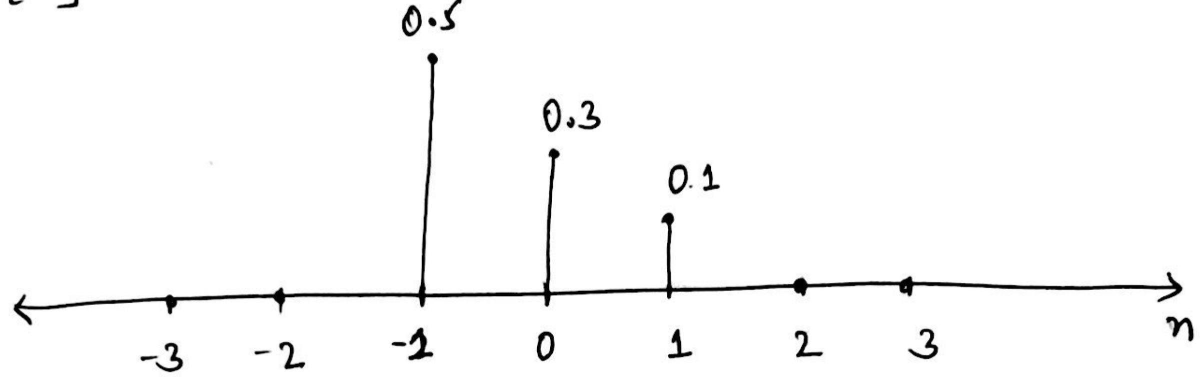
Ravitashaw Bathla (rb369)

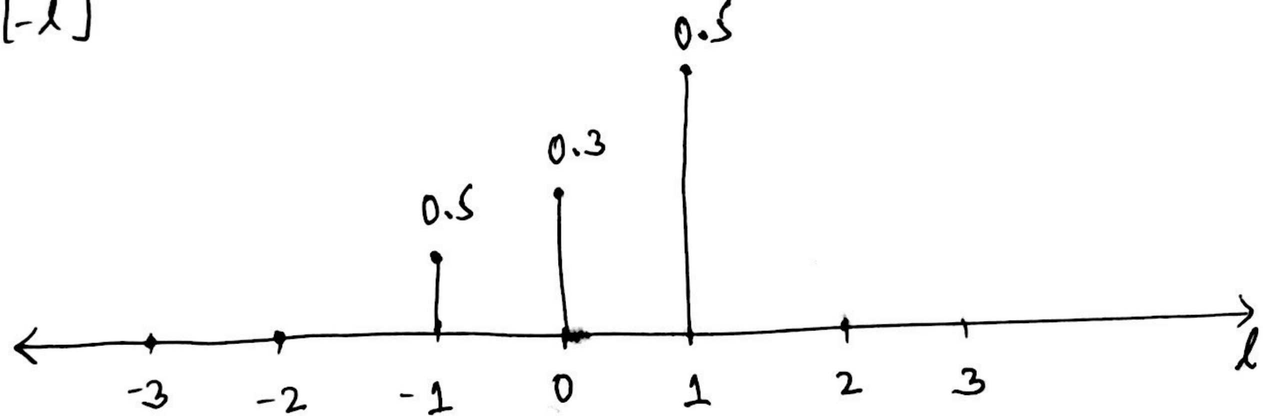
1) $$y[n] = x[n] * h[n] = \sum_{l=-\infty}^{\infty} x[l]\, h[n-l]$$

