We have learned how to obtain the scattering amplitude $\mathcal{M}(\underline{p}_1 s_1; \underline{p}_2 s_2; \underline{p}_3 s_3; \underline{p}_4 s_4) \equiv \mathcal{M}$ for a two-particle to two-particle scattering process using Feynman rules for tree-level Feynman diagrams.

In many experiments, the spins (polarizations) of the particles are not measured. For each incident spins, e.g $s_1 = \frac{1}{2}$, $s_2 = -\frac{1}{2}$, we sum up all the possible values of the final spins, e.g. $s_3 = \frac{1}{2}$, $s_4 = \frac{1}{2}$; $s_3 = \frac{1}{2}$, $s_4 = -\frac{1}{2}$; $s_3 = -\frac{1}{2}$, $s_4 = \frac{1}{2}$; $s_3 = -\frac{1}{2}$, $s_4 = -\frac{1}{2}$.

As each set of definite values of s_1, s_2, s_3, s_4 can be differentiated from other set of values, hence we compute total $|\mathcal{M}|^2 = \mathcal{M} \cdot \mathcal{M}^*$, not total $\mathcal{M}$.

So we sum up all final spin configurations and average over all initial spin configuration.

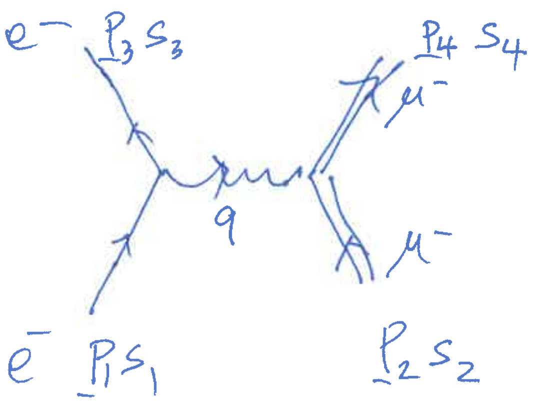
Total number of initial spin configurations $= 4$ for spin $\frac{1}{2}$ particles

Thus define

$$\langle |\mathcal{M}|^2 \rangle = \text{average over initial spins, sum over final spins}$$

$$= \frac{1}{4} \sum_{s_1 s_2 s_3 s_4} |\mathcal{M}|^2$$

As an example, consider $e^- \mu^- \to e^- \mu^-$



$$q^2 = (\underline{p}_1 - \underline{p}_3)^2$$
$$= (\underline{p}_4 - \underline{p}_2)^2$$

$$\mathcal{M} = -\frac{g^2}{q^2} \underbrace{\bar{u}(4)\gamma_\mu u(2)}_{\text{a number}} \cdot \underbrace{\bar{u}(3)\gamma^\mu u(1)}_{\text{number}}$$

$$|\mathcal{M}|^2 = \mathcal{M} \cdot \mathcal{M}^*$$

$$= \frac{g^4}{q^4} \bar{u}(4) \gamma_\mu u(2) \cdot \bar{u}(3) \gamma^\mu u(1) \cdot \left(\bar{u}(4) \gamma_\nu u(2) \cdot \bar{u}(3) \gamma^\nu u(1) \right)^*$$

$$= \frac{g^4}{q^4} \underline{\bar{u}(4) \gamma_\mu u(2)} \cdot \underline{\bar{u}(3) \gamma^\mu u(1)} \cdot \underline{\bar{u}(1) \gamma^\nu u(3)} \; \underline{\bar{u}(2) \gamma_\nu u(4)}$$

Completeness for the Dirac spinor $u^{(s)}(p)$:

$$\sum_s u^{(s)}(p) \, \bar{u}^{(s)}(p) = (\not{p} + mc)$$

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{4} \sum_{s_1 s_2 s_3 s_4} |\mathcal{M}|^2$$

$$\neq \frac{g^4}{4q^2} \sum_{\substack{s_1 s_2 \\ s_3 s_4}} \bar{u}(3) \gamma^\mu u(1) \cdot \bar{u}(1) \gamma^\nu u(3) \cdot \bar{u}(4) \gamma_\mu u(2) \, \bar{u}(2) \gamma_\nu u(4)$$

$$= \frac{g^4}{4q^2} \sum_{s_3 s_4} \bar{u}(3) \gamma^\mu (\not{p}_1 + m_1 c) \cdot \gamma^\nu u(3) \cdot \bar{u}(4) \gamma_\mu (\not{p}_2 + m_2 c) \cdot \gamma_\nu u(4)$$

