

CS2040 – Data Structures and Algorithms

Final Lecture – Revision

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Consultation Slots

- Enzo Kam
 - Thursday, 21 November 2-4 pm
 - Room: TBD
 - Roger Zimmermann
 - Friday, 22 November 2-4 pm
 - Room: TBD
- Please be courteous to your Lab TAs. They are all undergrad students and have exams too.

Assessments: Overview

Activities	Weightages
Tutorial + attendance/participation	3%
Lab attendance	2%
In-lab Assignments	15% (1.5%/problem)
Take-home Assignments	12% (1.5%/problem)
Midterm (open book+calculator)	20%
2 Online Quizzes (open book+calculator)	8% (4% each)
Final Exam (open book+calculator)	40%

- **Open Book** = allowed to bring in lectures notes, tutorials, quizzes, reference books or any piece of paper you want **but no internet** and **no offline (i.e., local) LLMs!**

Final Exam



Final Exam Scopes

- Entire semester with emphasis on the second half (10% vs 90%)
 - Lecture notes
 - Tutorials and labs

Final Exam Paper Format

- Exam Date: **Wednesday, 27 Nov 2024**
- Duration: **2 hours (5pm to 7pm)**
- Arrive: at **4:45pm**
- Format: similar to Midterm; Exemplify on your laptop
- Venue: MPSH1-A and MPSH1-B
 - If you are not present at the exam venue(s) you will get 0 for the exam.
- **Open Book** examination (hard copy and PDFs)
- No offline, local LLM!

Final Exam ...

- Please bring a laptop that is fully charged and can last, say, 2.5 hours on Exemplify.
- A few charging seats are available.

Review & Data Structures with Multiple Organizations



Week 13 Mix and Match

Basic Data Structures

- Arrays
 - Linked Lists
 - Map ADT (Hash Table)
 - Stacks and Queues
 - Trees
- We can combine them to implement different data structures for different applications.

Analysis of Array Implⁿ of List ADT

- Time complexity of the different list operations
 - Retrieval: *getItemAtIndex(int i), getFirst(), getLast()*
 - $O(1)$ – indexing into an array is constant time due to random access memory of the computer
 - Insertion: *addItemAtIndex(int i, int item), addFront(), addBack()*
 - Best case = $O(1)$ – if adding at the back and no need to enlarge array
 - Worst case = $O(n)$ – if adding to the front due to shifting all item to the right or adding to the back but need to enlarge the array so have to perform copying of n items to new array
 - Amortized analysis (adding at the back) = ? (find out during lecture)
 - Average case = $O(n)$ – on average need to shift $\frac{1}{2}(n)$ items to the right
 - Deletion: *removeItemAtIndex(int i), removeFront(), removeBack()*
 - Best case = $O(1)$ – if removing from the back
 - Worst case = $O(n)$ – if removing from the front due to shifting all items to the left
 - Average case = $O(n)$ – on average need to shift $\frac{1}{2}(n)$ items to the left

Analysis of **Linked List Implⁿ** of List ADT

- **Time complexity** of the different list operations
 - **Retrieval:** *getItemAtIndex(int i), getFirst(), getLast()*
 - Best case = $O(1)$ – accessing the first node, return the head
 - Worst case = $O(n)$ – accessing the last node, since you need to move all the way to the back from the head (n moves)
 - Average case = $O(n)$ – need to move about half way through the list to access any node on average so $\frac{1}{2}(n)$ iterations of the for loop
 - **Insertion:** *addItemAtIndex(int i, int item), addFront(), addBack()*
 - Best case = $O(1)$ – if adding at the front (don't have to worry about enlarging the list unlike array)
 - Worst case = $O(n)$ – if adding to the back due to having to move all the way to the back from the head (n moves)
 - Average case = $O(n)$ – on average need to make $\frac{1}{2}(n)$ moves

Analysis of **Linked List** Implⁿ of List ADT

- ❑ **Deletion**: *removeItemAtIndex(int i), removeFront(), removeBack()*
 - Best case = $O(1)$ – if removing from the front
 - Worst case = $O(n)$ – if removing from the back, again due to moving all the way to the back from the head
 - Average case = $O(n)$ – on average need to make $\frac{1}{2}(n)$ moves

- What about the **Space Complexity**?
 - ❑ We use as much space as there are nodes in the list so exactly $O(n)$ (plus some constant overhead to store each node which requires more space than a simple integer that is stored in an array)

Map ADT Operations

	Sorted List (Array impl. by sorting key)	Balanced BST	HashTable
Insert	$O(n)$	$O(\log n)$	$O(1)$ avg
Delete	$O(n)$	$O(\log n)$	$O(1)$ avg
Find	$O(\log n)$	$O(\log n)$	$O(1)$ avg

Note: Balanced Binary Search Tree (bBST) will be covered in later lectures.

- Hence, hash table supports the Map ADT in constant time on average for the above operations. It has many applications.

Map ADT – HashTable Summary

- How to hash? Criteria for good hash functions?

- How to **resolve collision**?

