2025. 1. 23 PC4245 L4 1

1. weak integration Feynman diagrams

2. Decays & Conservation laws

3. unification

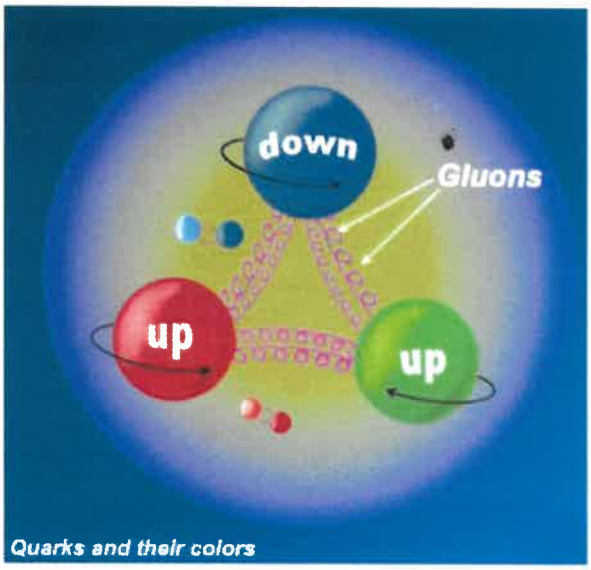
→ chapter 3. Relativistic kinematics

2025 1 14

https://phys.org/news/2025-01-approach-merges-theoretical-fundamentals-experimental.html

# New approach merges theoretical fundamentals with experimental studies of the proton's structure

by US Department of Energy




Artist's depiction of a proton's interior. In quantum chromodynamics, the constituent quarks come in three "colors," along with up and down "flavors." Also shown are virtual quark-antiquark pairs and the gluons that bind the quarks together. Credit: Ted Rogers

Protons and other subatomic particles that are subject to the strong nuclear force have a complex structure that involves even more fundamental constituents called quarks and gluons. These quarks and gluons bind under the influence of quantum chromodynamics (QCD). QCD is the theory of strong interaction of quarks and the role of color symmetry.

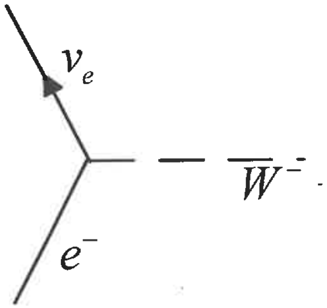
However, the mechanisms that lead to quarks and gluons combining to form the particles we see in nature are very mysterious and poorly understood. For example, virtual quarks and gluons constantly appear and disappear within

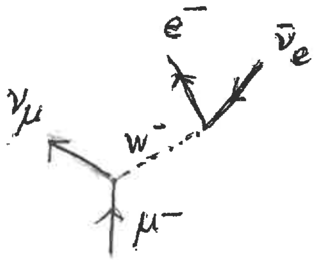
## ( c ) **Weak Interaction**

### Two kinds, charged and neutral vertices

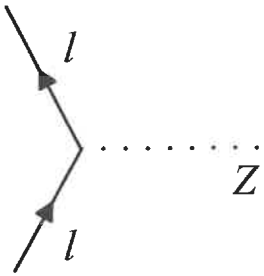
Leptons: primitive vertices connect members of the <u>same</u> generation
Lepton number is separately conserved for each Lepton generation, that is, $L_e$, $L_\mu$, $L_\tau$ separately conserved.

Charged vertex



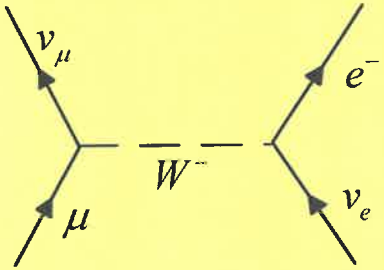




Neutral vertex



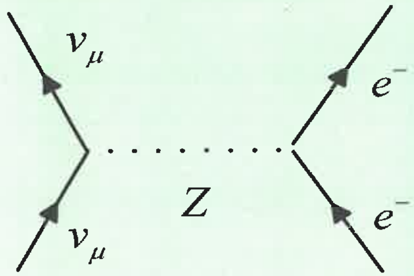


e.g.




