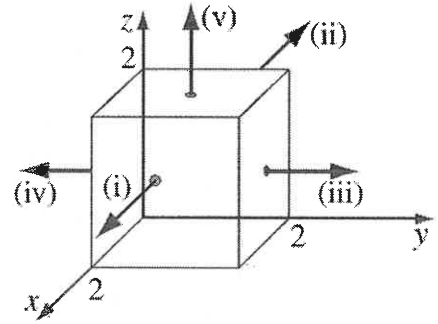
Example 1.7. Calculate the surface integral of $\mathbf{v} = 2xz\,\hat{\mathbf{x}} + (x+2)\,\hat{\mathbf{y}} + y(z^2-3)\,\hat{\mathbf{z}}$ over five sides (excluding the bottom) of the cubical box (side 2) in Fig. 1.23. Let "upward and outward" be the positive direction, as indicated by the arrows.



(i) $x=2$, $d\vec{a} = dy\,dz\,\hat{x}$

$$\int_{(i)} \vec{v}\cdot d\vec{a} = \int_0^2\int_0^2 2xz\,dy\,dz = \int_0^2\left[\int_0^2 (4z)\,dy\right]dz$$

$$= \int_0^2 8z\,dz = 4z^2\Big|_0^2 = 16$$

(ii) $x=0$, $d\vec{a} = dy\,dz\,(-\hat{x})$ $\Rightarrow \int_{(ii)} \vec{v}\cdot d\vec{a} = 0$

(iii) $y=2$, $d\vec{a} = dx\,dz\,\hat{y}$ $\Rightarrow \int_{(iii)} \vec{v}\cdot d\vec{a} = 12$

(iv) $y=0$, $d\vec{a} = dx\,dz\,(-\hat{y})$ $\Rightarrow \int_{(iv)} \vec{v}\cdot d\vec{a} = -12$

(v) $z=2$, $d\vec{a} = dx\,dy\,(\hat{z})$ $\Rightarrow \int_{(v)} \vec{v}\cdot d\vec{a} = 4$