

## PART-2

### PROBLEM-2

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We were asked to use this DB

& implement repeated two fold cross validation holdout.

We were also given the folds for each repetition to & the no. of repetitions given are 4.

We need to use  $a_1$  as the root.

We also considered the class C as nominal.

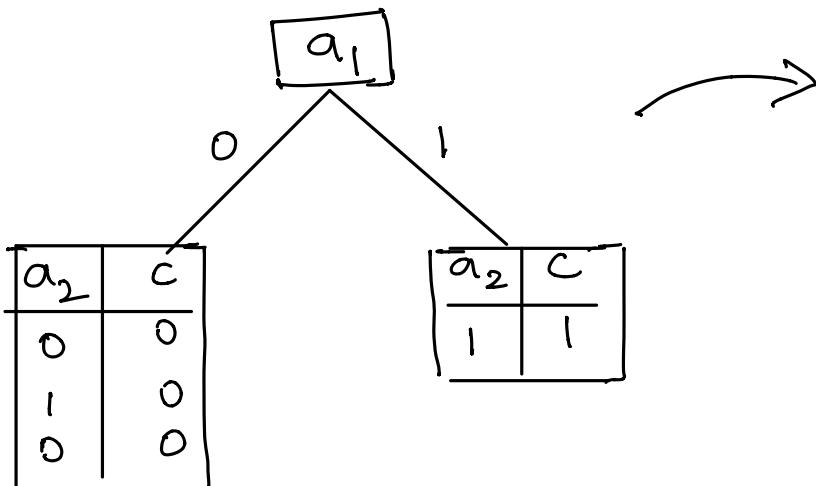
Step 1:

Repetition 1:

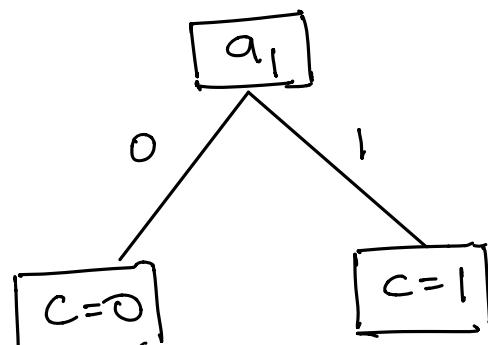
$$(1) \text{ train} = \{O_1, O_2, O_3, O_4\} \quad \text{test} = \{O_5, O_6, O_7, O_8\}$$

So following the stages.

Training : (Stage 1)



O	$a_1$	$a_2$	C
$O_1$	1	1	1
$O_2$	0	0	0
$O_3$	0	1	0
$O_4$	0	0	0
$O_5$	1	1	1
$O_6$	1	1	0
$O_7$	0	0	0
$O_8$	1	0	1



As the decision tree is completed.

Discriminant rules:

Rule 1 : IF  $a_1(x_i=0)$  THEN  $c(x_i=0)$

Rule 2 : IF  $a_1(x_i=1)$  THEN  $c(x_i=1)$

We'll try to resubstitute as the part of Stage 2.

$$\text{train} = \{o_1, o_2, o_3, o_4\}$$

The rule accuracy can be calculated as : 100%

$o_1$  is classified well using Rule 2.

$o_2, o_3, o_4$  are also classified well using Rule 1.

So stage 3:

$$\text{test} = \{o_5, o_6, o_7, o_8\}$$

$o_5, o_8$  are classified well using Rule 2.

$o_7$  is classified well using Rule 1.

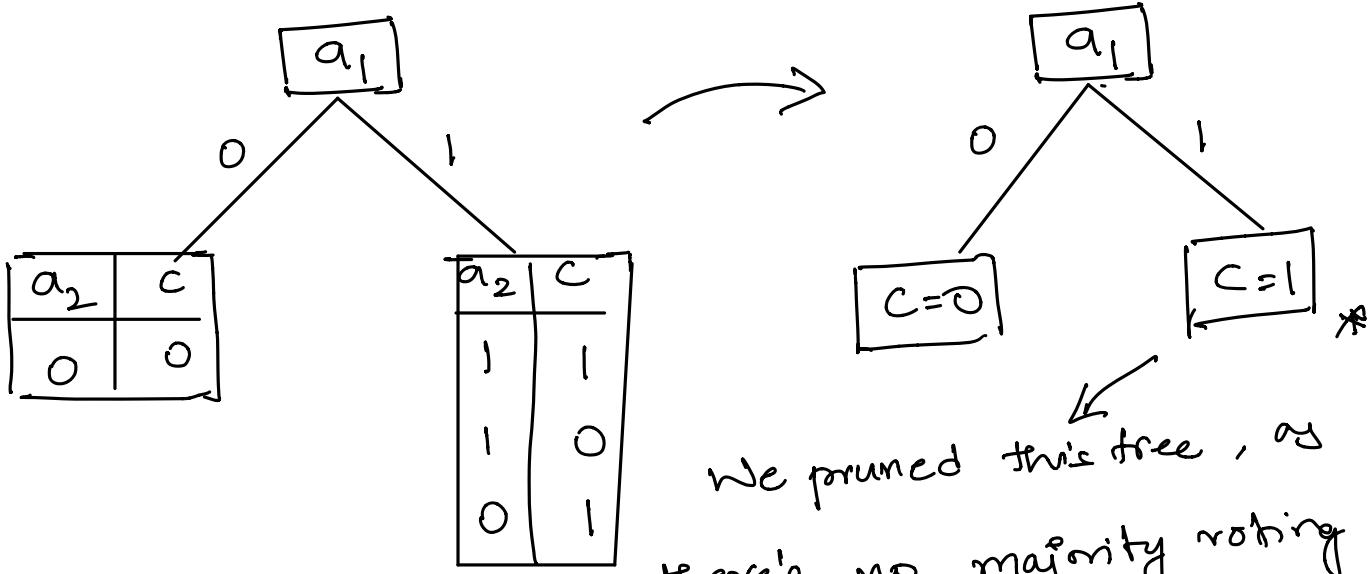
$o_6$  is misclassified from Rule 2.

