Chapter 5

Through the Looking Glass

> O, telescope, instrument of much knowledge, more precious than any sceptre, is not he who holds thee in his hand made king and lord of the works of God? – Johannes Kepler

Galileo was not the inventor of the telescope. But he was the first to use it to make scientific discoveries! Almost everything we know about the Universe comes from this family of devices. The quest to see better, see more and see further never ends. The James Webb Telescope, successor to the Hubble Space Telescope, was launched last Christmas (25 Dec 2021).

This chapter has been prepared with Koh Zhi Yong and Gerald Kang Joon Kiat.

Learning Objectives

At the end of this lesson, you will understand how distance and other physical quantities are measured in astronomy. You will also appreciate how the study of certain celestial objects help us to understand our Universe as a whole.

Contents

5.1 Pre-Lesson Homework

1. Visit NASA's Astronomy Picture of the Day. Look at 10 random pages. Select one that you really like. Why do you like the selected picture?

2. Read the following article: Miles, R., "A light history of photometry: from Hipparchus to Hubble Space Telescope", Journal of the British Astronomical Association, vol. 117, pp. 172–186, 2007.

5.2 Star Light, Star Bright

5.2.1 Luminosity-Distance Relationship

Inverse square law

You have probably noticed that far-away lights seem dimmer than those closer to you. This is because the light intensity decreases as the square of the distance between you and the light. The energy emitted by a light source per unit time, or power, is determined by the light source itself. Assuming the light is emitted isotropically[1], as the light waves travel away from the source, they are spread out over larger surface areas. As such, the light energy passing through a location per unit time per unit area, decreases with $1/d^2$. Since your eye, the light detector, is the same size regardless of how far you are from the light source, the light-capturing area is the same. But due to the reduced irradiance further away, the total energy per unit time caught by your eye is reduced, leading to the perception of a dimmer light.

A similar phenomenon is at play when we observe stars in the night sky. Stars are basically spherical light sources far away from us that emit light isotropically.

Go to Activity 1. Power of the Sun

Luminosity and Flux

The intrinsic luminosity L of a star quantifies how much light comes out of a source. It is measured in energy emitted per unit time.

The radiation flux F quantifies how much light is received by a detector. It measures the light energy per unit time per unit area.

The two quantities above are related by

$$F = \frac{L}{4\pi d^2}$$

where d is the distance from the source to the detector.

Strictly speaking, the flux from a star at any point can be defined to be dependent on the distance between the star and that point. However, for the purposes of observational astronomy, we are content to constrain our definition such that the point of observation is Earth.

Magnitude of Brightness

In observational astronomy, we measure the flux from the star as it reaches our detectors. The flux is quantified in a logarithmic scale called magnitude. This scale eases the process of comparison between different stars. The relative brightness between two stars, as observed on Earth, is quantified by **apparent magnitude** m, defined as follows

$$\frac{F_2}{F_1} = 100^{(m_1 - m_2)/5}$$

where F_1, F_2, are the radiant flux of star 1 and star 2 respectively, and m_1, m_2 are the apparent magnitude of star 1 and star 2 respectively. Note that by this definition, a brighter star will have a lower apparent magnitude, and that it is possible to have a negative magnitude. Also note that this definition merely establishes a way to compare relative brightness, but a zero point is not explicitly set by this equation.By convention, a star called Vega is taken as the zero point of the apparent magnitude scale. From there, the absolute magnitude of Vega is calculated (0.57) which indirectly defines the zero point of the absolute magnitude scale.

The apparent magnitude is a measure of the flux from a star as measured from Earth. Consequently, its measured value depends on the star's distance away from us. This additional confounding factor can be troublesome when we wish to discuss the properties of the star itself, and not how the star appears to us. Thus, we further define the

[1] evenly in all directions

Star	Apparent (Visual) Magnitude	Absolute Magnitude	Distance (pc)
Sun	-26.74		4.85×10^{-6}
Sirius	-1.46		2.64
Canopus	-0.74		95.0
α Centauri	-0.27		1.35
Arcturus	-0.05		11.3
Vega	0.03		7.67
Capella	0.08		13.2
Rigel	0.13		264
Procyon	0.34		3.37
Achernar	0.46		42.6
Betelgeuse	0.5		215
Hadar	0.61		120
Altair	0.76		5.21
Acrux	0.76		98.1
Aldebaran	0.85		19.9
Antares	0.96		169

Table 5.1: Table of magnitude of brightness of notable celestial objects. Fill in the Absolute Magnitude column and check your values.

absolute magnitude M as what the apparent magnitude would be, if the star were exactly 10 pc away from us.

$$\begin{aligned} 100^{(m-M)/5} &= \left(\frac{d}{10 \text{ pc}}\right)^2 \\ 10^{(m-M)/5} &= \frac{d}{10 \text{ pc}} \\ \frac{m-M}{5} &= \log_{10}\left(\frac{d}{10 \text{ pc}}\right) \\ m &= M + 5\log_{10}\left(\frac{d}{10 \text{ pc}}\right) \end{aligned} \tag{5.1}$$

where d is the distance between the star and us, measured in parsecs (pc).

So far, we have discussed the luminous properties of stars in general. Practically, we must also consider that light can have many different wavelengths. No star emits monochromatic light, nor do any emit light of uniform intensity across all wavelengths of the electromagnetic spectrum. Our detectors, however often operate in narrow bands of wavelengths. To account for the fact that stars emit light across a wide range of wavelengths, we define the apparent bolometric magnitude and absolute bolometric magnitude by applying a bolometric correction to the measured apparent magnitude and absolute magnitude respectively.

$$\begin{aligned} m_{bol} &= m_V + BC \\ M_{bol} &= M_V + BC \end{aligned}$$

where the subscript V stands for "visual" and *bol* stands for "bolometric".

The correction serves to account for light emitted outside of visible wavelengths. The more a star emits in, for example, ultraviolet or infrared wavelengths, the more negative the correction is (i.e. the bolometric magnitude is less than the apparent magnitude, indicating the star is "brighter" than it appears to us).

5.3 Cosmic distance ladder

The universe is a very large place. We have one star, the Sun, in our solar system. Our solar system is part of our galaxy, the Milky Way. Our galaxy is a part of yet even larger groups of galaxies. To get a sense of the scale, watch 3:49 to 7:30 of the following video: https://www.youtube.com/watch?v=Iy7NzjCmUf0&t=229s

Suffice to say, galaxies contain many stars, and the universe contains many galaxies. All of these entities are extremely far apart from each other by human standards. As a result, conducting distance measurements is extremely challenging. Here, we look at methods used to measure distances in cosmology.

5.3.1 The Astronomical Unit (A.U.)

Recall Kepler's third law which we have learn in the first chapter. It provides us with a simple formula to determine the relative size of the planetary orbits of different planets. By knowing the distance from Earth to our Sun as well as the corresponding periods of the planets by observation, one can in principle deduce the absolute scale of our solar system.

