

Name: Ting Jun Rui  
Student ID: A0179506W

**Question 1.** Using the Rayleigh criterion, estimate the theoretical diffraction limit for the angular resolution of a typical 20 cm (8 in) amateur telescope at 550 nm. Express your answer in arcseconds.

**Solution:** Using the Rayleigh criterion,

$$\theta \sim \frac{1.22\lambda}{D} = \frac{1.22(550 \times 10^{-9})}{20 \times 10^{-2}} = 3.355 \times 10^{-6} \text{ rad} \approx 0.692''. \quad (1)$$

■

**Question 2.** The New Technology Telescope (NTT) is operated by the European Southern Observatory at Cerro La Silla. This telescope was used as a testbed for evaluating the adaptive optics technology used in the VLT. The NTT has a 3.58 m primary mirror with a focal ratio of  $f/2.2$ .

- (a) Calculate the focal length of the primary mirror of the New Technology Telescope.
- (b) What is the value of the plate scale of the NTT?
- (c)  $\epsilon$  Bootes is a double star system whose components are separated by  $2.9''$ . Calculate the linear separation of the images on the primary mirror focal plane of the NTT.

**Solution:** (a) The focal ratio  $f/\#$  is defined as,

$$f/\# = \frac{f}{D} \quad \implies \quad f = 2.2 \times 3.58 = 7.876 \text{ m} \approx 7.88 \text{ m} \quad (2 \text{ s.f.}). \quad (2)$$

(b) The plate scale  $p$  is defined as,

$$p \sim \frac{206265}{7876} = 26.18906'' \approx 26.189'', \quad (3)$$

(c) and

$$s = \frac{\Delta\theta_{\varepsilon \text{ Boo}}}{p} = \frac{2.9}{26.18906} \approx 0.1107 \text{ mm} \approx 0.11 \text{ mm}. \quad (4)$$

■

**Question 3.** Suppose that a radio telescope receiver has a bandwidth of 50 MHz centered at 1.430 GHz. (1 GHz = 1000 MHz). Assume that, rather than being a perfect detector over the entire bandwidth, the receiver's frequency dependence is triangular, meaning that the sensitivity of the detector is 0% at the edges of the band and 100% at its center.

- (a) Assume that the radio dish is a 100% efficient reflector over the receiver's bandwidth and has a diameter of 100 m. Assume also that the source NGC 2558 (a spiral galaxy with an apparent visual magnitude of 13.8) has a constant spectral flux density of  $S = 2.5 \text{ mJy}$  over the detector bandwidth. Calculate the total power measured at the receiver.
- (b) Estimate the power emitted at the source in this frequency range if  $d = 100 \text{ Mpc}$ . Assume that the source emits the signal isotropically.

Notes:  $1 \text{ Mpc} = 3.086 \times 10^{22} \text{ m}$

Hints:

- The integral of a triangular function is the area of a triangle.
- Does the power emitted at the source depend on  $f_\nu$ ?

**Solution:** (a) Since the receiver has a triangular frequency dependence,

$$P = S_0 \left[ \pi \left( \frac{D}{2} \right)^2 \right] \frac{\Delta f}{2} = (2.5 \times 10^{-29}) \left[ \pi \left( \frac{100}{2} \right)^2 \right] \frac{50 \times 10^6}{2} \approx 4.909 \times 10^{-18} \text{ W} \approx 4.9 \times 10^{-18} \text{ W}. \quad (5)$$

(b) Using the inverse square law,

$$P_{\text{source}} = \frac{P_{\text{receiver}}}{\pi \left( \frac{D}{2} \right)^2} (4\pi d^2) = \frac{16 P_{\text{receiver}} d^2}{D^2} = \frac{16 (4.909 \times 10^{-18}) (100 \times 3.0857 \times 10^{22})^2}{100^2} \approx 7.479 \times 10^{28} \text{ W} \approx 7.5 \times 10^{28} \text{ W}. \quad (6)$$

■

**Question 4.** Consider an achromatic doublet. Let  $R_1$  and  $R_2$  be the radii of curvature of the converging lens and  $-R_2$  and  $R_3$  be the radii the curvature of the diverging lens.

- (a) Express the focal length for red light ( $f_r$ ) in terms of  $R_1$ ,  $R_2$ ,  $R_3$ , and the refractive indices of the crown and flint glasses.
- (b) Express the focal length for blue light ( $f_b$ ) in terms of  $R_1$ ,  $R_2$ ,  $R_3$ , and the refractive indices of the crown and flint glasses.
- (c) Let  $f_r = f_b$ . Find an equation relating  $R_1$ ,  $R_2$ ,  $R_3$ . Simplify the equation.

**Solution:** (a) For the crown (converging) lens made of crown glass with surfaces of radii  $R_1$  and  $R_2$ , it has a focal length  $f_{c, r}$  of,

$$\frac{1}{f_{c, r}} = (n_{c, r} - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right). \quad (7)$$

For the flint (diverging) lens with surfaces of radii  $-R_2$  and  $R_3$ , it has a focal length  $f_{f, r}$  of,

$$\frac{1}{f_{f, r}} = (n_{f, r} - 1) \left( \frac{1}{-R_2} - \frac{1}{R_3} \right) = (n_{f, r} - 1) \left( -\frac{1}{R_2} - \frac{1}{R_3} \right). \quad (8)$$

Thus, for red light

$$\begin{aligned} \frac{1}{f_r} &= \frac{1}{f_{c, r}} + \frac{1}{f_{f, r}} \\ &= (n_{c, r} - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) + (n_{f, r} - 1) \left( -\frac{1}{R_2} - \frac{1}{R_3} \right). \end{aligned} \quad (9)$$

(b) and for blue light,

$$\frac{1}{f_b} = (n_{c, b} - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) + (n_{f, b} - 1) \left( -\frac{1}{R_2} - \frac{1}{R_3} \right). \quad (10)$$

(c) Let  $f_r = f_b$ ,

$$\begin{aligned} &(n_{c, r} - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) + (n_{f, r} - 1) \left( -\frac{1}{R_2} - \frac{1}{R_3} \right) \\ &= (n_{c, b} - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) + (n_{f, b} - 1) \left( -\frac{1}{R_2} - \frac{1}{R_3} \right) \end{aligned} \quad (11)$$

$$\Rightarrow [(n_{c, r} - 1) - (n_{c, b} - 1)] \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = [(n_{f, b} - 1) - (n_{f, r} - 1)] \left( -\frac{1}{R_2} - \frac{1}{R_3} \right) \quad (12)$$

$$\Rightarrow (n_{c, r} - n_{c, b}) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = (n_{f, b} - n_{f, r}) \left( -\frac{1}{R_2} - \frac{1}{R_3} \right) \quad (13)$$

$$\therefore (n_{c, r} - n_{c, b}) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = (n_{f, r} - n_{f, b}) \left( \frac{1}{R_2} + \frac{1}{R_3} \right). \quad (14)$$

■