$$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$$

$$\mu^- + \nu_e \longrightarrow e^- + \nu_\mu$$



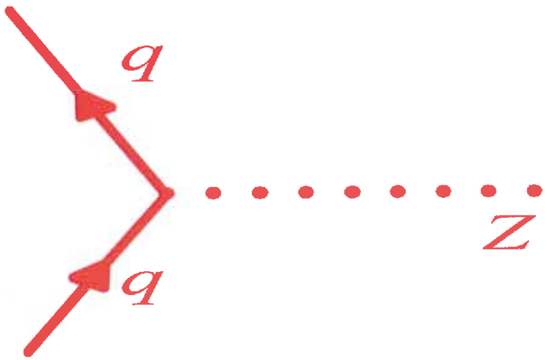

$$\nu_\mu + e^- \to \nu_\mu + e^-$$

Quarks

Flavour not conserved in weak interaction .

**Charged Vertex.**


**Neutral vertex**



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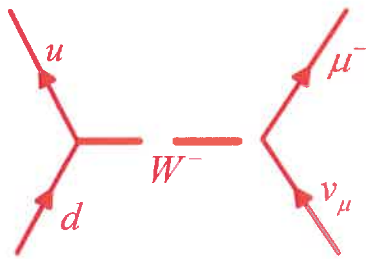
Quarks

Flavour not conserved  in weak interaction

**Charged Vertex**.

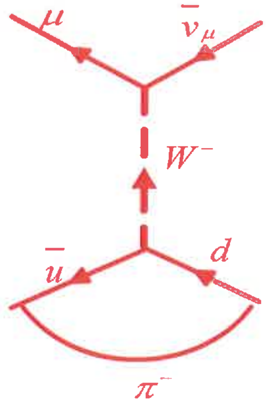

Semileptonic process $d + \nu_\mu \rightarrow u + \mu^-$



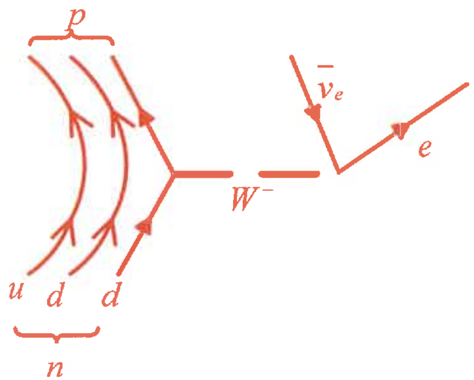

Not observable  due to quark confinement

But can be observed in

Decay of $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$

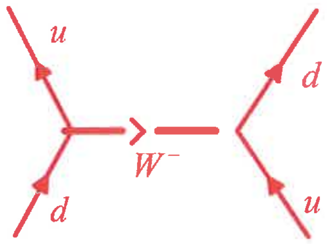



and neutron decay $n \rightarrow p + e^- + \bar{\nu}_e$




Two quarks *u, d* in neutron *n* not participating are called spectator quarks.

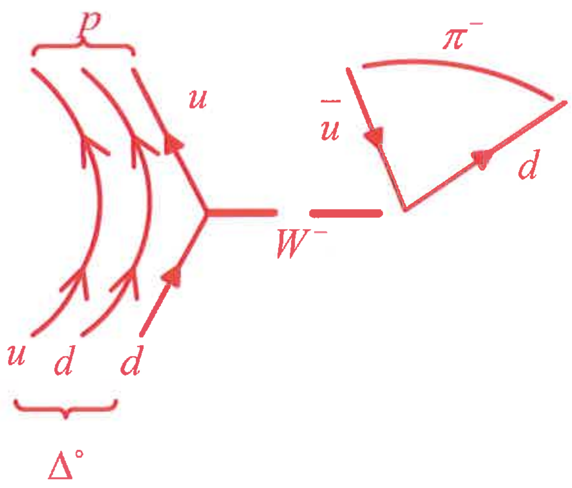
Hadronic decays



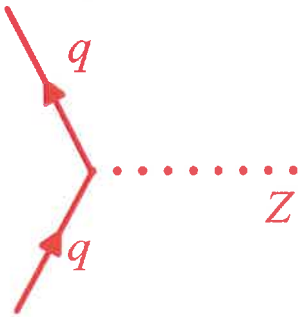

observed in

$\Delta^\circ(udd)$

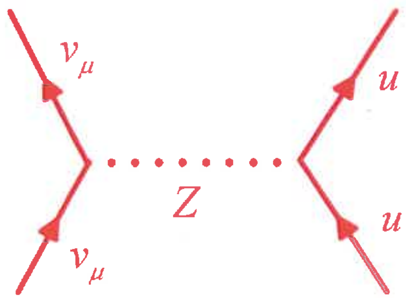
$\Delta^\circ \rightarrow p + \pi^-$




**Neutral vertex**




e.g.

