Chapter 24
Wave Optics

Quick Quizzes

1. (c). The fringes on the screen are equally spaced only at small angles where $\tan\theta \approx \sin\theta$ is a valid approximation.

2. (b). The space between successive bright fringes is proportional to the wavelength of the light. Since the wavelength in water is less than that in air, the bright fringes are closer together in the second experiment.

3. (b). The outer edges of the central maximum occur where $\sin\theta = \pm\lambda/a$. Thus, as a, the width of the slit, becomes smaller, the width of the central maximum will increase.

4. The compact disc. The tracks of information on a compact disc are much closer together than on a phonograph record. As a result, the diffraction maxima from the compact disc will be farther apart than those from the record.

Answers to Even Numbered Conceptual Questions

2. The wavelength of light is extremely small in comparison to the dimensions of your hand, so the diffraction of light around obstacles the size of your hand is totally negligible. However, sound waves have wavelengths that are comparable to the dimensions of the hand or even larger. Therefore, significant diffraction of sound waves occurs around hand sized obstacles.

4. The wavelength of light traveling in water would decrease, since the wavelength of light in a medium is given by $\lambda_n = \lambda/n$, where λ is the wavelength in vacuum and n is the index of refraction of the medium. Since the positions of the bright and dark fringes are proportional to the wavelength, the fringe separations would decrease.

6. Every color produces its own interference pattern, and we see them superimposed. The central maximum is white. The first maximum is a full spectrum with violet on the inside and red on the outside. The second maximum is also a full spectrum, with red in it overlapping with violet in the third maximum. At larger angles, the light soon starts mixing to white again.

8. (a) Two waves interfere constructively if their path difference is either zero or some integral multiple of the wavelength; that is, if the path difference is $m\lambda$, where m is an integer. (b) Two waves interfere destructively if their path difference is an odd multiple of one-half of a wavelength; that is, if the path difference equals $\left(m+\frac{1}{2}\right)\lambda = (2m+1)\frac{\lambda}{2}$.

10. The skin on the tip of a finger has a series of closely spaced ridges and swirls on it. When the finger touches a smooth surface, the oils from the skin will be deposited on the surface in the pattern of the closely spaced ridges. The clear spaces between the lines of deposited oil can serve as the slits in a crude diffraction grating and produce a colored spectrum of the light passing through or reflecting from the glass surface.

12. Suppose the index of refraction of the coating is intermediate between vacuum and the glass. When the coating is very thin, light reflected from its top and bottom surfaces will interfere constructively, so you see the surface white and brighter. Once the thickness reaches one-quarter of the wavelength of violet light in the coating, destructive interference for violet light will make the surface look red. Then other colors in spectral order (blue, green, yellow, orange, and red) will interfere destructively, making the surface look red, violet, and then blue. As the coating gets thicker, constructive interference is observed for violet light and then for other colors in spectral order. Even thicker coatings give constructive and destructive interference for several visible wavelengths, so the reflected light starts looking white again.

14. The reflected light is partially polarized, with the component parallel to the reflecting surface being the most intense. Therefore, the polarizing material should have its transmission axis oriented in the vertical direction in order to minimize the intensity of the reflected light from horizontal surfaces.

16. One way to produce interference patterns is to allow light to pass through very small openings. The opening between threads in a tautly stretched cloth like that in an umbrella is small enough for the effects to be observed.

18. Sound waves are longitudinal waves and cannot be polarized.

20. The first experiment. The separation between maxima is inversely proportional to the slit separation (see Eq. 24.5), so increasing the slit separation causes the distance between the two maxima to decrease.

22. The separations are greater in the second experiment when using red light having the longer wavelength.

Answers to Even Numbered Problems

2. **(a)** $1.77\ \mu\text{m}$ **(b)** $1.47\ \mu\text{m}$

4. 2.61 m

6. 1.5 mm

8. 3.00 cm

10. **(a)** $1.93\ \mu\text{m}$ **(b)** $\delta = 3\lambda$ **(c)** maximum

12. 1.73 km

14. **(a)** 123.4 nm **(b)** 81.58 nm

16. 193 nm

18. 233 nm

20. 290 nm

22. 8 (counting the zeroth order)

24. 6.5×10^{2} nm

26. 99.6 nm

28. $20.0 \times 10^{-6}\ (°\text{C})^{-1}$

30. **(a)** 2.3 mm **(b)** 4.5 mm

32. 91.2 cm

34. 0.227 mm

36. **(a)** 2 complete orders **(b)** 10.9°

38. **(a)** 13 orders **(b)** 1 order

40. 7.35°

42. 469 nm and 78.1 nm

44. 632.8 nm

46. $3/8$

48. 36.9°

50. 60.5°

52. **(a)** 54.7° **(b)** 63.4° **(c)** 71.6°

54. **(a)** $I/I_0 = 1/2$ **(b)** 54.7°

56. 432 nm

58. maxima at 0°, 29.1°, and 76.3°
minima at 14.1° and 46.8°

60. 113 dark fringes

62. **(a)** 0 **(b)** 0.25

68. 313 nm

70. 74 μm

Problem Solutions

24.1 $$\Delta y_{bright} = y_{m+1} - y_m = \frac{\lambda L}{d}(m+1) - \frac{\lambda L}{d}m = \frac{\lambda L}{d}$$

$$= \frac{\left(632.8\times10^{-9}\text{ m}\right)\left(5.00\text{ m}\right)}{0.200\times10^{-3}\text{ m}} = 1.58\times10^{-2}\text{ m} = \boxed{1.58\text{ cm}}$$

24.2 (a) For a bright fringe of order *m*, the path difference is $\delta = m\lambda$, where $m = 0, 1, 2, \ldots$ At the location of the third order bright fringe, $m = 3$ and

$$\delta = 3\lambda = 3(589\text{ nm}) = 1.77\times10^{3}\text{ nm} = \boxed{1.77\ \mu\text{m}}$$

(b) For a dark fringe, the path difference is $\delta = \left(m + \frac{1}{2}\right)\lambda$, where $m = 0,1,2,\ldots$

At the third dark fringe, $m = 2$ and

$$\delta = \left(2 + \frac{1}{2}\right)\lambda = \frac{5}{2}(589\text{ nm}) = 1.47\times10^{3}\text{ nm} = \boxed{1.47\ \mu\text{m}}$$

24.3 (a) The distance between the central maximum and the first order bright fringe is $\Delta y = y_{bright}\big|_{m=1} - y_{bright}\big|_{m=0} = \frac{\lambda L}{d}$, or

$$\Delta y = \frac{\lambda L}{d} = \frac{\left(546.1\times10^{-9}\text{ m}\right)\left(1.20\text{ m}\right)}{0.250\times10^{-3}\text{ m}} = 2.62\times10^{-3}\text{ m} = \boxed{2.62\text{ mm}}$$

(b) The distance between the first and second dark bands is

$$\Delta y = y_{dark}\big|_{m=1} - y_{dark}\big|_{m=0} = \frac{\lambda L}{d} = \boxed{2.62\text{ mm}}\ \text{as in (a) above.}$$

24.4 From $y_{dark}\big| = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right)$, the spacing between the first and second dark fringes is $\Delta y = \frac{\lambda L}{d}\left(\frac{3}{2} - \frac{1}{2}\right) = \frac{\lambda L}{d}$. Thus, the required distance to the screen is

$$L = \frac{(\Delta y)d}{\lambda} = \frac{\left(4.00\times10^{-3}\text{ m}\right)\left(0.300\times10^{-3}\text{ m}\right)}{460\times10^{-9}\text{ m}} = \boxed{2.61\text{ m}}$$

24.5 (a) From $d\sin\theta = m\lambda$, the angle for the $m = 1$ maximum for the sound waves is

$$\theta = \sin^{-1}\left(\frac{m}{d}\lambda\right) = \sin^{-1}\left[\frac{m}{d}\left(\frac{v_{sound}}{f}\right)\right] = \sin^{-1}\left[\frac{1}{0.300\text{ m}}\left(\frac{354\text{ m/s}}{2\,000\text{ Hz}}\right)\right] = \boxed{36.2°}$$

(b) For 3.00-cm microwaves, the required slit spacing is

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(3.00\text{ cm})}{\sin 36.2°} = \boxed{5.08\text{ cm}}$$

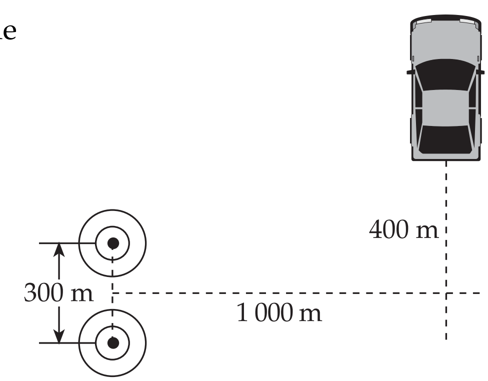
(c) The wavelength is $\lambda = \dfrac{d\sin\theta}{m}$; and if this is light, the frequency is

$$f = \frac{c}{\lambda} = \frac{mc}{d\sin\theta} = \frac{(1)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(1.00\times10^{-6}\text{ m}\right)\sin 36.2°} = \boxed{5.08\times10^{14}\text{ Hz}}$$

24.6 The position of the first order bright fringe for wavelength λ is $y_1 = \dfrac{\lambda L}{d}$

Thus, $$\Delta y_1 = \frac{(\Delta\lambda)L}{d} = \frac{\left[(700-400)\times10^{-9}\text{ m}\right](1.5\text{ m})}{0.30\times10^{-3}\text{ m}} = 1.5\times10^{-3}\text{ m} = \boxed{1.5\text{ mm}}$$

24.7 Note that, with the conditions given, the small angle approximation **does not work well**. That is, $\sin\theta$, $\tan\theta$, and θ are significantly different. The approach to be used is outlined below.



