PC3261
Classical Mechanics II

Worksheet 1

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Exercise 1.1. Find the particles's velocity and acceleration vectors. What are the magnitude and direction of the particle's acceleration?

Solution: Starting with the position vector of the particle undergoing uniform circular motion,

$$\mathbf{r}(t) = R\cos\omega t\hat{\mathbf{e}}_x + R\sin\omega t\hat{\mathbf{e}}_y. \tag{1}$$

From the definition of the velocity vector as the rate of change of the position vector w.r.t. time,

$$\begin{aligned}\mathbf{v}(t) \equiv \frac{d\mathbf{r}(t)}{dt} &= \frac{d}{dt}(R\cos\omega t\hat{\mathbf{e}}_x + R\sin\omega t\hat{\mathbf{e}}_y) = R(-\omega\sin\omega t\hat{\mathbf{e}}_x + \omega\cos\omega t\hat{\mathbf{e}}_y) \\ &= R\omega(-\sin\omega t\hat{\mathbf{e}}_x + \cos\omega t\hat{\mathbf{e}}_y).\end{aligned} \tag{2}$$

Similarly, from the definition of the acceleration vector as the rate of change of the velocity vector w.r.t. time,

$$\begin{aligned}\mathbf{a}(t) \equiv \frac{d\mathbf{v}(t)}{dt} &= \frac{d}{dt}[R\omega(-\sin\omega t\hat{\mathbf{e}}_x + \cos\omega t\hat{\mathbf{e}}_y)] = R\omega(-\omega\cos\omega t\hat{\mathbf{e}}_x - \omega\sin\omega t\hat{\mathbf{e}}_y) \\ &= -R\omega^2(\cos\omega t\hat{\mathbf{e}}_x + \sin\omega t\hat{\mathbf{e}}_y),\end{aligned} \tag{3}$$

which yields a magnitude,

$$\begin{aligned}a(t) = \sqrt{\mathbf{a}(t)\cdot\mathbf{a}(t)} &= \sqrt{R^2\omega^4\cos^2\omega t\,\underbrace{(\hat{\mathbf{e}}_x\cdot\hat{\mathbf{e}}_x)}_{1} + R^2\omega^4\sin^2\omega t\,\underbrace{(\hat{\mathbf{e}}_y\cdot\hat{\mathbf{e}}_y)}_{1}} \\ &= R\omega^2,\end{aligned} \tag{4}$$

and a direction,

$$\hat{\mathbf{a}}(t) = \frac{\mathbf{a}(t)}{a(t)} = -\cos\omega t\hat{\mathbf{e}}_x - \sin\omega t\hat{\mathbf{e}}_y, \tag{5}$$

pointing radially inwards towards the centre of the circular path.