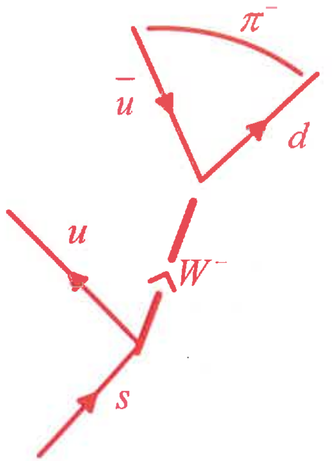



observed in

$\nu_\mu + p \rightarrow \nu_\mu + p$

Decays of quark by weak interaction can involve members of different generations

e.g. a strange quark can decay into an u-quark




The weak force not just couples members of the same generation

$$\binom{u}{d} \text{ or } \binom{c}{s} \text{ or } \binom{t}{b}$$

but **couples also members of different generations**

$$\begin{pmatrix} u \\ d' \end{pmatrix} or \begin{pmatrix} c \\ s' \end{pmatrix} or \begin{pmatrix} t \\ b' \end{pmatrix}$$ where $$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Cabibbo

## Kobayashi –Maskawa matrix

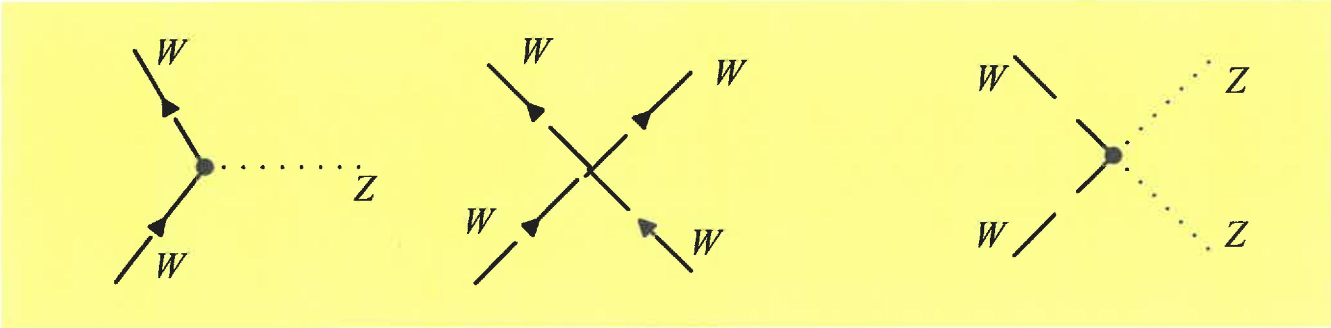
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9747-0.9759, & 0.218-0.224, & 0.001-0.007 \\ 0.218-0.224, & 0.9734-0.9752, & 0.030-0.058 \\ 0.003-0.019, & 0.029-0.058, & 0.9983-0.9996 \end{pmatrix}$$

