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Tutorial No:- 06 Group :- C

Questions:

1) If $\sigma_x = \sigma_y = \sigma$ and the angle between the lines of regressions is $\tan 3$. Find correlation.

2) Find the correlation coefficient between x & y when the lines of regression are
$2x - 9y + 6 = 0$
$x - 2y + 1 = 0$

3) Calculate the rank correlation coefficient from the following data

x	10	15	18	18	15	40
y	25	18	25	25	50	25

4) Calculate the correlation coefficient for the following height (in cms) inches) of fathers(x) and their sons (y)

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

Teacher's Sign:

5) Given $6y = 5x + 90$, $15x - y = 130$
$\sigma_x^2 = 16$ Find i) $\bar{x}$ & $\bar{y}$
ii) γ
iii) σ_y^2

1) →

The angle between two lines of regression

$$\tan\theta = \left(\frac{1-r^2}{r}\right) \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

Here,

angle = $\tan\theta = 3$

$\sigma_x = \sigma_y = \sigma$

$$\therefore \quad 3 = \left(\frac{1-r^2}{r}\right) \frac{\sigma \times \sigma}{\sigma^2 + \sigma^2}$$

$$3 = \frac{1-r^2}{r} \times \frac{\sigma^2}{2\sigma^2}$$

$$6r = 1 - r^2$$

$$r^2 + 6r - 1 = 0$$

$a = 1, \; b = 6, \; c = -1$

$$r = \frac{-6 \pm \sqrt{36+4}}{2}$$

$$= -3 \pm \sqrt{10}$$

$r = 0.162$ or -6.16

But

$-1 \leq r \leq 1$

$\therefore r = 0.162$

2)

Sol → Given lines of regressions are

$2x - 9y + 6 = 0$

$x - 2y + 1 = 0$

We know that, $r = \pm \sqrt{b_{xy} \cdot b_{yx}}$

Teacher's Sign.:

Assume first line $2x - 9y + 6 = 0$ in Y on X.

$$2x - 9y + 6 = 0$$
$$9y = 2x + 6$$
$$y = \frac{2}{9}x + \frac{6}{9}$$

$\therefore b_{yx} = \frac{2}{9}$

and 2nd line in X on Y

$$x - 2y + 1 = 0$$
$$x = 2y - 1$$
$$b_{xy} = 2$$

both b_{yx}, b_{xy} are +ve

$\therefore r = +\sqrt{b_{xy} \cdot b_{yx}}$

$$= +\sqrt{2 \times \frac{2}{9}}$$
$$= +\sqrt{\frac{4}{9}}$$
$$= \frac{2}{3}$$
$$= 0.667$$

which is in range $-1 \leq r \leq 1$

$\therefore$ correlation coefficient between x & y for given line is 0.67

Teacher's Sign:

3) Given data

x :	10	15	18	18	15	40
y :	25	18	25	25	50	25

Here, we have to calculate Rank correlation coefficient i.e R

$n = 6$

x	y	R_1	R_2	$d_i = R_1 \sim R_2$	d_i^2
10	25	6	3.5	2.5	6.25
15	18	4.5	6	1.5	2.25
18	25	2.5	3.5	1	1
18	25	2.5	3.5	1	1
15	50	4.5	1	3.5	12.25
40	25	1	3.5	2.5	6.25
					$\Sigma d_i^2 = 29$

We know that

For Repeated,

$$R = 1 - \frac{6\{\Sigma d_i^2 + \frac{1}{12}(m_1^3 - m_1)\}}{n^3 - n}$$

Here, $m_1 = m_2 = 2 \quad m_3 = 4$

So,

$$R = 1 - \frac{6\{\Sigma d_i^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \frac{1}{12}(m_3^3 - m_3)\}}{n^3 - n}$$

Teacher's Sign.:

$$R = 1 - \frac{6\left\{29 + \frac{1}{12}(2^3-2) + \frac{1}{12}(2^3-2) + \frac{1}{12}(4^3-4)\right\}}{6^3-6}$$

$$= 0$$

∴ Rank relation coefficient = 0

4) Calculate correlation coefficient

x :	65	66	67	67	68	69	70	72
y :	67	68	65	68	72	72	69	71

x → height of fathers, y → height of sons

$r = ?$

Calculation of r between x & y

Sr. No.	Demand in Qnt. $x - \bar{x}$, $\bar{x} = 68$			Price Per kg. $y - \bar{y}$, $\bar{y} = 69$			Product
	X	x	x^2	Y	y	y^2	xy
1	65	−3	9	67	−2	4	6
2	66	−2	4	68	−1	1	2
3	67	−1	1	65	−4	16	4
4	67	−1	1	68	−1	1	1
5	68	0	0	72	+3	9	0
6	69	+1	1	72	+3	9	3
7	70	+2	4	69	0	0	0
8	72	+4	16	71	+2	4	8
N=8	$\Sigma X = 544$		$\Sigma x^2 = 36$	$\Sigma Y = 552$,		$\Sigma y^2 = 44$	$\Sigma xy = +24$

Teacher's Sign.:

Now,

$$\bar{x} = \frac{544}{8} = 68,$$

$$\bar{y} = \frac{552}{8} = 69$$

$$\therefore r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 + \Sigma y^2}}$$

But, $\Sigma xy = 24$, $\Sigma x^2 = 36$, $\Sigma y^2 = 44$

$$\therefore r = \frac{24}{\sqrt{36 \times 44}}$$

$$= \frac{24}{39.80}$$

$$= 0.6030$$

5) Given:

$$6y = 5x + 90$$

$$15x - 8y = 130$$

$$\sigma_x^2 = 16$$

Find i) $\bar{x}$ & $\bar{y}$

ii) r

iii) σ_y^2

Since both LOR passes through $\bar{x}$ & $\bar{y}$

$$\therefore 6\bar{y} = 5\bar{x} + 90$$

i.e. $5\bar{x} - 6\bar{y} = -90$ — (1)

Teacher's Sign.:

& $15\bar{x} - 8\bar{y} = 130$

$\therefore \bar{x} = 30$

$\bar{y} = 40$

ii) r

$r = \pm\sqrt{byx \cdot byx}$

We Assume line $5x - 6y = -90$ in LOR Y on X

$6y = 5x + 90$

$y = \frac{5}{6}x + \frac{90}{6}$

$\therefore byx = \frac{5}{6}$

and 2nd line $15x - 8y = 130$ LOR in X on Y

$15x = 8y + 130$

$x = \frac{8}{15}y + \frac{130}{15}$

$\therefore bxy = \frac{8}{15}$

$\therefore r = \pm\sqrt{byx \times bxy} = +\sqrt{\frac{5}{6} \times \frac{8}{15}}$

$= \sqrt{\frac{4}{9}}$

$= \frac{2}{3} = 0.67$

$\therefore r = 0.67$

Teacher's Sign :

iii) $\sigma_x^2 = 16$

$\therefore \sigma_x = 4$

$\sigma_y^2 = ?$

We know that

R.C of y on x $b_{yx} = r \times \frac{\sigma_y}{\sigma_x}$

and R.C of x on y $b_{xy} = r \times \frac{\sigma_x}{\sigma_y}$

$\therefore b_{yx} = r \times \frac{\sigma_y}{\sigma_x}$

$\frac{5}{6} = \frac{2}{3} \times \frac{\sigma_y}{4}$

$\therefore \sigma_y = 5$

$\sigma_y^2 = 25$

Teacher's Sign :