As promising as it may sound, the distance from Earth to our Sun was not known during the time of Kepler. The Astronomical Unit (A.U.) is defined as the average distance between Earth and our Sun. It was first determined during the transit of Venus in 1761 with the help of a method proposed by the English astronomer Edmund Halley (although he never lived to see the remarkable phenomenon himself). Below we shall detail the simplified version of the method.

The general scheme of the method involves the use of parallax and requires two observers at different locations of Earth to record the transit independently before comparing their results.

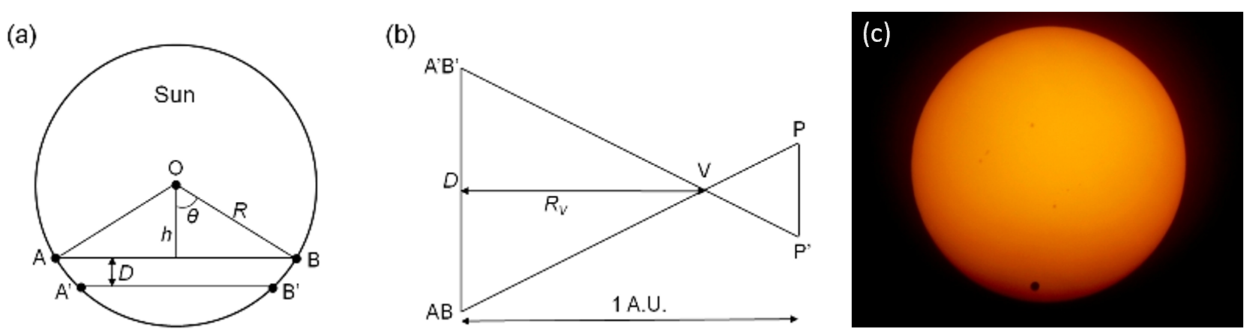

Figure 5.1: (a) Schematic of the transit of Venus as viewed from two different locations on Earth. (b) Trigonometric parallax applied to determine the astronomical unit. (c) Venus in transit on 6 June 2012, 12:08pm. Venus appeared as a small silhouette disc moving across the Sun. This picture was taken with a digital camera attached to a 104mm Newtonian telescope at the NUS multi-purpose field.

The figure above shows the schematic of the transit of Venus as observed from two different locations on Earth. Details of how the astronomical unit is derived from trasit-of-Venus observations can be found in the Appendix.

5.3.2 Stellar Parallax

The distance between us and nearby stars can be found by the stellar parallax method as sketched in the figure below. The star of interest is observed twice, with the second observation made six months after the first. The (different) positions of the star with respect to the fixed stars in the background were recorded. The reciprocal of the parallax angle (half the change in angular position) gives the distance of the star from Earth.

$$d = \frac{1 \text{ A.U.}}{\tan p \text{ (radians)}} \approx \frac{1 \text{ A.U.}}{p \text{ (radians)}} = \frac{1}{p'\text{''} \text{ (arc-seconds)}} \text{pc}$$

1 radian $= 206264.8''$ (arcseconds)

1 pc $= 2.062648 \times 10^5$ A.U. $= 3.08568 \times 10^{16}$ m

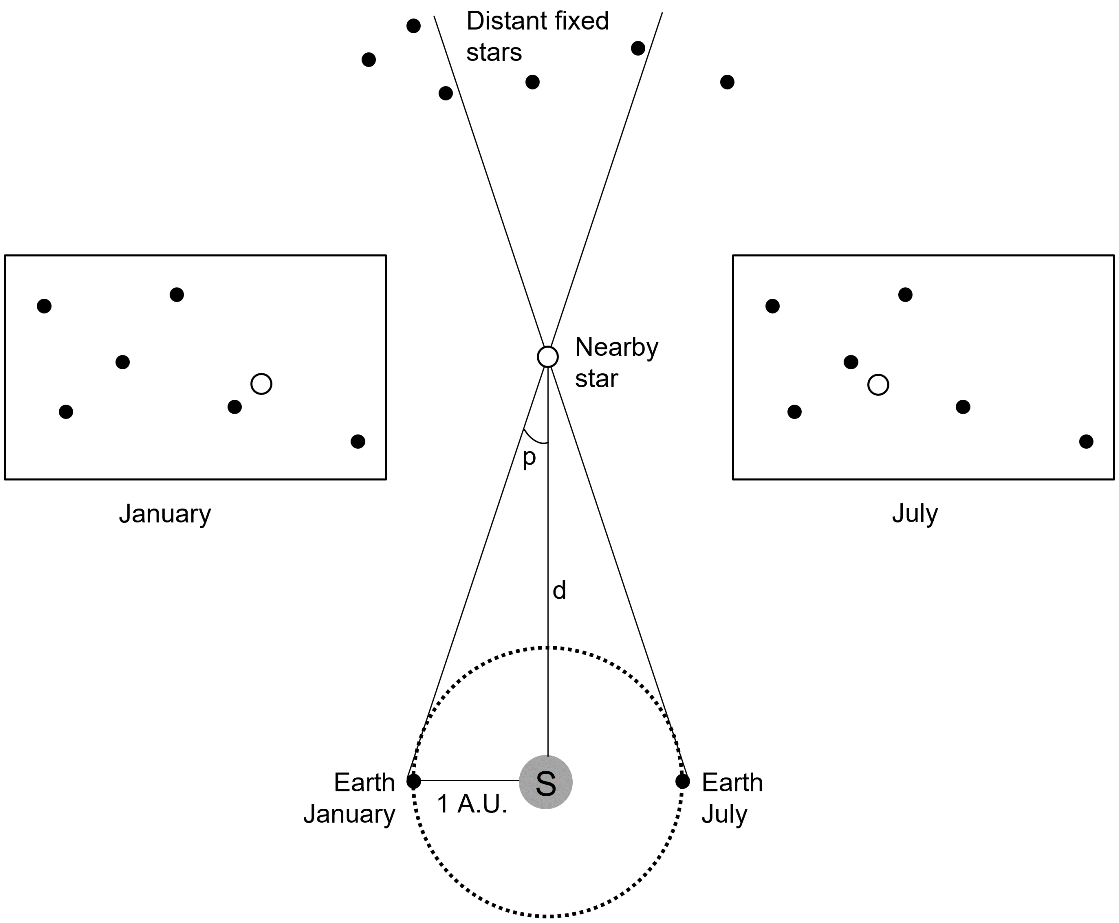

Figure 5.2: Stellar parallax. A nearby star will appear to be in different positions at different times of the year. Accurate observational data recording and simple geometrical calculations allow us to know the distance of the nearby star.

