PC3246
Astrophysics I

Tutorial 2

February 3, 2025

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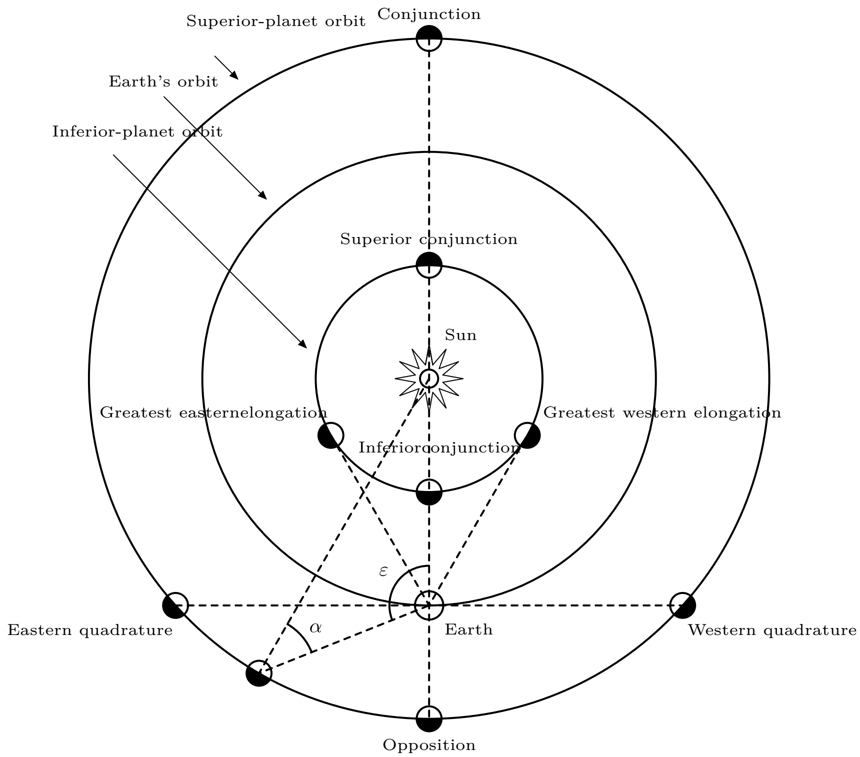
Question 1. Explain how observing an inferior planet at greatest elongation can be used to determine the planet's relative distance (i.e. the distance of the planet from the Sun relative to Earth's distance from the Sun).

Solution: First, we define the average distance the Earth and the Inner Planet are from the Sun as 1 and x astronomical unit(s) respectively where $x > 1$. Assuming that the orbits of Earth and the Inner Planet around the Sun are perfectly circular, we obtain the following geometric construction,



The Sun-Inner Planet-Earth triangle is a right triangle with the angle subtended between the Sun-Inner Planet and Inner Planet-Earth lines being a right angle. The angle θ subtended between the Inner Planet-Earth line and the Sun-Earth line can be observationally obtained by measuring the angular separation between the Inner Planet and the Sun as viewed from Earth. As such,

$$\sin\theta = \frac{x}{1} \qquad \Longrightarrow \qquad x = \sin\theta. \tag{1}$$

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Question 2. Consider a superior planet. State the relationship among its synodic period, sidereal period, and Earth's sidereal period. Explain the derivation of the relationship.

Solution: Let the synodic period be S, the sidereal period be T, and the Earth's sidereal period be T_E. Accordingly, we let the angular velocity of the superior period about the sun be ω and the angular velocity of the Earth about the Sun be ω_E. As such, the synodic period can be considered to be the time T needed for the angular displacement between the superior planet and Earth to be 2π. Hence,

$$2\pi = \omega S - \omega_\mathrm{E} S \qquad \Longrightarrow \qquad \frac{2\pi}{S} = \frac{2\pi}{T} - \frac{2\pi}{T_\mathrm{E}} \tag{2}$$

$$\therefore \quad \frac{1}{S} = \frac{1}{T} - \frac{1}{T_\mathrm{E}}. \tag{3}$$

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Question 3. Proxima Centauri (α Centauri C) is the closest star to the Sun and is a part of a triple star system. It has the epoch J2000.0 coordinates $(\alpha,\ \delta) = \left(14^{\,\mathrm{h}}29^{\,\mathrm{m}}42.95^{\,\mathrm{s}},\ -62^\circ 40' 46.1''\right)$. The brightest member of the system, Alpha Centauri (α Centauri A) has J2000.0 coordinates of $(\alpha,\ \delta) = \left(14^{\,\mathrm{h}}39^{\,\mathrm{m}}36.50^{\,\mathrm{s}},\ -60^\circ 50' 2.3''\right)$.

(a) What is the angular separation of Proxima Centauri and Alpha Centauri?

(b) If the distance to Proxima Centauri is 4.0 x 10'6 m, how far is the star from Alpha Centauri?

Solution: (a)

(b)

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Question 4. If two lenses of focal lengths f_1 and f_2 can be considered to have zero physical separation, then the effective focal length fo the combination of lenses is

$$\frac{1}{f_\mathrm{eff}} = \frac{1}{f_1} + \frac{1}{f_2}. \tag{4}$$

(a) Show that a compound lens system can be constructed from two lenses of different indices of refraction, $n_{1\lambda}$ and $n_{2\lambda}$, having the property that the resultant focal lengths of the compound lens at two specific wavelengths λ_1 and λ_2, respectively, can be made equal, or

$$f_{\mathrm{eff},\lambda_1} = f_{\mathrm{eff},\lambda_2}. \tag{5}$$

(b) Argue qualitatively that this condition does not guarantee that the focal length will be constant for all wavelengths.

Solution: (a)

(b)

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Question 5. Plate scale

(a) State the definition of the plate scale.

(b) State the relationship between the plate scale and the focal length. Explain the derivation of the relationship.

Solution: (a)

(b)

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Question 6. Illuminance and focal ratio

(a) State the definition of the illuminance.

(b) State the relationship among the illuminance and a lens' parameter(s). Explain the relationship.

(c) State the relationship among the focal ratio and a lens' parameter(s). A lens with a large focal ratio is said to be long or slow. Explain why.

Solution: (a)

(b)

(c)

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