$$h[n] = 0.5\,\delta[n+1] + 0.3\,\delta[n] + 0.1\,\delta[n-1]$$

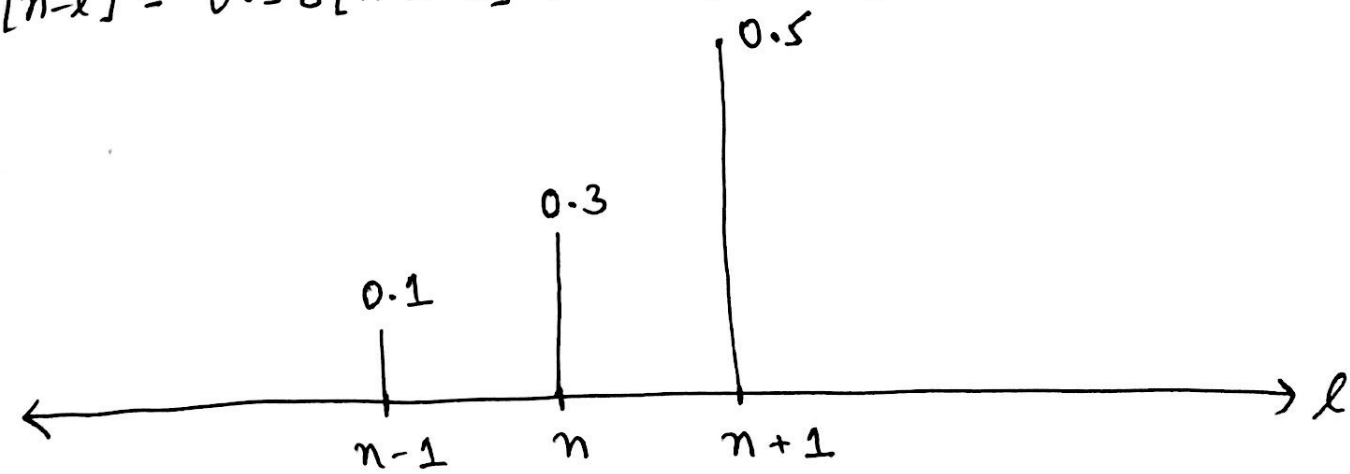
(a) $h[n]$



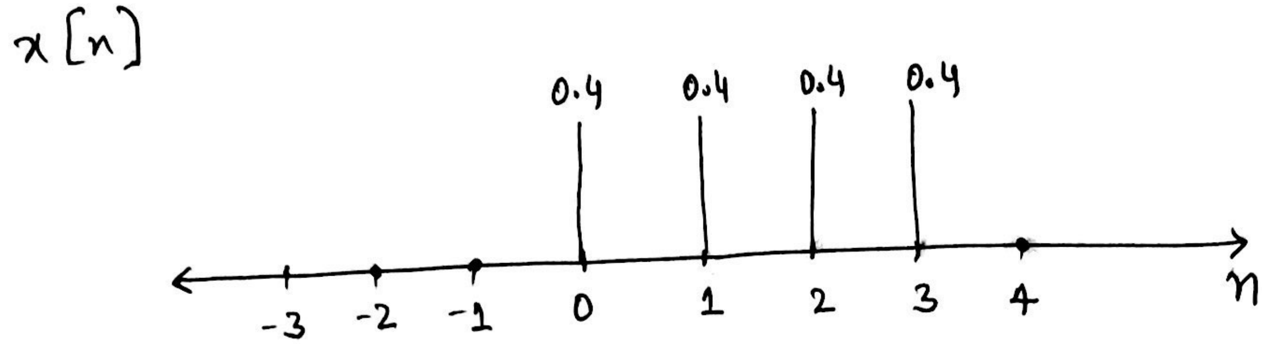
(b) $h[-l]$



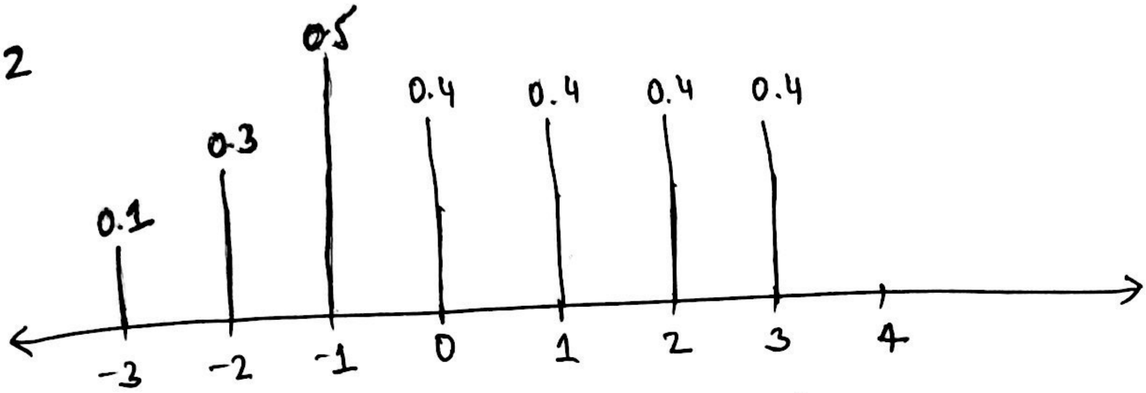
(c) $h[n-l] = 0.5\,\delta[n-l+1] + 0.3\,\delta[n-l+1] + 0.1\,\delta[n-l-1]$



