Prelab-10

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(rb369)

1)

(a) $y[n] = x[n] - x[n-1]$

$y(z) = X[z] - z^{-1} X[z]$

$= (1 - z^{-1}) X[z]$

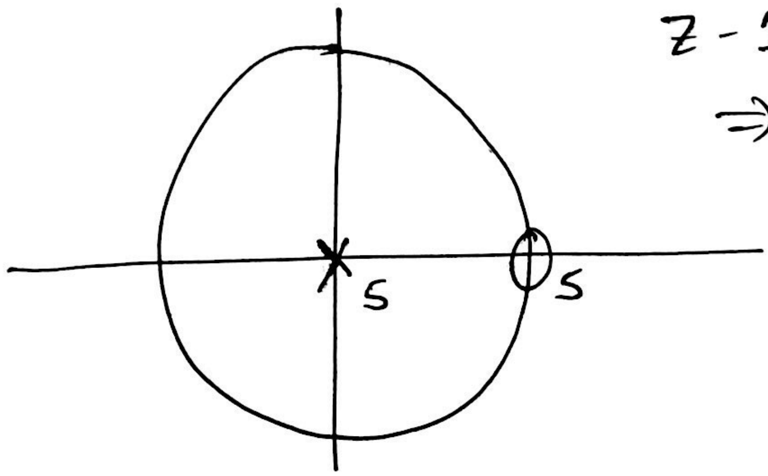
$\therefore H(z) = (1 - z^{-1})$ for 1 filter

$Y_5(z) = H(z) H(z) H(z) H(z) H(z) \cancel{X}(z)$

$= H(z)^5 X(z)$

$\therefore H_5(z) = (1 - z^{-1})^5 = \left(\frac{z-1}{z}\right)^5$

(b)



$z - 1 = 0$

$\Rightarrow z = 1$ (zero)

$z = 0$ at $z = 0$

(c) $Z = e^{j\omega}$

$$H(e^{j\omega}) = (1 - z^{-1})^5$$

$$= \left(\frac{z-1}{z}\right)^5$$

$$= \frac{(e^{j\omega} - 1)^5}{(e^{j\omega})^5}$$

(d) $H(z) = (1 - z^{-1})^5$

$$= 1 - 5z^{-1} + 10z^{-2} - 10z^{-3} + 5z^{-4} - z^{-5}$$

For first difference filter, the impulse response is same as the filter coefficient of $H(z)$

$\therefore\ b = [1 \quad -5 \quad 10 \quad -10 \quad 5 \quad -1]$

(2) $H(z) = (1 - z^{-2})(1 - 4z^{-2})$

$= 1 - 5z^{-2} + 4z^{-4}$

$H(e^{jw}) = 1 - 5e^{-2jw} + 4e^{-4jw}$

$x[n] = 100 - 70\delta[n] + 30\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$

Using Superposition

IN	OUT
$100 \longrightarrow$	$\mathcal{H}(0).100 = H(1).100 = 0$ (1)
$-70\delta[n] \longrightarrow$	$-70h[n] = -70\left(\delta[n] - 5\delta[n-2] + 4\delta[n-4]\right)$
	$= -70\delta[n] + 350\delta[n-2] - 280\delta[n-4]$ (2)

$30\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right) \longrightarrow 15\mathcal{H}\left(\frac{\pi}{2}\right)e^{j\pi/4}e^{j\pi/2 n} + 15\mathcal{H}\left(-\frac{\pi}{2}\right)e^{-j\frac{\pi}{4}}e^{-j\frac{\pi n}{2}}$

