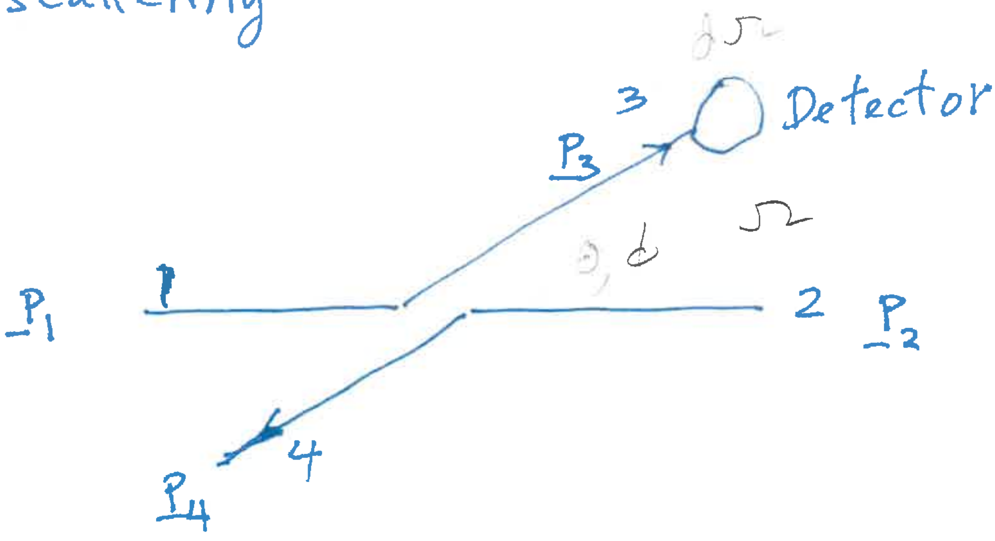
Consider 2 particles to 2 particles scattering



$$d\Omega = \frac{ds}{r^2}$$

Using the Fermi golden rule, the differential cross section can be written as (page 7) → previous lecture

$\mathcal{M}$ = scattering amplitude

$$d\sigma = \frac{s\,\hbar^2}{4\cdot\sqrt{(\underline{p}_1\cdot\underline{p}_2)^2 - (m_1 m_2 c^2)^2}} \cdot |\mathcal{M}|^2 \cdot$$

$$\cdot (2\pi)^4\, \delta^{(4)}(\underline{p}_1 + \underline{p}_2 - \underline{p}_3 - \underline{p}_4) \cdot$$

$$\prod_{j=3}^{4} \frac{d^4 p_j}{(2\pi)^4} (2\pi)\, \delta(\underline{p}_j^2 - m_j^2 c^2) \cdot \Theta(p_j^0)$$

Integrating away the energy p_j^0

We get

$$d\sigma = \frac{s\hbar^2}{4\sqrt{(\underset{\sim}{P}_1\cdot \underset{\sim}{P}_2)^2-(m_1 m_2 c^2)^2}} \cdot |\mathcal{M}|^2 \cdot$$

$$(2\pi)^4\, \delta^{(4)}(\underset{\sim}{P}_1+\underset{\sim}{P}_2-\underset{\sim}{P}_3-\underset{\sim}{P}_4)\cdot \prod_{j=3}^{4} \frac{d^3\underset{\sim}{p}_j}{(2\pi)^3}\, \frac{1}{2p_j^0}$$

$$= \frac{s\hbar^2}{4\cdot\sqrt{(\underset{\sim}{P}_1\cdot \underset{\sim}{P}_2)^2-(m_1 m_2 c^2)^2}}\, |\mathcal{M}|^2\cdot (2\pi)^4\, \delta^{(4)}(\underset{\sim}{P}_1+\underset{\sim}{P}_2-\underset{\sim}{P}_3-\underset{\sim}{P}_4)$$

$$\frac{d^3\underset{\sim}{p}_3}{(2\pi)^3}\, \frac{d^3\underset{\sim}{p}_4}{(2\pi)^3}\cdot \frac{1}{2p_3^0}\cdot \frac{1}{2p_4^0}$$