so the predictive accuracy would be  $3/4 = 75\%$ .

(2) Now for the second fold as training set.

$$\text{train} = \{o_5, o_6, o_7, o_8\} \quad \text{test} = \{o_1, o_2, o_3, o_4\}$$

so following the stages.



We pruned this tree, as there's no majority voting

present when  $a_2=1 \{1, 0\}$  whereas for  $a_2=0 \{1\}$ . So when  $a_2=1$ , we went forward by keeping  $c=1$  as it is seen first.

As the decision tree is completed.

Discriminant rules :

Rule 1 : IF  $a_1(x_i=0)$  THEN  $c(x_i=0)$

Rule 2 : IF  $a_1(x_i=1)$  THEN  $c(x_i=1)$

We'll try to resubstitute as the part of Stage 2.

$$\text{train} = \{0_5, 0_6, 0_7, 0_8\}$$

The rule accuracy can be calculated as :  $3/4 : 75\%$ .

$O_5$  is well classified using Rule 2

$O_6$  is misclassified from Rule 2.

$O_7$  is well classified using Rule 1.

$O_8$  is well classified using Rule 2.

So stage 3:

test :  $\{O_1, O_2, O_3, O_4\}$ .

$O_1$  is classified well using Rule 2.

$O_2, O_3, O_4$  are classified well using Rule 1.

so the predictive accuracy would be 100%.

so our classifier would be the union of these

rules.  $F_1$ .

Rule 1 : IF  $a_1(x, = 0)$  THEN  $c(x, = 0)$

Rule 2 : IF  $a_1(x, = 1)$  THEN  $c(x, = 1)$

Rule 3 : IF  $a_1(x, = 0)$  THEN  $c(x, = 0)$

Rule 4 : IF  $a_1(x, = 1)$  THEN  $c(x, = 1)$

After removing the repetitions.

It would be

Rule 1 : IF  $a_1(x_i=0)$  THEN  $c(x_i=0)$

Rule 2 : IF  $a_1(x_i=1)$  THEN  $c(x_i=1)$

The classifier's rule accuracy is  $\frac{100+75}{2} = 87.5\%$ .

So prediction accuracy is  $\frac{100+75}{2} = 87.5\%$ .

Repetition 2 :

