PC3246
Astrophysics I

Assignment 2

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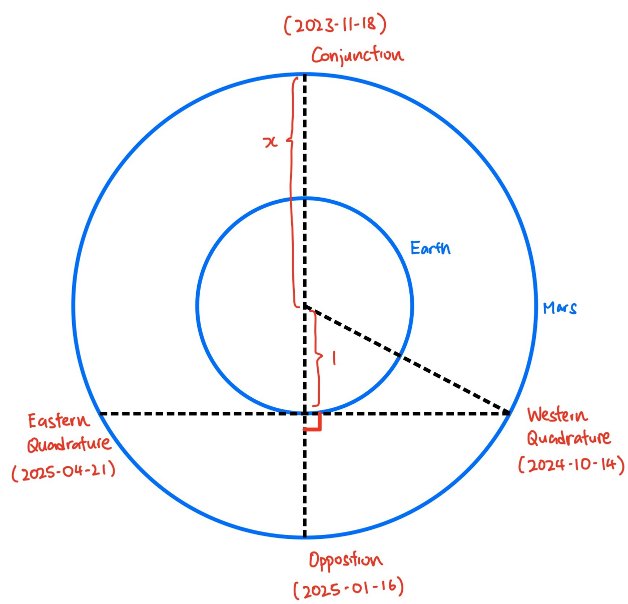
Question 1. Consider the planet Mars. Calculate its relative distance by using the following observational data. Do not use Kepler's laws of plaentary motion and Newton's law of universal gravitation (assume that they have yet to be discovered).
Hint: Consider the reference frame in which the Sun and Earth are not moving. Find the angle subtended at the Sun by two of the phenomena. Use trigonometry to calculate Mars' relative distance.

Date	Phenomenon
2023-11-18	Conjunction
2024-10-14	Western quadrature
2025-01-16	Opposition
2025-04-21	Eastern quadrature

Reference: https://eco.mtk.nao.ac.jp/cgi-bin/koyomi/cande/phenomena_en.cgi

Solution: First, we define the average distance the Earth and Mars are from the Sun as 1 and x astronomical unit(s) respectively where $x > 1$. Assuming that the orbits of Earth and Mars around the Sun is perfectly circular, we obtain the following geometric construction.



Where we define θ as the angle subtended between the Sun-Mars line when Mars is at conjunction (time $t_{\text{conjunction}}$) and the Sun-Mars line when Mars is at time t. As such,

$$\begin{cases} \theta(2023\text{-}11\text{-}18) = 0 \\ \theta(2025\text{-}01\text{-}16) = \dfrac{\pi}{2} \end{cases}. \tag{1}$$

Given the times at where Mars is in conjunction and opposition from Earth, we can obtain the synodic period of Mars as seen from Earth is

$$t_{\text{synodic}} = 2[t(2025\text{-}01\text{-}16) - t(2023\text{-}11\text{-}18)] = 425\,\text{d} \tag{2}$$

Additionally, we define the angle subtended between the Sun-Mars line when Mars is at opposition (time $t_{\text{opposition}}$) and the Sun-Mars line when Mars is at Eastern/Western quadrature (time $t_{\text{eastern quadrature}}/t_{\text{western quadrature}}$) as φ. As such,

$$\begin{cases} \varphi(2024\text{-}10\text{-}14) = \dfrac{\pi}{2} - \varphi \\ \varphi(2025\text{-}04\text{-}21) = \dfrac{\pi}{2} + \varphi \end{cases}. \tag{3}$$

Assuming that the

Question 2. Consider an observer at the latitude of 40°N and a star at the azimuth $A = 0°$ and altitude $h = 10°$. Find the star's A and h when it transits the upper meridian.

Solution:

Question 3. Consider an observer at the equator and a star at $A = 90°$ and $h = 0°$ at 8 pm after 4 months.

Solution:

Question 4. Consider an observer at the latitude of 20°N and a star at $A = 180°$ and $h = 60°$. If the vernal equinox is at its meridian two hours later, what are the declination and right ascension of the star?

Solution:

Question 5. Consider an observer at the equator and a star at $A = 70°$ and $h = 0°$. Calculate the star's A and h after 1 hour.
Hint: Assume that after 1 hour, the star is at point A. Draw a great circle passing through the zenith and point A. Let B be the point where the great circle intersects the horizon. Let the north celestial pole be point C. Consider the spherical triangle ABC. Assume that the radius of the celestial sphere is 1 unit. Hence,the lengths of side c and side a correspond to the altitude and azimuth of the star, respectively.

(a) What is the angle B?

(b) What is the angle C?

(c) What is the length of side b?

(d) By using a law of spherical trigonometry, calculate the length of side c.

(e) By using a law of spherical trigonometry, calculate the length of side a.

Solution: (a)

(b)

(c)

(d)

(e)