Collision resolution techniques:

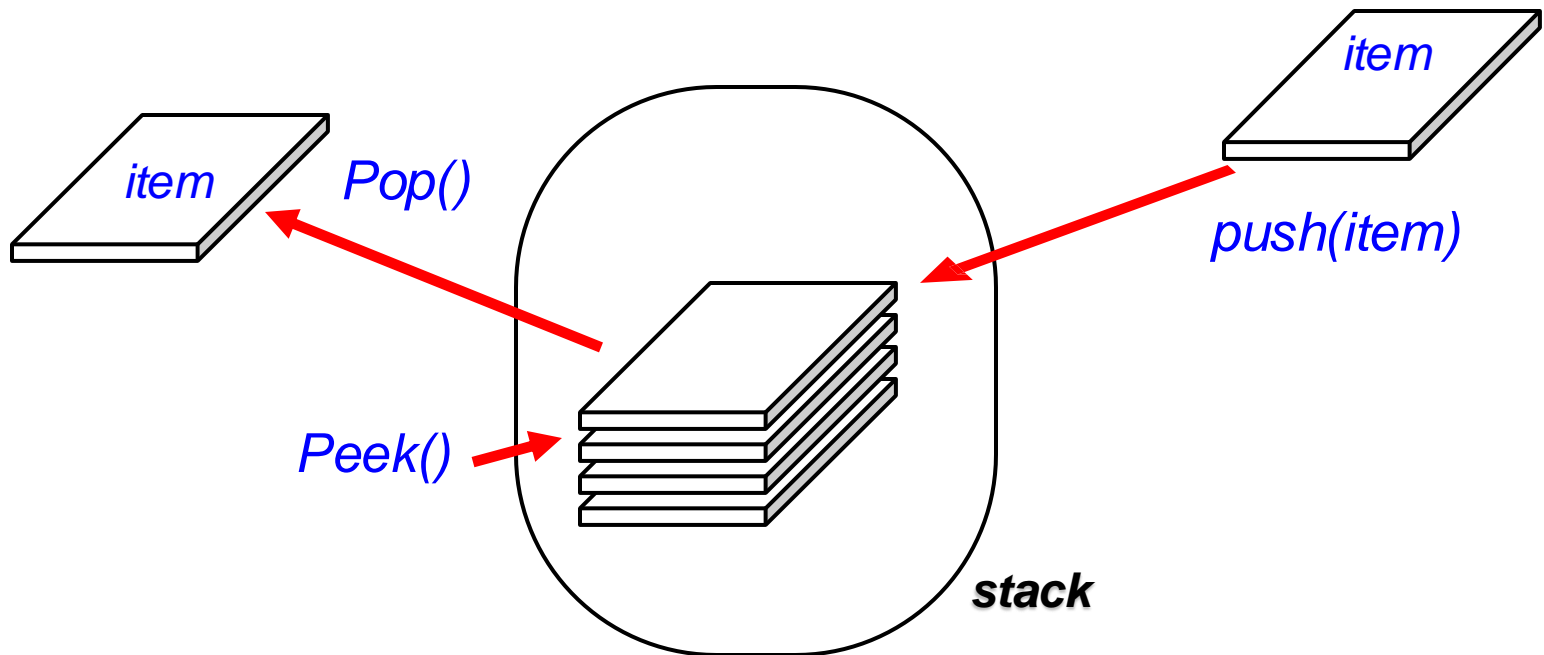
- ❑ separate chaining
- ❑ linear probing
- ❑ quadratic probing
- ❑ double hashing

- Problem on deletions

- **Primary** clustering and **secondary** clustering.

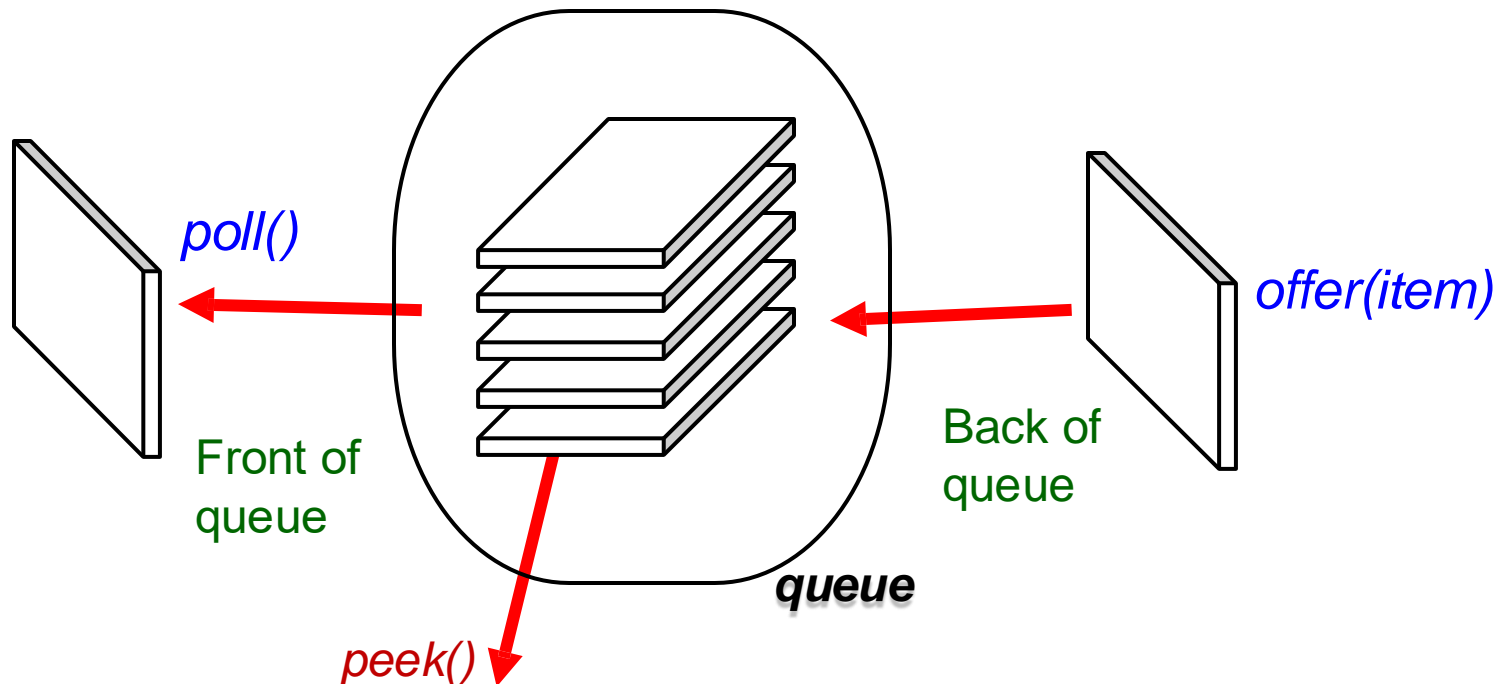
Stack ADT: Operations

- ❑ A **Stack** is a collection of data that is accessed in a **last-in-first-out** (LIFO) manner
- ❑ Major operations: “**push**”, “**pop**”, and “**peek**”.



