

§4.9—Derivatives of Exponential Functions

Example 1:

Sketch the graph of $f(x) = e^x$, then, on the same set of axes, sketch a possible graph of $f'(x)$. What do you notice? Confirm by sketching $f'(x)$ using your calculator's NDERIV capability.

Derivative of e^x

$$\frac{d}{dx}[e^x] = e^x. \text{ If } u \text{ is a differentiable function of } x, \text{ then } \frac{d}{dx}[e^u] = e^u \cdot u'$$

Example 2:

Find $\frac{dy}{dx}$ if $y = e^{(x+x^2)}$

Example 3:

Using your calculator, graph $f(x) = 2^x$ and $f'(x)$ using NDERIV. What do you notice? Do the same for $g(x) = 5^x$ and $g'(x)$.

Derivative of b^x

$$\frac{d}{dx}[b^x] = b^x \cdot \ln b. \text{ If } u \text{ is a differentiable function of } x, \text{ then } \frac{d}{dx}[b^u] = b^u \cdot \ln b \cdot u'$$

Example 4:

At what point on the graph of the function $y = 2^t - 3$ does the tangent line have a slope of 2?

Example 5:

Evaluate the following.

a) $\frac{d}{dx}[7^{-3/x}]$ b) $\frac{d}{dx}[e^{\ln x}]$ c) $\frac{d}{dt}[(e^{-t} + e^t)^2]$ d) $\frac{d}{dx}[x^2 \cdot 5^{\sin^2(4x)}]$ e) $\frac{d}{dx}[e^3 x]$

Example 6:

Find the coordinates of any points where $f(x) = xe^x$ has a horizontal tangent line.

Example 7:

Find the equation of the tangent line to $y = x^2 e^x - 2xe^x + 2e^x$ at $(1, e)$

Example 8:

Find $\frac{dy}{dx}$ for $e^{xy} + x^2 - y^2 = 10$

Example 9:

Find $\frac{d^2 y}{dx^2}$ for $y = (3 + 2x)5^{-3x}$

Example 10:

A glass of cold milk from the refrigerator is left on the counter on a hot summer day. Its temperature (in degrees Fahrenheit) after sitting on the counter t minutes is $y = 72 - 30(0.98)^t$. Answer the following by analyzing $\frac{dy}{dt}$.

- a) What is the temperature of the refrigerator? How can you tell?
- b) What is the temperature of the room? How can you tell?
- c) When is the milk warming up the fastest? How can you tell?
- d) Determine algebraically when the temperature of the milk reaches $55^\circ F$.
- e) At what rate is the milk warming when its temperature is $55^\circ F$? Answer in a complete sentence with appropriate units.

