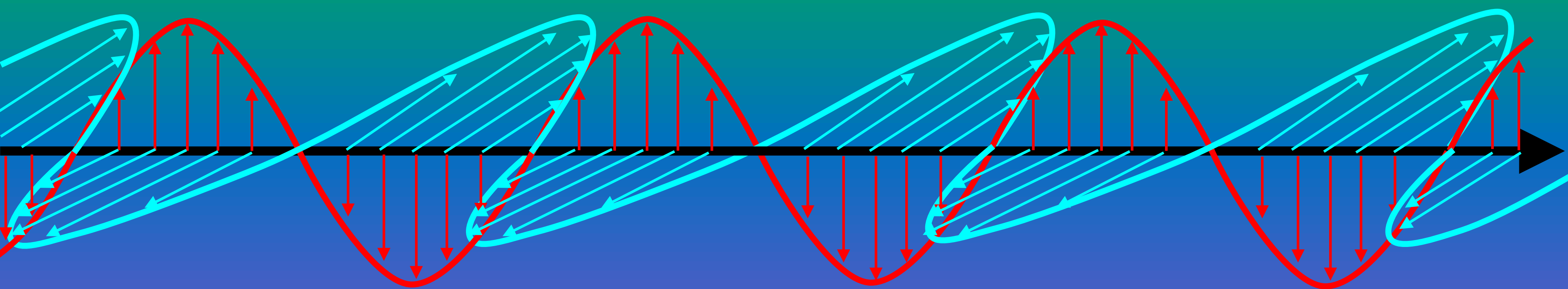
Electrodynamics



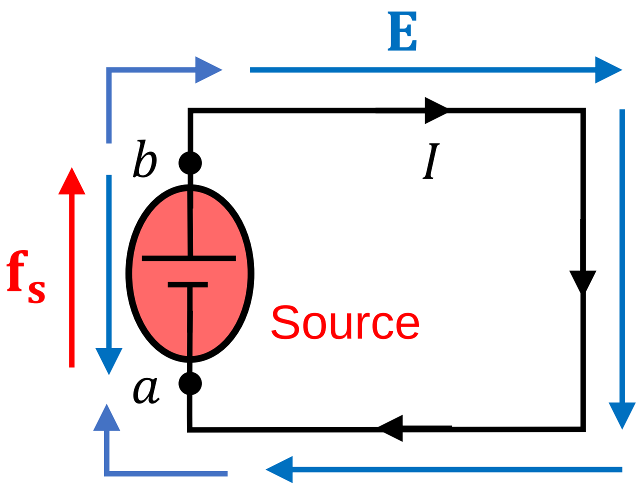
Electromotive force

- Electromotive "force", or emf ($\mathcal{E}$)
  - $\mathcal{E}$ defined as work done per unit charge by the source
  - Force experienced by unit charge in a circuit
    - $\mathbf{E}$ field pointing from the high potential to low potential
    - $\mathbf{f_s}$ pointing from the low to high potential in the source region
    - $\mathbf{f_s} = -\mathbf{E}$ for ideal sources
  - Relation of emf with potential difference

$$\Delta V = V(b) - V(a) = -\int_a^b \mathbf{E} \cdot d\mathbf{l}$$

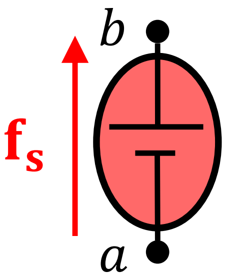
$$\mathcal{E} = \int_a^b \mathbf{f_s} \cdot d\mathbf{l} = -\int_a^b \mathbf{E} \cdot d\mathbf{l}$$

$$\Rightarrow \mathcal{E} = \Delta V$$



Electromotive force

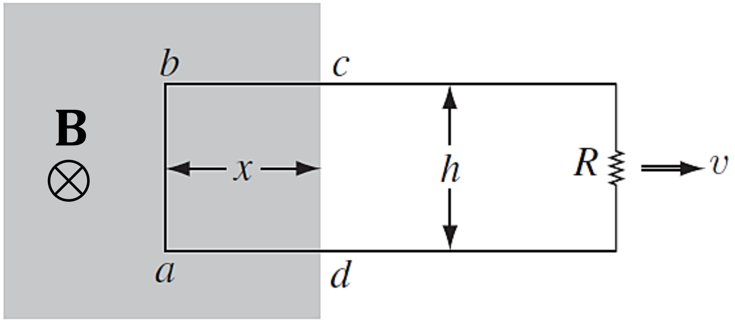
- Electromotive "force", or emf ($\mathcal{E}$)
  - Source: creator of $\mathbf{f_s}$ that initiates charge movement
    - Can be from chemical reaction, thermoelectricity, or piezoelectricity…
    - a 1.5 V battery = a battery giving an emf of 1.5V