(a) At the $m = 2$ maximum, $\delta = d\sin\theta = 2\lambda$,

$$\text{or } \lambda = \frac{d}{2}\sin\theta = \frac{d}{2}\left(\frac{y}{\sqrt{L^2+y^2}}\right)$$

$$\text{or } \lambda = \frac{(300\text{ m})}{2}\left[\frac{400\text{ m}}{\sqrt{(1\,000\text{ m})^2+(400\text{ m})^2}}\right] = \boxed{55.7\text{ m}}$$

(b) The next minimum encountered is the $m = 2$ minimum; and at that point,

$$\delta = d\sin\theta = \left(m + \frac{1}{2}\right)\lambda = \frac{5}{2}\lambda$$

or $\theta = \sin^{-1}\left(\frac{5\lambda}{2d}\right) = \sin^{-1}\left(\frac{5(55.7\text{ m})}{2(300\text{ m})}\right) = 27.7°$

Then, $y = (1\,000\text{ m})\tan 27.7° = 524\text{ m}$

so the car must travel an additional $\boxed{124\text{ m}}$

24.8 In a double-slit interference pattern the distance from the central maximum to the position of the m^{th} order bright fringe is given by

$$y_m = m\left(\frac{\lambda L}{d}\right)$$

where d is the distance between the splits and L is the distance to the screen. Thus, the spacing between the first- and second-order bright fringes is

$$\Delta y = y_2 - y_1 = [2-1]\left(\frac{\lambda L}{d}\right) = 1\left[\frac{(600\times10^{-9}\text{ m})(2.50\text{ m})}{0.050\times10^{-3}\text{ m}}\right] = 0.0300\text{ m} = \boxed{3.00\text{ cm}}$$

24.9 The path difference in the two waves received at the home is $\delta = 2d$, where d is the distance from the home to the mountain. Neglecting any phase change upon reflection, the condition for destructive interference is

$$\delta = \left(m + \frac{1}{2}\right)\lambda \text{ with } m = 0, 1, 2, \ldots$$

so $d_{\text{min}} = \frac{\delta_{\text{min}}}{2} = \left(0 + \frac{1}{2}\right)\frac{\lambda}{2} = \frac{\lambda}{4} = \frac{300\text{ m}}{4} = \boxed{75.0\text{ m}}$

24.10 The angular deviation from the line of the central maximum is given by

$$\theta = \tan^{-1}\left(\frac{y}{L}\right) = \tan^{-1}\left(\frac{1.80\text{ cm}}{140\text{ cm}}\right) = 0.737°$$

(a) The path difference is then

$$\delta = d\sin\theta = (0.150\text{ mm})\sin(0.737°) = 1.93\times10^{-3}\text{ mm} = \boxed{1.93\ \mu\text{m}}$$

(b) $$\delta = \left(1.93\times10^{-6}\text{ m}\right)\left(\frac{\lambda}{643\times10^{-9}\text{ m}}\right) = \boxed{3.00\,\lambda}$$

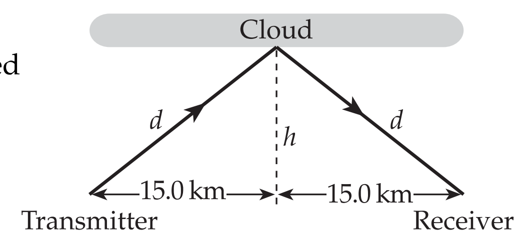
(c) Since the path difference for this position is a whole number of wavelengths, the waves interfere constructively and produce a $\boxed{\text{maximum}}$ at this spot.

24.11 The distance between the central maximum (position of A) and the first minimum is

$$y = \frac{\lambda L}{d}\left(m+\frac{1}{2}\right)\bigg|_{m=0} = \frac{\lambda L}{2d}$$

Thus, $d = \dfrac{\lambda L}{2y} = \dfrac{(3.00\text{ m})(150\text{ m})}{2(20.0\text{ m})} = \boxed{11.3\text{ m}}$

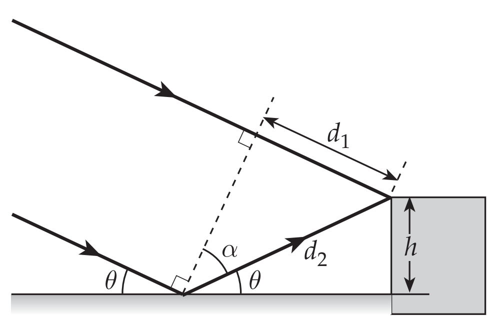
24.12 The path difference in the two waves received at the home is $\delta = 2d - 30.0\text{ km}$ where d is defined in the figure at the right. For minimum cloud height and (hence minimum path difference) to yield destructive interference, $\delta = \lambda/2$ giving



$$d_{\min} = \frac{1}{2}\left(30.0\text{ km} + \frac{\lambda}{2}\right) = 15.1\text{ km},\text{ and}$$

$$h_{\min} = \sqrt{d_{\min}^2 - (15.0\text{ km})^2} = \sqrt{(15.1\text{ km})^2 - (15.0\text{ km})^2} = \boxed{1.73\text{ km}}$$

24.13 As shown in the figure at the right, the path difference in the waves reaching the telescope is $\delta = d_2 - d_1 = d_2(1-\sin\alpha)$. If the first minimum $(\delta = \lambda/2)$ occurs when $\theta = 25.0°$, then



$$\alpha = 180° - (\theta + 90.0° + \theta) = 40.0°,\text{ and}$$

$$d_2 = \frac{\delta}{1-\sin\alpha} = \frac{(250\text{ m}/2)}{1-\sin 40.0°} = 350\text{ m}$$

Thus, $h = d_2\sin 25.0° = \boxed{148\text{ m}}$

24.14 With $n_{film} > n_{air}$, light reflecting from the front surface of the film (an air to film boundary) experiences a 180° phase shift, but light reflecting from the back surface (a film to air boundary) experiences no shift. The condition for constructive interference in the two reflected waves is then

$$2n_{film}t = \left(m + \frac{1}{2}\right)\lambda \qquad \text{with} \qquad m = 0, 1, 2, \ldots$$

For minimum thickness, $m = 0$, giving $\qquad t_{\min} = \dfrac{\lambda}{4n_{film}}$

(a) With $\lambda = 656.3\text{ nm}$ and $n_{film} = 1.330$, $\quad t_{\min} = \dfrac{656.3\text{ nm}}{4(1.330)} = \boxed{123.4\text{ nm}}$

(b) When $\lambda = 434.0\text{ nm}$ and $n_{film} = 1.330$, $t_{\min} = \dfrac{434.0\text{ nm}}{4(1.330)} = \boxed{81.58\text{ nm}}$

24.15 Light reflecting from the upper surface undergoes phase reversal while that reflecting from the lower surface does not. The condition for constructive interference in the reflected light is then

$$2t - \frac{\lambda_n}{2} = m\lambda_n\text{, or } t = \left(m + \frac{1}{2}\right)\frac{\lambda_n}{2} = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_{film}}, \; m = 0, 1, 2, \ldots$$

For minimum thickness, $m = 0$ giving

$$t = \frac{\lambda}{4n_{film}} = \frac{500\text{ nm}}{4(1.36)} = \boxed{91.9\text{ nm}}$$

24.16 With $n_{glass} > n_{air}$ and $n_{liquid} < n_{glass}$, light reflecting from the air-glass boundary experiences a 180° phase shift, but light reflecting from the glass-liquid boundary experiences no shift. Thus, the condition for destructive interference in the two reflected waves is

$$2n_{glass}t = m\lambda \qquad \text{where} \qquad m = 0, 1, 2, \ldots$$

For minimum (non-zero) thickness, $m = 1$ giving $\qquad t = \dfrac{\lambda}{2n_{glass}} = \dfrac{580\text{ nm}}{2(1.50)} = \boxed{193\text{ nm}}$

24.17 With $n_{coating} > n_{air}$ and $n_{coating} > n_{lens}$, light reflecting at the air-coating boundary experiences a phase reversal, but light reflecting from the coating-lens boundary does not. Therefore, the condition for destructive interference in the two reflected waves is

$$2n_{coating}t = m\lambda \qquad \text{where} \qquad m = 0, 1, 2, \ldots$$

For finite wavelengths, the lowest allowed value of m is $m = 1$. Then, if $t = 177.4 \text{ nm}$ and $n_{coating} = 1.55$, the wavelength associated with this lowest order destructive interference is

$$\lambda_1 = \frac{2n_{coating}t}{1} = 2(1.55)(177.4 \text{ nm}) = \boxed{550 \text{ nm}}$$