Queue ADT: Operations

- ❑ A **Queue** is a collection of data that is accessed in a **first-in-first-out** (FIFO) manner
- ❑ Major operations: “**poll**” (or “**dequeue**”), “**offer**” (or “**enqueue**”), and “**peek**”.



Stacks & Queues Summary

- We learn to create our own data structures from array and linked list
 - LIFO vs FIFO – a simple difference that leads to very different applications
 - Drawings can often help in understanding the different cases for operations on the Stack and Queue
- Stacks and Queues can be implemented with either **arrays** or **linked lists**.
- Queues can be **circular**.

Need to distinguish **full** from **empty**.

Full Case: $((B+1) \% \text{maxsize}) == F$

Empty Case: $F == B$



B F

Sorting Algorithms

- *Comparison based and Iterative algorithms*

1. Selection Sort
2. Bubble Sort
3. Insertion Sort

- *Comparison based and Recursive algorithms*

4. Merge Sort
5. Quick Sort

- *Non-comparison based*

6. Radix Sort

7. Comparison of Sort Algorithms

- In-place sort
- Stable sort

8. Use of Java Sort Methods

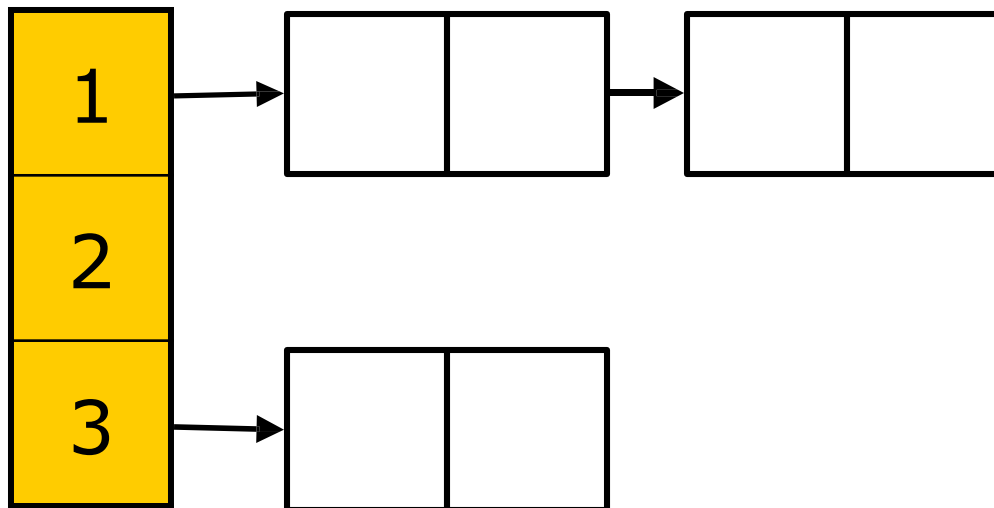
Summary of Sorting Algorithms

	Worst Case	Best Case	In-place?	Stable?
Selection Sort	$O(n^2)$	$O(n^2)$	Yes	No
Insertion Sort	$O(n^2)$	$O(n)$	Yes	Yes
Bubble Sort	$O(n^2)$	$O(n^2)$	Yes	Yes
Bubble Sort 2 (improved with flag)	$O(n^2)$	$O(n)$	Yes	Yes
Merge Sort	$O(n \log n)$	$O(n \log n)$	No	Yes
Radix Sort (non-comparison based)	$O(n)$ (see Notes 1)	$O(n)$	No	Yes
Quick Sort	$O(n^2)$	$O(n \log n)$	Yes	No

Notes: 1. $O(n)$ for Radix Sort is due to non-comparison based sorting.
2. $O(n \log n)$ is the best possible for comparison based sorting.

Mix-and-Match

- Array of Linked-Lists
 - E.g.: **Adjacency list** for representing graph
 - E.g.: **Hash table** with **separate chaining**



