Throw away the overall delta δ function for 4-momentum conservation, we get

$$-i\mathcal{M} = i g^2 \, \bar{u}(P_4\, s_4)\, \gamma^\nu \, u(P_2\, s_2) \frac{g_{\mu\nu}}{(P_1 - P_3)^2} \cdot \bar{u}(P_3, s_3)\, \gamma^\mu \, u(P_1\, s_1)$$

Multiplying by i to get $\mathcal{M}$, the scattering amplitude

$$\mathcal{M} = - g^2 \, \bar{u}(P_4, s_4)\, \gamma^\nu \, u(P_2, s_2) \cdot \frac{g_{\mu\nu}}{(P_1 - P_3)^2} \, \bar{u}(P_3, s_3)\, \gamma^\mu \, u(P_1, s_1)$$

If u and $\bar{u}$ are known explicitly, then $(\bar{u} \; \gamma^\nu \; u)$ is just a complex number

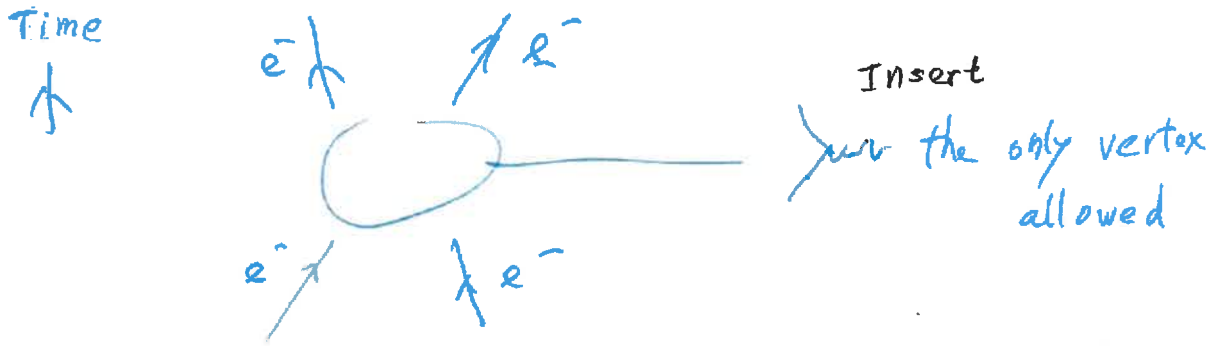
i.e. if all the u, $\bar{u}$ are known explicitly, then the scattering amplitude $\mathcal{M}$ is just a complex number.

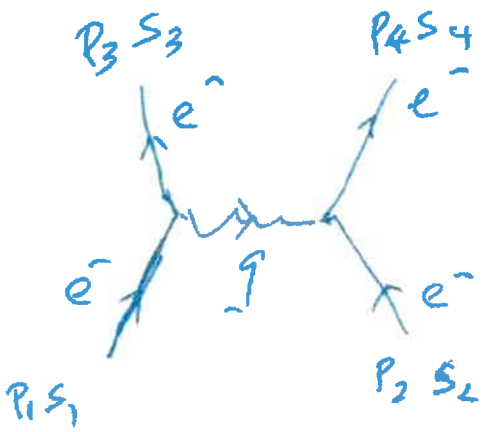
simplify the notations : $(P_1, s_1) \to (1)$, $(P_i, s_i) \to (i)$

$$\mathcal{M} = - g^2 \, \bar{u}(4)\, \gamma^\nu \, \bar{u}(2) \, \frac{g_{\mu\nu}}{(P_1 - P_3)^2} \cdot \bar{u}(3)\, \gamma^\mu \, u(1) .$$

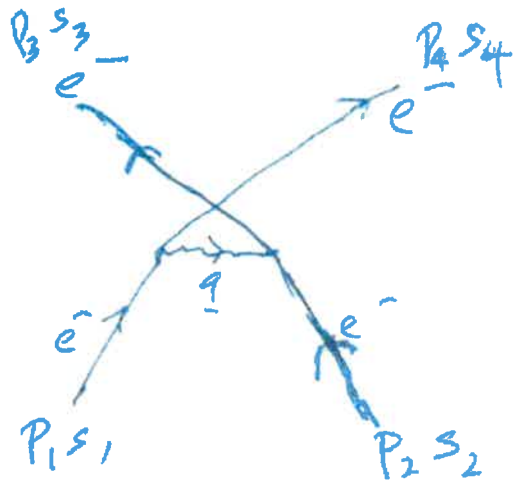
continue to get scattering amplitude of a physical process by using Feynman diagrams.

