PC#2245 2024 3.12 (11)

$$|K_L\rangle = \frac{|K_2\rangle + \varepsilon' |K_1\rangle}{\sqrt{1+|\varepsilon'|^2}}$$

$|\varepsilon'|$ small ~ 0

Another way to incorporate CP violation is to write

$$|K_I\rangle = \frac{1}{\sqrt{|p|^2+|q|^2}}\left(p|K^0\rangle - q|\bar{K}^0\rangle\right)$$

$$|K_{II}\rangle = \frac{1}{\sqrt{|p|^2+|q|^2}}\left(p|K^0\rangle + q|\bar{K}^0\rangle\right)$$

$|p| \sim 1$, $|q| \sim 1$ (use p, q instead of $\varepsilon, \varepsilon'$)

Note if $|p|=1$, $|q|=1$, then $|K_I\rangle = |K_1\rangle$
$|K_{II}\rangle = |K_2\rangle$

solve the S.E. all over again. Note CP is violated minutely at the beginning by assuming $|p| \sim 1$ but $|p| \neq 1$
$|q| \sim 1$ but $|q| \neq 1$

This treatment does not explain the decay of K_S, K_L, just accounts for CP violation.

To account for CP violation, and decay of K_S, K_L particles, we introduce the basis and an 'effective' Hamiltonian $\bar{H}$.

$$|K_{\substack{I\\II}}\rangle = \frac{1}{\sqrt{|p|^2+|q|^2}}\left(p|K^0\rangle \mp q|\bar{K}^0\rangle\right)$$

$$\bar{H}|K_I\rangle = E_I|K_I\rangle, \qquad \bar{H}|K_{II}\rangle = E_{II}|K_{II}\rangle$$

The state of the system at time t is

$$|\psi\rangle = a_I(t)|K_I\rangle + a_{II}(t)|K_{II}\rangle$$

and can be found by solving the S.E.

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \bar{H}|\psi\rangle, \qquad \bar{H} = H - \frac{i\Gamma}{2}$$

$\bar{H}$ = effective Hamiltonian

$$H^\dagger = H, \quad \Gamma^\dagger = \Gamma$$
$$\bar{H}^\dagger \neq \bar{H}$$

Instead of solving the S.E. we use evolution operator

$$|\psi(t)\rangle = e^{-i\bar{H}t/\hbar}|\psi(0)\rangle$$

$$e^{-i\bar{H}t/\hbar} = \text{evolution operator}$$

Let the state at time $t=0$ be

$$|\psi(0)\rangle = a_I(0)|K_I\rangle + a_{II}(0)|K_{II}\rangle$$
$$\equiv a_I|K_I\rangle + a_{II}|K_{II}\rangle$$

$a_I = a_I(0) \qquad a_{II} = a_{II}(0)$ constants

$$|\psi(t)\rangle = e^{-i\bar{H}t/\hbar}\,|\psi(0)\rangle$$

$$= a_I\, e^{-i\bar{H}t/\hbar}\,|k_I\rangle + a_{II}\, e^{-i\bar{H}t/\hbar}\,|k_{II}\rangle$$

$$= a_I\, e^{-iE_I t/\hbar}\,|k_I\rangle + a_{II}\, e^{-iE_{II} t/\hbar}\,|k_{II}\rangle$$

think

$$E_I = E_1 - i\frac{\Gamma_1}{2} = E_0 + A - \frac{i\Gamma_1}{2}$$

$$E_{II} = E_2 - i\frac{\Gamma_2}{2} = E_0 - A - \frac{i\Gamma_2}{2}$$

$$\Gamma \underset{\uparrow}{=} \begin{pmatrix} \Gamma_1 & 0 \\ 0 & \Gamma_2 \end{pmatrix}$$

see p. 13a

basis $|k_I\rangle, |k_{II}\rangle$

$$= e^{-iE_0 t/\hbar}\Big(a_I\, e^{-i(A - \frac{i\Gamma_1}{2})t/\hbar}\,|k_I\rangle$$

