# First Year Comp and Exam Solutions

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# Contents


**I Metrics** **3**

**1 Finals** **4**

1.1 Final 2010 . . . 4

1.1.1 Question 4 (Wolak) . . . 5

1.2 Comp 2007 Spring . . . 6

1.2.1 Question 5 (Wolak) . . . 6

1.3 Comp 2008 Spring . . . 7

1.3.1 Question 3 (?) . . . 7

**II Micro** **13**

**2 Finals** **14**

2.1 Final 2005 . . . 14

2.1.1 Question 3 (Bernheim) . . . 14

**3 Comps** **16**

3.1 Comp 2001 Spring . . . 16

3.1.1 Question 5 (Bernheim) . . . 16

3.2 Comp 2004 Spring . . . 18

3.2.1 Question 4 (Bernheim) . . . 18

3.3 Comp 2005 Fall . . . 19

3.3.1 Question 3 (Bernheim) . . . 20

3.3.2 Question 4 (Bernheim) . . . 21

3.4 Comp 2006 Spring . . . 23

3.4.1 Question 2 (Segall) . . . 23

**III Macro** **29**

**4 Finals** **30**

4.1 Final 2009 . . . 30

4.1.1 Question 1 (Piazzesi) . . . 30

**5 Comps** **33**
5.1 Comp 2009 Spring . . . . . . . . . . . . . . . . . . . . . . . . . . 33
5.1.1 Question 6 (Piazzesi) . . . . . . . . . . . . . . . . . . . . . . 33

# Part I

# Metrics

# Chapter 1

# Finals

## 1.1 Final 2010

PDF

### 1.1.1 Question 4 (Wolak)

#### 1.1.1.1 Part a

$$
\begin{aligned}
Y\Gamma + XB &= E \\
\begin{bmatrix} C_t & Y_t \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -\beta & 1 \end{bmatrix} + \begin{bmatrix} 1 & I_t \end{bmatrix} \begin{bmatrix} -\alpha & 0 \\ 0 & -1 \end{bmatrix} &= \begin{bmatrix} \epsilon_t & 0 \end{bmatrix} \\
\Pi &= -B\Gamma^{-1} \\
&= \frac{1}{1-\beta} \begin{bmatrix} \alpha & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ \beta & 1 \end{bmatrix} \\
&= \frac{1}{1-\beta} \begin{bmatrix} \alpha & \alpha \\ \beta & 1 \end{bmatrix} \\
\pi_{11} &= \frac{\alpha}{1-\beta} \\
\pi_{12} &= \frac{\beta}{1-\beta} \\
\pi_{21} &= \frac{\alpha}{1-\beta} \\
\pi_{22} &= \frac{1}{1-\beta} \\
V &= E\Gamma^{-1} \\
&= \frac{1}{1-\beta} \begin{bmatrix} \epsilon_t & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ \beta & 1 \end{bmatrix} \\
&= \frac{1}{1-\beta} \begin{bmatrix} \epsilon_t & \epsilon_t \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
Y &= X\Pi + V \\
\begin{bmatrix} C_t & Y_t \end{bmatrix} &= \begin{bmatrix} 1 & I_t \end{bmatrix} \begin{bmatrix} \frac{\alpha}{1-\beta} & \frac{\alpha}{1-\beta} \\ \frac{\beta}{1-\beta} & \frac{1}{1-\beta} \end{bmatrix} + \frac{1}{1-\beta} \begin{bmatrix} \epsilon_t & \epsilon_t \end{bmatrix}
\end{aligned}
$$

$$
Y_t = \frac{\alpha}{1-\beta} + \frac{1}{1-\beta} I_t + \frac{1}{1-\beta} \epsilon_t
$$

#### 1.1.1.2 Part b

$$\pi_{11} = \frac{\alpha}{1-\beta}$$
$$\pi_{12} = \frac{\beta}{1-\beta}$$
$$\pi_{21} = \frac{\alpha}{1-\beta}$$
$$\pi_{22} = \frac{1}{1-\beta}$$

$$\beta = \frac{\pi_{12}}{\pi_{22}} = 1 - \frac{1}{\frac{1}{1-\beta}} = 1 - \frac{1}{\pi_{22}}$$

$$Y_t = \frac{\alpha}{1-\beta} + \frac{1}{1-\beta} I_t + \frac{1}{1-\beta}\epsilon_t$$

$$\begin{aligned}
\hat{\pi}_{22} &= \frac{\frac{1}{T}\sum(Y_t-\bar{Y})(I_t-\bar{I})}{\frac{1}{T}\sum(I_t-\bar{I})^2} \\
Y_t - I_t &= C_t \\
\hat{\beta} &= 1 - 1/\frac{\frac{1}{T}\sum(Y_t-\bar{Y})(I_t-\bar{I})}{\frac{1}{T}\sum(I_t-\bar{I})^2} \\
&= 1 - \frac{\frac{1}{T}\sum(I_t-\bar{I})^2}{\frac{1}{T}\sum(Y_t-\bar{Y})(I_t-\bar{I})} \\
&= \frac{\frac{1}{T}\sum(C_t-\bar{C})(I_t-\bar{I})}{\frac{1}{T}\sum(Y_t-\bar{Y})(I_t-\bar{I})}
\end{aligned}$$

## 1.2 Comp 2007 Spring

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### 1.2.1 Question 5 (Wolak)

#### 1.2.1.1 Part a

$$\begin{aligned}
y_t &= \sigma_t \epsilon_t \\
y_t^2 &= \sigma_t^2 \epsilon_t^2 \\
\ln y_t^2 &= \ln \sigma_t^2 + \ln \epsilon_t^2 \\
\ln y_t^2 &= \lambda x_t + \eta_t + \ln \epsilon_t^2
\end{aligned}$$

We want an estimator for $\lambda$ of the form

$$\hat{\gamma} = \frac{\sum_{t=1}^{n} g(y_t)h(x_{t-j})}{\sum_{t=1}^{n} h(x_t)h(x_{t-j})}$$