Consider

$$\sum_{s_3} \bar{u}(3)\, \gamma^\mu (\not{p}_1 + m_1 c)\, \gamma^\nu \; u(3)$$

$$= \sum_{s_3} \bar{u}_a(3) \left(\gamma^\mu (\not{p}_1 + m_1 c) \gamma^\nu \right)_{ab} u_b(3)$$

$a, b = 1, 2, 3, 4$

$$= \sum_{s_3} \left(\gamma^\mu (\not{p}_1 + m_1 c) \gamma^\nu \right)_{ab} u_b(3)\, \bar{u}_a(3)$$

$$= \left(\gamma^\mu (\not{p}_1 + m_1 c) \gamma^\nu \right)_{ab} \left(\not{p}_3 + m_3 c \right)_{ba}$$

$$= \mathrm{Tr}\left[\gamma^\mu (\not{p}_1 + m_1 c) \gamma^\nu \cdot (\not{p}_3 + m_3 c) \right]$$

$$\therefore \langle |M|^2 \rangle = \frac{g^4}{4 q^2} \mathrm{Tr}\left[\gamma^\mu (\not{p}_1 + m_1 c) \gamma^\nu (\not{p}_3 + m_3 c) \right] \cdot$$

$$\cdot \mathrm{Tr}\left[\gamma_\mu (\not{p}_2 + m_2 c) \gamma_\nu (\not{p}_4 + m_4 c) \right]$$

$$\mathrm{Tr}\,\gamma^{\mu} = 0, \qquad \mu = 0, 1, 2, 3$$

$$\mathrm{Tr}(\gamma^{\mu}\gamma^{\nu}) = 4 g^{\mu\nu}$$

$$\mathrm{Tr}(\gamma^{\mu}\gamma^{\nu}\gamma^{\alpha}) = 0 \quad , \qquad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

$$\gamma^{5^2} = 1$$

$$\mathrm{Tr}(\gamma^{\mu}\gamma^{\nu}\gamma^{\alpha}\gamma^{\beta})$$

$$= 4(g^{\mu\nu}g^{\alpha\beta} + g^{\beta\mu}g^{\nu\alpha} - g^{\mu\alpha}g^{\nu\beta})$$

$$\mathrm{Tr}[\gamma^{\mu}(\not{p}_1 + m_1 c)\gamma^{\nu}(\not{p}_3 + m_3 c)]$$

$$= \mathrm{Tr}[\gamma^{\mu}\not{p}_1\gamma^{\nu}\not{p}_3 + \gamma^{\mu}\not{p}_1\gamma^{\nu}m_3 c + m_1 c\gamma^{\mu}\gamma^{\nu}\not{p}_3$$

$$+ m_1 m_3 c^2 \gamma^{\mu}\gamma^{\nu}]$$

$$= \mathrm{Tr}[\gamma^{\mu}\not{p}_1\gamma^{\nu}\not{p}_3 + m_1 \cdot m_3 c^2 \gamma^{\mu}\gamma^{\nu}]$$

$$\because \mathrm{Tr}\,\gamma^{\mu}\gamma^{\alpha}\gamma^{\nu} = 0$$

Using Casimir's trick, have computed

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{4} \alpha \sum_{s_1 s_2 s_3 s_4} (\qquad)$$

for the $e^- \mu^- \to e^- \mu^-$ process

Using formula from the previous lecture

$$\mathrm{Tr}\,[\gamma_\mu (\not{p}_4 + m_4 c)\gamma_\nu (\not{p}_2 + m_2 c)]$$

$$\overset{HW}{=} 4\,[p_{2\mu} p_{4\nu} + p_{2\nu} p_{4\mu} - g_{\mu\nu}(p_2 p_4 - m_2 m_4 c^2)]$$

