Prelab-4

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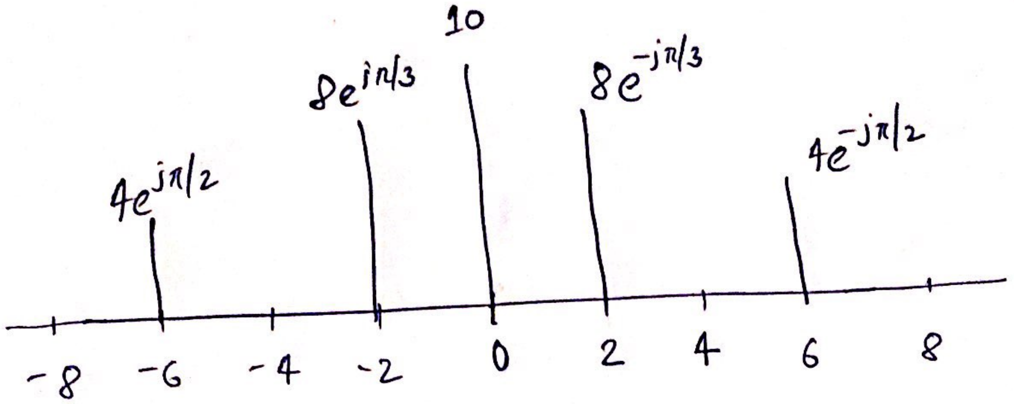
(rb6369)

1)



(a) $x(t) = 10 + 16\cos(4\pi t - \pi/3) + 8\cos(12\pi t - \pi/2)$

(b) Yes, $x(t)$ is periodic signal. The fundamental frequency of $x(t)$ is the gcd of the frequencies of the individual signal.

$\therefore \quad gcd(\pm 2, \pm 6) = \underline{\underline{2\,Hz}}$

(c) Negative frequency components are included for completeness, even though for real signals, there are conjugates of the corresponding positive frequency components.

2) $x(t) = 2\cos(6\pi t + \pi/4) - 10\cos(15\pi t) + 2\cos(24\pi t - \pi/3)$

(a) $$x(t) = \frac{2}{2} e^{j\pi/4} \cdot e^{j2\pi(3)t} + \frac{2}{2} e^{-j\pi/4} \cdot e^{-j2\pi(3)t} +$$

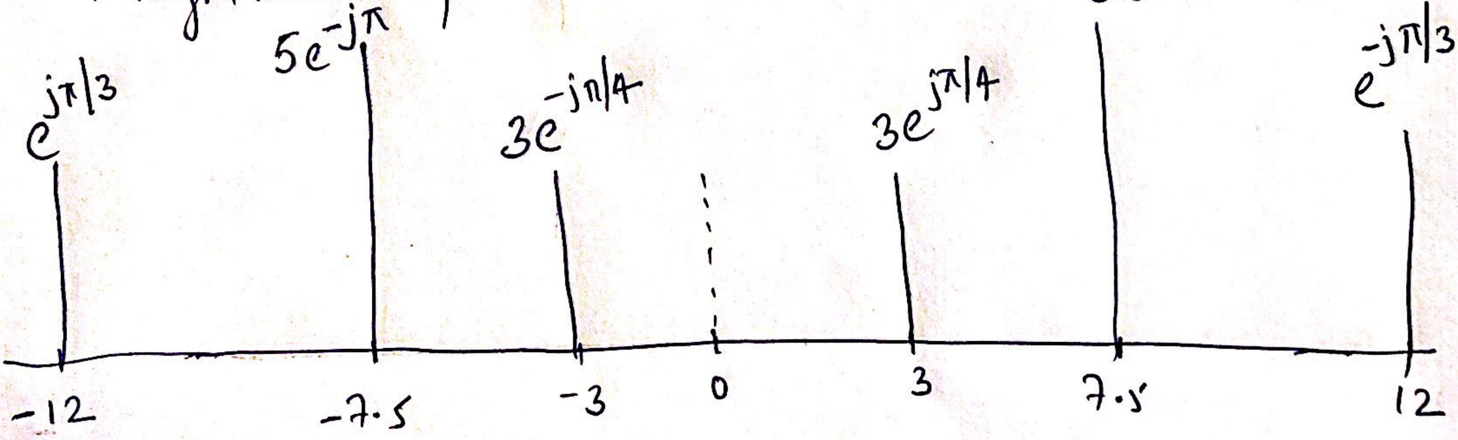
$$\frac{10}{2} e^{j\pi} \cdot e^{j2\pi\left(\frac{15}{2}\right)t} + \frac{10}{2} e^{-j\pi} \cdot e^{-j2\pi\left(\frac{15}{2}\right)t} +$$

$$\frac{2}{2} e^{-j\pi/3} \cdot e^{j2\pi(12)t} + \frac{2}{2} e^{j\pi/3} \cdot e^{-j2\pi(12)t}$$

