Prelab - 9

Ravitashaw Bathla
(rb3669)

1) $y[n] = 2x[n] - 2x[n-1] + 2x[n-2]$

(a) The filter coefficients are: $\{b_k\} = \{2, -2, 2\}$

$$\therefore \mathcal{H}(\hat{\omega}) = \sum_{k=0}^{2} b_k e^{-j\hat{\omega}k} = 2 - 2e^{-j\hat{\omega}} + 2e^{-2j\hat{\omega}}$$

$$= 2e^{-j\hat{\omega}}\left[e^{j\hat{\omega}} - 1 + e^{-j\hat{\omega}}\right]$$

$$= 2e^{-j\hat{\omega}}\left[-1 + 2\cos(\hat{\omega})\right]$$

$$= e^{-j\omega}\left[-2 + 4\cos(\hat{\omega})\right]$$

$$\therefore \boxed{R(e^{j\hat{\omega}}) = -2 + 4\cos(\hat{\omega})}$$

and $\boxed{n_0 = 1}$

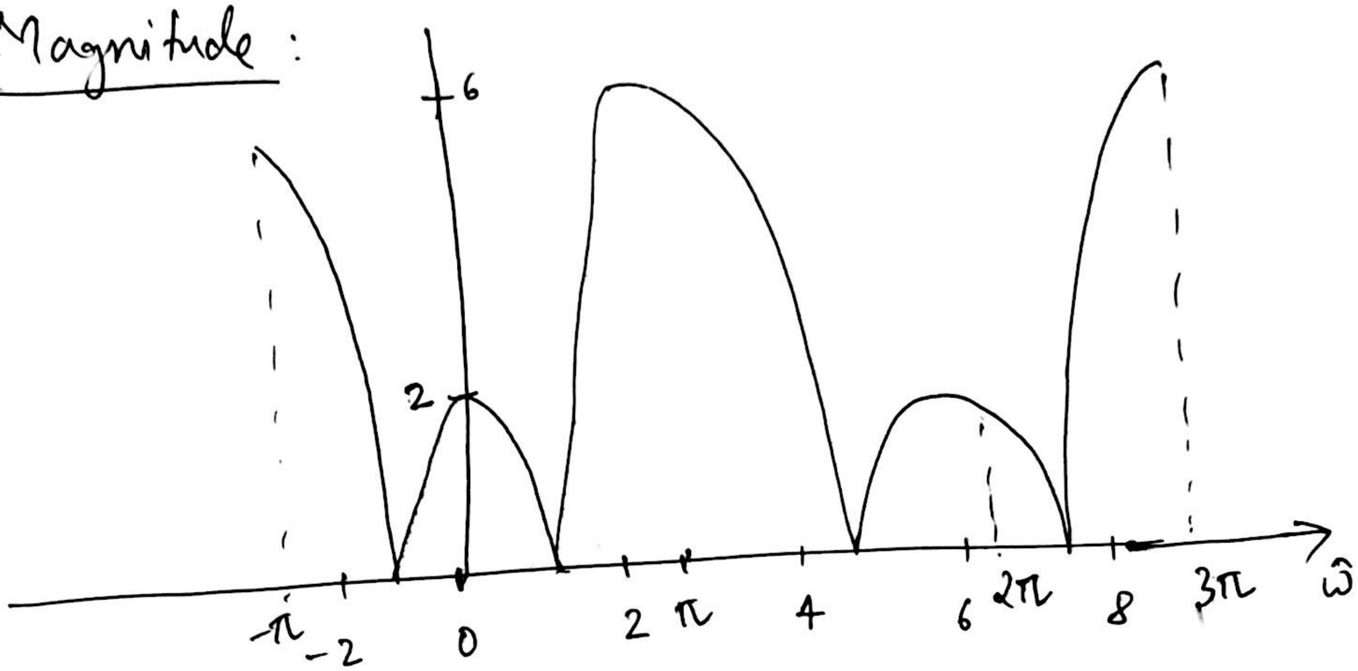
(b) $\mathcal{H}(\hat{\omega})$ always has period $= 2\pi$

$$\mathcal{H}(\hat{\omega} + 2\pi) = e^{-j(\hat{\omega}+2\pi)}\left(-2 + 4\cos(\hat{\omega} + 2\pi)\right)$$