(d) $$x[n] = \begin{cases} 0.4 & x = (0,1,2,3) \\ 0 & \text{elsewhere} \end{cases}$$

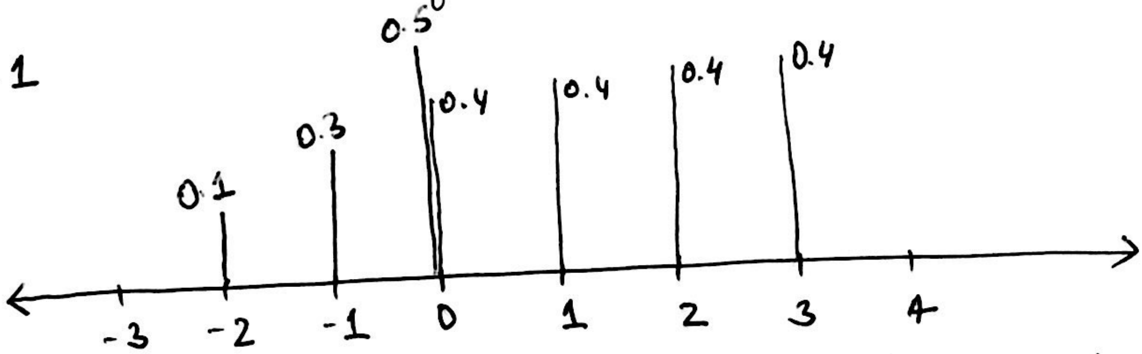


For, $n = -2$



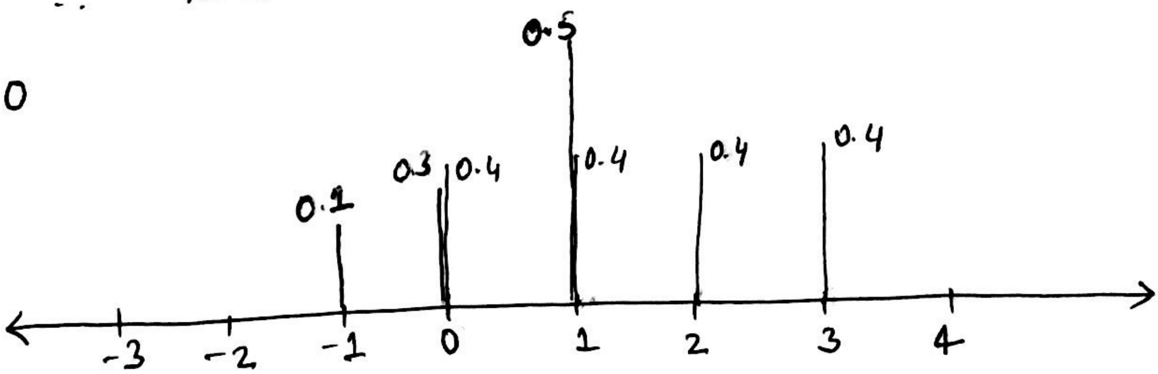
No, intersection, $\therefore$ for $n = -2$, $x * h = 0$

