(i) $\nabla \cdot \mathbf{E} = 0$, (iii) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$,

(ii) $\nabla \cdot \mathbf{B} = 0$, (iv) $\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$

$\Rightarrow$

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

Take the curl to (iii)

$$\nabla \times (\nabla \times \vec{E}) = \nabla \times \left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

$\nabla \cdot \vec{E} \to 0$; $\nabla \times \vec{B} \to \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$\Rightarrow \nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Take Curl to (iv)

$$\nabla \times (\nabla \times \vec{B}) = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}(\nabla \times \vec{E})$$

$$\nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$\nabla \cdot \vec{B} \to 0$

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$