Problem

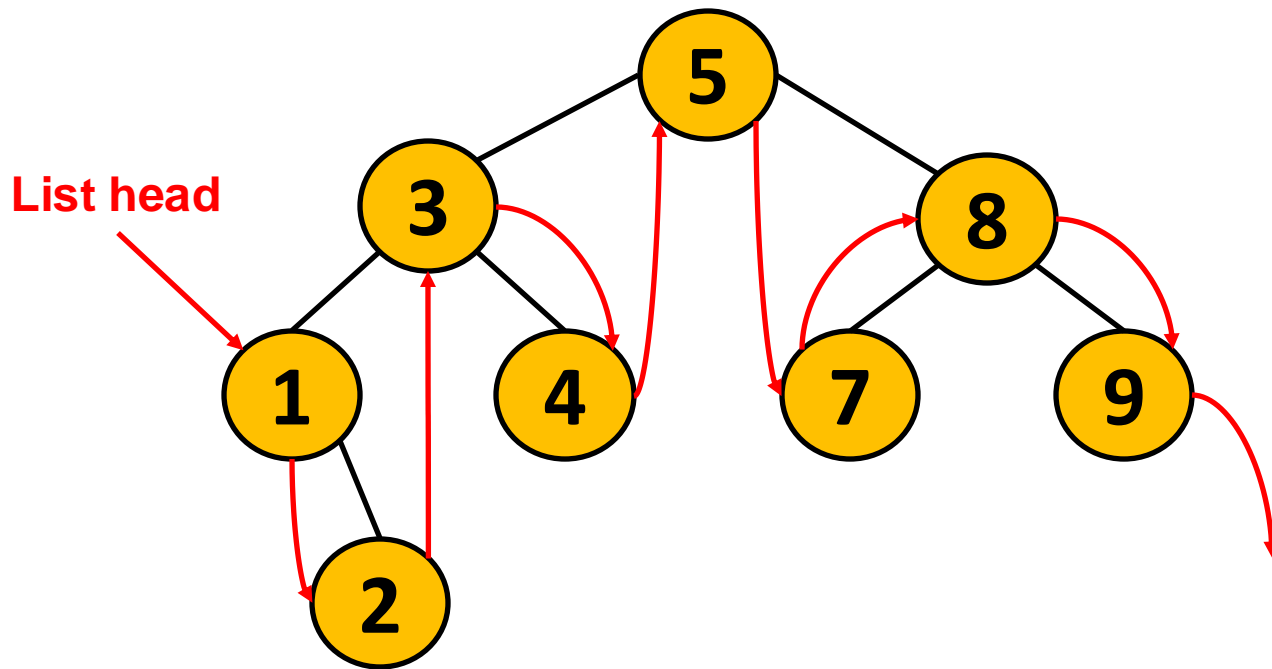
- ❑ Searching on an unsorted linked list is always $O(n)$
- ❑ How to improve it to $O(1)$?

Use hashing.

(i, j) as key and the hash value returned by hash function to be index to a hash table where (i, j) is stored together with the reference to the node in the linked list.

Mix-and-Match 2

- Binary Search Tree + Linked-List
- Can find the successors easily



Q: How to handle updates?

More Examples

- Suppose we need an ADT that support the following operations
 - `enqueue(item)`
 - `dequeue()`
 - `peek()`
 - `printInOrder()`

Use a Queue

- If we use a queue, we can support the queue operations efficiently $O(1)$.
- But to print the items in order, we need to first sort the items in the queue, which is $O(N \log N)$ time.

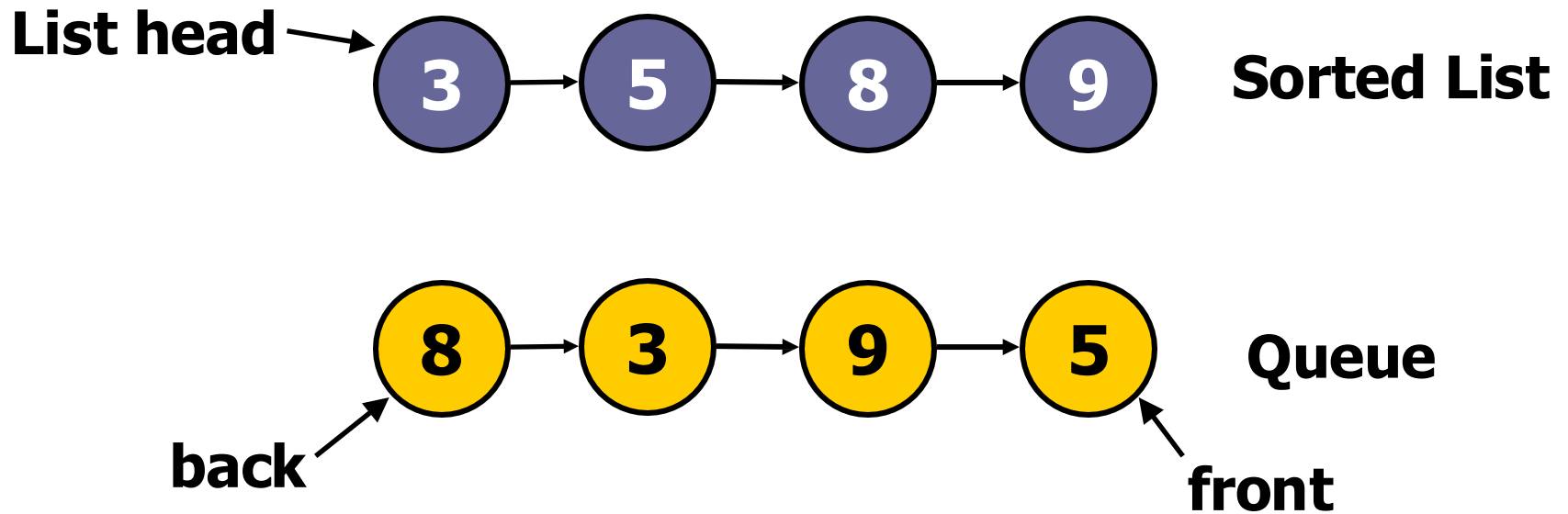
enqueue(item)	$O(1)$
dequeue()	$O(1)$
peek()	$O(1)$
printInOrder()	$O(N \log N)$

Use a **Sorted Linked List**

- We can reduce `printInOrder()` to $O(N)$ using a sorted linked list instead.
- But the queue operations are **not** supported.