5.3.3 Cepheid Variables

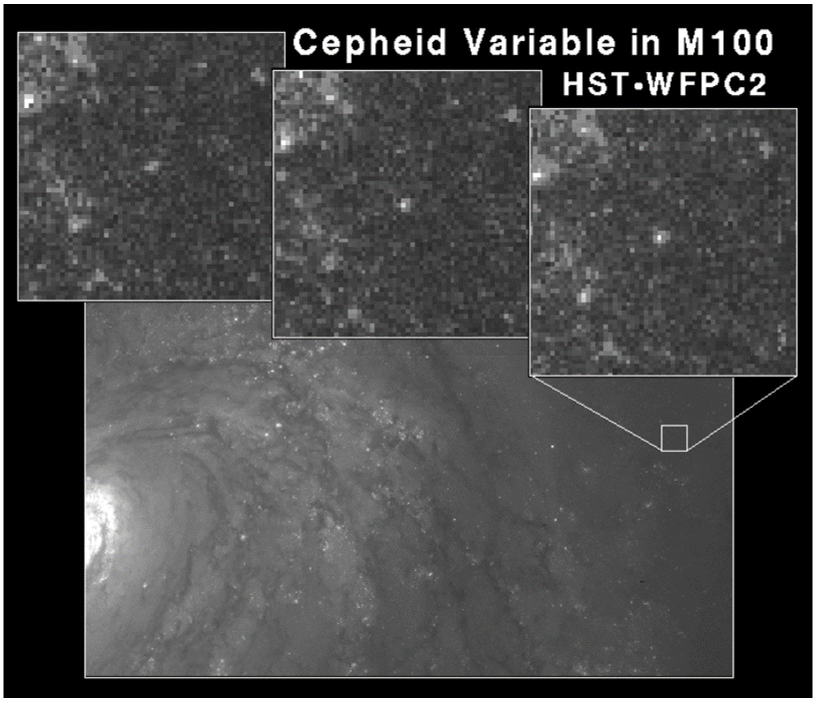

Figure 5.3: A Cepheid variable in M100 observed with the Hubble Space telescope. Image credit: Credit: NASA, HST, W. Freedman, R. Kennicutt, J. Mould

An independent method of distance measurement is required before we can assign absolute magnitudes to the stars we see in the sky. One such method relies on observing a type of star called a Cepheid variable. Cepheid variables are stars that contract and expand in size periodically, brightening and dimming periodically in the process. The first discovery of such an object was *o Ceti* in 1595. The brightness of this star varied over five orders of magnitudes with irregular periods of between 100 to 200 days. In 1784, a pulsating star with a regular period of 5 days and 9 hours was discovered. Other similar pulsating stars were subsequently found and were called Cepheid variables. Cepheids pulsate because the competing forces of gravity (inwards) and pressure (outwards) are far from equilibrium, similar to a wildly swinging pendulum.

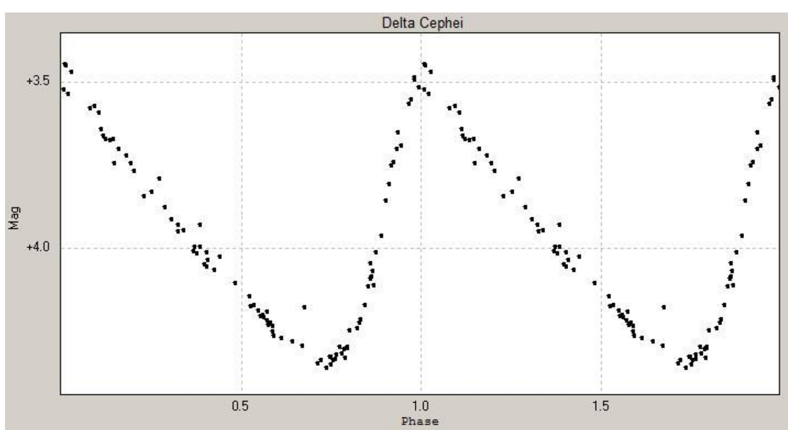

Figure 5.4: Variation of brightness of a typical Cepheid.

Several discoveries related to the properties of Cepheid variables made it possible to use them as independent measures of distance. The first discovery was a empirical relation between the pulsation period and the perceived brightness of the stars. Henrietta Swan Leavitt (1868 - 1921) found more than 2000 Cepheid variables, most of them located in the dwarf galaxy called the Small Magellanic Cloud (SMC). In 1912, she used a sample of 25 Cepheids in the SMC to show that the apparent magnitude of brightness varies linearly with the logarithm of the period of pulsation.

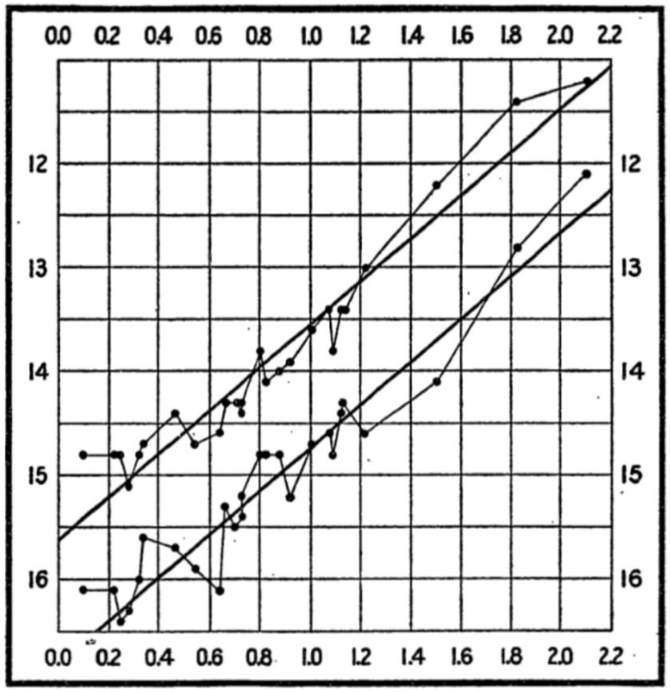

Figure 5.5: Plot of apparent (visual) magnitude of brightness (vertical axis) versus the logarithm of the period of pulsation. from a sample of 25 Cepheids in the SMC. The upper and lower graphs correspond to the maximum and minimum magnitudes respectively. Image credit: H. S Leavitt

Since the SMC has a diameter of about 7000 light years and its distance from us is much greater at about 200,000 light years, the stars in the SMC can be considered to be approximately the same distance from us. Hence, the **linear relation between absolute magnitudes and the logarithm of their pulsation periods of Cepheids** also holds.

