Discuss charge conjugation C

C P conservation

C P violation in kaon decays

photon and all those mesons that lie at the center of their Eightfold-Way diagrams: π^0, η, η'

ρ^0, ϕ, ω, ψ ... are eigenstates of charge conjugation operator C

For mesons, eigenvalue of $C = (-1)^{l+s}$ (can be shown)

l = orbital angular momentum, s = spin

For <u>pseudoscalar</u> mesons, $l=0$ and $s=0$ $\therefore \underline{C=1}$

e.g $\quad C\,\pi^0 = \pi^0$

For <u>vector</u> mesons, $l=0$, $s=1$, $\therefore \underline{C=-1}$

e.g $\quad C\,\omega = -\omega$

Not many hadrons are eigenstates of C.

Consider G-parity for hadrons,

combining C with isospin rotation

$$C\,e^{i\pi I_2}$$

$=$

Consider weak decays, e.g. pion decay

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

Pion weak decay

Pion decay violates charge conjugation

Consider the weak decay of π^-: $\pi^- \to \mu^- + \bar{\nu}_\mu$

In the frame at which the π^- is at rest,

RH (right hand) muon

RH antineutrino

(1)

Under charge conjugation C, we expect to get

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$

C changes internal quantum numbers only, C does not change angular momentum (spin polarization), so

Fig(1) becomes (helicity not affected by C, so no change in helicity under C)

(2)

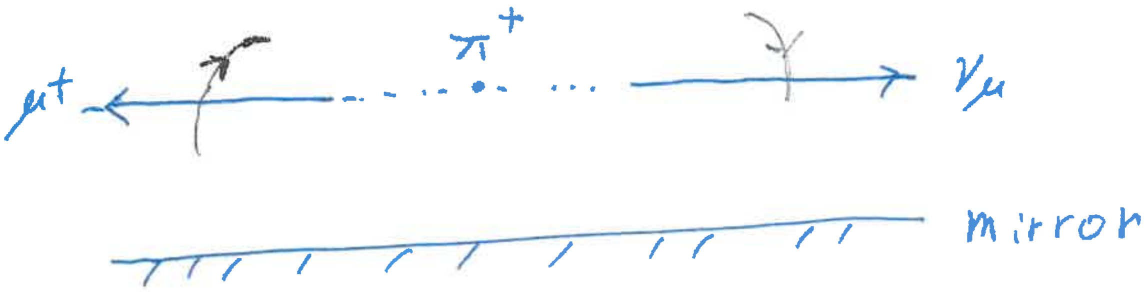
But in nature, we observe

(3)

This means the decay of π^- violates the charge conjugation since fig(2) ≠ fig (3).

If we apply parity operator π^P to fig (2), We can get fig (3)

place a mirror beneath fig (2)



we then get fig (3)

so applying C and P jointly ie CP, fig(1) becomes fig (3) which is realized in nature. CP is a symmetry obeyed by π^- decay

(23)

Table 4.6 Quantum numbers of some meson nonets

Orbital angular momentum	Net spin	J^{PC}	Observed Nonet $I = 1$	$I = \frac{1}{2}$	$I = 0$	Average mass (MeV/c^2)
$l = 0$	$s = 0$	0^{-+}	π	K	η, η'	400
	$s = 1$	1^{--}	ρ	K^*	ϕ, ω	900
$l = 1$	$s = 0$	1^{+-}	b_1	K_{1_B}	h_1, h_1	1200
	$s = 1$	0^{++}	a_0	K_0^*	f_0, f_0	1100
	$s = 1$	1^{++}	a_1	K_{1_A}	f_1, f_1	1300
	$s = 1$	2^{++}	a_2	K_2^*	f'_2, f_2	1400

It seemed peculiar that two otherwise identical particles should carry different parity. The alternative, suggested by Lee and Yang in 1956 was that τ and θ are really the *same particle* (now known as the K^+), and parity is simply not conserved in one of the decays. This idea prompted their search for evidence of parity invariance in the weak interactions and, when they found none, to their proposal for an experimental test.

