2024. 2. 20 11

Mirror reflection $\neq$ rotation

To discuss mirror reflection, more convenient to discuss space inversion

$$\underset{\sim}{x} \longrightarrow \underset{\sim}{x}' = -\underset{\sim}{x}$$

space inversion = mirror reflection + rotation [reflection commutes with rotation]

This is because reflection requires mirror position be specified whereas inversion no need.

Note : Rotations form a group $SO(3)$

Rotations together with reflections form a bigger group $O(3)$, orthogonal group.

space inversion can be represented by a 3×3 matrix

$$\mathcal{R}_- = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\underset{\sim}{x} \longrightarrow \underset{\sim}{x}' = \mathcal{R}_-[\underset{\sim}{x}] = -\underset{\sim}{x}$$

In 3-dim physical space, rotations form a group SO(3). If reflections included, then group is O(3) = orthogonal group in 3 dimensions.

Mirror Reflection → chirality = handedness.

Apply mirror reflection to a physical system, the state of the physical system may or may not change

In QM state is described by $|\Psi\rangle$ and mirror reflection is by an operator

A tran $\mathcal{R}_-$ in 3-dim. space induces an operator π in Hilbert space: π is defined:

$$\underset{\sim}{x} \rightarrow \underset{\sim}{x}' = \pi \underset{\sim}{x} \pi^{-1} = -\underset{\sim}{x} \rightarrow \pi \underset{\sim}{x} = -\underset{\sim}{x} \pi$$

$$\underset{\sim}{P} \rightarrow \underset{\sim}{P}' = \pi \underset{\sim}{P} \pi^{-1} = -\underset{\sim}{P}$$

$$\underset{\sim}{J} \rightarrow \underset{\sim}{J}' = \pi \underset{\sim}{J} \pi^{-1} = \underset{\sim}{J}$$

(Ballentine: QM, A modern approach WSPC)

In QM, π = parity operator

world scientific Publishing co.

From definition of π, we have

(i) π is unitary, linear

can show $\pi^2 = 1$ ← identity operator. HW

$\rightarrow \quad \pi^{-1} = \pi$

But $\quad \pi^{-1} = \pi^{+}$ (unitary)

i.e $\quad \pi = \pi^{+}$ i.e π is Hermitian

But π^2 = identity hence eigenvalues

$= +1$ or -1 (H w)

eigenvalue of the parity operator π is known as parity.

Note: We start with π as a transformation operator, but now it becomes an observable

(ii) π acts on state $|\psi\rangle$?

$$\pi : |\psi\rangle \rightarrow |\psi'\rangle = \pi|\psi\rangle$$

specifically $\quad |\underset{\sim}{x}\rangle \rightarrow |\underset{\sim}{x}'\rangle = \pi|\underset{\sim}{x}\rangle \overset{\text{can}}{\underset{\text{show}}{=}} |-\underset{\sim}{x}\rangle$

proof: Given $\quad \hat{\underset{\sim}{x}}|\underset{\sim}{x}\rangle = \underset{\sim}{x}|\underset{\sim}{x}\rangle$

$$\hat{x}(\pi|x\rangle) = -\pi\hat{x}|x\rangle$$

(by defn of π, $\pi x = -x\pi$ anticommutes)

$$= -\pi x|x\rangle = -x(\pi|x\rangle)$$

i.e. $\pi|x\rangle$ is an eigenstate of the operator $\hat{x}$ with eigenvalue $-x$

But $\hat{x}\,|-x\rangle = -x\,|-x\rangle$

$\Rightarrow \quad \pi|x\rangle = |-x\rangle$

$\rightarrow$ If π_c = coord. representation of π : $\langle x|\pi|\psi\rangle = \pi_c\langle x|\psi\rangle = \pi_c\psi(x)$

$\pi_c\,\psi(x) = \psi(-x)$ (as seen below lines)

As $\pi|x\rangle = |-x\rangle$, $\therefore$ $\langle x|\pi^\dagger = \langle -x|$ i.e. $\langle x|\pi = \langle -x|$ $\because \pi^\dagger = \pi$

Thus $\langle x|\pi|\psi\rangle = \langle -x|\psi\rangle = \psi(-x)$. $\langle x|\pi|\psi\rangle = \pi_c\langle x|\psi\rangle = \pi_c\psi(x)$

(iii)(a) If a physical system is in the eigenstate of π, then this physical system cannot have a dipole moment $\underset{\sim}{d} = q\underset{\sim}{x}$, q = charge

Electric

(HW)

Hint $\langle\psi|\underset{\sim}{x}|\psi\rangle = -\langle\psi|\pi\,\underset{\sim}{x}\,\pi^{-1}|\psi\rangle$

(b) If a physical system has a space inversion symmetry, and if the state of the physical system is a nondegenerate eigenstate of its Hamiltonian H, then the dipole moment $\underset{\sim}{d}$ of this system $= 0$

(iii) Proof:

The physical system is invariant under space inversion.
$\rightarrow [H, \pi] = 0.$

suppose $|\psi\rangle$ is a non-degenerate eigenstate of the Hamiltonian, then physical system in this state has zero electric dipole moment, $\langle\psi|\underset{\sim}{d}|\psi\rangle = 0$

Proof: Given $H|\psi\rangle = E|\psi\rangle$, E = nondegenerate
and $[\pi, H] = 0$

under parity operator π, $|\psi\rangle \rightarrow |\psi'\rangle = \pi|\psi\rangle$.

$$H|\psi'\rangle = H\pi|\psi\rangle = \pi H|\psi\rangle \quad \because [\pi, H] = 0$$
$$= \pi E|\psi\rangle = E\pi|\psi\rangle = E|\psi'\rangle$$

so $|\psi'\rangle$ and $|\psi\rangle$ have same energy value E.