HW

$$+ a_{II}\, e^{+i(A + i\frac{\Gamma_2}{2})t/\hbar}\,|k_{II}\rangle$$

Now express in terms of K^0, $\overline{K^0}$

$$|\psi(t)\rangle = e^{-iE_0 t/\hbar}\, N\Big[a_I\, e^{-i(A - \frac{i\Gamma_1}{2})t/\hbar}\cdot$$

$$(p|K^0\rangle - q|\overline{K^0}\rangle)$$

$$+ a_{II}\, e^{i(A + \frac{i\Gamma_2}{2})t/\hbar}(p|K^0\rangle + q|\overline{K^0}\rangle)\Big]$$

$$N \equiv \frac{1}{\sqrt{|p|^2 + |q|^2}}$$

$$\bar{H}\,|K_I\rangle \simeq E_I\,|K_I\rangle, \quad \bar{H} = H - \frac{i}{2}\Gamma$$

If put $|K_I\rangle = \frac{1}{\sqrt{|p|^2+|q|^2}}\left(p|K^0\rangle - q|\bar{K}^0\rangle\right)$, then approximately, $E_I \simeq E_1 - \frac{i\Gamma_1}{2}$

since $|p| \simeq 1$, $|q| \simeq 1$,

$H|K_1\rangle = E_1|K_1\rangle$, $|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$

To see this, look at the definition of $|K_I\rangle$ and $|K_1\rangle$

$$|K_I\rangle = N\left(p|K^0\rangle - q|\bar{K}^0\rangle\right), \quad N \equiv \frac{1}{\sqrt{|p|^2+|q|^2}}$$

Now
$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$$
$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$$

$\therefore$
$$|K^0\rangle = \frac{1}{\sqrt{2}}(|K_1\rangle + |K_2\rangle)$$
$$|\bar{K}^0\rangle = \frac{-1}{\sqrt{2}}(|K_1\rangle - |K_2\rangle)$$

$$\therefore |K_I\rangle = \frac{N}{\sqrt{2}}\left[(p+q)|K_1\rangle + (p-q)|K_2\rangle\right]$$

Thus
$$H|K_I\rangle = \frac{N}{2}\left[(p+q)E_1|K_1\rangle + (p-q)E_2|K_2\rangle\right]$$
$$\simeq \frac{N}{2}\left[(p+q)E_1|K_1\rangle\right] \quad \because (p-q)\simeq 0$$
$$= E_1\,|K_I\rangle \quad \because (p-q)\simeq 0$$

$$\bar{H}|K_I\rangle = \left(H - \frac{i\Gamma}{2}\right)|K_I\rangle \simeq \left(E_1 - \frac{i\Gamma_1}{2}\right)|K_I\rangle$$

Assume $|\psi(0)\rangle = |K^0\rangle$, $a_I = a_{II} = \frac{1}{2Np}$

HW

$$|\psi(t)\rangle = \frac{1}{2} e^{-iE_0 t/\hbar} \Big[\Big(e^{-i\frac{At}{\hbar}} e^{-\frac{\Gamma_1 t}{2\hbar}} + e^{i\frac{At}{\hbar}} e^{-\frac{\Gamma_2 t}{2\hbar}} \Big) |K^0\rangle$$

$$- \frac{q}{p} \Big(e^{-i\frac{At}{\hbar}} e^{-\frac{\Gamma_1 t}{2\hbar}} - e^{i\frac{At}{\hbar}} e^{-\frac{\Gamma_2 t}{2\hbar}} \Big) |\bar{K^0}\rangle \Big]$$

prob. of finding K^0 at time t

$$= \frac{1}{4} \Big[e^{-\frac{\Gamma_1 t}{\hbar}} + e^{-\frac{\Gamma_2 t}{\hbar}} + 2 \cos\frac{2At}{\hbar}\, e^{-\bar{\Gamma} t/\hbar} \Big]$$

$$\bar{\Gamma} \equiv \frac{\Gamma_1 + \Gamma_2}{2} \qquad \text{(HW)}$$

Prob of getting $\bar{K^0}$ at time t

$$= \frac{1}{4} \left|\frac{q}{p}\right|^2 \Big[e^{-\Gamma_1 t/\hbar} + e^{-\frac{\Gamma_2 t}{\hbar}} - 2\cos\frac{2At}{\hbar}\, e^{-\bar{\Gamma} t/\hbar} \Big]$$