$V_{ud}$ = coupling of $u$ to $d$

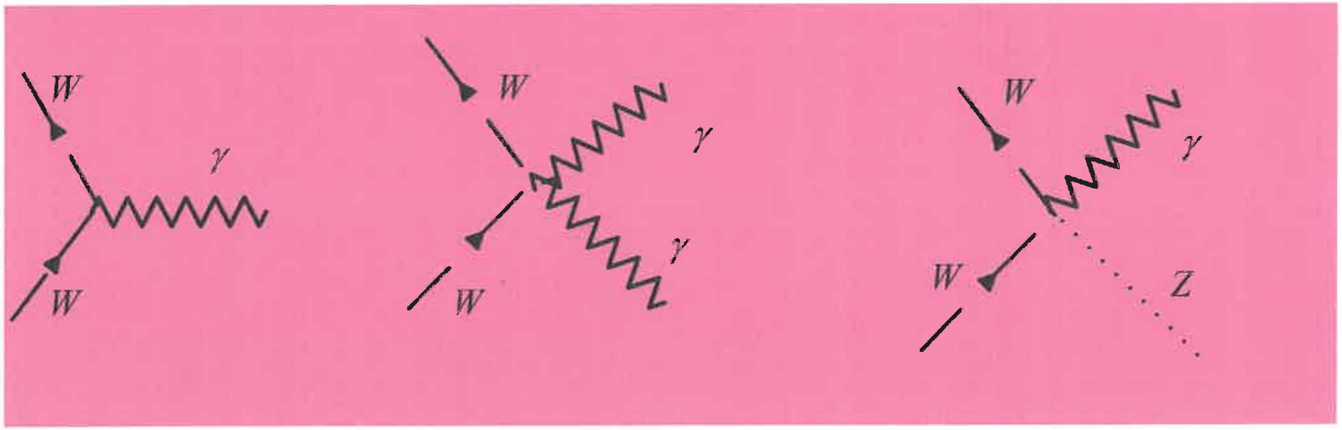
$V_{us}$ = coupling of $u$ to $s$

## (d) ***wk*** and ***em*** couplings of $W^{\pm}$ and $Z$

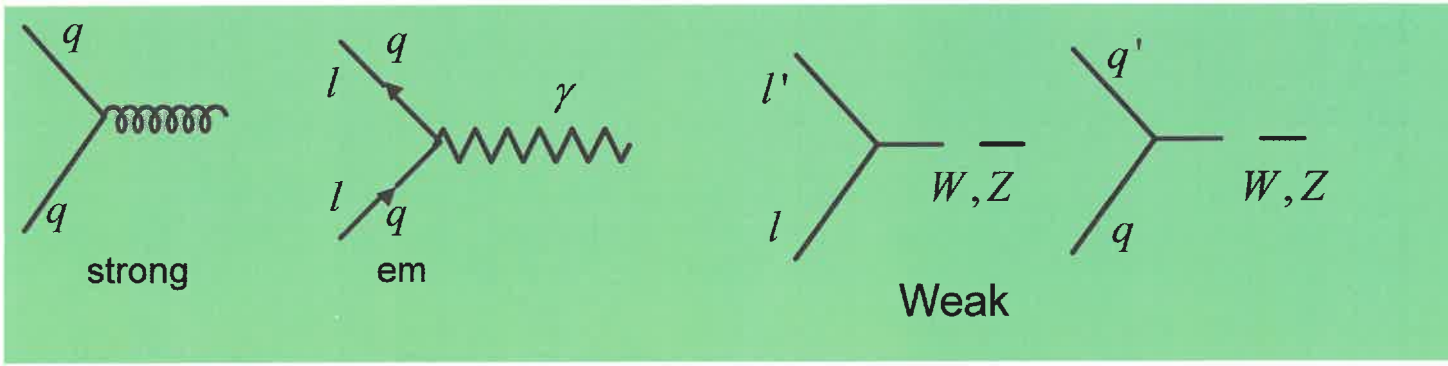
### Weak couplings




### Couplings involve photon $\gamma$




## Summary



# 1.5 Decay & Conservation Laws

## 1.5.1 Decay

(a) Every particle decays into lighter particles unless prevented by some conservation law

Stable particles : $e^-$ (lightest lepton, conservation of lepton number),

$p$ (lightest baryon, conservation of baryon number), neutrinos, photons (massless particles)

(b) Most particles exhibit several different decay modes
e.g.

| | | Branching ratio |
|---|---|---|
| $K^+ \rightarrow$ | $\mu^+ + \nu_\mu$ | 64% |
| | $\pi^+ + \pi^\circ$ | 21% |
| | $\pi^+ + \pi^+ + \pi^-$ | 6% |
| | $\pi^+ + \nu_e + \pi^\circ$ | 5% |

Each unstable species has a characteristic mean life time $\tau$

e.g.

$$\tau_{\mu} = 2.2 \times 10^{-6}\ \mathrm{s}$$
$$\tau_{\pi^+} = 2.6 \times 10^{-8}\ s$$
$$\tau_{\pi^\circ} = 8.3 \times 10^{-17} s$$

