

Name _____ Date _____ Period _____

Worksheet 6.3—The Fundamental Theorem of Calculus

Show all work on a separate sheet of paper. No calculator unless otherwise stated.

Multiple Choice

- (Calculator Permitted) What is the average value of $f(x) = \cos x$ on the interval $[1, 5]$?
 (A) -0.990 (B) -0.450 (C) -0.128 (D) 0.412 (E) 0.998
- If the average value of the function f on the interval $[a, b]$ is 10, then $\int_a^b f(x) dx =$
 (A) $\frac{10}{b-a}$ (B) $\frac{f(a)+f(b)}{10}$ (C) $10b-10a$ (D) $\frac{b-a}{10}$ (E) $\frac{f(a)+f(b)}{20}$
- (Calculator Permitted) Let $f(x) = \int_3^x \ln(2 + \sin t) dt$. If $f(3) = 4$, then $f(5) =$
 (A) 0.040 (B) 0.272 (C) 0.961 (D) 4.555 (E) 6.667
- What is $\lim_{h \rightarrow 0} \frac{1}{h} \int_x^{x+h} f(t) dt$?
 (A) 0 (B) 1 (C) $f'(x)$ (D) $f(x)$ (E) nonexistent
- What is the linearization of $f(x) = \int_{\pi}^x \cos^3 t dt$ at $x = \pi$?
 (A) $y = -1$ (B) $y = -x$ (C) $y = \pi$ (D) $y = x - \pi$ (E) $y = \pi - x$
- (Calculator Permitted) The area of the region enclosed between the graph of $y = \sqrt{1-x^4}$ and the x -axis is
 (A) 0.886 (B) 1.253 (C) 1.414 (D) 1.571 (E) 1.748

Short Answer

- Let f be a function such that $f''(x) = 6x + 12$.
 a) Find $f(x)$ if the graph of f is tangent to the line $4x - y = 5$ at the point $(0, -5)$
 b) Find the average value of $f(x)$ on the closed interval $[-1, 1]$.

8. Suppose f has a negative derivative for all values of x and that $f(1) = 0$. Which of the following statements must be true of the function

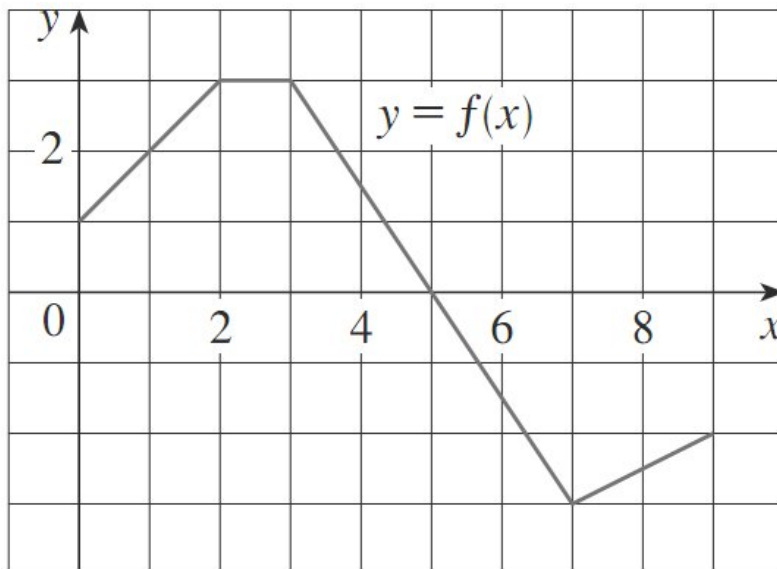
$$h(x) = \int_0^x f(t) dt ?$$

Give reasons for your answers.

