

Quiz 1

Please provide your class index number in addition to your full name.

1) In class, we discussed that for a Z-electron atom, we can approximate the potential felt by an electron using a central potential, which takes the following limits.

$$V_c(r) \simeq -\frac{e^2}{r} \quad \text{for large } r$$

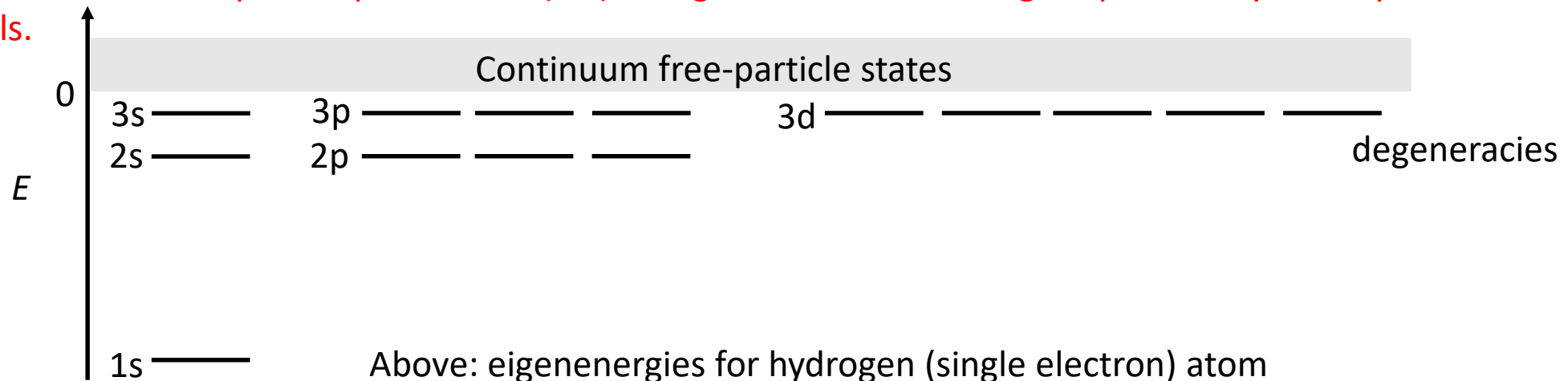
$$V_c(r) \simeq -\frac{Ze^2}{r} \quad \text{for small } r$$

Answer the following questions using the single particle approximation and central potential approximation.

- a) Given the single-particle spectrum for the H atom below, sketch a schematic of what the hierarchy of energy levels for 1s, 2s, 2p, 3s, 3p, and 3d orbitals might look like for a Ca atom ($Z = 20$).
- b) Explain your answer.

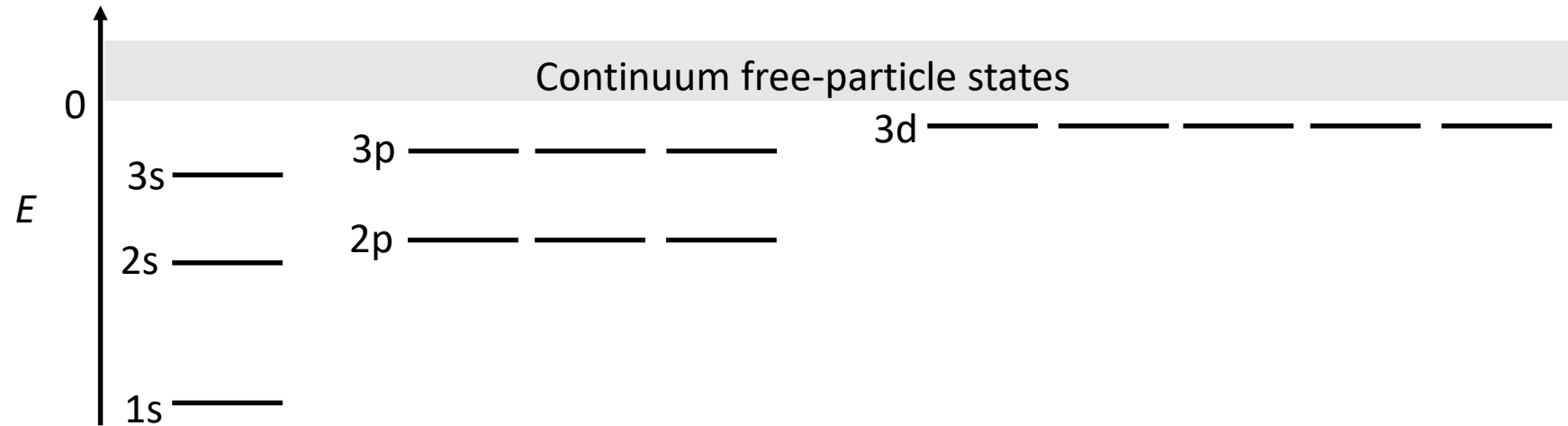
Hint: Are states with the same quantum number l , with different m , still degenerate? How about states with the same quantum number n , different l ?

