Time-dependent perturbation theory.

$n \neq m$

$$P_{n \leftarrow m} = \left(\frac{1}{\hbar}\right)^2 \left| \int_{t_0}^{t} \langle \psi_n^0 | V(t) | \psi_m^0 \rangle e^{\frac{i(E_n^0 - E_m^0)(t_1 - t_0)}{\hbar}} dt_1 \right|^2$$

time-dependence ⇒ gives us resonance condition for harmonic perturbation, frequency Ω

$P_{n \leftarrow m}$ is max. when $\Omega = \frac{E_n^0 - E_m^0}{\hbar}$.

BUT sometimes $P_{n \leftarrow m}$ can be zero.
selection rules.

For light-matter interactions:

relevant matrix elements for selection rules:

$$|\langle e | \hat{\varepsilon} \cdot \vec{p} | g \rangle| \quad \text{or} \quad |\langle e | \hat{\varepsilon} \cdot (q\vec{r}) | g \rangle|$$

Today: work out these selection rules for atoms with spherical harmonic (+ radial function) as eigenstates.

Selection rules tell us when matrix element must be zero. (by symmetry arguments)

(But there can be other matrix elements very close to zero.)

Consider

$|e\rangle, |g\rangle \longrightarrow |nlm\rangle$ eigenstates of a hydrogen atom.

When is $\langle e | \vec{r} | g \rangle = 0$?

Selection rules are: For $\langle e | \vec{r} | g \rangle \neq 0$ in the hydrogen atom,

we need $\Delta l = \pm 1$

and $\Delta m = 0, \pm 1$.

Eg. In 2p state.

$l = 1$, $m = \begin{cases} +1 \\ 0 \\ -1 \end{cases}$

Can we transition to: 3s state?
$l=0,\ m=0$

$(\Delta l = 0-1 = -1,\quad \Delta m = 0, \pm 1)$

Not forbidden by selection rules.

Can we transition from the 2p state to: 3d state?
$l=2$

Not forbidden.

Can we directly transition from the 1s state to the 3d state?
$l=0$ $\qquad$ $l=2$

No – forbidden.

Work out selection rules

[A] Spherical harmonic $Y_l^m(\theta,\phi) \rightarrow \begin{cases} \text{even if } l \text{ is even} \\ \text{odd if } l \text{ is odd.} \end{cases}$

(no need to remember)

(even/odd with respect to $\vec{r} \longrightarrow -\vec{r}$)

$$\langle e | \underset{\sim}{\vec{r}} | g \rangle = 0 \quad \text{if} \begin{cases} |e\rangle \text{ and } |g\rangle \text{ are both even} \\ \text{OR} \\ |e\rangle \text{ and } |g\rangle \text{ are both odd.} \end{cases}$$

odd operator

ie. $\langle e | \vec{r} | g \rangle = 0$ if Δl is even.

[B] Photon is a spin-1 particle.

$l_{photon} = 1$

$m = +1, 0, -1$

Addition of angular momentum

$\underset{\sim}{l_e}$

$l \qquad l_{total} = l + 1$

Addition of angular momentum,

Let $l_g = l_0$

We have $l_{photon} = 1$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \quad \begin{aligned} \max l^{total} &= l_0 + 1 \\ \min l^{total} &= |l_0 - 1| \end{aligned}$$

$$\Downarrow$$

$$\Delta l = -1, 0, +1$$

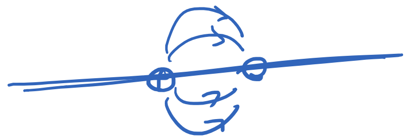
But from [A], If $\Delta l = 0$, $\langle e | \vec{r} | g \rangle = 0$.

$\Rightarrow$ $\Delta l = \pm 1$ is required for $\langle e | \vec{r} | g \rangle \neq 0$.
(but not sufficient)

$\boxed{\text{AND}}$ we also need $\Delta m = 0, \pm 1$ from addition of angular momentum.

(not in exam) excitonic interactions

bulk (3D) Si
(dielectric constant is larger in the bulk)



thin film Si

$$V(r) = -\frac{e^2}{4\pi\varepsilon r}$$

(not in this year's exam).

Adiabatic approx. — $V(t)$ is slow compared to time scale of the system — the system remain in its original state, which might change slightly because of $V(t)$.

Particle in a box.

Length a initially.

Slowly expand to αa
$(\alpha > 1)$

over a long interval lasting t seconds.

Eg If system were in state $|n\rangle$, assume it remains in state $|n\rangle$.

time.

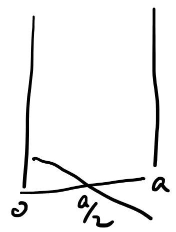
If before the expansion, the system is in state

$$\varphi_1(x) = \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a} \qquad 0 \le x \le a.$$

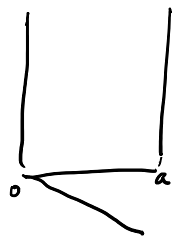
Then $\varphi_1 \xrightarrow[\text{(stays in state 1)}]{} \varphi_1(t) = \sqrt{\frac{2}{\alpha a}} \sin\left(\frac{\pi x}{\alpha a}\right)$

$$E_1 \longrightarrow E_1(t) = \frac{\hbar^2 \pi^2}{2m(\alpha a)^2}.$$

Tut 5 Q1



V(x) in (a)



V(x) in (b)
(shift downwards) by a constant.

Revision - general revision.
- next lecture - go through the rest of past year papers.

Background required from QM I.

- Postulation of QM.
measurements.
- expectation values
- uncertainty relations
- Schrödinger's equations. $\begin{cases} \text{time-independent} \\ \text{time-dependent.} \end{cases}$
- linear algebra., some calculus.
- standard egs
  - infinite square well
  - harmonic oscillator.

etc..

(Formula will be provided)

QM II (new topics)

- Angular momentum
- orbital, spin, total
- J^2, J_z, J_+, J_- and manipulations of commutator relations (formula sheet)
- addition of angular momentum.

+

Approximations for solving Schrödinger's equation.

- Born-Oppenheimer approximation
- Central potential approximation
- Single particle "
- Variational principle.
  different ways to use it.
- time-independent perturbation theory
  non-deg   degenerate
- time-dependent perturbation theory
  interaction picture, Heisenberg ", Schrödinger "
  resonance condition
  Fermi's Golden rule
  selection rules.
- Light-matter interaction
  - dipole approximation
  - rotating wave approximation

(- adiabatic approx.)

Tools

tensor products

Symmetries (how to apply)

&

identical & indistinguishabe

particles

(fermions , bosons)