0$$

$$\left(F_{\text{av}}\right)_x = \frac{m\left[\left(v_x\right)_f - \left(v_x\right)_i\right]}{\Delta t} = \frac{m\left[\left(-v\sin 60.0°\right) - \left(+v\sin 60.0°\right)\right]}{\Delta t}$$

$$= \frac{-2mv\sin 60.0°}{\Delta t} = \frac{-2(3.00\text{ kg})(10.0\text{ m/s})\sin 60.0°}{0.200\text{ s}} = -260\text{ N}$$

Thus, $\vec{\mathbf{F}}_{\text{av}} = \boxed{\text{260 N in the negative } x\text{-direction or perpendicular to the wall}}$

6.15 (a) $\Delta t = \dfrac{\Delta x}{v_{\text{av}}} = \dfrac{2(\Delta x)}{v_f + v_i} = \dfrac{2(1.20\text{ m})}{0 + 25.0\text{ m/s}} = \boxed{9.60 \times 10^{-2}\text{ s}}$

(b) $F_{\text{av}} = \dfrac{\Delta p}{\Delta t} = \dfrac{m(\Delta v)}{\Delta t} = \dfrac{(1\,400\text{ kg})(25.0\text{ m/s})}{9.60 \times 10^{-2}\text{ s}} = \boxed{3.65 \times 10^5\text{ N}}$

(c) $a_{\text{av}} = \dfrac{\Delta v}{\Delta t} = \dfrac{25.0\text{ m/s}}{9.60 \times 10^{-2}\text{ s}} = 260\text{ m/s}^2 = \left(260\text{ m/s}^2\right)\left(\dfrac{1\,g}{9.80\text{ m/s}^2}\right) = \boxed{26.6\,g}$

6.16 Choose the positive direction to be from the pitcher toward home plate.

(a) $\vec{\mathbf{I}} = \vec{\mathbf{F}}_{\text{av}}(\Delta t) = \Delta\vec{\mathbf{p}} = m\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i\right) = (0.15\text{ kg})\left[(-22\text{ m/s}) - (20\text{ m/s})\right]$

$\vec{\mathbf{I}} = \vec{\mathbf{F}}_{\text{av}}(\Delta t) = -6.3\text{ kg}\cdot\text{m/s}$ or $\boxed{6.3\text{ kg}\cdot\text{m/s toward the pitcher}}$

(b) $\vec{\mathbf{F}}_{\text{av}} = \dfrac{\vec{\mathbf{I}}}{\Delta t} = \dfrac{-6.3\text{ kg}\cdot\text{m/s}}{2.0 \times 10^{-3}\text{ s}} = -3.2 \times 10^3\text{ N}$

or $\vec{\mathbf{F}}_{\text{av}} = \boxed{3.2 \times 10^3\text{ N toward the pitcher}}$

6.17 Choose eastward as the positive x-direction.

$$\left(F_x\right)_{\text{av}} = \frac{\Delta p_x}{\Delta t} = \frac{m\left(\Delta v_x\right)}{\Delta t} = \frac{\left(1.6\times10^3\text{ kg}\right)\left[0-25\text{ m/s}\right]}{6.0\text{ s}} = -6.7\times10^3\text{ N}$$

or $\vec{\mathbf{F}}_{\text{av}} = \boxed{6.7\times10^3\text{ N westward}}$

6.18 We shall choose southward as the positive direction.

The mass of the man is $m = \dfrac{w}{g} = \dfrac{730\text{ N}}{9.80\text{ m/s}^2} = 74.5\text{ kg}$. Then, from conservation of momentum, we find

$$\left(m_{man}v_{man} + m_{book}v_{book}\right)_f = \left(m_{man}v_{man} + m_{book}v_{book}\right)_i \text{ or}$$

$$\left(74.5\text{ kg}\right)v_{man} + \left(1.2\text{ kg}\right)\left(-5.0\text{ m/s}\right) = 0+0 \text{ and } v_{man} = 8.1\times10^{-2}\text{ m/s}$$

Therefore, the time required to travel the 5.0 m to shore is

$$t = \frac{\Delta x}{v_{man}} = \frac{5.0\text{ m}}{8.1\times10^{-2}\text{ m/s}} = \boxed{62\text{ s}}$$

6.19 Requiring that total momentum be conserved gives

$$\left(m_{club}v_{club} + m_{ball}v_{ball}\right)_f = \left(m_{club}v_{club} + m_{ball}v_{ball}\right)_i$$

or $\left(200\text{ g}\right)\left(40\text{ m/s}\right) + \left(46\text{ g}\right)v_{ball} = \left(200\text{ g}\right)\left(55\text{ m/s}\right) + 0$

and $v_{ball} = \boxed{65\text{ m/s}}$

6.20 (a) The mass of the rifle is $m = \dfrac{w}{g} = \dfrac{30\text{ N}}{9.80\text{ m/s}^2} = 3.1\text{ kg}$. We choose the direction of the bullet's motion to be negative. Then, conservation of momentum gives

$$\left(m_{rifle}v_{rifle} + m_{bullet}v_{bullet}\right)_f = \left(m_{rifle}v_{rifle} + m_{bullet}v_{bullet}\right)_i$$

or $\left(3.1\text{ kg}\right)v_{rifle} + \left(5.0\times10^{-3}\text{ kg}\right)\left(-300\text{ m/s}\right) = 0+0$ and $v_{rifle} = \boxed{0.49\text{ m/s}}$

(b) The mass of the man plus rifle is $m = \dfrac{730 \text{ N}}{9.80 \text{ m/s}^2} = 74.5 \text{ kg}$. We use the same approach as in (a), to find $v = \left(\dfrac{5.0 \times 10^{-3} \text{ kg}}{74.5 \text{ kg}}\right)(300 \text{ m/s}) = \boxed{2.0 \times 10^{-2} \text{ m/s}}$

6.21 The velocity of the girl relative to the ice, v_{GI}, is $v_{GI} = v_{GP} + v_{PI}$ where
v_{GP} = velocity of girl relative to plank, and
v_{PI} = velocity of plank relative to ice. Since we are given that
$v_{GP} = 1.50 \text{ m/s}$, this becomes

$$v_{GI} = 1.50 \text{ m/s} + v_{PI} \tag{1}$$

(a) Conservation of momentum gives $m_G v_{GI} + m_P v_{PI} = 0$, or $v_{PI} = -\left(\dfrac{m_G}{m_P}\right) v_{GI}$ (2)

Then, Equation (1) becomes $\left(1 + \dfrac{m_G}{m_P}\right) v_{GI} = 1.50 \text{ m/s}$

or $v_{GI} = \dfrac{1.50 \text{ m/s}}{1 + \left(\dfrac{45.0 \text{ kg}}{150 \text{ kg}}\right)} = \boxed{1.15 \text{ m/s}}$

(b) Then, using (2) above, $v_{PI} = -\left(\dfrac{45.0 \text{ kg}}{150 \text{ kg}}\right)(1.15 \text{ m/s}) = -0.346 \text{ m/s}$

or $v_{PI} = \boxed{0.346 \text{ m/s directed opposite to the girl's motion}}$

6.22 Consider the thrower first, with velocity after the throw of $v_{thrower}$. Applying conservation of momentum yields

$$(65.0\ \text{kg})v_{thrower} + (0.045\,0\ \text{kg})(30.0\ \text{m/s}) = (65.0\ \text{kg} + 0.045\,0\ \text{kg})(2.50\ \text{m/s})$$

or $$v_{thrower} = \boxed{2.48\ \text{m/s}}$$

Now, consider the (catcher + ball), with velocity of $v_{catcher}$ after the catch. From momentum conservation,

$$(60.0\ \text{kg} + 0.045\,0\ \text{kg})v_{catcher} = (0.045\,0\ \text{kg})(30.0\ \text{m/s}) + (60.0\ \text{kg})(0)$$

or $$v_{catcher} = \boxed{2.25 \times 10^{-2}\ \text{m/s}}$$

6.23 The ratio of the kinetic energy of the Earth to that of the ball is

$$\frac{KE_E}{KE_b} = \frac{\frac{1}{2}m_E v_E^2}{\frac{1}{2}m_b v_b^2} = \left(\frac{m_E}{m_b}\right)\left(\frac{v_E}{v_b}\right)^2 \tag{1}$$

From conservation of momentum,

$$p_f = p_i = 0\text{, giving } m_E v_E + m_b v_b = 0 \text{ or } \frac{v_E}{v_b} = \boxed{-\frac{m_b}{m_E}}$$

Equation (1) then becomes $\dfrac{KE_E}{KE_b} = \left(\dfrac{m_E}{m_b}\right)\left(-\dfrac{m_b}{m_E}\right)^2 = \boxed{\dfrac{m_b}{m_E}}$

Using order of magnitude numbers, $\dfrac{KE_E}{KE_b} = \dfrac{m_b}{m_E} \sim \dfrac{1\ \text{kg}}{10^{25}\ \text{kg}}\ \boxed{\sim 10^{-25}}$

6.24 (a) B exerts a horizontal force on A.

(b) A exerts a force on B that is opposite in direction to the force B exerts on A.

(c) The force on A is equal in magnitude to the force on B, but is oppositely directed.

(d) Yes. The momentum of the system (the two skaters) is conserved because the net external force on the system is zero (neglecting friction).