For, $n = -1$



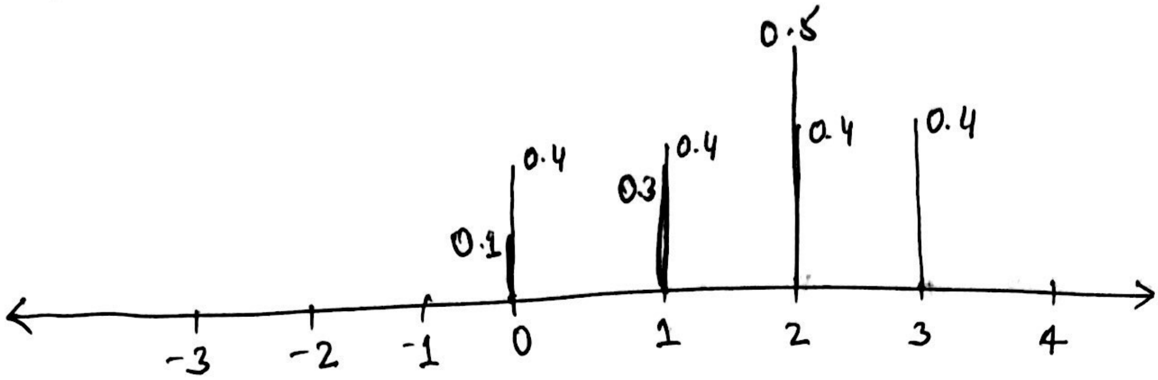
$\therefore \quad x * h = 0.5 \times 0.4 = 0.20$ for $n = -1$

for $n = 0$



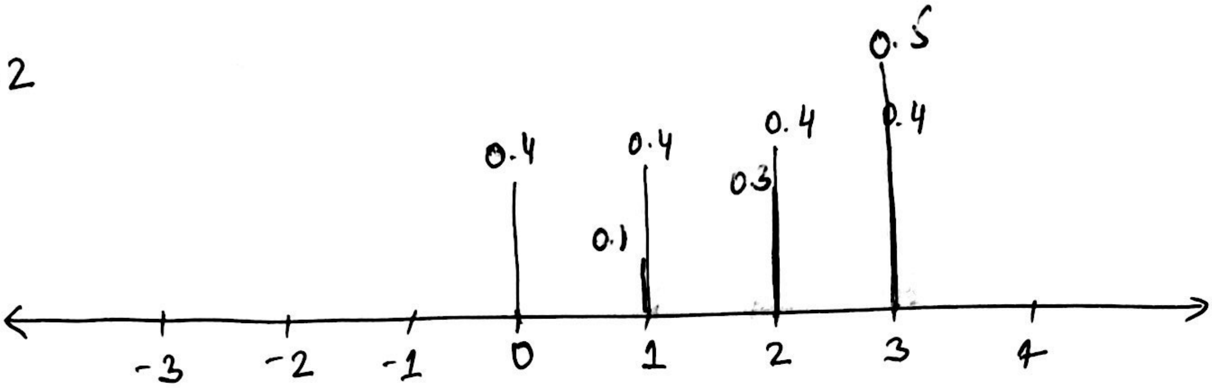
$\therefore x * h = (0.3 \times 0.4) + (0.5 \times 0.4) = 0.32$ for $n = 0$

for $n = 1$



$\therefore x * h = (0.1 \times 0.4) + (0.3 \times 0.4) + (0.4 \times 0.5) = 0.36$

for $n = 2$

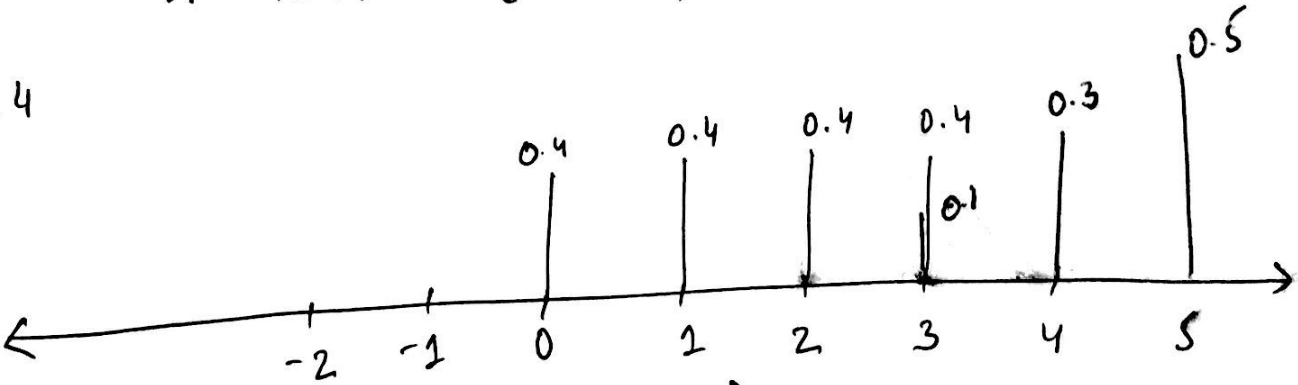


$\therefore x * h = (0.1 \times 0.4) + (0.4 \times 0.3) + (0.4 \times 0.5) = 0.36$