Note: $I = I_0\ e^{-t/\tau}$,

$\tau$ = time taken for ***I*** to decrease from $I_0$ to $I_0\ e^{-1}$

*t*$_{1/2}$ = time taken for ***I*** to reduce to $\frac{1}{2} I_o$,

$$\frac{1}{2} I_o = I_o\ e^{-(t_{1/2}/\tau)} \Rightarrow ln2 = (t_{1/2}/\tau)$$

( c ) Three Fundamental Decays:

Strong decay e.g. $\Delta^{++} \rightarrow p + \pi^{+}$ $\tau = 10^{-23} s$

***em*** decay e.g. $\pi^{\circ} \rightarrow \gamma + \gamma$ $\tau = 10^{-16} s$

***wk*** decay e.g. $\Sigma^{-} \rightarrow n + e^{-} + \bar{\nu}_e$ $\tau \sim 10^{-13} s$

Neutron decay $n \rightarrow p + e^{-} + \bar{\nu}_e$ $(\tau = 15 \min)$

$(d \rightarrow u + e^{-} + \bar{\nu}_e)$

(d) Kinematic Effect: the larger the mass difference between the original particle and the decay products, the more rapidly the decay occurs.

This is also known as phase space factor. It accounts for the enormous range of mean life time $\tau$ in ***wk*** decays.

## 1.5.2 CONSERVATION LAWS

(i) Spacetime symmetry

Homogeneity of space time $\rightarrow$ laws of physics are invariant under time and space translations $\rightarrow$

Conservation of spatial momentum $\underset{\sim}{p}$, Conservation of energy $E/c = p^0$

Isotropy of space time $\rightarrow$ laws of physics are invariant under rotations in space time.

In particular laws of physics are invariant under rotations in space $\rightarrow$
Conservation of angular momentum.
Invariant under rotation in space and time (Lorentz transformation), Lorentz Symmetry

Discrete Symmetry
Space inversion $\rightarrow$ conservation of parity
Time inversion T, no quantum number associated.
$T$ represented by anti-unitary operator.

$T^{+} = T^{-1}$ (unitary)
$T(c_1|\psi_1\rangle + c_2|\psi_2\rangle) = c_1^* T|\psi_1\rangle + c_2^* T|\psi_2\rangle$ (antilinear)
and $c_1^*$ is the complex conjugate of $c_i$, $i$=1, 2

## (ii) Internal Symmetry

### (1) $U(1)$ symmetry

A physical state of a physical system is represented by a vector $|\psi\rangle$ in Hilbert space up to a phase factor (assume normalization)

i.e. if $|\psi\rangle$ represents a physical state

then $e^{i\alpha}|\psi\rangle$ represents the same physical state, where $\alpha$=constant, phase whereas exp( $i\,\alpha$) is known as phase factor.

$\{e^{i\alpha_1}, e^{i\alpha_2} \ldots\}$ form a group, an Abelian group *U(1)*

Conservations of electric charge, baryon number and lepton number are due to the *U(1)* phase invariance.

For the electric charge case, can also let phase $\alpha$ be dependent on spacetime point $x^u$, namely $\alpha = \alpha(\underline{x})$ and one gets local gauge invariance

L 4

(2) The QCD Lagrangian is invariant under local *SU(3)* transformations. i.e. QCD has a local *SU(3)* symmetry. An *SU(3)* transformation is represented by a unitary 3 x 3 matrix with determinant=1.

SU(3) = special unitary group in three dimensions

(3) Approximate conservation of flavour. Quark flavour is conserved at a strong or electromagnetic vertex, but <u>not</u> at a weak vertex.

**OZI (Okubo, Zweig and Iizuka ) rule**
**Some <u>strong</u> decays are suppressed**

e.g.

$J/\psi = c\bar{c}$ bound state of charmed quarks has anomalously long lifetime

~$10^{-20}$sec
(Strong decay ~$10^{-23}$sec)

$\varphi$ Meson $(s\bar{s})$, $I^G(J^{pc}) = 0^-(1^{--})$
mass = 1020 $MeV$, Full width $\Gamma = 4MeV$, $(\tau\Gamma = \hbar)$ $\tau$=1.6 x $10^{-22}$s

Decay modes

| | |
|---|---|
| $K^+K^-$ | 50% |
| $K_l^0 K_s^0$ | 34% |
| $\rho\pi$ | 13% |
| $\pi^+\pi^-\pi^o$ | 2% |
| $\eta\gamma$ | 1% |

