Chapter 8

The Big Bang Theory

> We are now prepared to follow the course of cosmic evolution through the first three minutes. Events move much more swiftly at first than later, so it would not be useful to show pictures spaced at equal time intervals, like an ordinary movie. Instead, I will adjust the speed of our film to the falling temperature of the Universe... – Steven Weinberg

Learning Objectives

You have probably heard about the term "Big Bang" since you were a kid / toddler (in case you're still a kid). But what exactly is the Big Bang theory about, and how do we know it is true? We will find out in this chapter.

Learning flow

We will take this chapter in an "easy-mode". As the semester is ending, there will be no assessments related to this chapter. Deepavali and NUS wellness day typically falls around this time, as such some lessons may be cancelled. We will take it easy and see how far we go in these 1-2 weeks.

Contents

8.1 Pre-Lesson Homework

Steady State vs Big Bang

Do an internet search for the theories of Steady State vs Big Bang (avoid some of the trashy YouTube vids). Make a comparison between the theories and reflect upon the controversy that lasted on decades. Dig into the life (and science) of Fred Hoyle, an admirable scientist who probably should have won the Nobel Prize.

8.2 Temperature and Time

In this section, we find a relation between the temperature of the Universe and time. We assume here that the Universe started off with a (infinitely) hot Big Bang and time starts ticking at the moment of the Big Bang. In 1948, Ralph Alpher, Hans Bethe and George Gamow published a paper to discuss a model of a hot and dense expanding Universe where a burst of nuclear reactions occured, thus explaining the origins of chemical elements. As an interesting side note, while Alpher did most of the work, his supervisor Gamow included Bethe as a co-author without Bethe's knowledge, thinking that it is a good idea for a paper on cosmic beginnings be created by α (Alpher), β (Bethe) and γ (Gamow). The model proposed in the $\alpha - \beta - \gamma$ paper was subsequently corrected and modified but the key ideas remained. The early Universe was hot and dense, comprising of photons and relativistic particles in constant collision with each other. The mean free path of the particles was short and the Universe was in thermodynamic equilibrium. The radiation (photons and relativistic particles) is described by a blackbody spectrum with a characteristic temperature. The energy per unit volume having wavelengths between λ and $\lambda + d\lambda$ is given by

$$u_\lambda d\lambda = \frac{8\pi hc\lambda^{-5}}{\exp(\frac{hc}{\lambda kT}) - 1} d\lambda. \tag{8.1}$$

We had seen in the previous chapter that the energy density of the radiation field scales inversely to the fourth power of the cosmological scale factor.

$$\rho_r \propto S^{-4} \tag{8.2}$$

Thus the radiation energy density present (denoted by a subscript 0) is related to an earlier value by

$$S_0^4 u_0 d\lambda_0 = S^4 u_\lambda d\lambda \tag{8.3}$$

Since the wavelength is proportional to the scale factor due to cosmological redshift,

$$\begin{aligned} u_0 d\lambda_0 &= \frac{S^4}{S_0^4} \frac{8\pi hc\, S^{-5} S_0^5 \lambda_0^{-5}}{\exp(\frac{S_0 hc}{S\lambda_0 kT}) - 1} S S_0^{-1} d\lambda_0 \\ &= \frac{8\pi hc/\lambda_0^5}{\exp(\frac{hc}{\lambda_0 k}\frac{S_0}{ST}) - 1} d\lambda_0 \end{aligned} \tag{8.4}$$

which implies that

$$ST = S_0 T_0 \tag{8.5}$$

or

$$T \propto S^{-1}. \tag{8.6}$$

Hence if one solves the Friedmann equation in the radiation era, one will have the relation between temperature and time.
For the case of a flat radiation dominated Universe[1],

$$S \propto t^{1/2}. \tag{8.7}$$

Therefore

$$T \propto t^{-1/2} \tag{8.8}$$

The above temperature–time relation holds for closed and open radiation-dominated Universe as well. The interested reader may work this out yourself!