for $n = 3$

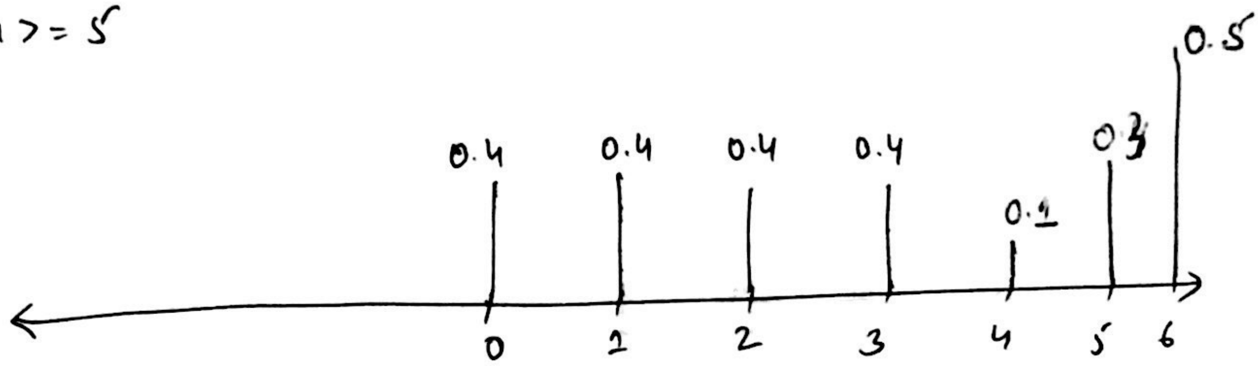
$\therefore x * h = (0.1 \times 0.4) + (0.3 \times 0.4) = 0.16$

for $n = 4$



$\therefore x * h = (0.1 \times 0.4) = 0.04$

for $n >= 5$



$\therefore \; x * h = 0$ since no intersection

$$x * h = \begin{cases} 0 & n \leq -2 \\ 0.20 & n = -1 \\ 0.32 & n = 0 \\ 0.36 & n = 1 \\ 0.36 & n = 2 \\ 0.16 & n = 3 \\ 0.04 & n = 4 \\ 0 & n \geq 5 \end{cases}$$

$$x * h = [\; 0.20 \quad 0.32 \quad 0.36 \quad 0.36 \quad 0.16 \quad 0.04 \;]$$

(2) (Contd. Next Page)

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2) (a) $y[n] = x[n] . \cos(0.4\pi n)$

<u>Additive</u>

$$T\{x_1[n] + x_2[n]\} = \cancel{x_1(n) . \cos(0.}$$
$$= (x_1[n] + x_2[n]) \cos(0.4\pi n)$$
$$= x_1[n] . \cos(0.4\pi n) + x_2[n] . \cos(0.4\pi n)$$
$$= T\{x_1[n]\} + T\{x_2[n]\}$$

Scaling $T\{c\, x[n]\} = c\, x[n] . \cos(0.4\pi n)$
$$= c\, T\{x[n]\}$$

$\therefore$ The system is $\boxed{\text{LINEAR}}$.

$$y[n-N] = x[n-N] \cos(0.4\pi(n-N))$$
$$T\{x[n-N]\} = x[n-N] . \cos(0.4\pi n)$$
$$y[n-N] \neq T\{x[n-N]\}$$

$\therefore$ $\boxed{\text{NOT TIME INVARIANT}}$

The signal $y[n]$ never precedes $x[n]$.

