Prelab-3

Ravitashaw Bathla
(rb369)

1) $x(t) = 3\cos\left(w_0 t - \frac{2}{3}\pi\right) + \cos(w_0 t)$

(a) $x(t) \Rightarrow 3e^{-\frac{2}{3}\pi j} + e^{0j} = 3e^{-\frac{2}{3}\pi j} + 1$

$$= 3\left[\cos\left(-\frac{2\pi}{3}\right) + j\sin\left(-\frac{2\pi}{3}\right)\right] + 1$$

$$= 3\left[\cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right)\right] + 1$$

$$= 3\left[-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right] + 1$$

$$= -\frac{3}{2} - j\frac{3\sqrt{3}}{2} + 1 = -\frac{1}{2} - j\frac{3\sqrt{3}}{2}$$

$$\Rightarrow r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} = 2.646$$

$$\theta = \tan^{-1}\left(\frac{3\sqrt{3}}{2} \times \frac{2}{1}\right) = \tan^{-1}(3\sqrt{3}) = 1.380 - \pi$$

$$= -1.76$$

$$\therefore 2.646e^{-j1.76}$$

$$\boxed{\therefore x(t) = 2.646\cos(w_0 t - 1.76)}$$

(b) $x(t) = \Re\{z(t)\}$

$$\therefore z(t) = 2.646e^{(-j1.76 + w_0 t)} = 2.646 \cdot e^{-j1.76} \cdot e^{jw_0 t}$$

2) (a) $x_1(t) = \sqrt{5}\cos(7t - \pi/3)$ Using Euler's formula

$$z_1(t) = \sqrt{5}\, e^{j(7t-\pi/3)} = \sqrt{5} e^{-j\pi/3} e^{j7t}$$

(b) $x_2(t) = \sqrt{5}\cos(7t + \pi)$, Using Euler's formula

$$z_2(t) = \sqrt{5}\, e^{j(7t+\pi)} = \sqrt{5}\, e^{j\pi} \cdot e^{j7t}$$

(c) $\mathcal{R}\{z(t)\} = \mathcal{R}\{z_1(t) + z_2(t)\} = \mathcal{R}\{z_1(t)\} + \mathcal{R}\{z_2(t)\}$.

$$= \sqrt{5} e^{-j\pi/3} e^{j7t} + \sqrt{5} e^{j\pi} e^{j7t}$$

$$= \sqrt{5} e^{7t} \left[e^{-j\pi/3} + e^{j\pi} \right]$$

$$= \sqrt{5} e^{7t} \left[\cos(-\pi/3) + j\sin\left(-\frac{\pi}{3}\right) + \cos(\pi) + j\sin\pi \right]$$

$$= \sqrt{5} e^{7t} \left[\frac{1}{2} - j\frac{\sqrt{3}}{2} - 1 + 0 \right]$$

$$= \sqrt{5} e^{7t} \left[-\frac{1}{2} - \frac{\sqrt{3}}{2} j \right] = \sqrt{5} e^{7t} \left(\sqrt{\frac{1}{2}^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \right)$$

$$\tan^{-1}\left(\frac{-\sqrt{3}/2}{-1/2}\right)$$

$$= \sqrt{5} e^{7t} \cdot e^{-\frac{2\pi}{3} j}$$

$$= \sqrt{5} e^{\left(7t - \frac{2\pi}{3} j\right)}$$

$$= \sqrt{5} \cos\left(7t - \frac{2\pi}{3}\right)$$

3) (a) $9\cos(\omega t + \phi) = Ae^{j8t + j\phi} + Ae^{-j8t + j2\pi/3}$

Using Inverse Euler's formula:

$$9\cos(\omega t + \phi) = \frac{9e^{j(\omega t+\phi)} + 9e^{-j(\omega t + \phi)}}{2}$$

$$= \frac{9}{2} e^{j\omega t + j\phi} + \frac{9}{2} e^{-j\omega t - \phi}$$

On comparing the given equation with the above equation.