Integrating away $\int d^3\underset{\sim}{p}_4$ by using the Dirac delta function $\delta^{(3)}(\underset{\sim}{p}_1+\underset{\sim}{p}_2-\underset{\sim}{p}_3-\underset{\sim}{p}_4)$,

$$d\sigma = \frac{s\hbar^2}{4\cdot\sqrt{(\underset{\sim}{P}_1\cdot \underset{\sim}{P}_2)^2-(m_1 m_2 c^2)^2}}\cdot |\mathcal{M}|^2 \cdot$$

$$\delta(p_1^0+p_2^0-p_3^0-p_4^0)\cdot \frac{d^3\underset{\sim}{p}_3}{(2\pi)^2}\cdot \frac{1}{4\,p_3^0\cdot p_4^0}$$

where $$\underset{\sim}{P}_4 = \underset{\sim}{P}_1 + \underset{\sim}{P}_2 - \underset{\sim}{P}_3$$

We assume the detector is detecting particle 3

So
$$d^3\underset{\sim}{P}_3 = |\underset{\sim}{P}_3|^2 \cdot d|\underset{\sim}{P}_3| \cdot d\Omega_{\underset{\sim}{P}_3}$$

and compute $\dfrac{d\sigma}{d\Omega_{\underset{\sim}{P}_3}}$.

We write

$$\frac{d\sigma}{d\Omega_{\underset{\sim}{P}_3}} = \frac{s\hbar^2}{4\cdot\sqrt{(\underline{P}_1\cdot\underline{P}_2)^2 - (m_1 m_2 c^2)^2}}\,.$$

$$\cdot \int \frac{|\underset{\sim}{P}_3|^2 \cdot d|\underset{\sim}{P}_3|}{(4\pi)^2\, P_3^0\, P_4^0} \cdot |\mathcal{M}|^2 \cdot \delta(P_1^0 + P_2^0 - P_3^0 - P_4^0)$$

$$\underset{\sim}{P}_4 = \underset{\sim}{P}_1 + \underset{\sim}{P}_2 - \underset{\sim}{P}_3$$
$$= -\underset{\sim}{P}_3 \quad (\text{CM frame})$$

Changing the integrating variable $|\underset{\sim}{P}_3|$ by defining

$$P^0 = P_3^0 + P_4^0,$$

$$dp^0 = dp_3^0 + dp_4^0$$

$$= \frac{|\mathbf{p}_3| \cdot d|\mathbf{p}_3|}{p_3^0} + \frac{|\mathbf{p}_3| \cdot d|\mathbf{p}_3|}{p_4^0}$$

$$\because \quad p_3^0 = \sqrt{\mathbf{p}_3^2 + m_3^2 c^2}$$

$$\mathbf{p}_4 = -\mathbf{p}_3$$

$$= \frac{p^0}{p_3^0 \cdot p_4^0} \cdot |\mathbf{p}_3| \cdot d|\mathbf{p}_3|$$

$$\therefore \quad \frac{dp^0}{p^0} = \frac{|\mathbf{p}_3| \cdot d|\mathbf{p}_3|}{p_3^0 \cdot p_4^0}$$

We get

$$\frac{d\sigma}{d\Omega} = \frac{s\hbar^2}{4 \cdot \sqrt{(\mathbf{p}_1 \cdot \mathbf{p}_2)^2 - (m_1 m_2 c^2)^2}} \cdot \frac{1}{(4\pi)^2} \cdot$$

$$\int |\mathbf{p}_3| \cdot \frac{dp^0}{p^0} \cdot |\mathcal{M}|^2 \cdot \delta(p_1^0 + p_2^0 - p^0)$$

Integrating $\int dp^0$,

$$\frac{d\sigma}{d\Omega} = \frac{s\hbar^2}{(8\pi)^2 \sqrt{(\underset{\sim}{P_1}\cdot\underset{\sim}{P_2})^2 - (m_1 m_2 c^2)^2}} \cdot \frac{|M|^2 \cdot |\underset{\sim}{P_3}|}{(P_1^0 + P_2^0)}$$

