Scoliomorph: A Spatial Orientation Analysis Library for Vertebral Body Point Cloud

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1 Introduction

This library provides tools to load STL files, compute pitch, roll, and yaw angles based on the principal axes, and visualize the results.

2 Data

The example STL files are extracted using the Auto3DSeg extension for 3D Slicer using the example dataset. The segmentation was exported as .stl and the files were inspected and extra vertices were cleaned up using Blender.

3 Method for Calculating the Spatial Orientation of Vertebral Bodies from a Point Cloud

This method computes the **pitch**, **roll**, and **yaw** angles of vertebral bodies using principal component analysis (PCA) applied to a point cloud representing the vertebral body. The steps are as follows:

3.1 Centroid Calculation

The centroid represents the geometric center of the point cloud. For a set of N points $\mathbf{P} = \{\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_N\}$, where each point $\mathbf{p}_i$ has coordinates (x_i, y_i, z_i), the centroid $\mathbf{c}$ is calculated as:

$$\mathbf{c} = \left(\frac{1}{N} \sum_{i=1}^{N} x_i, \frac{1}{N} \sum_{i=1}^{N} y_i, \frac{1}{N} \sum_{i=1}^{N} z_i \right)$$

3.2 Centering the Point Cloud

The point cloud is then centered by subtracting the centroid from each point, which shifts the origin of the coordinate system to the geometric center of the vertebral body:

$$\mathbf{P}_{\text{centered}} = \mathbf{P} - \mathbf{c}$$

3.3 Covariance Matrix Calculation

The covariance matrix $\mathbf{\Sigma}$ is computed from the centered points, capturing the variance and covariances between the X, Y, and Z axes:

$$\mathbf{\Sigma} = \frac{1}{N}\mathbf{P}_{\text{centered}}^{\top}\mathbf{P}_{\text{centered}}$$

This covariance matrix is a 3×3 matrix representing how the points are distributed along the three axes.

3.4 Eigenvalue and Eigenvector Decomposition

The covariance matrix is decomposed into eigenvalues and eigenvectors:

$$\mathbf{\Sigma v} = \lambda \mathbf{v}$$

Where $\mathbf{v}$ is an eigenvector representing a principal axis, and λ is its corresponding eigenvalue, indicating the amount of variance along that axis.

3.5 Sorting the Principal Axes

The eigenvectors are sorted in descending order based on their eigenvalues. The largest eigenvalue corresponds to the primary orientation of the vertebral body.

3.6 Calculating Pitch, Roll, and Yaw

The **pitch**, **roll**, and **yaw** angles are calculated from the eigenvectors, which represent the primary, secondary, and tertiary axes of the vertebral body:

- **Pitch** is the rotation around the **X-axis**:

$$\text{Pitch} = \arctan 2(v_{x,z}, v_{x,y}) \cdot \frac{180}{\pi}$$

- **Roll** is the rotation around the **Y-axis**:

$$\text{Roll} = \arctan 2(v_{y,z}, v_{y,x}) \cdot \frac{180}{\pi}$$

- **Yaw** is the rotation around the **Z-axis**:

$$\text{Yaw} = \arctan 2(v_{z,y}, v_{z,x}) \cdot \frac{180}{\pi}$$

Where $v_{x,y}, v_{x,z}, v_{y,x}, v_{y,z}, v_{z,x}, v_{z,y}$ are the components of the eigenvectors along the respective axes.

3.7 Projection of the Axes

Once the principal axes are calculated, they are projected onto the three primary planes:

- **YZ-plane**: This projection shows the **Pitch** (rotation around the X-axis) and how the vertebral body is tilted up or down.
- **XZ-plane**: This projection shows the **Roll** (rotation around the Y-axis), indicating the side-to-side tilting of the vertebral body.
- **XY-plane**: This projection shows the **Yaw** (rotation around the Z-axis), representing the sideways rotation (left or right turn) of the vertebral body.

3.8 Final Output

The method returns:

- **Pitch**, **Roll**, and **Yaw** angles (in degrees)
- The **centroid** of the point cloud
- The **principal axes** of the vertebral body

These results provide a precise measure of the spatial orientation of the vertebral body in the global coordinate system.

4 Results

To interpret the rotational angles, see Rotational Reference.