24.18 Since $n_{air} < n_{oil} < n_{water}$, light reflected from both top and bottom surfaces of the oil film experiences phase reversal, resulting in zero net phase difference due to reflections. Therefore, the condition for constructive interference in reflected light is

$$2t = m\lambda_n = m\frac{\lambda}{n_{film}}, \text{ or } t = m\left(\frac{\lambda}{2n_{film}}\right) \text{ where } m = 0, 1, 2, \ldots$$

Assuming that $m = 1$, the thickness of the oil slick is

$$t = (1)\frac{\lambda}{2n_{film}} = \frac{600 \text{ nm}}{2(1.29)} = \boxed{233 \text{ nm}}$$

24.19 There will be a phase reversal of the radar waves reflecting from both surfaces of the polymer, giving zero net phase change due to reflections. The requirement for destructive interference in the reflected waves is then

$$2t = \left(m + \frac{1}{2}\right)\lambda_n, \text{ or } t = (2m+1)\frac{\lambda}{4n_{film}} \text{ where } m = 0, 1, 2, \ldots$$

If the film is as thin as possible, then $m = 0$ and the needed thickness is

$$t = \frac{\lambda}{4n_{film}} = \frac{3.00 \text{ cm}}{4(1.50)} = \boxed{0.500 \text{ cm}}$$

This anti-reflectance coating could be easily countered by changing the wavelength of the radar—to 1.50 cm—now creating maximum reflection!

24.20 The transmitted light is brightest when the reflected light is a minimum (that is, the same conditions that produce destructive interference in the reflected light will produce constructive interference in the transmitted light). As light enters the air layer from glass, any light reflected at this surface has zero phase change. Light reflected from the other surface of the air layer (where light is going from air into glass) does have a phase reversal. Thus, the condition for destructive interference in the light reflected from the air film is $2t = m\lambda_n$, $m = 0, 1, 2, \ldots$

Since $\lambda_n = \dfrac{\lambda}{n_{film}} = \dfrac{\lambda}{1.00} = \lambda$, the minimum non-zero plate separation satisfying this condition is $d = t = (1)\dfrac{\lambda}{2} = \dfrac{580 \text{ nm}}{2} = \boxed{290 \text{ nm}}$

24.21 (a) For maximum transmission, we want destructive interference in the light reflected from the front and back surfaces of the film.

If the surrounding glass has refractive index greater than 1.378, light reflected from the front surface suffers no phase reversal, and light reflected from the back does undergo phase reversal. This effect by itself would produce destructive interference, so we want the distance down and back to be one whole wavelength in the film. Thus, we require that

$$2t = \lambda_n = \lambda/n_{film} \quad \text{or} \quad t = \frac{\lambda}{2n_{film}} = \frac{656.3 \text{ nm}}{2(1.378)} = \boxed{238 \text{ nm}}$$

(b) The filter will expand. As t increases in $2n_{film}t = \lambda$, so does $\boxed{\lambda \text{ increase}}$

(c) Destructive interference for reflected light happens also for λ in $2t = 2\lambda/n_{film}$, or

$$\lambda = n_{film}t = (1.378)(238 \text{ nm}) = \boxed{328 \text{ nm}} \text{ (near ultraviolet)}$$

24.22 Light reflecting from the lower surface of the air layer experiences phase reversal, but light reflecting from the upper surface of the layer does not. The requirement for a dark fringe (destructive interference) is then

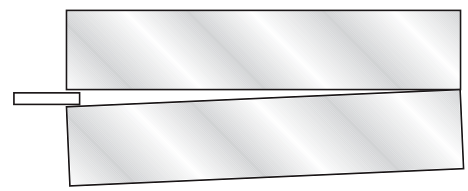
$$2t = m\lambda_n = m\left(\frac{\lambda}{n_{air}}\right) = m\lambda \text{, where } m = 0, 1, 2, \ldots$$

At the thickest part of the film $(t = 2.00\ \mu\text{m})$, the order number is

$$m = \frac{2t}{\lambda} = \frac{2\left(2.00\times10^{-6}\ \text{m}\right)}{546.1\times10^{-9}\ \text{m}} = 7.32$$

Since m must be an integer, $m = 7$ is the order of the last dark fringe seen. Counting the $m = 0$ order along the edge of contact, a total of $\boxed{\text{8 dark fringes}}$ will be seen.

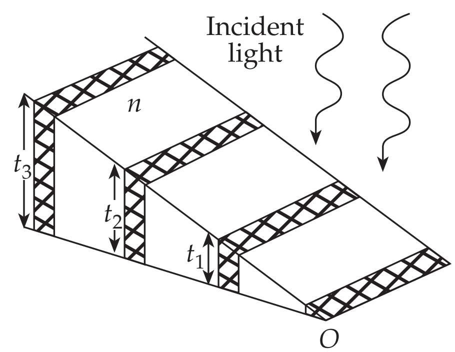
24.23 With a phase reversal upon reflection from the lower surface of the air layer and no phase change for reflection at the upper surface of the layer, the condition for destructive interference is



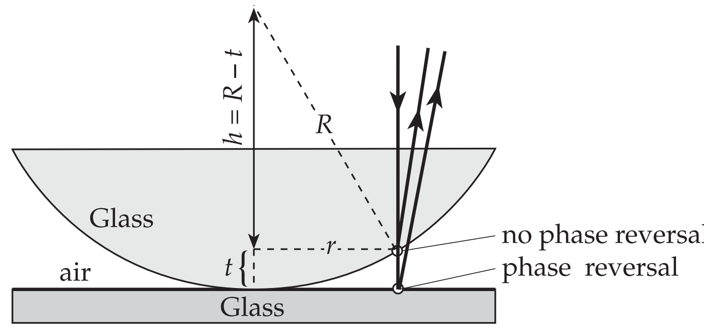
$$2t = m\lambda_n = m\left(\frac{\lambda}{n_{air}}\right) = m\lambda \text{, where } m = 0, 1, 2, \ldots$$

Counting the zeroth order along the edge of contact, the order number of the thirtieth dark fringe observed is $m = 29$. The thickness of the air layer at this point is $t = 2r$, where r is the radius of the wire. Thus,

$$r = \frac{t}{2} = \frac{29\lambda}{4} = \frac{29\left(600\times10^{-9}\ \text{m}\right)}{4} = \boxed{4.35\ \mu\text{m}}$$



24.24



From the geometry shown in the figure, $R^2 = (R-t)^2 + r^2$, or

$$t = R - \sqrt{R^2 - r^2}$$

$$= 3.0\text{ m} - \sqrt{(3.0\text{ m})^2 - (9.8\times 10^{-3}\text{ m})^2}$$

$$= 1.6\times 10^{-5}\text{ m}$$

With a phase reversal upon reflection at the lower surface of the air layer, but no reversal with reflection from the upper surface, the condition for a bright fringe is

$$2t = \left(m+\frac{1}{2}\right)\lambda_n = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_{air}} = \left(m+\frac{1}{2}\right)\lambda\text{, where } m = 0, 1, 2, \ldots$$

At the 50^{th} bright fringe, $m = 49$, and the wavelength is found to be

$$\lambda = \frac{2t}{m+1/2} = \frac{2(1.6\times 10^{-5}\text{ m})}{49.5} = 6.5\times 10^{-7}\text{ m} = \boxed{6.5\times 10^{2}\text{ nm}}$$

24.25 There is a phase reversal due to reflection at the bottom of the air film but not at the top of the film. The requirement for a dark fringe is then

$$2t = m\lambda_n = m\frac{\lambda}{n_{air}} = m\lambda\text{, where } m = 0, 1, 2, \ldots$$

At the 19^{th} dark ring (in addition to the dark center spot), the order number is $m = 19$, and the thickness of the film is

$$t = \frac{m\lambda}{2} = \frac{19(500\times 10^{-9}\text{ m})}{2} = 4.75\times 10^{-6}\text{ m} = \boxed{4.75\ \mu\text{m}}$$

24.26 With a phase reversal due to reflection at each surface of the magnesium fluoride layer, there is zero net phase difference caused by reflections. The condition for destructive interference is then

$$2t = \left(m + \frac{1}{2}\right)\lambda_n = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{film}}, \text{ where } m = 0, 1, 2, \ldots$$

For minimum thickness, $m = 0$, and the thickness is

$$t = (2m+1)\frac{\lambda}{4n_{film}} = (1)\frac{\left(550\times10^{-9}\text{ m}\right)}{4(1.38)} = 9.96\times10^{-8}\text{ m} = \boxed{99.6\text{ nm}}$$

24.27 There is a phase reversal upon reflection at each surface of the film and hence zero net phase difference due to reflections. The requirement for constructive interference in the reflected light is then

$$2t = m\lambda_n = m\frac{\lambda}{n_{film}}, \text{ where } m = 1, 2, 3, \ldots$$

With $t = 1.00\times10^{-5}\text{ cm} = 100\text{ nm}$, and $n_{film} = 1.38$, the wavelengths intensified in the reflected light are

$$\lambda = \frac{2n_{film}t}{m} = \frac{2(1.38)(100\text{ nm})}{m}, \text{ with } m = 1, 2, 3, \ldots$$

Thus, $\lambda = \boxed{276\text{ nm}, 138\text{ nm}, 92.0\text{ nm} \ldots}$

and $\boxed{\text{none of these wavelengths are in the visible spectrum}}$

24.28 As light emerging from the glass reflects from the top of the air layer, there is no phase reversal produced. However, the light reflecting from the end of the metal rod at the bottom of the air layer does experience phase reversal. Thus, the condition for constructive interference in the reflected light is $2t = \left(m + \frac{1}{2}\right)\lambda_{air}$.

