

Recap

constituents of matter { particle-like  
wave-like

standard models

quantum field theory

QED, QCD

Feynman diagrams for QED, QCD

Today Feynman diagrams for

weak interaction, quantum flavor dynamics  
(QFD)

5. Each virtual particle (internal line) is represented by the “propagator” (a function describes the propagation of the virtual particle). The virtual particles are responsible for the description of force fields through which interacting particles affect on another.

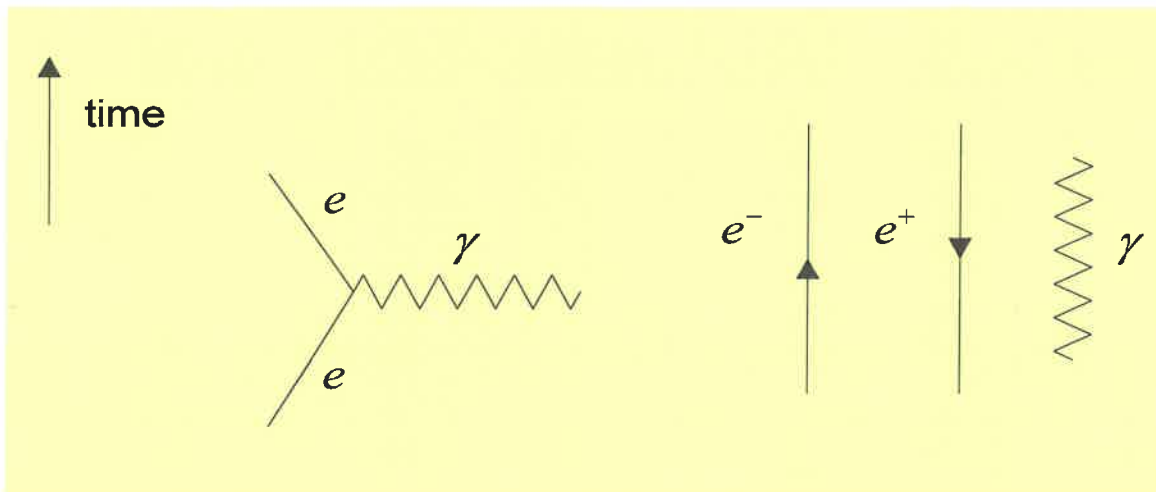
### (a) QED

Coupling constant  $\alpha_e = \frac{q_e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137}$

$q_e = 1.602 \times 10^{-19} \text{ Coul}, \hbar = 1.055 \times 10^{-34} \text{ Joule-Sec}$

$c = 2.998 \times 10^8 \text{ m/s}, \frac{1}{4\pi\epsilon_0} = 8.9875 \times 10^9$

All **em** phenomena are ultimately reducible to following elementary process (primitive vertex)



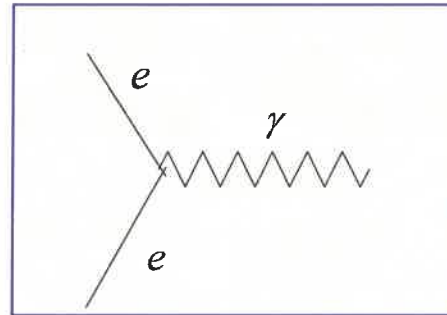
$$L = \bar{\psi}\gamma^\mu D_\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + m\bar{\psi}\psi$$

$$= \bar{\psi}\gamma^\mu\partial_\mu\psi - ie\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + m\bar{\psi}\psi$$

Interaction vertex  $\bar{\psi}\gamma^\mu\psi A_\mu = j^\mu A_\mu$   
and  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

All **em** processes can be described by patching together two or more of the primitive vertices.