Ex. We had spent considerable efforts to obtain a relation between temperature and time in the early Universe. This was done because the **processes** that happened during the Big Bang can be better understood

(a) by following the falling temperature as time evolves.

(b) by following the increasing size of the Universe as time evolves.

(c) by following the chronological order of events in a linear time scale.

(d) by following the decreasing density of the Universe as time evolves.

[1] You will work this out in Assignment 4 :)

Ex. "*Events move much more swiftly at first than later*" (Steven Weinberg, The First Three Minutes). Which of the following is a more likely interpretation of Weinberg's statement?

(a) As the temperature is hotter at earlier times, particles move faster and occurence of events are higher.

(b) As the rate of change of temperature is higher at ealier times, the particle composition of the Universe (which depends on temperature) changes more quickly at earlier times.

Ex. Given that

$$\rho_{rad} = \frac{aT^4}{c^2} \tag{8.9}$$

where a is the radiation constant and has the value 7.566×10^{-16} J m^{-3}K^{-4}. Show that

$$\begin{aligned} T(t) &= \left(\frac{3c^2}{32\pi Ga}\right)^{1/4} t^{-1/2} \\ &= (1.52 \times 10^{10}\text{Ks}^{1/2})t^{-1/2} \end{aligned} \tag{8.10}$$

8.3 Big Bang Nucleosynthesis

One of the success of the Big Bang theory is the model of primordial nucleo-synthesis which correctly predicts the abundance of helium and other light elements in the Universe. In this section we will outline the various nuclear reactions in the first three (and a half) minutes of the Big Bang. We shall begin our discussions at 10^{12}K or $t \sim 10^{-4}$s, for any earlier and hotter, the physics involved will be out of the scope of this course.

Our Universe just below 10^{12}K was a soup of particles comprising of photons (γ), electrons (e^-), positrons (e^+), neutrinos and their anti-particles ($\nu_e, \bar{\nu_e}$), protons (p^+) and neutrons (n). Apart from the massless photons, the speed of the particles follows the Maxwell-Boltzmann distribution[2]

$$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp\left(\frac{-mv^2}{2kT}\right). \tag{8.11}$$

The characteristic thermal energy at a temperature T, derived by evaluating the most probable speed in the distribution, is kT. At 10^{12}K, the characteristic energy

$$kT \approx 86\text{MeV} \tag{8.12}$$

is much larger than the energy difference between the proton and neutron

$$(m_p - m_n)c^2 = 1.293. \tag{8.13}$$

Ex. Plot the Maxwell Boltzmann distribution using $\frac{m}{k} = 10^{-4}$ and $T = 10^{10}$ with a computer. The plot

(a) is monotonously increasing

(b) is monotonously decreasing

(c) has 1 minimum point

(d) has 1 maximum point

Ex. Differentiate the Maxwell Boltzmann distribution with respect to v and set it to be zero to obtain the most probable speed in the distribution.

Because of the excessive amount of energy available, protons and neutrons constantly transforms from one to the other via the following equations

$$n + e^+ \rightleftharpoons p^+ + \bar{\nu_e} \tag{8.14}$$

$$n + \nu_e \rightleftharpoons p^+ + e^- \tag{8.15}$$

The ratio of the number density of neutrons and protons is given by the Boltzmann factor

$$\frac{n_n}{n_p} = \exp[-(m_p - m_n)c^2/kT] = 0.985. \tag{8.16}$$

This ratio was to drop as the temperature of the Universe falls.

[2] The Maxwell-Boltzmann distribution gives the probability distribution of speeds of a gas at a certain temperature. https://en.wikipedia.org/wiki/Maxwell%E2%80%93Boltzmann_distribution

Ex. What is the ratio of the number of neutrons to protons at 10^{11}K?