The second discovery provided a zero point to calibrate Leavitt's linear relation. This was first carried out in 1913 by Ejnar Hertzsprung (1873 - 1967), who measured the absolute distances of a few nearby Cepheid variables via statistical parallax, a modified stellar parallax technique that used the motion of the sun as a longer baseline together with measurements of average velocities of stars. Comparing the absolute distance with the apparent magnitude of a Cepheid variable, it was thus possible to determine its absolute magnitude. The absolute magnitude could then be correlated with its pulsation period, giving a reference point to use in conjunction with Leavitt's linear relation. A recent relationship found is

$$M_V = -2.76 \log_{10} P \; - 1.40$$

It was thus possible to obtain the absolute magnitud by measuring the pulsation period of a Cepheid variable. The apparent magnitudes of the Cepheids are measured observationally as well. With the apparent and absolute magnitudes known, we can then determine the distance to that Cepheid variable using Eq.(5.1). In this way, Cepheid variables are used as standard candles to measure extragalactic distances up to 26 Mpc.

For further distances, extremely bright objects known as supernovae are used as standard candles to measure distances up to 2200 Mpc. Further discussion on supernovae will be found in a later chapter.

5.4 Astro-Spectroscopy

Let us now discuss one of the most valuable tools in Science applied to the study of the Universe. This section introduces how astronomers determine the chemical compositions and physical properties of celestial objects. We will also look at how velocities of stars and galaxies in the radial direction are deduced by examining their spectra.

5.4.1 The Blackbody Spectrum

Have you seen a piece of material start to glow as it is heated to higher temperatures? Consider an iron rod being heated up. At first you will feel the heat radiating out of the iron. We are feeling the infra-red light that the iron emits. The iron will begin to glow in dull red at temperatures above 500°C. At even higher temperature, it changes to bright red to orange, then yellow and progressively white above 1300°C. From such observations, it seems that light is somewhat associated with temperatures. Indeed it does! In fact, light that are emitted from the iron are not limited to just visible or infrared but covers the entire electromagnetic spectrum.

Consider a hypothetical body that absorbs all incident electromagnetic radiation and reflects none. We termed this perfect absorber as a blackbody, since it will appear black when it does not reflect visible light. While such a body do not reflect light, it is capable of emitting (blackbody) radiation in all directions. The body emits electromagnetic energy as a result of the thermal agitation of electrons in its surface. When the body is in thermal equilibrium, it emits as much energy as it absorbs, thus making a blackbody both a perfect absorber and emitter.

The intensity of the electromagnetic energy emitted by a black body depends on the wavelengths (frequencies) and the body's thermal equilibrium temperature; the radiation it emits covers the entire range of the electromagnetic spectrum. The electromagnetic energy per unit volume per unit wavelength emitted by a black body at thermal equilibrium temperature T is given by

$$u(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{k_B T}\right) - 1}$$

where h is the Planck's constant, k_B is the Boltzmann's constant and c is the speed of light in vacuum.

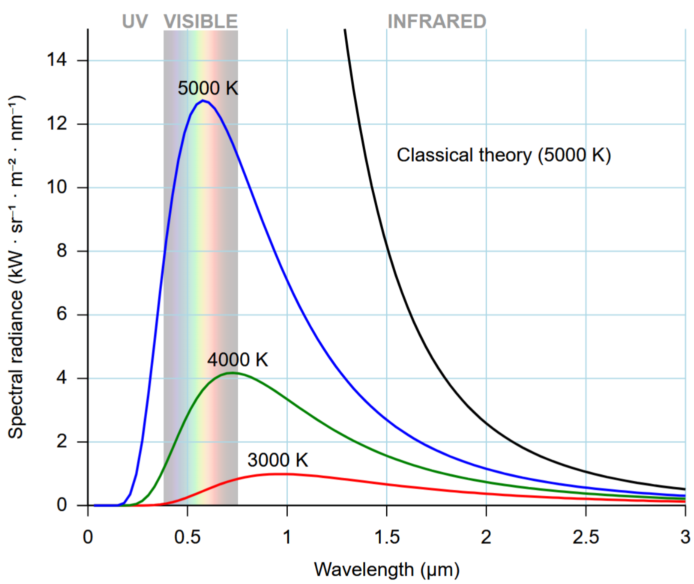

Figure 5.6: Blackbody spectra at various temperatures. Image credit: Darth Kule

The figure above shows the radiation emitted at thermal equilibrium by a blackbody at various temperatures. It has a well-defined continuous energy distribution over the entire range of wavelengths of the electromagnetic spectrum. For any specific wavelength, there is a corresponding energy density that depends neither on the chemical composition of the body nor its shape, but only on its surface temperature.

Blackbody spectrum simulation: https://phet.colorado.edu/en/simulations/blackbody-spectrum

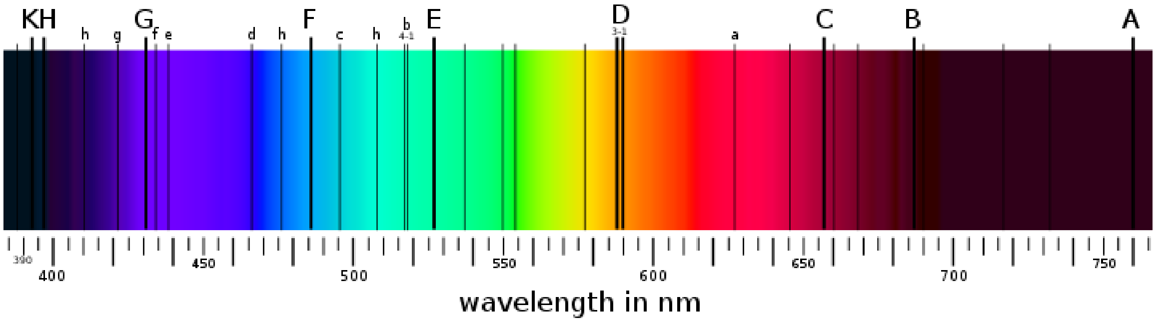

Figure 5.7: Notable Fraunhofer absorption lines in the solar spectrum. Image credit: Wikimedia commons

At a certain temperature T, the energy distribution peaks at a particular wavelength, which can be approximated from the Wien's Displacement Law

$$\lambda_{\text{peak}} = \frac{2.898 \times 10^{-3}\text{mK}}{T}.$$

Real materials emit electromagnetic energy at only a fraction of a blackbody. Like any materials, gases when heated and reached thermal equilibrium also emits electromagnetic radiation. Stars, which are huge dense masses of gases at their core, are therefore no exception. In fact, the interior of stars are good approximations of blackbodies because their hot gases are very opaque, that is, stellar material is nearly a perfect absorber of radiation. At thermal equilibrium, the interior of a star emits electromagnetic energy that has a continuous spectral energy density nearly similar to that of a black body. However, when the emitted radiation of a star's interior passes through the star's atmosphere, the relatively cooler less dense gases absorb parts of the wavelengths that are emitted. By studying these wavelengths, we can determine the chemical composition of the stellar atmosphere. Let us take a look at our closest example: The Sun.