- h is a twice-differentiable function of x .
 - h and dh/dx are both continuous.
 - The graph of h has a horizontal tangent at $x = 1$.
 - h has a local maximum at $x = 1$.
 - h has a local minimum at $x = 1$.
 - The graph of h has an inflection point at $x = 1$.
 - The graph of dh/dx crosses the x -axis at $x = 1$.
9. Find $\frac{dy}{dx}$
- $y = \int_{-\pi}^x \frac{2 - \sin t}{3 + \cos t} dt$
 - $y = \int_x^7 \sqrt{2m^4 + m + 1} dm$
 - $y = \int_{x^3}^5 \frac{\cos t}{t^2 + 1} dt$
 - $y = \int_{\sqrt{x}}^{x^3} \sqrt{u} \sin u du$
10. If $F(x) = \int_1^x f(t) dt$, where $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^4}}{u} du$, find $F''(2)$.
11. (Calculator Active) If $\frac{dy}{dx} = \sin^3 x$ and $y = 4$ when $x = 5$, construct an integral equation for y satisfying these conditions, then find $y(7)$, $y(0)$, and $y(-2)$.
12. Evaluate without a calculator, then verify using fnINT(
- $\int_2^{-1} 3^x dx$
 - $\int_{-2}^{-1} \frac{1}{x^2} dx$
 - $\int_0^1 (x^2 + \sqrt{x}) dx$
 - $\int_{\pi/6}^{5\pi/6} \csc^2 \theta d\theta$
 - $\int_0^4 \frac{1 - \sqrt{u}}{\sqrt{u}} du$
 - $\int_0^2 x(2 + x^5) dx$
 - $\int_0^1 \frac{4}{t^2 + 1} dt$
 - $\int_0^2 f(x) dx$ where $f(x) = \begin{cases} x^4, & 0 \leq x < 1 \\ x^5, & 1 \leq x \leq 2 \end{cases}$
13. Find the area of the region bounded by the x -axis and the curve $y = x^3 - 4x$ on $-2 \leq x \leq 2$
14. If $f(1) = 12$, $f'(x)$ is continuous, and $\int_1^4 f'(x) = 17$, what is the value of $f(4)$?
15. Find the average value of the following function on the given intervals. Verify with fnINT(
- $f(x) = \cos x$ on $[0, \pi/2]$
 - $f(x) = 1/x$ on $[1, 4]$
 - $y = \sec x \tan x$ on $[0, \pi/4]$

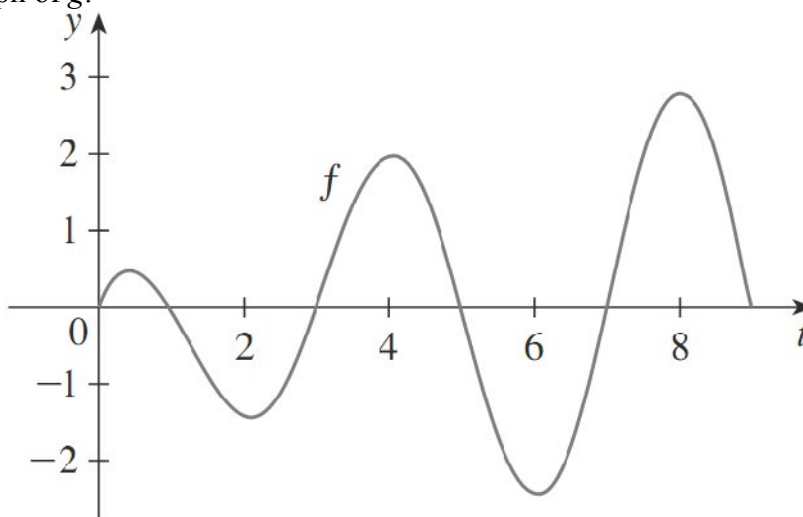
16. The graph of f is shown. If $F(x) = \int_2^x f(t) dt$, evaluate the following using areas to help you.

- a) $F(0)$ b) $F(2)$ c) $F(5)$ d) $F(7) - F(5)$ e) $F(9)$
 f) where does F have a maximum value? A minimum value?
 g) What is the average value of $F(x)$ on $[2, 9]$?



17. Let $g(x) = \int_0^x f(t) dt$, where f is the function whose graph is given below.

- a) At what values of x do the local maximum and local minimum of g occur?
 b) Where does g attain its absolute maximum value?
 c) On what intervals is g concave downward?
 d) Sketch the graph of g .



18. (Calculator Permitted) If a cup of coffee has temperature 95°C in a room where the temperature is 20°C , then, according to Newton's Law of Cooling, the temperature of the coffee after t minutes is $T(t) = 20 + 75e^{-t/50}$. What is the average temperature of the coffee during the first half hour? Show your integral set up. Include units in your final answer.