4.4.2
Charge Conjugation

Classical electrodynamics is invariant under a change in the sign of all electric charges; the potentials and fields reverse their signs, but there is a compensating charge factor in the Lorentz law, so the forces still come out the same. In elementary particle physics, we introduce an operation that generalizes this notion of 'changing the sign of the charge'. It is called *charge conjugation*, C, and it converts each particle into its antiparticle:

$$C|p\rangle = |\overline{p}\rangle \tag{4.54}$$

'Charge conjugation' is something of a misnomer, for C can be applied to a neutral particle, such as the neutron (yielding an antineutron), and it changes the sign of *all* the 'internal' quantum numbers – charge, baryon number, lepton number, strangeness, charm, beauty, truth – while leaving mass, energy, momentum, and spin untouched.

As with P, application of C *twice* brings us back to the original state:

$$C^2 = I \tag{4.55}$$

and hence the eigenvalues of C are ± 1. Unlike P, however, most of the particles in nature are clearly *not* eigenstates of C. For if $|p\rangle$ is an eigenstate of C, it follows that

$$C|p\rangle = \pm|p\rangle = |\overline{p}\rangle \tag{4.56}$$

$C|\gamma> = -|\gamma>$

(24)

so $|p\rangle$ and $|\bar{p}\rangle$ differ at most by a sign, which means that they represent the same physical state. Thus, *only those particles that are their own antiparticles can be eigenstates of* C. This leaves us the photon, as well as all those mesons that lie at the center of their Eightfold-Way diagrams: π^0, η, η', ρ^0, ϕ, ω, ψ, and so on. Because the photon is the quantum of the electromagnetic field, which changes sign under C, it makes sense that the photon's 'charge conjugation number' is -1. It can be shown [19] that a system consisting of a spin-$\frac{1}{2}$ particle and its antiparticle, in a configuration with orbital angular momentum l and total spin s, constitutes an eigenstate of C with eigenvalue $(-1)^{l+s}$. According to the quark model, the mesons in question are of precisely this form: for the pseudoscalars, $l = 0$ and $s = 0$, so $C = +1$; for the vectors, $l = 0$ and $s = 1$, so $C = -1$. (Often, as in Table 4.6, C is listed as though it were a valid quantum number for the entire supermultiplet; in *fact* it pertains only to the central members.)

π meson pseudoscalar

$C\,\pi^0 = \pi^0$

Charge conjugation is a multiplicative quantum number, and, like parity, it is conserved in the strong and electromagnetic interactions. Thus, for example, the π^0 decays into two photons:

$$\pi^0 \to \gamma + \gamma \tag{4.57}$$

(for n photons $C = (-1)^n$, so in this case $C = +1$ before and after), but it cannot decay into three photons. Similarly, the ω goes to $\pi^0 + \gamma$, but never to $\pi^0 + 2\gamma$. In the strong interactions, charge conjugation invariance requires, for example, that the energy distributions of the charged pions in the reaction

ω vector meson

$C\,\omega = -\omega$

$$p + \bar{p} \to \pi^+ + \pi^- + \pi^0 \tag{4.58}$$

should (on average) be identical [20]. On the other hand, charge conjugation is *not* a symmetry of the weak interactions: when applied to a neutrino (left-handed, remember), C gives a left-handed *anti*neutrino, which does not occur. So the charge-conjugated version of any process involving neutrinos is *not* a possible physical process. And purely hadronic weak interactions also show violations of C as well as P.

C: LH ν $\to$ LH $\bar{\nu}$

LH = left-handed

Because so few particles are eigenstates of C, its direct application in elementary particle physics is rather limited. Its power can be somewhat extended, if we confine our attention to the strong interactions, by combining it with an appropriate isospin transformation. Rotation by 180° about the number 2 axis in isospin space* will carry I_3 into $-I_3$, converting, for instance, a π^+ into a π^-. If we then apply the charge conjugation operator, we come back to π^+. Thus the charged pions are eigenstates of this combined operator, even though they are not eigenstates of C alone. For some reason the product transformation is called 'G-parity':

$$G = CR_2, \quad \text{where} \quad R_2 = e^{i\pi I_2} \tag{4.59}$$

* Some authors use the number 1 axis. Obviously, any axis in the 1–2 plane will do the job.