5.4.2 Stellar Classification

The first astro-spectroscopy was carried out more than two hundred years ago by William Hyde Wollaston (1766 – 1828), who noted dark lines in the spectrum of sunlight after passing through a prism. About a decade later, Joseph von Fraunhofer (1787 – 1826) repeated the experiment using a diffraction grating, and recorded about 600 discrete dark lines. Atomic physics at that time was not yet developed, so the nature of the lines remained a mystery.

Modern technologies have allowed us to collect the solar spectrum above Earth's atmosphere (which also causes absorption of wavelengths). Even so, some of the notable discrete dark lines still persist in the solar spectrum. This indicates that these lines are caused by the Sun's atmosphere. The Sun's interior emits a continuous blackbody spectrum of radiation. As the light passes through its cooler outer atmosphere, the atoms and molecules absorb photons at specific wavelengths to undergo electronic transitions. The absorbed wavelengths coincide with the emission spectrum of some heated elements, providing strong evidence that these elements are present in the solar atmosphere of our Sun.

The Fraunhofer absorption lines provide an insight into the chemical composition of our Sun's atmosphere. Extending to other stars, the absorption lines in a stellar spectrum serves as a fingerprint identification of elements present in the stellar atmosphere.

It was found that all stars have roughly the same chemical composition (therefore similar discrete dark lines in their stellar spectrum). However, the relative strength of the absorption lines varies between stars. Stars are therefore classified based on the relative strength of their absorption lines and the line strength mostly traces out the surface temperature of the star.

To see why, consider two extreme cases where the surface temperature of a star is too low or too high. A star with a low surface temperature will have few electrons in the $n = 2$ energy level. As a result the cool star will exhibit weak Balmer lines. On the other hand, a very hot star will mean that many atoms are fully ionised. With few bound-state electrons available, the absorption lines will be weak too. In the intermediary of the two extreme cases, the intensity of the spectral lines is determined by the ratio of the population of electrons in each state.

Therefore, by carefully analysing the intensity of the absorption lines in a stellar spectra, one can in principle determine the surface temperature of the star or at least narrow down its temperature range. The surface temperature of stars are commonly compared against other stellar properties, such as luminosity.

Most stars are generally classified under the Morgan Keenan (MK) system using the letters O, B, A, F, G, K, and M, with O type being the hottest to M type being the coolest. Each spectral type is also further subdivided using a numeric digit with 0 being the hottest and 9 being the coolest.

Spectral type	Characteristics	Temperature
M	These red stars are cool enough for simple molecules form. Spectrum involves lines from CN (cyanogen), TiO (titanium oxide), ZrO (zirconium oxide) and some neutral metals.	<3500 K
K	Spectral lines show mainly that of neutral and ionised metals.	3500 – 5000 K
G	Strong lines from ionized metals such as Ca^{2+}. Hydrogen lines are also present. This is the spectral type of our Sun!	5000 – 6000 K
F	More hydrogen lines than G.	6000 – 7500 K
A	White-blue in colour. Strongest hydrogen lines.	7500 – 10 000 K
B	Weak hydrogen lines, helium lines appear.	10 000 – 30 000 K
O	Blue stars. No hydrogen lines, mainly He^{2+}lines.	30 000 – 60 000 K

Table 5.2: Spectral classification of stars.

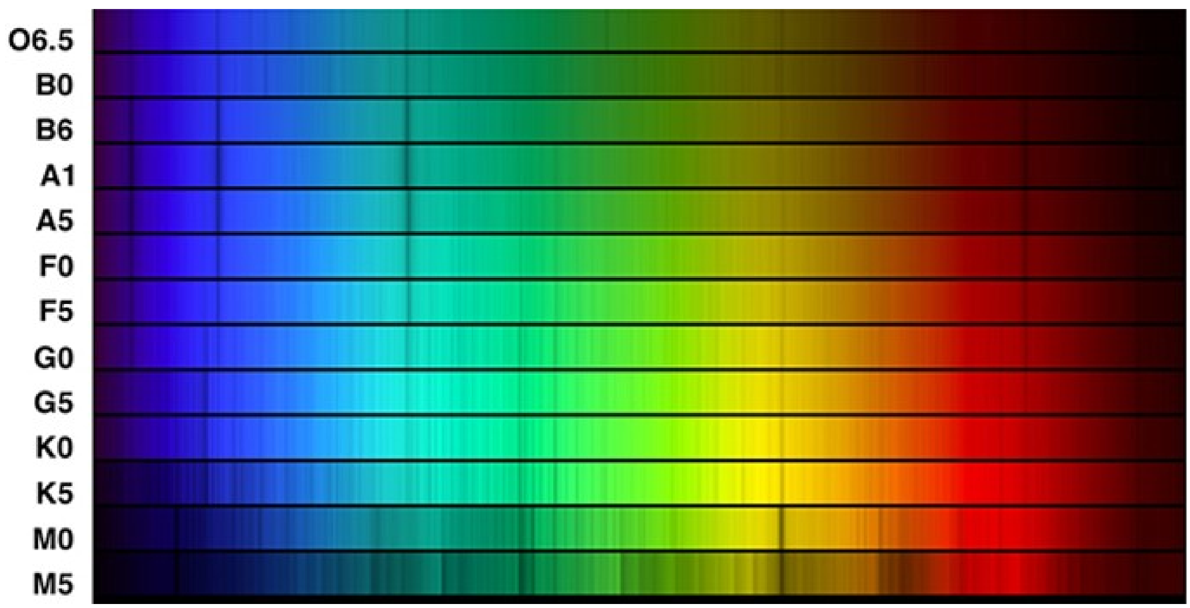

Figure 5.8: Stellar spectra of various spectral type stars. Image credit: KPNO 0.9-m Telescope, AURA, NOAO, NSF

5.4.3 Doppler effect

Go to Activity 2: Doppler rocket

The Doppler effect is commonly associated with astro-spectroscopy. Let's start off the discussion with an interesting phenomenon you have probably encountered. The pitch of an ambulance siren increases as it moves towards us while it decreases as it moves away from us. Such a phenomenon was first described by Christian Doppler and was eventually termed as the Doppler effect. When a source of sound and a listener is in relative motion relative to each other, the frequency of the sound heard is not the same as the source frequency.

To see why, let's first consider an observer and a source of sound. The source emits a sound wave with frequency, f ,and wavelength $\lambda = c/f$, where c is the speed of sound in air. To make things easier to visualise, consider the observer's motion in the horizontal direction.

