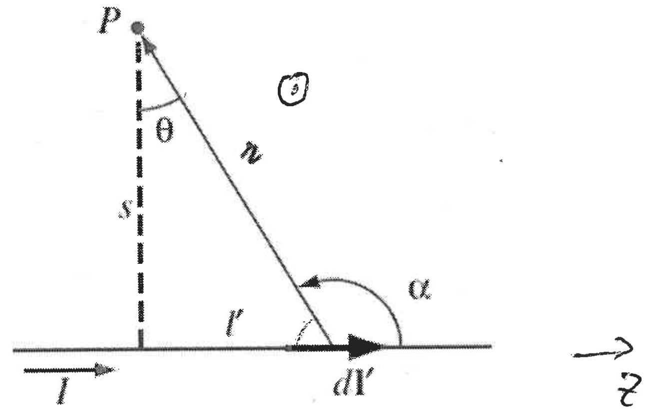
Example 5.5. Find the magnetic field a distance s from a long straight wire carrying a steady current I (Fig. 5.18).



$$\vec{B}(s,\phi,z) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}' \times \hat{\mathscr{r}}}{\mathscr{r}^2} \quad ①$$

$d\vec{l}' \times \hat{\mathscr{r}} \parallel \hat{\phi}$ for all $d\vec{l}'$ & $\hat{\mathscr{r}}$ on the line

$$d\vec{l}' \times \hat{\mathscr{r}} = \hat{\phi}\, dl' \sin\alpha = dl' \cos\theta\, \hat{\phi}$$

$$l' = s\tan\theta \Rightarrow dl' = \frac{s}{\cos^2\theta} d\theta \quad ②$$

$$s = \mathscr{r}\cos\theta \Rightarrow \mathscr{r} = \frac{s}{\cos\theta} \quad ③$$

② & ③ into ①

$$\vec{B}(s) = \hat{\phi} \frac{\mu_0}{4\pi} I \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{s}{\cos^2\theta} \cdot \cos\theta \cdot \left(\frac{s}{\cos\theta}\right)^{-2} d\theta$$

$$= \hat{\phi} \frac{\mu_0 I}{4\pi s} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\, d\theta = \frac{\mu_0 I}{2\pi s} \hat{\phi}$$