

Part -2

Problem -2.

Multi-Dimensional Association Rules.

Data Base $F = \{F_1, F_2, F_3, F_4, F_5\}$

$F_1 = \{\text{Bread, Endive}\}$

$F_2 = \{\text{Bread, Milk}\}$

$F_3 = \{\text{Bread, Milk, Endive}\}$

$F_4 = \{\text{Bread, Endive}\}$

$F_5 = \{\text{Endive}\}$

Additional information:

F_1 : livesin = SBrook, income = high

F_2 : income = high

F_3 : livesin = SBrook

F_4 : livesin = SBrook, income = high

F_5 : income = high

1. Combine the information to one transactional database.

$T = \{T_1, T_2, T_3, T_4, T_5\}$

Combined Transactional Data is represented as below

	I1 Bread	I2 Milk	I3 Endive	I4 livesin SBrook	I5 income high
T1	+	-	+	+	+
T2	+	+	-	-	+
T3	+	+	+	+	-
T4	+	-	+	+	+
T5	-	-	+	-	+
Sc	4	2	4	3	4

Writing transactions using numbers gives

$$T1 = \{1, 3, 4, 5\}$$

$$T2 = \{1, 2, 5\}$$

$$T3 = \{1, 2, 3, 4\}$$

$$T4 = \{1, 3, 4, 5\}$$

$$T5 = \{3, 5\}$$

2. Minimum Support = 3.

∴ I2 cannot be included in Frequent 1-item set

Frequent 1-item set L1:

$$L1 = \{\{1\}, \{3\}, \{4\}, \{5\}\}$$

Join step:

Joining L_{k-1} with itself will generate candidate set C_k .

∴ 2-item Candidate set C_2 for the given transactions
with support count is

$$\{\{1, 3\}(3), \{1, 4\}(3), \{1, 5\}(3), \{3, 4\}(3), \{3, 5\}(3), \{4, 5\}(2)\}$$

Prune Step:

Any $(k-1)$ itemset that is not frequent cannot be
a subset of a frequent k -itemset.

All the sets in C_2 are valid, so pruning is
not required.

$\{4, 5\}$ has 2 occurrences and min support is 3. So, it
cannot be included in Frequent 2-itemset.

Frequent 2-itemset L2:

$$L2 = \{\{1, 3\}, \{1, 4\}, \{1, 5\}, \{3, 4\}, \{3, 5\}\}$$

Join Step:

3-item Candidate set C3 with support count is

$$\{1, 3, 4\}, \{1, 3, 5\}, \{1, 4, 5\}, \{3, 4, 5\}$$

Prune Step:

sets $\{1, 4, 5\}$ and $\{3, 4, 5\}$ cannot be included for Frequent 3-itemset because all their corresponding possible subsets do not exist in Frequent 2-itemset.

Now we have, $\{1, 3, 4\}$ (3), $\{1, 3, 5\}$ (2)

$\{1, 3, 5\}$ has 2 occurrences and min support is 3. So, it cannot be included in Frequent 3-itemset.

Frequent 3-itemset L3:

$$L3 = \{\{1, 3, 4\}\}$$

END of Apriori

∴ Frequent itemset for Association rules is

$$FR = \{\{1\}, \{3\}, \{4\}, \{5\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{3, 4\}, \{3, 5\}, \{1, 3, 4\}\}$$

Single-Dimensional rules:

The rules formed involving only 1, 2, 3 are single dimensional.

From frequent itemset the rules formed out of $\{1, 3\}$ are single dimensional.

Two single-dimensional rules can be formed:

$\text{buys}(x, \text{"bread"}) \rightarrow \text{buys}(x, \text{"endive"})$

$\text{buys}(x, \text{"endive"}) \rightarrow \text{buys}(x, \text{"bread"})$

Multi-Dimensional rules are formed involving 1, 2, 3 and 4, 5.

From frequent itemset the rules formed out of $\{1, 3, 4\}$ are multi dimensional.

Inter-Dimensional rules:

All the rules formed out of $\{1, 3, 4\}$ will include 1 and 3, and we see the predicate

"buys" repeats in rules formed.

So, no inter-dimensional rules can be formed from $\{1, 3, 4\}$.

Considering 2-items Frequent sets $\{1, 4, 3\}, \{1, 5\}, \{3, 4\}, \{3, 5\}$.

Inter-Dimensional rules formed from these sets are:

$\text{buys}(x, \text{"bread"}) \rightarrow \text{livesin}(x, \text{"SBrook"})$

$\text{livesin}(x, \text{"SBrook"}) \rightarrow \text{buys}(x, \text{"bread"})$

$\text{buys}(x, \text{"endive"}) \rightarrow \text{livesin}(x, \text{"SBrook"})$

$\text{livesin}(x, \text{"SBrook"}) \rightarrow \text{buys}(x, \text{"endive"})$

$\text{buys}(x, \text{"bread"}) \rightarrow \text{income}(x, \text{"high"})$

$\text{income}(x, \text{"high"}) \rightarrow \text{buys}(x, \text{"bread"})$

$\text{buys}(x, \text{"endive"}) \rightarrow \text{income}(x, \text{"high"})$

$\text{income}(x, \text{"high"}) \rightarrow \text{buys}(x, \text{"endive"})$

These Inter-dimensional rules are formed by taking different combinations of elements from the applicable frequent set & forming two disjoint sets in

which one set of elements implies the other.

Hybrid-Dimensional rules:

All the rules formed out of {1,3,4} are Hybrid-Dimensional rules, because they must involve 1,3 that represent the same predicate "buys".

Some of the hybrid dimensional rules are:

$$\text{livesin}(x, \text{"SBrook"}) \wedge \text{buys}(x, \text{"bread"}) \rightarrow \text{buys}(x, \text{"endive"})$$

$$\text{livesin}(x, \text{"SBrook"}) \wedge \text{buys}(x, \text{"endive"}) \rightarrow \text{buys}(x, \text{"bread"})$$

$$\text{buys}(x, \text{"endive"}) \rightarrow \text{livesin}(x, \text{"SBrook"}) \wedge \text{buys}(x, \text{"bread"})$$

$$\text{buys}(x, \text{"bread"}) \rightarrow \text{livesin}(x, \text{"SBrook"}) \wedge \text{buys}(x, \text{"endive"})$$

$$\text{livesin}(x, \text{"SBrook"}) \rightarrow \text{buys}(x, \text{"endive"}) \wedge \text{buys}(x, \text{"bread"})$$

$$\text{buys}(x, \text{"endive"}) \wedge \text{buys}(x, \text{"bread"}) \rightarrow \text{livesin}(x, \text{"SBrook"})$$

These hybrid dimensional rules are formed by

taking different combinations of 1, 3, 5 and forming two disjoint sets in which one set of elements implies the other.