As the metal rod expands, the thickness of the air layer decreases. The increase in the length of the rod is given by

$$\Delta L = \left|\Delta t\right| = \left(m_i + \tfrac{1}{2}\right)\frac{\lambda_{air}}{2} - \left(m_f + \tfrac{1}{2}\right)\frac{\lambda_{air}}{2} = \left|\Delta m\right|\frac{\lambda_{air}}{2}$$

The order number changes by one each time the film changes from bright to dark and back to bright. Thus, during the expansion, the measured change in the length of the rod is

$$\Delta L = (200)\frac{\lambda_{air}}{2} = (200)\frac{\left(500 \times 10^{-9} \text{ m}\right)}{2} = 5.00 \times 10^{-5} \text{ m}$$

From $\Delta L = L_0 \alpha (\Delta T)$, the coefficient of linear expansion of the rod is

$$\alpha = \frac{\Delta L}{L_0(\Delta T)} = \frac{5.00 \times 10^{-5} \text{ m}}{(0.100 \text{ m})(25.0°\text{C})} = \boxed{20.0 \times 10^{-6} \ (°\text{C})^{-1}}$$

24.29 The distance on the screen from the center to either edge of the central maximum is

$$y = L\tan\theta \approx L\sin\theta = L\left(\frac{\lambda}{a}\right)$$

$$= (1.00 \text{ m})\left(\frac{632.8 \times 10^{-9} \text{ m}}{0.300 \times 10^{-3} \text{ m}}\right) = 2.11 \times 10^{-3} \text{ m} = 2.11 \text{ mm}$$

The full width of the central maximum on the screen is then

$$2y = \boxed{4.22 \text{ mm}}$$

24.30 (a) Dark bands occur where $\sin\theta = m(\lambda/a)$. At the first dark band, $m = 1$, and the distance from the center of the central maximum is

$$y_1 = L\tan\theta \approx L\sin\theta = L\left(\frac{\lambda}{a}\right)$$

$$= (1.5\text{ m})\left(\frac{600\times10^{-9}\text{ m}}{0.40\times10^{-3}\text{ m}}\right) = 2.25\times10^{-3}\text{ m} = \boxed{2.3\text{ mm}}$$

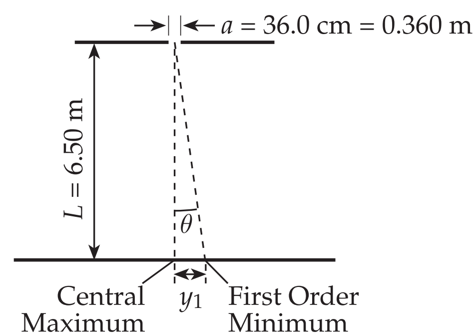
(b) The width of the central maximum is $2y_1 = 2(2.25\text{ mm}) = \boxed{4.5\text{ mm}}$

24.31 (a) Dark bands (minima) occur where $\sin\theta = m(\lambda/a)$. For the first minimum, $m = 1$ and the distance from the center of the central maximum is $y_1 = L\tan\theta \approx L\sin\theta = L(\lambda/a)$. Thus, the needed distance to the screen is

$$L = y_1\left(\frac{a}{\lambda}\right) = (0.85\times10^{-3}\text{ m})\left(\frac{0.75\times10^{-3}\text{ m}}{587.5\times10^{-9}\text{ m}}\right) = \boxed{1.1\text{ m}}$$

(b) The width of the central maximum is $2y_1 = 2(0.85\text{ mm}) = \boxed{1.7\text{ mm}}$

24.32 **Note:** The small angle approximation does not work well in this situation. Rather, you should proceed as follows.



At the first order minimum, $\sin\theta = \lambda/a$ or

$$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{5.00\text{ cm}}{36.0\text{ cm}}\right) = 7.98°$$

Then, $y_1 = L\tan\theta = (6.50\text{ m})\tan 7.98° = 0.912\text{ m} = \boxed{91.2\text{ cm}}$

24.33 The locations of the dark fringes (minima) mark the edges of the maxima, and the widths of the maxima equals the spacing between successive minima.

At the locations of the minima, $\sin\theta_m = m(\lambda/a)$ and

$$y_m = L\tan\theta_m \approx L\sin\theta_m = m\left[L\left(\frac{\lambda}{a}\right)\right]$$

$$= m\left[(1.20\text{ m})\left(\frac{500\times10^{-9}\text{ m}}{0.500\times10^{-3}\text{ m}}\right)\right] = m(1.20\text{ mm})$$

Then, $\Delta y = \Delta m(1.20\text{ mm})$ and for successive minima, $\Delta m = 1$.

Therefore, the width of each maxima, *other than the central maximum*, in this interference pattern is

$$\text{width} = \Delta y = (1)(1.20\text{ mm}) = \boxed{1.20\text{ mm}}$$

24.34 At the positions of the minima, $\sin\theta_m = m(\lambda/a)$ and

$$y_m = L\tan\theta_m \approx L\sin\theta_m = m\left[L(\lambda/a)\right]$$

Thus, $y_3 - y_1 = (3-1)\left[L(\lambda/a)\right] = 2\left[L(\lambda/a)\right]$

and $$a = \frac{2L\lambda}{y_3 - y_1} = \frac{2(0.500\text{ m})(680\times10^{-9}\text{ m})}{3.00\times10^{-3}\text{ m}} = 2.27\times10^{-4}\text{ m} = \boxed{0.227\text{ mm}}$$

24.35 The grating spacing is $d = \frac{1}{3\,660}\text{ cm} = \frac{1}{3.66\times10^5}\text{ m}$ and $d\sin\theta = m\lambda$

(a) The wavelength observed in the first-order spectrum is $\lambda = d\sin\theta$, or

$$\lambda = \left(\frac{1\text{ m}}{3.66\times10^5}\right)\left(\frac{10^9\text{ nm}}{1\text{ m}}\right)\sin\theta = \left(\frac{10^4\text{ nm}}{3.66}\right)\sin\theta$$

This yields: at 10.1°, $\lambda = \boxed{479\text{ nm}}$; at 13.7°, $\lambda = \boxed{647\text{ nm}}$;

and at 14.8°, $\lambda = \boxed{698\text{ nm}}$

(b) In the second order, $m = 2$. The second order images for the above wavelengths will be found at angles $\theta_2 = \sin^{-1}(2\lambda/d) = \sin^{-1}[2\sin\theta_1]$

This yields: for $\lambda = 479$ nm, $\theta_2 = \boxed{20.5°}$; for $\lambda = 647$ nm, $\theta_2 = \boxed{28.3°}$;

and for $\lambda = 698$ nm, $\theta_2 = \boxed{30.7°}$

24.36 (a) The longest wavelength in the visible spectrum is 700 nm, and the grating spacing is $d = \dfrac{1 \text{ mm}}{600} = 1.67 \times 10^{-3} \text{ mm} = 1.67 \times 10^{-6} \text{ m}$

Thus, $m_{\max} = \dfrac{d \sin 90.0°}{\lambda_{red}} = \dfrac{(1.67 \times 10^{-6} \text{ m}) \sin 90.0°}{700 \times 10^{-9} \text{ m}} = 2.38$

so $\boxed{\text{2 complete orders}}$ will be observed.

(b) From $\lambda = d \sin\theta$, the angular separation of the red and violet edges in the first order will be

$$\Delta\theta = \sin^{-1}\left[\frac{\lambda_{red}}{d}\right] - \sin^{-1}\left[\frac{\lambda_{violet}}{d}\right] = \sin^{-1}\left[\frac{700 \times 10^{-9} \text{ m}}{1.67 \times 10^{-6} \text{ m}}\right] - \sin^{-1}\left[\frac{400 \times 10^{-9} \text{ m}}{1.67 \times 10^{-6} \text{ m}}\right]$$

or $\Delta\theta = \boxed{10.9°}$

24.37 The grating spacing is $d = \dfrac{1 \text{ cm}}{4\,500} = \dfrac{1 \text{ m}}{4.50 \times 10^5}$. From $d \sin\theta = m\lambda$, the angular separation between the given spectral lines will be

$$\Delta\theta = \sin^{-1}\left[\frac{m\lambda_{red}}{d}\right] - \sin^{-1}\left[\frac{m\lambda_{violet}}{d}\right]$$

or

$$\Delta\theta = \sin^{-1}\left[\frac{m(656 \times 10^{-9} \text{ m})(4.50 \times 10^5)}{1 \text{ m}}\right] - \sin^{-1}\left[\frac{m(434 \times 10^{-9} \text{ m})(4.50 \times 10^5)}{1 \text{ m}}\right]$$

The results obtained are: for $m = 1$, $\Delta\theta = \boxed{5.91°}$; for $m = 2$, $\Delta\theta = \boxed{13.2°}$;

and for $m = 3$, $\Delta\theta = \boxed{26.5°}$. Complete orders for $m \geq 4$ are not visible.