All mesons that carry no strangeness (or charm, beauty, or truth) are eigenstates of G;* for a multiplet of isospin I the eigenvalue is given (Problem 4.36) by

$$G = (-1)^I C \tag{4.60}$$

where C is the charge conjugation number of the neutral member. For a single pion, $G = -1$, and for a state with n pions

$$G = (-1)^n \tag{4.61}$$

This is a very handy result, for it tells you how many pions can be emitted in a particular decay. For example, the ρ mesons, with $I = 1$, $C = -1$, and hence $G = +1$, can go to *two* pions, but not to three, whereas the ϕ, the ω, and the ψ (all $I = 0$) can go to *three*, but not to two.

4.4.3
CP

As we have seen, the weak interactions are not invariant under the parity transformation P; the cleanest evidence for this is the fact that the antimuon emitted in pion decay

parity of LH (4.62) = −1

$$\pi^+ \to \mu^+ + \nu_\mu \tag{4.62}$$

parity of RH (4.62) = (−1)·(−1) = +1

always comes out left-handed. Nor are the weak interactions invariant under C, for the charge-conjugated version of this reaction would be

$$\pi^- \to \mu^- + \bar{\nu}_\mu \tag{4.63}$$

with a left-handed muon, whereas in fact the muon always comes out *right*-handed. However, if we *combine* the two operations we're back in business: CP turns the left-handed antimuon into a *right*-handed muon, which is exactly what we observe in nature. Many people who had been shocked by the fall of parity were consoled by this realization; perhaps, it was the combined operation that our intuition had been talking about all along – maybe what we should have *meant* by the 'mirror image' of a right-handed electron was a left-handed *positron*.† If we had defined parity from the start to be what we now call CP, the trauma of parity violation might have been avoided (or at least postponed). It is too late to change the terminology

* K^+, for example, is *not* an eigenstate of G, for R_2 takes it to K^0, and C takes that to $\bar{K}^0$. The idea could be extended to the K's, by using an appropriate $SU(3)$ transformation in place of R_2, but since $SU(3)$ is not a very good symmetry of the strong forces, there is little percentage in doing so.

† Incidentally, we could perfectly well take electric charge to be a pseudoscalar in classical electrodynamics; **E** becomes a pseudovector and **B** a vector, but the results are all the same. It is really a matter of taste whether you say the mirror image of a plus charge is positive or negative. But it seems simplest to say the charge does *not* change, and this is the standard convention.

Discuss CP violation

All reactions obey CP symmetry (e.g. in the decay of π^- as illustrated in the previous figures) except the Kaons (containing strange quark s $K^0 = d\bar{s}$) decays, the B ($B^0 = d\bar{b}$, containing b quark) decays and possibly the D decay (containing c quark, $D^0 = c\bar{u}$) $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$

Kaons K^0 (strangeness +1), $\bar{K}^0$ (strangeness −1) are produced in strong interaction processes, e.g.

$$\pi^- p \rightarrow \Lambda^0 \; K^0$$

$$K^- p \rightarrow n \; \bar{K}^0$$

$$\pi^+ p \rightarrow p + K^+ + \bar{K}^0$$

$K^0 = d\bar{s}$

(see Feynman Lecture vol 3, p. 11-12 to 11-20)

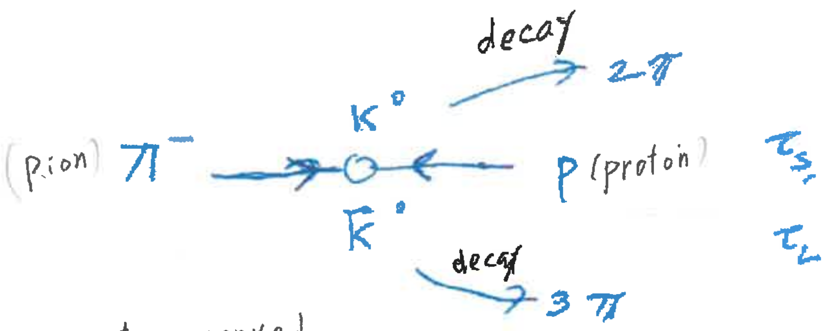
K^0 and $\bar{K}^0$ are eigenstates of H_{st} (strong interaction Hamiltonian) and H_{em}. Let $H_0 = H_{st} + H_{em}$ ← electromagnetic

$$H_0 |K^0\rangle = E_0 |K^0\rangle, \qquad H_0 |\bar{K}^0\rangle = E_0 |\bar{K}^0\rangle$$