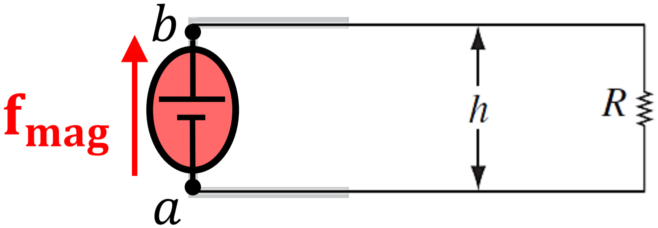


- Motional electromotive "force"
  - Moving wire through a magnetic field

  Lorentz force law

$$\mathcal{E} = \oint \mathbf{f}_{\text{mag}} \cdot d\mathbf{l} = vBh$$

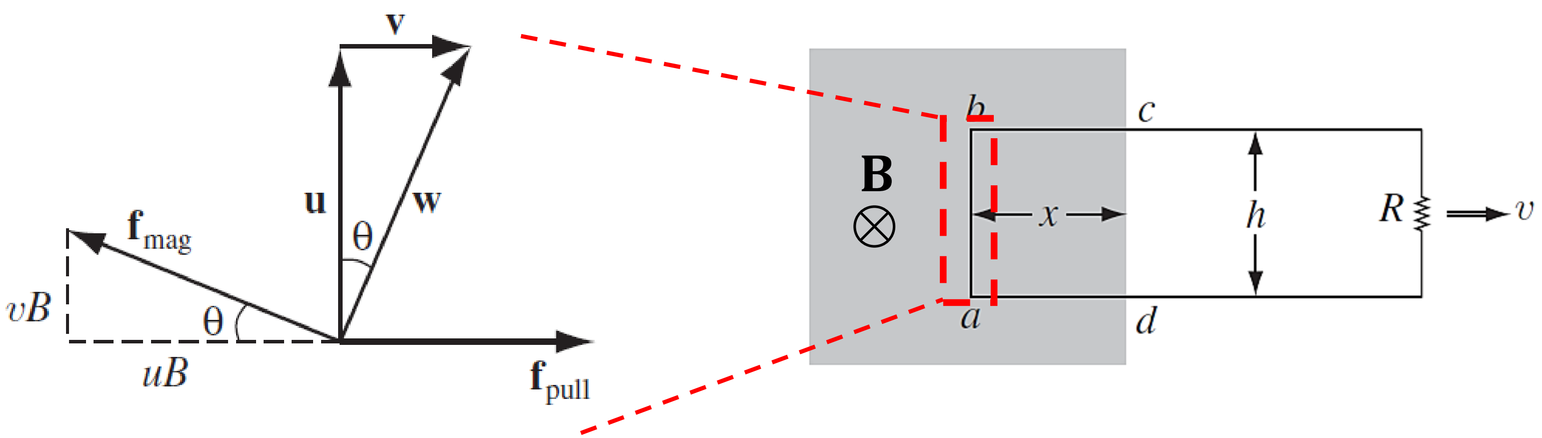




Electromotive force

- Motional electromotive "force"
  - It's not **B** field that is doing the work

$$\int \mathbf{f}_{\text{pull}} \cdot d\mathbf{l} = (uB)\left(\frac{h}{\cos\theta}\right)\sin\theta = vBh = \mathcal{E}$$



