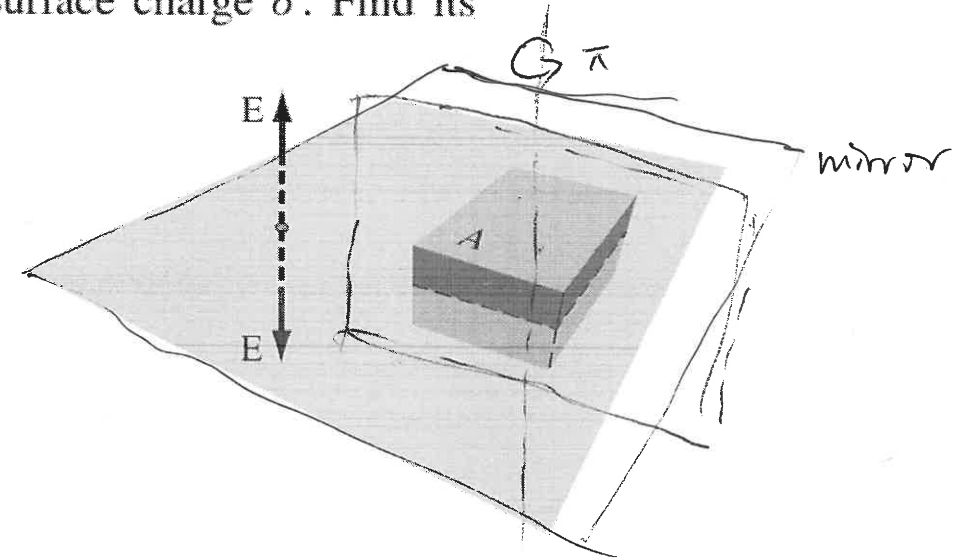
Example 2.5. An infinite plane carries a uniform surface charge σ. Find its electric field.



- translational symmetry + σ uniform.

  $\vec{E}$ constant everywhere

- rotational symmetry along $\hat{n}$ by 180°

  $\vec{E} \parallel \hat{n}$

- mirror symmetry wrt horizontal plane.

  $\vec{E}_{top} = -\vec{E}_{bottom}$

- Gauss's law. $\oint \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$

  area of box

  $Q_{enc} = A\sigma$

  $\oint \vec{E} \cdot d\vec{a} = \int_{top} \vec{E} \cdot d\vec{a} + \int_{bottom} \vec{E} \cdot d\vec{a}$

  $\begin{cases} \text{top} & d\vec{a} = \hat{n}\, dx\, dy \quad \vec{E} = E\hat{n} \\ \text{bottom} & \vec{E} = -E\hat{n} \quad d\vec{a} = -\hat{n}\, dx\, dy. \end{cases}$

  $= \int E\hat{n} \cdot \hat{n}\, da + \int E(-\hat{n})(-\hat{n})\, da = 2EA \Rightarrow E = \frac{\sigma}{2\varepsilon_0} \Rightarrow \vec{E} = \frac{\sigma}{2\varepsilon_0} \hat{n}$