24.38 (a) If $d=\dfrac{1\text{ cm}}{1\,500}=6.67\times10^{-4}\text{ cm}=6.67\times10^{-6}\text{ m}$, the highest order of $\lambda=500\text{ nm}$ that can be observed will be

$$m_{\max}=\frac{d\sin 90°}{\lambda}=\frac{\left(6.67\times10^{-6}\text{ m}\right)(1)}{500\times10^{-9}\text{ m}}=13.3\text{ or }\boxed{13\text{ orders}}$$

(b) If $d=\dfrac{1\text{ cm}}{15\,000}=6.67\times10^{-5}\text{ cm}=6.67\times10^{-7}\text{ m}$, then

$$m_{\max}=\frac{d\sin 90°}{\lambda}=\frac{\left(6.67\times10^{-7}\text{ m}\right)(1)}{500\times10^{-9}\text{ m}}=1.33\text{ or }\boxed{1\text{ order}}$$

24.39 The grating spacing is $d=\dfrac{1\text{ cm}}{5\,000}=2.00\times10^{-4}\text{ cm}=2.00\times10^{-6}\text{ m}$, and $d\sin\theta=m\lambda$ gives the angular position of a second order spectral line as

$$\sin\theta=\frac{2\lambda}{d}\quad\text{or}\quad\theta=\sin^{-1}\left(\frac{2\lambda}{d}\right)$$

For the given wavelengths, the angular positions are

$$\theta_1=\sin^{-1}\left[\frac{2\left(610\times10^{-9}\text{ m}\right)}{2.00\times10^{-6}\text{ m}}\right]=37.6°\quad\text{and}\quad\theta_2=\sin^{-1}\left[\frac{2\left(480\times10^{-9}\text{ m}\right)}{2.00\times10^{-6}\text{ m}}\right]=28.7°$$

If L is the distance from the grating to the screen, the distance on the screen from the central maximum to a second order bright line is $y=L\tan\theta$. Therefore, for the two given wavelengths, the screen separation is

$$\Delta y=L\left[\tan\theta_1-\tan\theta_2\right]$$

$$=(2.00\text{ m})\left[\tan(37.6°)-\tan(28.7°)\right]=0.445\text{ m}=\boxed{44.5\text{ cm}}$$

24.40 With 2 000 lines per centimeter, the grating spacing is

$$d = \frac{1}{2\,000}\ \text{cm} = 5.00\times10^{-4}\ \text{cm} = 5.00\times10^{-6}\ \text{m}$$

Then, from $d\sin\theta = m\lambda$, the location of the first order for the red light is

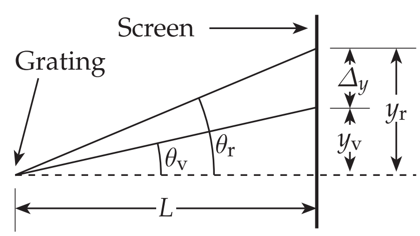
$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{(1)\left(640\times10^{-9}\ \text{m}\right)}{5.00\times10^{-6}\ \text{m}}\right] = \boxed{7.35°}$$

24.41 The grating spacing is $d = \frac{1\ \text{cm}}{2\,750} = \frac{10^{-2}\ \text{m}}{2\,750} = 3.636\times10^{-6}\ \text{m}$. From $d\sin\theta = m\lambda$, or $\theta = \sin^{-1}(m\lambda/d)$, the angular positions of the red and violet edges of the second-order spectrum are found to be

$$\theta_{\text{r}} = \sin^{-1}\left(\frac{2\lambda_{red}}{d}\right) = \sin^{-1}\left(\frac{2\left(700\times10^{-9}\ \text{m}\right)}{3.636\times10^{-6}\ \text{m}}\right) = 22.65°$$

and $$\theta_{\text{v}} = \sin^{-1}\left(\frac{2\lambda_{violet}}{d}\right) = \sin^{-1}\left(\frac{2\left(400\times10^{-9}\ \text{m}\right)}{3.636\times10^{-6}\ \text{m}}\right) = 12.71°$$

Note from the sketch at the right that $y_{\text{r}} = L\tan\theta_{\text{r}}$ and $y_{\text{v}} = L\tan\theta_{\text{v}}$, so the width of the spectrum on the screen is $\Delta y = L\left(\tan\theta_{\text{r}} - \tan\theta_{\text{v}}\right)$.



Since it is given that $\Delta y = 1.75\ \text{cm}$, the distance from the grating to the screen must be

$$L = \frac{\Delta y}{\tan\theta_{\text{r}} - \tan\theta_{\text{v}}} = \frac{1.75\ \text{cm}}{\tan(22.65°) - \tan(12.71°)}$$

or $$L = \boxed{9.13\ \text{cm}}$$

24.42 The grating spacing is $d = \dfrac{1 \text{ cm}}{1\,200} = 8.33 \times 10^{-4} \text{ cm} = 8.33 \times 10^{-6} \text{ m}$

Using $\sin\theta = \dfrac{m\lambda}{d}$ and the small angle approximation, the distance from the central maximum to the maximum of order m for wavelength λ is $y_m = L\tan\theta \approx L\sin\theta = (\lambda L/d)m$. Therefore, the spacing between successive maxima is $\Delta y = y_{m+1} - y_m = \lambda L/d$.

The longer wavelength in the light is found to be

$$\lambda_{long} = \frac{(\Delta y)d}{L} = \frac{(8.44 \times 10^{-3} \text{ m})(8.33 \times 10^{-6} \text{ m})}{0.150 \text{ m}} = \boxed{469 \text{ nm}}$$

Since the third order maximum of the shorter wavelength falls halfway between the central maximum and the first order maximum of the longer wavelength, we have

$$\frac{3\lambda_{short}L}{d} = \left(\frac{0+1}{2}\right)\frac{\lambda_{long}L}{d} \quad \text{or} \quad \lambda_{short} = \left(\frac{1}{6}\right)(469 \text{ nm}) = \boxed{78.1 \text{ nm}}$$

24.43 The grating spacing is $d = \dfrac{1 \text{ mm}}{400} = 2.50 \times 10^{-3} \text{ mm} = 2.50 \times 10^{-6} \text{ m}$

From $d\sin\theta = m\lambda$, the angle of the second-order diffracted ray is $\theta = \sin^{-1}(2\lambda/d)$.

(a) When the grating is surrounded by air, the wavelength is $\lambda_{air} = \lambda/n_{air} \approx \lambda$ and

$$\theta_a = \sin^{-1}\left(\frac{2\lambda_{air}}{d}\right) = \sin^{-1}\left[\frac{2(541 \times 10^{-9} \text{ m})}{2.50 \times 10^{-6} \text{ m}}\right] = \boxed{25.6^\circ}$$

(b) If the grating is immersed in water,

then $\lambda_n = \lambda_{water} = \dfrac{\lambda}{n_{water}} = \dfrac{\lambda}{1.333}$, yielding

$$\theta_b = \sin^{-1}\left(\frac{2\lambda_{water}}{d}\right) = \sin^{-1}\left[\frac{2(541 \times 10^{-9} \text{ m})}{(2.50 \times 10^{-6} \text{ m})(1.333)}\right] = \boxed{19.0^\circ}$$

(c) From $\sin\theta = \dfrac{m\lambda_n}{d} = \dfrac{m(\lambda/n)}{d}$, we have that $n\sin\theta = \dfrac{m\lambda}{d} = constant$ when m is kept constant. Therefore $\boxed{n_{air}\sin\theta_a = n_{water}\sin\theta_b}$, or the angles of parts (a) and (b) satisfy Snell's law.

24.44 When light of wavelength λ passes through a single slit of width a, the first minimum is observed at angle θ where

$$\sin\theta = \frac{\lambda}{a} \text{ or } \theta = \sin^{-1}\left(\frac{\lambda}{a}\right)$$

This will have no solution if $a < \lambda$, so the maximum slit width if no minima are to be seen is $a = \lambda = \boxed{632.8 \text{ nm}}$.

24.45 (a) From Brewster's law, the index of refraction is

$$n_2 = \tan\theta_p = \tan(48.0°) = \boxed{1.11}$$

(b) From Snell's law, $n_2 \sin\theta_2 = n_1 \sin\theta_1$, we obtain when $\theta_1 = \theta_p$

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_p}{n_2}\right) = \sin^{-1}\left(\frac{(1.00)\sin 48.0°}{1.11}\right) = \boxed{42.0°}$$

Note that when $\theta_1 = \theta_p$, $\theta_2 = 90.0° - \theta_p$ as it should.