$H(e^{j\hat{w}}) = 1 - 5e^{-2jw} + 4e^{-4jw}$

$H(e^{j\frac{\pi}{2}}) = 1 - 5e^{-j2\frac{\pi}{2}} + 4e^{-4j\frac{\pi}{2}}$

$= 1 - 5e^{-j\pi} + 4e^{-j2\pi}$

$= 1 + 5 + 4$

$= \underline{\underline{10}}$

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$\therefore$ IN | OUT

$$30\cos\left(\frac{\pi}{2}n+\frac{\pi}{4}\right) \longrightarrow 10\times 30\cos\left(\frac{\pi}{2}n+\frac{\pi}{4}\right)$$

$$= 300\cos\left(\frac{\pi}{2}n+\frac{\pi}{4}\right) \quad \text{--- (3)}$$

$y[n]$ is sum of eq (1) (2) & (3)

$$\therefore\ y[n] = 0 - 70\delta[n] + 350\delta[n-2] - 280\delta[n-4] + 300\cos\left(\frac{\pi}{2}n+\frac{\pi}{4}\right)$$

$$\therefore\ \boxed{y[n] = 70\delta[n] + 350\delta[n-2] - 280\delta[n-4] + 300\cos\left(\frac{\pi}{2}n+\frac{\pi}{4}\right)}$$

(3) $$x(t) = 4 + \cos(1000\pi t - \pi/4) - 3\cos(500\pi t)$$

$$H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$$

$$f_s = 2000 \text{ sps}$$

$$x[n] = 4 + \cos\left(\frac{1000}{2000}\pi n - \frac{\pi}{4}\right) - 3\cos\left(\frac{500}{2000}\pi n\right)$$

$$x[n] = 4 + \cos\left(\frac{\pi}{2}n - \frac{\pi}{4}\right) - 3\cos\left(\frac{\pi}{4}n\right)$$

$$H(z) = 1 - z^{-1} + z^{-2} - z^{-3}$$

$$= \left(1 - z^{-1}\right) + z^{-2}\left(1 - z^{-1}\right)$$

$$= \left(1 - z^{-1}\right)\left(1 + z^{-2}\right)$$

$$H(e^{jw}) = \left(1 - e^{-jw}\right)\left(1 + e^{-2jw}\right)$$

$$= e^{-j\frac{w}{2}}\left(e^{j\frac{w}{2}} - e^{-j\frac{w}{2}}\right) \cdot e^{-jw}\left(e^{jw} + e^{-jw}\right)$$

$$= e^{-j\frac{w}{2}}\left[2j \sin\left(\frac{w}{2}\right)\right] \cdot e^{-jw}\left[2\cos(w)\right]$$

$$= 4\sin\left(\frac{w}{2}\right) \cdot \cos(w) \cdot e^{-j\frac{w}{2}} \cdot e^{-jw} \cdot e^{j\pi/2}$$

$$H(e^{jw}) = 4\sin\left(\frac{w}{2}\right) \cdot \cos(w) \cdot e^{-j\frac{3w}{2}} \cdot e^{j\pi/2}$$

Since, input is a sum of cosine, we can calculate frequency response at each frequency from $x[n]$.

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At $\hat{\omega} = 0$,

$$H(e^{j\hat{\omega}}) = 4\cancel{\sin\left(\frac{0}{2}\right)}^{0} . \cos(0)\, e^{-j\frac{3\times 0}{2}} e^{j\pi/2} = 0.$$

At $\hat{\omega} = \pi/2$

$$H(e^{j\omega}) = 4\sin\left(\frac{\pi}{4}\right) . \cancel{\cos\left(\frac{\pi}{2}\right)}^{0} . e^{-j\frac{3\pi}{4}} . e^{j\frac{\pi}{2}} = 0$$