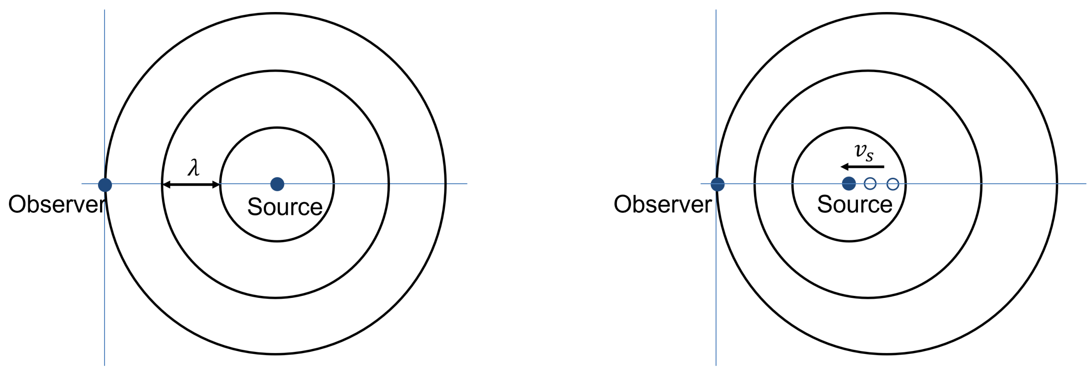

Figure 5.9: Left: An observer and a stationary source of sound. The wavefronts reaches the observer at intervals of $1/f = \lambda/c$. Right: The source moves towards the source. The wavefronts reaches the obsever at highere frequencies, and thus shorter wavelengths.

Now suppose the source moves, say at a constant velocity of v_s towards the observer. The frequency and wavelength of the sound wave as perceived by the observer is now different. In one period $T = 1/f$, the source moves by a distance of $v_s T = v_s/f$. The wavefronts towards to observer are compressed by this amount. The observer thus perceives a shorter wavelength, and experiences a higher frequency of recieving the wavefronts. Let λ' and f' be the wavelength and frequency measured by the observer. It is conventional in astronomy to take velocities as "recession velocities". This means sources that move away from us take on positive velocities. Here since the source moves towards us, $v_s < 0$. Hence the compressed wavelength is

$$\begin{aligned}
\lambda' &= \lambda + \frac{v_s}{f} \\
&= \lambda + v_s \frac{\lambda}{c} \\
\lambda' - \lambda &= \frac{v_s}{c}\lambda \\
\frac{\Delta\lambda}{\lambda} &= \frac{v_s}{c} \qquad (5.2)
\end{aligned}$$

where $\Delta\lambda$ is the shift in wavelength. Here the source is approaching the observer, so $v_s < 0$ and $\Delta\lambda$ is negative, meaning that the wavelengths received are shorter than the originally emitted. If the source is receeding away the observer, $v_s > 0, \Delta\lambda > 0$, the wavelengths received will be longer. The frequency will shift in a similar way:

$$\frac{\Delta f}{f} = -\frac{v_s}{c} \qquad (5.3)$$

The shifts in wavelengths and frequencies are called the Doppler shifts.

The Doppler effect of sound waves require the speed of sound wave (c), velocities of the source with respect to the observer (v_s) to be measured with respect to whatever medium they are travelling within. Electromagnetic waves, such as light, require no medium to propagate. Nonetheless, there is still a Doppler relation for electromagnetic waves (e.g. light) just by having knowledge of the relative velocity between the observer and the source. Here, we quote the result and shall not derive the relation:

$$f' = \sqrt{\frac{c - v_s}{c + v_s}}f$$

where c here is the speed of light. In the low (source) velocity limit where $v_s \ll c$, the above equation reduces to Eq.(5.3). In practical astronomy, this is almost always the case and hence the simpler formulae Eqs.(5.2) and (5.3) are usually used.

Generally all celestial objects are in relative motion with us, hence **all astronomical spectral lines are Doppler shifted**.

Astronomers define the redshift of a spectral line by

$$z = \frac{\Delta\lambda}{\lambda} \tag{5.4}$$

This term gives a intuitive picture of shifted spectral lines shfted towards higher wavelengths due to recession velocities. Together with Eq.(5.2) we see that the higher the (recession) velocity, the greater the redshift.

5.4.4 Binary stars and Exoplanets

The use of astro-spectroscopy has led to many interesting astronomical discoveries. Here, we briefly discuss how spectroscopy is used to detect binary stars and exoplanets.

A binary star system is a system of two stars that are gravitationally bound and orbit around each other about a common center of mass. While some binary star systems may be in principle resolvable by using a telescope, often the glare from the brighter star makes it difficult to observe the fainter star. In such situations, astro-spectroscopy is a useful tool in helping to detect binary star systems through the analysis of the Doppler effect on their stellar spectra. The absorption lines in the stellar spectra of each star are first blueshifted and then redshifted, as each star moves towards us and then away from us about their common center of mass, with the period of their common orbit. In some spectroscopic binaries, the spectral lines of both stars are visible and lines alternate between single and double. In other systems, only the spectrum of one star is observed and the lines periodically blueshift and redshift. By carefully analysing the Doppler shift of the spectral lines, it is possible to determine the radial velocity of the system and even the shape of the binary orbit.

Astro-spectroscopy also offers a useful indirect method of detecting exoplanets. If the exoplanet is gravitationally bound to a host star, both of them will orbit about a common center of mass. Again, we search for the Doppler shift (periodic blueshift and redshift) in the stellar spectrum of the host star as the star moves towards and away from us. However, compared to binary star systems, the spectral shift brought about by an exoplanet can be much less prominent and harder to detect if the planet is much smaller in size and orbits far from its host star, causing the radial velocity of the star-exoplanet system to be relatively much smaller compared to a binary star system. Therefore, this method is best used for detecting massive exoplanets orbiting close to their host stars, since this will allow for a much larger radial velocity of the system and thus a more observable Doppler shift in the spectral lines of the stellar spectra. The discovery of the first exoplanet was made in 1995 by Swiss astronomers Michael Mayor and Didier Queloz using astro-spectroscopy, who were both subsequently awarded the 2019 Nobel Prize in Physics. Since then, there have been a few different methods aside from spectroscopy used in detecting exoplanets and the number of verified exoplanets has surpassed 200.

5.4.5 Redshift of Galaxies

In 1912, Vesto Slipher examined the spectrum of our neighbour galaxy, Andromeda and reported a shift of the spectral lines. He argued that this (blue)shift of Andromeda's spectrum is most likely due to Andromeda moving towards us with a radial velocity of 300 km/s. He went on to analyse 15 other galaxies and found that some are approaching us and receding from us. Edwin Hubble and Milton Humason extended Slipher's work in the 1920-30s and found that most galaxies are moving aways from us. In addition to finding the redshifts and radial velocities, they used Cepheids in the galaxies they examine to determine the distance of these galaxies.