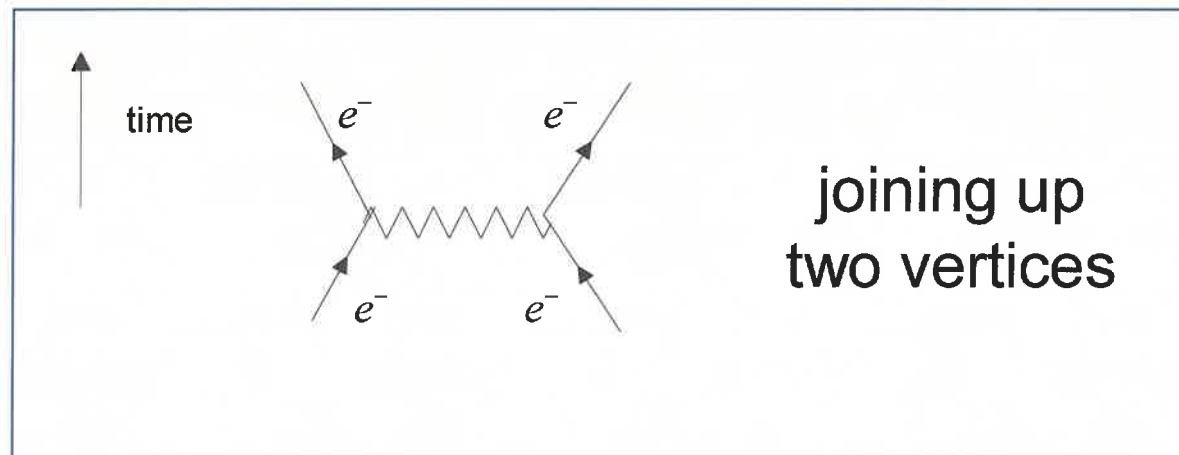
Note: The primitive QED vertex



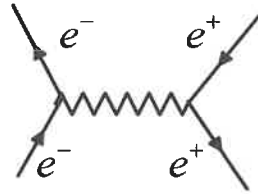
by itself does not represent a possible physical process as it violates the conservation of energy.

Some examples of electromagnetic interaction

### 1. Møller Scattering $e^-e^- \rightarrow e^-e^-$

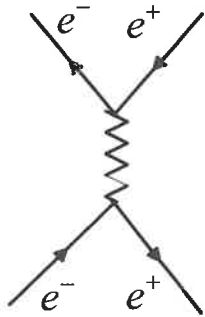


## 2. Bhabha Scattering $e^-e^+ \rightarrow e^-e^+$



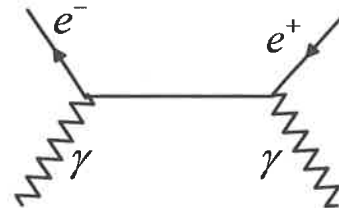
$e^-$  gives up a virtual photon which is absorbed by the positron  $e^+$

Particle line running backward in time (as indicated by the arrow) is interpreted as the corresponding antiparticle running forward.

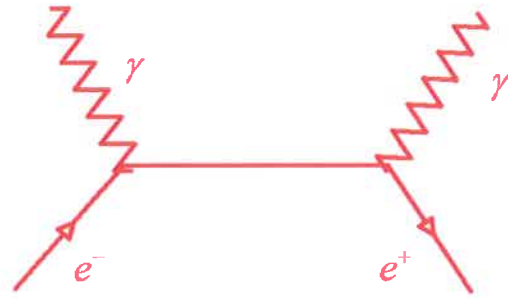


$e^+e^-$  annihilate to produce a virtual photon  $\gamma$  which then pair – produces  $e^+e^-$

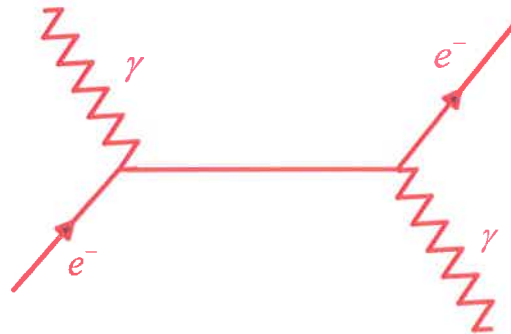
## 3. Pair Production $\gamma\gamma \rightarrow e^+e^-$