$\therefore$ $\boxed{\text{CAUSAL}}$

(b) $y[n] = x[n] - x[n-2]$

<u>Additive</u> $T\{x_1[n] + x_2[n]\} = x_1[n] + x_2[n] - x_1[n-2] - x_2[n-2]$
$$= x_1[n] - x_1[n-2] + x_2[n] - x[n-2]$$
$$= T\{x_1[n]\} + T\{x_2[n]\}$$

<u>Scaling</u> $T\{c\,x[n]\} = c\,x[n] - c\,x[n-2]$

$= c\left(x[n] - x[n-2]\right)$

$= c\,.T\{x[n]\}.$

$\therefore$ The system is $\boxed{\text{LINEAR}}$

$y[n-N] = x[n-N] - x[n-N-2]$

$T\{x[n-N]\} = x[n-N] - x[n-N-2]$

$y[n-N] \equiv T\{x[n-N]\}$

$\therefore \boxed{\text{TIME INVARIANT}}$

The signal $y[n]$ never precedes $x[n]$ as

$y[0] = x[0] - x[-2]$

$\therefore \boxed{\text{CAUSAL}}$

(c) $y[n] = |x[n]|$

$T\{x_1[n] + x_2[n]\} = |x_1[n] + x_2[n]|$

$\neq T\{x_1[n]\} + T\{x_2[n]\}$

$\therefore \boxed{\text{NOT LINEAR}}$

$T\{x[n-N]\} = |x[n-N]|$

$y[n-N] = |x[n-N]|$

$T[n-N] = y[n-N] \quad \therefore \boxed{\text{TIME INVARIANT}}$

The signal $y[n]$ never precedes $x[n]$

$\therefore$ $\boxed{\text{CAUSAL}}$

(d) $y[n] = A x[n] + B$

$$T\{x_1[n] + x_2[n]\} = A[x_1[n] + x_2[n]] + B$$

$$= A x_1[n] + B + A x_2[n]$$

$$\neq T\{x_1[n]\} + T\{x_2[n]\}$$

$\therefore$ $\boxed{\text{NOT LINEAR}}$

$$T\{x[n-N]\} = A x[n-N] + B$$

$$y[n-N] = A x[n-N] + B$$

$$T\{x[n-N]\} = y[n-N]$$

$\therefore$ $\boxed{\text{TIME INVARIANT}}$

The signal $y[n]$ never precedes $x[n]$

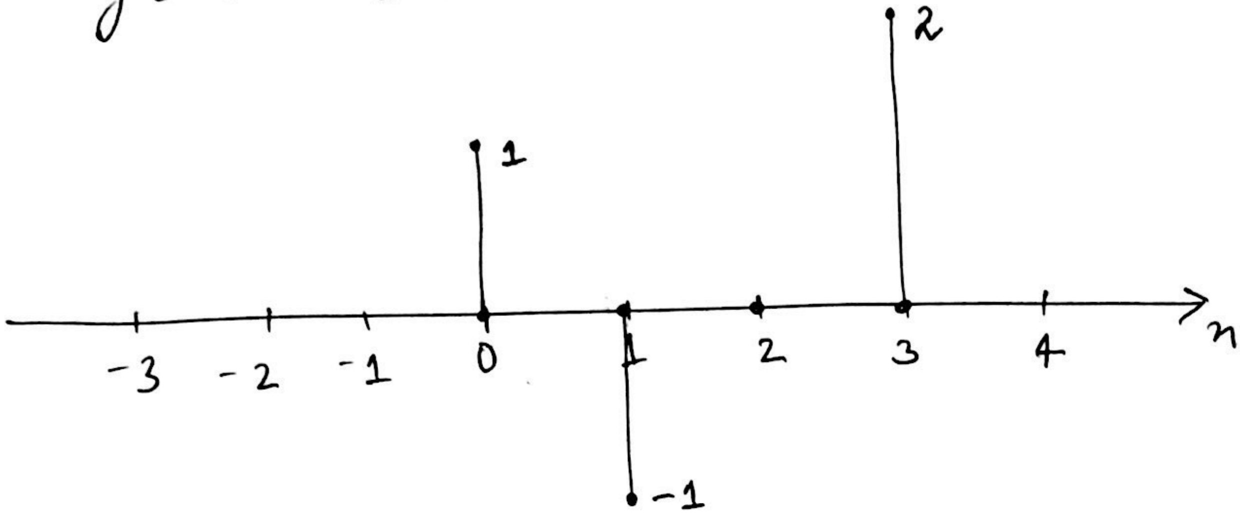
$\therefore$ $\boxed{\text{CAUSAL}}$.

