Telescopes

PROBLEM SET

1 For some point P in space, show that for any arbitrary closed surface surrounding P, the integral over a solid angle about P gives

$$\Omega_{\text{tot}} = \oint d\Omega = 4\pi.$$

2 The light rays coming from an object do not, in general, travel parallel to the optical axis of a lens or mirror system. Consider an arrow to be the object, located a distance p from the center of a simple converging lens of focal length f, such that $p > f$. Assume that the arrow is perpendicular to the optical axis of the system with the tail of the arrow located on the axis. To locate the image, draw two light rays coming from the tip of the arrow:

(i) One ray should follow a path *parallel* to the optical axis until it strikes the lens. It then bends toward the focal point of the side of the lens opposite the object.

(ii) A second ray should pass directly through the center of the lens undeflected. (This assumes that the lens is sufficiently thin.)

The intersection of the two rays is the location of the tip of the image arrow. All other rays coming from the tip of the object that pass through the lens will also pass through the image tip. The tail of the image is located on the optical axis, a distance q from the center of the lens. The image should also be oriented perpendicular to the optical axis.

(a) Using similar triangles, prove the relation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.$$

(b) Show that if the distance of the object is much larger than the focal length of the lens ($p \gg f$), then the image is effectively located on the focal plane. This is essentially always the situation for astronomical observations.

The analysis of a diverging lens or a mirror (either converging or diverging) is similar and leads to the same relation between object distance, image distance, and focal length.

3 Show that if two lenses of focal lengths f_1 and f_2 can be considered to have zero physical separation, then the effective focal length of the combination of lenses is

$$\frac{1}{f_{\text{eff}}} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Note: Assuming that the actual physical separation of the lenses is x, this approximation is strictly valid only when $f_1 \gg x$ and $f_2 \gg x$.

4 **(a)** Using the result of Problem 3, show that a compound lens system can be constructed from two lenses of different indices of refraction, $n_{1\lambda}$ and $n_{2\lambda}$, having the property that the resultant focal lengths of the compound lens at two specific wavelengths λ_1 and λ_2, respectively, can be made equal, or

$$f_{\text{eff},\lambda_1} = f_{\text{eff},\lambda_2}.$$

(b) Argue qualitatively that this condition does not guarantee that the focal length will be constant for all wavelengths.

5 Prove that the angular magnification of a telescope having an objective focal length of f_{obj} and an eyepiece focal length of f_{eye} is given by Eq. (9) when the objective and the eyepiece are separated by the sum of their focal lengths, $f_{\text{obj}} + f_{\text{eye}}$.

$$\boxed{m = \frac{f_{\text{obj}}}{f_{\text{eye}}}.} \tag{9}$$

6 The diffraction pattern for a single slit (Figs. 7 and 8) is given by

$$I(\theta) = I_0 \left[\frac{\sin(\beta/2)}{\beta/2} \right]^2,$$

where $\beta \equiv 2\pi D \sin\theta / \lambda$.

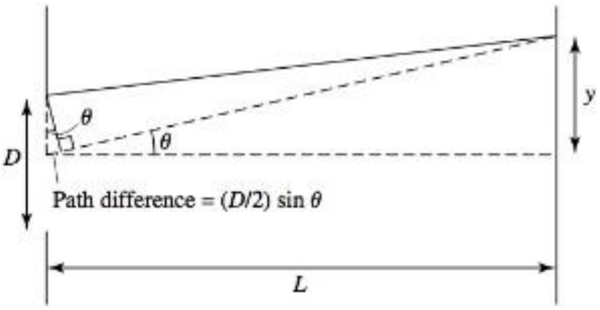

FIGURE 7 For a minimum to occur, the path difference between paired rays must be a half-wavelength.