(e) $\left(\Delta p_x\right)_{system} = \left(\Delta p_x\right)_A + \left(\Delta p_x\right)_B = 0 \quad \Rightarrow \quad m_A\left(v_A - 0\right) + m_B\left(v_B - 0\right) = 0$

or $v_A = -\left(\dfrac{m_B}{m_A}\right) v_B = -\left(\dfrac{\cancel{m_B}}{0.900\,\cancel{m_B}}\right)\left(2.00\ \text{m/s}\right) = -2.22\ \text{m/s}$

$\vec{\mathbf{v}}_A = \boxed{2.22\ \text{m/s in the direction opposite to } \vec{\mathbf{v}}_B}$

6.25 Consider a system consisting of arrow and target from the instant just before impact until the instant after the arrow emerges from the target. No external horizontal forces act on the system, so total horizontal momentum must be conserved, or

$$\left(m_a v_a + m_t v_t\right)_f = \left(m_a v_a + m_t v_t\right)_i$$

Thus, $$\left(v_a\right)_f = \frac{m_a\left(v_a\right)_i + m_t\left(v_t\right)_i - m_t\left(v_t\right)_f}{m_a}$$

$$= \frac{\left(22.5\ \text{g}\right)\left(+35.0\ \text{m/s}\right) + \left(300\ \text{g}\right)\left(-2.50\ \text{m/s}\right) - 0}{22.5\ \text{g}} = \boxed{1.67\ \text{m/s}}$$

6.26 For each skater, the impulse-momentum theorem gives

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{m\left(\Delta v\right)}{\Delta t} = \frac{\left(75.0\ \text{kg}\right)\left(5.00\ \text{m/s}\right)}{0.100\ \text{s}} = \boxed{3.75 \times 10^3\ \text{N}}$$

Since $F_{\text{av}} < 4500\ \text{N}$, there are $\boxed{\text{no broken bones}}$

6.27 (a) If M is the mass of a single car, conservation of momentum gives

$$\left(3M\right)v_f = M\left(3.00\ \text{m/s}\right) + \left(2M\right)\left(1.20\ \text{m/s}\right), \text{ or } v_f = \boxed{1.80\ \text{m/s}}$$

(b) The kinetic energy lost is $KE_{lost} = KE_i - KE_f$, or

$$KE_{lost} = \frac{1}{2}M\left(3.00\ \text{m/s}\right)^2 + \frac{1}{2}\left(2M\right)\left(1.20\ \text{m/s}\right)^2 - \frac{1}{2}\left(3M\right)\left(1.80\ \text{m/s}\right)^2$$

With $M = 2.00 \times 10^4\ \text{kg}$, this yields $KE_{lost} = \boxed{2.16 \times 10^4\ \text{J}}$

6.28 Let us apply conservation of energy to the block from the time just after the bullet has passed through until it reaches maximum height in order to find its speed V just after the collision.

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f \text{ becomes } \frac{1}{2}mV^2 + 0 = 0 + mgy_f$$

or $$V = \sqrt{2gy_f} = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(0.120 \text{ m}\right)} = 1.53 \text{ m/s}$$

Now use conservation of momentum from before until just after the collision in order to find the initial speed of the bullet, v.

$$\left(7.0\times10^{-3} \text{ kg}\right)v + 0 = \left(1.5 \text{ kg}\right)\left(1.53 \text{ m/s}\right) + \left(7.0\times10^{-3} \text{ kg}\right)\left(200 \text{ m/s}\right)$$

from which $v = \boxed{5.3\times10^2 \text{ m/s}}$

6.29 Let M = mass of ball, m = mass of bullet, v = velocity of bullet, and V = the initial velocity of the ball-bullet combination. Then, using conservation of momentum from just before to just after collision gives

$$\left(M+m\right)V = mv + 0 \quad \text{or} \quad V = \left(\frac{m}{M+m}\right)v$$

Now, we use conservation of mechanical energy from just after the collision until the ball reaches maximum height to find

$$0 + \left(M+m\right)g\,h_{\max} = \frac{1}{2}\left(M+m\right)V^2 + 0 \quad \text{or} \quad h_{\max} = \frac{V^2}{2g} = \frac{1}{2g}\left(\frac{m}{M+m}\right)^2 v^2$$

With the data values provided, this becomes

$$h_{\max} = \frac{1}{2\left(9.80 \text{ m/s}^2\right)}\left(\frac{0.030 \text{ kg}}{0.15 \text{ kg} + 0.030 \text{ kg}}\right)^2 \left(200 \text{ m/s}\right)^2 = \boxed{57 \text{ m}}$$

6.30 First, we will find the horizontal speed, v_{0x}, of the block and embedded bullet just after impact. After this instant, the block-bullet combination is a projectile, and we find the time to reach the floor by use of $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, which becomes

$$-1.00\ \text{m} = 0 + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t^2\text{, giving} \qquad t = 0.452\ \text{s}$$

Thus, $v_{0x} = \dfrac{\Delta x}{t} = \dfrac{2.00\ \text{m}}{0.452\ \text{s}} = 4.43\ \text{m/s}$

Now use conservation of momentum for the collision, with v_b = speed of incoming bullet:

$$\left(8.00\times10^{-3}\ \text{kg}\right)v_b + 0 = \left(258\times10^{-3}\ \text{kg}\right)\left(4.43\ \text{m/s}\right)\text{, so}$$

$$v_b = \boxed{143\ \text{m/s}} \qquad \text{(about 320 mph)}$$

6.31 When Gayle jumps on the sled, conservation of momentum gives

$$\left(50.0\ \text{kg} + 5.00\ \text{kg}\right)v_2 = \left(50.0\ \text{kg}\right)\left(4.00\ \text{m/s}\right) + 0\text{, or } v_2 = 3.64\ \text{m/s}$$

After Gayle and the sled glide down 5.00 m, conservation of mechanical energy gives

$$\frac{1}{2}\left(55.0\ \text{kg}\right)v_3^2 + 0 = \frac{1}{2}\left(55.0\ \text{kg}\right)\left(3.64\ \text{m/s}\right)^2 + \left(55.0\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\left(5.00\ \text{m}\right)$$

so $\qquad v_3 = 10.5\ \text{m/s}$

After her Brother jumps on, conservation of momentum yields

$$\left(55.0\ \text{kg} + 30.0\ \text{kg}\right)v_4 = \left(55.0\ \text{kg}\right)\left(10.50\ \text{m/s}\right) + 0\text{, and } v_4 = 6.82\ \text{m/s}$$

After all slide an additional 10.0 m down, conservation of mechanical energy gives the final speed as

$$\frac{1}{2}\left(85.0\ \text{kg}\right)v_5^2 + 0 = \frac{1}{2}\left(85.0\ \text{kg}\right)\left(6.82\ \text{m/s}\right)^2 + \left(85.0\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\left(10.0\ \text{m}\right)$$

or $\qquad v_5 = \boxed{15.6\ \text{m/s}}$

6.32 (a) Conservation of momentum gives $m_T v_{fT} + m_c v_{fc} = m_T v_{iT} + m_c v_{ic}$, or

$$v_{fT} = \frac{m_T v_{iT} + m_c\left(v_{ic} - v_{fc}\right)}{m_T}$$

$$= \frac{(9\,000 \text{ kg})(20.0 \text{ m/s}) + (1\,200 \text{ kg})\left[(25.0 - 18.0) \text{ m/s}\right]}{9000 \text{ kg}}$$

$$v_{fT} = \boxed{20.9 \text{ m/s East}}$$

(b) $$KE_{lost} = KE_i - KE_f = \left[\frac{1}{2}m_c v_{ic}^2 + \frac{1}{2}m_T v_{iT}^2\right] - \left[\frac{1}{2}m_c v_{fc}^2 + \frac{1}{2}m_T v_{fT}^2\right]$$

$$= \frac{1}{2}\left[m_c\left(v_{ic}^2 - v_{fc}^2\right) + m_T\left(v_{iT}^2 - v_{fT}^2\right)\right]$$

$$= \frac{1}{2}\left[(1\,200 \text{ kg})(625 - 324)\left(\text{m}^2/\text{s}^2\right) + (9\,000 \text{ kg})(400 - 438.2)\left(\text{m}^2/\text{s}^2\right)\right]$$

$$KE_{lost} = \boxed{8.68 \times 10^3 \text{ J, which becomes internal energy}}$$

Note: If 20.9 m/s were used to determine the energy lost instead of 20.9333, the answer would be very different. We keep extra digits in all intermediate answers until the problem is complete.

6.33 First, we use conservation of mechanical energy to find the speed of the block and embedded bullet just after impact:

$$\left(KE + PE_s\right)_f = \left(KE + PE_s\right)_i \text{ becomes } \frac{1}{2}(m+M)V^2 + 0 = 0 + \frac{1}{2}kx^2$$

and yields $$V = \sqrt{\frac{kx^2}{m+M}} = \sqrt{\frac{(150 \text{ N/m})(0.800 \text{ m})^2}{(0.0120 + 0.100) \text{ kg}}} = 29.3 \text{ m/s}$$

Now, employ conservation of momentum to find the speed of the bullet just before impact: $mv + M(0) = (m+M)V$,

or $$v = \left(\frac{m+M}{m}\right)V = \left(\frac{0.112 \text{ kg}}{0.0120 \text{ kg}}\right)(29.3 \text{ m/s}) = \boxed{273 \text{ m/s}}$$

6.34 (a) Using conservation of momentum, $\left(\Sigma\vec{\mathbf{p}}\right)_{after} = \left(\Sigma\vec{\mathbf{p}}\right)_{before}$, gives

$$\left[(4.0+10+3.0)\ \text{kg}\right]v = (4.0\ \text{kg})(5.0\ \text{m/s}) + (10\ \text{kg})(3.0\ \text{m/s}) + (3.0\ \text{kg})(-4.0\ \text{m/s})$$

Therefore, $v = +2.2\ \text{m/s}$, or $\boxed{2.2\ \text{m/s toward the right}}$

(b) $\boxed{\text{No}}$. For example, if the 10-kg and 3.0-kg mass were to stick together first, they would move with a speed given by solving

$$(13\ \text{kg})v_1 = (10\ \text{kg})(3.0\ \text{m/s}) + (3.0\ \text{kg})(-4.0\ \text{m/s}),\ \text{or}\ v_1 = +1.38\ \text{m/s}$$

Then when this 13 kg combined mass collides with the 4.0 kg mass, we have

$$(17\ \text{kg})v = (13\ \text{kg})(1.38\ \text{m/s}) + (4.0\ \text{kg})(5.0\ \text{m/s}),\ \text{and}\ v = +2.2\ \text{m/s}$$

just as in part (a).