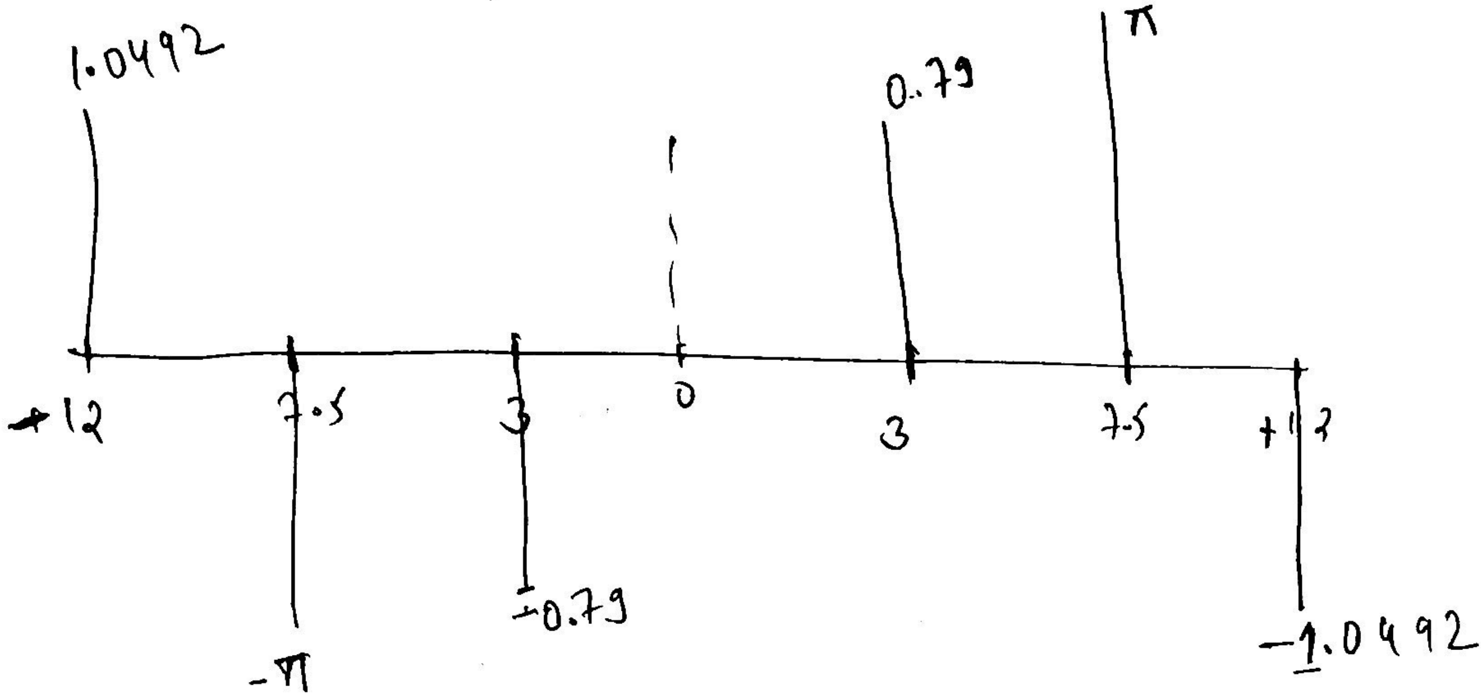
$$\therefore \left\{ \left(3, \tfrac{1}{2} \times 2 e^{j\pi/4}\right), \left(-3, \tfrac{1}{2} \cdot 2 \cdot e^{-j\pi/4}\right), \left(7.5, \tfrac{10}{2} e^{j\pi}\right), \left(-7.5, \tfrac{10}{2} e^{-j\pi}\right), \left(12, \tfrac{2}{2} e^{-j\pi/3}\right), \left(-12, \tfrac{2}{2} e^{j\pi/3}\right) \right\}$$

$$\equiv \left\{ \left(3, e^{j\pi/4}\right), \left(-3, e^{-j\pi/4}\right), \left(7.5, 5e^{j\pi}\right), \left(-7.5, 5e^{-j\pi}\right), \left(12, e^{-j\pi/3}\right), \left(-12, e^{j\pi/3}\right) \right\}.$$

(b) Magnitude Spectrum:



Phase Spectrum



(c) Yes, $x(t)$ is periodic.

