Prelab-2

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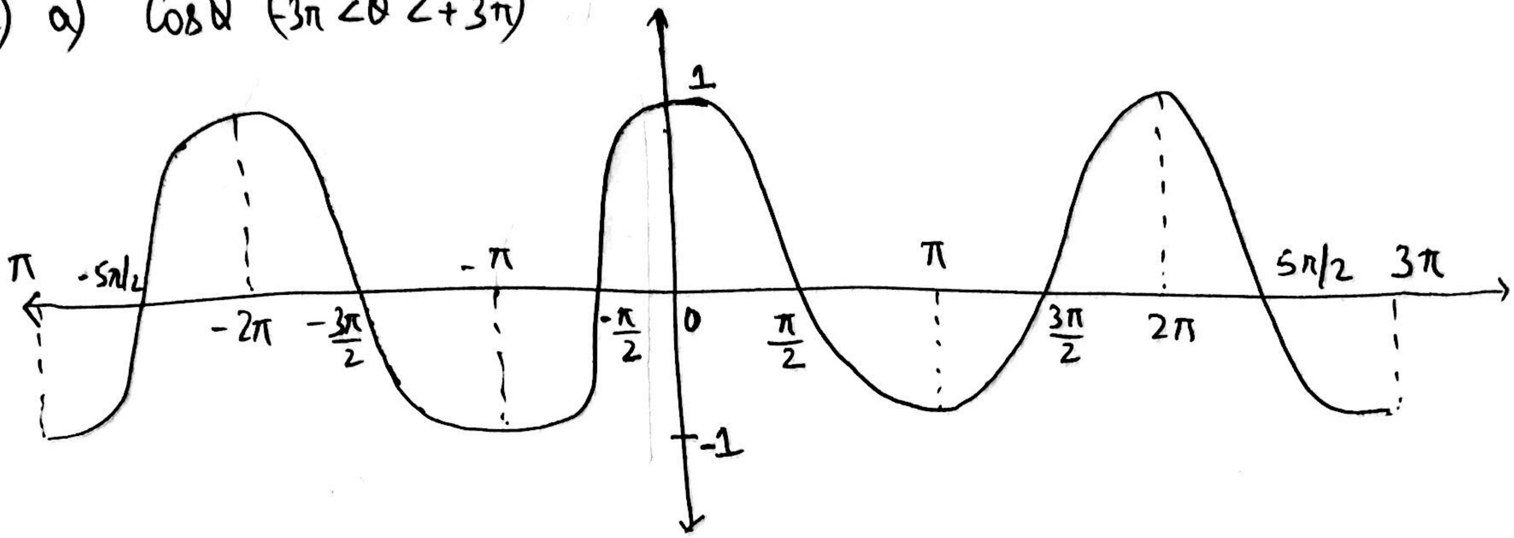