6.35 (a) From conservation of momentum,

$$(5.00\ \text{g})v_{1f} + (10.0\ \text{g})v_{2f} = (5.00\ \text{g})(20.0\ \text{cm/s}) + 0 \qquad (1)$$

Also for an elastic, head-on, collision, we have $v_{1i} + v_{1f} = v_{2i} + v_{2f}$, which becomes $20.0\ \text{cm/s} + v_{1f} = v_{2f}$. (2)

Solving (1) and (2) simultaneously yields

$$v_{1f} = \boxed{-6.67\ \text{cm/s}},\ \text{and}\ v_{2f} = \boxed{13.3\ \text{cm/s}}$$

(b) $KE_i = KE_{1i} + KE_{2i} = \frac{1}{2}\left(5.00\times10^{-3}\ \text{kg}\right)(0.200\ \text{m/s})^2 + 0 = 1.00\times10^{-4}\ \text{J}$

$$KE_{2f} = \frac{1}{2}m_2 v_{2f}^2 = \frac{1}{2}\left(10.0\times10^{-3}\ \text{kg}\right)\left(13.3\times10^{-2}\ \text{m/s}\right)^2 = 8.89\times10^{-5}\ \text{J},\ \text{so}$$

$$\frac{KE_{2f}}{KE_i} = \frac{8.89\times10^{-5}\ \text{J}}{1.00\times10^{-4}\ \text{J}} = \boxed{0.889}$$

6.36 Using conservation of momentum gives

$$(10.0\text{ g})v_{1f} + (15.0\text{ g})v_{2f} = (10.0\text{ g})(20.0\text{ cm/s}) + (15.0\text{ g})(-30.0\text{ cm/s}) \quad (1)$$

For elastic, head on collisions, $v_{1i} + v_{1f} = v_{2i} + v_{2f}$ which becomes

$$20.0\text{ cm/s} + v_{1f} = -30.0\text{ cm/s} + v_{2f} \quad (2)$$

Solving (1) and (2) simultaneously gives $v_{1f} = \boxed{-40.0\text{ cm/s}}$,

and $v_{2f} = \boxed{10.0\text{ cm/s}}$

6.37 Conservation of momentum gives

$$(25.0\text{ g})v_{1f} + (10.0\text{ g})v_{2f} = (25.0\text{ g})(20.0\text{ cm/s}) + (10.0\text{ g})(15.0\text{ cm/s}) \quad (1)$$

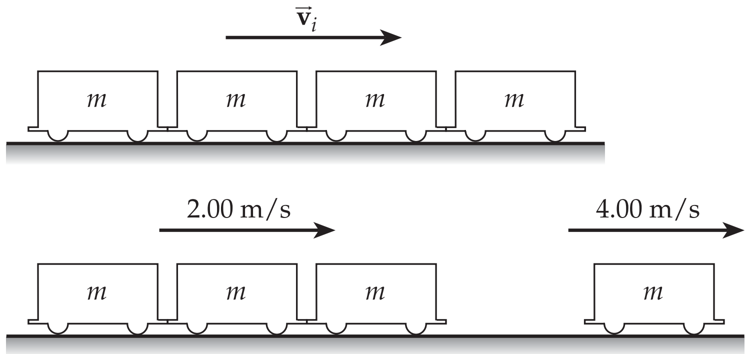
For head-on, elastic collisions, we know that $v_{1i} + v_{1f} = v_{2i} + v_{2f}$.

Thus, $20.0\text{ cm/s} + v_{1f} = 15.0\text{ cm/s} + v_{2f}$ (2)

Solving (1) and (2) simultaneously yields

$v_{1f} = \boxed{17.1\text{ cm/s}}$, and $v_{2f} = \boxed{22.1\text{ cm/s}}$

6.38 (a) The internal forces exerted by the actor do not change total momentum.



From conservation of momentum

$$(4m)v_i = (3m)(2.00 \text{ m/s}) + m(4.00 \text{ m/s})$$

$$v_i = \frac{6.00 \text{ m/s} + 4.00 \text{ m/s}}{4} = \boxed{2.50 \text{ m/s}}$$

(b) $W_{actor} = K_f - K_i = \frac{1}{2}\left[(3m)(2.00 \text{ m/s})^2 + m(4.00 \text{ m/s})^2\right] - \frac{1}{2}(4m)(2.50 \text{ m/s})^2$

$$W_{actor} = \frac{(2.50 \times 10^4 \text{ kg})}{2}\left[12.0 + 16.0 - 25.0\right](\text{m/s})^2 = \boxed{3.75 \times 10^4 \text{ J}}$$

6.39 We assume equal firing speeds v and equal forces F required for the two bullets to push wood fibers apart. These forces are directed opposite to the bullets displacements through the fibers.

When the block is held in the vise, $W_{net} = KE_f - KE_i$ gives

$$F(8.00 \times 10^{-2} \text{ m})\cos 180° = 0 - \frac{1}{2}(7.00 \times 10^{-3} \text{ kg})v^2$$

or $\frac{1}{2}(7.00 \times 10^{-3} \text{ kg})v^2 = (8.00 \times 10^{-2} \text{ m})F$ (1)

When a second 7.00-g bullet is fired into the block, now on a frictionless surface, conservation of momentum yields

$(1.014 \text{ kg})v_f = (7.00 \times 10^{-3} \text{ kg})v + 0$, or $v_f = \left(\frac{7.00 \times 10^{-3}}{1.014}\right)v$ (2)

Also, applying the work-energy theorem to the second impact,

$$F\,d\cos 180° = \frac{1}{2}(1.014\ \text{kg})v_f^2 - \frac{1}{2}\left(7.00\times10^{-3}\ \text{kg}\right)v^2 \tag{3}$$

Substituting (2) into (3), we obtain

$$-F\,d = \frac{1}{2}(1.014\ \text{kg})\left(\frac{7.00\times10^{-3}}{1.014}\right)^2 v^2 - \frac{1}{2}\left(7.00\times10^{-3}\ \text{kg}\right)v^2$$

or $$F\,d = \left[\frac{1}{2}\left(7.00\times10^{-3}\ \text{kg}\right)v^2\right]\left(1-\frac{7.00\times10^{-3}}{1.014}\right) \tag{4}$$

Finally, substituting (1) into (4) gives

$$F\,d = \left[\left(8.00\times10^{-2}\ \text{m}\right)F\right]\left(1-\frac{7.00\times10^{-3}}{1.014}\right)\text{, or } d = 7.94\times10^{-2}\ \text{m} = \boxed{7.94\ \text{cm}}$$

6.40 First, consider conservation of momentum and write

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Since $m_1 = m_2$, this becomes $v_{1i} + v_{2i} = v_{1f} + v_{2f}$. (1)

For an elastic head-on collision, we also have $v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right)$,

which may be written as $v_{1i} + v_{2i} = v_{1f} + v_{2f}$ (2)

Subtracting Equation (2) from (1) gives $v_{2f} = v_{1i}$ (3)

Adding Equations (1) and (2) yields $v_{1f} = v_{2i}$ (4)

Equations (3) and (4) show us that, under the conditions of equal mass objects striking one another in a head-on elastic collision, the two objects simply exchange velocities. Thus, we may write the results of the various collisions as

(a) $v_{1f} = \boxed{0}$, $v_{2f} = \boxed{1.50\ \text{m/s}}$

(b) $v_{1f} = \boxed{-1.00\ \text{m/s}}$, $v_{2f} = \boxed{1.50\ \text{m/s}}$

(c) $v_{1f} = \boxed{1.00\ \text{m/s}}$, $v_{2f} = \boxed{1.50\ \text{m/s}}$

6.41 Choose the $+x$-axis to be eastward and the $+y$-axis northward.

(a) First, we conserve momentum in the x direction to find

$$(185\text{ kg})V\cos\theta = (90\text{ kg})(5.0\text{ m/s}),\text{ or } V\cos\theta = \left(\frac{90}{185}\right)(5.0\text{ m/s}) \quad (1)$$

Conservation of momentum in the y direction gives

$$(185\text{ kg})V\sin\theta = (95\text{ kg})(3.0\text{ m/s}),\text{ or } V\sin\theta = \left(\frac{95}{185}\right)(3.0\text{ m/s}) \quad (2)$$

Divide equation (2) by (1) to obtain $\tan\theta = \dfrac{(95)(3.0)}{(90)(5.0)}$, and $\theta = \boxed{32°}$

Then, either (1) or (2) gives $V = 2.88\text{ m/s}$, which rounds to $V = \boxed{2.9\text{ m/s}}$

(b) $KE_{lost} = KE_i - KE_f$

$$= \frac{1}{2}\left[(90\text{ kg})(5.0\text{ m/s})^2 + (95\text{ kg})(3.0\text{ m/s})^2 - (185\text{ kg})(2.88\text{ m/s})^2\right]$$

$= \boxed{7.9\times10^2\text{ J}}$ converted into internal energy

6.42 Choose the $+x$-axis to be eastward and the $+y$-axis northward.

(a) Conserving momentum in the x direction gives

$$0 + (10.0\text{ kg})v_{2x} = (8.00\text{ kg})(15.0\text{ m/s}) + 0,\text{ or } v_{2x} = 12.0\text{ m/s}$$

Momentum conservation in the y direction yields

$$(8.00\text{ kg})(-4.00\text{ m/s}) + (10.0\text{ kg})v_{2y} = 0 + 0,\text{ or } v_{2y} = 3.20\text{ m/s}$$

After collision, $v_2 = \sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{154}\text{ m/s} = 12.4\text{ m/s}$

and $\theta = \tan^{-1}\left(\dfrac{v_{2y}}{v_{2x}}\right) = \tan^{-1}\left(\dfrac{3.20}{12.0}\right) = 14.9°$. Thus, the final velocity of the 10.0-kg mass is $\vec{\mathbf{v}}_2 = \boxed{12.4\text{ m/s at }14.9°\text{ N of E}}$

(b) $\dfrac{KE_{lost}}{KE_i} = \dfrac{KE_i - KE_f}{KE_i} = 1 - \dfrac{KE_f}{KE_i}$

$$= 1 - \left[\frac{(8.00)(-4.00)^2 + (10.0)\left(\sqrt{154}\right)^2}{(8.00)(15.0)^2 + 0} \right] = 0.0720$$

or $\boxed{7.20\%}$ of the original kinetic energy is lost in the collision.

