Table of Clebsch-Gordan coefficients

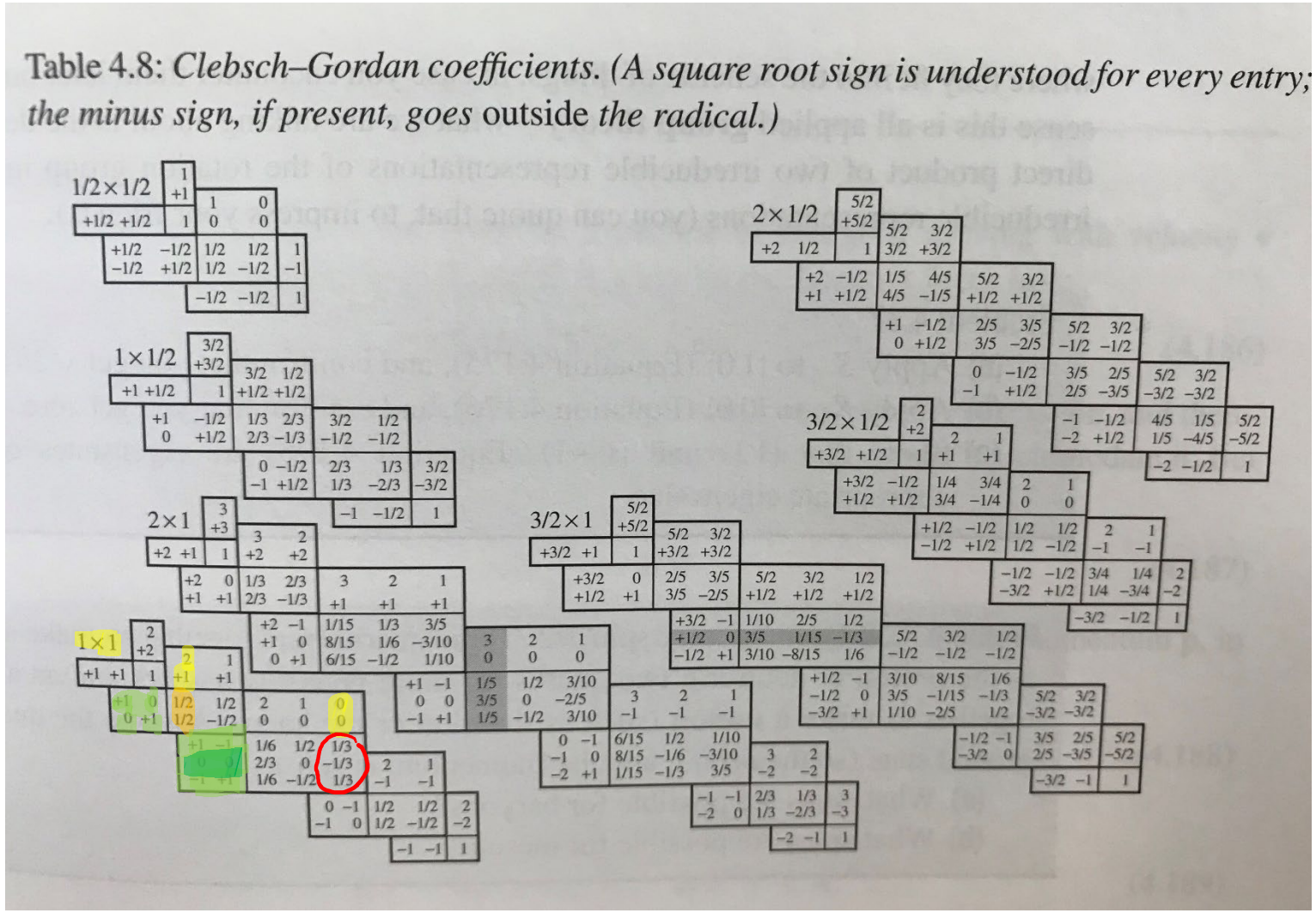

Table 4.8: *Clebsch–Gordan coefficients. (A square root sign is understood for every entry; the minus sign, if present, goes* outside *the radical.)*