If we clearly check the given fold is

$\text{test} = \{0_5, 0_6, 0_7, 0_8\}$ ,  $\text{test} = \{0_1, 0_2, 0_3, 0_4\}$

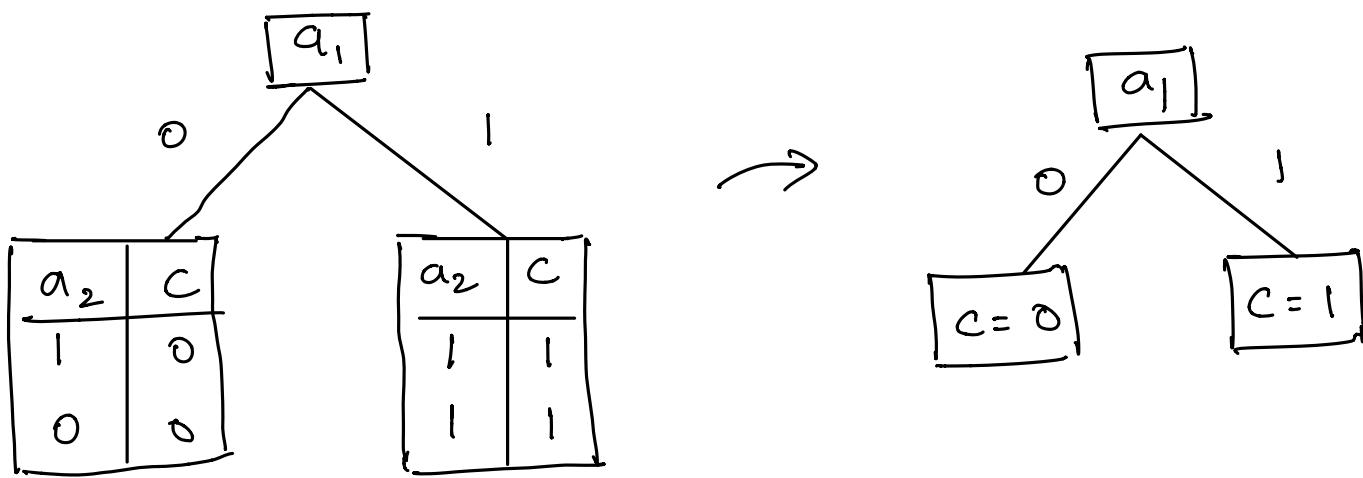
is exactly same as the 2<sup>nd</sup> fold in the previous repetition. So, similarly the 2<sup>nd</sup> fold in this case would be the same as the 1<sup>st</sup> fold in previous scenario. So the classifier would be the same.

$$\text{So } F_2 = F_1$$

Repetition 3 :

(1)  $\text{training} = \{0_1, 0_3, 0_5, 0_7\}$   $\text{test} = \{0_2, 0_4, 0_6, 0_8\}$ .

Training : Stage 1 :



Discriminant rules:

Rule 1 : IF  $a_1(x_1 = 0)$  THEN  $C(x_1 = 0)$

Rule 2 : IF  $a_1(x_1 = 1)$  THEN  $C(x_1 = 1)$

so the rule accuracy is obtained by substituting the training data.

$O_1 \& O_5$  are well classified using Rule 2.

$O_3 \& O_7$  are well classified using Rule 1.

So the rule accuracy is 100%.

so during stage 3 for calculating predictive accuracy

$O_2 \& O_4$  are well classified using Rule 1.

$O_8$  is well classified using Rule 2.

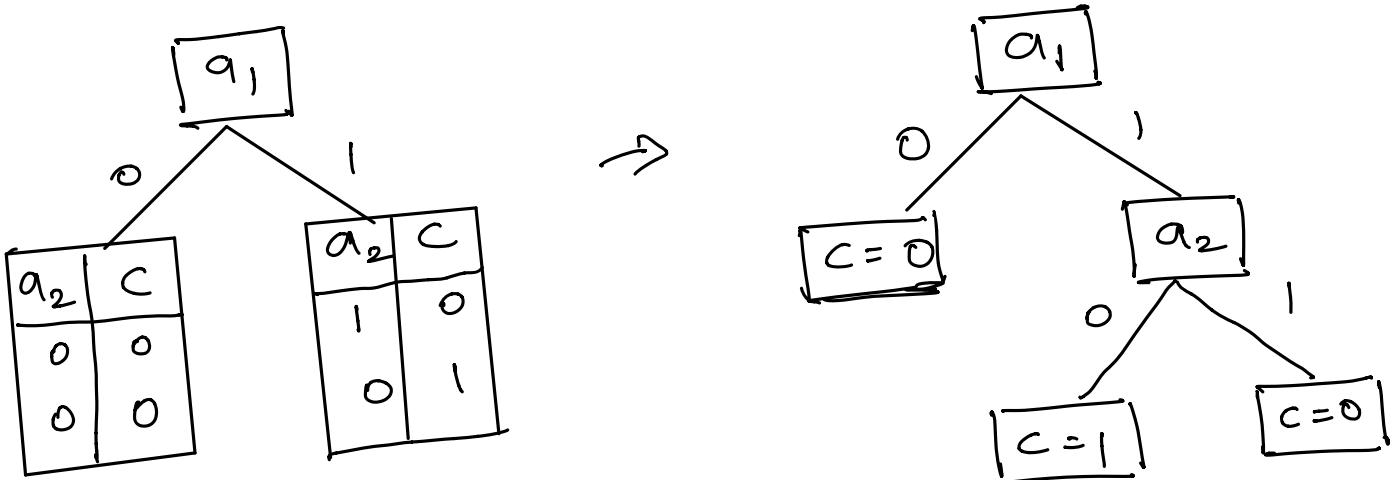
$O_6$  is misclassified from Rule 2.

so prediction accuracy = 75%.