$$A = \frac{9}{2} = 4.5$$

$$\omega = 8 \text{ rad/sec}$$

$$\phi = -\frac{2\pi}{3}$$

(b) $10\cos(9t - \pi/3) = A\cos(\omega t - \pi/2) + 5\cos(\omega t + \phi)$

$$\mathcal{R}\left\{10 e^{9tj - \pi/3 j}\right\} = \mathcal{R}\left\{A e^{\omega tj - \pi/2 j} + 5 e^{\omega tj + \phi j}\right\}$$

$$10 e^{9tj} \cdot e^{-\pi/3 j} = A e^{\omega tj} \cdot e^{-\pi/2 j} + 5 e^{\omega tj} \cdot e^{\phi j}$$

$$10 e^{9tj}\left[\cos\frac{\pi}{3} + j\sin\left(-\frac{\pi}{3}\right)\right] = A e^{\omega tj}\left[\cos\left(-\frac{\pi}{2}\right) + j\sin\left(-\frac{\pi}{2}\right)\right] + 5 e^{\omega tj}\left[\cos\phi + j\sin\phi\right]$$

$$10 \cancel{e^{9tj}}\left[\frac{1}{2} - j\frac{\sqrt{3}}{2}\right] = \cancel{e^{\omega tj}}\left[A(0 - j) + 5(\cos\phi + j\sin\phi)\right].$$

$$\boxed{\omega = 9}$$

$$5 - j5\sqrt{3} = (5\sin\phi - A)j + 5\cos\phi$$

Comparing real & imaginary part,

$$5 = 5\cos\phi$$

$$\Rightarrow \boxed{\phi = 0}$$

$$5\sin\phi - A = -5\sqrt{3}$$

$$5\sin(0) - A = -5\sqrt{3}$$

$$A = 5\sqrt{3}$$

$$\boxed{A = 8.66}$$

4) (a) $\cos 7t = A_1 \cos(7t + \phi_1) + A_2 \cos(7t + \phi_2)$

$$e^{j7t} = A_1 e^{j\phi_1} \cdot e^{7t} + A_2 e^{j\phi_2} \cdot e^{7t}$$

$$\boxed{1 = A_1 e^{j\phi_1} + A_2 e^{j\phi_2}}$$

$$\sin 7t = \cos\left(7t - \frac{\pi}{2}\right) = 2A_1 \cos(7t + \phi_1) + A_2 \cos(7t + \phi_2)$$

$$e^{j7t} \cdot e^{-j\pi/2} = 2A_1 e^{j\phi_1} e^{7t} + A_2 e^{j\phi_2} \cdot e^{7t}$$

$$\boxed{e^{-\pi/2} = 2A_1 e^{j\phi_1} + A_2 e^{j\phi_2}}$$

(b)

$$1 = Z_1 + Z_2$$

$$e^{-j\pi/2} = 2Z_1 + Z_2$$

(c)

$$Z_1 + Z_2 = 1$$

$$2Z_1 + Z_2 = e^{-j\pi/2}$$

$$-Z_1 = 1 - e^{-j\pi/2}$$

$$\Rightarrow Z_1 = e^{-j\pi/2} - 1 = \left(\cos(-\pi/2) + j \sin(-\pi/2)\right) - 1$$

$$= 0 - j - 1 = \underline{\underline{-1-j}}$$

$$\therefore Z_2 = 1 - Z_1 = 1 + 1 + j = \underline{\underline{2 + j}}$$

(d) $Z_1 = \underline{\underline{\sqrt{2}\, e^{-j3\pi/4}}}$, $Z_2 = \sqrt{5}\, e^{j \tan^{-1}(1/2)}$

$$= \underline{\underline{\sqrt{5}\, e^{j0.4636}}}$$

$\therefore$ $\underline{\underline{A_1 = \sqrt{2}}}$, $\underline{\underline{\phi_1 = -\frac{3\pi}{4}}}$, $\underline{\underline{A_2 = \sqrt{5}}}$, $\phi_2 = \underline{\underline{0.4636 \text{ rad/s}}}$

