PC3261: Classical Mechanics II
Assignment 1

1. [20 pts] A point particle is moving in the xy-plane parameterized by φ as follows:

$$x(\varphi) = a\varphi\cos\varphi\,, \qquad y(\varphi) = a\varphi\sin\varphi\,,$$

where $a > 0$ and $\varphi \geq 0$. Assume that the particle moves along the trajectory above with $\varphi(t) = \alpha t$ where α is a constant.

(a) Calculate the Cartesian components of the velocity $\mathbf{v}$, acceleration $\mathbf{a}$, the radius of curvature ρ and the radius of torsion $\sigma \equiv 1/\tau$ as a function of time t.

(b) Calculate the speed v as functions of time t.

2. [20 pts] A particle is projected vertically upwards with speed u_0 and moves under uniform gravity in a medium that exerts a resistance force proportional to the square of its speed and in which the particle's terminal speed is V_∞.

(a) Find the maximum height above the starting point attained by the particle and the time taken to reach that height.

(b) Show also that the speed of the particle when it returns to its starting point is $\dfrac{u_0 V_\infty}{\sqrt{u_0^2 + V_\infty^2}}$.

3. [30 pts] An electron of mass m and charge $-e$ is moving under the combined influence of a uniform electric field $E_0\,\hat{\mathbf{e}}_y$ and a uniform magnetic field $B_0\,\hat{\mathbf{e}}_z$. Initially, the electron is at the origin and is moving with velocity $u_0\,\hat{\mathbf{e}}_x$. Find the trajectory, $x(t)$, $y(t)$, $z(t)$, of the electron in its subsequent motion.
Remark: The general path is called a trochoid which becomes a cycloid in the special case. Cycloidal motion of motion of electrons is used in the magnetron vacuum tube which generates the microwaves in a microwave oven.

4. [30 pts] A small block of mass m glides under its own weight $\mathbf{W} = -mg\,\hat{\mathbf{e}}_z$ *frictionless* downward along a helical track

$$\mathbf{r}(t) = a\cos\phi(t)\,\hat{\mathbf{e}}_x + a\sin\phi(t)\,\hat{\mathbf{e}}_y + b\phi(t)\,\hat{\mathbf{e}}_z\,,$$

where a and b are positive constants. The block starts its motion with $\phi(0) = \phi_0$ and $\dot{\phi}(0) = 0$.

(a) Derive a second order ordinary differential equation for $\phi(t)$ governing the dynamics of the block. Solve for $\phi(t)$ and calculate the magnitude of the velocity $v(t)$ of the block as a function of time.

(b) Calculate the magnitude of the force $F(t)$ exerted on the block by the helical track as a function of time.

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