4. Pair Annihilation  $e^+e^- \rightarrow \gamma\gamma$

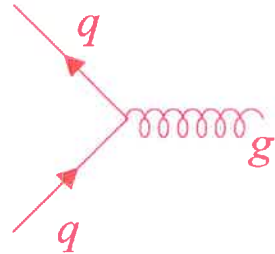


5. Compton Scattering  $e^- \gamma \rightarrow e^- \gamma$

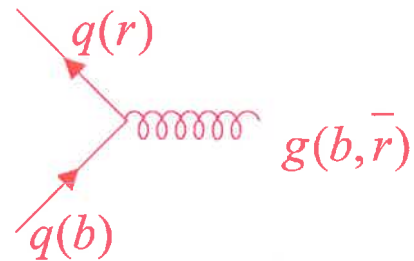


(b) **QCD**

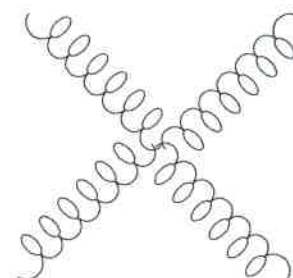
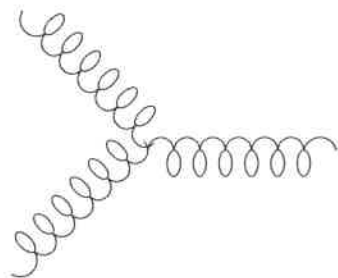
Only quarks and gluons involve basic vertices: Quark-gluon vertex  $q \rightarrow q + g$



More exactly



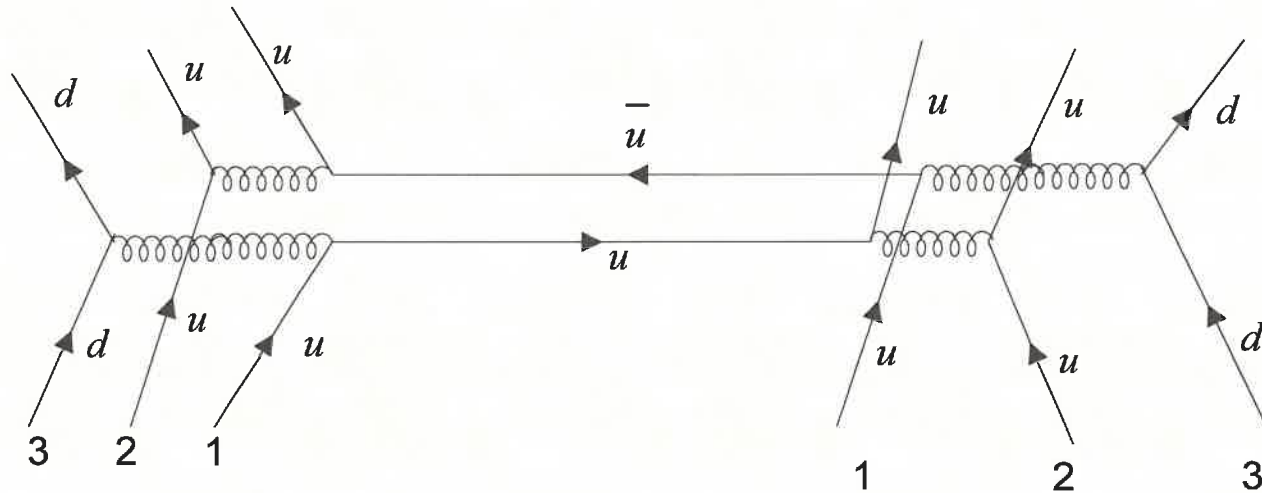
Gluon vertices



## Interaction between two proton

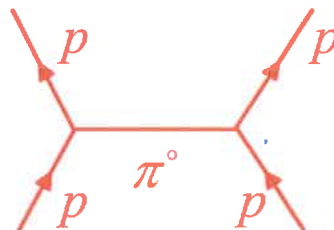
Nucleons (proton or neutron) interact by exchange of  $\pi$  mesons.

e.g.



First  $u$  quark of LH p interacts with  $d$  and then propagates to the RH p to become the  $u$  of the RH p and also interacts with the second  $u$  of the RH p.