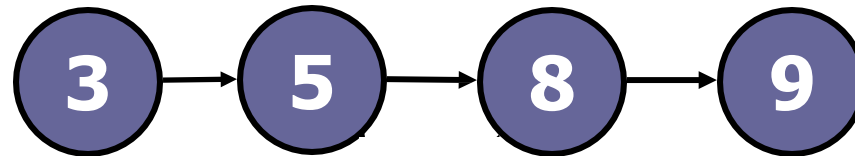
enqueue(item)	?
dequeue()	?
peek()	?
printInOrder()	$O(N)$

Use both: Queue + Sorted List ?

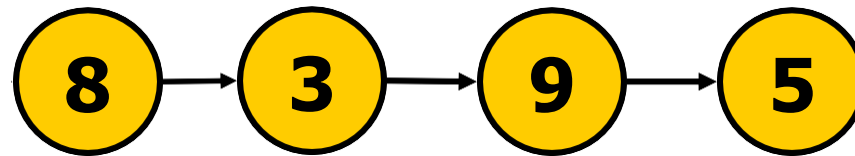


Trivial problem: Need to duplicate the data.

Enqueue(6)

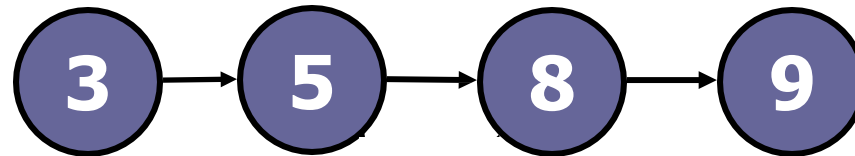


Sorted List



Queue

Enqueue(6)

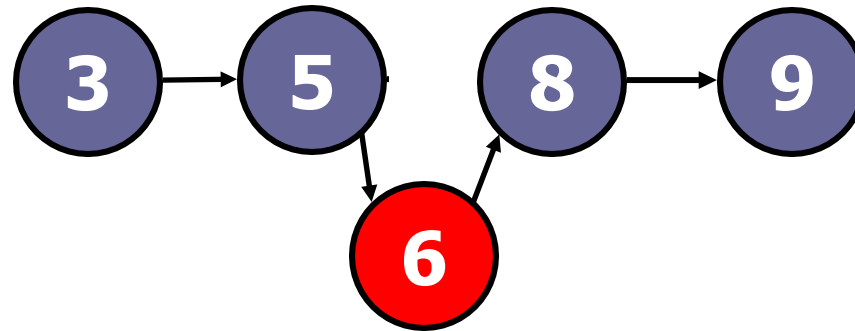


Sorted List



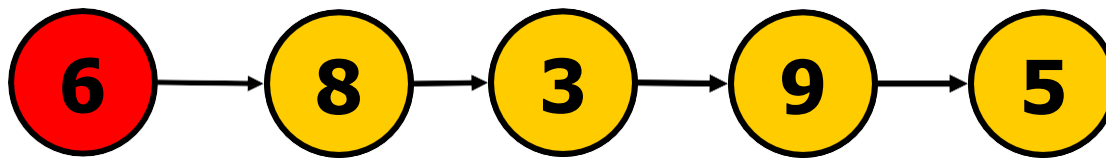
Queue

Enqueue(6)



Sorted List

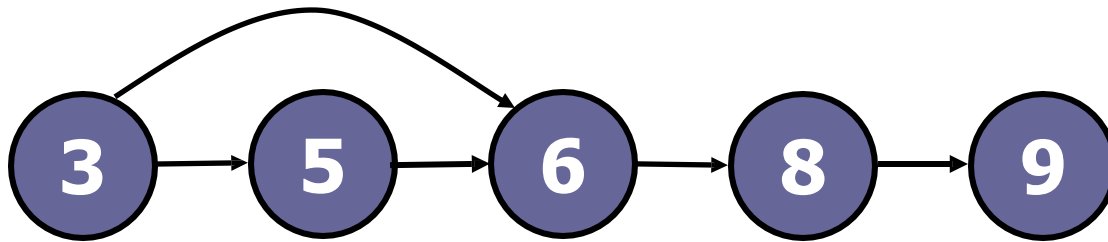
$O(N)$



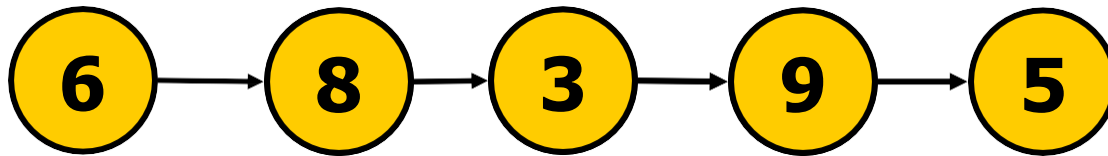
Queue

$O(1)$

Dequeue()

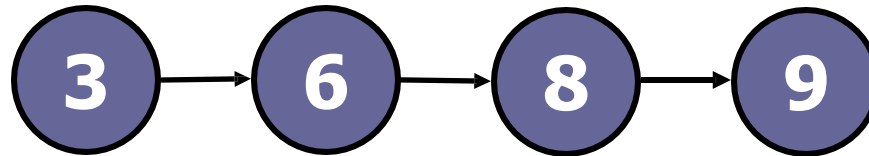


Sorted List



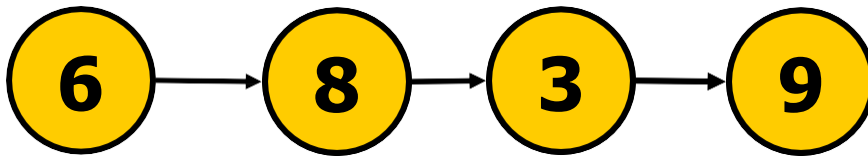
Queue

Dequeue()



Sorted List

$O(N)$



Queue

$O(1)$

Use Queue + Sorted List

But then **enqueue** and **dequeue** take linear time $O(N)$, because we have to look for the position of the item in the linked list to insert/delete. Too slow.