24.46 Unpolarized light incident on a polarizer contains electric field vectors at all angles to the transmission axis of the polarizer. Malus's law then gives the intensity of the transmitted light as $I = I_0\left(\cos^2\theta\right)_{\text{av}}$. Since the average value of $\cos^2\theta$ is $1/2$, the intensity of the light passed by the first polarizer is $I_1 = I_0/2$, where I_0 is the incident intensity.

Then, from Malus's law, the intensity passed by the second polarizer is

$$I_2 = I_1 \cos^2(30.0°) = \left(\frac{I_0}{2}\right)\left(\frac{3}{4}\right), \text{ or } \frac{I_2}{I_0} = \boxed{\frac{3}{8}}$$

24.47 The more general expression for Brewster's angle is (see Problem 51)

$$\tan\theta_p = n_2/n_1$$

(a) When $n_1 = 1.00$ and $n_2 = 1.52$, $\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.52}{1.00}\right) = \boxed{56.7°}$

(b) When $n_1 = 1.333$ and $n_2 = 1.52$, $\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.52}{1.333}\right) = \boxed{48.8°}$

24.48 The polarizing angle for light in air striking a water surface is

$$\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.333}{1.00}\right) = 53.1°$$

This is the angle of incidence for the incoming sunlight (that is, the angle between the incident light and the normal to the surface). The altitude of the Sun is the angle between the incident light and the water surface. Thus, the altitude of the Sun is

$$\alpha = 90.0° - \theta_p = 90.0° - 53.1° = \boxed{36.9°}$$

24.49 The polarizing angle is $\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.65}{1.00}\right) = 58.8°$

When the light is incident at the polarizing angle, the angle of refraction is

$$\theta_r = 90.0° - \theta_p = 90.0° - 58.8° = \boxed{31.2°}$$

24.50 The critical angle for total reflection is $\theta_c = \sin^{-1}\left(n_2/n_1\right)$. Thus, if $\theta_c = 34.4°$ as light attempts to go from sapphire into air, the index of refraction of sapphire is

$$n_{sapphire} = n_1 = \frac{n_2}{\sin\theta_c} = \frac{1.00}{\sin 34.4°} = 1.77$$

Then, when light is incident on sapphire from air, the Brewster angle is

$$\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.77}{1.00}\right) = \boxed{60.5°}$$

24.51 From Snell's law, the angles of incidence and refraction are related by $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

If the angle of incidence is the polarizing angle (that is, $\theta_1 = \theta_p$), the angles of incidence and refraction are also related by

$$\theta_p + \theta_2 + 90° = 180°\text{, or } \theta_2 = 90° - \theta_p$$

Substitution into Snell's law then gives

$$n_1 \sin\theta_p = n_2 \sin\left(90° - \theta_p\right) = n_2 \cos\theta_p \text{ or } \boxed{\tan\theta_p = n_2/n_1}$$

24.52 $I = I_0 \cos^2\theta \quad \Rightarrow \quad \theta = \cos^{-1}\left(\sqrt{\dfrac{I}{I_0}}\right)$

(a) $\dfrac{I}{I_0} = \dfrac{1}{3.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\sqrt{\dfrac{1}{3.00}}\right) = \boxed{54.7°}$

(b) $\dfrac{I}{I_0} = \dfrac{1}{5.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\sqrt{\dfrac{1}{5.00}}\right) = \boxed{63.4°}$

(c) $\dfrac{I}{I_0} = \dfrac{1}{10.0} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\sqrt{\dfrac{1}{10.0}}\right) = \boxed{71.6°}$

24.53 From Malus's law, the intensity of the light transmitted by the first polarizer is $I_1 = I_i \cos^2\theta_1$. The plane of polarization of this light is parallel to the axis of the first plate and is incident on the second plate. Malus's law gives the intensity transmitted by the second plate as $I_2 = I_1 \cos^2(\theta_2 - \theta_1) = I_i \cos^2\theta_1 \cos^2(\theta_2 - \theta_1)$. This light is polarized parallel to the axis of the second plate and is incident upon the third plate. A final application of Malus's law gives the transmitted intensity as

$$I_f = I_2 \cos^2(\theta_3 - \theta_2) = I_i \cos^2\theta_1 \cos^2(\theta_2 - \theta_1)\cos^2(\theta_3 - \theta_2)$$

With $\theta_1 = 20.0°$, $\theta_2 = 40.0°$, and $\theta_3 = 60.0°$, this result yields

$$I_f = (10.0 \text{ units})\cos^2(20.0°)\cos^2(20.0°)\cos^2(20.0°) = \boxed{6.89 \text{ units}}$$

24.54 (a) Using Malus's law, the intensity of the transmitted light is found to be

$$I = I_0 \cos^2(45°) = I_0\left(1/\sqrt{2}\right)^2, \text{ or } \boxed{I/I_0 = 1/2}$$

(b) From Malus's law, $I/I_0 = \cos^2\theta$. Thus, if $I/I_0 = 1/3$ we obtain

$$\cos^2\theta = 1/3 \text{ or } \theta = \cos^{-1}\left(1/\sqrt{3}\right) = \boxed{54.7°}$$

24.55 (a) If light has wavelength λ in vacuum, its wavelength in a medium of refractive index n is $\lambda_n = \lambda/n$. Thus, the wavelengths of the two components in the specimen are

$$\lambda_{n_1} = \frac{\lambda}{n_1} = \frac{546.1 \text{ nm}}{1.320} = \boxed{413.7 \text{ nm}}$$

and $$\lambda_{n_2} = \frac{\lambda}{n_2} = \frac{546.1 \text{ nm}}{1.333} = \boxed{409.7 \text{ nm}}$$

(b) The number of cycles of vibration each component completes while passing through the specimen are

$$N_1 = \frac{t}{\lambda_{n_1}} = \frac{1.000 \times 10^{-6} \text{ m}}{413.7 \times 10^{-9} \text{ m}} = 2.417$$

and $$N_2 = \frac{t}{\lambda_{n_2}} = \frac{1.000 \times 10^{-6} \text{ m}}{409.7 \times 10^{-9} \text{ m}} = 2.441$$

Thus, when they emerge, the two components are out of phase by $N_2 - N_1 = 0.024 \text{ cycles}$. Since each cycle represents a phase angle of 360°, they emerge with a phase difference of

$$\Delta\phi = (0.024 \text{ cycles})(360°/\text{cycle}) = \boxed{8.6°}$$

24.56 Bright lines occur at angles given by $\sin\theta = m(\lambda/d)$. Thus, if the $m = 4$ bright line of wavelength λ_1 and the $m = 5$ bright line of wavelength λ_2 occur at the same angle, we have

$$\sin\theta = 5\left(\frac{\lambda_2}{d}\right) = 4\left(\frac{\lambda_1}{d}\right), \text{ or } \lambda_2 = \left(\frac{4}{5}\right)\lambda_1 = \left(\frac{4}{5}\right)(540 \text{ nm}) = \boxed{432 \text{ nm}}$$

24.57 Dark fringes (destructive interference) occur where $d\sin\theta = (m + 1/2)\lambda$ for $m = 0, 1, 2, \ldots$ Thus, if the second dark fringe $(m = 1)$ occurs at

$$\theta = (18.0 \text{ min})\left(\frac{1.00°}{60.0 \text{ min}}\right) = 0.300°$$, the slit spacing is

$$d = \left(m + \frac{1}{2}\right)\frac{\lambda}{\sin\theta} = \left(\frac{3}{2}\right)\frac{(546 \times 10^{-9} \text{ m})}{\sin(0.300°)} = 1.56 \times 10^{-4} \text{ m} = \boxed{0.156 \text{ mm}}$$

24.58 The wavelength is $\lambda = \frac{v_{sound}}{f} = \frac{340 \text{ m/s}}{2\,000 \text{ Hz}} = 0.170 \text{ m}$

Maxima occur where $d\sin\theta = m\lambda$, or $\theta = \sin^{-1}\left[m(\lambda/d)\right]$ for $m = 0, 1, 2, \ldots$

Since $d = 0.350 \text{ m}$, $\lambda/d = 0.486$ which gives $\theta = \sin^{-1}(0.486m)$

For $m = 0, 1,$ and 2, this yields $\boxed{\text{maxima at } 0°, 29.1°, \text{ and } 76.3°}$

No solutions exist for $m \geq 3$ since that would imply $\sin\theta > 1$

Minima occur where $d\sin\theta = (m + 1/2)\lambda$ or $\theta = \sin^{-1}\left[(2m+1)\frac{\lambda}{2d}\right]$ for $m = 0, 1, 2, \ldots$

With $\lambda/d = 0.486$, this becomes $\theta = \sin^{-1}\left[(2m+1)(0.243)\right]$