Similarly the first  $u$  of RH p interacts with the  $d$  and goes to become a  $u$  of the LH p and also interacts with the second  $u$  of the LH p.



$$\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$$

The coupling constant  $\alpha_s$  decreases as interaction energy increases (short-range)

$$\alpha_{s\,eff} = \frac{\alpha_s}{\epsilon} \quad \epsilon = \text{dielectric constant}$$

known as asymptotic freedom.

$$\alpha_s(m_Z) = 0.112$$

$$m_Z = 91 \text{ GeV}/c^2$$

$$\alpha_s(m_\psi) = 0.2$$

$$m_\psi = 3.1 \text{ GeV}/c^2$$

$$\alpha_s(200 \text{ MeV}) \approx 1$$

For QCD  $\alpha_s$  increases as interaction energy decreases (long range)

known as infrared slavery.



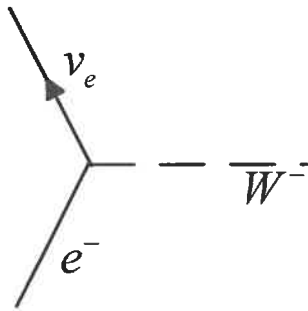
## (c) Weak Interaction

### Two kinds, charged and neutral vertices

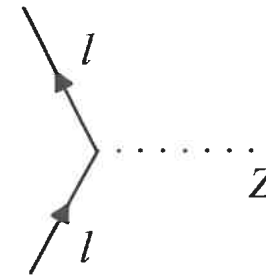
Leptons: primitive vertices connect members of the same generation

Lepton number is separately conserved for each Lepton generation, that is,  $L_e, L_\mu, L_\tau$  separately conserved.

Charged vertex

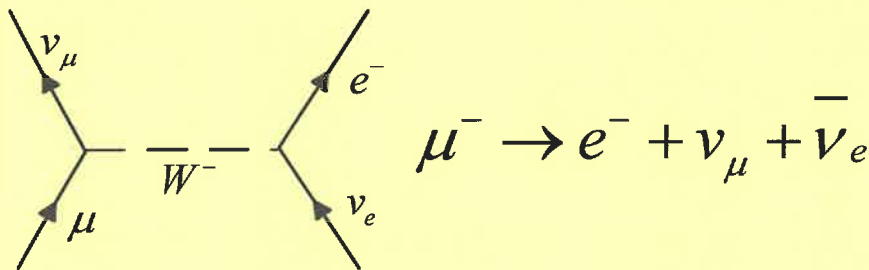
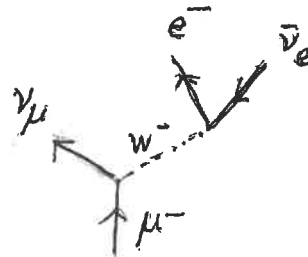


Neutral vertex



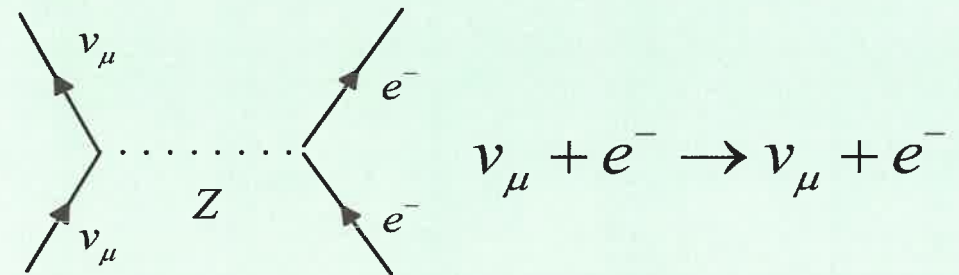
time  
↑

e.g.



$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$$

$$\mu^- + \nu_e \rightarrow e^- + \nu_\mu$$



$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

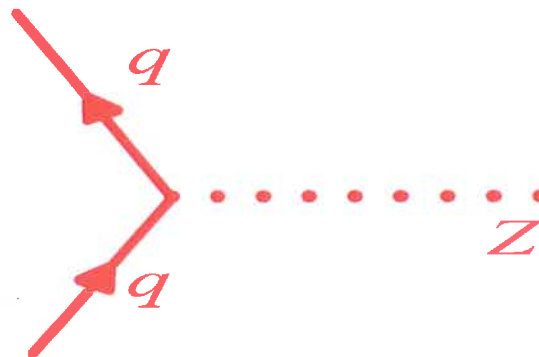
## Quarks

Flavour not conserved in weak interaction .