$2A = 3.5 \times 10^{-6}\, eV/c^2$, $\Gamma_1 = \frac{\hbar}{\tau_S}$, $\Gamma_2 = \frac{\hbar}{\tau_L}$

$\tau_S = 0.89 \times 10^{-10}\, s$, $\tau_L = 5.2 \times 10^{-8}\, s$

$\hbar = 6.582 \times 10^{-22}\, MeV \cdot s$

Plot probs. vs time (HW) for different values of A, Γ_1, Γ_2

Now remember that K^0 and $\bar{K}^0$ are each linear combinations of K_1 and K_2. In Eqs. (11.54) the amplitudes have been chosen so that at $t=0$ the $\bar{K}^0$ parts cancel each other out by interference, leaving only a K^0 state. But the $|K_1\rangle$ state *changes with time*, and the $|K_2\rangle$ state *does not*. After $t=0$ the interference of C_1 and C_2 will give finite amplitudes for both K^0 and $\bar{K}^0$.

What does all this mean? Let's go back and think of the experiment we sketched in Fig. 11-5. A π^- meson has produced a Λ^0 particle and a K^0 meson which is tooting along through the hydrogen in the chamber. As it goes along, there is some small but uniform chance that it will collide with a hydrogen nucleus. At first, we thought that strangeness conservation would prevent the K-particle from making a Λ^0 in such an interaction. Now, however, we see that that is not right. For although our K-particle *starts out* as a K^0—which cannot make a Λ^0—it does not *stay* this way. After a while, there *is some amplitude* that it will have flipped to the $\bar{K}^0$ state. We can, therefore, sometimes expect to see a Λ^0 produced along the K-particle track. The chance of this happening is given by the amplitude C_-, which we can [by using Eq. (11.50)) backwards] relate to C_1 and C_2. The relation is

$$C_- = \frac{1}{\sqrt{2}}(C_1 - C_2) = \frac{1}{2}(e^{-\beta t}e^{-i\alpha t} - 1).$$

As our K-particle goes along, the probability that it will "act like" a $\bar{K}^0$ is equal to $|C_-|^2$, which is

$$|C_-|^2 = \frac{1}{4}(1 + e^{-2\beta t} - 2e^{-\beta t}\cos\alpha t).$$

A complicated and strange result!

This, then, is the remarkable prediction of Gell-Mann and Pais: when a K^0 is produced, the chance that it will turn into a $\bar{K}^0$—as it can demonstrate by being able to produce a Λ^0—varies with time according to Eq. (11.56). This prediction came from using only sheer logic and the basic principles of the quantum mechanics—with no knowledge at all of the inner workings of the K-particle. Since nobody knows anything about the inner machinery, that is as far as Gell-Mann and Pais could go. They could not give any theoretical values for α and β. And nobody has been able to do so to this date. They were able to give a value of β obtained from the experimentally observed rate of decay into two π's ($2\beta = 10^{10}$ sec^{-1}), but they could say nothing about α.