$1/2 \times 1/2$

	$+1/2\ +1/2$	$+1/2\ -1/2$	$-1/2\ +1/2$	$-1/2\ -1/2$
1	1	0	0	
0		1/2	1/2	
1		1/2	−1/2	
−1				1

$1 \times 1/2$

	$+1\ +1/2$	$+1\ -1/2$	$0\ +1/2$	$0\ -1/2$	$-1\ +1/2$	$-1\ -1/2$
3/2	1					
1/2		1/3	2/3			
1/2		2/3	−1/3			
−1/2				2/3	1/3	
−1/2				1/3	−2/3	
−3/2						1

$2 \times 1/2$

	$+2\ 1/2$	$+1\ +1/2$	$+1\ -1/2$	$0\ +1/2$	$0\ -1/2$	$-1\ +1/2$	$-1\ -1/2$	$-2\ +1/2$	$-2\ -1/2$
5/2	1								
3/2		1/5	4/5						
3/2		4/5	−1/5						
1/2				2/5	3/5				
1/2				3/5	−2/5				
−1/2						3/5	2/5		
−1/2						2/5	−3/5		
−3/2								4/5	1/5
−3/2								1/5	−4/5
−5/2									1

$3/2 \times 1/2$

	$+3/2\ +1/2$	$+3/2\ -1/2$	$+1/2\ +1/2$	$+1/2\ -1/2$	$-1/2\ +1/2$	$-1/2\ -1/2$	$-3/2\ +1/2$	$-3/2\ -1/2$
2	1							
1		1/4	3/4					
1		3/4	−1/4					
0				1/2	1/2			
0				1/2	−1/2			
−1						3/4	1/4	
−1						1/4	−3/4	
−2								1

2×1

	$+2\ +1$	$+2\ 0$	$+1\ +1$	$+2\ -1$	$+1\ 0$	$0\ +1$	$+1\ -1$	$0\ 0$	$-1\ +1$
3	1								
2		1/3	2/3						
2		2/3	−1/3						
1				1/15	1/3	3/5			
1				8/15	1/6	−3/10			
0				6/15	−1/2	1/10			

$3/2 \times 1$

	$+3/2\ +1$	$+3/2\ 0$	$+1/2\ +1$	$+3/2\ -1$	$+1/2\ 0$	$-1/2\ +1$	$+1/2\ -1$	$-1/2\ 0$	$-3/2\ +1$
5/2	1								
3/2		2/5	3/5						
3/2		3/5	−2/5						
1/2				1/10	2/5	1/2			
1/2				3/5	1/15	−1/3			
−1/2				3/10	−8/15	1/6			
3							1/5	1/2	3/10
2							3/5	0	−2/5
1							1/5	−1/2	3/10
3									
2									
1									

1×1

	$+1\ +1$	$+1\ 0$	$0\ +1$	$+1\ -1$	$0\ 0$	$-1\ +1$	$0\ -1$	$-1\ 0$	$-1\ -1$
2	1								
1		1/2	1/2						
1		1/2	−1/2						
0				1/6	1/2	1/3			
0				2/3	0	−1/3			
0				1/6	−1/2	1/3			
2							1/2	1/2	
1							1/2	−1/2	
0									1

$5/2 \times 1/2$

	$+5/2\ +1/2$	$+3/2\ +1/2$	$+3/2\ -1/2$	$+1/2\ +1/2$	$+1/2\ -1/2$	$-1/2\ +1/2$	$-1/2\ -1/2$	$-3/2\ +1/2$	$-3/2\ -1/2$	$-5/2\ +1/2$	$-5/2\ -1/2$
3	1										
2		1/5	4/5								
2		4/5	−1/5								
1				2/5	3/5						
1				3/5	−2/5						
0						3/5	2/5				
0						2/5	−3/5				
−1								4/5	1/5		
−1								1/5	−4/5		
−2										1/2	1/2
−2										1/2	−1/2
−3											1