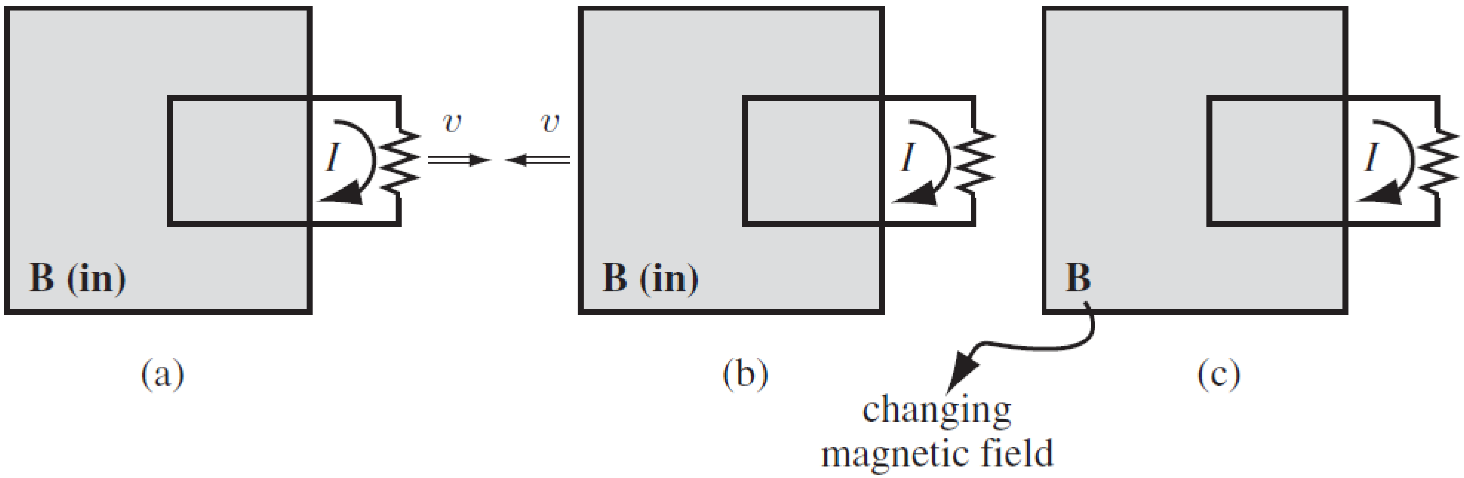
  - emf is minus the rate of change of flux

Apply to arbitrary-shaped loops through nonuniform fields

$$\mathcal{E} = \oint \mathbf{f}_{\text{mag}} \cdot d\mathbf{l} = vBh \xrightarrow{\Phi = Bhx} \boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$

Electromagnetic induction

- Michael Faraday's three experiments



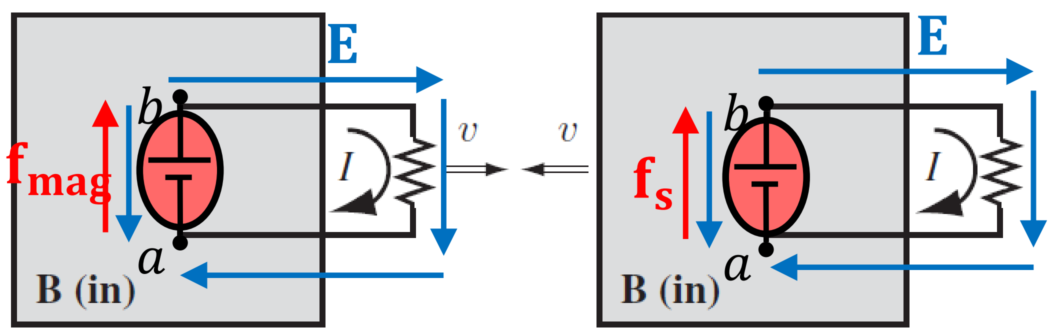
  - Case (a), pulling loops at speed $\mathbf{v}$ (same as previous scenario)
  - Case (b), pulling $\mathbf{B}$ field region at speed $-\mathbf{v}$
  - Case (c), everything still, but change $\mathbf{B}$ field flux at rate $d\Phi/dt$
  - For all cases, observed creation of emf with $\mathcal{E} = -\dfrac{d\Phi}{dt}$

Electromagnetic induction

- Michael Faraday's three experiments
  - Equivalence of cases (a) and (b) really that trivial?
    - Case (a): $\mathcal{E} = \oint \mathbf{f}_{\text{mag}} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$
    - Case (b): $\mathbf{f}_s$ must be electric field because magnetic force cannot be generated by static charge.
  - Therefore, case (b): $\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}$

$$\Rightarrow \boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}}$$

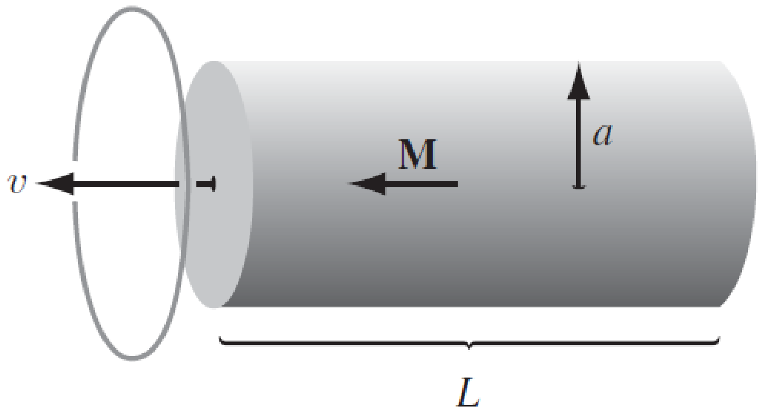
Changing magnetic field induces an electric field!