$\rightarrow$

$$\langle |\mathcal{M}|^2 \rangle = \frac{g^4}{(p_1 - p_3)^4} \frac{1}{4} \mathrm{Tr}[\gamma^\mu (\not{p}_1 + m_1 c)\gamma^\nu (\not{p}_3 + m_3 c)] \cdot \mathrm{Tr}[\gamma_\mu (\not{p}_2 + m_2 c) \cdot \gamma_\nu (\not{p}_4 + m_4 c)]$$

$$\overset{HW}{=} \frac{4 g^4}{(p_1 - p_3)^4} [p_{2\mu} p_{4\nu} + p_{2\nu} p_{4\mu} - g_{\mu\nu}(p_2 p_4 - m_2 m_4 c^2)] \cdot$$

$$\cdot [p_1^\mu p_3^\nu + p_1^\nu p_3^\mu - g^{\mu\nu}(p_1 p_3 - m_1 m_3 c^2)]$$

$$\overset{HW}{=} \frac{8 g^4}{(p_1 - p_3)^4} \Big[(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_2 \cdot p_3)(p_1 p_4)$$

$$- (p_2 \cdot p_4) m_1 m_3 c^2 - (p_1 \cdot p_3) m_2 m_4 c^2$$

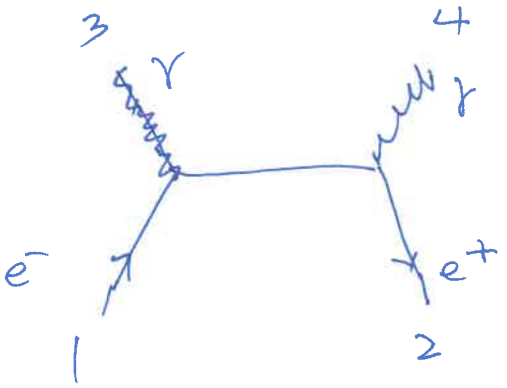
$$+ 2 (m_1 m_2 m_3 m_4) c^4\Big]$$

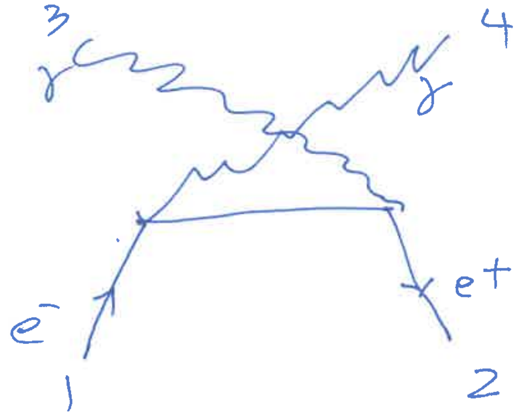
$$m_1 = m_3 = m_e$$

$$m_2 = m_4 = m_\mu$$

Griffith Probl. 7.40

Example. Pair annihilation $e^+e^- \to \gamma\gamma$





$$\mathcal{M} = g^2 \Big[\bar{v}(2)\, \not{\epsilon}^*(4) \frac{1}{\not{p}_1 - \not{p}_3 - mc} \not{\epsilon}^*(3)\, u(1)$$

$$+ \bar{v}(2)\, \not{\epsilon}^*(3) \frac{1}{\not{p}_1 - \not{p}_4 - mc} \not{\epsilon}^*(4)\, u(1) \Big]$$

Suppose the e^+e^- pair is in a singlet configuration, that is, both e^+ and e^- each has a definite spin value, then one can show

$$\mathcal{M}_{singlet} = -2\sqrt{2} \cdot i \cdot g^2 \left(\underset{\sim}{\epsilon}_3^* \wedge \underset{\sim}{\epsilon}_4^* \right)_3 , \qquad \underset{\sim}{\epsilon}_3 = \underset{\sim}{\epsilon}(\underset{\sim}{P}_3)$$

Griffiths Prob. 7.40 chapter 7

From eq (7.158)

$$\mathcal{M}_{\text{singlet}} = -2\cdot\sqrt{2}\cdot i\, g_e^2\, (\underset{\sim}{\varepsilon}_3^* \wedge \underset{\sim}{\varepsilon}_4^*)_z$$

$$\underset{\sim}{\varepsilon}_3 = \underset{\sim}{\varepsilon}(\underset{\sim}{p}_3)$$

$$|\mathcal{M}_{\text{singlet}}|^2 = 8\, g_e^4\, (\underset{\sim}{\varepsilon}_3 \wedge \underset{\sim}{\varepsilon}_4)_z \cdot (\underset{\sim}{\varepsilon}_3 \wedge \underset{\sim}{\varepsilon}_4)_z^*$$