6.43 Choose the $+x$-axis to be eastward and the $+y$-axis northward.

If v_i is the initial northward speed of the 3000-kg car, conservation of momentum in the y direction gives

$$0 + (3000\text{ kg})v_i = (5000\text{ kg})\left[(5.22\text{ m/s})\sin 40.0°\right], \text{ or } v_i = \boxed{5.59\text{ m/s}}$$

Observe that knowledge of the initial speed of the 2000-kg car was unnecessary for this solution.

6.44 We use conservation of momentum for both northward and eastward components.

For the eastward direction: $M(13.0\text{ m/s}) = 2MV_f \cos 55.0°$

For the northward direction: $Mv_{2i} = 2MV_f \sin 55.0°$

Divide the northward equation by the eastward equation to find:

$$v_{2i} = (13.0\text{ m/s})\tan 55.0°$$

$$= \left[(13.0\text{ m/s})\left(\frac{2.237\text{ mi/h}}{1\text{ m/s}}\right)\right]\tan 55.0° = \boxed{41.5\text{ mi/h}}$$

Thus, the driver of the north bound car was untruthful.

6.45 Choose the x-axis to be along the original line of motion.

(a) From conservation of momentum in the x direction,

$$m(5.00\ \text{m/s})+0=m(4.33\ \text{m/s})\cos 30.0°+mv_{2f}\cos\theta$$

or $v_{2f}\cos\theta=1.25\ \text{m/s}$ (1)

Conservation of momentum in the y direction gives

$$0=m(4.33\ \text{m/s})\sin 30.0°+mv_{2f}\sin\theta\text{, or } v_{2f}\sin\theta=-2.16\ \text{m/s} \qquad (2)$$

Dividing (2) by (1) gives $\tan\theta=\dfrac{-2.16}{1.25}=-1.73$ and $\theta=-60.0°$

Then, either (1) or (2) gives $v_{2f}=2.50\ \text{m/s}$, so the final velocity of the second ball is $\vec{\mathbf{v}}_{2f}=\boxed{2.50\ \text{m/s at -60.0°}}$

(b) $KE_i=\dfrac{1}{2}mv_{1i}^2+0=\dfrac{1}{2}m(5.00\ \text{m/s})^2=m(12.5\ \text{m}^2/\text{s}^2)$

$$KE_f=\frac{1}{2}mv_{1f}^2+\frac{1}{2}mv_{2f}^2$$

$$=\frac{1}{2}m(4.33\ \text{m/s})^2+\frac{1}{2}m(2.50\ \text{m/s})^2=m(12.5\ \text{m}^2/\text{s}^2)$$

Since $KE_f=KE_i$, this is an $\boxed{\text{elastic collision}}$

6.46 The recoil speed of the subject plus pallet after a heartbeat is

$$V=\frac{\Delta x}{\Delta t}=\frac{6.00\times10^{-5}\ \text{m}}{0.160\ \text{s}}=3.75\times10^{-4}\ \text{m/s}$$

From conservation of momentum, $mv-MV=0+0$, so the mass of blood leaving the heart is

$$m=M\left(\frac{V}{v}\right)=(54.0\ \text{kg})\left(\frac{3.75\times10^{-4}\ \text{m/s}}{0.500\ \text{m/s}}\right)=4.05\times10^{-2}\ \text{kg}=\boxed{40.5\ \text{g}}$$

6.47 From the Impulse-momentum theorem:

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}(\Delta t) = \Delta\vec{\mathbf{p}} = m\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i\right) \quad \Rightarrow \quad \vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}(\Delta t)}{m}$$

(a) For the first 1.5-s interval, $\vec{\mathbf{v}}_i = 0$ and $\vec{\mathbf{F}} = +3.0\text{ N}$

so $$\vec{\mathbf{v}}_f = 0 + \frac{(+3.0\text{ N})(1.5\text{ s})}{0.50\text{ kg}} = \boxed{+9.0\text{ m/s}}$$

(b) For the next 3.0-s interval, $\vec{\mathbf{v}}_i = +9.0\text{ m/s}$ and $\vec{\mathbf{F}} = -4.0\text{ N}$

giving $$\vec{\mathbf{v}}_f = 9.0\text{ m/s} + \frac{(-4.0\text{ N})(3.0\text{ s})}{0.50\text{ kg}} = \boxed{-15\text{ m/s}}$$

6.48 First, we use conservation of mechanical energy to find the speed of m_1 at B just before collision. This gives $\frac{1}{2}m_1 v_1^2 + 0 = 0 + m_1 g h_i$,

or $$v_1^2 = \sqrt{2gh_i} = \sqrt{2\left(9.80\text{ m/s}^2\right)(5.00\text{ m})} = 9.90\text{ m/s}$$

Next, we apply conservation of momentum and knowledge of elastic collisions to find the velocity of m_1 at B just after collision.

From conservation of momentum, with the second object initially at rest,

we have $m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + 0$, or $v_{2f} = \dfrac{m_1}{m_2}\left(v_{1i} - v_{1f}\right)$ (1)

For head-on elastic collisions, $v_{1f} + v_{1i} = v_{2f} + v_{2i}$. Since $v_{2i} = 0$ in this case, this becomes $v_{2f} = v_{1f} + v_{1i}$ and combining this with (1) above we obtain

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} = \left(\frac{5.00 - 10.0}{5.00 + 10.0}\right)(9.90\text{ m/s}) = -3.30\text{ m/s}$$

Finally, use conservation of mechanical energy for m_1 after the collision to find the maximum rebound height. This gives $0 + m_1 g h_{\max} = \frac{1}{2}m_1 v_{1f}^2 + 0$

or $$h_{\max} = \frac{v_{1f}^2}{2g} = \frac{(-3.30\text{ m/s})^2}{2\left(9.80\text{ m/s}^2\right)} = \boxed{0.556\text{ m}}$$

6.49 Choose the positive direction to be the direction of the truck's initial velocity.

Apply conservation of momentum to find the velocity of the combined vehicles after collision:

$$(4\,000\text{ kg}+800\text{ kg})V=(4\,000\text{ kg})(+8.00\text{ m/s})+(800\text{ kg})(-8.00\text{ m/s})$$

which yields $V=+5.33\text{ m/s}$

Use the impulse-momentum theorem, $I=F_{\text{av}}(\Delta t)=\Delta p=m(v_f-v_i)$, to find the magnitude of the average force exerted on each driver during the collision.

Truck Driver:

$$|F_{\text{av}}|=\frac{m|v_f-v_i|_{truck}}{\Delta t}=\frac{(80.0\text{ kg})|5.33\text{ m/s}-8.00\text{ m/s}|}{0.120\text{ s}}=\boxed{1.78\times10^3\text{ N}}$$

Car Driver:

$$|F_{\text{av}}|=\frac{m|v_f-v_i|_{car}}{\Delta t}=\frac{(80.0\text{ kg})|5.33\text{ m/s}-(-8.00\text{ m/s})|}{0.120\text{ s}}=\boxed{8.89\times10^3\text{ N}}$$

6.50 If the pendulum bob barely swings through a complete circle, it arrives at the top of the arc (having risen a vertical distance of 2ℓ) with essentially zero velocity.

From conservation of mechanical energy, we find the minimum velocity of the bob at the bottom of the arc as $(KE+PE_g)_{bottom}=(KE+PE_g)_{top}$, or $\frac{1}{2}MV^2=0+Mg(2\ell)$. This gives $V=2\sqrt{g\ell}$ as the needed velocity of the bob just after the collision.

Conserving momentum through the collision then gives the minimum initial velocity of the bullet as

$$m\left(\frac{v}{2}\right)+M\left(2\sqrt{g\ell}\right)=mv+0\text{, or } v=\boxed{\frac{4M}{m}\sqrt{g\ell}}$$

6.51 Note that the initial velocity of the target particle is zero (that is, $v_{2i} = 0$).

From conservation of momentum, $m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + 0$ (1)

For head-on elastic collisions, $v_{1f} + v_{1i} = v_{2f} + 0$ (2)

Solving (1) and (2) simultaneously yields the final velocities as

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} \text{ and } v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i}$$

(a) If $m_1 = 2.0$ g, $m_2 = 1.0$ g, and $v_{1i} = 8.0$ m/s, then

$$v_{1f} = \boxed{\frac{8}{3} \text{ m/s}} \text{ and } v_{2f} = \boxed{\frac{32}{3} \text{ m/s}}$$

(b) If $m_1 = 2.0$ g, $m_2 = 10$ g, and $v_{1i} = 8.0$ m/s, we find

$$v_{1f} = \boxed{-\frac{16}{3} \text{ m/s}} \text{ and } v_{2f} = \boxed{\frac{8}{3} \text{ m/s}}$$

(c) The final kinetic energy of the 2.0 g particle in each case is:

Case (a): $KE_{1f} = \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2}\left(2.0 \times 10^{-3} \text{ kg}\right)\left(\frac{8}{3} \text{ m/s}\right)^2 = 7.1 \times 10^{-3} \text{ J}$

Case (b): $KE_{1f} = \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2}\left(2.0 \times 10^{-3} \text{ kg}\right)\left(-\frac{16}{3} \text{ m/s}\right)^2 = 2.8 \times 10^{-2} \text{ J}$

Since the incident kinetic energy is the same in cases (a) and (b), we observe that $\boxed{\text{the incident particle loses more kinetic energy in case (a)}}$

6.52 Use conservation of mechanical energy, $\left(KE + PE_g\right)_B = \left(KE + PE_g\right)_A$, to find the speed of the bead at point B just before it collides with the ball. This gives $\frac{1}{2} m v_{1i}^2 + 0 = 0 + m g y_A$, or $v_{1i} = \sqrt{2 g y_A} = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(1.50 \text{ m}\right)} = 5.42 \text{ m/s}$

Conservation of momentum during the collision gives

$$(0.400\ \text{kg})v_{1f} + (0.600\ \text{kg})v_{2f} = (0.400\ \text{kg})(5.42\ \text{m/s}) + 0$$

or $$v_{1f} + 1.50v_{2f} = 5.42\ \text{m/s} \tag{1}$$

For a head-on elastic collision, we have $v_{2f} + v_{2i} = v_{1f} + v_{1i}$, which gives

$$v_{2f} = v_{1f} + 5.42\ \text{m/s} \tag{2}$$

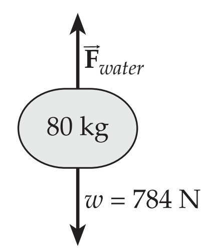
Solving (1) and (2) simultaneously, the velocities just after collision are

$$v_{1f} = -1.08\ \text{m/s} \text{ and } v_{2f} = 4.34\ \text{m/s}$$

Now, we use conservation of the mechanical energy of the ball after collision to find the maximum height the ball will reach. This gives

$$0 + Mgy_{\max} = \frac{1}{2}Mv_{2f}^2 + 0, \text{ or } y_{\max} = \frac{v_{2f}^2}{2g} = \frac{(4.34\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{0.960\ \text{m}}$$

6.53 We first find the speed of the diver when he reaches the water by using $v_y^2 = v_0^2 + 2a_y(\Delta y)$. This becomes



$$v_y^2 = 0 + 2(-9.80\ \text{m/s}^2)(-3.0\ \text{m}), \text{ and yields } v_y = -\sqrt{59}\ \text{m/s}$$

The negative sign indicates the downward direction.