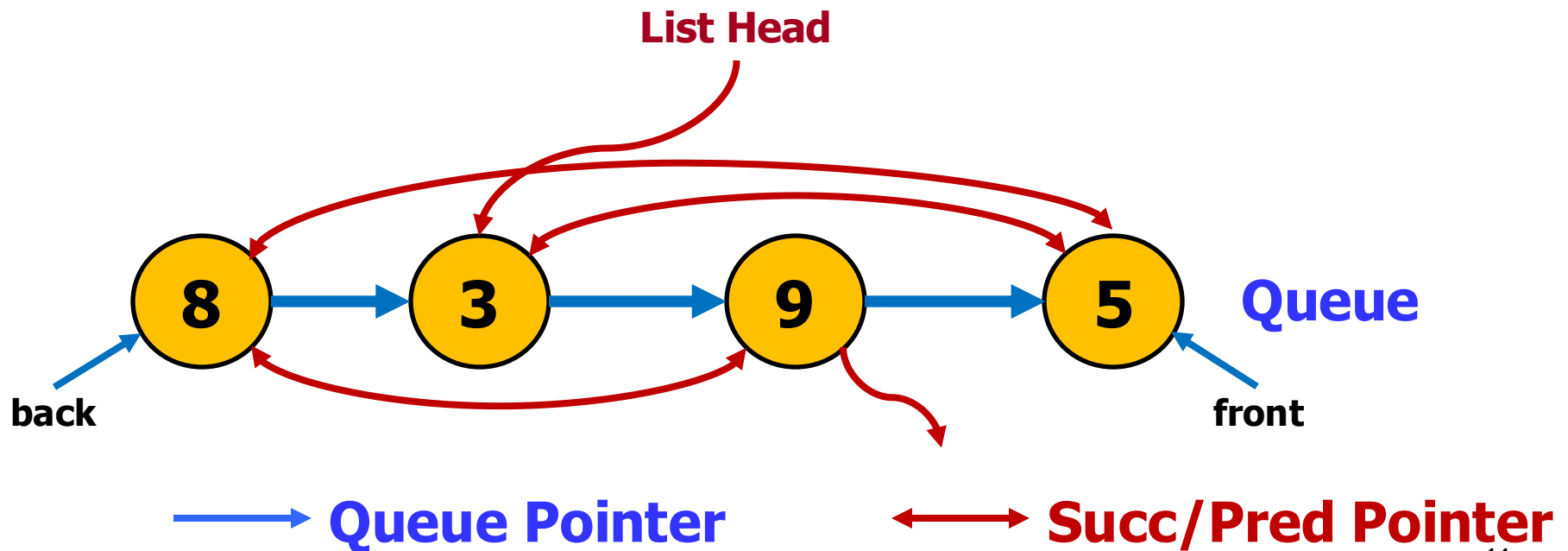
enqueue(item)	$O(N)$
dequeue()	$O(N)$
peek()	$O(1)$
printInOrder()	$O(N)$

Q: Can we improve them?

Improvement:

Queue combines with DLinked List

- Only store **one copy** of each item
- Each node have 2 sets of pointers:
 - One for **queue** and one for a **doubly linked list**



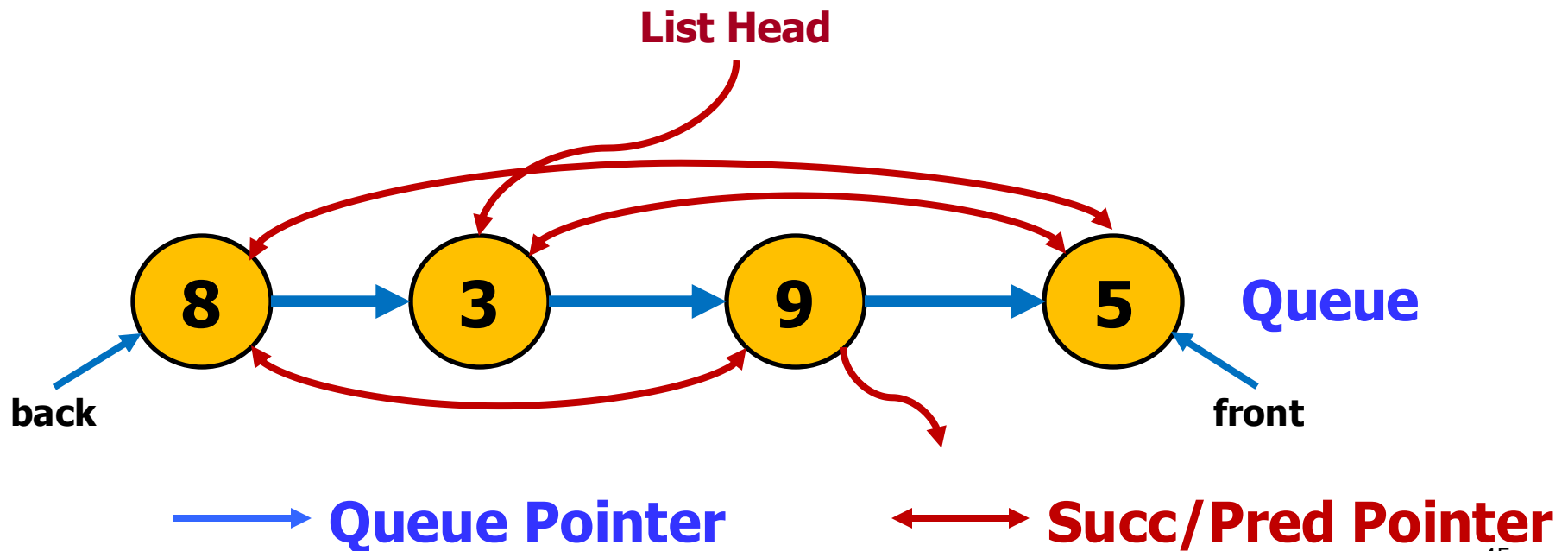
Combine Queue and DLinked List

- Dequeue of a doubly linked list can be done in $O(1)$ time.