The gcd of $(\pm 3, \pm 7.5, \pm 12)$ is $1.5\ Hz$

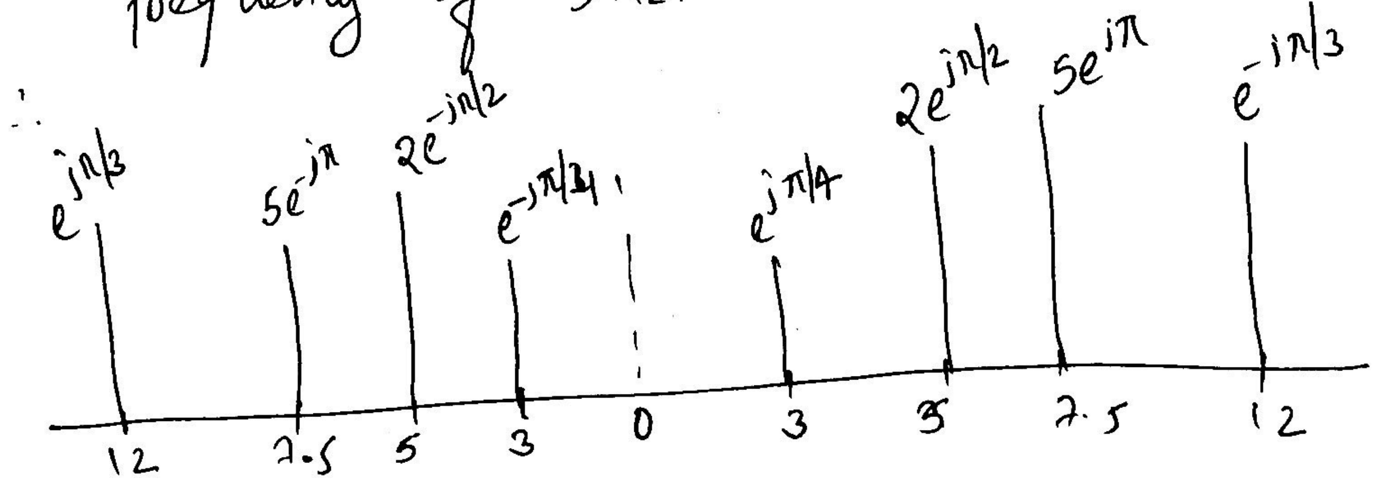
$\therefore$ fundamental period is $1/1.5 = 0.66\ sec$

(d) $y(t) = x(t) + 4\cos(10\pi t + \pi/2)$

$y(t) = x(t) + 2e^{j\pi/2}e^{2\pi(5)t} + 2e^{-j\pi/2}e^{-2\pi(5)t}$

There is a new signal in spectrum with frequency of 5 Hz.

$\therefore$



The gcf of $(\pm 3, \pm 5, \pm 7.5, \pm 12)$ is 0.5

$\therefore$ The ~~freq~~ fundmental frequency is 0.5 Hz.

$\therefore$ fundamental peiod $= \frac{1}{0.5} = \underline{\underline{2 \text{ sec}}}$

3) (a) $x(t) = \sin^3(27\pi t)$

$$= \left(\frac{1}{2j} e^{j27\pi t} - \frac{1}{2j} e^{-27\pi t}\right)^3$$

Using $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$$x(t) = \frac{j}{8}\left(e^{j81\pi t} - 3e^{j54\pi t - j27\pi t} + 3e^{j27\pi t - j54\pi t} - e^{-j81\pi t}\right)$$

$$x(t) = \frac{1}{8} e^{j\pi/2} e^{j81\pi t} + \frac{3}{8} e^{-j\pi/2} e^{j27\pi t} + \frac{3}{8} e^{j\pi/2} e^{-j27\pi t} + \frac{1}{8} e^{-j\pi/2} e^{-j81\pi t}$$

$$x(t) = \underline{\frac{1}{4}\cos(81\pi t + \pi/2) + \frac{3}{4}\cos(27\pi t - \pi/2)}$$