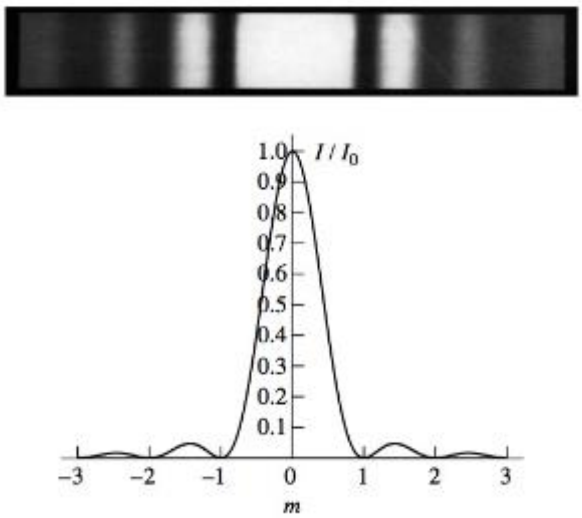

FIGURE 8 The diffraction pattern produced by a single slit. (Photograph from Cagnet, Francon, and Thrierr, *Atlas of Optical Phenomena*, Springer-Verlag, Berlin, 1962.)

(a) Using l'Hôpital's rule, prove that the intensity at $\theta = 0$ is given by $I(0) = I_0$.

(b) If the slit has an aperture of 1.0 μm, what angle θ corresponds to the first minimum if the wavelength of the light is 500 nm? Express your answer in degrees.

7 **(a)** Using the Rayleigh criterion, estimate the angular resolution limit of the human eye at 550 nm. Assume that the diameter of the pupil is 5 mm.

(b) Compare your answer in part (a) to the angular diameters of the Moon and Jupiter. You may find the data in Appendix helpful.

(c) What can you conclude about the ability to resolve the Moon's disk and Jupiter's disk with the unaided eye?

8 **(a)** Using the Rayleigh criterion, estimate the theoretical diffraction limit for the angular resolution of a typical 20-cm (8-in) amateur telescope at 550 nm. Express your answer in arcseconds.

(b) Using the information in Appendix: Solar System Data, estimate the minimum size of a crater on the Moon that can be resolved by a 20-cm (8-in) telescope.

(c) Is this resolution limit likely to be achieved? Why or why not?

9 The New Technology Telescope (NTT) is operated by the European Southern Observatory at Cerro La Silla. This telescope was used as a testbed for evaluating the adaptive optics technology used in the VLT. The NTT has a 3.58-m primary mirror with a focal ratio of $f/2.2$.

(a) Calculate the focal length of the primary mirror of the New Technology Telescope.

(b) What is the value of the plate scale of the NTT?

(c) ϵ Bootes is a double star system whose components are separated by 2.9″. Calculate the linear separation of the images on the primary mirror focal plane of the NTT.

10 When operated in "planetary" mode, HST's WF/PC 2 has a focal ratio of $f/28.3$ with a plate scale of $0.0455''\ \text{pixel}^{-1}$. Estimate the angular size of the field of view of one CCD in the planetary mode.

11 Suppose that a radio telescope receiver has a bandwidth of 50 MHz centered at 1.430 GHz (1 GHz = 1000 MHz). Assume that, rather than being a perfect detector over the entire bandwidth, the receiver's frequency dependence is triangular, meaning that the sensitivity of the detector is 0% at the edges of the band and 100% at its center. This filter function can be expressed as

$$f_\nu = \begin{cases} \dfrac{\nu}{\nu_m - \nu_\ell} - \dfrac{\nu_\ell}{\nu_m - \nu_\ell} & \text{if } \nu_\ell \le \nu \le \nu_m \\[2ex] -\dfrac{\nu}{\nu_u - \nu_m} + \dfrac{\nu_u}{\nu_u - \nu_m} & \text{if } \nu_m \le \nu \le \nu_u \\[2ex] 0 & \text{elsewhere.} \end{cases}$$

(a) Find the values of ν_ℓ, ν_m, and ν_u.

(b) Assume that the radio dish is a 100% efficient reflector over the receiver's bandwidth and has a diameter of 100 m. Assume also that the source NGC 2558 (a spiral galaxy with an apparent visual magnitude of 13.8) has a constant spectral flux density of $S = 2.5$ mJy over the detector bandwidth. Calculate the total power *measured* at the receiver.

(c) Estimate the power emitted at the source in this frequency range if $d = 100$ Mpc. Assume that the source emits the signal isotropically.

12 What would the diameter of a single radio dish need to be to have a collecting area equivalent to that of the 27 telescopes of the VLA?