Q: How?

- However, enqueue is still $O(N)$. Why? E.g., enqueue 4?

A: Need to find the insertion point in the DLinked List



Combine Queue and DLinked List

- Dequeue of a doubly linked list can be done in $O(1)$ time.
Q: How?
- However, enqueue is still $O(N)$. Why? E.g. enqueue 4?

enqueue(item)	$O(N)$
dequeue()	$O(1)$
peek()	$O(1)$
printInOrder()	$O(N)$

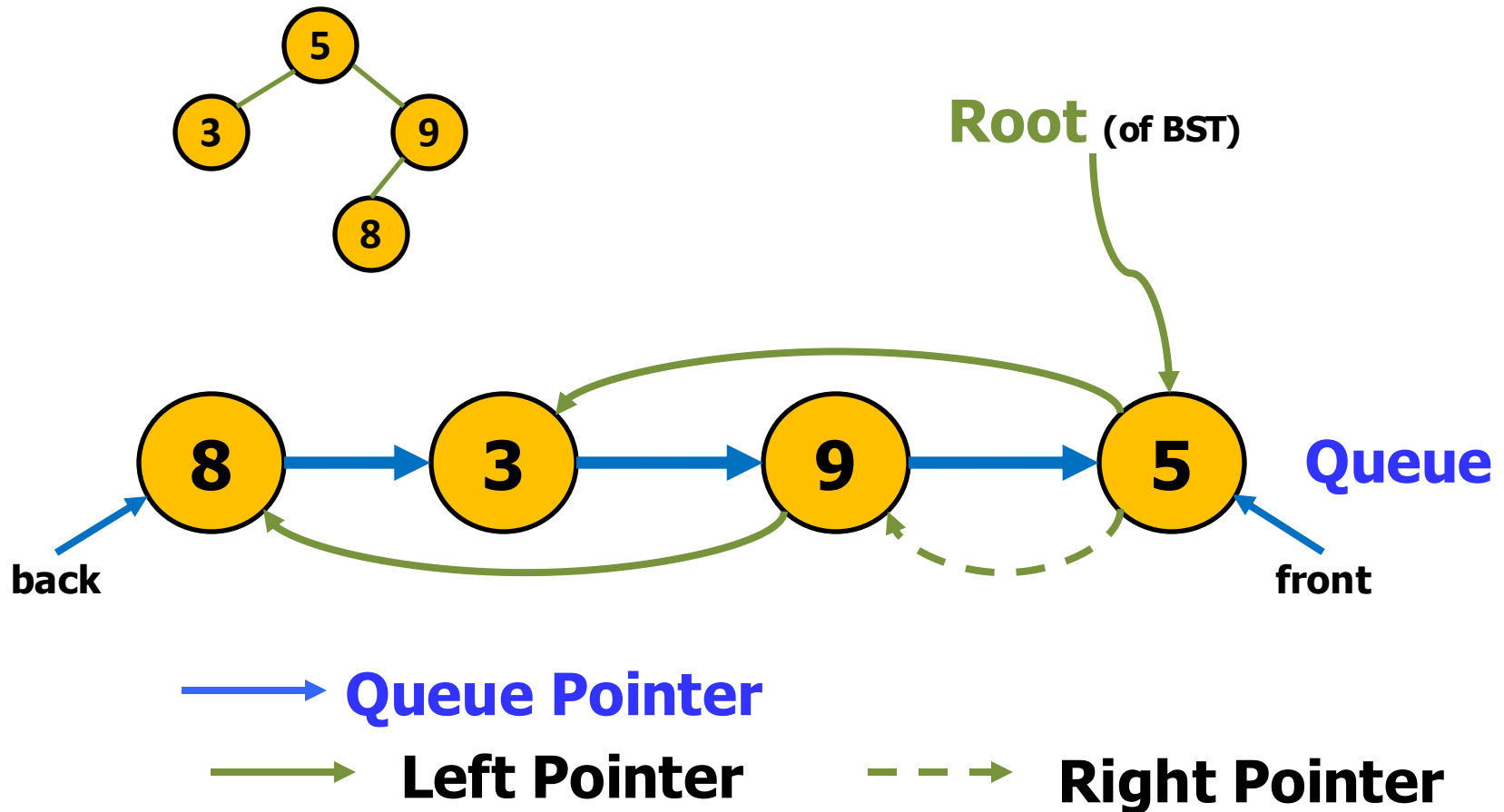
Q: Can we improve it?

Combine Queue and BST

- We can improve enqueue to $O(\log N)$ by combining a queue with a BST instead of a linked list.

More improvement:

Queue combines with **BST**



Combine **Queue** and **BST**

- But now **dequeue** also takes $O(\log N)$.

enqueue(item)	$O(\log N)$
dequeue()	$O(\log N)$
peek()	$O(1)$
printInOrder()	$O(N)$

Q: Is there a way to make dequeue $O(1)$?