$3/2 \times 1/2$

	$+3/2\ +1/2$	$+3/2\ -1/2$	$+1/2\ +1/2$	$+1/2\ -1/2$	$-1/2\ +1/2$	$-1/2\ -1/2$	$-3/2\ +1/2$	$-3/2\ -1/2$
2	1							
1		1/4	3/4					
1		3/4	−1/4					
0				1/2	1/2			
0				1/2	−1/2			
−1						3/4	1/4	
−1						1/4	−3/4	
−2								1

2×1

	$+2\ +1$	$+2\ 0$	$+1\ +1$	$+2\ -1$	$+1\ 0$	$0\ +1$	$+1\ -1$	$0\ 0$	$-1\ +1$
3	1								
2		1/3	2/3						
2		2/3	−1/3						
1				1/15	1/3	3/5			
1				8/15	1/6	−3/10			
0				6/15	−1/2	1/10			
3							1/5	1/2	3/10
2							3/5	0	−2/5
1							1/5	−1/2	3/10
3									
2									
1									

$3/2 \times 1$

	$+3/2\ +1$	$+3/2\ 0$	$+1/2\ +1$	$+3/2\ -1$	$+1/2\ 0$	$-1/2\ +1$	$+1/2\ -1$	$-1/2\ 0$	$-3/2\ +1$
5/2	1								
3/2		2/5	3/5						
3/2		3/5	−2/5						
1/2				1/10	2/5	1/2			
1/2				3/5	1/15	−1/3			
−1/2				3/10	−8/15	1/6			
3							6/15	1/2	1/10
2							8/15	−1/6	−3/10
1							1/15	−1/3	3/5
3									
2									
1									

$2 \times 1/2$

	$+2\ 1/2$	$+1\ +1/2$	$+1\ -1/2$	$0\ +1/2$	$0\ -1/2$	$-1\ +1/2$	$-1\ -1/2$	$-2\ +1/2$	$-2\ -1/2$
5/2	1								
3/2		1/5	4/5						
3/2		4/5	−1/5						
1/2				2/5	3/5				
1/2				3/5	−2/5				
−1/2						3/5	2/5		
−1/2						2/5	−3/5		
−3/2								4/5	1/5
−3/2								1/5	−4/5
−5/2									1

$3/2 \times 1/2$

	$+3/2\ +1/2$	$+3/2\ -1/2$	$+1/2\ +1/2$	$+1/2\ -1/2$	$-1/2\ +1/2$	$-1/2\ -1/2$	$-3/2\ +1/2$	$-3/2\ -1/2$
2	1							
1		1/4	3/4					
1		3/4	−1/4					
0				1/2	1/2			
0				1/2	−1/2			
−1						3/4	1/4	
−1						1/4	−3/4	
−2								1

Eg. $l_1 = 1, l_2 = 1$

$$|l=2, m=1\rangle = \sqrt{\tfrac{1}{2}}\,|m_1=1, m_2=0\rangle + \sqrt{\tfrac{1}{2}}\,|m_1=0, m_2=1\rangle$$

$$|l=0, m=0\rangle = \sqrt{\tfrac{1}{3}}\,|m_1=1, m_2=-1\rangle - \sqrt{\tfrac{1}{3}}\,|m_1=0, m_2=0\rangle + \sqrt{\tfrac{1}{3}}\,|m_1=-1, m_2=1\rangle$$

