Prelab-6

Ravitashaw Bathla
(rb369)

1) $x(t) = 6\cos(5\pi t - \pi/4) \xrightarrow[f_s]{} x[n]$

(a) $f_s = 7$ samples/sec

Input frequency = 2.5 Hz

$\therefore f_s > 2(2.5) \Rightarrow$ $\boxed{\text{Oversampled}}$

$x[n] = 6\cos\left(5\pi\left(\frac{n}{7}\right) - \frac{\pi}{4}\right) = 6\cos\left(\frac{5\pi}{7}n - \frac{\pi}{4}\right)$

$$\boxed{x[n] = 6\cos\left(\frac{5\pi}{7}n - \frac{\pi}{4}\right)}$$

(b) $f_s = 4$ samples/sec

Input frequency = 2.5 Hz

$\therefore f_s < 2(2.5) \Rightarrow$ $\boxed{\text{Undersampled}}$

$x[n] = 6\cos\left(5\pi\left(\frac{n}{4}\right) - \frac{\pi}{4}\right) = 6\cos\left(\frac{5}{4}\pi n - \frac{\pi}{4}\right)$

$= 6\cos\left(2\pi n - \frac{3\pi n}{4} - \frac{\pi}{4}\right) = \boxed{6\cos\left(\frac{3\pi n}{4} + \frac{\pi}{4}\right)}$

This is $\boxed{\text{Folding}}$ because $\pi < \frac{5}{4}\pi n < 2\pi$

(c) $f_s = 2$ sample/sec

$\therefore f_s < 2(2.5) \Rightarrow$ $\boxed{\text{Undersampled}}$

$$x[n] = 6\cos\left(5\pi\left(\frac{n}{2}\right) - \frac{\pi}{4}\right)$$

$$= 6\cos\left(\frac{5\pi n}{2} - \frac{\pi}{4}\right) = 6\cos\left(2\pi n + \frac{\pi n}{2} - \frac{\pi}{4}\right)$$

$$= \boxed{6\cos\left(\frac{\pi n}{2} - \frac{\pi}{4}\right)}$$

Here, $\frac{5\pi n}{2} > 2\pi$, $\therefore$ It is $\boxed{\text{aliasing}}$

2) (a) $y[n] = 4\cos(0.4\pi n - \pi/3)$

→ If No aliasing:

$$y(t) = 4\cos(0.4\pi(2500)t - \pi/3)$$

$$= 4\cos(\underline{1000\pi t} - \pi/3)$$

If folding:

$$y(t) = 4\cos((0.4\pi - 2\pi)2500t - \pi/3)$$

$$= 4\cos(-1.6\pi \times 2500t - \pi/3)$$

$$= 4\cos(\underline{4000\pi t} + \pi/3)$$

(b) $y[n] = 8\cos(1.2\pi n - \pi/2)$

If No aliasing, $y(t) = 8\cos(1.2\pi \times 2500 \times t - \pi/2)$

$$= 8\cos(\underline{3000\pi t} - \pi/2)$$

If folding, $y(t) = 8\cos((1.2\pi - 2\pi)2500 - \pi/2)$

$$= 8\cos(\underline{2000\pi t} + \pi/2)$$

(c) $y[n] = 2\cos(2.6\pi n - \pi/4) =$ 3

if aliasing is assumed

$$y(t) = 2\cos(2.6\pi \times 2500\,t - \pi/4)$$

$$= \underline{\underline{2\cos(6500\pi t - \pi/4)}}$$

if aliasing is performed,

$$y[n] = 2\cos(0.6\pi n - \pi/4)$$

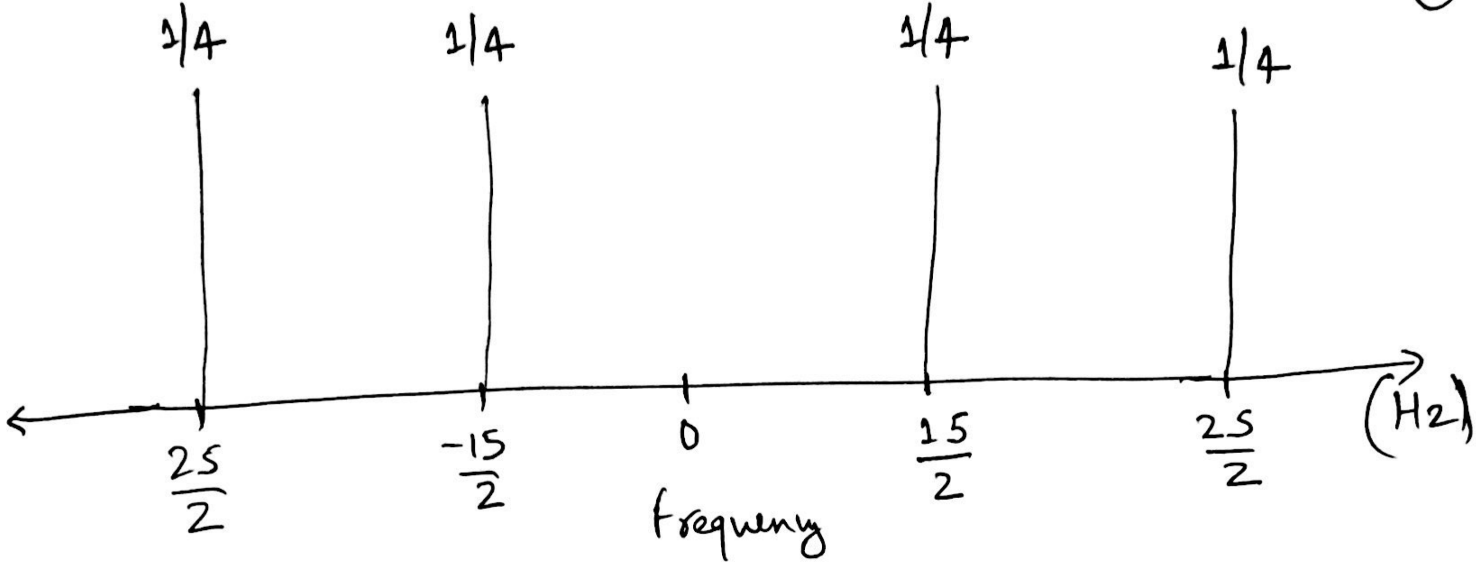
$$y(t) = 2\cos(0.6\pi \times 2500\,t - \pi/4)$$

$$= \underline{\underline{2\cos(1500\pi t - \pi/4)}}$$

3) (a) $x(t) = \cos(5\pi t) \cdot \cos(20\pi t)$

$$= \left(\frac{e^{i5\pi t} + e^{-i5\pi t}}{2}\right)\left(\frac{e^{i20\pi t} + e^{-i20\pi t}}{2}\right)$$

$$= \frac{1}{4}e^{i25\pi t} + \frac{1}{4}e^{-i15\pi t} + \frac{1}{4}e^{i15\pi t} + \frac{1}{4}e^{-i25\pi t}$$



(b) Minimum Sampling Rate

$$\geq 2 \times (\text{Highest Frequency})$$

$$\geq 2 \times \left(\frac{25\pi}{2\pi}\right)$$

$$\geq \underline{\underline{25\,Hz}}$$

(c) $$x(t) = \cos(2\pi t)\sin(4\pi t)\cos(8\pi t)$$

$$= \left(\frac{e^{i2\pi t} + e^{-i2\pi t}}{2}\right)\left(\frac{e^{i4\pi t} - e^{-i4\pi t}}{2i}\right)\left(\frac{e^{i8\pi t} + e^{-i8\pi t}}{2}\right)$$

$$= \frac{-1i}{8}\left[\left(e^{i6\pi t} - e^{-i6\pi t} - e^{-i2\pi t} + e^{i2\pi t}\right)\left(e^{i8\pi t} + e^{-i8\pi t}\right)\right]$$

$$= \frac{1}{8} e^{-\pi/2}\left[e^{i14\pi t} - e^{i2\pi t} - e^{i6\pi t} + e^{i10\pi t} + e^{-i2\pi t} + e^{-i14\pi t} - e^{-i10\pi t} + e^{-i6\pi t}\right]$$

frequencies: $(\pm 1, \pm 3, \pm 5, \pm 7)$

$$\text{Minimum Sampling Rate} \geq 2 \times f_{max}$$
$$\geq 2 \times 7$$
$$\geq 14 \text{ Hz}$$