But E is nondegenerate, $\therefore |\psi'\rangle$ and $|\psi\rangle$ must be the same state, $\therefore |\psi'\rangle = k|\psi\rangle$, k = constant $\neq 0$

under π, $\underset{\sim}{d} \rightarrow \underset{\sim}{d}' = \pi\underset{\sim}{d}\pi^{-1} = -\underset{\sim}{d} \quad \because \underset{\sim}{d} = q\underset{\sim}{x}$

As the physical system has space inversion symmetry, the expectation value is unchanged under π.

so $\langle\psi'|\underset{\sim}{d}'|\psi'\rangle = \langle\psi|\underset{\sim}{d}|\psi\rangle$

$$LHS = \langle\psi'|\underset{\sim}{d}'|\psi'\rangle = kk^*\langle\psi|-\underset{\sim}{d}|\psi\rangle$$
$$= -|k|^2\langle\psi|\underset{\sim}{d}|\psi\rangle$$

$$RHS = \langle\psi|\underset{\sim}{d}|\psi\rangle$$

$$\therefore (1+|k|^2)\langle\psi|\underset{\sim}{d}|\psi\rangle = 0$$
$$\text{i.e. } \langle\psi|\underset{\sim}{d}|\psi\rangle = 0$$

(iv) Intrinsic Parity for a particle

In strong and electromagnetic interactions, parity is conserved i.e. parity is a good quantum number. So it is useful to assign parity quantum numbers for particles participating in strong and electromagnetic interactions, so hadrons are assigned parity but not lepton.

Familiar example from quantum mechanics course is hydrogen atom. State of H atom $\psi(\underset{\sim}{x}) = \psi(r, \theta, \phi)$

$= \text{constant} . \text{ Laguerre polynomial} \cdot \text{spherical harmonics}$

$= f(r)\, Y_\ell^m(\theta, \phi)$.

Under space inversion $\underset{\sim}{x} \to -\underset{\sim}{x}$, $(r, \theta, \phi) \to (r, \pi-\theta, \phi+\pi)$

$\psi(r, \theta, \phi) = \text{const } f(r) Y_\ell^m(\theta, \phi) \to Y_\ell^m(\pi-\theta, \phi+\pi) = (-)^\ell\, Y_\ell^m(\theta, \phi)$

i.e. the state of the H atom is an eigenstate of the parity operator π with eigenvalue $(-1)^\ell$ = parity, $\ell = 0, 1, 2, \cdots$

Along the same line, hadrons can have intrinsic parity. By convention, nucleon (p, n) has parity +1

→ quark has parity +1, antiquark parity −1.

Mesons made out of quark, antiquark,

parity of meson $= (+1)(-1)\cdot(-1)^\ell = (-)^{\ell+1}$, $\ell = 0, 1, 2 \cdots$

ℓ = relative orbital motion of quark and antiquark

Parity of photon = −1 $\because$ momentum $\underset{\sim}{p}$ and $\underset{\sim}{p} - q\underset{\sim}{A}$

$\underset{\sim}{A} \to$ photon field

under space inversion $\underset{\sim}{p} \to -\underset{\sim}{p}$

Baryons made out of 3 quarks,

parity of a baryon

$= \pi(q)\,\pi(q)\,\pi(q)\,\pi(\text{relative motion})$

$= \pi(\text{relative motion})$, $\quad \pi(q) = +1$

For 2 particle relative motion, the parity is $(-1)^{\ell}$, ℓ = orbital quantum number of the relative motion of the 2 particles

For 3 particle in relative motion, the parity of the relative motion is not so simple.

particles 1, 2 $\rightarrow \ell$
particles 2, 3 $\rightarrow \ell'$
particles 1, 3 $\rightarrow \ell''$

For integer j, the rank j tensor that trasforms as $(-1)^{j+1}$ under space inversion is a pseudo tensor. $j = 0, 1, 2, \ldots$

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Table 4.5 Scalars and vectors under parity

Scalar	:	$P(s) = s$
Pseudoscalar	:	$P(p) = -p$
Vector (or polar vector)	:	$P(\mathbf{v}) = -\mathbf{v}$
Pseudovector (or axial vector)	:	$P(\mathbf{a}) = \mathbf{a}$

SO(3) has one casimir operator $\underset{\sim}{J}^2$

O(3), the orthogonal group, has two Casimir opeators, $\underset{\sim}{J}^2$ and π

just as they are classified by spin, charge, isospin, strangeness, and so on. According to quantum field theory, the parity of a fermion (half-integer spin) must be *opposite* to that of the corresponding antiparticle, while the parity of a boson (integer spin) is the *same* as its antiparticle. We take the *quarks* to have *positive* intrinsic parity, so the antiquarks are negative.* The parity of a composite system in its ground state is the *product* of the parities of its constituents (we say that parity is a 'multiplicative' quantum number, in contrast to charge, strangeness, and so on, which are 'additive').† Thus the baryon octet and decuplet have positive parity, $(+1)^3$, whereas the pseudoscalar and vector meson nonets have negative parity, $(-1)(+1)$. (The prefix 'pseudo' tells you the parity of the particles.) For an excited state (of two particles) there is an extra factor of $(-1)^l$, where l is the orbital angular momentum [18]. Thus, in general, the mesons carry a parity of $(-1)^{l+1}$ (see Table 4.6). Meanwhile, the *photon* is a *vector* particle (it is represented by the vector potential A^μ); its spin is 1 and its intrinsic parity is -1. $\underset{\sim}{P} - q\underset{\sim}{A}$

The mirror symmetry of strong and electromagnetic interactions means that parity is conserved in all such processes. Originally, everyone took it for granted that the same goes for the weak interactions as well. But a disturbing paradox arose in the early fifties, known as the 'tau–theta puzzle'. Two strange mesons, called at the time τ and θ, appeared to be identical in every respect – same mass, same spin (zero), same charge, and so on – except that one of them decayed into two pions and the other into three pions, states of opposite parity:

$\theta, \tau \to K^0, \bar{K}^0$

$$
\begin{aligned}
K^+ \quad & \theta^+ \to \pi^+ + \pi^0 && (P = (-1)^2 = +1) \\
K^+ \quad & \tau^+ \to \left\{ \begin{matrix} \pi^+ + \pi^0 + \pi^0 \\ \pi^+ + \pi^+ + \pi^- \end{matrix} \right\} && (P = (-1)^3 = -1)
\end{aligned}
\tag{4.53}
$$

* This choice is completely arbitrary; we could just as well do it the other way around. Indeed, in principle we could assign positive parity to some quark flavors and negative to others. This would lead to a different set of hadronic parities, but the *conservation* of parity would still hold. The rule stated here is obviously the *simplest*, and it leads to the conventional assignments.