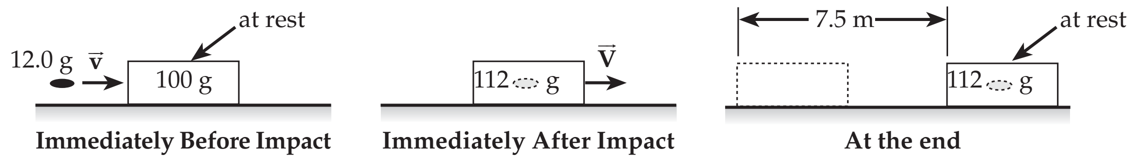
Next, we use the impulse-momentum theorem to find the resistive force exerted by the water as the diver comes to rest.

$$I = F_{net}(\Delta t) = \Delta p = m(v_f - v_i), \text{ or}$$

$$(F_{water} - 784\ \text{N})(2.0\ \text{s}) = (80\ \text{kg})\left[0 - \left(-\sqrt{59}\ \text{m/s}\right)\right], \text{ which gives}$$

$$F_{water} = 784\ \text{N} + \left(\frac{80\sqrt{59}}{2}\right)\text{N} = \boxed{1.1 \times 10^3\ \text{N}\ \ (\text{upward})}$$

6.54



Using the work-energy theorem from immediately after impact to the end gives:

$$W_{net} = F_{friction}\, s\cos 180° = KE_{end} - KE_{after}$$

or, $$-\left[\mu_k (M+m)\, g\right]s = 0 - \frac{1}{2}(M+m)V^2 \text{ and } V = \sqrt{2\mu_k\, g\, s}$$

Then, using conservation of momentum from immediately before to immediately after impact gives $mv + 0 = (M+m)V$, or

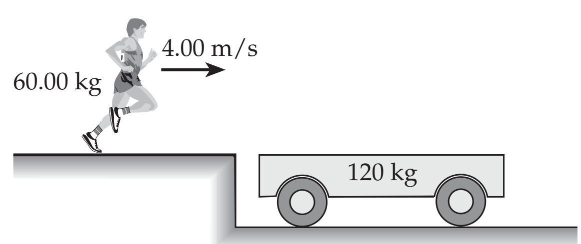
$$v = \left(\frac{M+m}{m}\right)V = \left(\frac{M+m}{m}\right)\sqrt{2\mu_k\, g\, s} = \left(\frac{112 \text{ g}}{12.0 \text{ g}}\right)\sqrt{2(0.650)\left(9.80 \text{ m/s}^2\right)(7.5 \text{ m})}$$

$$v = \boxed{91 \text{ m/s}}$$

6.55 (a) Using conservation of momentum,

$$(60.0 \text{ kg} + 120 \text{ kg})v_f = (60.0 \text{ kg})(4.00 \text{ m/s}) + 0,$$

or $$v_f = \boxed{1.33 \text{ m/s}}$$



(b) $$\Sigma F_y = n - (60.0 \text{ kg})\left(9.80 \text{ m/s}^2\right) = 0$$

so the normal force is $n = 588$ N

and $$f_k = \mu_k n = (0.400)(588 \text{ N}) = \boxed{235 \text{ N}}$$

(c) Apply the impulse-momentum theorem to the person:

$$I = F_{av}(\Delta t) = \Delta p = m\left(v_f - v_i\right)$$

so $$\Delta t = \frac{m\left(v_f - v_i\right)}{-f_k} = \frac{(60.0 \text{ kg})(1.33 \text{ m/s} - 4.00 \text{ m/s})}{-235 \text{ N}} = \boxed{0.681 \text{ s}}$$

(d) $\Delta p_{person} = m\left(v_f - v_i\right) = \left(60.0 \text{ kg}\right)\left(1.33 \text{ m/s} - 4.00 \text{ m/s}\right) = \boxed{-160 \text{ N}\cdot\text{s}}$

$\Delta p_{cart} = M\left(v_f - 0\right) = \left(120 \text{ kg}\right)\left(1.33 \text{ m/s} - 0\right) = \boxed{+160 \text{ N}\cdot\text{s}}$

(e) $\Delta x_{person} = v_{\text{av}}\left(\Delta t\right) = \left(\dfrac{v_f + v_i}{2}\right)\left(\Delta t\right)$

$$= \left(\frac{1.33 \text{ m/s} + 4.00 \text{ m/s}}{2}\right)\left(0.681 \text{ s}\right) = \boxed{1.82 \text{ m}}$$

(f) $\Delta x_{cart} = v_{\text{av}}\left(\Delta t\right) = \left(\dfrac{v_f + 0}{2}\right)\left(\Delta t\right) = \left(\dfrac{1.33 \text{ m/s}}{2}\right)\left(0.681 \text{ s}\right) = \boxed{0.454 \text{ m}}$

(g) $\Delta KE_{person} = \dfrac{1}{2}m\left(v_f^2 - v_i^2\right)$

$$= \frac{\left(60.0 \text{ kg}\right)}{2}\left[\left(1.33 \text{ m/s}\right)^2 - \left(4.00 \text{ m/s}\right)^2\right] = \boxed{-427 \text{ J}}$$

(h) $\Delta KE_{cart} = \dfrac{1}{2}M\left(v_f^2 - 0\right) = \dfrac{\left(120 \text{ kg}\right)}{2}\left[\left(1.33 \text{ m/s}\right)^2 - 0\right] = \boxed{107 \text{ J}}$

(i) Equal friction forces act through different distances on person and cart to do different amounts of work on them. This is a perfectly inelastic collision in which the total work on both person and cart together is –320 J, which becomes +320 J of internal energy.

6.56 (a) Let v_{1i} and v_{2i} be the velocities of m_1 and m_2 just before the collision. Then conservation of energy gives:

$$v_{1i} = -v_{2i} = \sqrt{2gh} = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(5.00 \text{ m}\right)} = \boxed{9.90 \text{ m/s}}$$

(b) From conservation of momentum:

$$\left(2.00\right)v_{1f} + \left(4.00\right)v_{2f} = \left(2.00\right)\left(9.90 \text{ m/s}\right) + \left(4.00\right)\left(-9.90 \text{ m/s}\right)$$

or $\left(2.00\right)v_{1f} + \left(4.00\right)v_{2f} = -19.8 \text{ m/s}$ (1)

For an elastic head-on collision, $v_{1f} + v_{1i} = v_{2f} + v_{2i}$, giving

$v_{1f} + 9.90 \text{ m/s} = v_{2f} - 9.90 \text{ m/s}$, or $v_{2f} = v_{1f} + 19.8 \text{ m/s}$ (2)

Solving (1) and (2) simultaneously gives

$$v_{1f} = \boxed{-16.5 \text{ m/s}}, \text{ and } v_{2f} = \boxed{3.30 \text{ m/s}}$$

(c) Applying conservation of energy to each block after the collision gives:

$$h_{1f} = \frac{v_{1f}^2}{2g} = \frac{(-16.5 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{13.9 \text{ m}}$$

and $h_{2f} = \dfrac{v_{2f}^2}{2g} = \dfrac{(3.30 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{0.556 \text{ m}}$

6.57 (a) Use conservation of mechanical energy to find the speed of m_1 just before collision. This gives

$$v_{1i} = \sqrt{2gh_1} = \sqrt{2(9.80 \text{ m/s}^2)(2.50 \text{ m})} = 7.00 \text{ m/s}$$

Apply conservation of momentum from just before to just after the collision:

$$(0.500 \text{ kg})v_{1f} + (1.00 \text{ kg})v_{2f} = (0.500 \text{ kg})(7.00 \text{ m/s}) + 0$$

or $\quad v_{1f} + 2v_{2f} = 7.00 \text{ m/s} \quad$ (1)

For a head-on elastic collision, $v_{1f} + v_{1i} = v_{2f} + v_{2i}$

which becomes $v_{1f} - v_{2f} = -7.00 \text{ m/s} \quad$ (2)

Solving (1) and (2) simultaneously yields

$$v_{1f} = \boxed{-2.33 \text{ m/s}}, \text{ and } v_{2f} = \boxed{4.67 \text{ m/s}}$$

(b) Apply conservation of mechanical energy to m_1 after the collision to find

$$h_1' = \frac{v_{1f}^2}{2g} = \frac{(-2.33 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{0.277 \text{ m}} \text{ (rebound height)}$$

(c) From $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, with $v_{0y} = 0$, the time for m_2 to reach the floor after it flies horizontally off the table is found to be

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-2.00 \text{ m})}{-9.80 \text{ m/s}^2}} = 0.639 \text{ s}$$