From Griffiths and Schroeter, Introduction to QM

Table of Clebsch-Gordan coefficients

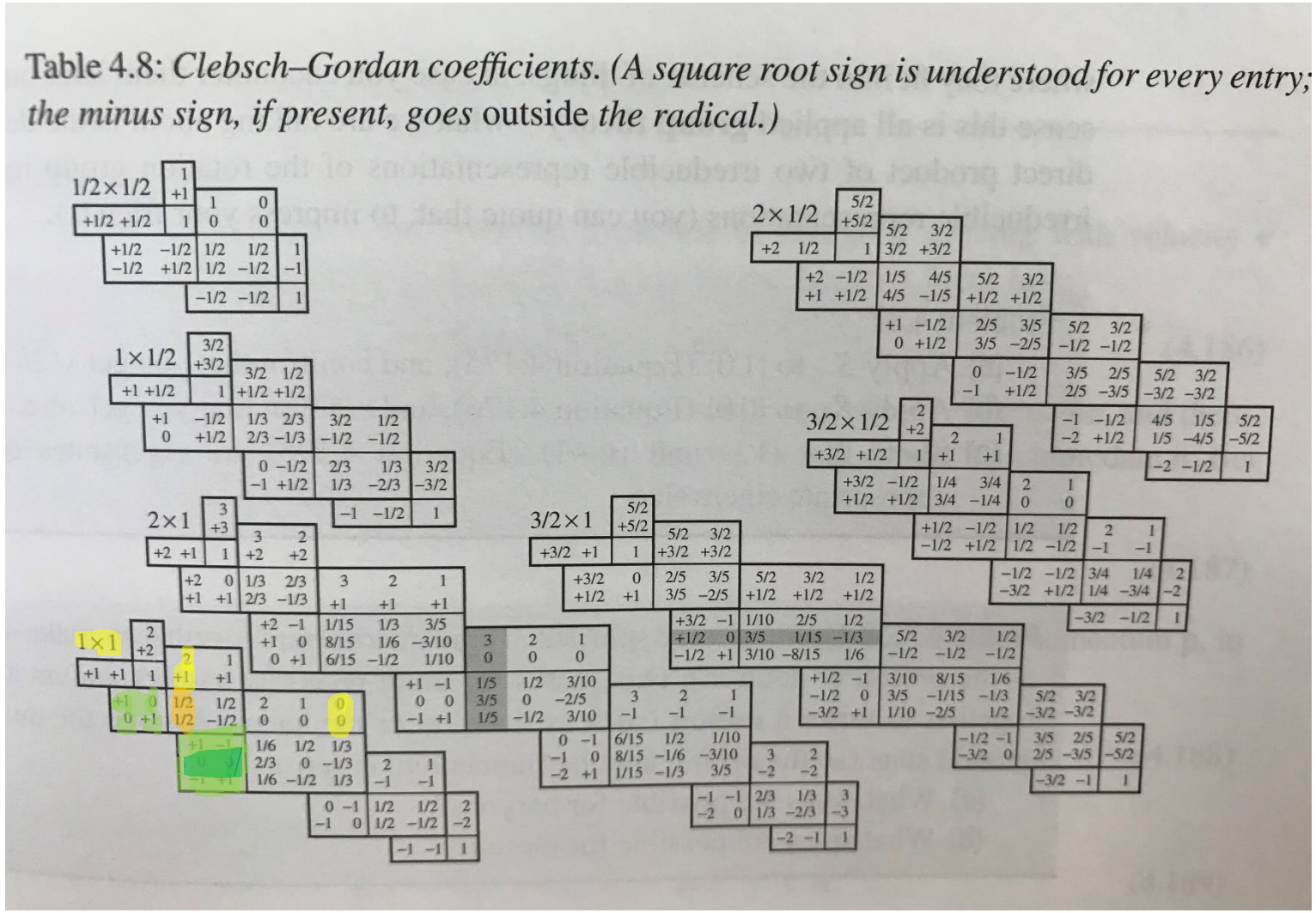

Table 4.8: *Clebsch–Gordan coefficients. (A square root sign is understood for every entry; the minus sign, if present, goes* outside *the radical.)*