(ii) $e^- e^- \to e^- e^-$ Möller scattering





(i)



(ii)

2 diagrams $\longrightarrow$ 2 amplitudes $\mathcal{M}_{(i)}$, $\mathcal{M}_{(ii)}$

For diagram (i), it is like the $e^- \mu^- \to e^- \mu^-$, so we copy the result from previous example

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$$\mathcal{M}_{(i)} = -g^2\, \bar{u}(4)\gamma^\nu u(2) \frac{g_{\mu\nu}}{(\underline{p}_1-\underline{p}_3)^2} \bar{u}(3)\gamma^\mu u(1)$$

$(\underline{p}_1-\underline{p}_3)^2$ = momentum transfer square $\equiv \underline{q}^2 = t$

$$= \frac{-g^2}{q^2}\, \bar{u}(4)\gamma^\mu u(2)\cdot \bar{u}(3)\gamma_\mu u(1).$$

$\gamma_\mu \equiv g_{\mu\nu}\gamma^\nu$

$$\mathcal{M}_{(ii)} = \frac{-g^2}{q'^2}\, \bar{u}(3)\gamma^\mu u(2)\cdot \bar{u}(4)\gamma_\mu u(1)$$

(H.W)

$\mathcal{M}_{(ii)}$ can be obtained from $\mathcal{M}_{(i)}$ $\underline{q}'^2 = (\underline{p}_1-\underline{p}_4)^2$

by interchanging either the 2 outgoing particles or incoming

$$\mathcal{M} = \mathcal{M}_{(i)} - \mathcal{M}_{(ii)} \quad (\text{'$-$' not '$+$', see note below})$$

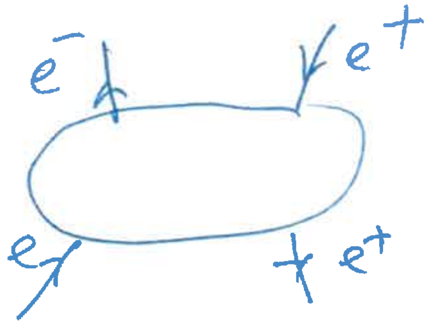
$$= -g^2\Big[\bar{u}(4)\gamma^\mu u(2)\frac{1}{q^2}\bar{u}(3)\gamma_\mu u(1)$$

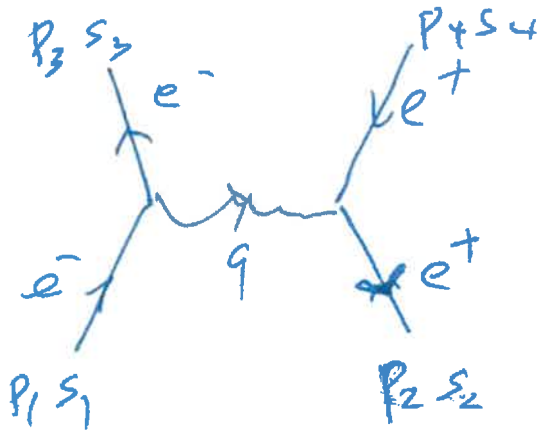
$$- \bar{u}(3)\gamma^\mu u(2)\cdot\frac{1}{q'^2}\bar{u}(4)\gamma_\mu u(1)\Big]$$

$\underline{q}^2 = (\underline{p}_1-\underline{p}_3)^2$, $\underline{q}'^2 = (\underline{p}_1-\underline{p}_4)^2$