$\underset{\sim}{P_3} = -\underset{\sim}{P_4}$ (CM frame)

$p^0 = P_3^0 + P_4^0 = P_1^0 + P_2^0$

Only unknown is $|\underset{\sim}{P_3}|$

We can find $|\underset{\sim}{P_3}|$ by using

$$P_1^0 + P_2^0 = P_3^0 + P_4^0$$

$$= \sqrt{\underset{\sim}{P_3}^2 + m_3^2 c^2} + \sqrt{\underset{\sim}{P_3}^2 + m_4^2 c^2}$$

As $(P_1^0 + P_2^0)$ is fixed and known, so can get $|\underset{\sim}{P_3}|$ from the above relation

$$\underset{\sim}{P_3}^2 = \frac{\left(K^2 + (m_4^2 - m_3^2)c^2\right)^2}{4K^2} - m_4^2 c^2$$

$$K \equiv P_1^0 + P_2^0$$

Chapter 7 QED

Part I

QED study interaction of a charged particle with a photon

1. Equation of motion for the charged particle (as a free particle)

2. Equation of motion for the photon (as a free particle)

3. Interaction between the charged particle and the photon

classically, interaction of electric current $\underset{\sim}{j}$ and $\underset{\sim}{E}$, $\underset{\sim}{B}$

Chapter 7 Griffiths QED Part I

Basically we study the dynamics of charged particles with electromagnetic field; Interaction of a charged particle e.g. electron with a photon in QED

Classically, equation of motion is needed for particles

Particles obey Newton's law $\underset{\sim}{F} = \frac{d\underset{\sim}{p}}{dt}$ (1687)

The electromagnetic field $(\underset{\sim}{E}, \underset{\sim}{B})$ obeys the Maxwell eqn (1865)

$$\nabla \cdot \underset{\sim}{E} = \frac{\rho}{\epsilon_0}$$ ρ = charge density

$$\nabla \wedge \underset{\sim}{E} = -\frac{\partial \underset{\sim}{B}}{\partial t}$$

$$c^2 (\nabla \wedge \underset{\sim}{B}) = \frac{\underset{\sim}{j}}{\epsilon_0} + \frac{\partial \underset{\sim}{E}}{\partial t}$$ $\underset{\sim}{j}$ = current density

$$\nabla \cdot \underset{\sim}{B} = 0$$

Lorentz force equation $\underset{\sim}{F} = q(\underset{\sim}{E} + \underset{\sim}{v} \wedge \underset{\sim}{B})$

The above 3 sets of equations answer all problems of charged particles interacting with $\underset{\sim}{E}, \underset{\sim}{B}$.

Quantum mechanically, the Newton eqn is replaced by Schrödinger equation or by the Dirac equation if including relativistic effect. The Maxwell equation can be taken over quantum mechanically by using the gauge field $A_\mu(\underset{\sim}{x})$

To study the interaction of a photon with an electron, first find free photon solution (plane wave) and also free electron solution (plane wave). After free particle solutions are obtained, we solve the interaction (Hamiltonian) by using Feynman diagramatic technique, basically a perturbation method.

Now first put Maxwell's equations in relativistically covariant form:

$$\underset{\sim}{E} = -\nabla V - \frac{\partial \underset{\sim}{A}}{\partial t} \qquad V = \text{electric potential}$$

By convention $(\nabla V)^i = \partial_i V = \frac{\partial V}{\partial x^i}$, $\therefore E^i = -\frac{\partial V}{\partial x^i} - \frac{\partial A^i}{\partial t}$ $\qquad \underset{\sim}{A} = \text{vector potential}$

$$\underset{\sim}{B} = \nabla \wedge \underset{\sim}{A}$$

$$B^i = (\nabla \wedge \underset{\sim}{A})^i = \varepsilon^{ijk} \partial_j A^k = -\varepsilon^{ijk} \partial_j A_k$$

put V and $\underset{\sim}{A}$ together as a <u>4-vector</u>, A_μ

$$\underline{A} = \left(\frac{V}{c}, \underset{\sim}{A}\right) = (A^0, \underset{\sim}{A})$$