(d) $$v(t) = \cos(2\pi t) + \sin(4\pi t) + \cos(8\pi t)$$

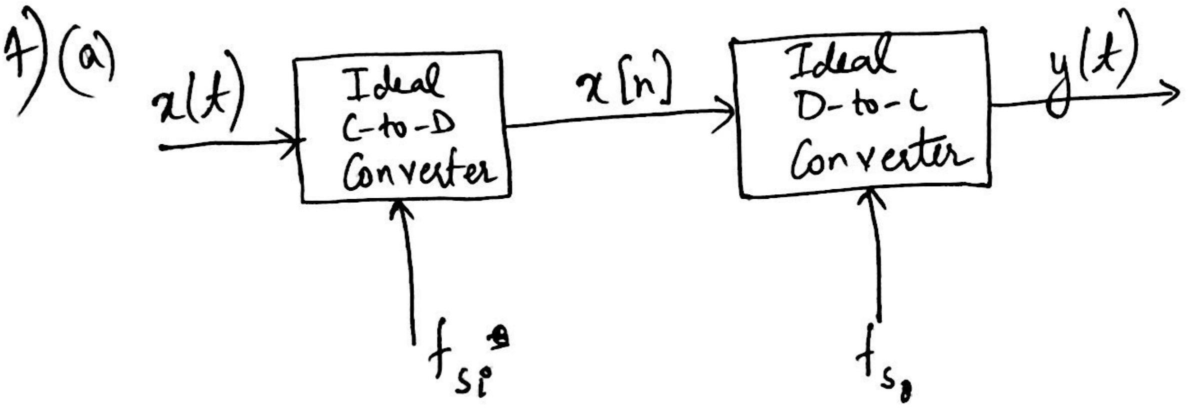
$$= \frac{e^{i2\pi t} + e^{-i2\pi t}}{2} + \frac{e^{i4\pi t} - e^{-i4\pi t}}{2j} + \frac{e^{i8\pi t} + e^{-i8\pi t}}{2}$$

$$= \frac{1}{2} e^{i2\pi t} \quad \frac{1}{2} e^{-i2\pi t} - \frac{1}{2} e^{i\pi/2} \cdot e^{i4\pi t} + \frac{1}{2} e^{-i\pi/2} \cdot e^{i4\pi t}$$
$$+ \frac{1}{2} e^{i8\pi t} + \frac{1}{2} e^{-i8\pi t}$$

frequencies: $(\pm 1, \pm 2, \pm 4)$

$$\text{Minimum Sampling Rate} \geq 2 \times f_{max}$$
$$\geq 2 \times 4$$
$$\geq 8 \text{ Hz}$$

7) (a)



Where,

$$x(t) = 6\cos(200\pi t + 0.2\pi) + 4\cos(800\pi t + 0.7\pi)$$

$$x[n] = 6\cos\left(200\pi\left(\frac{n}{360}\right) + 0.2\pi\right) + 4\cos\left(800\pi\left(\frac{n}{360}\right) + 0.7\pi\right)$$

$$= 6\cos\left(\frac{5}{9}\pi n + 0.2\pi\right) + 4\cos\left(2\pi n + \frac{2\pi n}{9} + 0.7\pi\right)$$

$$= 6\cos\left(\frac{5}{9}\pi n + 0.2\pi\right) + 4\cos\left(\frac{2\pi n}{9} + 0.7\pi\right)$$

f_s = 360 samples/sec

$$y(t) = 6\cos\left(360t \cdot \frac{5\pi}{9} + 0.2\pi\right) + 4\cos\left(\frac{2\pi}{9} \cdot 360t + 0.7\pi\right)$$