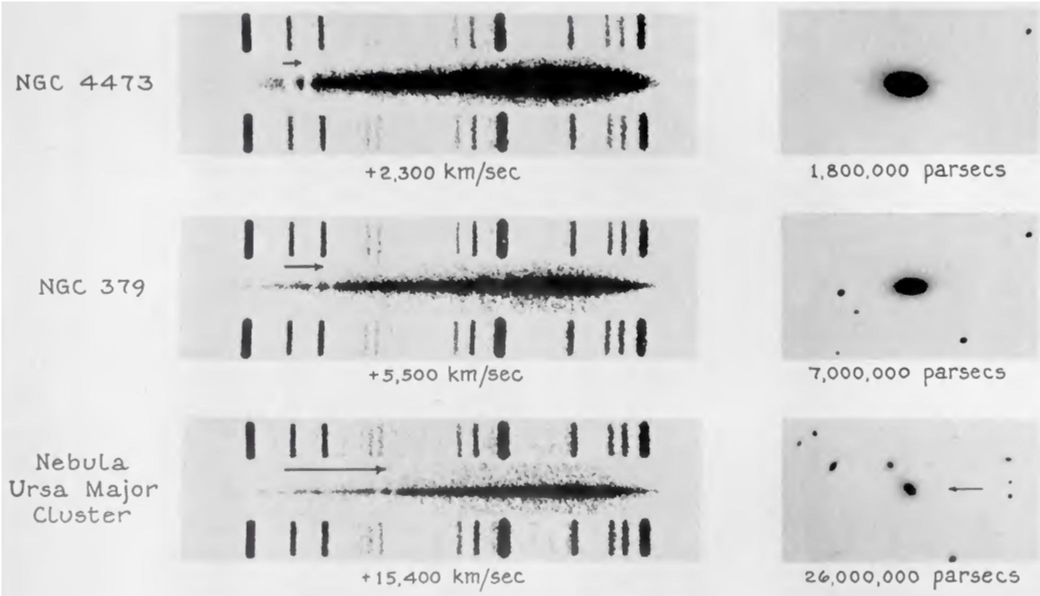

Figure 5.10: Photographs of some galaxies and their spectra captured by Milton Humason using the 100 inch Mount Wilson Telescope. As shown the fainter the galaxy (thus further), the more red-shifted its spectrum is.

In 1929, Hubble published a result that changed the view of the Universe. He found that with the exception of a few nearby galaxies whose spectra are blueshifted and thus movng towards us, all other galaxies have redshifted spectra and move away from us. Even mose surprisingly, **the further away the galaxy, the faster the receeding velocity.** The proportional relationship between distance and recession velocity of a galaxy, now known as the Hubble's law

$$v = H_0 d. \tag{5.5}$$

What kind of force is pulling the Universe apart? Gravity is an attractive force, it will not do this. Electric forces cannot be at play as that will require the galaxies to be highly charged. Nuclear forces do not work in long range. What could be the cause of this phenomenon?

Even now we do not truly know the "cause" of the phenomenon. The best "explanation" is to *assume that the Universe is expanding*, and model the expansion based on our existing theories. What is the meaning of an expanding Universe? We will dig into the this in the second half of the course.

5.5 In-class Activities

Activity 1: Power of the Sun and the Inverse Square Law

In this activity, you will estimate the power of the Sun by using your feelings and performing a simple calculation.
First recall the feeling of standing under a hot sun. If it is bright and sunny now, go out and experience it for a while!
Position yourself close to a 100W incandescent light bulb. Cover your eyes with a blindfold or solar filter spectacles.
Ask one of your group-mate to switch on the bulb and move it towards you slowly.
Signal “stop” when the heat from the bulb feels like the heat from the hot sun.
Ask another group member to measure and record the distance from the bulb to you.
Repeat the experiment.

Student	Distance (m)
#1	
#2	
#3	

The power of the radiation from Sun and the light bulb is related to the intensity you feel by

$$\text{Intensity} = \frac{\text{Power}}{4\pi r^2}$$

where r is the distance between the light source and the detector (you!).
Discuss the above equation in the geometrical sense.
How far is the Sun?
Estimate the power of the Sun with your data.

Activity 2: Doppler Rocket

1. Play with the Doppler rocket toy. (Careful, don’t hurt anyone!!)

2. Understand qualitatively the phenomenon. Sketching some wave forms will help!

3. What are some applications of the Doppler effect

4. Explain the comic below

EDWIN HUBBLE'S CAR

5.6 Discussion Questions

1. Bats use echolation to determine distance. Can you think of a similar way to measure astronomical distances? Can we use this to find method the Astronomical Unit?

2. Read about Mira "the wonderful" star in

https://astrobob.areavoices.com/2019/10/08/meet-mira-a-wonderful-and-astonishing-star/and

https://www.skyandtelescope.com/astronomy-news/miras-marvelous-tail/

and answer the following questions.

(a) Assuming that Mira is 100 parsecs away from us, calculate the absolute magnitude of Mira when it is at it's brightest and dimmest. In terms of luminousity, how much brighter or dimmer is Mira as compared to the Sun?

(b) "*Mira is plunging through our part of the Milky Way with an unusually large space velocity, 130 kilometers per second.*" Is this velocity in the radial direction?

3. Refer to Figure 1 of Kaspi et al.[2] Compare the spectrums of PG 1351, PG 1354 and PG 1512 and discuss. From the spectrum of PG1512, what is the wavelength of the photons corresponding to the most prominent peak emitted from the quasar? (Obtain information on the redshift from Table 1 in the paper.)

4. Given the redshift of a particular quasar to be $z = 2.504$, what is the observed wavelength of the Hydrogen Lyα line?

5. In radio astronomy, spectral lines are usually displayed in a very different way. This is due to the unique way of data collection where the radio telescope/spectrometer scans a very narrow band of wavelegths at a time. Radio astronomers thus need to know exactly what they want to look for. This can be done by looking up the rest frame frequencies of their objects of interest. An example is as follows:

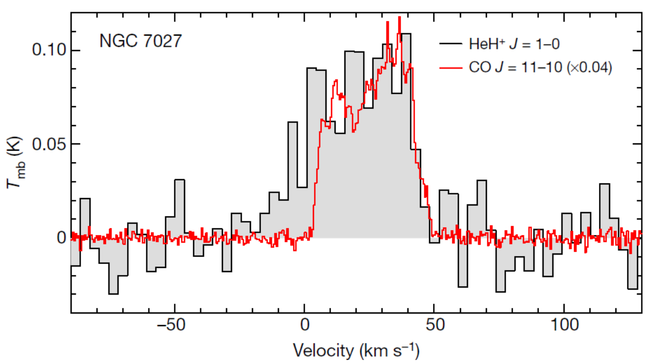

Figure 5.11: Spectra of HeH^+ $J = 1 \rightarrow 0$ and CO $J = 11 \rightarrow 10$ rotational energy levels transitions in NGC 7027.

Note that the horizontal axis is given in terms of velocity. This velocity is the velocity of the source and thus if related to Doppler redshifts.