† There is less to this distinction than meets the eye; in a sense, it results from a notational anomaly. Scrupulous consistency would require that we write the parity operator in exponential form, $P = e^{i\pi K}$, with the operator K playing a role analogous to, say, spin (Equation 4.28). The eigenvalues of K would be 0 and 1, corresponding to $+1$ and -1 for P, and multiplication of parities would correspond to addition of K.

The problem of θ, τ particles was resolved by Lee and Yang in 1956, proposing parity is not conserved i.e. broken, in weak interaction

Parity and θ, τ puzzle

How to test whether parity is conserved ?

Do experiment.

(1) Prepare a physical system (object) or a reaction process which has a handedness (chirality)

(2) Investigate the mirror image of this chiral object or reaction. Thus a LH reaction occurs as a RH reaction in the mirror world

LH = left hand, RH = right hand.

(3) If mirror events = physical events (events in physical world), then parity is conserved

(4) If mirror event $\neq$ physical event, then, as a phenomenon, parity is not conserved (i.e. broken).

(5) However the nonconservation of parity in the events (phenomena) may be due to the initial choice of solutions, or it may be due to dynamics

(6) If it can be attributed to the initial choice of solutions, we can still claim parity is conserved. (E.g. human heart position on the left side of the chest.)

otherwise, the nonconservation is due the dynamics

$$[\pi, H] \neq 0$$

and we say parity is broken

Discuss downfall of parity conservation i.e reflection symmetry broken.

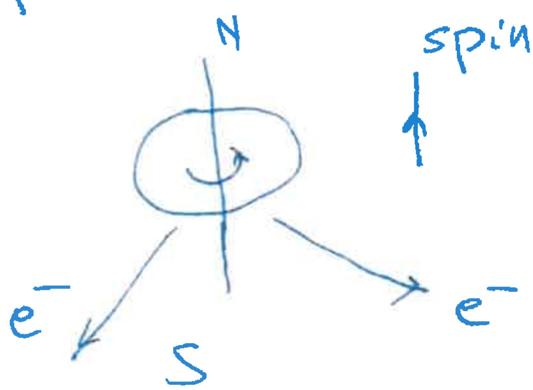
C.S. Wu experiment 1956

$$^{60}_{27}Co \rightarrow ^{60}_{28}Ni + e^- + \bar{\nu}_e$$

(basically, $n \rightarrow p + e^- + \bar{\nu}_e$

or $d \rightarrow u + e^- + \bar{\nu}_e$)

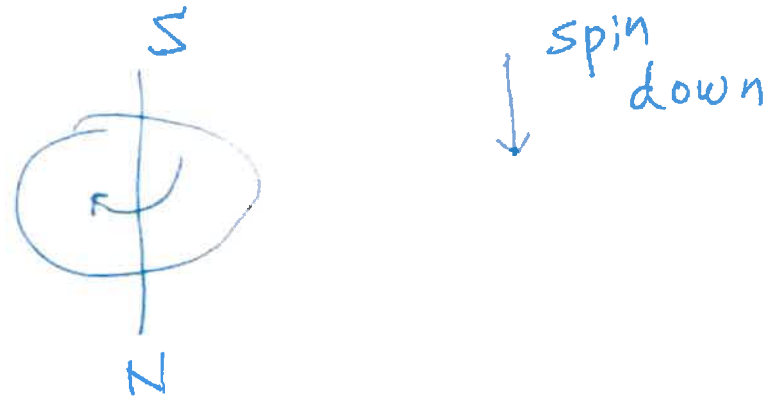
The cobalt 60 nuclei were cooled to very low temperature so that their spins were aligned up



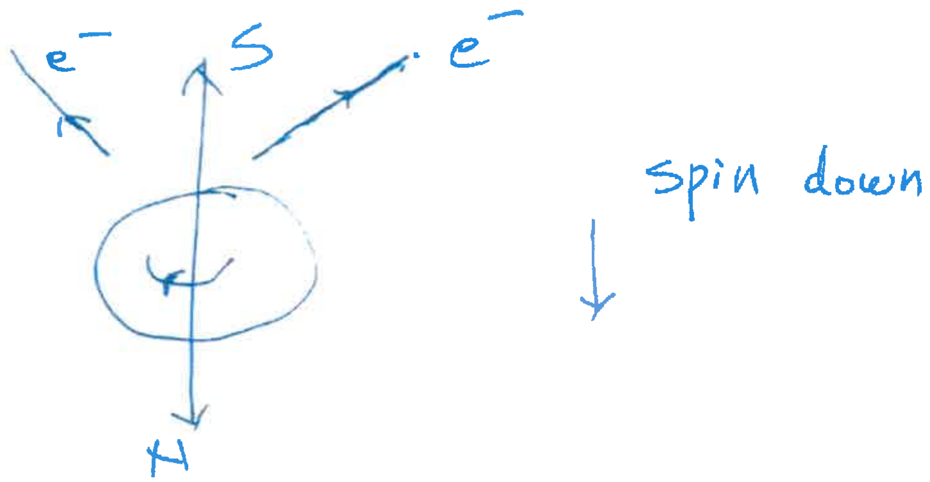
the electrons were detected in the 'southerly' direction, opposite to the ^{60}Co spin up direction.

Repeat the experiment.

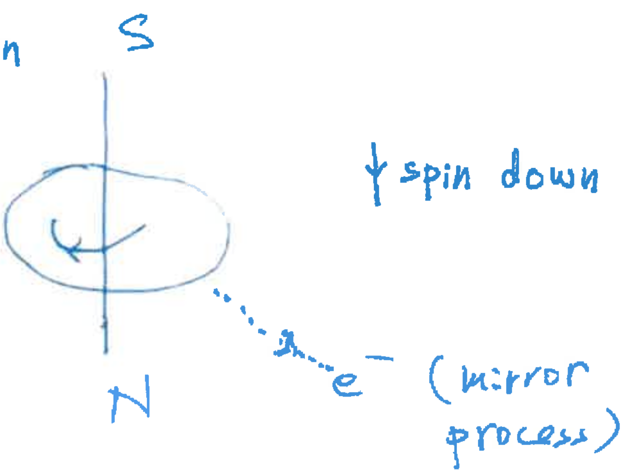
Invert spins of the ^{60}Co nuclei to the 'down' direction.