So we can use

$$g(y_t) = \ln y_t^2$$

Checking

$$\begin{aligned} E(x_t x_{t-1}) &= E(\frac{1}{2}\xi_{t-1}^2) = \frac{1}{2} \\ E(\ln y_t^2 x_{t-1}) &= E(\lambda x_t \xi_{t-1} + \frac{1}{2}\xi_{t-2}\lambda x_t) \\ &= \frac{1}{2}\lambda \end{aligned}$$

To get

$$\begin{aligned} \hat{\gamma} &= \frac{\sum_{t=1}^{n}(\ln y_t^2)(x_{t-1})}{\sum_{t=1}^{n}(x_t)(x_{t-1})} \\ &\xrightarrow{p} \frac{E(\ln y_t^2 x_{t-1})}{E(x_t x_{t-1})} \\ &= \frac{E(\ln y_t^2 x_{t-1})}{\frac{1}{2}} \\ &= \frac{\frac{1}{2}\lambda}{\frac{1}{2}} \\ &= \lambda \end{aligned}$$

## 1.3 Comp 2008 Spring

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### 1.3.1 Question 3 (?)

There are four cases:

| y_1i | y_2i |
|---|---|
| 0 | 0 |
| 0 | 1 |
| 1 | 0 |
| 1 | 1 |

Write out the unconditional likelihood function for a single observation using the following notation. This notation enables you to write all four cases in one summation:

$$\begin{aligned} P\left(Y_{1i}=y_{1i},Y_{2i}=y_{2i};p,\rho\right) &= (y_{1i})(y_{2i})P(Y_{1i}=1,Y_{2i}=1) \\ &+ (y_{1i})(1-y_{2i})P(Y_{1i}=1,Y_{2i}=0) \\ &+ (1-y_{1i})(y_{2i})P(Y_{1i}=0,Y_{2i}=1) \\ &+ (1-y_{1i})(1-y_{2i})P(Y_{1i}=0,Y_{2i}=0) \end{aligned}$$

$$\begin{aligned} P\left(Y_{1i}=y_{1i},Y_{2i}=y_{2i};p,\rho\right) &= (y_{1i})(y_{2i})P(\epsilon_{1i}\leq p,\epsilon_{2i}\leq p) \\ &+ (y_{1i})(1-y_{2i})P(\epsilon_{1i}\leq p,\epsilon_{2i}> p) \\ &+ (1-y_{1i})(y_{2i})P(\epsilon_{1i}> p,\epsilon_{2i}\leq p) \\ &+ (1-y_{1i})(1-y_{2i})P(\epsilon_{1i}> p,\epsilon_{2i}> p) \end{aligned}$$

Note that the probability is a joint probability. It is only separately if the \epsilon are independent. Because they are distributed normally, we can use correlation to IFF independence. Thus, they are only independent if $\rho = 0$.

We can rewrite this for convenience as

$$\begin{aligned} P\left(Y_{1i}=y_{1i},Y_{2i}=y_{2i};p,\rho\right) &= I_{1i}\Phi_{1i} \\ &+ I_{2i}\Phi_{2i} \\ &+ I_{3i}\Phi_{3i} \\ &+ I_{4i}\Phi_{4i} \end{aligned}$$

#### 1.3.1.1 Part a

$$\begin{aligned} P\left(Y_{1i}=y_{1i},Y_{2i}=y_{2i};p,\rho\right) &= \sum_{j=1}^{4} I_{1i}\Phi_{1i} \\ P\left(Y_{1}=y_{1},Y_{2}=y_{2};p,\rho\right) &= \prod_{i=1}^{n}\sum_{j=1}^{4} I_{1i}\Phi_{1i} \end{aligned}$$

Now write the log-likelihood using the observed data.

Useful trick: note that we can pull the indicator function out of the log because in each observation, only one of the above cases is actually realized. Thus, the log of sum of these different cases is only going to be the log of one of these cases. We therefore simplify things by pulling the indicator out of the log and moving the log inside the sum.

$$
\begin{aligned}
L\left(p, \rho ; \underline{Y_1}, \underline{Y_2}\right) &= \sum_{i=1}^{n} \ln\Big(\sum_{j=1}^{4} I_{1i}\Phi_{1i}\Big) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{4} I_{1i}\ln(\Phi_{1i})
\end{aligned}
$$

The above equation is the log-likelihood whether or not they are indepedent.

Say we assumed $\rho = 0$ and they are independent. Now we only need two terms to write out the possibilities:

$$
\begin{aligned}
P\left(Y_{1i} = y_{1i}, Y_{2i} = y_{2i}; p, \rho\right) &= P\left(Y_{1i} = y_{1i}\right)P(Y_{2i} = y_{2i}) \\
&= \left[y_{1i}P(\epsilon_{1i} \le p) + (1 - y_{1i})P(\epsilon_{1i} > p)\right] \\
&\times \left[y_{2i}P(\epsilon_{2i} \le p) + (1 - y_{2i})P(\epsilon_{2i} > p)\right] \\
&= \left[y_{1i}\Phi(p) + (1 - y_{1i})(1 - \Phi(p)\right]\left[y_{2i}\Phi(p) + (1 - y_{2i})(1 - \Phi(p)\right]
\end{aligned}
$$

Therefore, under the assumption that \rho=0 and thus the epsilons are independent then we do have the correct likelihood function (normalized):

$$
Q_n(b) = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{2} y_{1i}\ln(\Phi(b)) + (1 - y_{1i})\ln((1 - \Phi(b))
$$

#### 1.3.1.2 Part b

$$
\hat{b} = \text{argmax } Q_n(b)
$$

Want to know if

$$
\hat{b} \xrightarrow{b} p
$$

We assumpted independence to get the likelihood function. Let's first name what this converges to. Call it $Q(b)$:

$$
\begin{aligned}
Q_n(b) &\xrightarrow{p} \lim \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{2} y_{1i}\ln(\Phi(b)) + (1 - y_{1i})\ln((1 - \Phi(b)) \\
&\xrightarrow{p} E\left[\sum_{j=1}^{2} y_{1i}\ln(\Phi(b)) + (1 - y_{1i})\ln((1 - \Phi(b))\right] \\
&= Q(b)
\end{aligned}
$$

The question is would this converge if $\rho \neq 0$ and the errors were not indepenent?