### Charged Vertex.



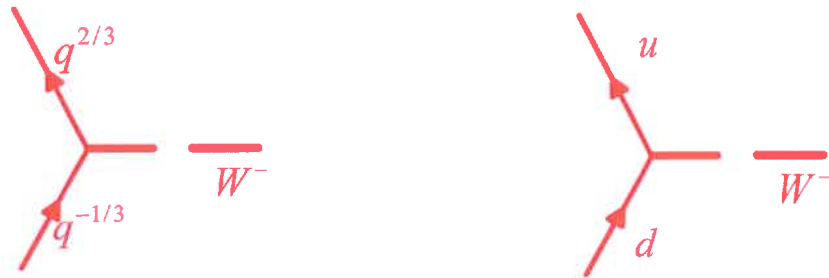
### Neutral vertex



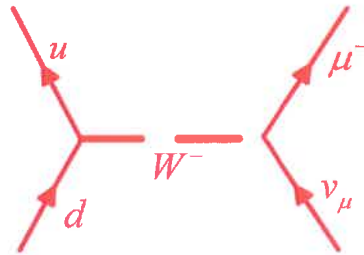
## Quarks

Flavour not conserved in weak interaction

**Charged Vertex.**



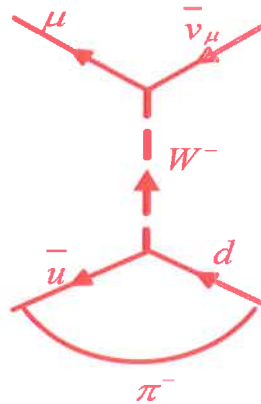
Semileptonic process  $d + \nu_\mu \rightarrow u + \mu^-$



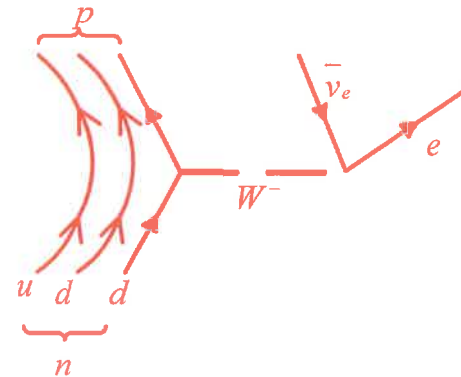
Not observable due to quark confinement

But can be observed in

Decay of  $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$

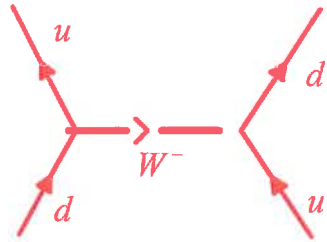


and neutron decay  $n \rightarrow p + e^- + \bar{\nu}_e$



Two quarks  $u, d$  in neutron  $n$  not participating are called spectator quarks.

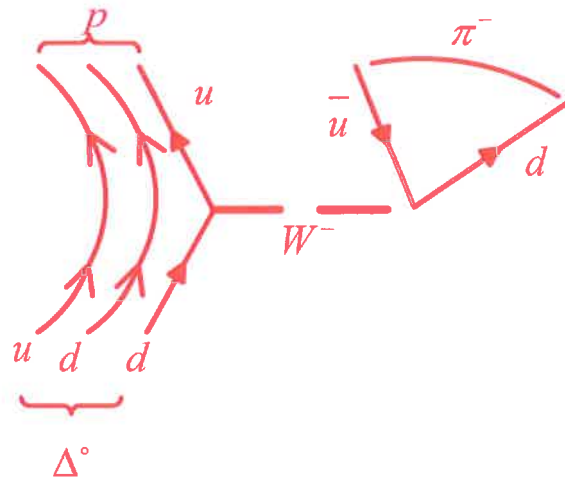
## Hadronic decays



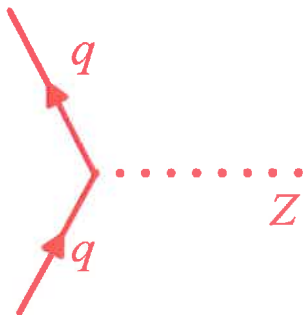
observed in

$$\Delta^0(udd)$$

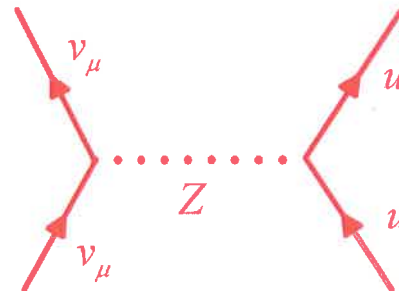
$$\Delta^0 \rightarrow p + \pi^-$$



## Neutral vertex



e.g.

