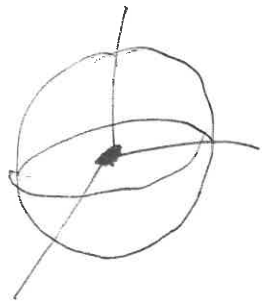
prove $\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi \delta^3(\vec{r})$



$$LHS = \int_V \nabla \cdot \left(\frac{\hat{r}}{r^2}\right) d\tau$$

$$= \oint_S \frac{\hat{r}}{r^2} \cdot d\vec{a}$$

$$d\vec{a} = \hat{r}\, d\theta\, d\phi$$

$$= \hat{r}\, r^2 \sin\theta\, d\theta\, d\phi$$

$$= \int\int \frac{\hat{r}}{r^2} \cdot \hat{r}\, r^2 \sin\theta\, d\theta\, d\phi$$

$$= 4\pi$$

$$RHS = \iiint 4\pi\, \delta^3(\vec{r})\, d\tau = 4\pi \quad \checkmark \text{ proved}$$