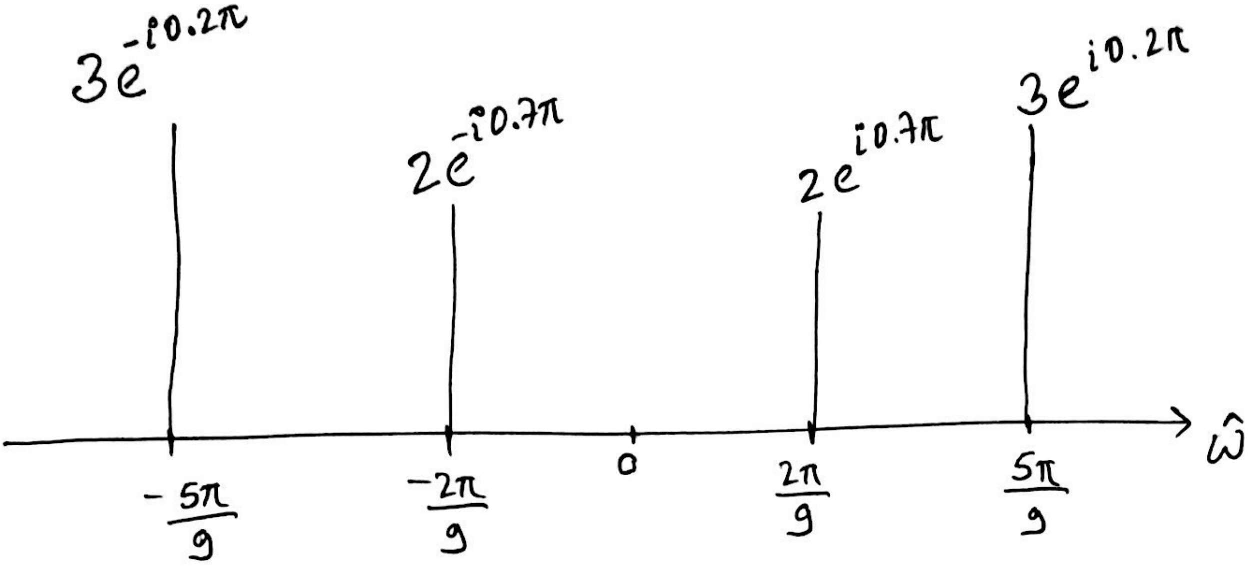
$$= 6\cos\left(200\pi t + 0.2\pi\right) + 4\cos\left(80\pi t + 0.7\pi\right)$$

(b) No, the output signal $y(t)$, not equal to $x(t)$ becausing of aliasing.

(c) $$x[n] = 6\cos\left(\frac{5}{9}\pi n + 0.2\pi\right) + 4\cos\left(\frac{2\pi n}{9} + 0.7\pi\right)$$

$$= 3 \cdot e^{i0.2\pi} \cdot e^{i\frac{5}{9}\pi n} + 3e^{-i0.2\pi} e^{-i\frac{5}{9}\pi n}$$

$$+ 2e^{i0.7\pi} e^{i\frac{2\pi n}{9}} + 2e^{-i0.7\pi} \cdot e^{-i\frac{2\pi n}{9}}$$



(d)

$$y(t) = 6\cos\left(360t \cdot \frac{5\pi}{9} + 0.2\pi\right) + 4\cos\left(\frac{2\pi}{9} \cdot 360t + 0.7\pi\right)$$

$$= 6\cos(200\pi t + 0.2\pi) + 4\cos(80\pi t + 0.7\pi)$$

(assuming no aliasing occured)

5)

$$x(t) = \sum_{K=-10}^{K} (j8\pi k)\, e^{j5\pi k}$$

(a) Minimum Sampling Rate $\geq 2 \times f_{max}$

$$\geq 2 \times \left(\frac{5\pi}{2\pi}\right) K_{max}$$

$$\geq 2 \times 2.5 \times 10$$

$$\geq \underline{\underline{50\ Hz}}$$

(b) $\quad y(t) = x(t)\cos(22\pi t)$ (8)

$$= \left(\sum_{K=-10}^{10} (j8\pi k)\, e^{j5\pi kt}\right)\left(\frac{e^{j22\pi t} + e^{-j22\pi t}}{2}\right)$$

$$= \frac{1}{2}\sum_{K=-10}^{10} (j8\pi k)\, e^{j(5\pi kt + 22\pi t)} + e^{+j(5\pi kt - 22\pi t)}$$