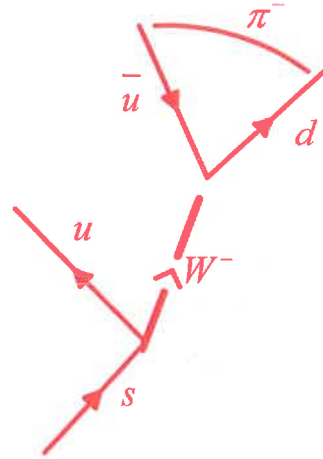


observed in

$$\nu_\mu + p \rightarrow \nu_\mu + p$$

Decays of quark by weak interaction can involve members of different generations

e.g. a strange quark can decay into an u-quark



The weak force not just couples members of the same generation

$$\begin{pmatrix} u \\ d \end{pmatrix} \text{ or } \begin{pmatrix} c \\ s \end{pmatrix} \text{ or } \begin{pmatrix} t \\ b \end{pmatrix}$$

but **couples also members of different generations**

$$\begin{pmatrix} u \\ d' \end{pmatrix} \text{ or } \begin{pmatrix} c \\ s' \end{pmatrix} \text{ or } \begin{pmatrix} t \\ b' \end{pmatrix}$$

where

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Cabibbo

## Kobayashi –Maskawa matrix

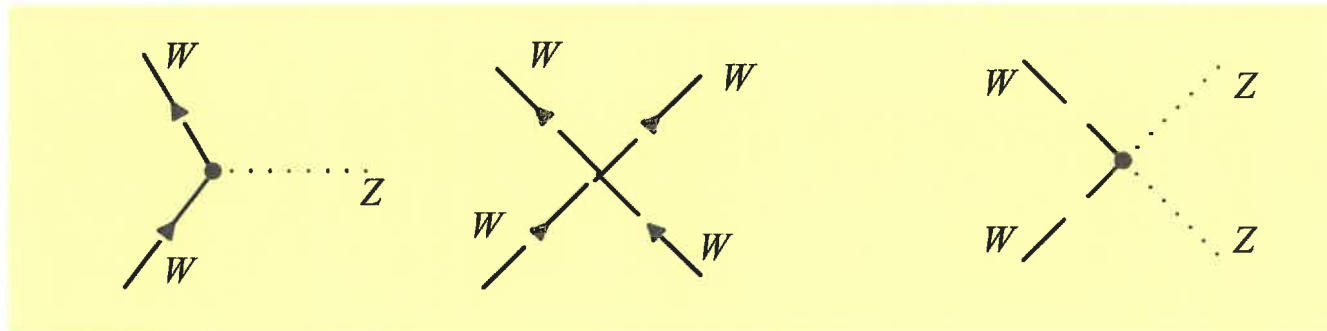
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9747 - 0.9759, & 0.218 - 0.224, & 0.001 - 0.007 \\ 0.218 - 0.224, & 0.9734 - 0.9752, & 0.030 - 0.058 \\ 0.003 - 0.019, & 0.029 - 0.058, & 0.9983 - 0.9996 \end{pmatrix}$$