Combine Queue and BST

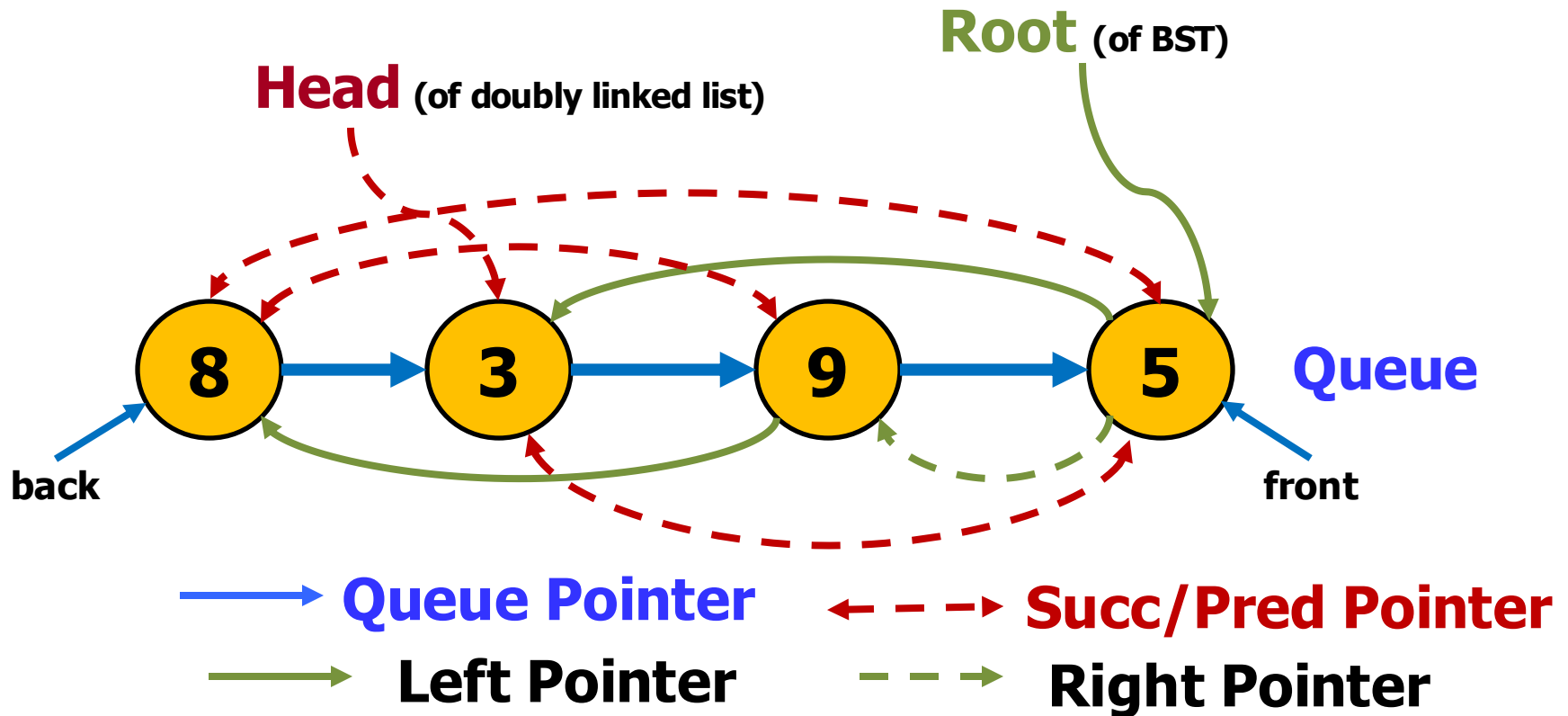
enqueue(item)	$O(\log N)$
dequeue()	$O(1)$?
peek()	$O(1)$
printInOrder()	$O(N)$

Q: Is there a way to make dequeue $O(1)$?

Yes, use another doubly linked list, so that finding the replacement for BST deletion can be done in $O(1)$ instead of $O(\log N)$.

More improvement: combine **Queue** + **BST** + **DList**

- Use another doubly linked list.



Combine queue + BST + DList

enqueue(item)	$O(\log N)$
dequeue()	$O(1)$
peek()	$O(1)$
printInOrder()	$O(N)$

Recall: use another doubly linked list, so that finding the replacement for BST deletions can be done in $O(1)$ instead of $O(\log N)$. Why?

Improvement summary

- use a queue and a linked list
- combine queue with doubly linked list
- combine queue and BST
- combine queue, BST, and doubly linked list

Q: Which improvement should be used?

Depends on the application.

E.g., it depends how often certain operations are executed.

End of Mix and Match

CS2040 Objectives

- Give an introduction to data structures and algorithms for constructing **efficient** computer programs.
- Emphasize on **data abstraction** issues (through **ADTs**) in the code development.
- Emphasize on **efficient implementations** of chosen data structures and algorithms.

CS2040 Objectives

- Include **arrays, lists, stacks, queues, hash tables,** and **BST/AVL trees, heaps, graphs** together with their algorithms (insert, delete, find, etc.).
- Simple algorithmic paradigms, such as **sorting** and **search** algorithms and **greedy** algorithms were introduced.
- Elementary **analysis of algorithmic complexities** were taught.

What is Next?

- Continue to program 😊!
- For non-CS, take more CS modules
 - CS2103 – Software Engineering
 - CS3230 – Design and Analysis of Algorithms
 - CS3217/CS3216 – Software development on Modern Platforms
 - CS3233 – Competitive Programming
 - CS3247/CS4213 – Game Development
 - Others: AI, Cybersecurity, Blockchain, etc.

What is Next?

- ❑ Do a minor in CS
- ❑ Do a major in CS
- ❑ Transfer to CS
- ❑ Create the next software for the whole world/universe 😊.
- ❑ Be a TA

Student Feedback Exercise

- Give your honest feedback to all in the teaching team.
- Not a chance to get back at them.
- Help them to help yourself.
- Help them to help your juniors.