3)

$$x_1[n] = \delta[n] - \delta[n-1] \;,\; y[n] = \delta[n] - \delta[n-1] + 2\delta[n-3]$$

$$x_2[n] = \cos(\pi n/2) \;,\; y[n] = 2\cos(\pi n/2 - \pi/4)$$

(a) $$y[n] = \delta[n] - \delta[n-1] + 2\delta[n-3]$$



(b) $$x_3[n] = 7\delta[n] - 7\delta[n-2]$$

$$x_1[n] = \delta[n] - \delta[n-1]$$

$$\therefore \quad x_1[n-1] = \delta[n-1] - \delta[n-2] \qquad \text{(L.T.I system)}$$

$$\therefore \quad x_3[n] = 7\left(x_1[n] + x_1[n-1]\right)$$

for LTI system,

$$7x_1[n] = 7\delta[n] - 7\delta[n-1] + 14\delta[n-3]$$

$$7x_1[n-1] = 7\delta[n-1] - 7\delta[n-2] + 14\delta[n-4]$$

Adding above two,

$$7x_1[n] + 7x_1[n-1] = 7\delta[n] - 7\delta[n-2] + 14\delta[n-3] + 14\delta[n-4]$$

$$\therefore \boxed{y_3[n] = 7\delta[n] - 7\delta[n-2] + 14\delta[n-3] + 14\delta[n-4]}$$

4)

$$S_1 : y_1[n] = x_1[n] + x_1[n-1]$$

$$S_2 : y_2[n] = x_2[n] - x_2[n-1]$$

$$S_3 : y_3[n] = x_3[n-1] - x_3[n-2]$$

(a) $h_i : [n]$

$$H_1(\hat{w}) = e^{j\hat{w}(0)} + e^{j\hat{w}(-1)} = e^{-j\hat{w}(1/2)}\left[e^{jw(1/2)} + e^{-jw(1/2)}\right]$$

$$= e^{-j\hat{w}/2} 2.\cos(\hat{w}/2)$$

$$\boxed{h_1[n] = [1, 1]}$$

$$H_2(\hat{w}) = e^{j\hat{w}(0)} - e^{jw(-1)} = e^{-jw(1/2)}\left[e^{jw(1/2)} - e^{-jw(1/2)}\right]$$

$$= e^{-jw/2} \cdot 2 \cdot \sin(\hat{w}/2)$$

$$\boxed{h_2[n] = [1, -1]}$$

$$H_3(\hat{w}) = e^{jw(-1)} - e^{jw(-2)} = e^{-jw(3)}\left[e^{jw(1/2)} - e^{jw(-1/2)}\right]$$

$$= e^{-jw(3/2)} 2. \sin(\hat{w}/2)$$

$$\boxed{h_3[n] = [0, 1, -1]}$$

(b)

$$h[n] = h_1[n] * h_2[n] * h_3[n]$$

$$= [1, 1][1, -1][0, 1, -1]$$

$$= [1, 0, -1][0, 1 - 1]$$

$$h[n] = [0, 1, -1, -1, 1]$$

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$$H[\hat{w}] = 0 \cdot e^{j\hat{w}(0)} + 1\, e^{j\hat{w}(-1)} - 1\, e^{j\hat{w}(-2)} - 1\, e^{j\hat{w}(-3)} + e^{j\hat{w}(-4)}$$

(c)