Again the electrons were emitted opposite to the nuclear spin direction



The image process as shown was not detected.



→ Parity is broken

If the image process (e^- emitted in the same direction as the nuclear spin) were observed, then in physical world both original process and image process occur.

That means one cannot differentiate between the physical world and the mirror world.

That would mean parity would be conserved.

C S Wu experiment clearly indicates parity is broken.

Note that we did point out before mirror reflection can be done with a rotation in a higher dimensional space. E. g. reflection in 2-dim. can be realized as rotation in 3-dim

2-dim.

p can be rotated to q in 3-dimensional space.

$${}^{60}_{27}Co \rightarrow {}^{60}_{28}Ni + e^- + \bar{\nu}_{e^-}$$

$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$d \rightarrow u + e^- + \bar{\nu}_e$$

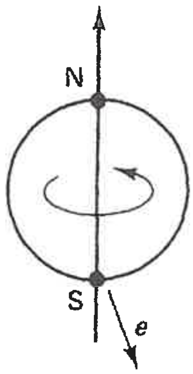


Fig. 4.7 In the beta decay of cobalt 60, most electrons are emitted in the direction opposite to the nuclear spin.

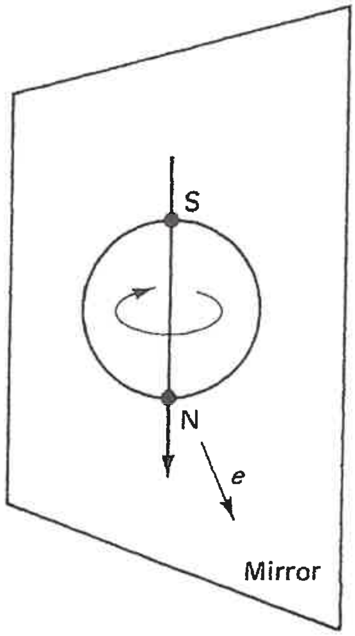


Fig. 4.8 Mirror image of Figure 4.7: Most electrons are emitted parallel to the nuclear spin.

The asymmetric distribution of the electrons emitted in the ${}^{60}Co$ decays indicates parity is broken

see in Chapter 9, it is in fact 'maximal'. Nor is it limited to beta decay in cobalt; once you *look* for it, parity violation is practically the *signature* of the weak force. It is most dramatically revealed in the behavior of the neutrino. In the theory of angular momentum, the axis of quantization is, by convention, the z axis. Of course, the orientation of the z axis is completely up to us, but if we are dealing with a particle traveling through the laboratory at velocity $\mathbf{v}$, a natural choice suggests itself: why not pick the *direction of motion* as the z axis? The value of m_s/s for this axis is called the *helicity* of the particle. Thus a particle of spin $\frac{1}{2}$ can have a helicity of $+1(m_s = \frac{1}{2})$ or $-1(m_s = -\frac{1}{2})$; we call the former 'right-handed' and the latter 'left-handed.'* The difference is not terribly profound, however, because it is not Lorentz-invariant. Suppose I have a right-handed electron going to the right (Figure 4.9a), and someone else looks at it from an inertial system traveling to the right at a speed *greater* than v. From *his* perspective, the electron is going to the *left* (Figure 4.9b); but it is still spinning the same way, so this observer will say it's a *left*-handed electron. In other words, you can convert a right-handed electron into a left-handed one simply by changing your frame of reference. That's what I mean, when I say the distinction is not Lorentz-invariant.

But what if we applied that same reasoning to a *neutrino* – taken, for the moment, to be *massless*, so it travels at the speed of light, and hence there *is* no observer traveling faster? It is *impossible* to 'reverse the direction of motion' of a (massless) neutrino by getting into a faster-moving reference system, and therefore the helicity

* In Chapter 9, I shall introduce a technical distinction between 'handedness' and *helicity*, but for the moment I will use the terms interchangeably.

Read: Scientific American, April 1957 volume 196, number 4, pages 45-53 (1957): The overthrow of parity

Further discussion on

Parity

Our physical right hand (RH) becomes a Left hand (LH) in the mirror world.

If in our physical world, we all have only one hand, the RH, then we can use hand to differentiate between a physical world and a mirror world. As soon as we see only RH human beings we immediately know we are in physical world whereas if we see only LH human beings, we are in the mirror world. Because we can distinguish RH and LH distinctly, we say space inversion (mirror reflection) is not a symmetry.

However in our physical world, we have both RH and LH equally. In other words, RH and its image LH, or LH and its image RH, both appear in our physical world (and hence also equally in the mirror world). We therefore cannot use our hand as an indicator to say whether we are in the physical world or in the mirror world. As far as hands are concerned, physical world and mirror world are the same. We say space inversion is a perfect symmetry (symmetry $\Rightarrow$ loss of information)

On the other hand, we can use the position of a human heart in a human being to differentiate a physical world and a mirror world because in the physical world almost all human hearts are on the left hand side of the chest, whereas the other way in the mirror world. So phenomenologically human heart indicates that space inversion is a broken symmetry. However human heart is based on the electromagnetic interaction which is described by the Maxwell equation. Maxwell's equation remains the same under space inversion. So although as a phenomenon human heart breaks space inversion symmetry, its dynamics still obeys space inversion symmetry

CS Wu experiment involves a physical process the weak decay of Co^{60} to Ni^{60} ($n \to p + e^- + \bar{\nu}_e$ or $d \to u + e^- + \bar{\nu}_e$) As a physical process (the decay of Co^{60}), the e^- is mostly detected in a direction opposite to the nucleus (Co^{60}) spin direction (momentum of the electron $e^- = \underset{\sim}{p}$, and the spin $\underset{\sim}{s}$ of the Co^{60} in opposite directions) The mirror process in which $\underset{\sim}{p}$ and $\underset{\sim}{s}$ same directions are hardly seen in our physical world so the decay process of Co^{60} is an indicator of broken space inversion symmetry. This broken space inversion symmetry is not only as a phenomenon, but also dynamical. This is because the decay process involves weak interaction. so we say weak interaction breaks the space inversion symmetry. We can use weak interaction process as an indicator to determine whether we are in a physical world ($\underset{\sim}{p}$, $\underset{\sim}{s}$ opposite) or in a mirror world ($\underset{\sim}{p}$, $\underset{\sim}{s}$ same direction).

