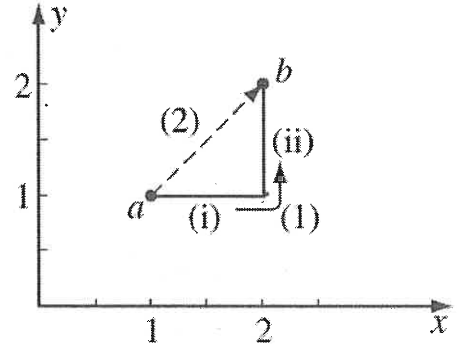
Example 1.6. Calculate the line integral of the function $\mathbf{v} = y^2\,\hat{\mathbf{x}} + 2x(y+1)\,\hat{\mathbf{y}}$ from the point $\mathbf{a} = (1, 1, 0)$ to the point $\mathbf{b} = (2, 2, 0)$, along the paths (1) and (2) in Fig. 1.21. What is $\oint \mathbf{v} \cdot d\mathbf{l}$ for the loop that goes from $\mathbf{a}$ to $\mathbf{b}$ along (1) and returns to $\mathbf{a}$ along (2)?



Path 1 consists of 2 parts (i) & (ii)

(i) $d\vec{l} = dx\,\hat{x}$, $y = 1$

$$\int_{(i)} \vec{v} \cdot d\vec{l} = \int_1^2 y^2\, dx = \int_1^2 dx = 1$$

(ii) $$\int_{(ii)} \vec{v} \cdot d\vec{l} = \int_1^2 2x(y+1)\, dy = 4\int_1^2 (y+1)\, dy = 4\left(\frac{y^2}{2} + y\right)\Big|_1^2 = 10$$

$x = 2$

Path 1 $$\int_{\text{path } 1} \vec{v} \cdot d\vec{l} = \underline{11}$$

Path 2 $x = y$ $dx = dy$ $d\vec{l} = dx\,\hat{x} + dy\,\hat{y}$

$$\int_{\text{path } 2} \vec{v} \cdot d\vec{l} = \int_1^2 y^2\, dx + \int_1^2 2x(y+1)\, dy = \int_1^2 x^2\, dx + \int_1^2 2x(x+1)\, dx = \underline{10}$$