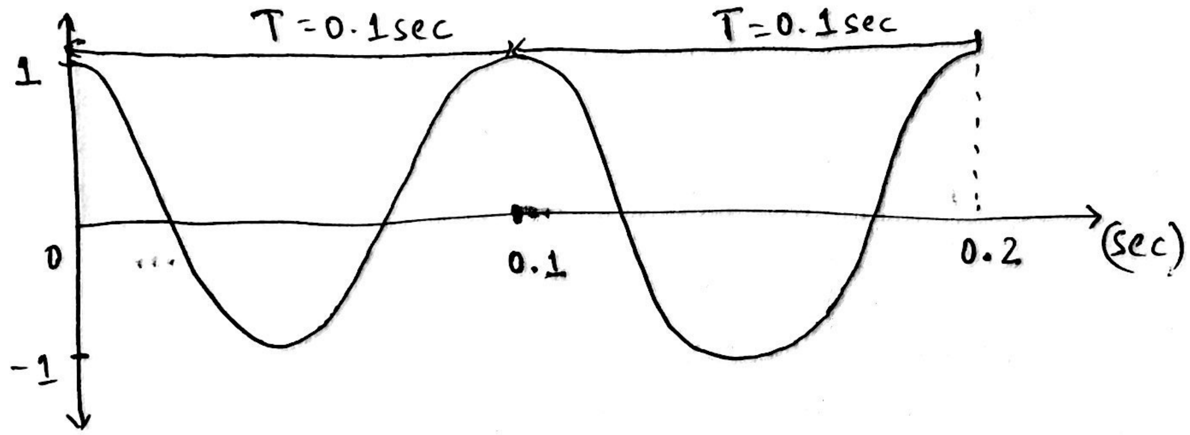
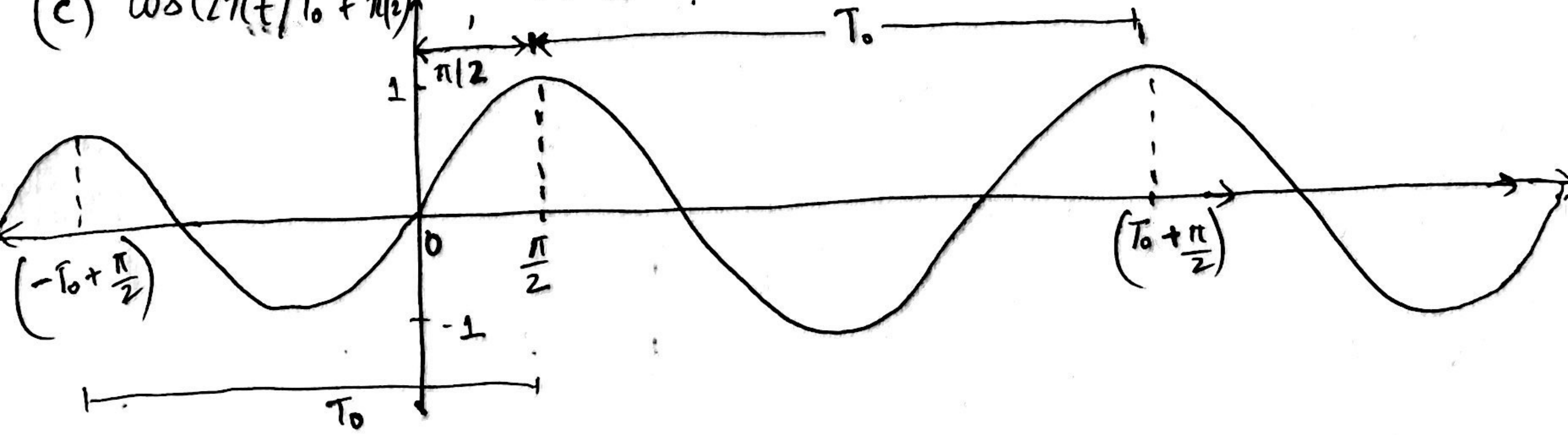
1) a) $\cos\theta \; (-3\pi < \theta < +3\pi)$