$$= 8\, g_e^4\, \varepsilon_{3ij}\, \varepsilon_{3i}\, \varepsilon_{4j} \cdot \varepsilon_{3\ell m}\, \varepsilon_{3\ell}^*\, \varepsilon_{4m}^*$$

$$\varepsilon_{3i} = \varepsilon_i(\underset{\sim}{p}_3)$$

Completeness relation for polarization

$$\sum_{s=1,2} \varepsilon_i^{(s)}(\underset{\sim}{p})\, \varepsilon_j^{(s)*}(\underset{\sim}{p}) = \delta_{ij} - \hat{p}_i \hat{p}_j$$

$$\hat{p}_i = \frac{p_i}{|\underset{\sim}{p}|}$$

$\langle |\mathcal{M}|^2 \rangle$ = average over initial spins, sum over final spins

$$= \frac{1}{4} \sum_{s_1 s_2 s_3 s_4} |\mathcal{M}|^2$$

As initial configuration is already in a singlet state, so

$$\langle |M|^2 \rangle = \sum_{s_3 s_4} |M|^2$$

$$= 8 g_e^4 \, \varepsilon_{3ij} \, \varepsilon_{3lm} \left(\delta_{il} - \hat{P}_{3i} \hat{P}_{3l} \right) \left(\delta_{jm} - \hat{P}_{4j} \hat{P}_{4m} \right)$$

$$= 8 g_e^4 \Big[\varepsilon_{3lm} \, \varepsilon_{3lm} - \varepsilon_{3lj} \, \varepsilon_{3lm} \hat{P}_{4j} \hat{P}_{4m}$$

$$- \varepsilon_{3im} \, \varepsilon_{3lm} \hat{P}_{3i} \hat{P}_{3l} + \varepsilon_{3ij} \, \varepsilon_{3lm} \hat{P}_{3i} \hat{P}_{3l} \hat{P}_{4j} \hat{P}_{4m} \Big]$$

$$\varepsilon_{3lm} \, \varepsilon_{3lm} = \varepsilon_{31m} \, \varepsilon_{31m} + \varepsilon_{32m} \, \varepsilon_{32m}$$

$$= \varepsilon_{312} \, \varepsilon_{312} + \varepsilon_{321} \, \varepsilon_{321} = 1 \cdot 1 + (-1) \cdot (-1) = 2$$

$$\varepsilon_{3lj} \, \varepsilon_{3lm} \hat{P}_{4j} \hat{P}_{4m}$$

$$= \left(\delta_{33} \delta_{jm} - \delta_{3m} \delta_{j3} \right) \hat{P}_{4j} \hat{P}_{4m}$$

$$= \hat{P}_{4j} \hat{P}_{4j} - \hat{P}_{43} \hat{P}_{43}$$

From eq (7.136) Griffiths chapter 7

$\hat{\underset{\sim}{P}}_3 = (0, 0, 1)$, $\quad \hat{\underset{\sim}{P}}_4 = (0, 0, -1)$

$$\therefore \quad \varepsilon_{3\ell j}\, \varepsilon_{3\ell m}\, \hat{P}_{4j}\, \hat{P}_{4m} = \hat{\underset{\sim}{P}}_4^{\,2} - \hat{P}_{43}\, \hat{P}_{43} = 1 - 1 = 0$$

$$\varepsilon_{3im}\, \varepsilon_{3\ell m}\, \hat{P}_{3i}\, \hat{P}_{3\ell} = \hat{P}_{3\ell}\, \hat{P}_{3\ell} - \hat{P}_{33}\, \hat{P}_{33} = 1 - 1 = 0$$

$$(\hat{\underset{\sim}{P}}_3 \wedge \hat{\underset{\sim}{P}}_4)_3 = \varepsilon_{3ij}\, \hat{P}_{3i}\, \hat{P}_{4j} = \varepsilon_{31j}\, \hat{P}_{31}\, \hat{P}_{4j} + \varepsilon_{32j}\, \hat{P}_{32}\, \hat{P}_{4j}$$
$$= 0$$