Also recall that there is a repulsive potential $\sim l(l+1)/r^2$; eg. s orbitals have a higher probability density near the nucleus than p and d orbitals.



Eigenenergies for multi-electron atoms

1a)

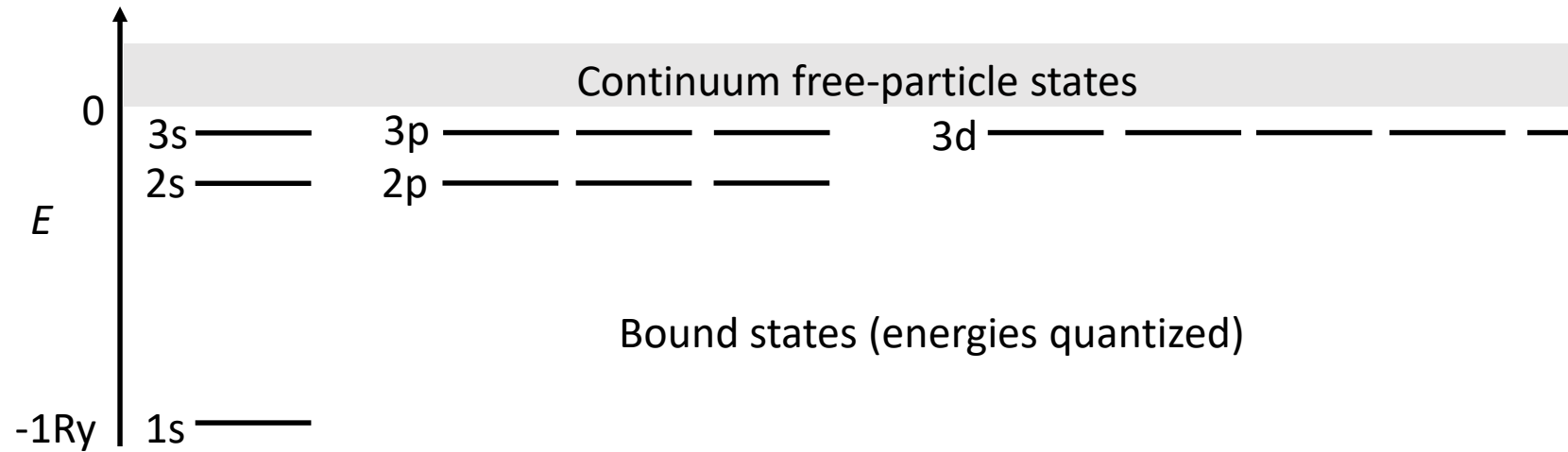
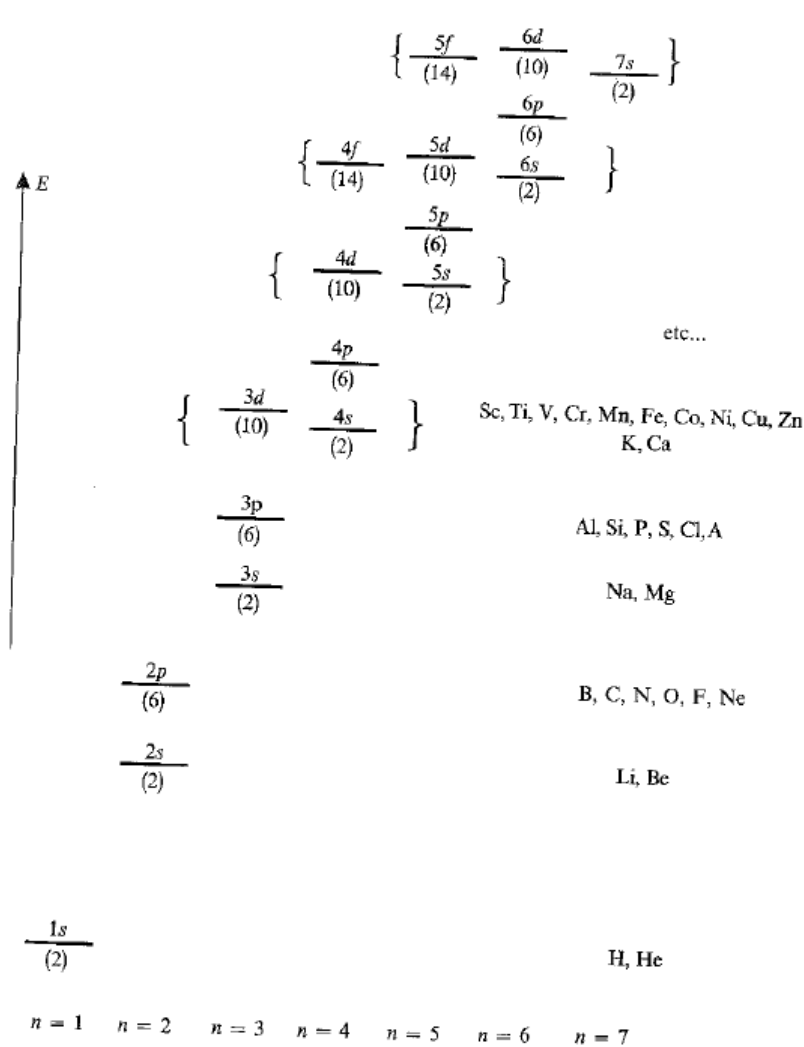


1b)

Compared to the hydrogen atom, the energy degeneracy is lifted for a given n but different l (eg. $2s$ is now more stable than $2p$), because the $2p$ orbitals are less penetrating (farther from the nucleus than the $2s$ orbitals), due to the repulsive potential $l(l+1)/r^2$. While in the H atom, the $2s$ and $2p$ orbitals have the same energy, the screening from other electrons in a multi-electron atom further weakens the strength of the Coulomb attraction felt by the $2p$ electron compared to the $2s$ electron. Therefore, the $2p$ orbital energy is now higher than the $2s$ orbital energy. Similar for the $3s$, $3p$ and $3d$ orbital energies.

Orbitals with the same quantum number l but different quantum number m are still degenerate, because the Hamiltonian does not depend on m .

Eigenenergies for multi-electron atoms



Above: eigenenergies for hydrogen (single electron) atom

- Compared to the hydrogen atom, we see that the energy degeneracy is lifted for a given n (eg. $2s$ is now more stable than $2p$)
- Also we see that $4s$ is more stable than $3p$.

Points to note

- In general, stronger attraction **stabilizes** the electron, implying a **more negative energy** for the single particle state.
- We can also use the term “shielding” instead of “screening”.