The horizontal distance traveled in this time is

$$\Delta x = v_{0x}t = (4.67 \text{ m/s})(0.639 \text{ s}) = \boxed{2.98 \text{ m}}$$

(d) After the 0.500 kg mass comes back down the incline, it flies off the table with a horizontal velocity of 2.33 m/s. The time of the flight to the floor is 0.639 s as found above and the horizontal distance traveled is

$$\Delta x = v_{0x}t = (2.33 \text{ m/s})(0.639 \text{ s}) = \boxed{1.49 \text{ m}}$$

6.58 Use conservation of mechanical energy to find the velocity, v, of Tarzan just as he reaches Jane. This gives $v = \sqrt{2gh_i} = \sqrt{2(9.80 \text{ m/s}^2)(3.00 \text{ m})} = 7.67 \text{ m/s}$

Now, use conservation of momentum to find the velocity, V, of Tarzan + Jane just after the collision. This becomes $(M+m)V = Mv + 0$, or

$$V = \left(\frac{M}{M+m}\right)v = \left(\frac{80.0 \text{ kg}}{140 \text{ kg}}\right)(7.67 \text{ m/s}) = 4.38 \text{ m/s}$$

Finally, use conservation of mechanical energy from just after he picks her up to the end of their swing to determine the maximum height, H, reached. This yields

$$H = \frac{V^2}{2g} = \frac{(4.38 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{0.980 \text{ m}}$$

6.59 (a) The momentum of the system is initially zero and remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have $m_2 v_{wedge} + m_1 v_{block} = 0$, or

$$v_{wedge} = -\left(\frac{m_1}{m_2}\right)v_{block} = -\left(\frac{0.500}{3.00}\right)(4.00 \text{ m/s}) = \boxed{-0.667 \text{ m/s}}$$

(b) Using conservation of energy as the block slides down the wedge, we have $\left(KE + PE_g\right)_i = \left(KE + PE_g\right)_f$ or

$$0 + m_1 gh = \frac{1}{2}m_1 v_{block}^2 + \frac{1}{2}m_2 v_{wedge}^2 + 0$$

Thus, $h = \dfrac{1}{2g}\left[v_{block}^2 + \left(\dfrac{m_2}{m_1}\right)v_{wedge}^2\right]$

$$= \frac{1}{19.6\ \text{m/s}^2}\left[\left(4.00\ \text{m/s}\right)^2 + \left(\frac{3.00}{0.500}\right)\left(-0.667\ \text{m/s}\right)^2\right] = \boxed{0.952\ \text{m}}$$

6.60 (a) Let m be the mass of each cart. Then, if v_0 is the initial velocity of the red cart, applying conservation of momentum to the collision gives

$$mv_b + mv_r = mv_0 + 0\text{, or } v_b + v_r = v_0 \qquad (1)$$

where v_b and v_r are the velocities of the blue and red carts after collision.

In a head-on elastic collision, we have $v_{2f} + v_{2i} = v_{1f} + v_{1i}$ which reduces to

$$v_b - v_r = v_0 \qquad (2)$$

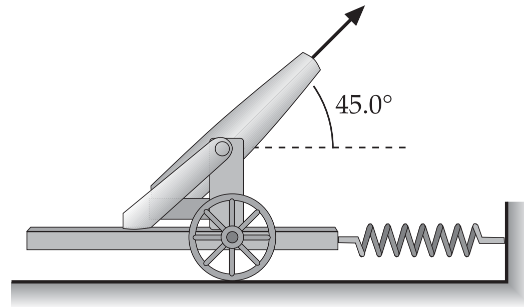
Solving (1) and (2) simultaneously gives $v_r = \boxed{0}$, and $v_b = \boxed{3.00\ \text{m/s}}$

(b) Using conservation of mechanical energy for the blue cart-spring system, $\left(KE + PE_s\right)_f = \left(KE + PE_s\right)_i$ becomes

$$0 + \frac{1}{2}kx^2 = \frac{1}{2}mv_b^2 + 0$$

or $$x = \sqrt{\frac{m}{k}}\,v_b = \sqrt{\frac{0.250\ \text{kg}}{50.0\ \text{N/m}}}\left(3.00\ \text{m/s}\right) = \boxed{0.212\ \text{m}}$$

6.61 (a) Use conservation of the component of momentum in the horizontal direction from just before to just after the cannon firing.



$(\Sigma p_x)_f = (\Sigma p_x)_i$ gives

$$m_{shell}(v_{shell}\cos 45.0°) + m_{cannon}v_{recoil} = 0\text{, or}$$

$$v_{recoil} = -\left(\frac{m_{shell}}{m_{cannon}}\right)v_{shell}\cos 45.0°$$

$$= -\left(\frac{200\text{ kg}}{5\,000\text{ kg}}\right)(125\text{ m/s})\cos 45.0° = \boxed{-3.54\text{ m/s}}$$

(b) Use conservation of mechanical energy for the cannon-spring system from right after the cannon is fired to the instant when the cannon comes to rest.

$$(KE + PE_g + PE_s)_f = (KE + PE_g + PE_s)_i$$

$$0 + 0 + \frac{1}{2}kx^2_{\max} = \frac{1}{2}m_{cannon}\,v^2_{recoil} + 0 + 0$$

$$x_{\max} = \sqrt{\frac{m_{cannon}\,v^2_{recoil}}{k}} = \sqrt{\frac{(5000\text{ kg})(\text{-}3.54\text{ m/s})^2}{2.00\times 10^4\text{ N/m}}} = \boxed{1.77\text{ m}}$$

(c) $|F_{\max}| = k\,x_{\max} = (2.00\times 10^4\text{ N/m})(1.77\text{ m}) = 3.54\times 10^4\text{ N}$

(d) No. The rail exerts a vertical external force (the normal force) on the cannon and prevents it from recoiling vertically. Momentum is not conserved in the vertical direction. The spring does not have time to stretch during the cannon firing. Thus, no external horizontal force is exerted on the system (cannon plus shell) from just before to just after firing. Momentum is conserved in the horizontal direction during this interval.

6.62 Conservation of the x-component of momentum gives

$$(3m)v_{2x}+0=-mv_0+(3m)v_0\text{, or } v_{2x}=\frac{2}{3}v_0 \tag{1}$$

Likewise, conservation of the y-component of momentum gives

$$-mv_{1y}+(3m)v_{2y}=0\text{, and } v_{1y}=3v_{2y} \tag{2}$$

Since the collision is elastic, $(KE)_f=(KE)_i$, or

$$\frac{1}{2}mv_{1y}^2+\frac{1}{2}(3m)\left(v_{2x}^2+v_{2y}^2\right)=\frac{1}{2}mv_0^2+\frac{1}{2}(3m)v_0^2 \tag{3}$$

Substituting (1) and (2) into (3) yields

$$9v_{2y}^2+3\left(\frac{4}{9}v_0^2+v_{2y}^2\right)=4v_0^2\text{, or } v_{2y}=v_0\frac{\sqrt{2}}{3}$$

(a) The particle of mass m has final speed $v_{1y}=3\,v_{2y}=\boxed{v_0\sqrt{2}}$

and the particle of mass $3m$ moves at

$$v_2=\sqrt{v_{2x}^2+v_{2y}^2}=\sqrt{\frac{4}{9}v_0^2+\frac{2}{9}v_0^2}=\boxed{v_0\sqrt{\frac{2}{3}}}$$

(b) $\theta=\tan^{-1}\left(\dfrac{v_{2y}}{v_{2x}}\right)=\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right)=\boxed{35.3^\circ}$

6.63 Let particle 1 be the neutron and particle 2 be the carbon nucleus. Then, we are given that $m_2=12m_1$.

(a) From conservation of momentum $m_2v_{2f}+m_1v_{1f}=m_1v_{1i}+0$. Since $m_2=12m_1$, this reduces to $12v_{2f}+v_{1f}=v_{1i}$. (1)

For a head-on elastic collision, $v_{2f}+v_{2i}=v_{1f}+v_{1i}$

Since $v_{2i} = 0$, this becomes $v_{2f} - v_{1f} = v_{1i}$ (2)

Solve (1) and (2) simultaneously to find

$$v_{1f} = -\frac{11}{13}v_{1i}\text{, and } v_{2f} = \frac{2}{13}v_{1i}$$

The initial kinetic energy of the neutron is $KE_{1i} = \frac{1}{2}m_1 v_{1i}^2$, and the final kinetic energy of the carbon nucleus is

$$KE_{2f} = \frac{1}{2}m_2 v_{2f}^2 = \frac{1}{2}(12m_1)\left(\frac{4}{169}v_{1i}^2\right) = \frac{48}{169}\left(\frac{1}{2}m_1 v_{1i}^2\right) = \frac{48}{169}KE_{1i}$$

The fraction of kinetic energy transferred is $\frac{KE_{2f}}{KE_{1i}} = \frac{48}{169} = \boxed{0.28}$

(b) If $KE_{1i} = 1.6 \times 10^{-13}$ J, then

$$KE_{2f} = \frac{48}{169}KE_{1i} = \frac{48}{169}\left(1.6 \times 10^{-13}\text{ J}\right) = \boxed{4.5 \times 10^{-14}\text{ J}}$$

The remaining energy $1.6 \times 10^{-13}\text{ J} - 4.5 \times 10^{-14}\text{ J} = \boxed{1.1 \times 10^{-13}\text{ J}}$ stays with the neutron.

6.64 Choose the positive x-axis in the direction of the initial velocity of the cue ball. Let v_i be the initial speed of the cue ball, v_c be the final speed of the cue ball, v_T be the final speed of the target, and θ be the angle the target's final velocity makes with the x-axis.