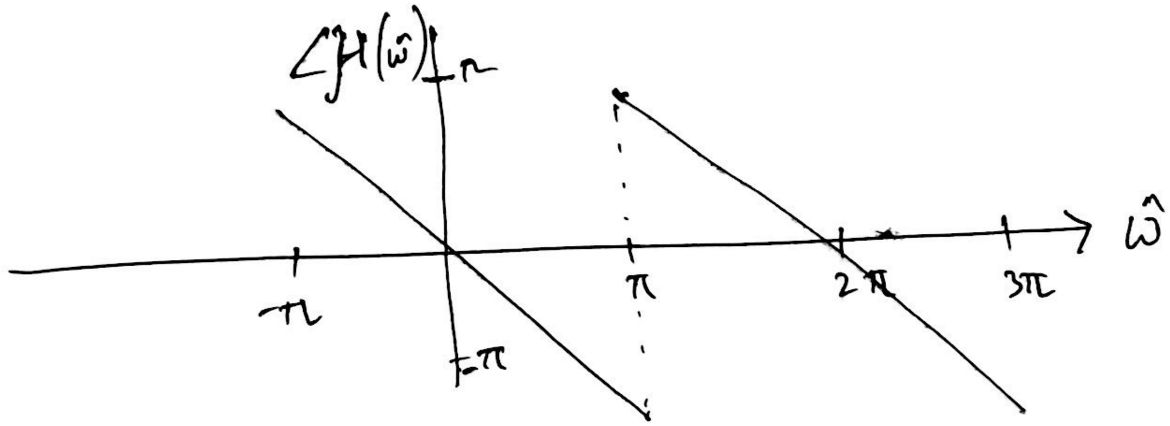
$$= e^{-j\hat{\omega}}\left(-2 + 4\cos(\hat{\omega})\right) = \mathcal{H}(\hat{\omega})$$

$\because e^{j2\pi} = 1 \therefore$ Cosine has <u>period $= 2\pi$</u>.

(c) Magnitude:



Phase



(d) Output Response is zero, when amplitude is zero.

$$\therefore \quad -2 + 4\cos(\hat{\omega}) = 0$$

$$\cos(\hat{\omega}) = \frac{1}{2}$$

$$\therefore \quad \cancel{\cos}\ \hat{\omega} = \cos^{-1}(1/2) = \pm\pi/3$$

Also, $\cancel{\cos(-\pi/3) = 1/2}$ $\therefore$ $\boxed{\omega = \pm\pi/3}$

(e) $x[n] = \cos(0.5\pi n)$

$$= \frac{1}{2}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right)$$

$$y[n] = H(\hat{w}) \cdot x[n]$$

$$= \frac{1}{2} H\left(\frac{\pi}{2}\right) e^{j\frac{\pi}{2}n} + \frac{1}{2} H\left(\frac{-\pi}{2}\right) e^{-j\frac{\pi}{2}n}$$

$$= \frac{1}{2}\left[-2 \cdot e^{j\frac{\pi}{2}n}\right] + \frac{1}{2}\left[-2 \cdot e^{-j\frac{\pi}{2}}\right]$$

$$= -e^{-j\frac{\pi}{2}n} - e^{j\pi n/2} \qquad (\cos(\pi/2) = 0)$$

$$y[n] = -1\left[e^{j\frac{\pi}{2}n} + e^{-j\pi n/2}\right]$$

$$\underline{\underline{y[n] = -\cos(0.5\pi n)}}$$

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2) (a) $y[n] = x[n] - x[n-2]$

Impulse response is when $x[n] = \delta[n]$

$$\therefore \boxed{h[n] = \delta[n] - \delta[n-2]}$$

Frequency Response :

$$y[n] = x[n] - x[n-2]$$

$$H(e^{j\hat{\omega}}) = 1 - e^{-2j\hat{\omega}}$$

$$= e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} - e^{-j\hat{\omega}}\right)$$

$$= e^{-j\omega}\left(2\sin(\hat{\omega})\right)$$

$$\boxed{H(e^{j\hat{\omega}}) = e^{-j\omega}[2\sin(\hat{\omega})]}$$

(b) $$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}}\left(2 + 2\cos(\hat{\omega})\right)$$

$$= e^{-j\omega}\left(2 + 2\left(\frac{e^{j\hat{\omega}} + e^{-j\hat{\omega}}}{2}\right)\right)$$

$$= e^{-j\omega}\left[2 + e^{j\hat{\omega}} + e^{-j\hat{\omega}}\right]$$

$$= 2e^{-j\omega} + 1 + e^{-2j\omega}$$

$$\therefore \{b_k\} = \{1, 2, 1\}$$

Difference Equation.

$$\boxed{y[n] = x[n] + 2x[n-1] + x[n-2]}$$

Impulse Response, $x[n] = \delta[n]$

$$\boxed{h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]}$$

(c) $y = \text{conv}([0, 2, 0, 2], x)$

$b_k = \{0, 2, 0, 2\}$

$\therefore$ $\boxed{\text{Difference Eq}^n = 2x[n-1] + 2x[n-3]}$

$\boxed{\text{Impulse Response} = 2\delta[n-1] + 2\delta[n-3]}$

Frequency Response =

$$H(e^{j\hat{\omega}}) = 2e^{-j\hat{\omega}} + 2e^{-3j\hat{\omega}}$$

$$= 2e^{-2j\hat{\omega}}[e^{j\hat{\omega}} + e^{-j\hat{\omega}}]$$

$$= 2e^{-2j\hat{\omega}}[2\cos(\omega)]$$

$$\boxed{H(e^{j\hat{\omega}}) = e^{-2j\omega} \cdot 4\cos(\omega)}$$

3) ① $\delta[n] \longrightarrow \delta[n] - \delta[n-3]$

② $\cos(2\pi n/3) \longrightarrow 0$

③ $\cos(\pi n/3 + \pi/2) \longrightarrow 2\cos(\pi n/3 + \pi/2)$

(a) $x[n] = 3\delta[n] - 2\delta[n-2] + \delta[n-3]$

for LTI system, using property of linearity and time invariance on eq-(1).