(a) Using velocity as the horizontal axis may seem to be a strange choice. Can you think of a reason why this is useful in context of collecting a spectrum of a nebula (cloud of gas in between stars)? Clue: this paper is on the "Astrophysical detection of the funny molecule HeH^+". So why is the CO spectrum doing there?

(b) Given that the spectral resolution of the HeH^+ spectrum is 3.6 km s^{-1}. Find the spectral resolution in terms of MHz. Clue: Use the formulae related to Doppler redshift.

6. In Hubble's 1929 paper, he deduced the famous equation

$$v = H_0 d$$

from the velocities and distance of several spiral galaxies. Perform a statistical analysis to determine the goodness-of-fit of his relation on his data (given below).

[2] S. Kaspi et al., Astrophys. J. 533, 631 (2000)

Object	r (distance in units of 10^6 parsecs)	v (measured velocities in km/s)
S. Mag.	0.032	170
L. Mag.	0.034	290
N.G.C.6822	0.214	-130
N.G.C.598	0.263	-70
N.G.C.221	0.275	-185
N.G.C.224	0.275	-220
N.G.C.5457	0.45	200
N.G.C.4736	0.5	290
N.G.C.5194	0.5	270
N.G.C.4449	0.63	200
N.G.C.4214	0.8	300
N.G.C.3031	0.9	-30
N.G.C.3627	0.9	650
N.G.C.4826	0.9	150
N.G.C.5236	0.9	500
N.G.C.1068	1	920
N.G.C.5055	1.1	450
N.G.C.7331	1.1	500
N.G.C.4258	1.4	500
N.G.C.4151	1.7	960
N.G.C.4382	2	500
N.G.C.4472	2	850
N.G.C.4486	2	800
N.G.C.4649	2	1090

5.7 Appendix

Determination of Astronomical Unit using the Transit of Venus

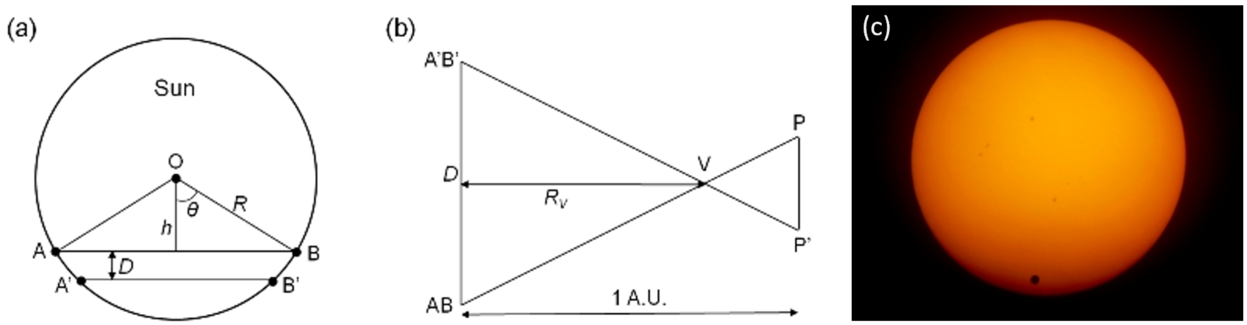

Figure 5.12: (a) Schematic of the transit of Venus as viewed from two different locations on Earth. (b) Trigonometric parallax applied to determine the astronomical unit. (c) Venus in transit on 6 June 2012, 12:08pm. Venus appeared as a small silhouette disc moving across the Sun. This picture was taken with a digital camera attached to a 104mm Newtonian telescope at the NUS multi-purpose field.

The figure above shows the schematic of the transit of Venus as observed from two different locations on Earth. All lengths depicted on the figure are measured in terms of visual angles and equivalently in terms of A.U. since they are lengths on the Sun's disk as observed from Earth. Suppose an observer (P) sees Venus cross the Sun's disk from A to B while another observer (P') sees the transit happens from A' to B'. The two paths are separated by the distance D, whose measurement is crucial for determining the A.U.

From the similar triangles in (b),

$$\frac{PP'(\text{in km})}{(1-R_V)(\text{in A.U.})} = \frac{D(\text{in A.U.})}{R_V(\text{in A.U.})}$$

Here R_V is the radius of Venus' orbit around the Sun, which can be found in terms of the A.U. using Kepler's third law. Rearranging the terms, we have

$$1\text{A.U.} = \frac{R_V PP'}{(1-R_V)D}\text{km} \tag{5.6}$$

We can determine D through fundamental geometrical reasoning and accurate measurements of the transit times recorded by the two observers. The transit time as measured by observer P is proportional to the chord AB.

$$t_{AB} = k\text{AB} = 2kR\sin\theta$$

with k the proportionality constant. We note that the angle subtended by the chord A'B' is slightly smaller than that subtended by the chord AB, by say 2δ. We can write the transit time as measured by observer P'.

$$\begin{aligned} t'_{A'B'} &= 2kRsin(\theta-\delta) \\ &= 2kR(\sin\theta\cos\delta - \sin\delta\cos\theta) \\ &\approx 2kR(\sin\theta - \sin\delta\cos\theta) \\ &= t_{AB} - 2kR\sin\delta\cos\theta \end{aligned}$$

From the above equations we can obtain $\sin\delta$ which will be useful later

$$\begin{aligned} t_{AB} - t'_{A'B'} &= 2kR\sin\delta\cos\theta \\ \sin\delta &= \frac{t_{AB} - t'_{A'B'}}{2kR\cos\theta} \\ &= \frac{t_{AB} - t'_{A'B'}}{t_{AB}}\frac{\sin\theta}{\cos\theta} \end{aligned}$$

In the third step, we have used the small angle ($\delta \ll 1$) approximation $\cos\delta \approx 1$ on the first term.

The distance between the two paths D can be written and approximated as

$$\begin{aligned}
D &= h' - h \\
&= R\cos(\theta - \delta) - R\cos\theta \\
&= R(\cos\theta\cos\delta + \sin\theta\sin\delta) - R\cos\theta \\
&= R(\cos\theta + \sin\theta\sin\delta) - Rcos \\
&= R\sin\theta\sin\delta \\
&= \frac{R\sin^2\theta}{\cos\theta}\frac{t_{AB} - t'_{A'B'}}{t_{AB}}
\end{aligned}$$

Putting this in 5.6,

$$1\text{A.U.} = \frac{R_V PP'}{(1 - R_V)}\frac{\cos\theta}{R\sin^2\theta}\frac{t_{AB}}{t_{AB} - t'_{A'B'}}\text{km}$$

The above is a simplified calculation to bring out the essence of the method to determine the astronomical unit. With the use of radar to accurately measure the astronomical unit, this method is now obselete. Nevertheless the transit of Venus is still a spectacular and rare event to observe. The previous two transit happened in 2004 and 2012. The next transit of Venus will happen in 2117!