Introduce electromagnetic field tensor $F_{\mu\nu}$,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \mu, \nu = 0, 1, 2, 3$$

$$= \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}$$

Check $E^i = c F^{i0}$ $\qquad$ (HW)

$$B^i = -\frac{1}{2} \varepsilon^{ijk} F_{jk}$$

$$g^{00} = +1, \; g^{11} = -1 = g^{22} = g^{33}$$

The 4 Maxwell equations become

$$\partial_\mu F^{\mu\nu} = j^\nu \qquad (j^0 = \rho c) \quad \underline{j} = (j^0; \underset{\sim}{j})$$

$$\ldots \qquad (1)$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \ldots \quad \text{sourceless}$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2!} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

$\tilde{F}$ = dual of F

Look for free photon solution from eq (1), i.e. want to find $A_\mu(\underline{x})$ for a free photon, $A_\mu(\underline{x})$ = gauge field

First note equation (1) has a gauge degree of freedom because a new gauge field $A'_\mu(\underline{x})$ defined by

$$A'_\mu(\underline{x}) = A_\mu(\underline{x}) + \partial_\mu \lambda(\underline{x})$$

$\lambda(\underline{x})$ = smooth function

can lead to the same $F_{\mu\nu}$:

$$F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu$$

$$= F_{\mu\nu} + \partial_\mu \partial_\nu \lambda(\underline{x}) - \partial_\nu \partial_\mu \lambda(\underline{x})$$

$$= F_{\mu\nu}$$

Can introduce conditions to make $A_\mu(\underline{x})$ unique. First impose Lorentz condition

$\underset{\sim}{P} = \frac{\hbar}{i}\nabla$

$$\partial_\mu A^\mu = 0 \longrightarrow \partial_\mu \partial^\mu \lambda(\underset{\sim}{x}) = 0$$

$P_\mu = i\hbar\partial_\mu \longrightarrow P \cdot A = 0$

still not sufficient to specify $A^\mu(x)$ uniquely.

Next use coulomb gauge condition to make

$$A^0 = 0$$

$\underset{\sim}{P} \cdot \underset{\sim}{A} = 0$ $\quad \nabla \cdot \underset{\sim}{A} = 0$ coulomb gauge

With the Lorentz condition and coulomb gauge, $\underset{\sim}{A}$ has 2 independent components

The free photon equation

$$\partial_\mu F^{\mu\nu} = 0, \quad \partial_\mu \tilde{F}^{\mu\nu} = 0$$

Look at $\partial_\mu F^{\mu\nu} = 0$

$$\partial_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu) = 0 \qquad \partial_\mu = \frac{\partial}{\partial x^\mu}, \quad \partial^\mu = \frac{\partial}{\partial x_\mu}$$

$$\partial_\mu \partial^\mu A^\nu \underline{- \partial_\mu \partial^\nu A^\mu} = 0 \qquad \text{HW}$$

$$\hookrightarrow \partial^\nu \partial_\mu A^\mu = 0$$

$\because \partial_\mu A^\mu = 0$ (Lorentz condition)

$$\rightarrow \partial_\mu \partial^\mu A^\nu = 0$$

$$\rightarrow \Box^2 A^\nu = 0, \quad \cdots (2)$$

$\Box^2 =$ D'Alembertian

$\equiv \partial_\mu \partial^\mu$

$= \left(\frac{\partial}{\partial x^0}\right)^2 - \left(\frac{\partial}{\partial x^1}\right)^2 - \left(\frac{\partial}{\partial x^2}\right)^2 - \left(\frac{\partial}{\partial x^3}\right)^2$

solution is Ansatz

$$A_\mu(x) = \text{Const} \cdot e^{-i\underset{\sim}{P}\cdot\underset{\sim}{x}/\hbar} \cdot \varepsilon_\mu(\underset{\sim}{P}) \quad \cdots (3)$$