PC3246
Astrophysics I

Assignment 2

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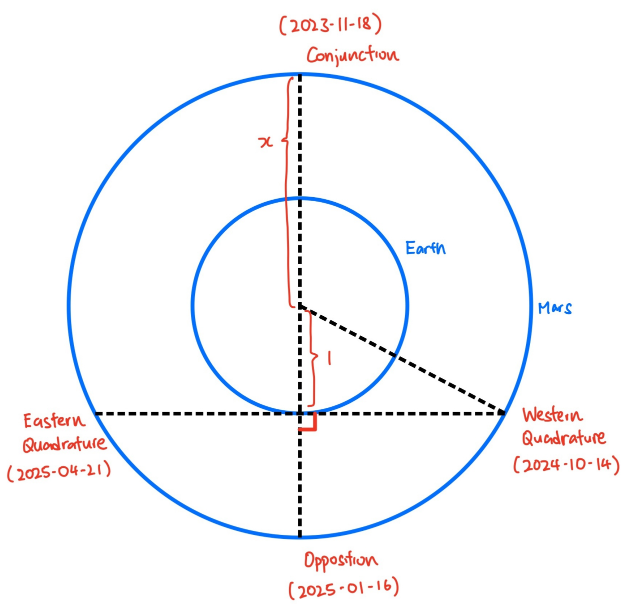
Question 1. Consider the planet Mars. Calculate its relative distance by using the following observational data. Do not use Kepler's laws of planetary motion and Newton's law of universal gravitation (assume that they have yet to be discovered).
Hint: Consider the reference frame in which the Sun and Earth are not moving. Find the angle subtended at the Sun by two of the phenomena. Use trigonometry to calculate Mars' relative distance.

Date	Phenomenon
2023-11-18	Conjunction
2024-10-14	Western quadrature
2025-01-16	Opposition
2025-04-21	Eastern quadrature

Reference: https://eco.mtk.nao.ac.jp/cgi-bin/koyomi/cande/phenomena_en.cgi

Solution: First, we define the average distance the Earth and Mars are from the Sun as 1 and x astronomical unit(s) respectively where $x > 1$. Assuming that the orbits of Earth and Mars around the Sun is perfectly circular, we obtain the following geometric construction.



Consider Mars' motion around the Sun in the fixed Sun-Earth stationary frame, Mars should have a constant angular velocity. Hence,

$$\omega = \frac{2\pi}{t_{\text{synodic}}} = \frac{\Delta\theta}{\Delta t} \qquad \Longrightarrow \qquad \Delta\theta \propto \Delta t. \tag{1}$$

As such,

$$\Delta\theta = \omega\Delta t = \frac{2\pi}{t_{\text{synodic}}}\Delta t = \frac{2\pi}{t_{\text{synodic}}}(t_{\text{eastern quadrature}} - t_{\text{western quadrature}}), \tag{2}$$

where if we consider the Sun-Earth-Mars (western quadrature) right triangle such that the angle subtended between the Sun-Mars line and the Sun-Earth line is φ, we have

$$\varphi = \frac{\Delta\theta}{2} = \frac{\pi}{t_{\text{synodic}}}(t_{\text{eastern quadrature}} - t_{\text{western quadrature}}) \qquad \text{and} \qquad \cos\varphi = \frac{1}{x}. \tag{3}$$

Given the date where Mars is in conjunction and in opposition with respect to Earth,

$$t_{\text{synodic}} = 2(t_{\text{opposition}} - t_{\text{conjunction}}). \tag{4}$$

we thus have

$$\begin{aligned} x &= \frac{1}{\cos\left[\frac{\pi}{2(t_{\text{opposition}} - t_{\text{conjunction}})}(t_{\text{eastern quadrature}} - t_{\text{western quadrature}})\right]} \\ &= \frac{1}{\cos\left[\frac{\pi}{2(425\,\text{d})}(189\,\text{d})\right]} \approx 1.3059\,\text{AU} \\ &\approx 1.31\,\text{AU} \quad (3\text{ s.f.}) \end{aligned} \tag{5}$$

Alternatively, if we choose $t_{\text{synodic}} = 780\,\text{d}$, we have,

$$\begin{aligned} x &= \frac{1}{\cos\left[\frac{\pi}{t_{\text{synodic}}}(t_{\text{eastern quadrature}} - t_{\text{western quadrature}})\right]} = \frac{1}{\cos\left[\frac{\pi}{780\,\text{d}}(189\,\text{d})\right]} \approx 1.3812\,\text{AU} \\ &\approx 1.38\,\text{AU} \quad (3\text{ s.f.}) \end{aligned} \tag{6}$$

Notes: Regardless of what approach we use to determine the value of x, if we work only using 4 specific points in Mars' trajectory about a fixed Sun-Earth reference frame, we will always obtain an underestimate when using the eastern/western quadrature as this does not take into account when Mars enters apparent retrograde motion when viewed from Earth.

Question 2. Consider an observer at the latitude of 40°N and a star at the azimuth $A = 0°$ and altitude $h = 10°$. Find the star's A and h when it transits the upper meridian.

Solution: Since the star has an azimuth of $A = 0°$ with an altitude $h < h_{\text{NCP}}$, the star is currently at lower culmination north of the prime vertical with $h = 10°$. Given a declination angle,

$$\delta = h_{\text{lower culmination}} - (\lambda - 90°) = 10° - (40° - 90°) = 60° \tag{7}$$

At upper culmination, the star will have an altitude of,

$$h_{\text{upper culmination}} = (\lambda + 90°) - \delta = (40° + 90°) - 60° = 70° \tag{8}$$

and an azimuth $A_{\text{upper culmination}} = 0°$ since the star will remain north of the prime vertical.