We are taking a sample average across the i observations. Important: we know that there are correlations across $j$, but not across $i$. Thus, we are able to use the regular LLN even if $\rho \neq 0$.

Now let's compute what $Q(b)$ actually is. Note that the expected value of a indicator function is the probability that it is realized.

$$
\begin{aligned}
E\left[\sum_{j=1}^{2} y_{1i}\ln(\Phi(b)) + (1-y_{1i})\ln((1-\Phi(b))\right] &= \sum_{j=1}^{2} E(y_{1i})\ln(\Phi(b)) + E(1-y_{1i})\ln((1-\Phi(b)) \\
&= \sum_{j=1}^{2} \Phi(p)\ln(\Phi(b)) + (1-\Phi(p))\ln((1-\Phi(b))
\end{aligned}
$$

Nothing inside of the summation depends on j anymore, so re-write:

$$
= 2\left[\Phi(p)\ln(\Phi(b)) + (1-\Phi(p))\ln((1-\Phi(b))\right]
$$

Last, but not least, we want to actually solve for the argmax b:

$$
\begin{aligned}
\frac{\partial Q(b)}{\partial b} &= 2\frac{\Phi(p)}{\Phi(b)}\phi(b) - \frac{(1-\Phi(p))}{(1-\Phi(b))}\phi(b) = 0 \\
&= 2\phi(b)\left\{\frac{\Phi(p)}{\Phi(b)} - \frac{(1-\Phi(p))}{(1-\Phi(b))}\right\} = 0
\end{aligned}
$$

We know that the PDF of a normal has support on the full R, so the question is when does the second term equal 0 as you vary b. The answer is that the first term is always decreasing as b increases and the second term is always increasing as b increases. Thus, they must have a single point of intersection. That point is at $b = p$.

Thus we have that

$$
\hat{b} = \text{argmax } Q_n(b) \xrightarrow{p} \text{argmax } Q(b)
$$

Therefore, the estimator, $\hat{b}$, is consistent for the true maximizer, $p$.

#### 1.3.1.3 Part c

$$
\begin{aligned}
(\hat{b}_1, \hat{b}_2) &= \text{argmax } Q_n(b_1, b_2) \\
&= \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{2} y_{1i}\ln(\Phi(b_j)) + (1-y_{1i})\ln((1-\Phi(b_j)) \\
Q(b_1, b_2) &= \sum_{j=1}^{2} \Phi(p_j)\ln(\Phi(b_j)) + (1-\Phi(p_j))\ln((1-\Phi(b_j))
\end{aligned}
$$

Note that this is the same as before except that the expectation of $y_{1i}$ is now $\Phi(p_j)$.

Take first order conditions

$$\frac{\partial Q}{\partial b_j} = \phi(b_j)\left\{\frac{\Phi(p_j)}{\Phi(b_j)} - \frac{1-\Phi(p_j)}{1-\Phi(b_j)}\right\} = 0$$

So they are still good.

#### 1.3.1.4 Part d

As we saw in part a, this likelihood function only works if they $\rho = 0$. The problem is that in general this not true. Thus, we cannot simply jump to the information matrix. We need to instead do ABA.

$$\sqrt{n}(\hat{b} - b) \to_d N(0, H_0^{-1} V_0 H_0^{-1})$$

$$\begin{aligned}
Var(\frac{1}{\sqrt{n}} S_n(p)) &= V_0 \\
E(H_n(p)) &= H_0 \\
S_n &= \frac{\partial}{\partial} Q_n(b) \\
H_n &= \frac{\partial}{\partial b \partial b} Q_n(b)
\end{aligned}$$

Computing derivatives is messy.

This is the MLE trick:

We already computed the first order condition

$$\begin{aligned}
\frac{\partial Q_n}{\partial b_j} &= \frac{1}{n}\sum_{i=1}^{n}\left\{\frac{y_{ij}\phi(b_j)}{\Phi(b_j)} - \frac{(1-y_{1j})\phi(b_j)}{1-\Phi(b_j)}\right\} = 0 \\
&= \phi(b_j)\left\{\frac{\bar{y}}{\Phi(b_j)} - \frac{(1-\bar{y})}{1-\Phi(b_j)}\right\} = 0
\end{aligned}$$

Because nothing depends on $i$ except $y$. Again, we can see that to have both of the terms in brackets equal to 1 we need to set $\Phi(b_j) = \bar{y}_j$.

Given that $\Phi(b_j) = \bar{y}_j$ is strictly increasing, we can invert to get:

$$\hat{b} = \Phi^{-1}(\bar{y}_j)$$

Now we can use the delta method to get the asymptotic distribution. The idea is to do the delta method with the function $g = \Phi^{-1}$ and use what we already know about $Y$ being distributed as Bernoulli.

$$\sqrt{n}\left( \begin{array}{c} \hat{b}_1 \\ \hat{b}_2 \end{array} - \begin{array}{c} b_1 \\ b_2 \end{array} \right) \to_d N(0, \cdot)$$

$$\sqrt{n}\left( \begin{array}{c} \bar{Y}_1 \\ \bar{Y}_2 \end{array} - \begin{array}{c} E(y_1) \\ E(y_2) \end{array} \right) \to_d N(0, \Omega)$$

$$\begin{array}{c} b_1 \\ b_2 \end{array} = G\left( \begin{array}{c} \bar{Y}_1 \\ \bar{Y}_2 \end{array} \right) = \left( \begin{array}{c} \Phi^{-1}(\bar{Y}_1) \\ \Phi^{-1}(\bar{Y}_2) \end{array} \right)$$

$$\frac{\partial G}{\partial y_1} = \begin{pmatrix} \frac{1}{\phi(\Phi^{-1}(\bar{y}_1))} \\ 0 \end{pmatrix}$$

$$\frac{\partial G}{\partial y_2} = \begin{pmatrix} \frac{1}{\phi(\Phi^{-1}(\bar{y}_2))} \\ 0 \end{pmatrix}$$