Hence

$$\langle |\mathcal{M}|^2 \rangle = \langle |\mathcal{M}_{singlet}|^2 \rangle$$

$$= 16\, g_e^4$$

To show the helicity operator $S(\underset{\sim}{p}) = \frac{\hbar}{2} \underset{\sim}{\Sigma} \cdot \frac{\underset{\sim}{p}}{|\underset{\sim}{p}|}$

commutes with the Dirac Hamiltonian

$$H = c \underset{\sim}{\alpha} \cdot \underset{\sim}{p} + \beta m c^2, \quad \underset{\sim}{\alpha} = \begin{pmatrix} 0 & \underset{\sim}{\sigma} \\ \underset{\sim}{\sigma} & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[S(\underset{\sim}{p}), H] = \frac{\hbar}{2} \frac{1}{|\underset{\sim}{p}|} [\underset{\sim}{\Sigma} \cdot \underset{\sim}{p}, H]$$

$$= \frac{\hbar}{2|\underset{\sim}{p}|} \left[\begin{pmatrix} \underset{\sim}{\sigma} \cdot \underset{\sim}{p} & 0 \\ 0 & \underset{\sim}{\sigma} \cdot \underset{\sim}{p} \end{pmatrix}, \begin{pmatrix} m c^2 & c \underset{\sim}{\sigma} \cdot \underset{\sim}{p} \\ c \underset{\sim}{\sigma} \cdot \underset{\sim}{p} & -m c^2 \end{pmatrix} \right]$$

$$= \frac{\hbar}{2|\underset{\sim}{p}|} \cdot \left\{ \begin{pmatrix} m c^2 \underset{\sim}{\sigma} \cdot \underset{\sim}{p} & c (\underset{\sim}{\sigma} \cdot \underset{\sim}{p})^2 \\ c (\underset{\sim}{\sigma} \cdot \underset{\sim}{p})^2 & -m c^2 \underset{\sim}{\sigma} \cdot \underset{\sim}{p} \end{pmatrix} - \right.$$

$$\begin{pmatrix} m c^2 \underset{\sim}{\sigma} \cdot \underset{\sim}{p} & c (\underset{\sim}{\sigma} \cdot \underset{\sim}{p})^2 \\ c (\underset{\sim}{\sigma} \cdot \underset{\sim}{p})^2 & -m c \underset{\sim}{\sigma} \cdot \underset{\sim}{p} \end{pmatrix} = 0$$

For the chirality operator $\gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, it

commutes with the massless Dirac Hamiltonian

2 (c)

$$H = c\,\underset{\sim}{\alpha}\cdot\underset{\sim}{p} + \beta mc^2$$

$$= \begin{pmatrix} mc^2 & c\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p} \\ c\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p} & -mc^2 \end{pmatrix}$$

$$[\gamma^5, H] = \gamma^5 H - H\gamma^5$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} mc^2 & c\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p} \\ c\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p} & -mc^2 \end{pmatrix} -$$

$$\begin{pmatrix} mc^2 & c\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p} \\ c\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p} & -mc^2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} c\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p} & -mc^2 \\ mc^2 & c\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p} \end{pmatrix} - \begin{pmatrix} c\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p} & mc^2 \\ -mc^2 & c\,\underset{\sim}{\sigma}\cdot\underset{\sim}{p} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -2mc^2 \\ -2mc^2 & 0 \end{pmatrix} \neq 0$$

For massless fermion, $m = 0$, $\therefore [\gamma^5, H] = 0$

As $(s(\underset{\sim}{p}))^2 = \left(\frac{\hbar}{2}\right)^2$, $\therefore$ eigenvalues of $s(\underset{\sim}{p}) = \pm\frac{\hbar}{2}$

As $\gamma^{5\,2} = 1$, $\therefore$ eigenvalues of $\gamma^5 = \pm 1$

From the definition of the helicity operator $s(\underset{\sim}{p})$, it depends on the direction of the momentum $\underset{\sim}{p}$. Helicity is a kinematic entity, in different inertial frames, $\underset{\sim}{p}$ can change, hence $s(\underset{\sim}{p})$ is not the same, $s(\underset{\sim}{p})$ is not invariant under Lorentz transformation. In one frame, $s(\underset{\sim}{p}) = +\frac{\hbar}{2}$ whilst in another frame $s(\underset{\sim}{p}) = -\frac{\hbar}{2}$. $s(\underset{\sim}{p})$ is a scalar wrt to spatial rotations but not wrt Lorentz transformations.

