

Consider cross product between

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

$$\vec{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$

Note that

$$\begin{aligned}\hat{x} \times \hat{x} &= 0 \\ \hat{x} \times \hat{y} &= \hat{z} \\ \hat{y} \times \hat{z} &= \hat{x} \\ \hat{z} \times \hat{x} &= \hat{y}\end{aligned}$$

Method 1

$$\begin{aligned}\vec{A} \times \vec{B} &= (A_x \hat{x} + A_y \hat{y} + A_z \hat{z}) \times \\ &\quad (B_x \hat{x} + B_y \hat{y} + B_z \hat{z}) \\ &= A_x B_y \underbrace{\hat{x} \times \hat{y}}_{\hat{z}} + A_x B_z \underbrace{\hat{x} \times \hat{z}}_{-\hat{y}} + \dots\end{aligned}$$

Method 2

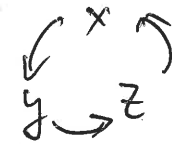
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\begin{aligned}&= \hat{x} A_y B_z + \hat{y} A_z B_x + \hat{z} A_x B_y \\ &\quad - \hat{x} A_z B_y - \hat{y} A_x B_z - \hat{z} A_y B_x\end{aligned}$$

Method 3

= "rotation"

$$\vec{A} \times \vec{B}$$



$$\begin{aligned}&= \hat{x} (A_y B_z - A_z B_y) \\ &\quad + \hat{y} (A_z B_x - A_x B_z) \\ &\quad + \hat{z} (A_x B_y - A_y B_x)\end{aligned}$$