Evaluate at the true expected value

$$E(y_{ji}) = \Phi(p_j)$$

$$G_0 = \begin{pmatrix} \frac{1}{\phi(p_1)} & \\ & \frac{1}{\phi(p_2)} \end{pmatrix}$$

$$\begin{aligned} Var(y_1) &= \Phi(p_1)(1 - \Phi(p_1) \\ Var(y_2) &= \Phi(p_2)(1 - \Phi(p_2) \\ Cov(y_1, y_2) &= E(Y_1 Y_2) - E(Y_1)E(Y_2) \\ &= P(Y_1 = 1, Y_2 = 1) - \Phi(p_1)\Phi(p_2) \end{aligned}$$

$$\begin{aligned} \Omega_0 = CLT &\begin{pmatrix} Var(y_1) & Cov(y_1, y_2) \\ Cov(y_1, y_2) & Var(y_2) \end{pmatrix} \\ = &\begin{pmatrix} \Phi(p_1)(1 - \Phi(p_1) & P(Y_1 = 1, Y_2 = 1) - \Phi(p_1)\Phi(p_2) \\ P(Y_1 = 1, Y_2 = 1) - \Phi(p_1)\Phi(p_2) & \Phi(p_1)(1 - \Phi(p_1) \end{pmatrix} \end{aligned}$$

Delta method

$$\sqrt{n}\left( G\left( \begin{array}{c} \bar{Y}_1 \\ \bar{Y}_2 \end{array} \right) - G\left( \begin{array}{c} E(\bar{Y}_1) \\ E(\bar{Y}_2) \end{array} \right) \right) \to_d N(0, G_0 \Omega_0 G_0)$$

We don't really know what that last covariance thing is so no reason to plug it into the thing.

# Part II

# Micro

# Chapter 2

# Finals

## 2.1 Final 2005

### 2.1.1 Question 3 (Bernheim)

#### 2.1.1.1 Part a

i.

By Rubinstein's bargaining game, we know that in the final period the proposer, $i$, would propose

$$(1, 0, 0)$$

Because the only alternative is for everyone to get 0 then everyone would agree.

ii.

$$(1/3, 1/3, 1/3)$$

iii.

The person doing the proposing wants to offer the exact amount such that the other two guys are indifferent between accepting the offer and their expected utility in the next round. Thus, in the first round, the proposer should offer

$$\delta * \frac{1}{3} = proposal_1$$

because in the coming round, each person has an expected utility.

iv.

If we assume that the people accept their offer when it is the same as their expected discounted utility then they have expected value of the game:

$$E = \frac{1}{3}(1 - \frac{2}{3}\delta) + \frac{2}{3}\delta\frac{1}{3}$$

#### 2.1.1.2 Part b

For any finite game, the first round will have an offer of

$$\frac{1}{3}\delta$$

for all other players

#### 2.1.1.3 Part c

i.

No, the proposer would still take everything and the others would still accept since this is the same to them as everyone getting zero.

ii.

The proposer would offer

$$\frac{2}{3}\delta$$

to one other person.

iii.

$$E = \frac{1}{3}(1 - \frac{1}{3}\delta) + \frac{1}{2}(\frac{2}{3}\delta\frac{1}{3})$$

iv.

# Chapter 3

# Comps

## 3.1 Comp 2001 Spring

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### 3.1.1 Question 5 (Bernheim)

First, think always write out the strategy sets for player $i$:

i. Trade always

ii. Trade if and only if in my state, $\omega_i$

iii. Do not trade if and only if in my state, $\omega_i$ (trade in the other states that he can't differentiate)

iv. Never trade

The payoff matrix thus look like

| player 1/player2 | Trade always | Trade if w_2 | Dont trade if w_2 | Never |
|---|---|---|---|---|
| Trade always | 4-9p,4-9p | (-2p,p) | 4-7p,4-10p | (0,0) |
| Trade if w_1 | p,-2p | (0,0) | p,-2p | (0,0) |
| Not trade if w_1 | 4-10p,4-7p | -2p,p | 4-8p,4-8p | (0,0) |
| Never | (0,0) | (0,0) | (0,0) | (0,0) |

#### 3.1.1.1 Part a

We can immediately see that not trading in all states will provide the same payoff regardless of what the other person does. Thus, we cannot have a strictly dominant strategy.

For player 1

i. In state 1: accepting weakly dominants rejecting

ii. In state 2/3: neither strategy domiants (rejecting weakly dominants in state 2 and accepting weakly dominants in state 3)

For player 2

i. In state 1/3: neither strategy domiants (rejecting weakly domiannts in state 1 and accepting weakly dominants in state 3)

ii. In state 2: rejecting weakly dominants.

So the weakly dominanted strategies are: {Do not trade if and only if you are $\omega_i$, Never trade}. They are dominated by {Always trade, Trade if and only if you are in $\omega_i$}.

#### 3.1.1.2 Part b

Say p=2/5

| player 1/player2 | Trade always | Trade if w_2 | ~~Dont trade if w_2~~ | ~~Never~~ |
|---|---|---|---|---|
| Trade always | 2/5,2/5 | -4/5,2/5 | ~~6/5,0~~ | ~~(0,0)~~ |
| Trade if w_1 | 2/5,-4/5 | 0,0 | ~~2/5,-4/5~~ | ~~(0,0)~~ |
| ~~Not trade if w_1~~ | ~~0,6/5~~ | ~~-4/5,2/5~~ | ~~4-8p,4-8p~~ | ~~(0,0)~~ |
| ~~Never~~ | ~~0,0~~ | ~~0,0~~ | ~~0,0~~ | ~~(0,0)~~ |

We know that the weakly dominanted strategies are: {Do not trade if and only if you are $\omega_i$, Never trade}.

