Student Name:

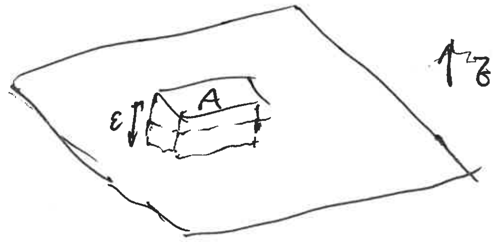
SIS ID (starts with letter "e"):

1. Knowing the relation $\nabla \cdot \boldsymbol{H} = -\nabla \cdot \boldsymbol{M}$ between the auxiliary field $\boldsymbol{H}$ and the magnetization $\boldsymbol{M}$, prove that the perpendicular components satisfy

$$H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}})$$

across any surface.

$$\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}$$



$$\Rightarrow \int_V \nabla \cdot \vec{H}\, d\tau = -\int_V \nabla \cdot \vec{M}$$ where V is the Gaussian pillbox that straddles the surface

From divergence theorem

$$\oint_S \vec{H} \cdot d\vec{a} = -\oint_S \vec{M} \cdot d\vec{a}$$

Within the Gaussian pillbox, side surface contributions to the integral $\to 0$ as thickness $\varepsilon \to 0$, so only the top and bottom caps of the box contribute.

$$\Rightarrow \vec{H}_{\text{above}} \cdot A\hat{z} + \vec{H}_{\text{below}} \cdot A(-\hat{z}) = -\vec{M}_{\text{above}} \cdot A \cdot \hat{z} - \vec{M}_{\text{below}} \cdot A(-\hat{z})$$

$$\Rightarrow (H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}})\hat{z} = -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}})\hat{z}$$

$$\Rightarrow H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}})$$