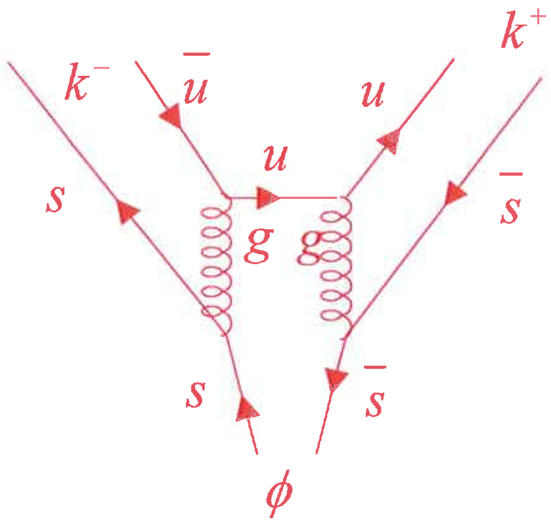
Clearly, $\phi$ meson decays more often into $K^+K^-$

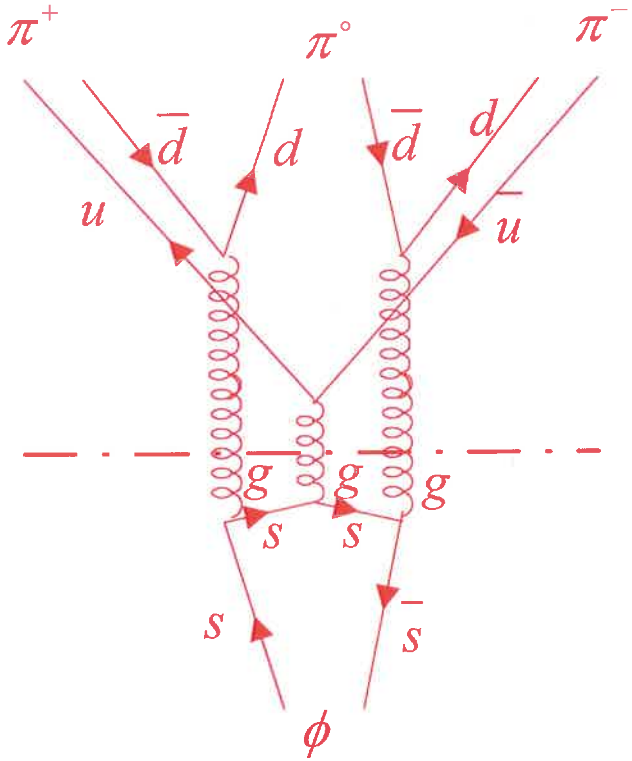
$\phi \to K^+ + K^-$ mass of $(K^+ + K^-) = 990\,MeV/c^2$

than into $3\pi$'s

$\phi \to \pi^+ + \pi^- + \pi^0$ mass of $(\pi^+ + \pi^- + \pi^0) = 415\,MeV/c^2$

quark diagrams




**OZI rule:**

If the diagram can be cut in two by slicing only gluon lines (and not cutting open any external lines), the process is suppressed.

Qualitatively OZI rule is related to the asymptotic freedom.

In an OZI suppressed diagram the gluons have higher energy than those in the OZI - allowed diagram. (More gluons imply higher energy, higher energy so strong interaction coupling constant is smaller, meaning process less likely to occur).

$J/\psi$ $\quad I^G(J^P) = 0^-(1^-)$

mass = 3100 MeV/c$^2$, $\Gamma$=0.093 MeV

Decay modes

| | |
|---|---|
| $e^+e^-$ | 6.0% |
| $\mu^+\mu^-$ | 6.0% |
| hadrons | 88% |

$J/\psi \to 3\pi$ OZI - suppressed

$J/\psi \to D^+ + D^-(D^\circ + \overline{D}^\circ)$ charmed nonstrange Mesons

mass of D $\approx$ 1869 MeV/c$^2$. $D^+$ (c $\overline{d}$); $D^0$ (c $\overline{u}$)

Kinematically forbidden since a pair of two charmed D mesons has larger total mass than the J particle. **Hence this J particle has a longer life time than the $\phi$ meson.**