(a) 0.96

(b) 0.86

(c) 0.76

(d) 0.66

At $T = 10^{10}$K, $n_n/n_p = 0.223$. At this temperature, the rates of reactions of $n \leftrightarrow p^+$ drops drastically for two reasons. Firstly the energy of neutrinos had fell for them to become much less interacting and unable to initiate reactions in Eqs. (8.14) and (8.15). Secondly, as the temperature continues to drop from 10^{10}K, energetic photons that are able to fuel the positron–electron pair production $2\gamma \leftrightarrow e^+ + e^-$ are depleting significantly (see Ex. below). This means that more electron and positrons annihilate each other without being replaced. As a result the ratio of 223 neutrons to 1000 protons was almost frozen.

Almost, but not entirely. The n_n/n_p ratio now falls gradually due to a slow beta decay process

$$n \to p^+ + e^- + \bar{\nu_e} \tag{8.17}$$

with a decay half-life of $\tau_{1/2} = 617$s.

Ex. Assuming that the photons and matter particles are in thermodynamic equilibrium, calculate the characteristic energy of the photons at 10^{10}K and compare with the mass of an electron–positron pair.

Ex. Show that beta decay of the neutron results in a drop of n_n/n_p from 0.223 to 0.164 as the Universe cools from 10^{10} K to 10^9 K.

The proton p^+ is also known as the hydrogen ^{1}H nucleus. The next stable nucleus to form is $^4_2$He, the second lightest element in the Periodic Table. The formation of $^4_2$He could not possibly be the product of two protons and two neutrons colliding simultaneously since such events will be too rare. The efficient reactions that produce $^4_2$He include

$$\begin{aligned} p^+ + n &\rightleftharpoons {}^2_1\mathrm{H} + \gamma \\ {}^2_1\mathrm{H} + {}^2_1\mathrm{H} &\rightleftharpoons {}^3_1\mathrm{H} + p^+ \\ {}^3_1\mathrm{H} + {}^2_1\mathrm{H} &\rightleftharpoons {}^4_2\mathrm{He} + n \end{aligned} \tag{8.18}$$

and

$$\begin{aligned} p^+ + n &\rightleftharpoons {}^2_1\mathrm{H} + \gamma \\ {}^2_1\mathrm{H} + {}^2_1\mathrm{H} &\rightleftharpoons {}^3_2\mathrm{He} + n \\ {}^3_2\mathrm{He} + {}^2_1\mathrm{H} &\rightleftharpoons {}^4_2\mathrm{He} + p^+ \end{aligned} \tag{8.19}$$

Ex. Compare the binding energy of $^2_1$H nucleus and the characteristic energy of photons at 10^9 K.

Notice that in both the reaction schemes above, deuterium ${}^2_1$H needs to be first produced. Above 10^9K, the energetic radiation would have dissociated any ${}^2_1$H nucleus formed, preventing and subsequent reactions. This is know as the deuterium bottleneck. Just below the 10^9K, the deuterium bottleneck is cleared and almost all the neutrons we had started with before 10^9K were cooked into helium ${}^4_2$He. Our sample of 164 n and 1000 p^+ becomes 82 ${}^4_2$He and 836 p^+ (or ^{1}H). The mass fraction of ${}^4_2$He in the Universe is thus estimated to be $4(82)/[836 + 4(82)] = 0.28$. This is consistent with observations which finds the primordial abundance of ${}^4_2$He to be 24%. This is important as it was shown in separate studies that stella nucleo-synthesis could not have produced so much helium. The fact that Big Bang nucleo-synthesis quantitatively accounts for the helium abundance scores the triumph against the steady state theory, which relied on the stars to produce all the elements.