Perfect symmetry implies loss of information.

But to discuss physics, we still needs to assign convention to 'break' the symmetry.

Consider a sphere. We need to assign its 'north' (N) or 'south' (S) to describe the sphere fully. That is, N, S is a convention.

This N, S convention can be assigned rigidly (globally) or non-rigidly (flexibly, or locally)

However it appears that space inversion symmetry, supposed is a perfect symmetry, the convention of left hand (L) or right hand (R) can only be assigned rigidly (globally). Theoretically we still do not know how to assign L, R convention locally.

We do not know how to localize or 'gauge' the space inversion symmetry

PC 4245

Handedness is a property of a physical system or a physical process under mirror reflection or space inversion. If under mirror reflection, the physical system or physical process remains the same, we say the system has no handedness, otherwise, it is either left or right handed. E. g. sphere has no handedness, a screw is right-handed.

Helicity is defined as the projection of the spin of the physical system along its momentum direction.

For a spin $\frac{1}{2}$ particle, its helicity is defined as

$$h(\vec{p}) = \frac{\vec{\Sigma} \cdot \vec{p}}{\vec{p}}$$

where $\frac{\hbar}{2}\vec{\Sigma}$ is spin operator of the particle.

Helicity is a **kinematic** property for a system in motion, this system must also have a nonzero spin.

Chirality is defined as an **intrinsic** property of an elementary particle. For a spin ½ particle, chirality is given by the Dirac matrix γ^5

Conceptually the two are different, helicity not the same as chirality.

However it is often stated that the two are the same if they are massless. This is easy to prove for spin 1/2 system by using the Dirac equation.

Handedness as a term is applicable to helicity and chirality, by usage convention.

Charge conjugation

Originally charge conjugation means +ve electric charge changes to −ve electric charge and vice versa

Charge conjugation C is a symmetry transformation for the Maxwell equations

$$\nabla \cdot \underset{\sim}{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \wedge \underset{\sim}{E} = -\frac{\partial \underset{\sim}{B}}{\partial t}$$

$$c^2 (\nabla \wedge \underset{\sim}{B}) = \frac{\underset{\sim}{j}}{\epsilon_0} + \frac{\partial \underset{\sim}{E}}{\partial t}$$

$$\nabla \cdot \underset{\sim}{B} = 0$$

Under C, $\rho \to -\rho$, $\underset{\sim}{j} \to -\underset{\sim}{j}$

$\underset{\sim}{E} \to -\underset{\sim}{E}$ $\quad \underset{\sim}{B} \to -\underset{\sim}{B}$ $\because \underset{\sim}{F} = q(\underset{\sim}{E} + \underset{\sim}{v} \wedge \underset{\sim}{B})$

Maxwell's equations remain unchanged.

Extend charge conjugation to flip sign of all internal quantum numbers

Apply charge conjugation C to elementary particles

Few particles are eigenstates of C

To be eigenstate of C, the particle must be neutral. Yet neutron is not an eigenstate of C, neutron ≠ antineutron

(20)

Table 4.6 Quantum numbers of some meson nonets

Orbital angular momentum	Net spin	J^{PC}	Observed Nonet $I=1$	$I=\frac{1}{2}$	$I=0$	Average mass (MeV/c^2)
$l=0$	$s=0$	0^{-+}	π	K	η, η'	400
	$s=1$	1^{--}	ρ	K^*	ϕ, ω	900
$l=1$	$s=0$	1^{+-}	b_1	K_{1_B}	h_1, h_1	1200
	$s=1$	0^{++}	a_0	K_0^*	f_0, f_0	1100
	$s=1$	1^{++}	a_1	K_{1_A}	f_1, f_1	1300
	$s=1$	2^{++}	a_2	K_2^*	f'_2, f_2	1400

It seemed peculiar that two otherwise identical particles should carry different parity. The alternative, suggested by Lee and Yang in 1956 was that τ and θ are really the *same particle* (now known as the K^+), and parity is simply not conserved in one of the decays. This idea prompted their search for evidence of parity invariance in the weak interactions and, when they found none, to their proposal for an experimental test.

4.4.2
Charge Conjugation

Classical electrodynamics is invariant under a change in the sign of all electric charges; the potentials and fields reverse their signs, but there is a compensating charge factor in the Lorentz law, so the forces still come out the same. In elementary particle physics, we introduce an operation that generalizes this notion of 'changing the sign of the charge'. It is called *charge conjugation, C*, and it converts each particle into its antiparticle:

$$C|p\rangle = |\bar{p}\rangle \tag{4.54}$$

'Charge conjugation' is something of a misnomer, for C can be applied to a neutral particle, such as the neutron (yielding an antineutron), and it changes the sign of *all* the 'internal' quantum numbers – charge, baryon number, lepton number, strangeness, charm, beauty, truth – while leaving mass, energy, momentum, and spin untouched.

As with P, application of C *twice* brings us back to the original state:

$$C^2 = I \tag{4.55}$$

and hence the eigenvalues of C are ± 1. Unlike P, however, most of the particles in nature are clearly *not* eigenstates of C. For if $|p\rangle$ is an eigenstate of C, it follows that

$$C|p\rangle = \pm|p\rangle = |\bar{p}\rangle \tag{4.56}$$

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so $|p\rangle$ and $|\overline{p}\rangle$ differ at most by a sign, which means that they represent the same physical state. Thus, *only those particles that are their own antiparticles can be eigenstates of C*. This leaves us the photon, as well as all those mesons that lie at the center of their Eightfold-Way diagrams: π^0, η, η', ρ^0, ϕ, ω, ψ, and so on. Because the photon is the quantum of the electromagnetic field, which changes sign under C, it makes sense that the photon's 'charge conjugation number' is -1. It can be shown [19] that a system consisting of a spin-$\frac{1}{2}$ particle and its antiparticle, in a configuration with orbital angular momentum l and total spin s, constitutes an eigenstate of C with eigenvalue $(-1)^{l+s}$. According to the quark model, the mesons in question are of precisely this form: for the pseudoscalars, $l = 0$ and $s = 0$, so $C = +1$; for the vectors, $l = 0$ and $s = 1$, so $C = -1$. (Often, as in Table 4.6, C is listed as though it were a valid quantum number for the entire supermultiplet; in *fact* it pertains only to the central members.)