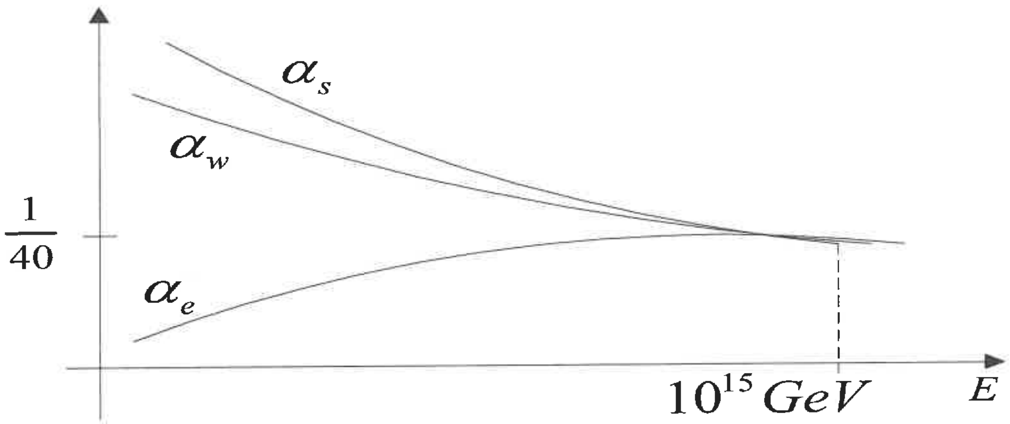
# Planck Scale

Strong coupling constant $\alpha_s$ decreases at short distances (very high energy collisions)
Weak coupling $\alpha_w$ also decreases but at a slower rate.

Electromagnetic coupling constant $\alpha_e$ increases as energy increases

[Note: the relative weakness of the weak force is due to the large mass of $W^{\pm}$, $Z$; its intrinsic strength is greater than that of the ***em*** force.]

From the present functional form of the running coupling constants, $\alpha_s$, $\alpha_w$, and $\alpha_e$ converge at around $10^{15}$ *GeV* (Planck energy scale).




$At\ 10^{-19}\, m,$

$$\alpha_s = \frac{1}{10}$$

$$\alpha_w = \frac{1}{27}$$

$$\alpha_e = \frac{1}{129}$$

$g\bar{\psi}\gamma^{\mu}T^{a}\psi A^{a}_{\mu}$
$= g j^{\mu a} A^{a}_{\mu}$

$A^{a}_{\mu}$ = Yang-Mills field
= Non-Abelian gauge field.
$j^{\mu a}$ = Non-Abelian current density.

$\psi(x)$ = matter field

## Our Universe according to Wilkison Microwave Anistropy Probe (WMAP) 2003

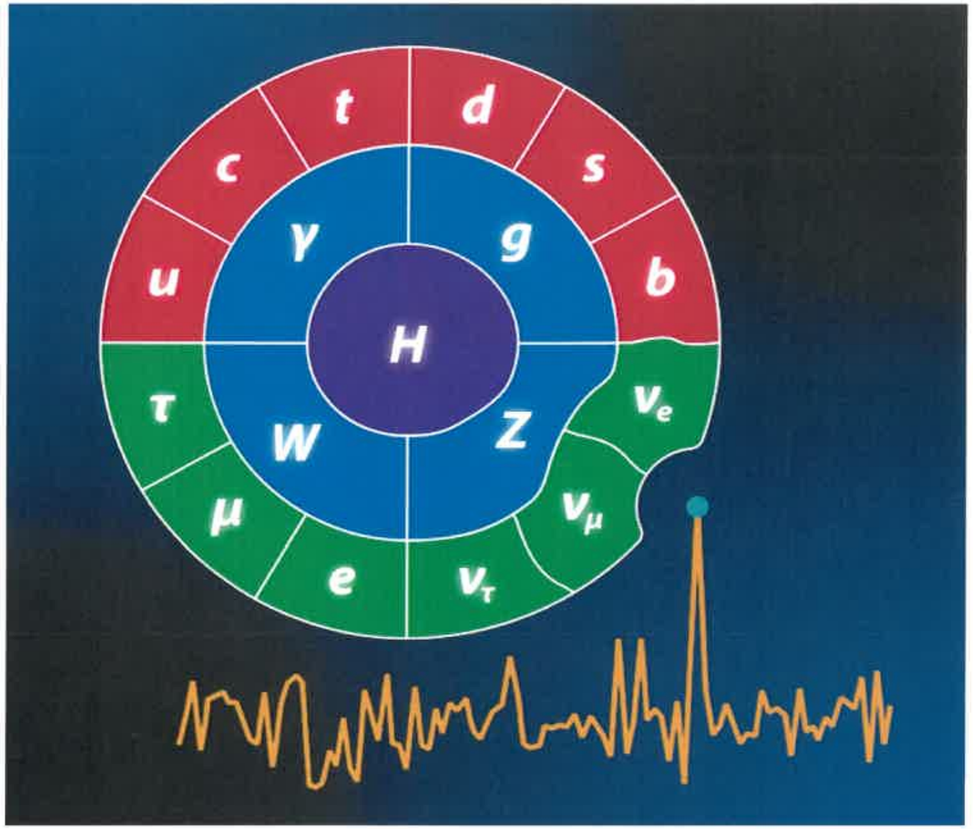
- Age: 13.7 billion years
- Shape: Flat
- Age when first light appeared:200 Million years
- Contents: 4% ordinary matter, 23% dark matter, nature unknown; 73% dark energy, nature unknown
- Hubble constant (expansion rate):71km/sec/megaparsec