Nyquist Sampling Rate $= 2 \times f_{max}$

$$= 2 \times \left(\frac{5k\pi + 22\pi}{2\pi}\right)$$

for $K_{max} = 10$

$$= 2 \times \frac{(50+22)\pi}{2\pi}$$

$$= 2 \times \frac{72\pi}{2\pi} = \underline{\underline{72\,Hz}}$$

(c) $\quad v(t) = x(t) + x(t - 0.02)$

$$= \sum_{K=10}^{-10} (j8\pi k)\, e^{j5\pi kt} + \sum_{K=10}^{-10} (j8\pi k)\, e^{j5\pi kt} \cdot e^{-j\pi k}$$

There is no change in frequency from time shift

$\therefore$ Nyquist Sampling Rate $= 2 \times f_{max}$

$$= 2 \times \frac{5\pi \times 10}{2\pi} = \underline{\underline{50\,Hz}}$$

6) (a)

$f_s = 8000$ samples/sec

$x[n] = \cos(0.2\pi n)$

$\omega = 0.2\pi \times f_s = 1600\pi$

$f_0 = \frac{1600\pi}{2\pi} = 800 \text{ Hz}.$

$f_{input} = 800 \pm n\,8000 \geq 10000$

$= 800 \pm 16000$

$= (16800) \text{ or } (15,200)$

∴ The smallest fequency greater than 10K is 15200Hz

∴ $x(t) = \cos(15200 \times 2\pi t) = \underline{\underline{\cos(30,400\pi t)}}$

(b)

$x[n] = \cos(0.25\pi n)$

$x(t) = \cos(510\pi t)$ $\qquad f_s < 130$ samples/sec

$\hat{\omega} = \frac{\omega}{f_s}$

$510\pi = f_s(\pm 0.25\pi \pm 2\pi n)$

$f_s = \frac{510\pi}{2\pi n \pm 0.25\pi} = \frac{2040}{\pm 1 + 2 \times 4n} = \frac{2040}{\pm 1 + 8n}$

∴ $n = 1 \Rightarrow \frac{2040}{7}$ or $\frac{2040}{9}$

$= 291.2$ $\qquad 226.6$

$n = 2 \Rightarrow \frac{2040}{15}$ or $\frac{2040}{17} = \underline{\underline{120 \text{ Hz}}}$

$= 136$

$\therefore$ The largest possible sampling rate is $\underline{120 Hz}$

(c) $$y(t) = \cos(240\pi t)$$

$$x(t) = \cos(60\pi t)$$

$$\hat{w} = \frac{w}{f_s}$$

$$\frac{240\pi}{f_s} = \frac{\pm 60\pi}{f_s} + 2\pi n$$

Aliasing

$$\frac{180\pi}{f_s} = 2\pi n$$

$$\boxed{f_s = \frac{90}{n}}$$

$n = 1,\ f_s = 90$

folding

$$\frac{300\pi}{f_s} = 2\pi n$$

$$\boxed{f_s = \frac{150}{n}}$$

$n = 1\ ,\ f_s = 150$

$\therefore$ The maximum frequency we get is $\underline{150 Hz}$

7) (a) $a[n] = b[n]$ and no aliasing.

$$\therefore \frac{w_a}{f_{sa}} = \frac{w_b}{f_{sb}}$$

$$w_a = \frac{30\pi \times 3\cancel{0}}{8\cancel{0}} = \frac{90}{8}\pi = \underline{11.25\pi \text{ rad/se}}$$

(b) $a[n] = b[n]$

$$\therefore \frac{W_a}{f_{sa}} = 2\pi n \pm \frac{W_b}{f_{sb}}$$

$$\frac{W_a}{30} = 2\pi n \pm \frac{W_b}{80}$$

also, $\omega_a = W_b$

$$\therefore \frac{W_a}{30} = 2\pi n \pm \frac{W_a}{80}$$

Aliasing (+) OR folding (−)

