Homework #3

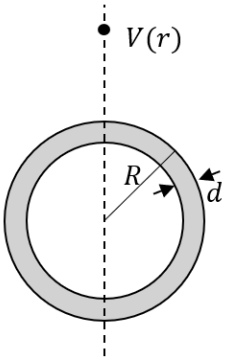
(Due on Canvas by Thu, Oct. 24)

1. Multipole expansion

Consider a spherical shell with thickness d and radius R (from center to outer surface) carrying a non-uniform volume charge density (in the spherical coordinate) as $\rho(r,\theta,\varphi) = \sigma_0 \cos\theta\,/d$, where σ_0 is a constant.

(1) Use the multipole expansion to find the monopole, dipole, and quadrupole contributions to the electrostatic potential $V(r)$. You can assume the point of interest is outside the sphere and located on the z-axis.

(2) Try to justify your findings in (1) based on a physics picture rather than on mathematical grounds. (Hint: consider the $d \to 0$ limit and try connecting this case to a textbook example we worked out in class.)



2. Separation of variables in the spherical coordinate

An uncharged conducting sphere of radius a is coated with a thick insulating shell (with dielectric constant ε_r) out to radius b. This object is now placed in an otherwise uniform electric field $E_0\hat{z}$.

(1) Find the electrostatic potential everywhere in space. (Hint: you can refer to the technique adopted in textbook Example 4.7.)

(2) Calculate the surface charge density at the metal-insulator interface and at the insulator-vacuum interface.

3. Bound charges

Consider a dielectric sphere (with susceptibility χ_e) with radius R, and we manually embed a point charge q_f at its center.

(1) Calculate the electric field inside and outside the sphere.

(2) Calculate the polarization in the sphere.

(3) Calculate the volume bound charge density and the surface bound charge density of the dielectric.

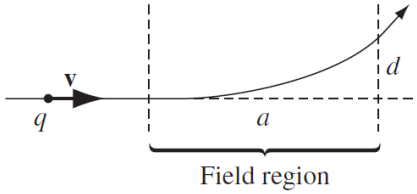
4. Charged particle in a magnetic field

A particle of charge q enters a region of uniform magnetic field **B** pointing into the page with width a. The field deflects the particle a distance d above the original line of flight, as shown below.

(1) Is the charge positive or negative?

(2) Calculate the momentum of the particle.

(3) Suppose the charged particle is a proton with energy $E = 5$ MeV, and $B = 1.5$ Tesla, calculate the Lorentz force experienced by the proton.



5. Steady current

There are two layers of concentric conducting shells (with radius a and b), and we fill the region in between with a media with conductivity σ. If we apply a voltage V across the two conductors, what would be the effective resistance of this geometry? (Effective resistance defined as $R = V/I$, where I is the total current)