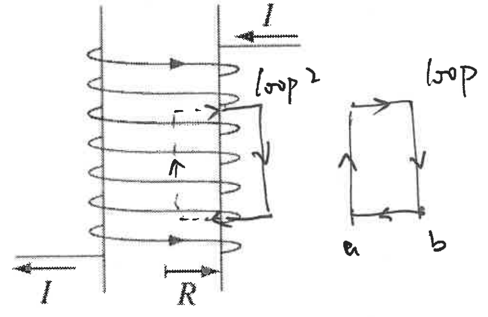
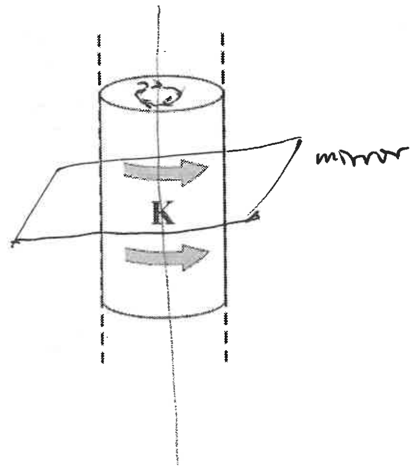
Example 5.9. Find the magnetic field of a very long solenoid, consisting of n closely wound turns per unit length on a cylinder of radius R, each carrying a steady current I (Fig. 5.34). And calculate the vector potential $\vec{A}$.



$$\vec{B}(\vec{r}) = \cancel{B_s(s,\phi,z)\hat{s}} + \cancel{B_\phi(s,\phi,z)\hat{\phi}} + B_z(s,\cancel{\phi},\cancel{z})\hat{z}$$

- Translational symmetry along z $\Rightarrow$ $\vec{B}$ uniform along z



- Horizontal mirror plane

$\Rightarrow B_s = 0, \quad B_\phi = 0$

- Rotational symmetry $\Rightarrow B_z(s)\hat{z} = \vec{B}(\vec{r})$

Ampère's law : $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$\Rightarrow B(a)L - B(b)L = 0 \Rightarrow B(a) = B(b) = 0$ (Loop 1)

$\Rightarrow BL = \mu_0 I \cdot n \cdot L$

$$\Rightarrow \quad \vec{B} = \begin{cases} \mu_0 n I \hat{z} & s<R \\ 0 & s>R \end{cases}$$

$A_s = 0$ because $\nabla \cdot \vec{A} = 0$

$A_z = 0$ because of horizontal mirror

$\vec{A} \parallel \hat{\phi}$

$$\oint \vec{A}\cdot d\vec{l} = A_\phi \cdot 2\pi s = \int \vec{B}\cdot d\vec{a} = B\cdot \pi s^2 = \mu_0 n I \pi s^2 \quad (s<R)$$

$$\oint \vec{A}\cdot d\vec{l} = A_\phi \cdot 2\pi s = B\cdot \pi R^2$$

$$\Rightarrow \vec{A} = \begin{cases} \dfrac{\mu_0 n I}{2} s \,\hat{\phi} & (s<R) \\[2ex] \dfrac{\mu_0 n I R^2}{2s} \,\hat{\phi} & (s>R) \end{cases}$$

Top-down view