While the simple account presented above gives a reasonably good estimate to the helium-4 abundance, detailed numerical simulations were made to give a more accurate quantitative abundance of helium-4 and other light elements (see figures 2 and 3 in [**?**]). The abundance of light elements other than helium-4 is strongly dependent on the baryon[3] number density. This can be qualitatively understood easily for deuterium. Other than reactions with itself as shown in Eqs. (8.18) and (8.19), deuterium also reacts with protons through

$$ {}^2_1\mathrm{H} + p \rightarrow {}^3_2\,\mathrm{He} + \gamma \tag{8.20} $$

If the baryon number density is high, the reaction is fast, pushing more deuterium to be converted to ${}^3_2$He (which subsequently converts to ${}^4_2$He), resulting in a low deuterium abundance. Conversely a low baryon density will result in more deuterium left over. Astronomical observations of the deuterium abundance is hence of great importance as it gives information on the baryon number density. Current observations finds the relative abundance of the light elements to be

$$ \begin{aligned} & {}^1_1\mathrm{H} &:&\ {}^2_1\mathrm{H} &:&\ {}^3_2\mathrm{He} &:&\ {}^4_2\mathrm{He} &:&\ {}^7_3\mathrm{Li} \\ = & \ 0.75 &:&\ 10^{-5} &:&\ 10^{-5} &:&\ 0.25 &:&\ 10^{-9} \end{aligned} \tag{8.21} $$

[3]A class of particles that includes protons and neutrons.

8.4 Cosmic Microwave Background Radiation

The big bang model is one that the universe began as a small and radiation dominated "fireball". The space expands, diluting the number density of photons and stretching the the wavelength of each photon. The cosmological redshift that occurred since time immemorial will have stretched the super-short wavelengths of highly energetic (gamma) photons at big bang to super-long wavelengths of cool (radio) photons today.

Going with the above logic, if one can find the cool radio wavelengths photons, one essentially finds the relics of the big bang.

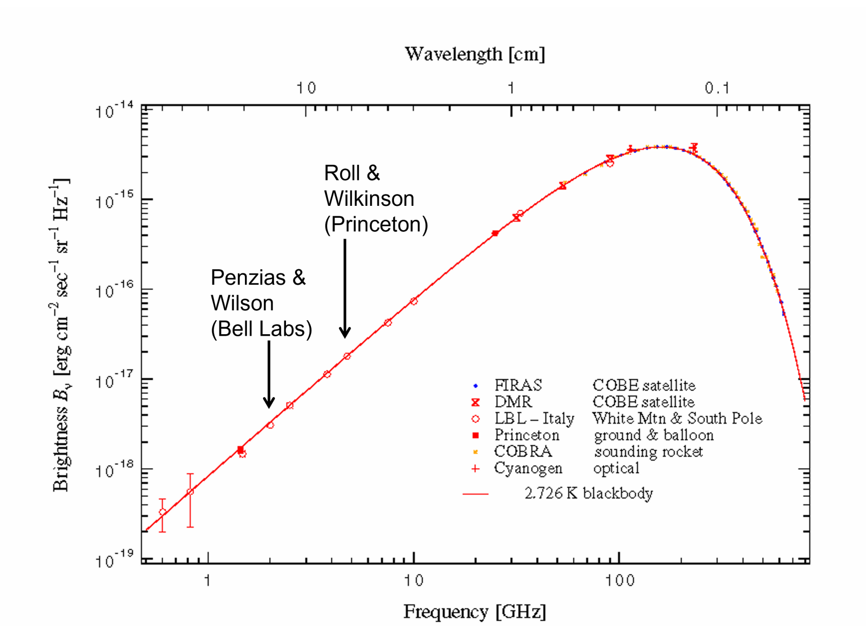

Figure 8.1: The cosmic microwave background (CMB) spectrum. Data (dots and crosses) measured by various radio telescopes are put together to form a full spectrum. The solid line represents the theoretical blackbody spectrum of an object at 2.73 K. Adapted from G. Smoot