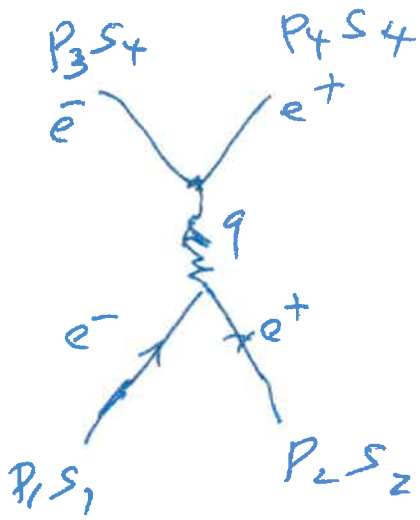
Note: Diagram (i) and (ii) are different, e.g the outgoing electron $(\underline{p}_4, s_4)$ can be from different interaction vertices. But (ii) and (i) can be obtained each other by exchange of identical particles (fermions), hence $(-)$ sign

(iii) $e^- e^+ \to e^- e^+$ Bhabha scatt.





(i)



(ii)

Compute $\mathcal{M}_{(i)}$

$$\bar{v}(2)\, i g \gamma^\nu\, v(4)\, \frac{-i g_{\mu\nu}}{q^2}\, (2\pi)^4 \delta^{(4)}(p_1 - q - p_3)\, \bar{u}(3)\, i g \gamma^\mu\, u(1)$$

$$\int \frac{d^4 q}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(q + p_2 - p_4) = ?$$ HW

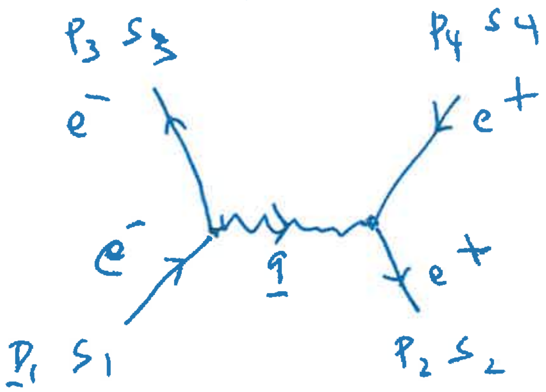
$$\mathcal{M}_{(i)} = \frac{-g^2}{(p_1 - p_3)^2} \left[\bar{u}(3) \gamma^\mu u(1)\right] \left[\bar{v}(2) \gamma_\mu v(4)\right]$$

HW

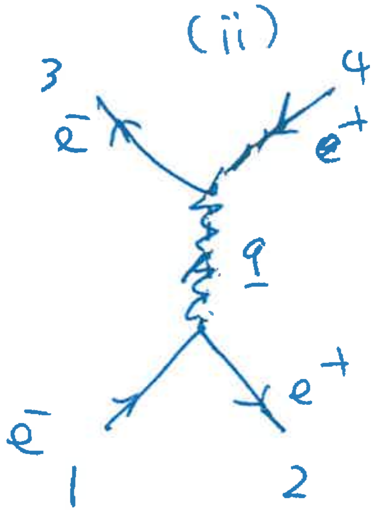
Time

Feynman diagram

(i)



(ii)



Find scatt. amp $\mathcal{M}(i)$

number

$$\int \frac{d^4\underline{q}}{(2\pi)^4}\left(\bar{v}(2)\, ig\gamma^\nu\, v(4)\right)\frac{-ig_{\mu\nu}}{\underline{q}^2}\left(\bar{u}(3)\, ig\gamma^\mu\, u(1)\right)$$

$$(2\pi)^4\,\delta^{(4)}(\underline{p}_1-\underline{p}_3-\underline{q})\cdot(2\pi)^4\,\delta^{(4)}(\underline{q}+\underline{p}_2-\underline{p}_4)$$

$$= i g^2 (2\pi)^4\,\delta^{(4)}(\underline{p}_1-\underline{p}_3-\underline{p}_4+\underline{p}_2)\cdot$$

