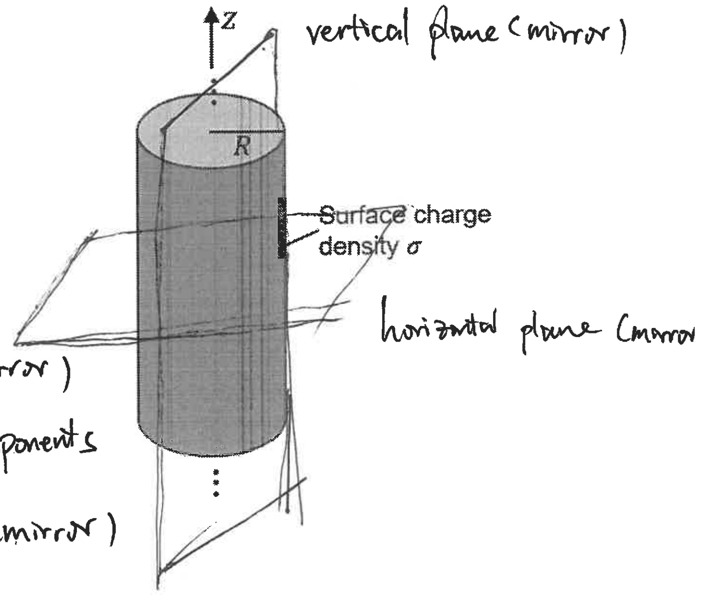
Student Name:

SIS ID (starts with letter "e"):

1. Consider an infinitely long cylindrical tube (with radius R) carrying a surface charge density of σ, use the Gauss's law to calculate the electric field inside ($s < R$) and outside the tube ($s > R$). Please clearly list the symmetry arguments which facilitate application of the Gauss's law.

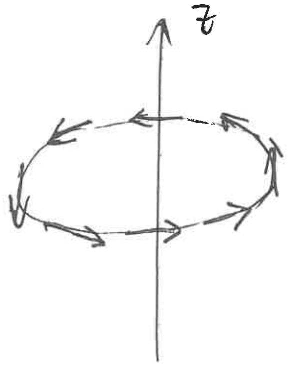


- Translational symmetry along z
  $\Rightarrow \vec{E}$ uniform along $\hat{z}$
- rotational symmetry about axis z
  $\Rightarrow \vec{E}$ uniform along $\hat{\phi}$
- reflection about horizontal plane (mirror)
  $\Rightarrow \vec{E}$ must not contain $\hat{z}$ components
- reflection about vertical plane (mirror)
  $\Rightarrow \vec{E}$ must not contain $\hat{\phi}$ components

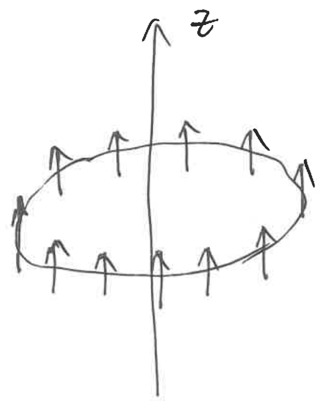
$$\vec{E} = E_s(s, \cancel{\phi}, \cancel{z})\,\hat{s} + \cancel{E_\phi(s, \phi, z)\,\hat{\phi}} + \cancel{E_z(s, \phi, z)\,\hat{z}}$$

$$= E(s)\,\hat{s}$$

$E_\phi(s, \phi, z)\,\hat{\phi}$



$E_z(s, \phi, z)\,\hat{z}$



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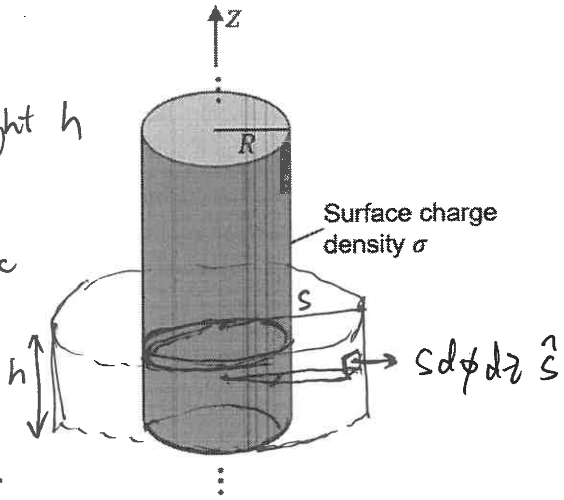
- $s > R$, choose Gaussian surface as a cylinder with radius s, height h

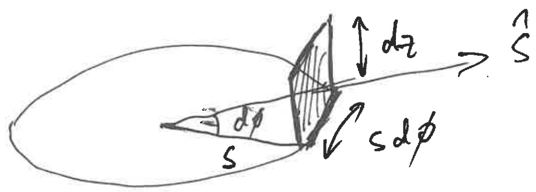


Gauss's law $\oint \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$

$$\oint \vec{E}\cdot d\vec{a}$$

$$= \underbrace{\oint_{top} \vec{E}\cdot d\vec{a} + \oint_{bottom} \vec{E}\cdot d\vec{a}}_{} + \oint_{side} \vec{E}\cdot d\vec{a}$$

$$\vec{E}\cdot d\vec{a} = E(s)\,\hat{s}\cdot(\pm\hat{z})\,da = 0$$



$$= \oint_{side} E(s)\,\hat{s}\cdot\hat{s}\,da$$

$$= \oint_{side} E(s)\,da = \int_0^h \left(\int_0^{2\pi} s E(s)\,d\phi\right) dz$$

$$= s E(s) \int_0^h \left(\int_0^{2\pi} d\phi\right) dz = 2\pi h s E(s)$$

$$\frac{1}{\varepsilon_0} Q_{enc} = \frac{1}{\varepsilon_0} \underbrace{(2\pi R h)}_{\text{Area of side surface of inner cylinder carrying charge}} \sigma$$

$$\Rightarrow 2\pi h s E(s) = \frac{1}{\varepsilon_0} 2\pi R h \sigma \Rightarrow E(s) = \frac{R\sigma}{s\varepsilon_0}$$

$$\Rightarrow \vec{E} = \frac{R\sigma}{\varepsilon_0 s} \cdot \hat{s}$$

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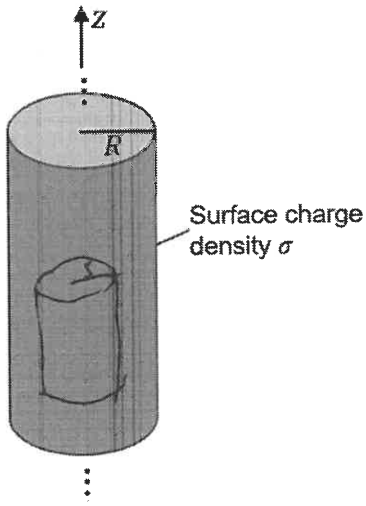

- $s<R$, choose Gaussian surface as a cylinder with radius s. height h

$$\oint \vec{E}\cdot d\vec{a} = \cdots = 2\pi h s\, E(s)$$

$$\frac{1}{\varepsilon_0} Q_{enc} = 0$$

$$\Rightarrow \quad E(s) = 0$$

$$\Rightarrow \quad \vec{E} = 0$$