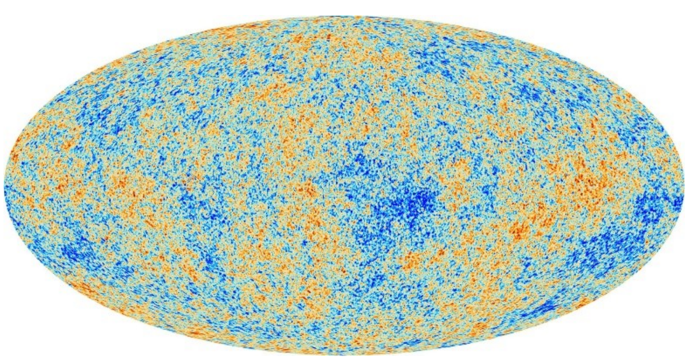

Figure 8.2: The map of comic brackground radiation across the universe as seen from the Planck satellite. The variation in temperature across the universe is shown using an orange–blue colour scale. The standard deviation of the temperature is in the order of milliKelvins. Source: ESA

Go to Activities 1, 2, 3 and 4.

In 1965, Arno Penzias and Robert Wilson were working on statellite communications in Bell Laboratories. They used a radio antenna operating at 4.08 GHz or 7.3 cm, and found that no matter how careful they are in eliminating noise and accounting for radio sources, there is still an unaccountable excess signal that corresponds to a blackbody of 3.5 K.

Meanwhile at Princeton (which coincidentally is in the same state – New Jersey), Robert Dicke and Jim Peebles derived in theory that if the Universe started of in a Big Bang, the blackbody radiation from the last scattering surface will be redshifted to the microwave regime. This microwave radiation can possibly be detected. Dicke began to work with David Wilkinson and Peter Roll to construct a suitable radio receiver for this purpose.

Before Dicke's group obtained any data from their new equipment, they began writing a paper to report to their theoretical prediction and intention to measure the cosmic radiation. Penzias got to know about the Princeton's group imminent paper through a mutual friend. Thinking that this is likely the explanation for their excess signal, Penzias immediately called Dicke to understand their work. Dicke sent him a copy of the manuscript he was preparing and soon the two groups met up at Bell Labs, where Penzias and Wilson showed the Princeton's group their setup and demonstrated the collection of the cosmic radiation.

The two groups agreed to write their results in two seperate consecutive papers, to be published side-by-side. Dicke, Peebles, Wilkinson and Roll will send in the manuscript they had prepared with the additional note about Penzias and Wilson's discovery. The paper was published in Astrophysical Journal vol.142, p.414, with the title **Cosmic Black-Body Radiation**.

Penzias and Wilson will report on their experimental findings of the excess antenna temperature of 3.5 K, and cite Dicke et. al.'s paper as a possible explanation for their discovery. The paper was published in Astrophysical Journal vol.142, p.419 with an unassuming title – **A Measurement of Excess Antenna Temperature at 4080 Mc/s**.

Penzias and Wilson eventually won the Physics Nobel prize in 1978.

The papers by Dicke et al. and Penzias and Wilson can be downloaded in the following:

https://ui.adsabs.harvard.edu/abs/1965ApJ...142..414D/abstract

https://ui.adsabs.harvard.edu/abs/1965ApJ...142..419P/abstract

8.5 In-class Activities – Blackbody Radiation

When light enters a material, it can be reflected, absorbed ot transmitted.

A blackbody can be imagined to be a perfect material that do not transmit nor reflect light. It absorbs all incoming light.

This does not mean that the blackbody itself do not emit light. In fact a blackbody will emit light as long as it's tmeperature is not zero.

Blackbody emission is broadband. The intensity profile (over a range of wavelengths) is given by

$$I_\lambda = A\frac{2hc^2}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda kT}\right)-1} \tag{8.22}$$

where $h = 6.63\times10^{-34}\mathrm{m^2kgs^{-1}}$, $c = 3.0\times10^{8}\mathrm{ms^{-1}}$, $k = 1.38\times10^{-23}\mathrm{m^2kgs^{-2}K^{-1}}$, λ is the wavelength in meters (**not** nm), T is the temperature in Kelvins, and A is a geometrical scaling factor.