Question 3. Consider an observer at the equator and a star at $A = 90°$ and $h = 0°$ at 8 pm after 4 months.

Solution: Since the observer is at the equator, the point $(A = 0°,\ h = 0°)$ coincides with the north celestial pole, for a star viewed by an observer at the equator lying due east,

$$\delta = 0° \qquad \text{and} \qquad \text{HA} = -6^{\,\text{h}}. \tag{9}$$

As such, after 4 months at 8 pm,

$$\delta' = \delta = 0° \qquad \text{and} \qquad \text{HA}' = -6^{\,\text{h}} + 8^{\,\text{h}} = 2^{\,\text{h}}, \tag{10}$$

and thus a local observer will observe the star at.

$$A = 270° \qquad \text{and} \qquad h = 60°. \tag{11}$$

Question 4. Consider an observer at the latitude of 20°N and a star at $A = 180°$ and $h = 60°$. If the vernal equinox is at its meridian two hours later, what are the declination and right ascension of the star?

Solution: Since the star has an azimuth of $A = 180°$, the star is currently at upper culmination south of the prime vertical with an altitude $h = 60°$ and a declination angle,

$$\delta = (\lambda + 90°) - (180° - h_{\text{upper culmination}}) = (20° + 90°) - (180° - 60°) = -10°. \tag{12}$$

Consider the local sidereal time (LST),

$$\text{LST} = \text{HA}_{\Upsilon} = -2^{\,\text{h}}, \tag{13}$$

and the hour angle of the star,

$$\text{HA} = 0^{\,\text{h}}, \tag{14}$$

which gives,

$$\alpha = \text{LST} - \text{HA} = -2^{\,\text{h}} - 0^{\,\text{h}} = -2^{\,\text{h}}. \tag{15}$$

Question 5. Consider an observer at the equator and a star at $A = 70°$ and $h = 0°$. Calculate the star's A and h after 1 hour.
Hint: Assume that after 1 *hour, the star is at point* A*. Draw a great circle passing through the zenith and point* A*. Let* B *be the point where the great circle intersects the horizon. Let the north celestial pole be point* C*. Consider the spherical triangle* ABC*. Assume that the radius of the celestial sphere is* 1 *unit. Hence,the lengths of side* c *and side* a *correspond to the altitude and azimuth of the star, respectively.*

(a) What is the angle B?

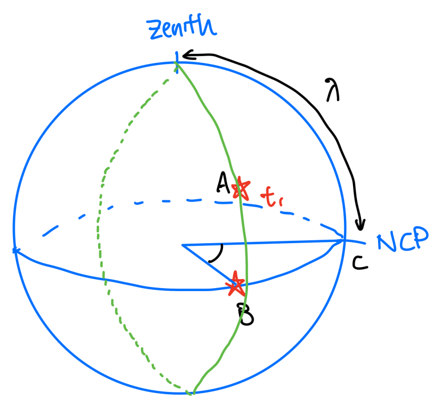
(b) What is the angle C?

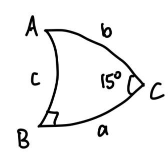
(c) What is the length of side b?

(d) By using a law of spherical trigonometry, calculate the length of side c.

(e) By using a law of spherical trigonometry, calculate the length of side a.

Solution: Consider the following celestial sphere construction with spherical triangle ABC bound by A (position of star 1 h later), B, C (NCP):





(a)

Since the great circle containing arc AB passes through the zenith, a pole of the horizon plane of the observer, $B = 90°$.

(b)

Since the observer is at the equator, the point $(A = 0°,\ h = 0°)$ coincides with the north celestial pole, $C = 90°$, C is equal in value to the change in hour angle of the star. Hence,

$$C = -5^{\,\mathrm{h}} - \left(-6^{\,\mathrm{h}}\right) = 15°. \tag{16}$$

(c)

Similarly, when the star is at $(A = 70^\circ,\ h = 0^\circ)$, the star has a declination angle,

$$\delta = 90^\circ - 70^\circ = 20^\circ. \tag{17}$$

Thus, since b is the angular distance between the position of the star 1 hour later and the NCP,

$$b = 90^\circ - \delta = 90^\circ - 20^\circ = 70^\circ. \tag{18}$$

(d)

By the law of sines for spherical triangles,

$$\frac{\sin c}{\sin C} = \frac{\sin b}{\sin B} \tag{19}$$

$$\implies \quad \begin{aligned} c &= \arcsin\left(\frac{\sin b \sin C}{\sin B}\right) = \arcsin\left(\frac{\sin 70^\circ \sin 15^\circ}{\sin 90^\circ}\right) = 14.08^\circ \\ &\approx 14^\circ \end{aligned}. \tag{20}$$

(e)

By the cotangent four-part formula,

$$\cos \mathrm{IS} \cos \mathrm{IA} = \cot \mathrm{OS} \sin \mathrm{IS} - \cot \mathrm{OA} \sin \mathrm{IA}, \tag{21}$$

which applied to spherical triangle ABC, gives,

$$\cos a \cos C = \cot b \sin a - \cot B \sin C \tag{22}$$

$$\implies \quad \begin{aligned} a &= \arctan\left(\frac{\cos C}{\cot b}\right) = \arctan\left(\frac{\cos 15^\circ}{\cot 70^\circ}\right) \approx 69.35^\circ \\ &\approx 69^\circ. \end{aligned} \tag{23}$$