We can see that the Nash equilibria are: (Trade always, Trade Always) with payoff (4-9p,4-9p) and (Trade if $\omega_1$, Trade if $\omega_2$) with payoff (0,0).

The expected total surplus is 8-18p in the first case and 0 in ths second.

#### 3.1.1.3 Part c

If $p \in (2/5, 1/2)$ then

| player 1/player2 | Trade always | Trade if w_2 | ~~Dont trade if w_2~~ | ~~Never~~ |
|---|---|---|---|---|
| Trade always | <-1/2,<-1/2 | >-1/2,<1/2 | ~~4-7p,4-10p~~ | ~~(0,0)~~ |
| Trade if w_1 | <1/2,>-1 | (0,0) | ~~p,-2p~~ | ~~(0,0)~~ |
| ~~Not trade if w_1~~ | ~~4-10p,4-7p~~ | ~~-2p,p~~ | ~~4-8p,4-8p~~ | ~~(0,0)~~ |
| ~~Never~~ | ~~(0,0)~~ | ~~(0,0)~~ | ~~(0,0)~~ | ~~(0,0)~~ |

So, we have only one Nash eqm which is the lower right corner: (Trade if $\omega_1$, Trade if $\omega_2$) with payoff (0,0).

#### 3.1.1.4 Part d

Go back to the big payoff matrix

| player 1/player2 | Trade always | Trade if w_2 | Dont trade if w_2 | Never |
|---|---|---|---|---|
| Trade always | 4-9p,4-9p | (-2p,p) | 4-7p,4-10p | (0,0) |
| Trade if w_1 | p,-2p | (0,0) | p,-2p | (0,0) |
| Not trade if w_1 | 4-10p,4-7p | -2p,p | **4-8p,4-8p** | (0,0) |
| Never | (0,0) | (0,0) | (0,0) | (0,0) |

The socially optimal way to go is to be (Don't trade if $\omega_1$, Don't trade if $\omega_2$), which would maximize the total surplus to 8-16p. The only state in which this occurs is when $\omega_3$. Otherwise, someone isn't trading and they both get 0.

Unfortunately, if I know that you were going to do this dont trade unless in the recognizable state, then I would deviate to trade always because I get a +1p advantage.

## 3.2 Comp 2004 Spring

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### 3.2.1 Question 4 (Bernheim)

#### 3.2.1.1 Part a

First, consider if neither firm quoted a price. This could not possibly be a Pure Strategy Nash Eqm because then one firm would diverge to p=v, where they could make a positive profit since v>c+k.

Second, consider if one firm quoted some price between c+k and v. In that case, the second firm would deviate to also quote a price just below the first firms price and above c+k.

Finally, consider if both firms propose a price. We know that they must be offering the same price, otherwise if one was lower than the other then the high priced firm would be making -k profit and thus shouldn't be selling in the first place. We also know that if they both are quoting a price then there is a probability of 1/2 that the customer buys from the other guy. Thus, we need price to be at least c+2k. We know that if they sell then they would get c+2k, but if they weren't picked then they would lose k for having advertised. Thus, the lowest price they would both propose would be:

$$
\begin{aligned}
&0 = \frac{1}{2}(2k) + \frac{1}{2}(-k) \\
&p \geq c + 2k
\end{aligned}
$$

The problem is that there is still room for deviation. Each firm is going to try to go down just a little bit so that they for sure get the customer at the price

$$c + 2k > p > c + k$$

Thus, there can be not PSNE because these guys are all deviating.

#### 3.2.1.2 Part b

F puts no weight above v because this is above the customer's reservation value, and thus the customer will not purchase the product that price.

F puts no weight below c+k because that is the cost incurred by the firm when selling the item. Selling below that level would give the firm a negative profit, and thus, the firm would be better off not selling anything.

We know that the firm has the option to not sell in the support of his strategy set. This option has expected payoff of 0. Thus, because a mixed strategy equilibrium must have all of the payoffs equal then we know that the equilibrium payoff is 0.

Next, we calculate the probability that the other firm makes an offer of p or less:

$$(1 - \pi F(p))$$

Then we calculate out expected payoff at that price given we are making an offer:

$$(1 - \pi F(p))(p - c) - k$$

Note that the k goes on the end because we pay that regardless of whetehr or not we make the sale (again, all this is conditional on us making an offer).

Using our knowledge that the expected payoff must be 0, then all of the prices in the support must be 0:

$$\begin{aligned} (1 - \pi F(p))(p - c) - k &= 0 \\ \frac{1 - \frac{k}{p-c}}{\pi} &= F(p) \\ \frac{p - c - k}{\pi(p - c)} &= F(p) \end{aligned}$$

To get \pi, we recognize that

$$\begin{aligned} F(c + k) &= 0 \\ F(v) &= 1 \end{aligned}$$

Plugging into the over equation we get

$$\pi = \frac{v - c - k}{v - k}$$

## 3.3 Comp 2005 Fall

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### 3.3.1 Question 3 (Bernheim)

#### 3.3.1.1 Part a

i.

Need the loss in utility during the first endowed period from giving up g units to be less than the utility gained from receiving g units in the unendowed period.

$$u(W) - u(W-g) < \delta(u(g) - u(0))$$

More succintly, we can write

$$u'(W) < \delta u'(g)$$

To find the socially optimal point, just think about the points on a concave curve: expand the distance between u(0) and u(g) by increasing g until it touches u(W-g):

$$u'(W-g) = \delta u'(g)$$

ii.