$$\therefore \boxed{y[n] = x[n-1] - x[n-2] - x[n-3] + x[n-4]}.$$

5) $$x_1[n] = 3u[n]$$

$$y_1(n) = \delta[n] - 3\delta[n-1] - 3\delta[n-2]$$

(a) $$x_2[n] = 3u[n] - 6u[n-2]$$

we know that, $$x_1(n) = 3u[n]$$

$$U[n] = \frac{1}{3} x_1[n]$$

$$\therefore x_2[n] = \frac{3}{3} x_1(n) - \frac{6}{3} x_1[n-2]$$

$$= x_1(n) - 2x_1[n-2]$$

Since, it is an LTI

$$y_2[n] = y_1(n) - 2y_1[n-2]$$

$$= \delta[n] - 3\delta[n-1] - 3\delta[n-2]$$

$$- 2\delta[n-2] + 6\delta[n-3] + 6\delta[n-4]$$

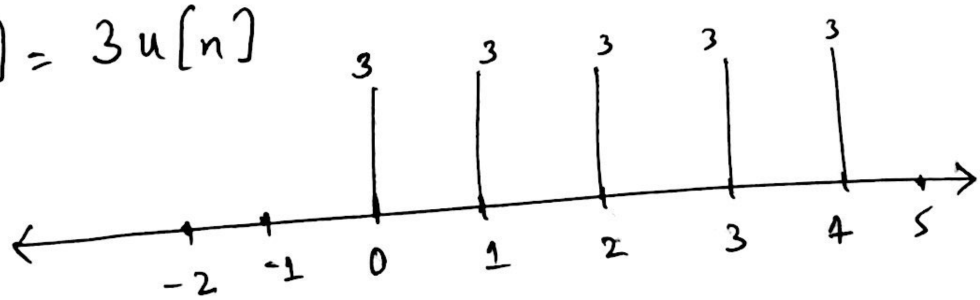
$$\boxed{y_2(n) = \delta[n] - 3\delta[n-1] - 5\delta[n-2] + 6\delta[n-3] + 6\delta[n-4]}$$

$$y_2 = \begin{cases} 0 & n < 0 \\ 1 & n = 0 \\ -3 & n = 1 \\ -5 & n = 2 \\ 6 & n = 3 \text{ or } 4 \\ 0 & n \geq 5 \end{cases}$$

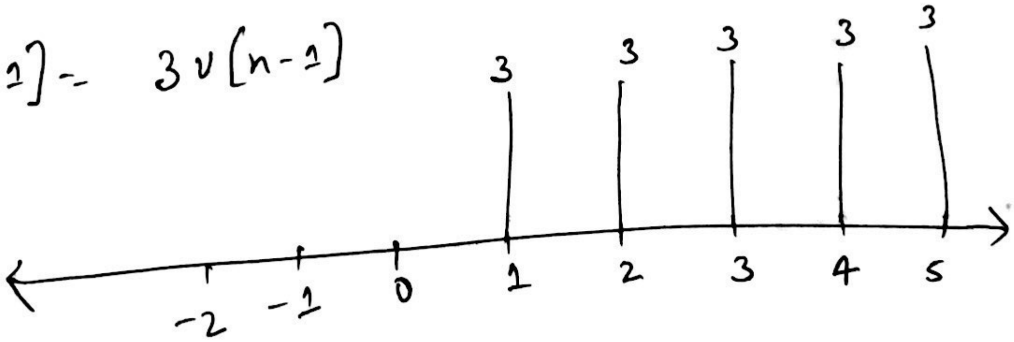
(b) Unit step function ; $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$

Unit Impulse function, $\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$

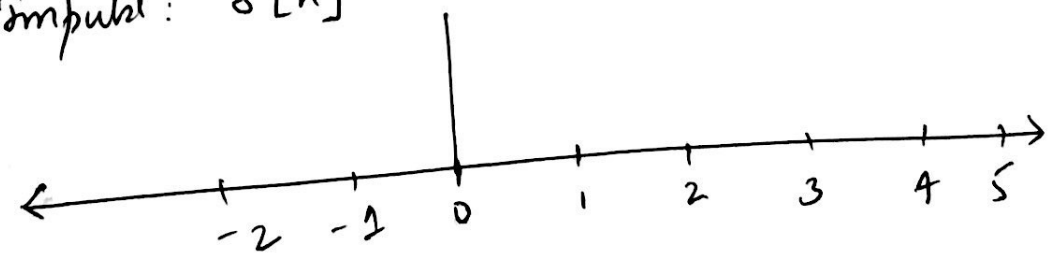
$x_1[n] = 3u[n]$



$x_1[n-1] = 3u[n-1]$



Unit Impulse : $\delta[n]$



we get

$$\delta[n] = \frac{1}{3} x_1[n] - \frac{1}{3} x_1[n-1]$$

$$\boxed{\delta[n] = u[n] - u[n-1]}$$

(c) from b, we know that

Unit impulse, $\delta[n] = u[n] - u[n-1]$

$\therefore$ Impulse Response, $h[n] = \delta[n]$ for LTI system

$$\boxed{\therefore h[n] = u[n] - u[n-1]}$$

because, $\delta[n] \rightarrow h[n]$

$$h[n] = x[n] . \delta[n]$$

$$y[n] = x[n] . h[n].$$

$$\Rightarrow \boxed{h[n] = \delta[n] = u[n] - u[n-1]}$$