At $\hat{\omega} = \pi/4$

$$H(e^{j\omega}) = 4\sin\left(\frac{\pi}{8}\right) . \cos\left(\frac{\pi}{4}\right) . e^{-j\frac{3\pi}{8}} . e^{j\frac{\pi}{2}}$$

$$= 4.\ (0.382)\left(\frac{1}{\sqrt{2}}\right) . e^{j\frac{4\pi - 3\pi}{8}}$$

$$= (2\sqrt{2})\,(0.382)\, e^{j\pi/8}$$

$$= 1.08239 . e^{j\pi/8}$$

$$|H(e^{j\omega})| = 1.08239, \quad \text{phase} = \pi/8$$

$$\therefore\ y[n] = 4(0) + (0)\cos\left(\frac{\pi}{2}n - \frac{\pi}{4}\right) - 3(1.08239)\cos\left(\frac{\pi}{4}n + \frac{\pi}{8}\right)$$

$$= -3.24717 . \cos\left(\frac{\pi}{4}n + \frac{\pi}{8}\right)$$

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$$\therefore\ y[t] = -3.24717 \cos\left(\frac{\pi}{4}(2000)t + \frac{\pi}{8}\right)$$

$$\boxed{y[t) = -3.24717 \cos(500\pi t + \pi/8)}$$

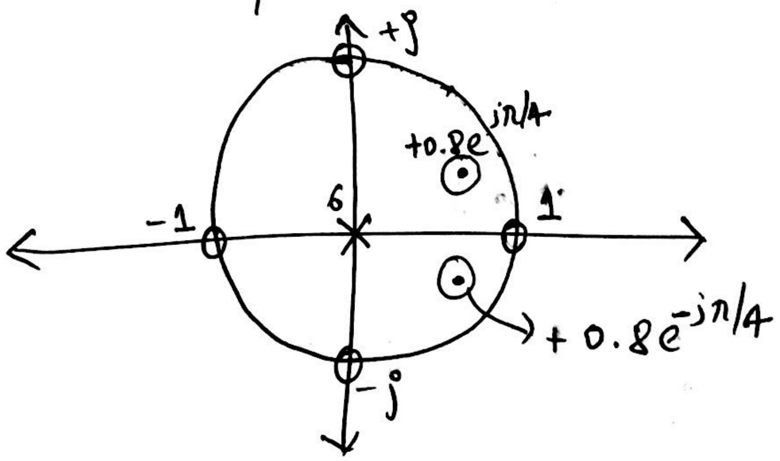
4) $H_1(z) H_2(z) = H(z) = (1 - z^{-2})\left(1 - 0.8e^{j\frac{\pi}{4}} z^{-1}\right)\left(1 - 0.8e^{-j\frac{\pi}{4}} z^{-1}\right)(1 + z^{-2})$

(a) The zeros of H(Z) are factors of H(Z)

$\therefore\ \pm 1,\ 0.8e^{\pm j\frac{\pi}{4}}$ and $\pm j$

The poles of H(Z) are 0 and there are

6 poles



(b) $y_1[n] = x[n] - x[n-4]$

$H_1[n] = \delta[n] - \delta[n-4] = 1 - z^{-4}$

$= (1 + z^{-2})(1 - z^{-2})$ [from $a^2 - b^2 = (a+b)(a-b)$]

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$$H(z) = (1-z^{-2})\left(1-0.8e^{j\pi/4}z^{-1}\right)\left(1-0.8e^{-j\pi/4}z^{-1}\right)(1+z^{-2})$$

$$= (1-z^{-2})(1+z^{-2})\left[1-0.8\left[e^{j\pi/4}+e^{-j\pi/4}\right]z^{-1}+(0.64)z^{-2}\right]$$

$$H(z) = H_1(z)\left[1-1.6\cos\left(\frac{\pi}{4}\right)z^{-1}+0.64z^{-1}\right]$$

$$\therefore \quad H_2(z) = \frac{H(z)}{H_1(z)} = \left[1-1.6\cos\left(\frac{\pi}{4}\right)z^{-1}+0.64z^{-1}\right]$$