5) $z(t) = Ze^{j2\pi t}, \quad Z = e^{j\pi/4}$

(a) $\dfrac{dz(t)}{dt} = \dfrac{d(Ze^{j2\pi t})}{dt} = Z(j2\pi)e^{j2\pi t}$

$= 2\pi Z j \cdot e^{j2\pi t}$

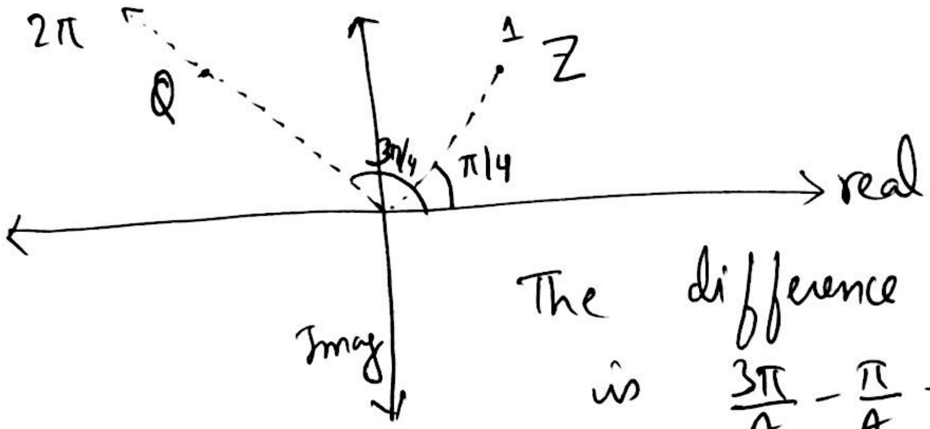
$\therefore \boxed{Q = 2\pi Z j}$

$Q = 2\pi e^{j\pi/4} j$

$= 2\pi j\left[\cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right)\right]$

$= 2\pi\left[j\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right] = \underline{2\pi e^{+3\pi/4 j}}$

(b)



The difference b/w Q & Z is $\frac{3\pi}{4} - \frac{\pi}{4} = \underline{\frac{\pi}{2}}$

(c) $\mathcal{R}\left\{\frac{d(z(t))}{dt}\right\} = \mathcal{R}\left\{Q \cdot e^{j2\pi t}\right\} = \mathcal{R}\left\{2\pi e^{\frac{3\pi}{4}j} \cdot e^{j2\pi t}\right\}$

$= 2\pi\cos\left(2\pi t + \frac{3}{4}\pi\right)$

On the other side,

$$\frac{d}{dt}\Re\{z(t)\} = \frac{d}{dt}\Re\{Z e^{j2\pi t}\}$$

$$= \frac{d}{dt}\Re\{e^{j\pi/4} e^{j2\pi t}\}$$

$$= \frac{d}{dt}\{\cos(2\pi t + \pi/4)\}$$

$$= 2\pi\left[-\sin(2\pi t + \pi/4)\right]$$

$$= +2\pi\cos\left(2\pi t + \frac{\pi}{4} + \frac{\pi}{2}\right) \quad \because \cos(\pi/2+\theta) = -\sin\theta$$

$$= 2\pi\cos\left(2\pi t + \frac{3\pi}{4}\right)$$

Both the sides are equal, therefore the condition is true. It is also true that this condition will be True for any complex signal.

(d) $$\int_{-0.5}^{0.5} z(t)\,dt = \int_{-0.5}^{0.5}\left(e^{j\pi/4} \cdot e^{j2\pi t}\right)dt$$

$$= \frac{e^{j\pi/4}}{2\pi j} \times e^{j2\pi t}\Big|_{-0.5}^{0.5} = \frac{e^{j\pi/4}}{2\pi j}\left[e^{j\pi} - e^{-j\pi}\right]$$

$$= \frac{e^{j\pi/4}}{2\pi j}\left[\cos\pi + j\sin\pi - \cos(-\pi) \div j\sin(-\pi)\right]$$

$$= \frac{e^{j\pi/4}}{2\pi j}\left[\cancel{\cos\pi} + j\underbrace{\sin\pi}_{0} - \cancel{\cos\pi} + j\underbrace{\sin(\pi)}_{0}\right]$$

$$= \frac{e^{j\pi/4}}{2\pi j} \times 0 = 0.$$

The area under the curve for ~~the~~ one complete time period ($T = 1$ sec) is always zero.