Activity 1: Stefan-Boltzmann Law

The total intensity of the radiation over all wavelength is given by the integral

$$I = \int_0^\infty I_\lambda d\lambda$$

Show that

$$I \propto T^4$$

(Hint: do a bit of rescaling by letting $\frac{hc}{kT} \equiv \frac{1}{T'}$, and perform the integration with a software.)

Activity 2: Wein's Law

The peak of the blackbody emission profile can be found by finding its derivative and equate to 0, i.e. $\dfrac{dI_\lambda}{d\lambda} = 0$.

Show that

$$\lambda_{\text{mode}}T = 0.29 \text{ cm K}$$

(Hint: after you differentiate, let $\lambda T = x$)

Show that the maximum wavelength corresponds to photon energy of the order of kT.

Activity 3: Finding the temperature of a blackbody emission

Download the spectrum of an incandescent light bulb collected by an optical spectrometer in LumiNUS.
Using A and T as your fitting parameter, fit the data with Eq.(8.22)
What are the values of A and T from your fitting?

Try also fitting the data with

$$\tilde{I}_\lambda = A\frac{2hc^2}{\lambda^5}\exp\left(-\frac{hc}{\lambda kT}\right) \tag{8.23}$$

Does it fit well?

Eq.(8.23) is known as the **Wien's Tail** of the distribution. The approximated expression for works well for wavelengths much lower than the mode wavelength.

Activity 4: Rayleigh-Jeans

When $E = \dfrac{hc}{\lambda} \ll kT$,

$$I_\lambda \approx 2ckT\lambda^{-4} \tag{8.24}$$

This is the approximated expression for the other end of the spectrum known as the Rayleigh-Jeans side.
Show Eq.(8.24)

8.6 Discussion Questions

1. The baryon to photon ratio

The number of integer-spin particles (such as photons) occupying a quantum state with momentum $\mathbf{p}$ is given by the Bose-Einstein statistics

$$f(\mathbf{p}) = \left[\exp\left(\frac{E(\mathbf{p}) - \mu}{kT}\right) - 1\right]^{-1} . \tag{8.25}$$

Define the number density n to be

$$n = \frac{g}{(2\pi)^3} \int \frac{4\pi p^2 dp}{\hbar^3} f(p) . \tag{8.26}$$

(a) Show that for the case of photons ($E = pc\,,\ \mu = 0\,,\ g = 2$),

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} \left(\frac{kT}{\hbar c}\right)^3$$

where $\zeta(3) = 1.202$, ζ is the the Riemann zeta function defined as

$$\zeta(m) = \sum_{n=1}^{\infty} n^{-m}$$

Hint: You may like to use $1/(e^x - 1) = e^{-x} + e^{-2x} + e^{-3x} + e^{-4x} + ...$ to evaluate the integral. Or you can use any software (eg. MATHEMATICA) if you wish to.

(b) Using $T_0 = 2.735\,\mathrm{K}$, evaluate the present photon number density $n_{\gamma,0}$.

(c) The ratio of baryon to photon number is of great astrophysical importance. Defined as

$$\eta = \frac{n_B}{n_\gamma} , \tag{8.27}$$

this quantity do not change since Big Bang nucleosynthesis. Taking $\rho_{c,0} = 3H_0^2/8\pi G = 1.88 \times 10^{-26} h^2\,\mathrm{kg\ m^{-3}}$, write η in terms of $\Omega_{B,0}$. (Knowing this helps: $\Omega_{B,0} = \rho_{B,0}/\rho_{c,0} = n_{B,0} m_p/\rho_{c,0}$. Also, note that here, h is a numerical parameter related to the Hubble constant)

(d) Astronomical observations on deuterium abundance together with numerical simulations of Big Bang nucleosynthesis contrains the value of η to be

$$\eta < 10^{-9} \tag{8.28}$$

Show that this sets a limit for $\Omega_{B,0}$. Discuss. (You may use $h = 0.72 \pm 0.08$ for your discussion.)