(b) There are two signals with

$$\omega_1 = 81\pi \qquad\qquad \omega_2 = 27\pi$$

$$2\pi f_1 = 81\pi \qquad\qquad 2\pi f_2 = 27\pi$$

$$f_1 = \frac{81}{2} \qquad\qquad f_2 = \frac{27}{2}$$

$$f_1 = 3 \times \frac{27}{2}$$

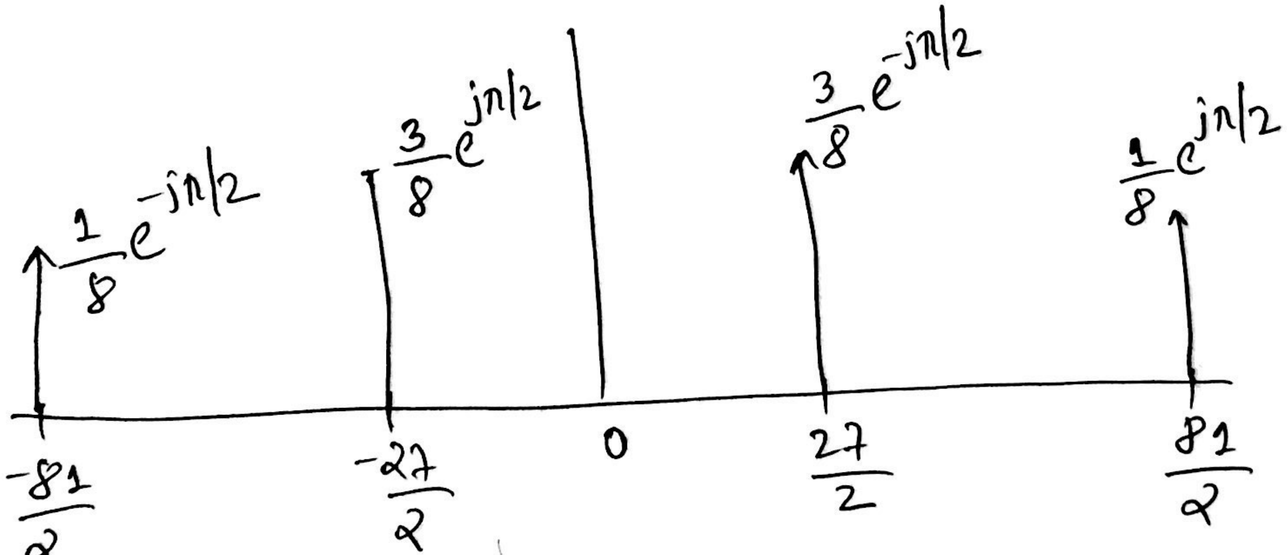
$\therefore$ The gcd of f_1 and f_2 is

$$\frac{27}{2}\ \text{Hz}$$

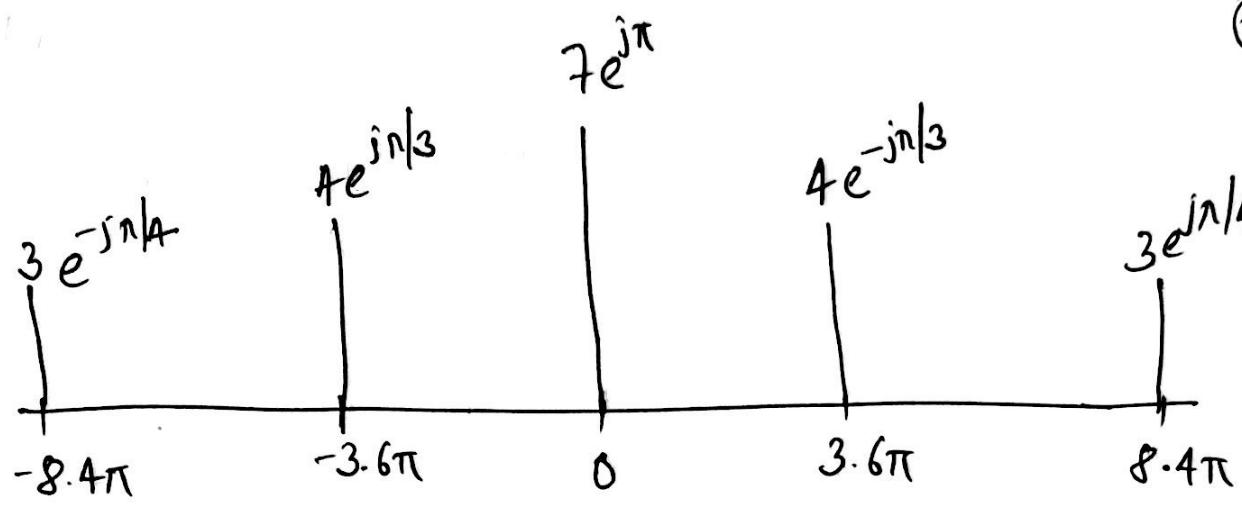
Hence, fundamental frequency is $\frac{27}{2}$ Hz