Say that deviations from the optimal gift are punished by the next generation not giving anything. Under the social optimum, each generation gets

$$u(W-g) + \delta u(g)$$

If you give anything else and get punished then you receive

$$u(W-g') + \delta u(0)$$

Because the utility is strictly increasing, we know that this is less than

$$u(W) + \delta u(0)$$

But we already showed in part a of this question that this is less optimal than the sharing scheme. Thus, deviating must be less optimal than maintaining the social convention over time.

iii. The above answer shows that given any social convention that makes everyone better off, adherence, counterintuitively, does not depend on the $\delta$. Instead, it depends on the social convention being more beneficial than doing nothing.

iv.

No this would not be possible because everyone would deviate to not following the convention. They would all stop giving the next generation, so that they would receive nothing in return. Their utility wuld revert to

$$u(W) + \delta u(0)$$

which by assumption is better than whatever sharing program they had in place before. Thus, we can also see that this does not depend on the $\delta$.

v.

In the finite case, this all breaks down. Why would the last generation give the previous generation? Without some reciprocity, sharing with their parents is strictly dominanted. But if the last generation won't give to their parents, then their parents won't give to the previous generation either because

$$u(W) - \delta u(0) > u(W - g) - \delta u(0)$$

Thus, going up the chain, no one gives to their parents and the whole world reverts to being selfish.

#### 3.3.1.2 Part b

***NEED TO FILL IN***

### 3.3.2 Question 4 (Bernheim)

#### 3.3.2.1 Part a

Let $v_A$ be the value to the consumer.

Let $v_B$ be the value to the consumer.

Let $p_A$ be the price quoted by firm A.

Let $p_B$ be the price quoted by firm B.

Know that $v_A \geq v_B$ by assumption.

The comsumer wants the best deal, which we will define as

$$v_i - p_i$$

Start by rulling out $p_B > c$. We know this because the consumer will pick from the company that has the largest spread between value and price. Thus, if $p_B > c$, then firm B would lower their price slightly in an effort to get the consumer while not losing much revenue. So, $p_B = c$

Next we can rule out $v_A - p_A > v_B - p_B$. If that were the case then firm A would keep moving up $p_A$ to increase revenue without losing the consumer.

So now we have settled that

$$v_A - p_A = v_B - p_B$$

Thus, we get that

$$p_A = v_A - v_B + p_B$$

$$p_B = c$$

and there is complete indifference between these two firms. So the consumer can flip a coin and buy from either.

#### 3.3.2.2 Part b

i.

Then firm A is maximizing given knownledge that $p_B = c$ in the second stage and that $v_A = v_B$. Thus, the second stage is really an equilibrium such that

$$\begin{aligned} v_A + f(d) - p_A &= v_B - p_B \\ v_A + f(d) - p_A &= v_B - c \end{aligned}$$

so firm A would set its price in the second stage as

$$p_A = v_A + f(d) - v_B + c$$

Thus, firm A has profits

$$\begin{aligned} \pi_A &= p_A - c - d \\ &= v_A + f(d) - v_B + c - c - d \\ &= v_A + f(d) - v_B - d \\ &= f(d) - d \end{aligned}$$

with the last line since $v_A = v_B$ in this version.

ii.

The question is asking us to maximize the profit with respect to advertising, $d$:

$$\begin{aligned} \frac{\partial}{\partial d}\pi_A &= \frac{\partial}{\partial d}(f(d) - d) \\ 0 &= f'(d) - 1 \\ f'(d) &= 1 \end{aligned}$$

In the specific case of $f(d) = \sqrt{d}$:

$$\begin{aligned}\frac{1}{2}d^{-\frac{1}{2}} &= 1 \\ d &= (2)^{-2} \\ &= \frac{1}{4}\end{aligned}$$

#### 3.3.2.3 Part c

i.

$$\begin{aligned}\pi_A &= \max\{f(d_A) - f(d_B), 0\} - d_A \\ \pi_B &= \max\{f(d_B) - f(d_A), 0\} - d_B\end{aligned}$$

ii.

There couldn't be a Nash equilibrium where both firms have the same level of advertising expenditure because if there was then they would both have profits that were negative

$$\begin{aligned}\pi_A &= max\{0, 0\} - d_A \\ \pi_B &= max\{0, 0\} - d_B\end{aligned}$$

So they wouldn't even operate then.

iii.

***NEED TO DO***

## 3.4 Comp 2006 Spring

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### 3.4.1 Question 2 (Segall)

#### 3.4.1.1 Part a

Definition: a normal good is one for which consumption is increasing with wealth.

We want to find increasing difference in $(s_i, e_i)$, so that we can use Topkis to say that the optimal $s_i$ is increasing in the strong set order.

Our maximization problem is

$$\begin{aligned}&\max u(c_i, s_i) \\ &\qquad s.t c_i + ps_i = \frac{1}{2}p + e_i\end{aligned}$$

We can rewrite without the c_i:

$$\max u(e_i + \frac{1}{2}p - ps_i, s_i)$$

So what we want to find is that when $e_i' > e_i$ then:

$$u(e_i' - \frac{p}{2}, 1) - u(e_i' + \frac{p}{2}, 0) > u(e_i - \frac{p}{2}, 1) - u(e_i + \frac{p}{2}, 0)$$

We have that u is concave with respect to its first arguement. We can use that to say

$$u(e_i' - \frac{p}{2}, 1) - u(e_i - \frac{p}{2}, 1) > u(e_i' + \frac{p}{2}, 1) - u(e_i + \frac{p}{2}, 1)$$

We also know by the fact that $u(c, 1) - u(c, 0)$ is strictly positive that

$$u(e_i' + \frac{p}{2}, 1) - u(e_i + \frac{p}{2}, 1) > u(e_i' + \frac{p}{2}, 0) - u(e_i + \frac{p}{2}, 0)$$

Thus, we have ID in $(s_i, e_i)$ and have shown that diamonds are normal.