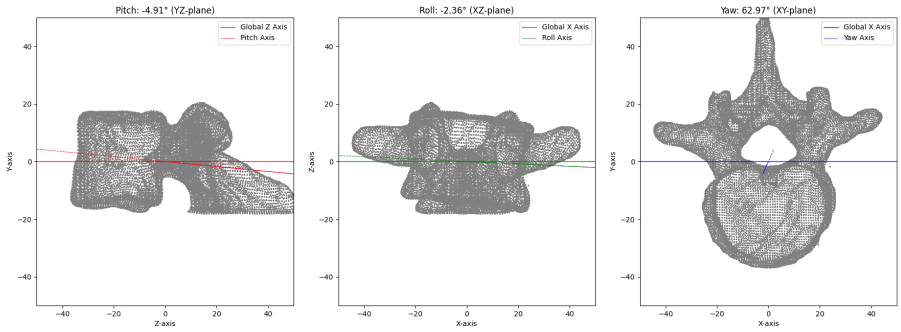

Figure 1: L3 Vertebral Body Example 1

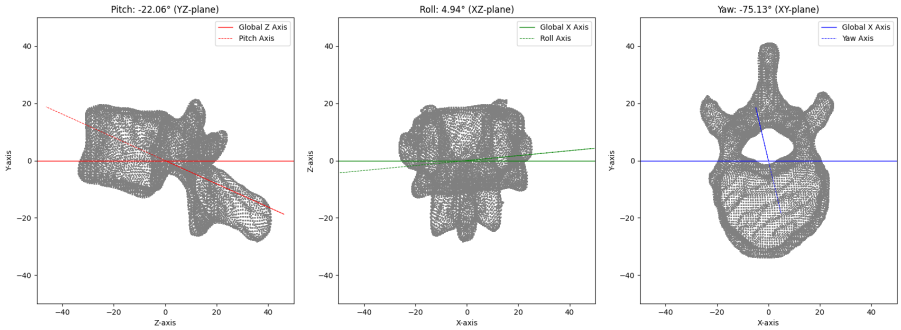

Figure 2: T12 Vertebral Body Example 2

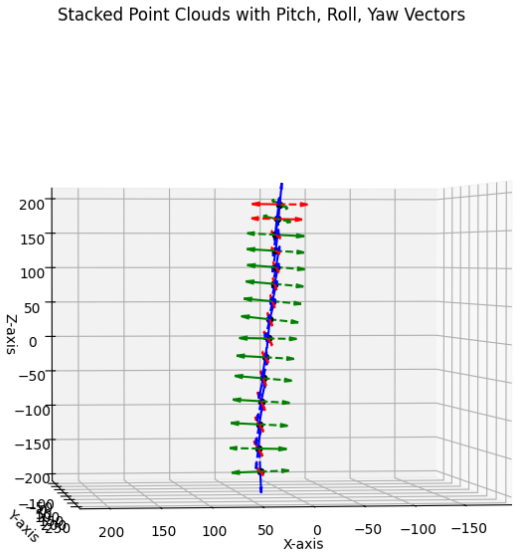

Figure 3: Column View 1

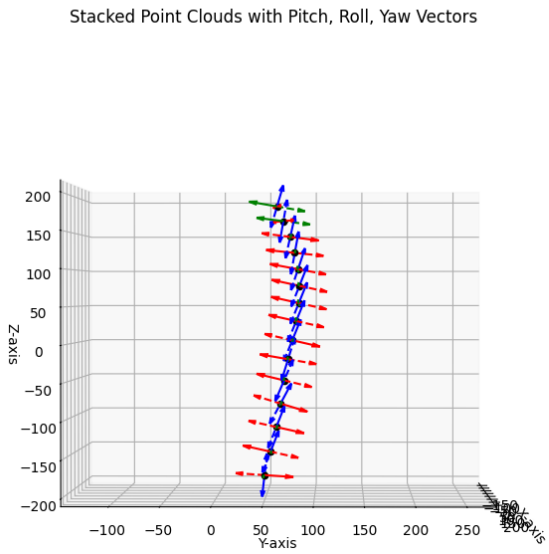

Figure 4: Column View 2

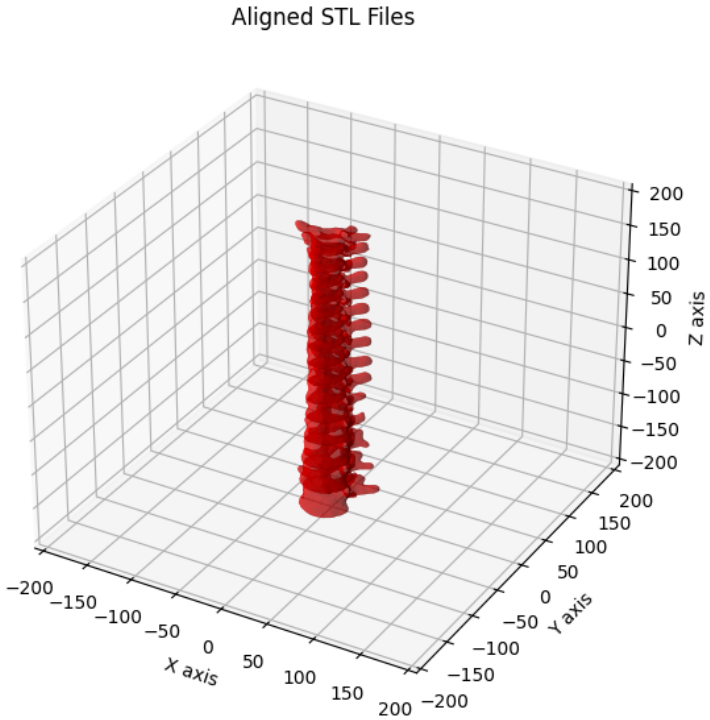

Figure 5: Column Stack View