Aliasing (+)

$$\frac{W_a}{30} - \frac{W_a}{80} = 2\pi n$$

$$\frac{5\cancel{0} W_a}{24\cancel{0}} = 2\pi n$$

$$W_a = \frac{48\pi}{5} n$$

$$= 9.6\pi n$$

$0, 9.6\pi, 19.2\pi, 28.8\pi, \ldots \ldots 86.4\pi, 96\pi$

folding (−)

$$\frac{W_a}{30} + \frac{W_a}{80} = 2\pi n$$

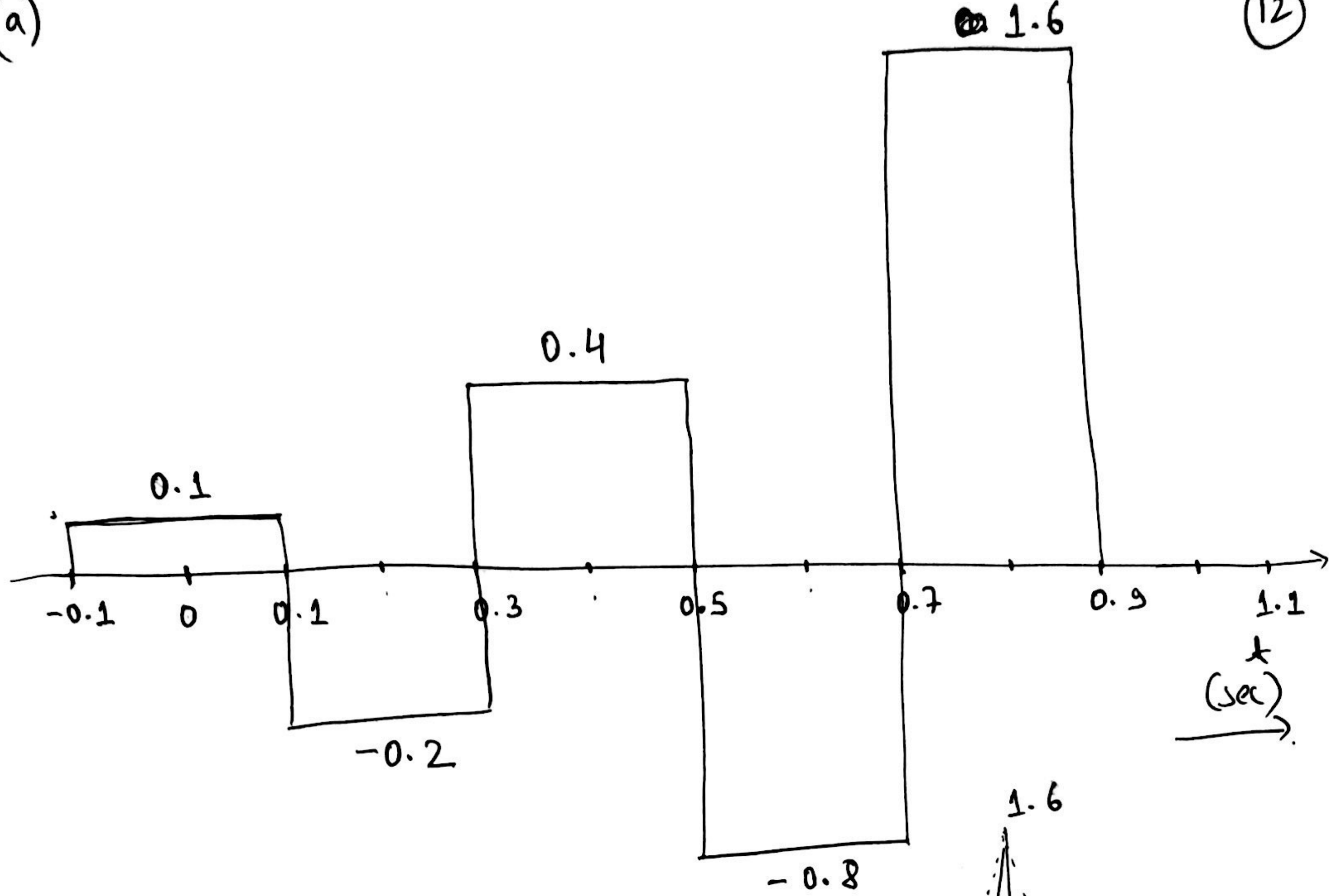
$$\frac{13\cancel{0} W_a}{24\cancel{0}} = 2\pi n$$