π meson
peudoscalar
$C\,\pi^0 = \pi^0$

Charge conjugation is a multiplicative quantum number, and, like parity, it is conserved in the strong and electromagnetic interactions. Thus, for example, the π^0 decays into two photons:

$$\pi^0 \to \gamma + \gamma \tag{4.57}$$

(for n photons $C = (-1)^n$, so in this case $C = +1$ before and after), but it cannot decay into three photons. Similarly, the ω goes to $\pi^0 + \gamma$, but never to $\pi^0 + 2\gamma$. In the strong interactions, charge conjugation invariance requires, for example, that the energy distributions of the charged pions in the reaction

ω vector
meson
$C\,\omega = -\omega$

$$p + \overline{p} \to \pi^+ + \pi^- + \pi^0 \tag{4.58}$$

should (on average) be identical [20]. On the other hand, charge conjugation is *not* a symmetry of the weak interactions: when applied to a neutrino (left-handed, remember), C gives a left-handed *anti*neutrino, which does not occur. So the charge-conjugated version of any process involving neutrinos is *not* a possible physical process. And purely hadronic weak interactions also show violations of C as well as P.

Because so few particles are eigenstates of C, its direct application in elementary particle physics is rather limited. Its power can be somewhat extended, if we confine our attention to the strong interactions, by combining it with an appropriate isospin transformation. Rotation by 180° about the number 2 axis in isospin space* will carry I_3 into $-I_3$, converting, for instance, a π^+ into a π^-. If we then apply the charge conjugation operator, we come back to π^+. Thus the charged pions are eigenstates of this combined operator, even though they are not eigenstates of C alone. For some reason the product transformation is called 'G-parity':

$$G = CR_2, \quad \text{where} \quad R_2 = e^{i\pi I_2} \tag{4.59}$$

* Some authors use the number 1 axis. Obviously, any axis in the 1–2 plane will do the job.

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All mesons that carry no strangeness (or charm, beauty, or truth) are eigenstates of G;* for a multiplet of isospin I the eigenvalue is given (Problem 4.36) by

$$G = (-1)^I C \tag{4.60}$$

where C is the charge conjugation number of the neutral member. For a single pion, $G = -1$, and for a state with n pions

$$G = (-1)^n \tag{4.61}$$

This is a very handy result, for it tells you how many pions can be emitted in a particular decay. For example, the ρ mesons, with $I = 1$, $C = -1$, and hence $G = +1$, can go to *two* pions, but not to three, whereas the ϕ, the ω, and the ψ (all $I = 0$) can go to *three*, but not to two.

4.4.3 *CP*

As we have seen, the weak interactions are not invariant under the parity transformation P; the cleanest evidence for this is the fact that the antimuon emitted in pion decay

parity of LH (4.62) $= -1$

$$\pi^+ \to \mu^+ + \nu_\mu \tag{4.62}$$

Parity of RH 4.62) $= (-1)\cdot(-1) = +1$

always comes out left-handed. Nor are the weak interactions invariant under C, for the charge-conjugated version of this reaction would be

$$\pi^- \to \mu^- + \bar{\nu}_\mu \tag{4.63}$$

with a left-handed muon, whereas in fact the muon always comes out *right*-handed. However, if we *combine* the two operations we're back in business: CP turns the left-handed antimuon into a *right*-handed muon, which is exactly what we observe in nature. Many people who had been shocked by the fall of parity were consoled by this realization; perhaps, it was the combined operation that our intuition had been talking about all along – maybe what we should have *meant* by the 'mirror image' of a right-handed electron was a left-handed *positron*.† If we had defined parity from the start to be what we now call CP, the trauma of parity violation might have been avoided (or at least postponed). It is too late to change the terminology

* K^+, for example, is *not* an eigenstate of G, for R_2 takes it to K^0, and C takes that to $\overline{K}^0$. The idea could be extended to the K's, by using an appropriate $SU(3)$ transformation in place of R_2, but since $SU(3)$ is not a very good symmetry of the strong forces, there is little percentage in doing so.

† Incidentally, we could perfectly well take electric charge to be a pseudoscalar in classical electrodynamics; **E** becomes a pseudovector and **B** a vector, but the results are all the same. It is really a matter of taste whether you say the mirror image of a plus charge is positive or negative. But it seems simplest to say the charge does *not* change, and this is the standard convention.

photon and all those mesons that lie at the center of their Eightfold-Way diagrams : π^0, η, η' ρ^0, ϕ, ω, ψ ... are eigenstates of charge conjugation operator C

For mesons, eigenvalue of $C = (-1)^{l+s}$

l = orbital angular momentum, s = spin

For pseudoscalar mesons, $l = 0$ and $s = 0$ $\therefore C = 1$

e.g $C \pi^0 = \pi^0$

For vector mesons, $l = 0$, $s = 1$, $\therefore C = -1$

e.g $C \omega = -\omega$

Not many hadrons are eigenstates of C.