#### 3.4.1.2 Part b

Player 1 gets the diamond because we just showed that there are increasing differences in endowment and the diamond. Increasing differences says that with higher endowment, the optimal consumption of diamond is weakly increasing. Thus, we would expect that the first person with his larger endowment will end up with the diamond.

#### 3.4.1.3 Part c

We want to solve for

$$(c_1, s_1), (c_2, s_2), p$$

We have the following maximization problem

$$\begin{aligned} &\max U(c_i, s_i) \\ &\quad s.t. \\ &\qquad ps_i + c_i = \frac{1}{2}p + e_i \end{aligned}$$

We know from part b that the first consumer will get the diamond, so we are really solving for

$$(c_1, 1), (c_2, 0), p$$

We can plug into the budget constraints to get

$$c_1 + p = \frac{1}{2}p + e_1$$
$$c_2 = \frac{1}{2}p + e_2$$

or rearranging

$$c_1 = e_1 - \frac{1}{2}p$$
$$c_2 = e_2 + \frac{1}{2}p$$

To characterize the equilbrium, we can use what we know about the increasing differencees

$$u(e_1 - \frac{1}{2}p, 1) \geq u(e_1 + \frac{1}{2}p, 0)$$
$$u(e_2 + \frac{1}{2}p, 0) \geq u(e_2 - \frac{1}{2}p, 1)$$

For all $x > 0$

$$u(e - x, 1) - u(e + x, 0)$$

is strcitly increasing in $e$.

#### 3.4.1.4 Part d

We know that

$$e_1 \geq e_2$$

That means that we can explore both cases: $e_1 > e_2$ and $e_1 = e_2$.

We know that any equilibrium must satisfy the condition

$$u(e_1 - \frac{1}{2}p, 1) \geq u(e_1 + \frac{1}{2}p, 0)$$
$$u(e_2 + \frac{1}{2}p, 0) \geq u(e_2 - \frac{1}{2}p, 1)$$

Or written differently

$$u(e_1 - \frac{1}{2}p, 1) - u(e_1 + \frac{1}{2}p, 0) \geq 0$$
$$0 \geq u(e_2 - \frac{1}{2}p, 1) - u(e_2 + \frac{1}{2}p, 0)$$

Combined to get

$$u(e_1 - \frac{1}{2}p, 1) - u(e_1 + \frac{1}{2}p, 0) \geq u(e_2 - \frac{1}{2}p, 1) - u(e_2 + \frac{1}{2}p, 0)$$

Thus, multiple equilibria

#### 3.4.1.5 Part e

First, we figure out if the possibilities frontier is convex or concave by using the inverse function theorem

$$u_2(\bar{e} - x, 1) = (\bar{e} - g^{-1}(u_1), 1)$$
$$u_1(x, D) = g(x)$$
$$x = g^{-1}(u_1)$$

$$\frac{\partial u_2}{\partial u_1} = u_c(\bar{e} - x, 1)[-\frac{\partial g^{-1}(u_1)}{\partial u_1}]$$
$$= -\frac{u_c(\bar{e} - x, 1)}{u_c(x, 0)}$$

so we have concave.
We still need to know if

$$u(0, 1) = u_2$$
$$u(\bar{e}, 0) = u_1$$

It can't be true that

$$u(0, 1) > u(\bar{e}, 0)$$

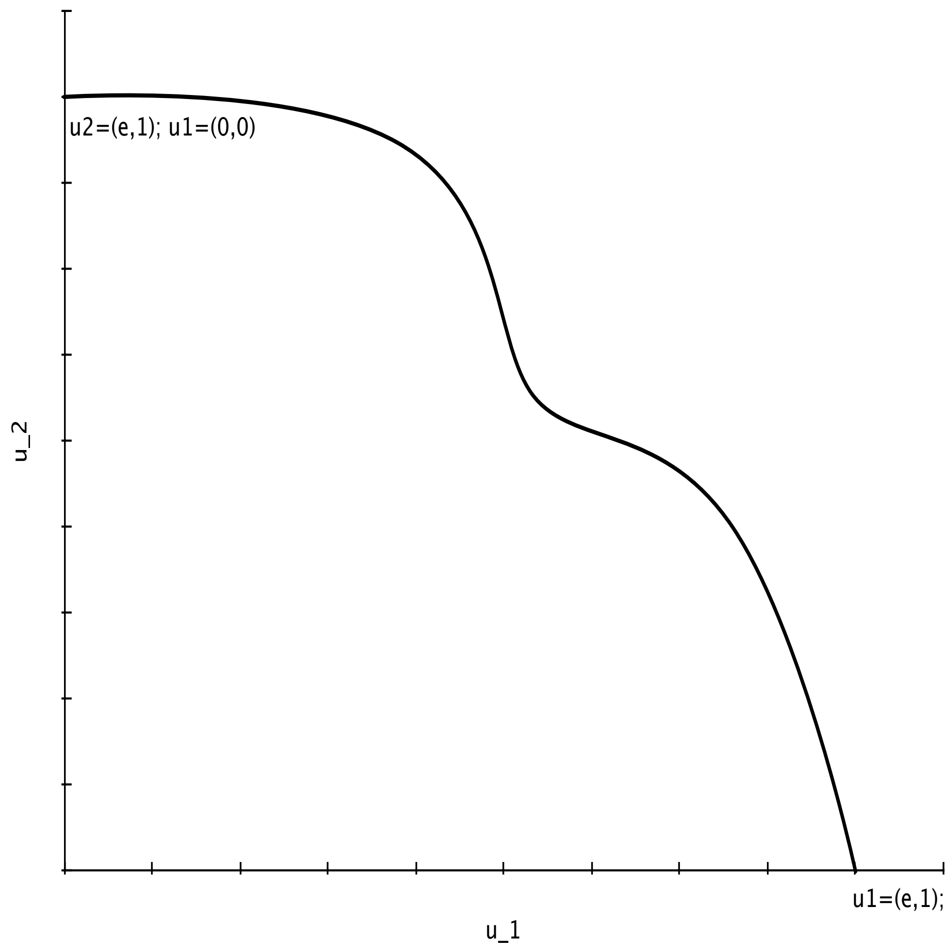
because we have an equilibrium so they are willing to trade.
Thus we get that either $u_2 = u_1$ or $u_2 < u_1$.
How do we know that there is a kink in the graph?
The marginal rate of substitution between them is

$$-\frac{u_c(\bar{e} - x, 1)}{u_c(x, 0)}$$

and

$$u_c(\bar{e},0) < u_c(0,0) < u_c(0,1)$$

and that has absolute value greater than 1.