$V_{ud}$  = coupling of  $u$  to  $d$

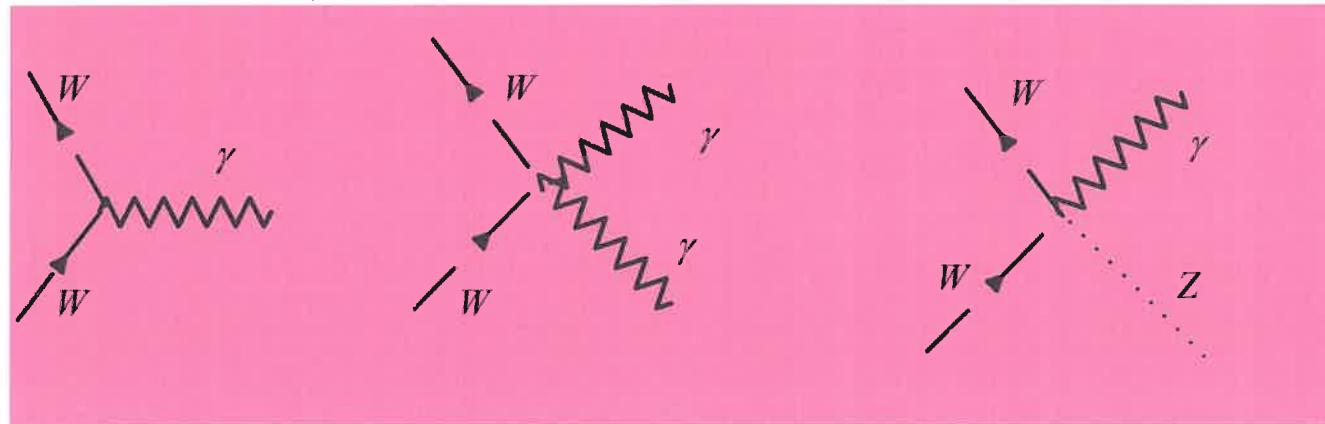
$V_{us}$  = coupling of  $u$  to  $s$

(d) **wk** and **em** couplings of  $W^\pm$  and  $Z$

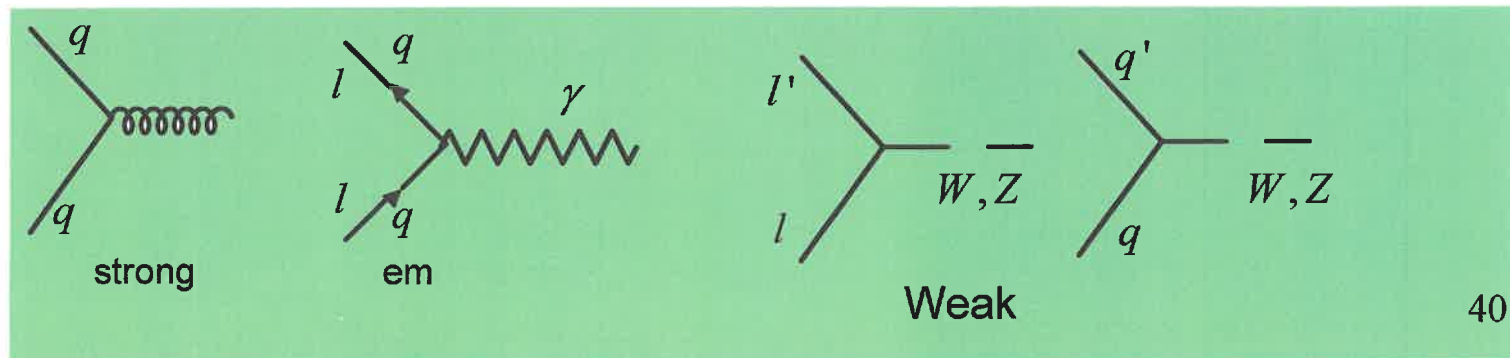
Weak couplings



Couplings involve photon  $\gamma$



## Summary





## 1.5 Decay & Conservation Laws

### 1.5.1

- (a) Every particle decays into lighter particles unless prevented by some conservation law

Stable particles :  $e^-$  (lightest lepton, conservation of lepton number),

$p$  (lightest baryon, conservation of baryon number), neutrinos, photons (massless particles)

- (b) Most particles exhibit several different decay modes  
e.g.