Time Period is $\frac{1}{f} = \underline{\underline{\frac{2}{27}\ \text{sec}}}$

(c)



4)



(a) fundament frequency =

$$GCD(\pm 3.6\pi, \pm 8.4\pi) = \underline{1.2\pi \text{ rad/sec}}$$

(b) Fundamental Period ⇒

$$2\pi f = w$$

$$2\cancel{\pi} f = 1.2\cancel{\pi}$$

$$f = 0.6$$

$$\therefore T_0 = \frac{1}{f} = \frac{10}{6} = \underline{\frac{5}{3} \text{ sec}}$$

(c) The DC val signal is

$$\underline{7e^{j\pi}}$$

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(d) The fundamental frequency is 1.2π: the values are
∴ For all non-zero fourier coefficients,

K	ω	a_K		Amp	phase
0	0	$7e^{j\pi}$	$= -7 + 0j$	7	0
+3	3.6π	$4e^{-j\pi/3}$	$= 2 - 2\sqrt{3}j$	4	$\frac{2\pi}{3}$
−3	-3.6π	$4e^{j\pi/3}$	$= 2 + 2\sqrt{3}j$	4	$\pi/3$
+7	8.4π	$3e^{j\pi/4}$	$= \frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}j$	3	$\pi/4$
−7	-8.4π	$3e^{-j\pi/4}$	$= \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}j$	3	$\frac{3\pi}{4}$

5) $x(t) = 3\cos(2000\pi t + \pi/4) + \cos(4000\pi t + \pi/2)$

(a) $y(t) = \left[3\cos(2\pi 10^3 \pi t + \pi/4) + \cos(2\pi . 2 . 10^3 t + \pi/2) + A\right]$
$\cos(2\pi . 620 . 10^3 t)$

from the trigonometry, we can use equation

$$\cos\theta_1 . \cos\theta_2 = \frac{1}{2}\cos(\theta_1+\theta_2) + \frac{1}{2}\cos(\theta_1-\theta_2)$$

with, $A = 2$

$$\therefore y(t) = \frac{2}{2}\cos(2\pi . 620 . 10^3 t)$$
$$+ \frac{3}{2}\cos(2\pi . 620 . 10^3 t + 2\pi . 10^3 t + \pi/4)$$
$$+ \frac{3}{2}\cos(2\pi . 620 . 10^3 t - 2\pi . 10^3 t + \pi/4)$$
$$+ \frac{1}{2}\cos(2\pi . 620 . 10^3 t + 2\pi . 2 . 10^3 t + \pi/2)$$
$$+ \frac{1}{2}\cos(2\pi . 620 . 10^3 t - 2\pi . 2 . 10^3 t - \pi/2)$$

$\therefore$ 3 $A_3 = \frac{2}{2} = 1$, $\omega_3 = 2\pi . 620 . 10^3 t$, $\phi_3 = 0$

