Probability

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→ Study of Uncertainity.

① **Random Experiment :**
It is a process for which the Outcome is Uncertain.
Ex: Tossing a coin
Rolling a Dice

② **Sample Space :**
It is a Set of all the possible outcomes of a Random Experiment.
Ex: ① RE → Tossing a coin
S.S → {H, T}

② RE → Rolling a dice
S.S → {1, 2, 3, 4, 5, 6}

③ **Event :** A subset ~~of~~ a Sample Space.

Ex : RE → Rolling a dice
S.S → {1, 2, 3, 4, 5, 6}
Even number → {2, 4, 6}
OR
E_1

Q; What is the probability of getting an Even Number on a dice Roll?
R.E → Rolling a dice
S.S → {1, 2, 3, 4, 5, 6}
E → Even → {2, 4, 6}

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$$\boxed{p(E) = \frac{\text{Favorable No. of outcomes}}{\text{Total No. of outcomes}}}$$

$$\boxed{p(E) = \frac{|E|}{|S.S|}}$$

Conditional probability

Q) what is the probability of getting atleast one Head when 2 coins are tossed?

R.E → 2 coins tossed

S.S → {HH, HT, TH, TT}

Atleast one Head, E_1 → {HH, HT, TH}

$p(E_1) = \frac{3}{4} = 0.75$

Q) what is the probability of getting atleast one Head Given that on the first coin toss you got a Tail?

S.S → $\{\cancel{HH}, \cancel{HT}, \underbrace{TH, TT}_{S.S}\}$

E → {TH}

$p(E) = \frac{1}{2}$

Q) What is the prob. of getting an odd no. when a dice is rolled, Given that there is prime No:

R.E → Rolling a dice

S.S $\rightarrow \{\cancel{1}, 2, 3, \cancel{4}, 5, \cancel{6}\} \rightarrow \{2, 3, 5\}$

Event $\rightarrow$ Getting an odd no. $\{1, 3, 5\}$

$$p(\text{Event}) = \frac{|E|}{|SS|} = \frac{2}{3}$$

Formula for conditional probability:

$$\boxed{p(A/B) = \frac{p(A\cap B)}{p(B)} = \frac{\frac{|A\cap B|}{|SS|}}{\frac{|B|}{|SS|}}} \quad \text{—— (1)}$$

Example: R.E $\rightarrow$ Rolling a dice

S.S $\rightarrow \{1, 2, 3, 4, 5, 6\}$

A $\rightarrow$ Getting odd no. $\{1, 3, 5\}$

B $\rightarrow$ prime no. $\{2, 3, 5\}$

$A\cap B \rightarrow \{3, 5\}$

$$p(A|B) = \frac{p(A\cap B)}{p(B)} = \frac{\frac{|A\cap B|}{|SS|}}{\frac{|B|}{|SS|}} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}$$

what if ? $p(B|A) = \dfrac{p(B\cap A)}{p(A)}$

$\boxed{\because A\cap B = B\cap A}$

$$p(B|A) = \frac{p(A\cap B)}{p(A)}$$

$$p(A\cap B) = p(B|A) * p(A) \quad \text{—— (2)}$$

If we submitted Equation (2) in Equation (1)

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$$\boxed{p(A|B) = \frac{p(B|A) * p(A)}{p(B)}}$$ ← Bayes' Theorem.

Mutually Exclusive Events

Two Events are called as mutually Exclusive, if $A \cap B = \phi$

Independent Events

Two Events are not dependent on One another are called as Independent.

consider that A & B are independent } $p(A|B) = p(A)$

On the other hand, if A & B are Dependent }

→ $p(A|B) = \frac{p(A \cap B)}{p(B)}$

OR

→ $p(A|B) = \frac{p(B|A) * p(A)}{p(B)}$

probability Axioms : (Rules)

① $0 \leq p(E) \leq 1$

Reason: Event is a subset of a sample space.

Denominator is always Greater than (OR) Equal to Numerator → $p(E) = \frac{|E|}{|SS|}$

② $P(S.S) = \frac{|SS|}{|SS|} = 1$

E.g; R.E → Tossing a coin

S.S → {H, T}

A → Getting Either H (OR) T → {H, T}

B → Getting 5 → ϕ

$p(A) = \frac{|A|}{|SS|} = \frac{2}{2} = 1$

③ $P(A \cup B) = ?$

find the p(A) (OR) p(B) occuring ?

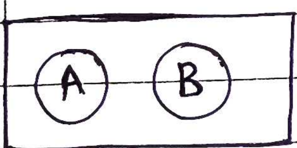
$p(A \cup B)$

Independent

$A \cap B = \phi$

(Mutually Exclusive) MEE

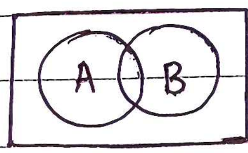
$p(A \cup B) = p(A) + p(B)$



$A \cap B \neq \phi$ Not MEE

(Not Mutually Exclusive)

$p(A \cup B) = p(A) + p(B) - P(A \cap B)$



probability

single Event (or) uni-variate

$p(A) = \frac{|A|}{|SS|}$ $\qquad$ $p(B) = \frac{|B|}{|SS|}$

** Marginal probability

Two Events (or) Bi-variate

$p(A|B) = \frac{P(A \cap B)}{p(B)}$

** conditional probability

Given that (B) has already occured, what is the probability of (A)

$p(A \cup B)$

* what is the probability of either (A) occuring (or) either (B) occuring

$p(A \cap B)$

** Joint probability

what is the probability of A & B occuring together?

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probability

single Event (or) uni-variate

$p(A) = \frac{|A|}{|SS|}$

** Marginal probability

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Two Events (or) Bi-variate (i.e; Event A, B)

$p(A \cup B)$

Mutually Exclusive

$\boxed{A \cap B = \phi}$

$p(A \cup B) = p(A) + p(B)$

Not Mutually Exclusive

$\boxed{A \cap B \neq \phi}$

$p(A \cup B) = p(A) + p(B) - p(A \cap B)$

Joint probability / $p(A \cap B)$

(A & B are Not mutually Exclusive)

Dependent

using conditional probability

$p(A \cap B) = p(A|B) * p(B)$

Independent

using conditional probability

$p(A \cap B) = p(A) * p(B)$

Conditional probability / $p(A|B)$

Dependent

$$p(A|B) = \frac{p(A \cap B)}{p(B)}$$

(or)

$$p(A|B) = \frac{p(B/A) * p(A)}{p(B)}$$

Independent

$p(A|B) = p(A)$

Formulae :

$\boxed{\text{Marginal probability}}$ → $p(A) = \frac{|A|}{|S|}$

Single Event

① $p(A \cup B)$
- if $A \cap B \neq \phi$ (not MEE) → $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- if $A \cap B = \phi$ (MEE) → $p(A \cup B) = p(A) + p(B)$

② $p(A|B)$ — Given that

Conditional probability
- if A & B are mutually Exclusive → $P(A|B) = 0$
- if A & B are not mutually Exclusive
  - Dependent: $p(A|B) = \frac{p(A \cap B)}{p(B)}$ (OR) Baye's Theorem: $P(A|B) = \frac{P(B|A) * p(A)}{p(B)}$
  - Independent: $p(A|B) = p(A)$

③ $p(A \cap B)$

Joint probability
- if $A \cap B = \phi$ (Mutually Exclusive) → $p(A \cap B) = 0$
- if $A \cap B \neq \phi$ (Not mutually Exclusive)
  - Dependent: $p(A \cap B) = p(A|B) * p(A)$
  - Independent: $p(A \cap B) = p(A) * p(B)$.