$$y[n] = 3[\delta[n] - \delta[n-3]] - 2[\delta[n-2) - \delta[n-5]] + [\delta[n-3] - \delta[n-6]]$$

$$= 3\delta[n] - 3\delta[n-3] - 2\delta[n-2] + 2\delta[n-5] + \delta[n-3] - \delta[n-6]$$

$$\boxed{\therefore y[n] = 3\delta[n] - 2\delta[n-2] - 2\delta[n-3] + 2\delta[n-5] - \delta[n-6]}$$

(b) $x[n] = \cos(\pi(n-3)/3)$

$H(\pi/3) = 2$ (no phase)

$$\therefore \boxed{y[n] = 2\cos(\pi(n-3)/3)}$$

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(c) $h[n] = \delta[n] - \delta[n-3]$ (eq-1)

$b_k = \{1, 0, 0, -1\}$

$H(e^{j\hat{\omega}}) = H(\hat{\omega}) = 1 - e^{-j\hat{\omega}(3)}$

$H(\pi/2) = 1 - e^{-j3\pi/2} = 1 + j$

$\therefore$ $\cancel{H(e^{j\pi/2})}$ $\boxed{H[e^{j\pi/2}] \neq 0}$

4) S_1 : $y_1[n] = x_1[n] - x_1[n-1]$

S_2 : $y_2[n] = x_2[n] + x_2[n-2]$

S_3 : $y_3[n] = x_3[n-1] + x_3[n-2]$

(a) $H_1(\hat{\omega}) = e^{jw(0)} - e^{-jw}$, $b_1 = [1, -1]$

$= \underline{\underline{1 - e^{-j\hat{\omega}}}}$

$H_2(\hat{\omega}) = e^{jw(0)} + e^{-j\hat{\omega}(2)}$, $b_2 = [1, 0, +1]$

$\underline{\underline{= 1 + e^{-2j\hat{\omega}}}}$

$H_3(\hat{\omega}) = \underline{\underline{e^{-jw} + e^{-2jw}}}$ $b_3 = [0, 1, 1]$

(b) $H(\hat{\omega}) = H_1(\hat{\omega}) \times H_2(\hat{\omega}) \times H_3(\hat{\omega})$

$\Rightarrow \quad b_K = b_1 * b_2 * b_3$

$= [1, -1] [1, 0, 1] [0, 1, 1]$

$= [0, 1, 0, 0, 0, -1]$

$\quad\;\; 0 \;\; 1 \;\; 2 \;\; 3 \;\; 4 \;\; 5$

$$\boxed{\therefore\; H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} - e^{-5j\hat{\omega}}}$$

(c) Impulse Response cannot be multiplied in cascade systems. However, frequency response can be combined. using multiplication or convolution.

$H = H_1 * H_2 \times H_3$

$h = h_1 * h_2 * h_3$

(5)

$$x(t) = 7 + 8\cos(1000\pi t) + 9\cos(1600\pi t + 0.7\pi)$$

$$h[n] = \sum_{K=0}^{4} \delta[n-k]$$

$$x[n] = 7 + 8\cos[\hat{\omega}_1 n] + 9\cos[\hat{\omega}_2 n + 0.7\pi]$$

$\hat{\omega}_0 = 0$ $\hat{\omega}_1 = \pi/4$ $\hat{\omega}_2 = 2\pi/5$

$$\therefore \quad x[n] = 7 + 8\cos(0.25\pi n) + 9\cos(0.4\pi n + 0.7\pi)$$

$$H(e^{j\omega}) = \sum_{n=0}^{4} e^{-j\omega n} = \frac{1-e^{-j5\hat{\omega}}}{1-e^{-j\omega}}$$

$$= e^{-j2\hat{\omega}} \frac{\sin(5\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$$

W_0,

$$H(e^{j0}) = e^{-j2(0)} \frac{\sin(0)}{\sin(0)} = 5 \longrightarrow \left(\text{Using Lopital Rule}\right)$$

$$H(e^{j0.25\pi}) = e^{-0.5\pi} \frac{\sin(5\pi/8)}{\sin(\pi/8)} = 2.414\, e^{-j0.5\pi}$$

$$H(e^{j0.4\pi}) = e^{-0.8\pi} \frac{\sin(\pi)}{\sin(\pi/5)} = 0$$

$y[n] = H(e^{j\hat{\omega}}) \cdot x[n]$

$y[n] = 7(5) + 8(2.414)\cos(0.25\pi n - \pi/2)$

$n = t f_s = 4000t$

$$\boxed{\therefore\ y(t) = 35 + 19.312\cos(1000\pi n - \pi/2)}$$