For $m = 0$ and 1, we find $\boxed{\text{minima at } 14.1° \text{ and } 46.8°}$

No solutions exist for $m \geq 2$ since that would imply $\sin\theta > 1$

24.59 The source and its image, located 1.00 cm below the mirror, act as a pair of coherent sources. This situation may be treated as double-slit interference, with the slits separated by 2.00 cm, if it is remembered that the light undergoes a phase reversal upon reflection from the mirror. The existence of this phase change causes the conditions for constructive and destructive interference to be reversed. Therefore, dark bands (destructive interference) occur where

$$y = m(\lambda L/d) \text{ for } m = 0, 1, 2, \ldots$$

The $m = 0$ dark band occurs at $y = 0$ (that is, at mirror level). The first dark band above the mirror corresponds to $m = 1$ and is located at

$$y = (1)\left(\frac{\lambda L}{d}\right) = \frac{(500 \times 10^{-9} \text{ m})(100 \text{ m})}{2.00 \times 10^{-2} \text{ m}} = 2.50 \times 10^{-3} \text{ m} = \boxed{2.50 \text{ mm}}$$

24.60 Assuming the glass plates have refractive indices greater than that of both air and water, there will be a phase reversal at the reflection from the lower surface of the film but no reversal from reflection at the top of the film. Therefore, the condition for a dark fringe is

$$2t = m\lambda_n = m\left(\lambda/n_{film}\right) \text{ for } m = 0, 1, 2, \ldots$$

If the highest order dark band observed is $m = 84$ (a total of 85 dark bands counting the $m = 0$ order at the edge of contact), the maximum thickness of the wedge is

$$t_{\max} = \frac{m_{\max}}{2}\left(\frac{\lambda}{n_{film}}\right) = \frac{84}{2}\left(\frac{\lambda}{1.00}\right) = 42\lambda$$

When the film consists of water, the highest order dark fringe appearing will be

$$m_{\max} = 2t_{\max}\left(\frac{n_{film}}{\lambda}\right) = 2\left(42\lambda\right)\left(\frac{1.333}{\lambda}\right) = 112$$

Counting the zeroth order, a total of $\boxed{113}$ dark fringes are now observed.

24.61 With $n_{air} < n_{oil} < n_{water}$, there is a 180° phase shift in light reflecting from each surface of the oil film. In such a case, the conditions for constructive and destructive interference are reversed from those valid when a phase shift occurs at only one surface. Thus, in this case, the condition for constructive interference is

$$2n_{oil}t = m\lambda \qquad \text{where} \qquad m = 0, 1, 2\ldots$$

If $n_{oil} = 1.25$ and $\lambda = 525\text{ nm}$, thicknesses of the oil film producing constructive interference are

$$t = m\left[\frac{\lambda}{2n_{oil}}\right] = m\left[\frac{525\text{ nm}}{2(1.25)}\right] = m\left(210\text{ nm}\right) \quad \text{or} \quad \boxed{\text{any positive integral multiple of 210 nm}}$$

24.62 From Malus's law, the intensity of the light transmitted by the first polarizer is $I_1 = I_i\cos^2\theta_1$. The plane of polarization of this light is parallel to the axis of the first plate and is incident on the second plate. Malus's law gives the intensity transmitted by the second plate as $I_2 = I_1\cos^2\left(\theta_2 - \theta_1\right) = I_i\cos^2\theta_1\cos^2\left(\theta_2 - \theta_1\right)$. This light is polarized parallel to the axis of the second plate and is incident upon the third plate. A final application of Malus's law gives the transmitted intensity as

$$I_f = I_2\cos^2\left(\theta_3 - \theta_2\right) = I_i\cos^2\theta_1\cos^2\left(\theta_2 - \theta_1\right)\cos^2\left(\theta_3 - \theta_2\right)$$

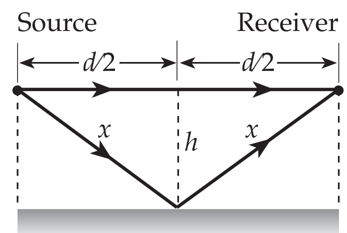
(a) If $\theta_1 = 45°$, $\theta_2 = 90°$, and $\theta_3 = 0°$, then

$$I_f/I_i = \cos^2 45° \cos^2(90° - 45°)\cos^2(0° - 90°) = \boxed{0}$$

(b) If $\theta_1 = 0°$, $\theta_2 = 45°$, and $\theta_3 = 90°$, then

$$I_f/I_i = \cos^2 0° \cos^2(45° - 0°)\cos^2(90° - 45°) = \boxed{0.25}$$

24.63 In the figure at the right, observe that the path difference between the direct and the indirect paths is



$$\delta = 2x - d = 2\sqrt{h^2 + (d/2)^2} - d$$

With a phase reversal (equivalent to a half-wavelength shift) occurring on the reflection at the ground, the condition for constructive interference is $\delta = (m + 1/2)\lambda$, and the condition for destructive interference is $\delta = m\lambda$. In both cases, the possible values of the order number are $m = 0, 1, 2, \ldots$.

(a) The wavelengths that will interfere constructively are $\lambda = \dfrac{\delta}{m + 1/2}$. The longest of these is for the $m = 0$ case and has a value of

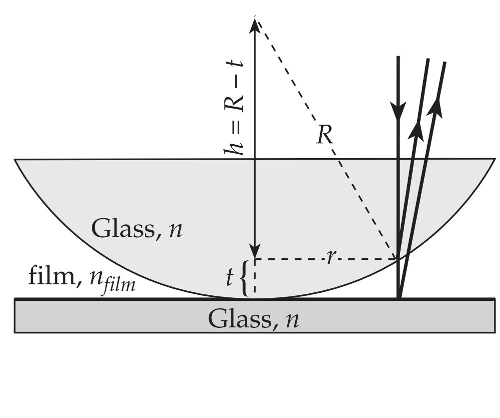
$$\lambda = 2\delta = 4\sqrt{h^2 + (d/2)^2} - 2d$$

$$= 4\sqrt{(50.0\text{ m})^2 + (300\text{ m})^2} - 2(600\text{ m}) = \boxed{16.6\text{ m}}$$

(b) The wavelengths that will interfere destructively are $\lambda = \delta/m$, and the largest finite one of these is for the $m = 1$ case. That wavelength is

$$\lambda = \delta = 2\sqrt{h^2 + (d/2)^2} - d = 2\sqrt{(50.0\text{ m})^2 + (300\text{ m})^2} - 600\text{ m} = \boxed{8.28\text{ m}}$$

24.64 There will be a phase reversal associated with the reflection at one surface of the film but no reversal at the other surface of the film. Therefore, the condition for a dark fringe (destructive interference) is



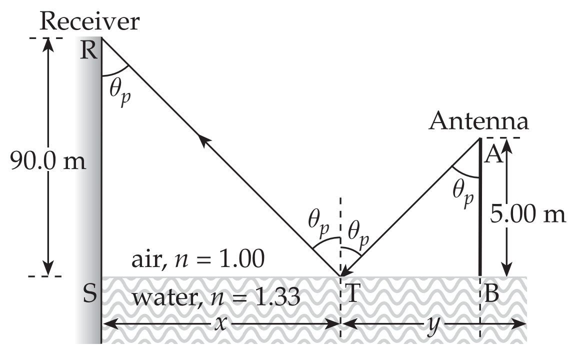
$$2t = m\lambda_n = m\left(\frac{\lambda}{n_{film}}\right) \quad m = 0, 1, 2, \ldots$$

From the figure, note that $R^2 = r^2 + (R-t)^2 = r^2 + R^2 - 2Rt + t^2$ which reduces to $r^2 = 2Rt - t^2$. Since t will be very small in comparison to either r or R, we may neglect the term t^2, leaving $r \approx \sqrt{2Rt}$.

For a dark fringe, $t = \dfrac{m\lambda}{2n_{film}}$ so the radii of the dark rings will be

$$r \approx \sqrt{2R\left(\frac{m\lambda}{2n_{film}}\right)} = \boxed{\sqrt{\frac{m\lambda R}{n_{film}}}} \quad \text{for } m = 0, 1, 2, \ldots$$

24.65 If the signal from the antenna to the receiver station is to be completely polarized by reflection from the water, the angle of incidence where it strikes the water must equal the polarizing angle from Brewster's law. This is given by



$$\theta_p = \tan^{-1}\left(\frac{n_{\text{water}}}{n_{\text{air}}}\right) = \tan^{-1}(1.33) = 53.1°$$

From the triangle RST is the sketch, the horizontal distance from the point of refection, T, to shore is given by

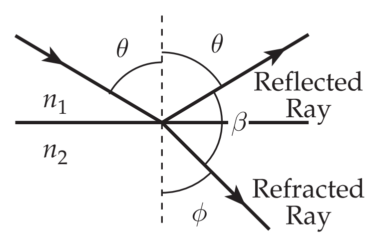
$$x = (90.0 \text{ m})\tan\theta_p = (90.0 \text{ m})(1.33) = 120 \text{ m}$$

and from triangle ABT, the horizontal distance from the antenna to this point is

$$y = (5.00 \text{ m})\tan\theta_p = (5.00 \text{ m})(1.33) = 6.65 \text{ m}$$

The total horizontal distance from ship to shore is then $x + y = 120 \text{ m} + 6.65 \text{ m} = \boxed{127 \text{ m}}$

24.66 (a) From Snell's law, $n_1 \sin\theta = n_2 \sin\phi$ with θ and ϕ defined as shown in the figure at the right. However, $\phi = 180° - (\theta + \beta)$ so Snell's law becomes $n_1 \sin\theta = n_2 \sin\left[180° - (\theta + \beta)\right]$.



