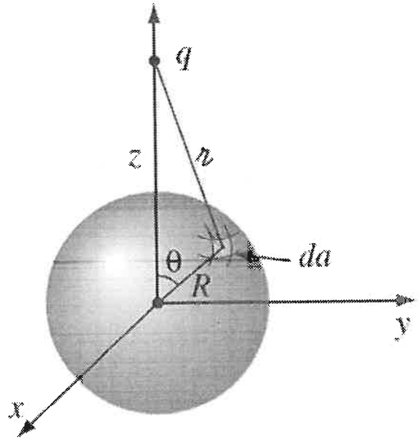
$$V(\mathbf{r}) = \frac{1}{4\pi R^2} \oint_{\text{sphere}} V\, da$$ where area is a sphere centered at $(x, \underbrace{y}_{z})$

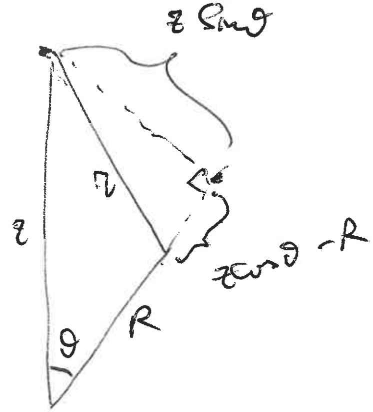


$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{z}$$

$$V_{ave} = \frac{1}{4\pi R^2} \oint \frac{1}{4\pi\varepsilon_0} \frac{q}{\mathscr{r}}\, da$$

$$\mathscr{r}^2 = z^2 + R^2 - 2zR\cos\theta \quad \text{(law of cosines)}$$

$$da = R^2 \sin\theta\, d\theta\, d\phi$$



$$V_{ave} = \frac{1}{4\pi R^2} \frac{q}{4\pi\varepsilon_0} \int_0^{2\pi}\int_0^{\pi} \frac{R^2 \sin\theta}{\sqrt{z^2+R^2-2zR\cos\theta}}\, d\theta\, d\phi$$

$$= \frac{1}{4\pi R^2} \frac{q}{4\pi\varepsilon_0} \cdot 2\pi \int_{(z-R)^2}^{(z+R)^2} \frac{R}{2z} u^{-\frac{1}{2}}\, du$$

$$u \equiv z^2 + R^2 - 2zR\cos\theta$$

$$du = 2zR\sin\theta\, d\theta$$

$$= \frac{1}{4\pi R^2} \frac{q}{4\pi\varepsilon_0} \cdot 2\pi \frac{R}{2z}\, 2u^{\frac{1}{2}} \Big|_{(z-R)^2}^{(z+R)^2} = \frac{1}{4\pi\varepsilon_0 z} \frac{1}{4R} \cdot 2\,(z+R-z+R) = \frac{q}{4\pi\varepsilon_0 z}$$