#### Branching ratio

$K^+ \rightarrow$	$\mu^+ + \nu_\mu$	64%
	$\pi^+ + \pi^0$	21%
	$\pi^+ + \pi^+ + \pi^-$	6%
	$\pi^+ + \nu_e + \pi^0$	5%

Each unstable species has a characteristic mean life time  $\tau$

e.g. 
$$\begin{aligned}\tau_{\mu} &= 2.2 \times 10^{-6} \text{ s} \\ \tau_{\pi^+} &= 2.6 \times 10^{-8} \text{ s} \\ \tau_{\pi^0} &= 8.3 \times 10^{-17} \text{ s}\end{aligned}$$

Note:  $I = I_0 e^{-t/\tau},$

$\tau$  = time taken for  $I$  to decrease from  $I_0$  to  $I_0 e^{-1}$

$t_{1/2}$  = time taken for  $I$  to reduce to  $\frac{1}{2} I_0,$

$$\frac{1}{2} I_0 = I_0 e^{-(t_{1/2}/\tau)} \Rightarrow \ln 2 = (t_{1/2}/\tau)$$

( c ) Three Fundamental Decays:

Strong decay e.g.  $\Delta^{++} \rightarrow p + \pi^+$   $\tau = 10^{-23} s$

**em** decay e.g.  $\pi^0 \rightarrow \gamma + \gamma$   $\tau = 10^{-16} s$

**wk** decay e.g.  $\Sigma^- \rightarrow n + e^- + \bar{\nu}_e$   $\tau \sim 10^{-13} s$

Neutron decay  $n \rightarrow p + e^- + \bar{\nu}_e$  ( $\tau = 15 \text{ min}$ )  
( $d \rightarrow u + e^- + \bar{\nu}_e$ )

- (d) Kinematic Effect: the larger the mass difference between the original particle and the decay products, the more rapidly the decay occurs.

This is also known as phase space factor. It accounts for the enormous range of mean life time  $\tau$  in **wk** decays.

## 1.5.2 CONSERVATION LAWS

### (i) Spacetime symmetry

Homogeneity of space time  $\rightarrow$  laws of physics are invariant under time and space translations  $\rightarrow$

Conservation of spatial momentum  $\vec{p}$ , Conservation of energy  $E/c = p^0$

Isotropy of space time  $\rightarrow$  laws of physics are invariant under rotations in space time.

In particular laws of physics are invariant under rotations in space  $\rightarrow$

Conservation of angular momentum.

Invariant under rotation in space and time (Lorentz transformation), Lorentz Symmetry

### Discrete Symmetry

Space inversion  $\rightarrow$  conservation of parity

Time inversion  $T$ , no quantum number associated.

$T$  represented by anti-unitary operator.

$$T^\dagger = T^{-1} \text{ (unitary)}$$

$$T(c_1|\psi_1\rangle + c_2|\psi_2\rangle) = c_1^* T|\psi_1\rangle + c_2^* T|\psi_2\rangle \text{ (antilinear)}$$

and  $c_i^*$  is the complex conjugate of  $c_i$ ,  $i=1, 2$

## (ii) Internal Symmetry

### (1) $U(1)$ symmetry

A physical state of a physical system is represented by a vector  $|\psi\rangle$  in Hilbert space up to a phase factor (assume normalization)

i.e. if  $|\psi\rangle$  represents a physical state

then  $e^{i\alpha}|\psi\rangle$  represents the same physical state, where  $\alpha$ =constant, phase  
whereas  $\exp(i\alpha)$  is known as phase factor.

$\{e^{i\alpha_1}, e^{i\alpha_2} \dots\}$  form a group, an Abelian group  $U(1)$

Conservations of electric charge, baryon number and lepton number are due to the  $U(1)$  phase invariance.

For the electric charge case, can also let phase  $\alpha$  be dependent on spacetime point  $x^\mu$ , namely  $\alpha = \alpha(\underline{x})$  and one gets local gauge invariance

(2) The QCD Lagrangian is invariant under local  $SU(3)$  transformations. i.e. QCD has a local  $SU(3)$  symmetry. An  $SU(3)$  transformation is represented by a unitary  $3 \times 3$  matrix with determinant=1.

$SU(3)$  = special unitary group in three dimensions

(3) Approximate conservation of flavour. Quark flavour is conserved at a strong or electromagnetic vertex, but not at a weak vertex.

**OZI (Okubo, Zweig and Iizuka ) rule**  
**Some strong decays are suppressed**

e.g.

$J/\psi = c\bar{c}$  bound state of charmed quarks has anomalously long lifetime

$\sim 10^{-20}$ sec

(Strong decay  $\sim 10^{-23}$ sec)