Experimentally (i) K^0 oscillates to $\bar{K}^0$ and (ii) they decay into 2π or 3π via weak interaction

The decays have 2 different life times

$$\tau_s = 0.89 \times 10^{-10} \text{ s}, \qquad \tau_L = 5.2 \times 10^{-8} \text{ s}$$



strangeness not conserved

$S = +1$ $\quad K^0 \rightleftharpoons \bar{K}^0$ $\quad$ strangeness $S = -1$

The oscillation K^0 to $\bar{K}^0$ and vice versa
is due to decay channels 2π, 3π

$$K^0 \underset{3\pi}{\overset{2\pi}{\rightleftharpoons}} \bar{K}^0$$

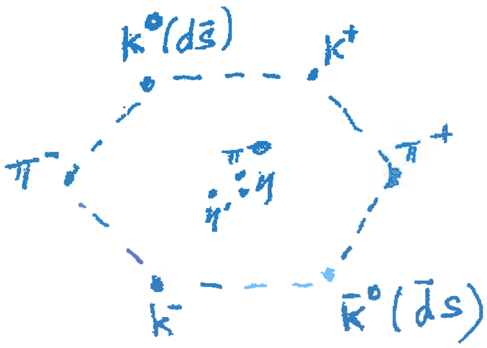
see Feynman Lecture vol III Eq (11.43)

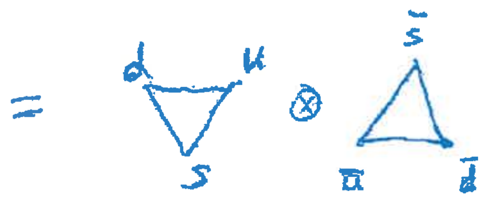
and is explained by weak interaction, H_{wk}

Look at quark content :