Apply the given identity, realizing that

$$\sin[-(\theta+\beta)] = -\sin(\theta+\beta)$$

and $\cos[-(\theta+\beta)] = \cos(\theta+\beta)$, to obtain

$$n_1 \sin\theta = n_2\left[\sin(180°)\cos(\theta+\beta) - \cos(180°)\sin(\theta+\beta)\right] = n_2 \sin(\theta+\beta)$$

Apply the identity once again to get

$$n_1 \sin\theta = (n_2 \cos\beta)\sin\theta + (n_2 \sin\beta)\cos\theta,$$

or $(n_1 - n_2\cos\beta)\sin\theta = (n_2\sin\beta)\cos\theta$

Since $\sin\theta/\cos\theta = \tan\theta$, this simplifies to $\boxed{\tan\theta = \dfrac{n_2 \sin\beta}{n_1 - n_2\cos\beta}}$

(b) If $\beta = 90°$, $n_1 = 1$, and $n_2 = n$, the above result reduces to

$\boxed{\tan\theta = n}$, which is Brewster's law.

24.67 In the single slit diffraction pattern, destructive interference (or minima) occur where $\sin\theta = m(\lambda/a)$ for $m = 0, \pm 1, \pm 2, \ldots$. The screen locations, measured from the center of the central maximum, of these minima are at

$$y_m = L\tan\theta_m \approx L\sin\theta_m = m(\lambda L/a)$$

If we assume the first-order maximum is halfway between the first- and second-order minima, then its location is

$$y = \frac{y_1 + y_2}{2} = \frac{(1+2)(\lambda L/a)}{2} = \frac{3\lambda L}{2a}$$

and the slit width is

$$a = \frac{3\lambda L}{2y} = \frac{3(500\times10^{-9}\text{ m})(1.40\text{ m})}{2(3.00\times10^{-3}\text{ m})} = 3.50\times10^{-4}\text{ m} = \boxed{0.350\text{ mm}}$$

24.68 Phase reversals occur in the reflections at both surfaces of the oil layer, so there is zero net phase difference due to reflections. The condition for constructive interference is then

$$2t = m\lambda_n = m\left(\frac{\lambda_{bright}}{n_{film}}\right) \tag{1}$$

and the condition for destructive interference is

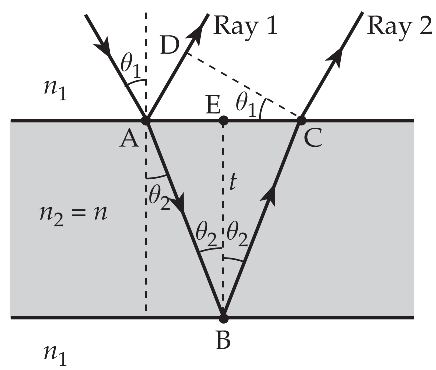
$$2t = \left(m+\frac{1}{2}\right)\lambda_n = \left(m+\frac{1}{2}\right)\left(\frac{\lambda_{dark}}{n_{film}}\right) \tag{2}$$

Solving for the order number, m, in equation (1),

and substituting into equation (2) gives the film thickness as

$$t = \frac{\lambda_{dark}}{4n_{film}\left(1-\lambda_{dark}/\lambda_{bright}\right)} = \frac{500\text{ nm}}{4(1.20)(1-500\text{ nm}/750\text{ nm})} = \boxed{313\text{ nm}}$$

24.69 (a) Assuming that $n > n_1$ in the figure at the right, Ray 1 undergoes a phase reversal as it reflects at point A, but Ray 2 has no reversal as it reflects at B. Therefore, the condition for the two rays to interfere constructively is that the difference in their *optical path lengths* be an odd number of half-wavelengths,



or $\delta = (m+1/2)\lambda$ for $m = 0, 1, 2, \ldots$

The difference in the optical path lengths is

$$\delta = n\left(\overline{AB}+\overline{BC}\right) - n_1\,\overline{AD} = 2n\,\overline{AB} - n_1\,\overline{AD}$$

But, $\overline{AB} = \dfrac{\overline{AE}}{\sin\theta_2}$ and $\overline{AD} = 2\,\overline{AE}\sin\theta_1$, so $\delta = 2\,\overline{AE}\left(\dfrac{n}{\sin\theta_2} - n_1\sin\theta_1\right)$

Now, observe that

$\overline{AE} = t\tan\theta_2$, and $n_1\sin\theta_1 = n\sin\theta_2$ (from Snell's law)

Thus, $$\delta = 2nt\tan\theta_2\left(\frac{1}{\sin\theta_2} - \sin\theta_2\right) = \frac{2nt}{\cos\theta_2}\left(1-\sin^2\theta_2\right) = 2nt\cos\theta_2$$

The condition for constructive interference is then

$$\boxed{2nt\cos\theta_2 = (m+1/2)\lambda}$$

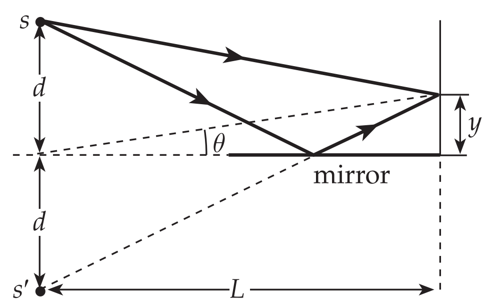
(b) When $\theta_1 = 30.0°$, then

$$\theta_2 = \sin^{-1}\left(\frac{n_1\sin\theta_1}{n}\right) = \sin^{-1}\left(\frac{(1.00)\sin 30.0°}{1.38}\right) = 21.2°$$

For minimum thickness, $m = 0$ which gives a thickness of

$$t = \frac{(0+1/2)\lambda}{2n\cos\theta_2} = \frac{590\text{ nm}}{4(1.38)\cos 21.2°} = \boxed{115\text{ nm}}$$

24.70 The indirect ray suffers a phase reversal as it reflects from the mirror, but there is no reversal for the direct ray. Therefore, the condition for constructive interference, with the two sources separated by distance 2*d*, is



$$\delta = (2d)\sin\theta = \left(m+\frac{1}{2}\right)\lambda \text{ for } m = 0, 1, 2, \ldots$$

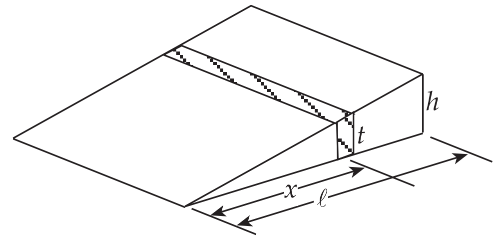
The location of these maxima on the screen is given by

$$y_m = L\tan\theta \approx L\sin\theta = \frac{\lambda L}{2d}\left(m+\frac{1}{2}\right)$$

For the first bright fringe, $m = 0$, giving

$$y = \frac{\lambda L}{2d}\left(0+\frac{1}{2}\right) = \frac{\lambda L}{4d} = \frac{(620\times10^{-9}\text{ m})(1.20\text{ m})}{4(2.5\times10^{-3}\text{ m})} = 7.4\times10^{-5}\text{ m} = \boxed{74\ \mu\text{m}}$$

24.71 The refractive index, n, of the wedge material is greater than that of the surrounding air. Thus, when illuminated from above, light reflecting from the upper surface of the wedge experiences a 180° phase shift while light reflecting from the bottom surface experiences no shift. The condition for constructive interference of light reflecting from the thin film of transparent material is then



$$2nt = \left(m + \frac{1}{2}\right)\lambda \text{ where } m = 0, 1, 2, \ldots$$

and the condition for destructive interference is $\quad 2nt = m\lambda$ where $m = 0, 1, 2, \ldots$

The thickness, t, at distance x from the edge of the wedge is found using the similar triangles shown in the above sketch. Observe that

$$\frac{t}{x} = \frac{h}{\ell} \qquad \text{or} \qquad t = x\left(\frac{h}{\ell}\right)$$

The condition for constructive interference or a bright fringe then becomes

$$2nx\left(\frac{h}{\ell}\right) = \left(m + \frac{1}{2}\right)\lambda \qquad \text{or} \qquad \boxed{x = \frac{\lambda\ell}{2hn}\left(m + \frac{1}{2}\right) \quad \text{where} \quad m = 0, 1, 2, \ldots}$$

and the condition for destructive interference or a dark fringe becomes

$$2nx\left(\frac{h}{\ell}\right) = m\lambda \qquad \text{or} \qquad \boxed{x = \frac{\lambda\ell m}{2hn} \quad \text{where} \quad m = 0, 1, 2, \ldots}$$