Conservation of momentum in the x-direction gives

$$mv_T\cos\theta + mv_c\cos 30.0° = 0 + mv_i\text{, or } v_T\cos\theta = v_i - v_c\cos 30.0° \quad (1)$$

From conservation of momentum in the y-direction,

$$mv_T\sin\theta - mv_c\sin 30.0° = 0 + 0\text{, or } v_T\sin\theta = v_c\sin 30.0° \quad (2)$$

Since this is an elastic collision, kinetic energy is conserved, giving

$$\frac{1}{2}mv_T^2 + \frac{1}{2}mv_c^2 = \frac{1}{2}mv_i^2\text{, or } v_T^2 = v_i^2 - v_c^2 \quad (3)$$

(b) To solve, square equations (1) and (2). Then add the results to obtain $v_T^2 = v_i^2 - 2v_i v_c \cos 30.0° + v_c^2$. Substitute this into equation (3) and simplify to find

$$v_c = v_i \cos 30.0° = (4.00 \text{ m/s})\cos 30.0° = \boxed{3.46 \text{ m/s}}$$

Then, equation (3) yields $v_T = \sqrt{v_i^2 - v_c^2}$, or

$$v_T = \sqrt{(4.00 \text{ m/s})^2 - (3.46 \text{ m/s})^2} = \boxed{2.00 \text{ m/s}}$$

(a) With the results found above, equation (2) gives

$$\sin\theta = \left(\frac{v_c}{v_T}\right)\sin 30.0° = \left(\frac{3.46}{2.00}\right)\sin 30.0° = 0.866 \text{, or } \theta = 60.0°$$

Thus, the angle between the velocity vectors after collision is

$$\phi = 60.0° + 30.0° = \boxed{90.0°}$$

6.65 The deceleration of the incident block is $a = -\frac{f_k}{m} = -\frac{\mu_k(mg)}{m} = -\mu_k g$

Therefore, $v^2 = v_0^2 + 2a(\Delta x)$ gives the speed of the incident block just before collision as $v = \sqrt{v_0^2 - 2\,\mu_k g\,d}$

Conservation of momentum from <u>just</u> before to <u>just</u> after collision gives

$$mv_1 + (2m)v_2 = mv \text{, or } 2v_2 + v_1 = v \qquad (1)$$

where v_1 and v_2 are the speeds of the two blocks <u>just</u> after collision.

Since this is a head-on elastic collision, $v_{2f} + v_{2i} = v_{1f} + v_{1i}$

which becomes $v_2 - v_1 = v$. (2)

Adding equations (1) and (2) yields $v_2 = \frac{2}{3}v = \frac{2}{3}\sqrt{v_0^2 - 2\,\mu_k g\,d}$

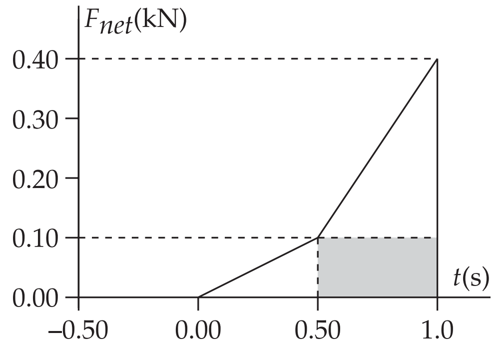
Note that the mass canceled in the calculation of the deceleration above. Thus, the second block will have the same deceleration after collision as the incident block had before. Then, $v_f^2 = v_i^2 + 2a(\Delta x)$ with $v_f = 0$ gives the stopping distance for the second block as $0 = v_2^2 + 2(-\mu_k g)D$, or

$$D = \frac{v_2^2}{2\mu_k g} = \frac{2}{9\mu_k g}\left(v_0^2 - 2\mu_k g\, d\right) = \boxed{\frac{2v_0^2}{9\mu_k g} - \frac{4d}{9}}$$

6.66 Observe from Figure P6.66, the platform exerts a 0.60-kN to support the weight of the standing athlete prior to $t = 0.00$ s. From this, we determine the mass of the athlete:

$$m = \frac{w}{g} = \frac{0.60\text{ kN}}{g} = \frac{600\text{ N}}{9.8\text{ m/s}^2} = 61\text{ kg}$$

For the interval $t = 0.00$ s to $t = 1.0$ s, we subtract the 0.60-kN used to counterbalance the weight to get the *net* upward force exerted on the athlete by the platform during the jump. The result is shown in the force-versus-time graph at the right. The net impulse imparted to the athlete is given by the area under this graph. Note that this area can be broken into two triangular areas plus a rectangular area.



The net upward impulse is then:

$$I = \frac{1}{2}(0.50\text{ s})(100\text{ N}) + \frac{1}{2}(0.50\text{ s})(300\text{ N}) + (0.50\text{ N})(100\text{ N}) = 150\text{ N}\cdot\text{s}$$

The upward velocity v_i of the athlete as he lifts off of the platform (at $t = 1.0$ s) is found from

$$I = \Delta p = mv_i - mv_0 = mv_i - 0 \;\Rightarrow\; v_i = \frac{I}{m} = \frac{150\text{ N}\cdot\text{s}}{61\text{ kg}} = 2.5\text{ m/s}$$

The height of the jump can then be found from $v_f^2 = v_i^2 + 2a_y\Delta y$ (with $v_f = 0$) to be

$$\Delta y = \frac{v_f^2 - v_i^2}{2a_y} = \frac{0-(2.5\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = \boxed{0.31\text{ m}}$$

6.67 (a) The owner's $\boxed{\text{claim should be denied}}$. Immediately prior to impact, the total momentum of the two-car system had a northward component and an eastward component. Thus, after impact, the wreckage moved in a northeasterly direction and could not possibly have damaged the owner's property on the southeast corner.

(b) From conservation of momentum:

$$(p_x)_{after} = (p_x)_{before} \quad \Rightarrow \quad (m_1 + m_2)v_x = m_1(v_{1i})_x + m_2(v_{2i})_x$$

$$\text{or} \quad v_x = \frac{m_1(v_{1i})_x + m_2(v_{2i})_x}{m_1 + m_2} = \frac{(1\,300 \text{ kg})(30.0 \text{ km/h}) + 0}{1\,300 \text{ kg} + 1\,100 \text{ kg}} = 16.3 \text{ km/h}$$

$$(p_y)_{after} = (p_y)_{before} \quad \Rightarrow \quad (m_1 + m_2)v_y = m_1(v_{1i})_y + m_2(v_{2i})_y$$

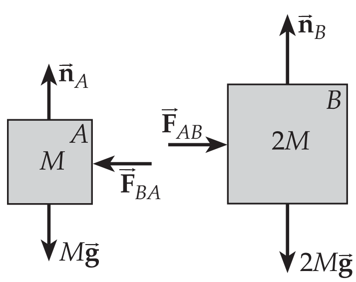
$$\text{or} \quad v_y = \frac{m_1(v_{1i})_y + m_2(v_{2i})_y}{m_1 + m_2} = \frac{0 + (1\,100 \text{ kg})(20.0 \text{ km/h})}{1\,300 \text{ kg} + 1\,100 \text{ kg}} = 9.17 \text{ km/h}$$

Thus, the velocity of the wreckage immediately after impact is

$$v = \sqrt{v_x^2 + v_y^2} = 18.7 \text{ km/h} \text{ and } \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}(0.564) = 29.4°$$

or $\vec{\mathbf{v}} = \boxed{18.7 \text{ km/h at } 29.4° \text{ north of east, consistent with part (a)}}$

6.68 (a)



(b) From Newton's third law, the force $\vec{\mathbf{F}}_{BA}$ exerted by *B* on *A* is at each instant equal in magnitude and opposite in direction to the force $\vec{\mathbf{F}}_{AB}$ exerted by *A* on *B*.

(c) There are no horizontal external forces acting on System *C* which consists of both blocks. The forces $\vec{\mathbf{F}}_{BA}$ and $\vec{\mathbf{F}}_{AB}$ are internal forces exerted on one part of System *C* by another part of System *C*.

Thus, $\quad \Sigma\vec{\mathbf{F}}_{external} = \dfrac{\Delta\vec{\mathbf{p}}_C}{\Delta t} = 0 \quad \Rightarrow \quad \boxed{\Delta\vec{\mathbf{p}}_C = 0}$

This gives $\quad (\vec{\mathbf{p}}_C)_f = (\vec{\mathbf{p}}_C)_i = (\vec{\mathbf{p}}_A)_i + (\vec{\mathbf{p}}_B)_i \quad$ or $\quad (M + 2M)V = M(+v) + 0$

so the velocity of the combined blocks after collision is $V = +v/3$

The change in momentum of A is then:

$$\Delta\vec{\mathbf{p}}_A = \left(\vec{\mathbf{p}}_A\right)_f - \left(\vec{\mathbf{p}}_A\right)_i = MV - Mv = M\left(\frac{v}{3} - v\right) = \boxed{-2Mv/3}$$

and the change in momentum for B is:

$$\Delta\vec{\mathbf{p}}_B = \left(\vec{\mathbf{p}}_B\right)_f - \left(\vec{\mathbf{p}}_B\right)_i = 2MV - 0 = 2M\left(\frac{+v}{3}\right) = \boxed{+2Mv/3}$$

(d) $$\Delta KE = \left(KE_C\right)_f - \left[\left(KE_A\right) + \left(KE_B\right)\right]_i = \tfrac{1}{2}(3M)\left(\frac{v}{3}\right)^2 - \left[\tfrac{1}{2}Mv^2 + 0\right] = -\frac{1}{3}Mv^2$$

Thus, $\boxed{\text{kinetic energy is not conserved in this inelastic collision}}$

6.69 (a) The speed v_i of both balls just before the basketball reaches the ground may be found from $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ as

$$v_i = \sqrt{v_{0y}^2 + 2a_y\Delta y} = \sqrt{0 + 2\left(-9.80\ \text{m/s}^2\right)\left(-1.20\ \text{m/s}\right)} = \boxed{4.85\ \text{m/s}}$$

(b) Immediately after the basketball rebounds from the floor, it and the tennis ball meet in an elastic collision. The velocities of the two ball just before collision are:

For the tennis ball: $v_{1i} = -v_i = -4.85\ \text{m/s}$

For the basketball: $v_{2i} = +v_i = +4.85\ \text{m/s}$

We determine the velocity of the tennis ball immediately after this elastic collision as follows:

Momentum conservation gives

$$\left(57.0\ \text{g}\right)v_{1f} + \left(590\ \text{g}\right)v_{2f} = \left(57.0\ \text{g}\right)\left(-4.85\ \text{m/s}\right) + \left(590\ \text{g}\right)\left(+4.85\ \text{m/s}\right)$$

which reduces to $$v_{2f} = 4.38\ \text{m/s} - \left(9.66\times10^{-2}\right)v_{1f} \qquad (1)$$

Elastic Collision $\Rightarrow$ $v_{1f} - v_{2f} = -\left(v_{1i} - v_{2i}\right) = -\left(-4.85\ \text{m/s} - 4.85\ \text{m/s}\right) = +9.70\ \text{m/s}$

and substituting from Equation (1) gives $\left(1 + 9.66\times10^{-2}\right)v_{1f} = 9.70\ \text{m/s} + 4.38\ \text{m/s}$

or $v_{1f} = \dfrac{14.1\ \text{m/s}}{1.10} = +12.8\ \text{m/s}$

The vertical displacement of the tennis ball during its rebound is given by $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ as

$$\Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (12.8\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = \boxed{8.41\ \text{m}}$$

6.70 Ignoring the force of gravity during the brief collision time, we use the conservation of momentum to obtain:

$$(0.45\ \text{kg})v_{bf} + (60\ \text{kg})v_{pf} = (0.45\ \text{kg})(-25\ \text{m/s}) + (60\ \text{kg})(4.0\ \text{m/s})$$

or $\quad v_{pf} = 3.8\ \text{m/s} - (7.5\times10^{-3})v_{bf} \quad$ (1)

Also, elastic collision $\Rightarrow\ v_{bf} - v_{pf} = -(v_{bi} - v_{pi}) = -(-25\ \text{m/s} - 4.0\ \text{m/s})$

or $\quad v_{bf} = 29\ \text{m/s} + v_{pf} \quad$ (2)

Substituting Equation (1) into Equation (2) yields

$$v_{bf} = \frac{29\ \text{m/s} + 3.8\ \text{m/s}}{1 + 7.5\times10^{-3}} = \boxed{33\ \text{m/s}}$$

The average acceleration of the ball during the collision is

$$a_{\text{av}} = \frac{v_{bf} - v_{bi}}{\Delta t} = \frac{33\ \text{m/s} - (-25\ \text{m/s})}{20\times10^{-3}\ \text{s}} = \boxed{2.9\times10^3\ \text{m/s}^2}$$

6.71 Use conservation of mechanical energy from the instant an ice cube is released until it reaches the end of the frictionless track to find its speed as it leaves the track:

$$\tfrac{1}{2}mv^2 + mgy_f = 0 + mgy_i \ \Rightarrow\ v = \sqrt{2g(y_i - y_f)} = \sqrt{2(9.80\ \text{m/s}^2)(1.50\ \text{m})} = 5.42\ \text{m/s}$$

The initial velocity components of the ice cube for the projectile phase of its trip are then:

$$v_{0x} = (5.42\ \text{m/s})\cos 40.0° = 4.15\ \text{m/s} \quad \text{and} \quad v_{0y} = (5.42\ \text{m/s})\sin 40.0° = 3.49\ \text{m/s}$$

At the highest point on its trajectory, $v_y = 0$ and the velocity of the ice cube just before impact with the wall is $v_i = v_{0x} = 4.15\ \text{m/s}$. The velocity it rebounds from the wall with is then $v_f = -v_i/2 = -(4.15\ \text{m/s})/2 = -2.08\ \text{m/s}$

From the impulse-momentum theorem, we find

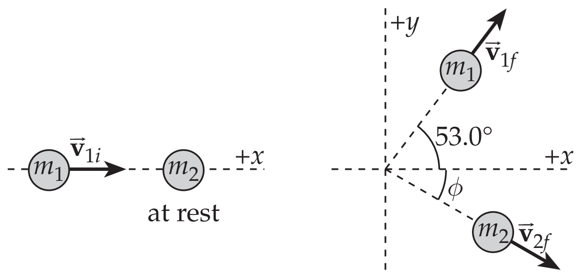
$$\vec{\mathbf{I}} = \vec{\mathbf{F}}_{\text{av}}\Delta t = \Delta\vec{\mathbf{p}} = \left(5.00\times10^{-3}\ \text{kg}\right)\left[-2.08\ \text{m/s} - 4.15\ \text{m/s}\right] = -3.12\times10^{-2}\ \text{kg}\cdot\text{m/s}$$

If the cubes strike the wall, one after the other, at the rate of 10.0 cubes per second, the collision of each cube with the wall has a duration of $\Delta t = 0.100$ s and the average force the wall exerts on the cubes is

$$\vec{\mathbf{F}}_{\text{av}} = \frac{\vec{\mathbf{I}}}{\Delta t} = \frac{-3.12\times10^{-2}\ \text{kg}\cdot\text{m/s}}{0.100\ \text{s}} = -0.312\ \text{N} \quad \text{or} \quad 0.312\ \text{N toward the left}$$

Thus, the reaction force the ice cubes exert on the wall is $\left(\vec{\mathbf{F}}_{\text{av}}\right)_{\substack{on\\wall}} = \boxed{0.312\ \text{N to the right}}$

6.72 (a)



Just Before Collision **Just After Collision**

The situations just before and just after the collision are shown above. Conserving momentum in both the x and y directions gives

$$\left(p_y\right)_f = \left(p_y\right)_i \;\Rightarrow\; m_1v_{1f}\sin 53° - m_2v_{2f}\sin\phi = 0 \;\text{ or }\; m_2v_{2f}\sin\phi = m_1v_{1f}\sin 53° \tag{1}$$

$$\left(p_x\right)_f = \left(p_x\right)_i \;\Rightarrow\; m_1v_{1f}\cos 53° + m_2v_{2f}\cos\phi = m_1v_{1i} + 0$$

or $$m_2v_{2f}\cos\phi = m_1v_{1i} - m_1v_{1f}\cos 53° \tag{2}$$

Dividing Equation (1) by Equation (2) yields

$$\tan\phi = \frac{v_{1f}\sin 53°}{v_{1i} - v_{1f}\cos 53°} = \frac{(1.0\ \text{m/s})\sin 53°}{(2.0\ \text{m/s}) - (1.0\ \text{m/s})\cos 53°} = 0.57 \;\text{ or }\; \boxed{\phi = 30°}$$

Equation (1) then gives: $$v_{2f} = \frac{m_1v_{1f}\sin 53°}{m_2\sin\phi} = \frac{(0.20\ \text{kg})(1.0\ \text{m/s})\sin 53°}{(0.30\ \text{kg})\sin 30°} = \boxed{1.1\ \text{m/s}}$$

(b) The fraction of the incident kinetic energy lost in this collision is

$$\frac{|\Delta KE|}{KE_i} = \frac{KE_i - KE_f}{KE_i} = 1 - \frac{KE_f}{KE_i} = 1 - \frac{\frac{1}{2}(0.20\ \text{kg})(1.0\ \text{m/s})^2 + \frac{1}{2}(0.30\ \text{kg})(1.1\ \text{m/s})^2}{\frac{1}{2}(0.20\ \text{kg})(2.0\ \text{m/s})^2}$$

$$\frac{|\Delta KE|}{KE_i} = \boxed{0.32} \text{ or } \boxed{32\%}$$

6.73 (a) First, we use conservation of mechanical energy from the instant after the shot embeds itself in m_3 until the combined system comes to rest (with spring compressed). This gives $(KE + PE_s)_i = (KE + PE_s)_f$, or $\frac{1}{2}(m_2 + m_3)V^2 + 0 = 0 + \frac{1}{2}kx_f^2$ and the velocity of the combined system just after impact is

$$V = \sqrt{\frac{kx_f^2}{m_2 + m_3}} = \sqrt{\frac{(4\,500\ \text{N/m})(0.500\ \text{m})^2}{10.0\ \text{kg} + 7\,990\ \text{kg}}} = 0.375\ \text{m/s}$$

Taking toward the right as positive, and using conservation of momentum from just before to just after the collision between m_2 and m_3, we find $m_2 v_2 + 0 = (m_2 + m_3)V$

$$\text{or } v_2 = \left(1 + \frac{m_3}{m_2}\right)V = \left(1 + \frac{7\,990\ \text{kg}}{10.0\ \text{kg}}\right)(-0.375\ \text{m/s}) = -300\ \text{m/s} \quad \boxed{300\ \text{m/s to the left}}$$

(b) Now consider the "collision" between the shot and the cannon. Applying conservation of momentum to this event yields $m_2 v_2 + m_1 v_1 = (m_1 + m_2)(0)$

$$\text{or} \quad v_{recoil} = v_1 = -\left(\frac{m_2}{m_1}\right)v_2 = -\left(\frac{10.0\ \text{kg}}{800\ \text{kg}}\right)(-300\ \text{m/s}) = \boxed{3.75\ \text{m/s to the right}}$$

(c) Applying the work-energy theorem to the sliding cannon gives

$$-f_k d = 0 - \tfrac{1}{2}m_1 v_1^2 \text{ or } d = \frac{m_1 v_1^2}{2f_k} = \frac{\cancel{m_1} v_1^2}{2(\mu_k \cancel{m_1} g)} = \frac{(3.75\ \text{m/s})^2}{2(0.600)(9.80\ \text{m/s}^2)} = \boxed{1.20\ \text{m}}$$

6.74 (a) Apply conservation of momentum in the vertical direction to the squid-water system from the instant before to the instant after the water is ejected. This gives

$$m_s v_s + m_w v_w = (m_s + m_w)(0) \text{ or } v_s = -\left(\frac{m_w}{m_s}\right)v_w = -\left(\frac{0.30\ \text{kg}}{0.85\ \text{kg}}\right)(-20\ \text{m/s}) = \boxed{7.1\ \text{m/s}}$$

(b) Apply conservation of mechanical energy to the squid from the instant after the water is ejected until the squid reaches maximum height to find:

$$0 + m_s g y_f = \tfrac{1}{2} m_s v_s^2 + m g y_i \quad \text{or} \quad \Delta y = \frac{v_s^2}{2g} = \frac{(7.1\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = \boxed{2.5\ \text{m}}$$