We have plotted the function of Eq. (11.56) for two values of α in Fig. 11-6. You can see that the form depends very much on the ratio of α to β. There is no $\bar{K}^0$ probability at first; then it builds up. If α is large, the probability would have large oscillations. If α is small, there will be little or no oscillation—the probability will just rise smoothly to 1/4

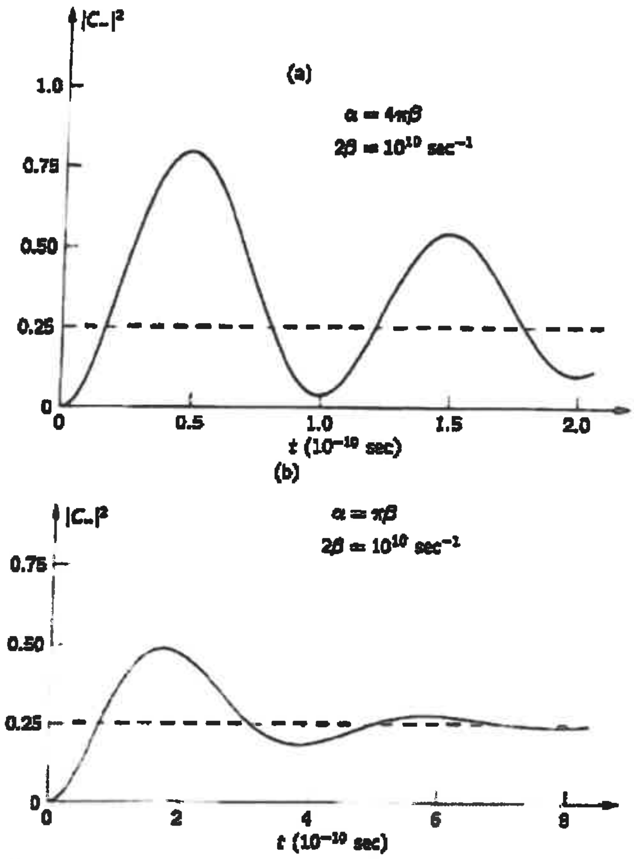

Fig. 11-6. The function of Eq. (11.56): (a) for $\alpha = 4\pi\beta$, (b) for $\alpha = \pi\beta$ (with $2\beta = 10^{10}$ sec^{-1}).

Now, typically, the K-particle will be travelling at a constant speed near the speed of light. The curves of Fig. 11-6 then also represent the probability along the track of observing a $\bar{K}^0$—with typical distances of several centimeters. You can see why this prediction is so remarkably peculiar. You produce a single particle and instead of just disintegrating, it does something else. Sometimes it disintegrates, and other times it turns into a different kind of a particle. Its characteristic

Recap

Kaon decays and CP violation

1964 Experiment. Cronin - Fitch

57 ft $\simeq$ 17.4 m

source K^0, $\overline{K^0}$ — K_S, K_L ⟶ K_L

strong interaction — weak decay

Mostly 3-pion decay
seen 45 2-pion decay
among 22700 decays

Theoretical understanding.

Hamiltonian $H = H_{st} + H_{em} + H_{wk} \equiv H_0 + H_{wk}$

Kaons K^0, $\overline{K^0}$ eigenstates of H_0 but not H;

H_{wk} causes the oscillation $K^0 \rightleftharpoons \overline{K^0}$

Construct common eigenstates of H and CP

$$|K_{\substack{1\\2}}\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle \mp |\overline{K^0}\rangle\right)$$

K_1 same CP as 2 pions, K_2 as 3 pions

CP is conserved. $K_1 \sim K_S$, $K_2 \sim K_L$

Although k_1 has same CP ($=+1$) quantum number as 2 pions and k_2 same CP ($=-1$) as 3 pions, k_1 and k_2 are eigenstates of H and hence cannot decay.

Now modify the states $|k_1\rangle$, $|k_2\rangle$ and the Hamiltonian H

To account for CP violation, introduce

$$|K_{\substack{I\\II}}\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \left(p\,|K^0\rangle \mp q\,|\bar{K}^0\rangle \right)$$

$$|p| \simeq 1, \qquad |p| \neq 1; \quad |q| \simeq 1, \quad |q| \neq 1$$

two parameters p, q are used.