Consider G-parity for hadrons,

combining C with isospin rotation

$$C e^{i\pi I_2}$$

$=$

Consider weak decays, e.g. pion decay

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

Pion decay violates charge conjugation

Consider the weak decay of π^-: $\pi^- \to \mu^- + \bar{\nu}_\mu$

In the frame at which the π^- is at rest,

RH (right hand) muon

RH antineutrino

(1)

Under charge conjugation C, we expect to get

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$

C changes internal quantum numbers only, C does not change angular momentum (spin polarization), so Fig(1) becomes

(2)

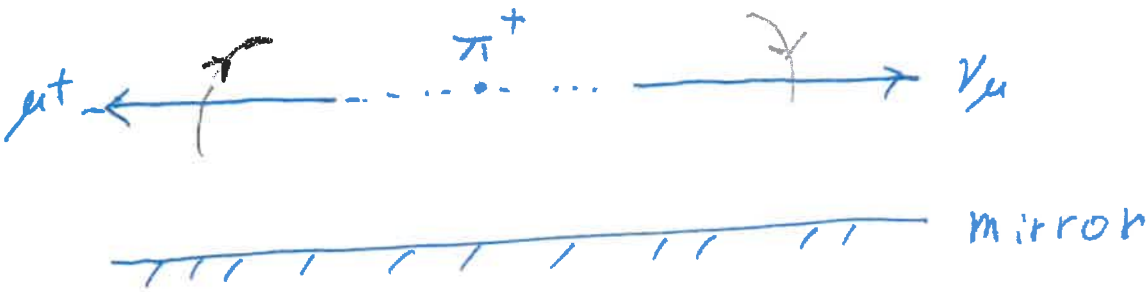
But in nature, we observe

(3)

This means the decay of π^- violates the charge conjugation since fig(2) $\neq$ fig (3).

If we apply parity operator $\overset{P}{\pi}$ to fig (2), we can get fig (3)

place a mirror beneath fig (2)



we then get fig (3)

so applying C and P jointly ie CP, fig(1) becomes fig (3) which is realized in nature. CP is a symmetry obeyed by π^- decay

Discuss CP violation

All reactions obey CP symmetry (e.g. in the decay of π^- as illustrated in the previous figures) except the Kaons (containing strange quark s $K^\circ = d\bar{s}$) decays, the B ($B^\circ = d\bar{b}$, containing b quark) decays and possibley the D decay (containing c quark, $D^\circ = c\bar{u}$) $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$

Kaons K° (strangeness +1), $\bar{K}^\circ$ (strangeness −1) are produced in strong interaction processes, e.g.

$$\pi^- p \to \Lambda^\circ \; K^\circ \qquad K^\circ = d\bar{s}$$

$$K^- p \to n \; \bar{K}^\circ$$

$$\pi^+ p \to p + K^+ + \bar{K}^\circ$$

(see Feynman Lecture Vol 3, p. 11-12 to 11-20)

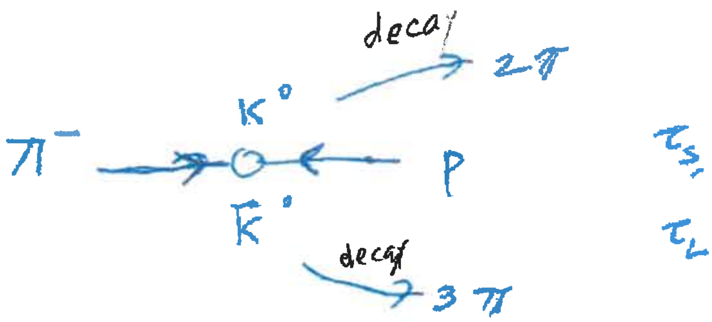
K° and $\bar{K}^\circ$ are eigenstates of H_{st} (strong interaction Hamiltonian) and H_{em}. Let $H_\circ = H_{st} + H_{em}$ ← electromagnetic

$$H_\circ |K^\circ\rangle = E_\circ |K^\circ\rangle, \qquad H_\circ |\bar{K}^\circ\rangle = E_\circ |\bar{K}^\circ\rangle$$

Experimentally (i) K° oscillates to $\bar{K}^\circ$ and (ii) they decay into 2π or 3π via weak interaction

The decays have 2 different life times

$$\tau_S = 0.89 \times 10^{-10} \text{ s}, \qquad \tau_L = 5.2 \times 10^{-8} \text{ s}$$



strangeness
$S=+1$ $K^0 \rightleftharpoons \bar{K}^0$ strangeness $S=-1$

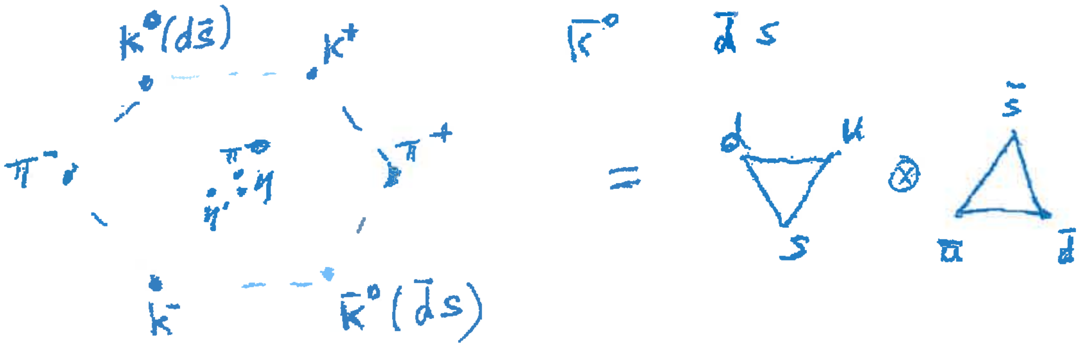
The oscillation K^0 to $\bar{K}^0$ and vice versa
is due to decay channels 2π, 3π

$$K^0 \underset{3\pi}{\overset{2\pi}{\rightleftarrows}} \bar{K}^0$$

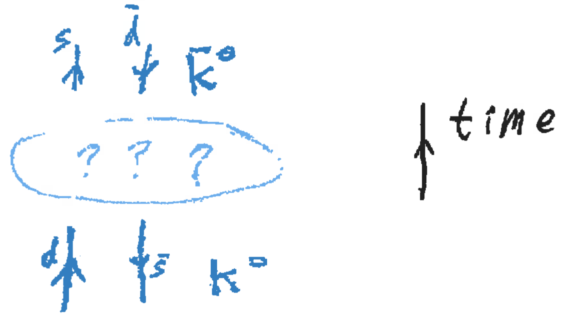
see Feynman Lecture vol III Eq(11.43)

and is explained by weak interaction, H_{wk}
Look at quark content:

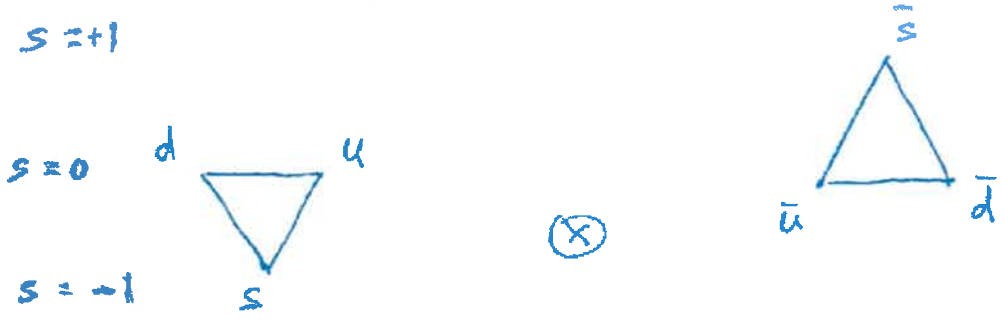
K^0 $d\bar{s}$
$\bar{K}^0$ $\bar{d}s$



Oscillation via Feynman diagram
start with external lines



27a



K^0

$= \qquad d\bar{s} \qquad u\bar{s}$

$d\bar{u} \qquad d\bar{d},\ u\bar{u},\ s\bar{s} \qquad u\bar{d}$

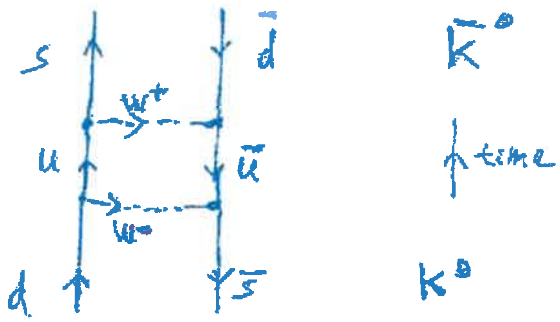
$s\bar{u} \qquad s\bar{d} \quad \bar{K}^0$

Get the relevant weak interaction vertex (see 2nd lecture PowerPoint)

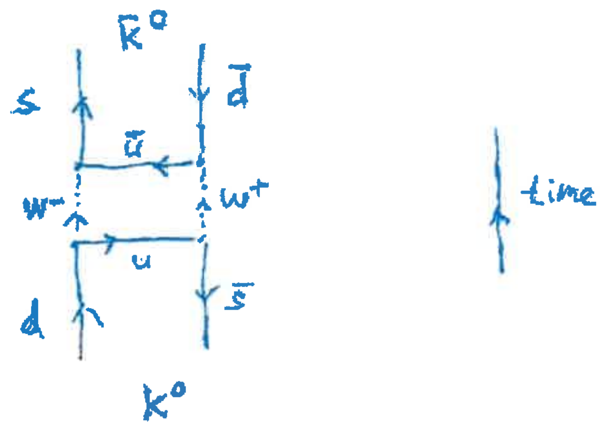
$W^{\pm}$ = intermediate bosons

Insert vertices to complete the diagram

an example is



similarly



The above two diagrams illustrates the oscillations of $K^0 \rightleftharpoons \bar{K}^0$

We now proceed to explain the two decay modes: 2 π's and 3 π's and also two lifetimes τ_s, τ_L

K^0, $\overline{K^0}$ are produced by strong interaction. Immediately they decay by weak interaction. One can say the kaons appear as particles K^0, $\overline{K^0}$ when interact strongly, but appear as particles K_s, K_L when interact weakly

Gellman & Pais (1955) proposed to use linearly superposed states, K_s, K_L are linearly superposed from K^0, $\overline{K^0}$

$$|K_s\rangle \sim |K^0\rangle - |\overline{K^0}\rangle \quad \rightarrow \tau_s$$

$$|K_L\rangle \sim |K^0\rangle + |\overline{K^0}\rangle \quad \rightarrow \tau_L$$

Applying charge conjugation C and parity operato P

$$CP\,|K_s\rangle \sim CP\,(|K^0\rangle - |\overline{K^0}\rangle)$$

Note: Previously we denote parity operator (space inversion operator) by π.

Now

$$CP|K^0\rangle = C(-)|K^0\rangle = -C|K^0\rangle = -|\bar{K}^0\rangle$$

$$CP|\bar{K}^0\rangle = C(-)|\bar{K}^0\rangle = -C|\bar{K}^0\rangle = -|K^0\rangle$$

hence

$$CP|K_S\rangle \sim +|K_S\rangle \qquad \text{HW}$$

$$CP|K_L\rangle \sim -|K_L\rangle \qquad \text{HW}$$

i.e.

$|K_S\rangle$ eigenstate of CP with eigenvalue +1

$|K_L\rangle$ eigenstate of CP with eigenvalue −1

With the introduction of kaons as K_S, K_L in weak decay, we can account for the 2πs and 3πs decays.

For 2 pions, $CP|\pi^0\pi^0\rangle = C(-1)(-1)|\pi^0\pi^0\rangle$

$$= C|\pi^0\pi^0\rangle = |\pi^0\pi^0\rangle$$

$$CP|\pi^+\pi^-\rangle = C(-1)(-1)|\pi^+\pi^-\rangle = C|\pi^+\pi^-\rangle = |\pi^-\pi^+\rangle$$

Similarly for 3 pions, $CP = (-1)(-1)(-1) = -1$.

If CP is conserved in weak decay, then

$K_S \rightarrow 2$ pions and $K_L \rightarrow 3$ pions

To check CP conservation, the kaons after production can be separated out as K_S and K_L particles

1964 Cronin-Fitch

source 57 ft away $\approx 17.4\,m$

$*$ $\longrightarrow$ Detectors

K^0, $\overline{K^0}$ are produced

only K_L remained (no more K_S)

found 45 2-pion decays among 22700 decays.

If CP is conserved, K_L can only decay to 3 pions. So this experiment indicates CP is not conserved in kaon decays.

However the violation of CP conservation is very small

$$\frac{45}{22700} \sim 2 \times 10^{-3}$$

We proceed to account for $K^0 \rightleftarrows \overline{K^0}$ oscillation and CP violation of kaon decays by treating the kaon as a 2-state system, see Feynman lecture vol 3, chapter 11.