$$\bar{v}(2)\,\gamma^\nu\, v(4)\,\frac{g_{\mu\nu}}{(\underline{p}_1-\underline{p}_3)^2}\,\bar{u}(3)\,\gamma^\mu\, u(1)$$

$$\mathcal{M}_{(i)} = -g^2\,\bar{v}(2)\,\gamma_\mu\, V(4)\cdot\frac{1}{(\underline{p}_1-\underline{p}_3)^2}\,\bar{u}(3)\,\gamma^\mu\, u(1)$$

$$\gamma_\mu \equiv g_{\mu\nu}\gamma^\nu$$

$$\gamma_0=\gamma^0,\quad \gamma_i=-\gamma^i,\quad i=1,2,3$$

$$\int \frac{d^4q}{(2\pi)^4} \; \bar{u}(3)\, ig\gamma^\nu\, V(4) \cdot \frac{-ig_{\mu\nu}}{q^2}\; \bar{V}(2)\, ig\gamma^\mu\, u(1)$$

$$(2\pi)^4\, \delta^{(4)}(\underline{p}_1 - \underline{q} + \underline{p}_2)\,(2\pi)^4\, \delta^{(4)}(\underline{q} - \underline{p}_3 - \underline{p}_4)$$

$$= (2\pi)^4\, \delta^{(4)}(\underline{p}_1 + \underline{p}_2 - \underline{p}_3 - \underline{p}_4)\; ig^2\, \bar{u}(3)\gamma^\nu V(4) \cdot \frac{g_{\mu\nu}}{\underbrace{q^2}_{\underline{p}_1 + \underline{p}_2}}$$

$$\bar{V}(2)\, \gamma^\mu\, u(1)$$

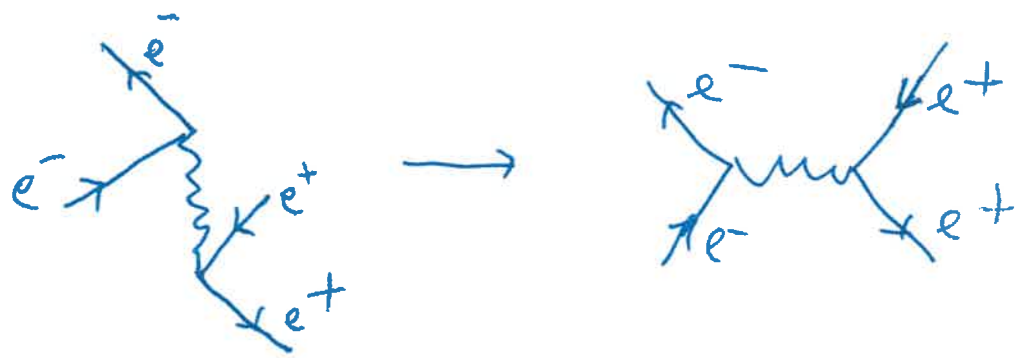
$$\rightarrow \quad \mathcal{M}_{(ii)} = -g^2\, \bar{u}(3)\, \gamma_\mu\, V(4) \cdot \frac{1}{(\underline{p}_1 + \underline{p}_2)^2} \cdot$$

$$\bar{V}(2)\gamma^\mu u(1)$$

should we add $\mathcal{M}_{(ii)}$ to $\mathcal{M}_{(i)}$ or should we subtract?

This depends on whether the two diagrams can be obtained from each other by (i) interchanging the two incoming identical particles, or (ii) interchanging the two outgoing identical particles, or (iii) interchanging an incoming e^- with an outgoing e^+ (antiparticle) or vice versa

In diagram (ii), interchange outgoing e^+ with incoming e^-



That means the first diagram can be obtained from the 2nd diagram by using <u>crossing symmetry</u>.

Can show 2nd diagram can be obtained from 1st diagram by crossing symmetry (HW)

So the total scatt. amp. is

$$\mathcal{M} = \mathcal{M}_{(i)} - \mathcal{M}_{(ii)}$$

Now do

(iv) $e^- \gamma \to e^- \gamma$