To account for decays, introduce an 'effective' Hamiltonian

$$\bar{H} = H - \frac{i}{2}\Gamma, \qquad H^\dagger = H, \qquad \Gamma^\dagger = \Gamma$$

$$\bar{H}^\dagger \neq \bar{H}$$

k_I may be identified with the particle K_S

k_{II} may be identified with the particle K_L

Instead of solving the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = \bar{H} |\Psi\rangle$$

we make use of the evolution operator $e^{-i\bar{H}t/\hbar}$

to find the general state $|\Psi(t)\rangle$,

$$|\Psi(t)\rangle = e^{-i\bar{H}t/\hbar} |\Psi(0)\rangle$$

Using $|K_I\rangle$ and $|K_{II}\rangle$ as a basis, can write

$$|\Psi(0)\rangle = a_I |K_I\rangle + a_{II} |K_{II}\rangle,$$

the coefficients a_I, a_{II} are constants

thus

$$|\Psi(t)\rangle = e^{-iE_0 t/\hbar} \Big[a_I e^{-i(A - \frac{i\Gamma_1}{2})t/\hbar} |K_I\rangle + a_{II} e^{+i(A + i\frac{\Gamma_2}{2})t/\hbar} |K_{II}\rangle \Big]$$

$$= N e^{-iE_0 t/\hbar} \Big[\Big(a_I e^{-i(A - \frac{i\Gamma_1}{2})t/\hbar} + a_{II} e^{i(A + i\frac{\Gamma_2}{2})t/\hbar} \Big) p |K^0\rangle - \Big(a_I e^{-i(A - \frac{i\Gamma_1}{2})t/\hbar} - a_{II} e^{i(A + \frac{i\Gamma_2}{2})t/\hbar} \Big) q |\bar{K^0}\rangle \Big]$$

$$N = \frac{1}{\sqrt{(|p|^2 + |q|^2)}}$$

CP violation

Hilbert space

K^0, $\overline{K^0}$

$H_0 = H_{st} + H_{em}$

K^0, $\overline{K^0}$ eigenstates of H_0

$\xrightarrow[\text{on}]{H_{wk}}$

oscillation

$K^0 \rightleftharpoons \overline{K^0}$

K_1, K_2

$H = H_0 + H_{wk}$

$|K_{1,2}\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle \mp |\overline{K^0}\rangle)$

$\rightarrow$

$|K_1\rangle$, $|K_2\rangle$ are eigenstates of CP, also eigenstates of H with eigenvalues $E_1 = E_0 + A$, $E_2 = E_0 - A$ respectively. (analogous to coupled pendulum)

Decay $\xrightarrow[\text{on}]{-\frac{i\Gamma}{2}}$

K^0, $\overline{K^0}$

K_1, K_2

K_I, K_{II}

$\bar{H} = H - \frac{i}{2}\Gamma$

K_I, K_{II} eigenstates of $\bar{H}$

$$|K_{I,II}\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \cdot (p\,|K^0\rangle \mp q\,|\overline{K^0}\rangle)$$

$|K_I\rangle$, $|K_{II}\rangle$ not eigenstates of CP

$E_I = E_1 - \frac{i\Gamma_1}{2}$, $E_{II} = E_2 - \frac{i\Gamma_2}{2}$

Today:

Time reversal symmetry U_T antilinear

Kramer's theorem

Electric dipole moment.

L. Ballentine Quantum Mechani[cs]

Time reversal transformation

1. Definition of time reversal transformation $\mathcal{U}_T$ in QM.

2 (i) $\mathcal{U}_T$ unitary and antilinear = antiunitary

(ii) $\mathcal{U}_T^2 = 1$, $\mathcal{U}_T^2 = -1$

3 Kramer theorem

For a physical system having a time reversal symmetry, the eigenvalue of its Hamiltonian is doubly degenerate

4. If time reversal is a perfect symmetry then electric dipole moment of a fundamental particle vanishes

$$\langle \underset{\sim}{d} \rangle = 0, \qquad \underset{\sim}{d} : \text{electric dipole moment}$$