5 $A_5 = \frac{1}{2}$ $\omega_5 = 2\pi . 622 . 10^3 t$, $\phi_5 = \pi/2$

4 $A_4 = \frac{3}{2}$ $\omega_4 = 2\pi\ 621\ 10^3 t$, $\phi_4 = \pi/4$

2 $A_2 = \frac{3}{2}$ $\omega_2 = 2\pi . 619 . 10^3 t$, $\phi_2 = -\pi/4$

1 $A_1 = \frac{1}{2}$ $\omega_1 = 2\pi . 618 . 10^3 t$, $\phi_1 = -\pi/2$

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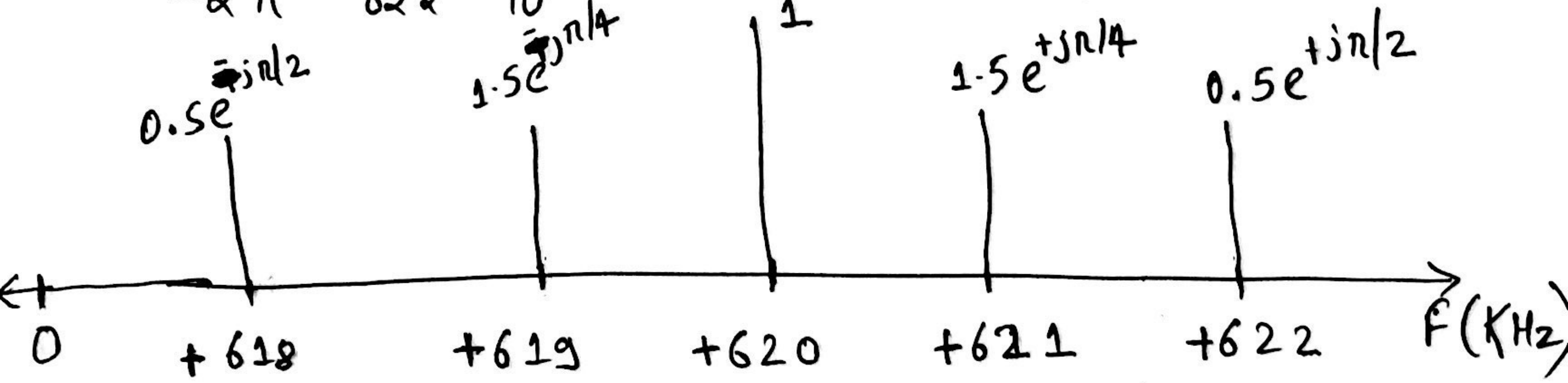
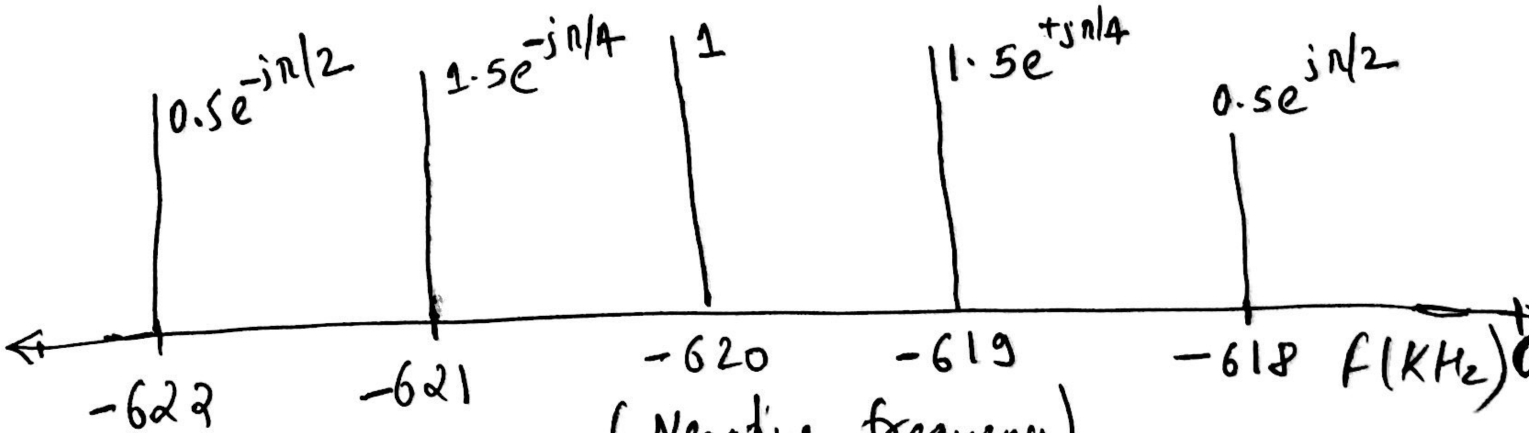
(b)

frequency (rad/sec)	Complex Amplitude
$2\pi \cdot 618 \cdot 10^3$	$0.5\, e^{-j\pi/2}$
$-2\pi\, 618\, 10^3$	$0.5\, e^{+j\pi/2}$
$2\pi \cdot 619\, 10^3$	$1.5\, e^{-j\pi/4}$
$-2\pi \cdot 619\, 10^3$	$1.5\, e^{j\pi/4}$
$2\pi\, 620\, 10^3$	1
$-2\pi\, 620\, 10^3$	1
$2\pi \cdot 621\, 10^3$	$1.5\, e^{j\pi/4}$
$-2\pi\, 621\, 10^3$	$1.5\, e^{-j\pi/4}$
$2\pi\, 622\, 10^3$	$0.5\, e^{j\pi/2}$
$-2\pi\, 622\, 10^3$	$0.5\, e^{-j\pi/2}$





(Negative frequency)