K^0 : $d\bar{s}$

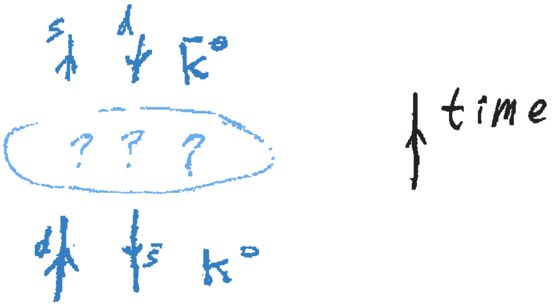
$\bar{K}^0$: $\bar{d}s$

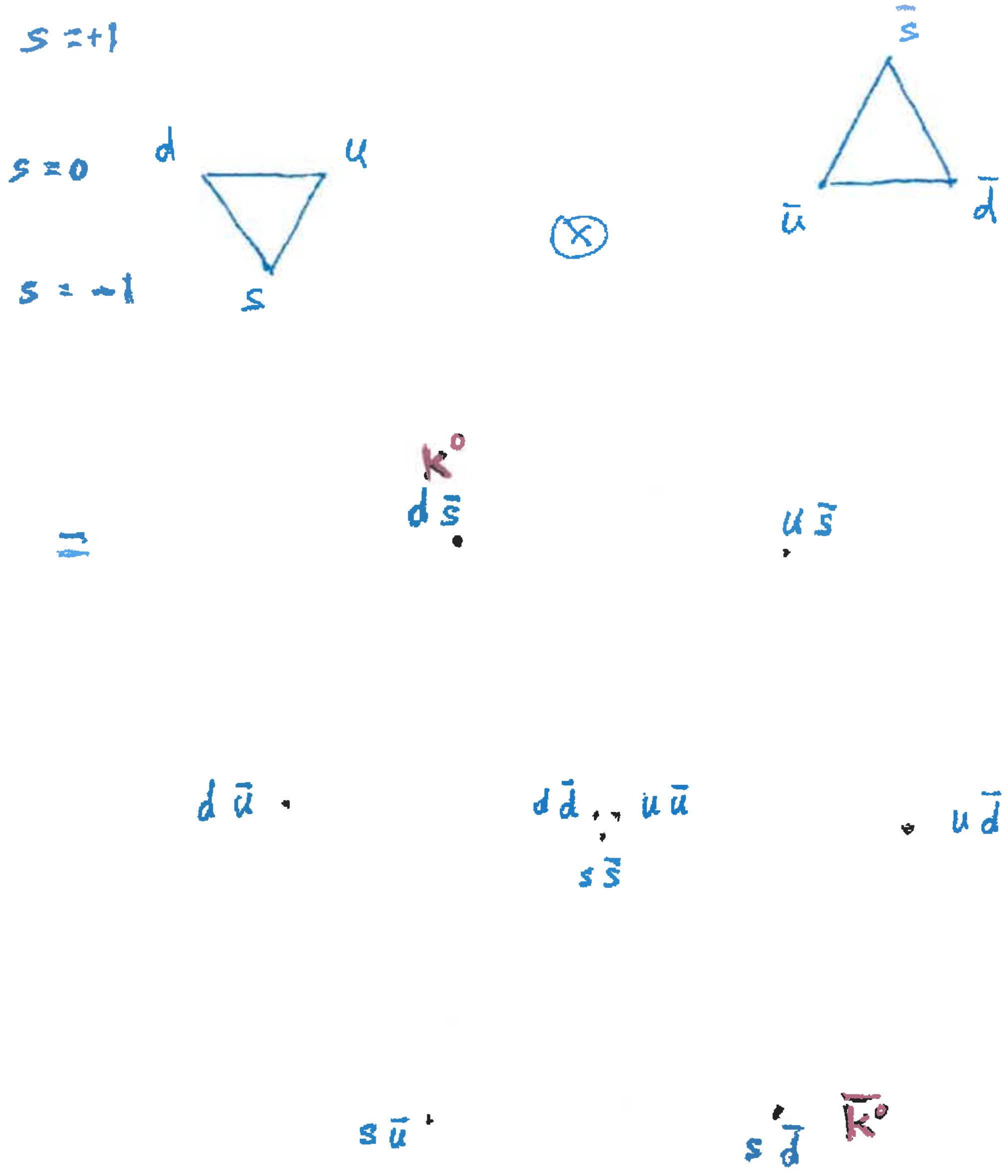




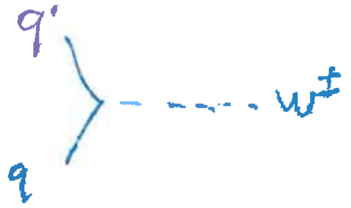
Oscillation via Feynman diagram

start with external lines



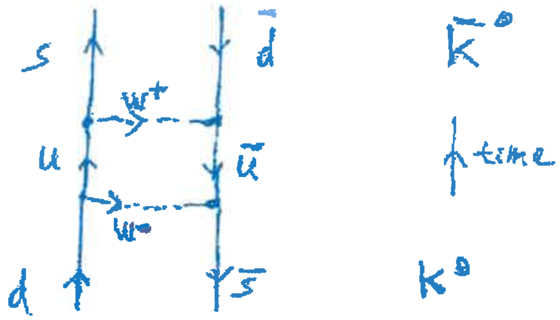

s = +1
s = 0
s = −1
d
u
s
⊗
s̄
ū
d̄
=
K°
ds̄
us̄
dū
dd̄, uū
ss̄
ud̄
sū
sd̄
K̄°

Get the relevant weak interaction vertex (see 2nd lecture PowerPoint)