(b) $\cos(20\pi t)$, $T = 2\pi / 20\pi = 0.1$ sec



(c) $\cos(2\pi t / T_0 + \pi/2)$, $T = T_0$



2) $x(t) = A\cos(\omega_0 t + \phi)$

$A = 4$

$\omega_0 t = \frac{2\pi}{T} t$,

1st peak occurs at 2ms $\therefore$ $\phi = 2\times10^{-3}$ sec

Distance between 2 peaks = $12 - 2 = 10\times10^{-3}$ sec

$\therefore \omega_0 = \frac{2\pi}{10\times10^{-3}} = 200\pi$

$\therefore x(t) = \underline{\underline{4\cos(200\pi t + 0.002)}}$

3) Euler's formula :-

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

$$e^{j\theta} = 1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \frac{(j\theta)^5}{5!} + \cdots$$

$$= 1 + j\theta - \frac{\theta^2}{2!} - \frac{j\theta^3}{3!} + \frac{\theta^4}{4!} + \frac{j\theta^5}{5!} - \frac{\theta^6}{6!} - \frac{j\theta^7}{7!} + \cdots$$

$$= \underbrace{\left[1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots\right]}_{\cos\theta} + j\underbrace{\left[\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots\right]}_{\sin\theta}$$

$$= \cos\theta + j\sin\theta$$

$$\therefore \boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

4) (a) $\cos(\theta_1 + \theta_2) = \cos\theta_1 . \cos\theta_2 - \sin\theta_1 \sin\theta_2$

A/c to Euler's formula:

$$\cos(\omega t) = \text{Real}[e^{j\omega t}]$$

$$\therefore \cos(\theta_1 + \theta_2) = \text{Real}\left\{e^{j(\theta_1+\theta_2)}\right\} = \text{Real}\left\{e^{j\theta_1} . e^{j\theta_2}\right\}$$

$$= \text{Real}\left\{(\cos\theta_1 + j\sin\theta_2)(\cos\theta_2 + j\sin\theta_2)\right\}$$

$$= \text{Real}\left\{[\cos\theta_1 . \cos\theta_2 + j\sin\theta_1 \cos\theta_2 + j\sin\theta_2 \cos\theta_1 - \sin\theta_1 \sin\theta_2]\right\}$$

$$= \text{Real}\left\{(\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) + j(\sin\theta_1 \cos\theta_2 + \sin\theta_2 \cos\theta_1)\right\}$$

$$= (\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2)$$

$$\therefore \boxed{\cos(\theta_1 + \theta_2) = \cos\theta_1 . \cos\theta_2 - \sin\theta_1 \sin\theta_2}$$

(b) $\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$

A/c to Euler's formula:

$$\cos(\omega t) = \text{Real}\{e^{j\omega t}\}$$

$$\therefore \cos(\theta_1 - \theta_2) = \text{Real}\left\{e^{j(\theta_1 - \theta_2)}\right\} = \text{Real}\left\{e^{j\theta_1} . e^{-j\theta_2}\right\}$$

$$= \text{Real}\left\{(\cos\theta_1 + j\sin\theta_1)(\cos(-\theta_2) + j\sin(-\theta_2))\right\}$$

$$= \text{Real}\left\{(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 - j\sin\theta_2)\right\}$$

$$= \text{Real}\left\{(\cos\theta_1\cos\theta_2 + j\sin\theta_1\cos\theta_2 - j\sin\theta_2\cos\theta_1 - j^2\sin\theta_1\sin\theta_2)\right\}$$

$$= \text{Real}\left\{\left[\cos\theta_1 . \cos\theta_2 + j(\sin\theta_1 . \cos\theta_2 - \sin\theta_2\cos\theta_1) - (\sqrt{-1})^2 \sin\theta_1 . \sin\theta_2\right]\right\}$$

$$= \text{Real}\left\{(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + j(\sin\theta_1 . \cos\theta_2 - \sin\theta_2 \cos\theta_1)\right\}$$

$$= \cos\theta_1\cos\theta_2 + \sin\theta_1 . \sin\theta_2$$

$$\therefore \boxed{\cos(\theta_1 - \theta_2) = \cos\theta_1 . \cos\theta_2 + \sin\theta_1 . \sin\theta_2}$$

5) (a) $3e^{j\pi/3} + 4e^{-j\pi/6}$

$$= 3\left(\cos(\pi/3) + j\sin(\pi/3)\right) + 4\left(\cos(-\pi/6) + j\sin(-\pi/6)\right)$$

$$= 3\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + 4\left(\frac{\sqrt{3}}{2} - \frac{j}{2}\right) = \left(\frac{3}{2} + \frac{4\sqrt{3}}{2}\right) + j\left(\frac{3\sqrt{3}}{2} - 2\right)$$

$$= (1.5 + 3.464) + j(2.598 - 2) = \underline{\underline{4.964 + j0.598}}$$

Polar form: $4.999\, e^{j\tan^{-1}\left(\frac{0.598}{4.964}\right)} \simeq \underline{\underline{5.\, e^{j(0.11988)}}}$