Chirality is an intrinsic property of a particle. In weak interaction, particles participate as chiral particles, $\frac{1}{2}(1 \pm \gamma^5)\psi$ instead of just ψ.

Note: $\gamma^5\left[\frac{1}{2}(1 \pm \gamma^5)\psi\right] = \pm\frac{1}{2}\left[(1 \pm \gamma^5)\psi\right]$

We can show that for a Dirac bispinor u,

$$\gamma^5 u = \underset{\sim}{\Sigma} \cdot \frac{\underset{\sim}{p}}{|\underset{\sim}{p}|} u$$

if the Dirac particle is massless.

Consider a Dirac bispinor $u(\underset{\sim}{p})$

$$\not{p}\, u(\underset{\sim}{p}) = mc\, u(\underset{\sim}{p})$$

and write $u(\underset{\sim}{p}) = \begin{pmatrix} u_A \\ u_B \end{pmatrix}.$

Thus $\not{p}\, u(\underset{\sim}{p}) = mc\, u(\underset{\sim}{p})$ can be written as

$$(\gamma^0 p^0 - \underset{\sim}{\gamma} \cdot \underset{\sim}{p})\, u(\underset{\sim}{p}) = mc\, u(\underset{\sim}{p})$$

or

$$\begin{pmatrix} p^0 - mc & -\underset{\sim}{\sigma} \cdot \underset{\sim}{p} \\ \underset{\sim}{\sigma} \cdot \underset{\sim}{p} & -p^0 - mc \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix} = 0$$

or

$$(p^0 - mc)\, u_A = \underset{\sim}{\sigma} \cdot \underset{\sim}{p}\, u_B$$

$$\underset{\sim}{\sigma} \cdot \underset{\sim}{p}\, u_A = (p^0 + mc)\, u_B$$

We want to show, for a massless Dirac particle, $\gamma^5 u(\underset{\sim}{p})$ is same as $S(\underset{\sim}{p})\, u(\underset{\sim}{p})$ apart from a constant. Now

$$\gamma^5 u(\underset{\sim}{p}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix} = \begin{pmatrix} u_B \\ u_A \end{pmatrix}.$$

Consider $S(\boldsymbol{p})\, u(\boldsymbol{p}) = \frac{\hbar}{2} \boldsymbol{\Sigma} \cdot \hat{\boldsymbol{p}}\; u(\boldsymbol{p})$, $\quad \hat{\boldsymbol{p}} = \frac{\boldsymbol{p}}{|\boldsymbol{p}|}$

$$= \frac{\hbar}{2} \begin{pmatrix} \boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}} & 0 \\ 0 & \boldsymbol{\sigma}\cdot\boldsymbol{p} \end{pmatrix} u(\boldsymbol{p})$$

$$= \frac{\hbar}{2} \begin{pmatrix} \boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\; u_A \\ \boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\; u_B \end{pmatrix}$$

$$= \frac{\hbar}{2} \begin{pmatrix} \frac{p^0 + mc}{|\boldsymbol{p}|} u_B \\ \frac{p^0 - mc}{|\boldsymbol{p}|} u_A \end{pmatrix} = \frac{\hbar}{2|\boldsymbol{p}|} \begin{pmatrix} (p^0 + mc)\, u_B \\ (p^0 - mc)\, u_A \end{pmatrix}$$

$$= \frac{\hbar}{2|\boldsymbol{p}|} p^0 \begin{pmatrix} u_B \\ u_A \end{pmatrix}$$ for $m = 0$ (massless)

$$= \frac{\hbar}{2} \begin{pmatrix} u_B \\ u_A \end{pmatrix}$$ $p^0 = |\boldsymbol{p}|$ for $m = 0$

$$\therefore\ \gamma^5 u(\boldsymbol{p}) = \frac{S(\boldsymbol{p})}{\left(\frac{\hbar}{2}\right)} u(\boldsymbol{p}) = \boldsymbol{\Sigma}\cdot\hat{\boldsymbol{p}}\; u(\boldsymbol{p})$$

For for a massless Dirac particle,

$$\gamma^5 u(\boldsymbol{p}) = \boldsymbol{\Sigma}\cdot\hat{\boldsymbol{p}}\; u(\boldsymbol{p})$$

Note this does not mean $\gamma^5 = \boldsymbol{\Sigma}\cdot\hat{\boldsymbol{p}}$ for $m = 0$.