Electromagnetic induction

- How to figure out the direction of the induced current?
  - Lenz's law: nature abhors a change in magnetic flux
    - The induced current will flow in such a direction that the flux it produces tends to cancel the change of flux

Example 7.5. A long cylindrical magnet of length L and radius a carries a uniform magnetization $\mathbf{M}$ parallel to its axis. It passes at constant velocity v through a circular wire ring of slightly larger diameter (Fig. 7.22). Graph the emf induced in the ring, as a function of time.



Maxwell's correction to EM equations

- Inconsistency within the current EM equations

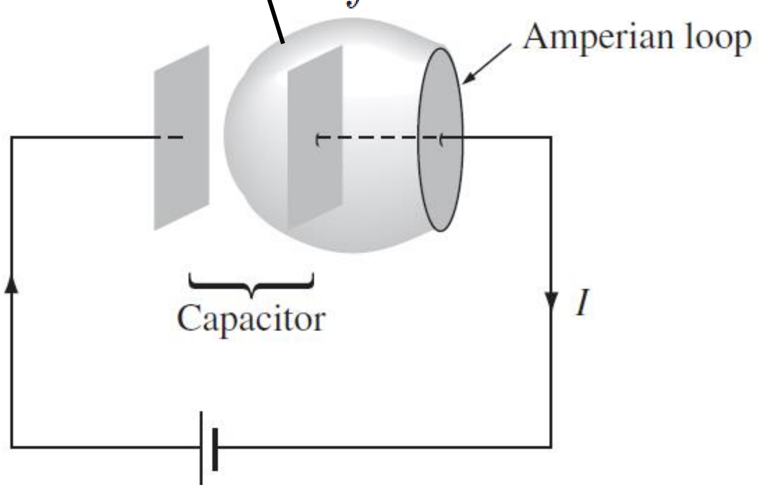
$$
\begin{aligned}
&\text{(i)} \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho && \text{(Gauss's law)},\\
&\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0 && \text{(no name)},\\
&\text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} && \text{(Faraday's law)},\\
&\text{(iv)} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} && \text{(Ampère's law)}.
\end{aligned}
$$

  - Take the divergence of (iv):

    $$\nabla \cdot (\nabla \times \mathbf{B}) = \mu_0 (\nabla \cdot \mathbf{J})$$

    - Left: must be zero (divergence of a curl vanishes), right: does not have to be zero because in electrodynamics $\nabla \cdot \mathbf{J} = -\partial\rho/\partial t$

Contradiction with $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$



Maxwell's correction to EM equations

- Maxwell's equations

$$\text{(i)} \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \qquad \text{(Gauss's law),}$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0 \qquad \text{(no name),}$$

$$\text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \text{(Faraday's law),}$$

$$\text{(iv)} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \underline{\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}} \qquad \text{(Ampère's law with } \underline{\text{Maxwell's correction}}\text{).}$$

  - Now take the divergence of (iv):

$$\nabla \cdot (\nabla \times \mathbf{B}) = \mu_0 \left[\nabla \cdot \mathbf{J} + \varepsilon_0 \frac{\partial (\nabla \cdot \mathbf{E})}{\partial t} \right] = \mu_0 \left[\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} \right] = 0 \quad \checkmark$$

  - The correction term is named "the displacement current" $\mathbf{J}_d \equiv \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$
    - Changing electric field induces a magnetic field

Electromagnetic wave

- Maxwell's equations in vacuum

$$\text{(i)} \quad \nabla \cdot \mathbf{E} = 0, \quad \text{(iii)} \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\text{(ii)} \quad \nabla \cdot \mathbf{B} = 0, \quad \text{(iv)} \quad \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

  - Take the curl of (iii): $\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \dfrac{\partial^2 \mathbf{E}}{\partial t^2}$
  - Take the curl of (iv): $\nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \dfrac{\partial^2 \mathbf{B}}{\partial t^2}$
  - Solution to these wave equations:

$$\tilde{\mathbf{E}}(z, t) = \tilde{\mathbf{E}}_0 e^{i(kz - \omega t)}, \quad \tilde{\mathbf{B}}(z, t) = \tilde{\mathbf{B}}_0 e^{i(kz - \omega t)}$$

    - With speed of light $v = \dfrac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m/s}$