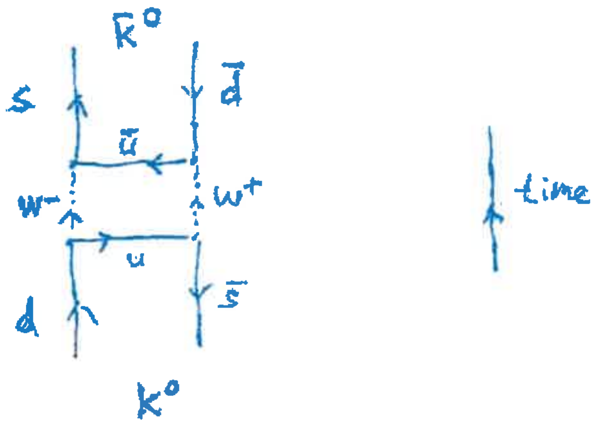


$W^\pm$: intermediate bosons

Insert vertices to complete the diagram, an example is



similarly



The above two diagrams illustrates the oscillations of $K^0 \rightleftarrows \bar{K}^0$

We now proceed to explain the two decay modes: 2π's and 3π's and also two lifetimes τ_S, τ_L

K^0, $\overline{K^0}$ are produced by strong interaction.
Immediately they decay by weak interaction.
One can say the kaons appear as particles K^0, $\overline{K^0}$ when interact strongly, but appear as particles K_S, K_L when interact weakly

Gellmann & Pais (1955) proposed to use linearly superposed states, K_S, K_L are linearly superposed from K^0, $\overline{K^0}$

$$|K_S\rangle \sim |K^0\rangle - |\overline{K^0}\rangle \rightarrow \tau_S$$

$$|K_L\rangle \sim |K^0\rangle + |\overline{K^0}\rangle \rightarrow \tau_L$$

Applying charge conjugation C and parity operato P

$$CP|K_S\rangle \sim CP\left(|K^0\rangle - |\overline{K^0}\rangle\right)$$

Note: Previously we denote parity operator (space inversion operator) by π.

Now

$$CP|K^0\rangle = C(-)|K^0\rangle = -C|K^0\rangle = -|\bar{K}^0\rangle$$

$$CP|\bar{K}^0\rangle = C(-)|\bar{K}^0\rangle = -C|\bar{K}^0\rangle = -|K^0\rangle$$

hence

$$CP|K_S\rangle \sim +|K_S\rangle \qquad \text{HW}$$

$$CP|K_L\rangle \sim -|K_L\rangle \qquad \text{HW}$$

i.e.

$|K_S\rangle$ eigenstate of CP with eigenvalue +1

$|K_L\rangle$ eigenstate of CP with eigenvalue −1

With the introduction of kaons as K_S, K_L in weak decay, we can account for the 2πs and 3πs decays.

For 2 pions, $CP|\pi^0\pi^0\rangle = C(-1)(-1)|\pi^0\pi^0\rangle$

$$= C|\pi^0\pi^0\rangle = |\pi^0\pi^0\rangle$$

$$CP|\pi^+\pi^-\rangle = C(-1)(-1)|\pi^+\pi^-\rangle = C|\pi^+\pi^-\rangle = |\pi^-\pi^+\rangle$$

Similarly for 3 pions, $CP = (-1)(-1)(-1) = -1$.

If CP is conserved in weak decay, then

$K_S \rightarrow 2$ pions and $K_L \rightarrow 3$ pions

To check CP conservation, the kaons after production can be separated out as K_S and K_L particles

1964 Cronin-Fitch

source

57 ft away $\approx 17.4\,m$

Detectors

K^0, $\overline{K^0}$ are produced

Only K_L remained (no more K_S)

found 45 2-pion decays among 22700 decays.

If CP is conserved, K_L can only decay to 3 pions.
So this experiment indicates CP is not conserved in kaon decays.

However the violation of CP conservation is very small

$$\frac{45}{22700} \sim 2 \times 10^{-3}$$

We proceed to account for $K^0 \rightleftharpoons \overline{K^0}$ oscillation and CP violation of kaon decays by treating the kaon as a 2-state system, see Feynman lecture vol 3, chapter 11.