(2)

$$\text{training} = \{o_2, o_4, o_6, o_8\} \quad \text{test} = \{o_1, o_3, o_5, o_7\}$$

Training: Stage 1:



Discriminant Rules:

Rule 1: IF  $a_1(x_1=0)$  THEN  $C(x_1=0)$

Rule 2: IF  $a_1(x_1=1)$  AND  $a_2(x_2=0)$  THEN  
 $C(x_1=1)$

Rule 3: IF  $a_1(x_1=1)$  AND  $a_2(x_2=1)$  THEN  
 $C(x_1=0)$ .

For stage 2 by resubstitution, the rule accuracy is

$o_2$  &  $o_4$  are well classified using Rule 1.

$o_6$  is well classified using Rule 3.

$o_8$  is well classified using Rule 2.

So Rule Accuracy is 100%.

Prediction Accuracy during stage 3 is

$O_3 \& O_7$  are well classified using Rule 1.

$O_1 \& O_5$  are misclassified from Rule 1.

So predictive accuracy is 50%.

So the combined rules of our classifier  $F_3$  would be without repetitions.

Rule 1: IF  $a_1(x_1=0)$  THEN  $c(x_1=0)$

Rule 2: IF  $a_1(x_1=1) \text{ AND } a_2(x_2=0)$  THEN  
 $c(x_1=1)$

Rule 3: IF  $a_1(x_1=1) \text{ AND } a_2(x_2=1)$  THEN  
 $c(x_1=0)$ .

Final Rule accuracy =  $\frac{100+100}{2} = 100\%$ .

Predictive accuracy would be =  $\frac{50+75}{2} = 62.5\%$ .

Repetition 4: The folds same as repetition 3 & hence the classifier would be the same to

$$F_4 = F_3$$

Step 2 :

After calculating the predictive & rule accuracy in the previous step.

We know that  $F_1 = F_2$  &  $F_3 = F_4$ .

So let's compare  $F_1$  &  $F_3$ .

	Rules Accuracy	Predictive Accuracy
$F_1$	87.5 %.	87.5 %.
$F_3$	100 %.	62.5 %.

Based on these details, Rule Accuracy is almost similar for both these classifiers but the predictive accuracy of  $F_3$  is very low compared to  $F_1$ .

So I would pick  $F_1$  or  $F_2$  as my classifier  $F$ . Let it be  $F_1$ .

so  $F_1 = F$ .

Step 3 :

We need to construct the bagged ensemble classifier  $F^*$ .

$F^*$  contains  $F_1, F_2, F_3 \cup F_4$ . Since  $F_1 = F_2$

$\cup F_3 = F_4$ . The majority voting is calculated

just between  $F_1 \cup F_3$ .

Given data.

	0	$a_1$	$a_2$
$o_1$	0	0	1
$o_2$	0	0	0
$o_3$	1	0	0
$o_4$	1	1	1

Since our  $F$  is  $F_1$

the rules of this classifier are:

$F$  : Rule 1 : IF  $a_1(x_i=0)$  THEN  $c(x_i=0)$

Rule 2 : IF  $a_1(x_i=1)$  THEN  $c(x_i=1)$

And our  $F^*$  consists of  $F_1, F_2$  ( $F_1=F_2$ )  $\cup F_3, F_4$   
( $F_3=F_4$ )

$F^*$  :  $F_1$

Rule 1 : IF  $a_1(x_i=0)$  THEN  $c(x_i=0)$

Rule 2 : IF  $a_1(x_i=1)$  THEN  $c(x_i=1)$

$F_3$

Rule 1: IF  $a_1(x_1=0)$  THEN  $c(x_1=0)$

Rule 2: IF  $a_1(x_1=1)$  AND  $a_2(x_2=0)$  THEN  
 $c(x_1=1)$

Rule 3: IF  $a_1(x_1=1)$  AND  $a_2(x_1=1)$  THEN  
 $c(x_1=0)$ .

Let's first classify using  $F$ .

$O_1$  is classified 0 using Rule 1.

$O_2$  is classified 0 using Rule 1.

$O_3$  is classified 1 using Rule 2.

$O_4$  is classified 1 using Rule 2.

for  $F^*$

$F_1$  is classified similar to  $F$  as above.

where as  $F_3$ .

$O_1$  is classified 0 using Rule 1.

$O_2$  is classified 0 using Rule 1.

$O_3$  is classified 1 using Rule 2.

$O_4$  is classified 0 using Rule 3.

Since we need to take majority vote.

$F_1 \approx F_3$  for  $\hat{F}^*$ . The final result would be -

$O_1 \approx O_2$  are classified as 0.

$O_3 \approx O_4$  are classified as 1.

( $O_4$  doesn't have a majority vote, so we went with 1 as it's seen first).

	$F$	$\hat{F}^*$
$O_1$	0	0
$O_2$	0	0
$O_3$	1	1
$O_4$	1	1

So based on our results, both these classifiers provided the same results.