13 How much must the pointing angle of a two-element radio interferometer be changed in order to move from one interference maximum to the next? Assume that the two telescopes are separated by the diameter of Earth and that the observation is being made at a wavelength of 21 cm. Express your answer in arcseconds.

14 Assuming that ALMA is completed with the currently envisioned 50 antennas, how many unique baselines will exist within the array?

15 The technical specifications for the planned SIM PlanetQuest mission call for the ability to resolve two point sources with an accuracy of better than 0.000004″ for objects as faint as 20th magnitude in visible light. This will be accomplished through the use of optical interferometry.

(a) Assuming that grass grows at the rate of 2 cm per week, and assuming that SIM could observe a blade of grass from a distance of 10 km, how long would it take for SIM to detect a measurable change in the length of the blade of grass?

(b) Using a baseline of the diameter of Earth's orbit, how far away will SIM be able to determine distances using trigonometric parallax, assuming the source is bright enough? (For reference, the distance from the Sun to the center of the Milky Way Galaxy is approximately 8 kpc.)

(c) From your answer to part (b), what would the apparent magnitude of the Sun be from that distance?

(d) The star Betelgeuse (in Orion) has an absolute magnitude of -5.14. How far could Betelgeuse be from SIM and still be detected? (Neglect any effects of dust and gas between the star and the spacecraft.)

16 (a) Using data available in the text or on observatory websites, list the wavelength ranges (in cm) and photon energy ranges (in eV) covered by the following telescopes: VLA, ALMA, SIRTF, JWST, VLT/VLTI, Keck/Keck Interferometer, HST, IUE, EUVE, Chandra, CGRO.

(b) Graphically illustrate the wavelength coverage of each of the telescopes listed in part (a) by drawing a horizontal bar over a horizontal axis like the one shown in Fig. 25.

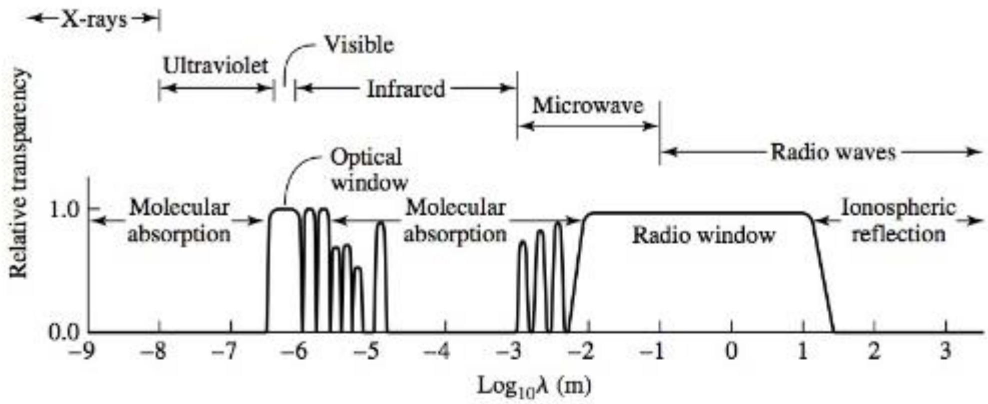

FIGURE 25 The transparency of Earth's atmosphere as a function of wavelength.

(c) Using photon energies rather than wavelengths, create a graphic similar to the one in part (b).

COMPUTER PROBLEM

17 Suppose that two identical slits are situated next to each other in such a way that the axes of the slits are parallel and oriented vertically. Assume also that the two slits are the same distance from a flat screen. Different light sources of identical intensity are placed behind each slit so that the two sources are incoherent, which means that double-slit interference effects can be neglected.

(a) If the two slits are separated by a distance such that the central maximum of the diffraction pattern corresponding to the first slit is located at the second minimum of the second slit's diffraction pattern, plot the resulting superposition of intensities (i.e., the total intensity at each location). Include at least two minima to the left of the central maximum of the leftmost slit and at least two minima to the right of the central maximum of the rightmost slit. *Hint:* Refer to the equation given in Problem 6 and plot your results as a function of β.

(b) Repeat your calculations for the case when the two slits are separated by a distance such that the central maximum of one slit falls at the location of the first minimum of the second (the Rayleigh criterion for single slits).

(c) What can you conclude about the ability to resolve two individual sources (the slits) as the sources are brought progressively closer together?