$1/2 \times 1/2$

m_1	m_2	$j=1$	$j=0$
$+1/2$	$+1/2$	1	0
$+1/2$	$-1/2$	$1/2$	$1/2$
$-1/2$	$+1/2$	$1/2$	$-1/2$
$-1/2$	$-1/2$	1	

$1 \times 1/2$

m_1	m_2	$j=3/2$	$j=1/2$
$+1$	$+1/2$	1	
$+1$	$-1/2$	$1/3$	$2/3$
0	$+1/2$	$2/3$	$-1/3$
0	$-1/2$	$2/3$	$1/3$
-1	$+1/2$	$1/3$	$-2/3$
-1	$-1/2$	1	

$2 \times 1/2$

m_1	m_2	$j=5/2$	$j=3/2$
$+2$	$+1/2$	1	
$+1$	$+1/2$	$1/5$	$4/5$
$+1$	$-1/2$	$4/5$	$-1/5$
0	$+1/2$	$2/5$	$3/5$
0	$-1/2$	$3/5$	$2/5$
-1	$+1/2$	$3/5$	$-2/5$
-1	$-1/2$	$1/5$	$4/5$
-2	$+1/2$	$1/5$	$-4/5$
-2	$-1/2$	1	

$3/2 \times 1/2$

m_1	m_2	$j=2$	$j=1$
$+3/2$	$+1/2$	1	
$+1/2$	$+1/2$	$1/4$	$3/4$
$+1/2$	$-1/2$	$3/4$	$-1/4$
$-1/2$	$+1/2$	$1/2$	$1/2$
$-1/2$	$-1/2$	$3/4$	$1/4$
$-3/2$	$+1/2$	$1/4$	$-3/4$
$-3/2$	$-1/2$	1	

2×1

m_1	m_2	$j=3$	$j=2$	$j=1$
$+2$	$+1$	1		
$+2$	0	$1/3$	$2/3$	
$+1$	$+1$	$2/3$	$-1/3$	
$+2$	-1	$1/15$	$1/3$	$3/5$
$+1$	0	$8/15$	$1/6$	$-3/10$
0	$+1$	$6/15$	$-1/2$	$1/10$
$+1$	-1	$1/5$	$1/2$	$3/10$
0	0	$3/5$	0	$-2/5$
-1	$+1$	$1/5$	$-1/2$	$3/10$
0	-1	$6/15$	$1/2$	$1/10$
-1	0	$8/15$	$-1/6$	$-3/10$
-2	$+1$	$1/15$	$-1/3$	$3/5$
-1	-1	$2/3$	$1/3$	
-2	0	$1/3$	$-2/3$	
-2	-1	1		

$3/2 \times 1$

m_1	m_2	$j=5/2$	$j=3/2$	$j=1/2$
$+3/2$	$+1$	1		
$+3/2$	0	$2/5$	$3/5$	
$+1/2$	$+1$	$3/5$	$-2/5$	
$+3/2$	-1	$1/10$	$2/5$	$1/2$
$+1/2$	0	$3/5$	$1/15$	$-1/3$
$-1/2$	$+1$	$3/10$	$-8/15$	$1/6$
$+1/2$	-1	$3/10$	$8/15$	$1/6$
$-1/2$	0	$3/5$	$-1/15$	$-1/3$
$-3/2$	$+1$	$1/10$	$-2/5$	$1/2$
$-1/2$	-1	$1/2$	$1/2$	
$-3/2$	0	$1/2$	$-1/2$	
$-3/2$	-1	1		

1×1

m_1	m_2	$j=2$	$j=1$	$j=0$
$+1$	$+1$	1		
$+1$	0	$1/2$	$1/2$	
0	$+1$	$1/2$	$-1/2$	
$+1$	-1	$1/6$	$1/2$	$1/3$
0	0	$2/3$	0	$-1/3$
-1	$+1$	$1/6$	$-1/2$	$1/3$
0	-1	$1/2$	$1/2$	
-1	0	$1/2$	$-1/2$	
-1	-1	1		

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