***To see a World in a Grain of Sand***
***And a Heaven in A Wild Flower***
***Hold Infinity in the palm of your hand***
***And Eternity in an hour***

**W. Blake (1757-1827)**

# M.C. Escher

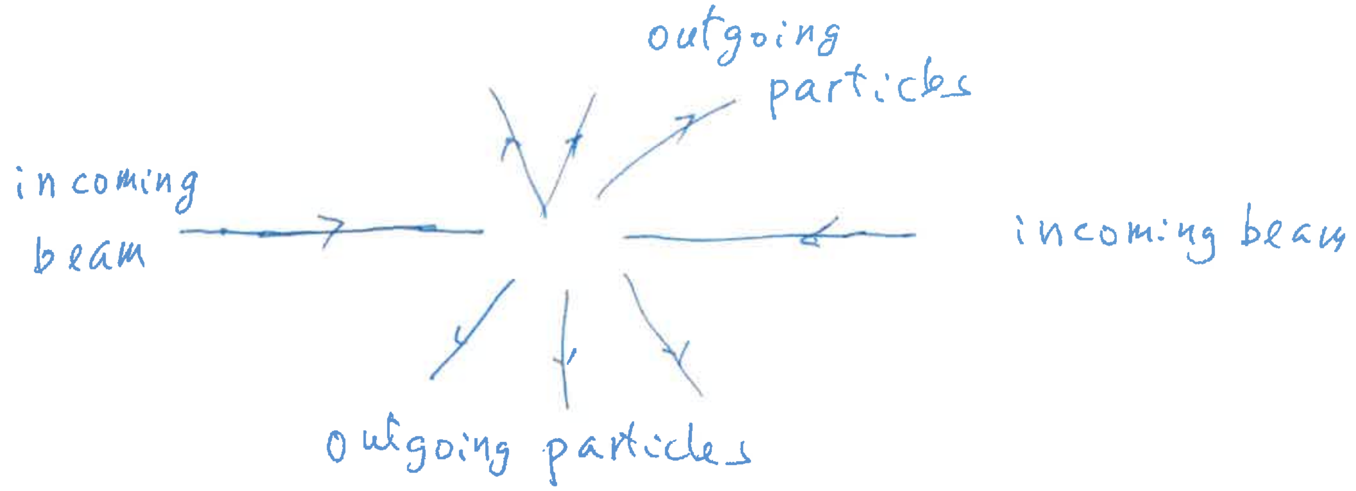
(Dutch graphic artist,
1898- 1972)




APS/Alan Stonebraker
The colored circle represents the standard model of particle physics, which describes the Higgs boson (purple), the force-carrying particles (blue), the quarks (red), and the leptons (green). Particle physicists hope that an anomaly (shown as a data peak) will pierce through the standard model's long-standing dominance, so that a new, more comprehensive theory can develop.

# Relativistic Kinematics.

In particle physics, particle reactions involve high energy, e.g in the collider, violent




collisions. Thus the reactions are relativistic

We review special relativity in 4-vector notations and study simple examples in high energy collisions.

Special Relativity:

Frames of reference

Postulates of special Relativity

Galilean and Lorentz transformations