$$W_a = \frac{48}{13}\pi n$$

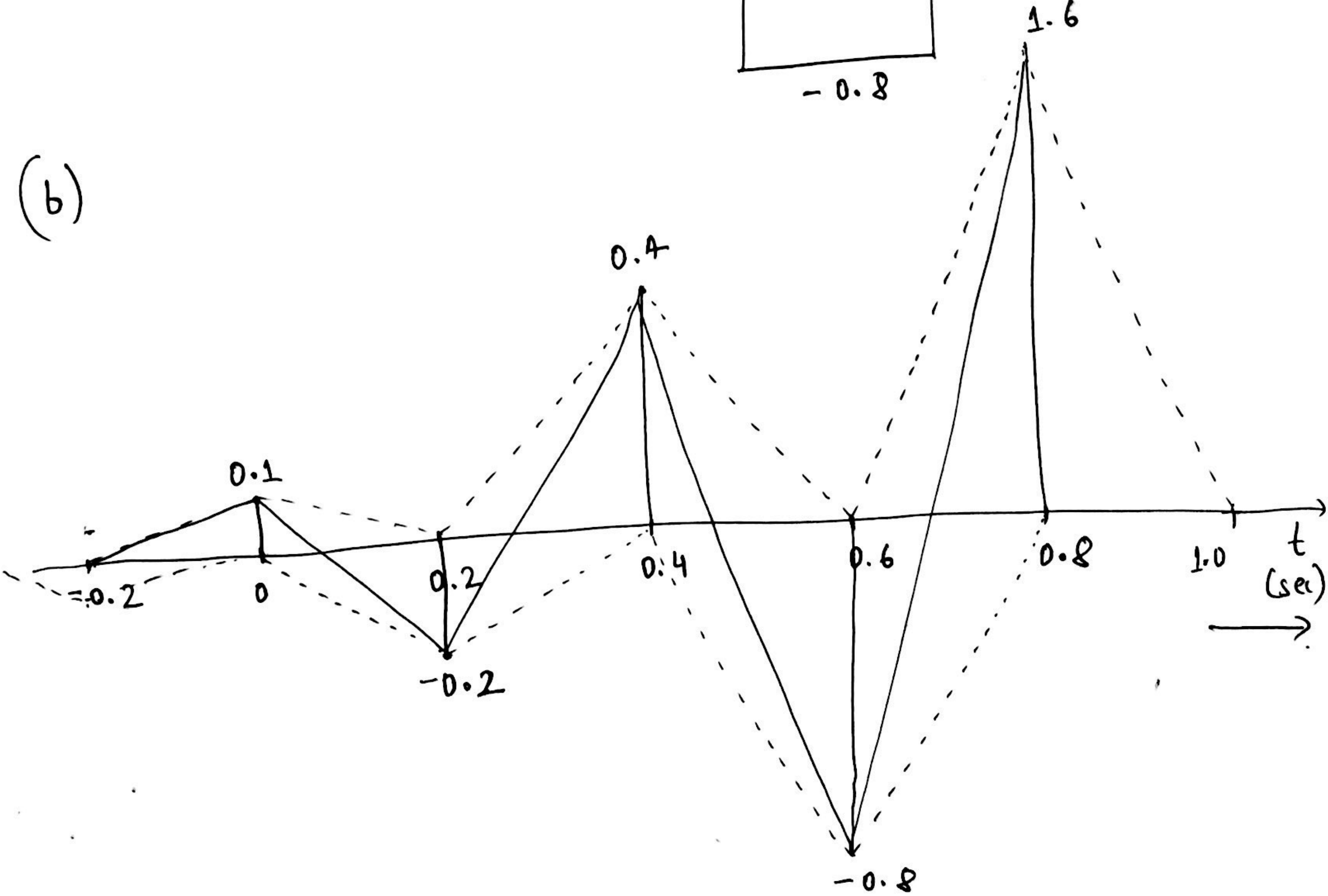
$$= 3.69\pi n$$

$0, 3.69\pi, 7.38\pi, 11.07\pi, \ldots \ldots 95.94\pi, 99.63\pi$

8) (a)


1.6
0.4
0.1
-0.1
0
0.1
0.3
0.5
0.7
0.9
1.1
t
(sec)
-0.2
-0.8

(b)


1.6
0.4
0.1
-0.2
0
0.2
0.4
0.6
0.8
1.0
t
(sec)
-0.2
-0.8