#### 3.4.1.6 Part f

If you look at the kind then you can see that any convex combination that has the connecting line going over the indent will provide better expected utility.

$$\begin{aligned} E(u) &= \frac{1}{2}u(c,1) + \frac{1}{2}u(e_1+e_2-c,0) \\ c &\in [0, e_1+e_2] \end{aligned}$$

To find the max allocation just look for the slope = -1.

$$u_c(c, 1) = u_c(\bar{e} - c, 0)$$

# Part III

# Macro

# Chapter 4

# Finals

## 4.1 Final 2009

### 4.1.1 Question 1 (Piazzesi)

#### 4.1.1.1 Part 1

We know that

$$q = D\psi$$

We also know that under complete markets we can invert D to get q.
Since we are only looking for the price of the Boyle contract we can just write it out

$$\begin{aligned} q_B &= \text{price of contract for Susan Boyle wins} \\ &= 100 * \psi_B \end{aligned}$$

#### 4.1.1.2 Part 2

We know that with complete markets we have a representative agent.
We also know that

So, we can write the

$$\psi_s = p_s c_s^{-\gamma}$$

#### 4.1.1.3 Part 3

We know that state prices are the price of a single unit of cosumption in any state is

$$\psi_s$$

Thus, the price of a riskless bond is

$$\psi_0 = \sum_s^S \psi_s = \sum_s^S p_s c_s^{-\gamma} = \frac{1}{R^f}$$

#### 4.1.1.4 Part 4

We know that these funny risk neutral probabilities can be written

$$p_s^* = \frac{\psi_s}{\sum \psi_s} = \frac{\psi_s}{\psi_0}$$

because the price of a unit of consumption in state s is $\psi_s$, so $\psi_s/\psi_0$ will be a probability of sorts.

We can then write this using the price of a risk-free bond

$$p_s^* = \frac{\psi_s}{\sum \psi_s} = \frac{\psi_s}{\psi_0} = \frac{p_s c_s^{-\gamma}}{\sum_s^S p_s c_s^{-\gamma}}$$

Thus, we know that the price of a Susan Boyle contract is

$$p_s^* * D_{js} * \psi_0 = \psi_s$$

$$p_s^\star * 100 * \psi_0 = p_s^\star * 100 * \frac{1}{R^f}$$

#### 4.1.1.5 Part 5

In terms of preferences, if the guy is actually risk neutral so that $\gamma = 0$, then of course the risk neutral probabilities are also the real probabilities.

Alternatively, if the aggregate endowment is the same in both states such that $e_1 = e_2 = c$

$$\begin{aligned}
p_s^* &= \frac{p_s c^{-\gamma}}{\sum_s^S p_s c^{-\gamma}} \\
&= \frac{p_s c^{-\gamma}}{c^{-\gamma} \sum_s^S p_s} \\
&= \frac{p_s}{1} \\
&= p_s
\end{aligned}$$

#### 4.1.1.6 Part 6

The price of the asset is

$$\psi_s = p_s^* * 100 * \psi_0$$

If we divide by 100:

$$\begin{aligned}
\frac{\psi_s}{100} &= p_s^* * \psi_0 \\
&= p_s^* * \frac{1}{R^f}
\end{aligned}$$

If the time between now and the realization period is very short then there is no discounting and thus we are really looking at

$$\frac{\psi_s}{100} = p_s^*$$

Setting aggregate consumption equal in both states (which makes sense if the time period is very short)

$$\frac{\psi_s}{100} = p_s^* = p_s$$

so the price of the asset divided by 100 is equal to the probability of the realization of that state.

# Chapter 5

# Comps

## 5.1 Comp 2009 Spring

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### 5.1.1 Question 6 (Piazzesi)

#### 5.1.1.1 Part 1

$$
\begin{aligned}
g(\alpha) &= E_0\left[\sum \beta^t \frac{(c_t+\alpha x)^{1-\gamma}}{1-\gamma}\right] \\
&= \begin{cases} E_0\left[\sum \beta^t \frac{(c_t+\alpha x)^{1-\gamma}}{1-\gamma}\right] & \gamma \neq 1 \\ E_0\left[\sum \beta^t \log(c_t+\alpha x)\right] & \gamma = 1 \end{cases} \\
g'(\alpha) &= \begin{cases} E_0\left[\sum \beta^t (c_t+\alpha x_t)^{-\gamma} x_t\right] & \gamma \neq 1 \\ E_0\left[\sum \beta^t \frac{1}{c_t+\alpha x_t} x_t\right] & \gamma = 1 \end{cases} \\
g'(0) &= \begin{cases} E_0\left[\sum \beta^t (c_t)^{-\gamma} x_t\right] & \gamma \neq 1 \\ E_0\left[\sum \beta^t \frac{1}{c_t} x_t\right] & \gamma = 1 \end{cases} \\
&= E_0\left[\sum \beta^t (c_t)^{-\gamma} x_t\right]
\end{aligned}
$$

We know that the Riesz representation is

$$\delta U(c^\star; x) = E_0 \sum^{T} \pi_t x_t$$

So

$$\pi_t = \beta^t (c_t)^{-\gamma}$$