(b) $(\sqrt{3} - j3)^{10}$

$= (\sqrt{3})^{10} (1 - \sqrt{3}j)^{10} = 3^5 \left[2 e^{-\frac{\pi}{3}j} \right]^{10}$

$= 3^5 . 2^{10} e^{-j\frac{10\pi}{3}}$

$= 243 \times 1024 \times e^{-j\frac{10}{3}\pi} = 248832 e^{j(4\pi - \frac{20\pi}{3})}$

$= \underline{248832 e^{\frac{2\pi}{3}j}}$

Rectangular form:
$248832 \left(\cos\frac{2\pi}{3} + j \sin\frac{2\pi}{3} \right)$

$= \underline{-124416 + j215488.512}$

(c) $(\sqrt{3} - 3j)^{-1} = \dfrac{1}{\sqrt{3}(1 - \sqrt{3}j)} \dfrac{(1 + \sqrt{3}j)}{(1 + \sqrt{3}j)}$

$= \dfrac{1}{4\sqrt{3}} (1 + \sqrt{3}j) = \underline{\dfrac{1}{4\sqrt{3}} + \dfrac{1}{4} j}$

Polar form:

$r = \sqrt{\left(\dfrac{1}{4\sqrt{3}}\right)^2 + \left(\dfrac{1}{4}\right)^2} = \dfrac{1}{2\sqrt{3}}$

$\theta = \tan^{-1}\left(\dfrac{1/4}{1/4\sqrt{3}}\right) = \tan^{-1}(\sqrt{3}) = \pi/3$

$\therefore = \underline{\dfrac{1}{2\sqrt{3}} e^{j\pi/3}}$

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(d) $\left(\sqrt{3} - j3\right)^{1/3}$

Let, $x = \sqrt{3} - j3$, and solving for x first

$\Rightarrow \sqrt{3} - j3 = \sqrt{3}\left(1 - j\sqrt{3}\right)$

$= \sqrt{3}\left(2e^{j\tan^{-1}(-\sqrt{3})}\right)$

$= 2\sqrt{3}\, e^{-\frac{\pi}{3}j}$

Now, $x^{1/3} \Rightarrow \left(2\sqrt{3}e^{-\frac{\pi}{3}j}\right)^{1/3}$

$= 2^{1/3} \cdot 3^{1/2\times 3} \cdot e^{-\frac{\pi}{9}j}$

$= \underline{1.51308\, e^{-\frac{\pi}{9}j}} = \underline{1.51308\, e^{\frac{16}{9}\pi j}}$

Rectangular form $= 1.51308\left(\cos\left(\frac{16}{9}\pi\right) + j\sin\left(\frac{16}{9}\pi\right)\right)$

$= \underline{1.16 - 0.97j}$

(e) $\mathcal{R}\{je^{-j\pi/3}\} = \mathcal{R}\left\{j\left(e^{j\frac{2\pi}{3}}\right)\right\}$

$= \mathcal{R}\left\{j\left[\cos\left(\frac{2\pi}{3}\right) + j\sin\left(\frac{2\pi}{3}\right)\right]\right\} = \mathcal{R}\left\{j\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\right\}$

$= \mathcal{R}\left\{-\frac{\sqrt{3}}{2} - \frac{j}{2}\right\} = -\frac{\sqrt{3}}{2}$

$\therefore$ Polar form $= -\frac{\sqrt{3}}{2}e^{0\cdot j} = -\frac{\sqrt{3}}{2}$

Rectangular form $= -\frac{\sqrt{3}}{2}(\cos 0 + j\sin 0) = \underline{-\frac{\sqrt{3}}{2}}$