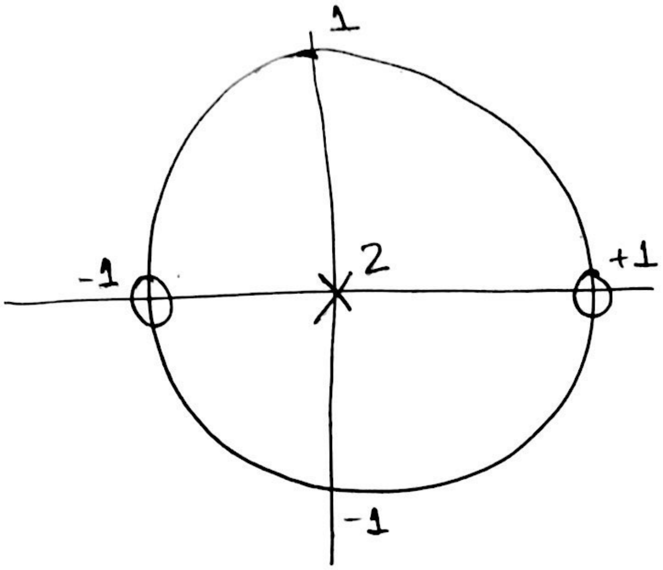
$$\therefore \quad H_1(z) = (1+z^{-2})(1-z^{-2}) = \underline{\underline{1-z^{-4}}}$$

$$\underline{\underline{H_2(z) = \left[1-1.6\cos\left(\frac{\pi}{4}\right)z^{-1}+0.64z^{-1}\right]}}$$

(5)

Contd.

(5)



(a) $$H(z) = \frac{\beta (z-1)(z+1)}{z^2}$$

$$= \beta (1-z^{-1})(1+z^{-1})$$

$$= \beta (1-z^{-2})$$

$$H(e^{j\omega}) = \beta (1 - e^{-j\omega 2})$$

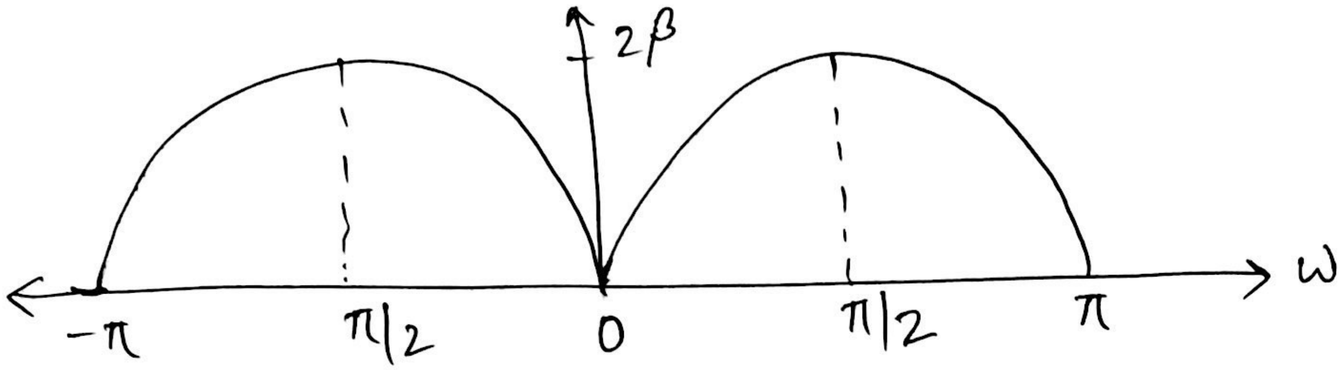
(b) $$H(e^{j\omega}) = \beta e^{-j\omega} (e^{j\omega} - e^{-j\omega})$$

$$= \beta e^{-j\omega} 2j \sin(\omega)$$

$$|H(e^{j\omega})| = 2\beta |\sin(\omega)|$$

Maximum of $\sin(\hat{\omega})$ is at $\pi/2$

$\therefore$ Max Value = $\underline{\underline{2\beta}}$



(c) for maximum value of frequency response to be equal to 1

Max Value = $2\beta = 1$, where β is scaling constant

$\therefore \boxed{\beta = \frac{1}{2}}$

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