PC3246
Astrophysics I

Assignment 2

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Question 1. Using the Rayleigh criterion, estimate the theoretical diffraction limit for the angular resolution of a typical 20 cm (8 in) amateur telescope at 550 nm. Express your answer in arcseconds.

Solution: Using the Rayleigh criterion,

$$\begin{aligned}\theta &\sim \frac{1.22\lambda}{D} = \frac{1.22(550 \times 10^{-9})}{20 \times 10^{-2}} = 3.355 \times 10^{-6}\text{rad} \\ &\approx 0.692''.\end{aligned} \tag{1}$$

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Question 2. The New Technology Telescope (NTT) is operated by the European Southern Observatory at Cerro La Silla. This telescope was used as a testbed for evaluating the adaptive optics technology used in the VLT. The NTT has a 3.58 m primary mirror with a focal ratio of $f/2.2$.

(a) Calculate the focal length of the primary mirror of the New Technology Telescope.

(b) What is the value of the plate scale of the NTT?

(c) ε Bootes is a double star system whose components are separated by $2.9''$. Calculate the linear separation of the images on the primary mirror focal plane of the NTT.

Solution: **(a)** The focal ratio $f/\#$ is defined as,

$$\begin{aligned} f/\# = \frac{f}{D} \qquad \implies \qquad f &= 2.2 \times 3.58 = 7.876\,\text{m} \\ &\approx 7.88\,\text{m} \quad (2\text{ s.f.})\end{aligned} \tag{2}$$

(b) The plate scale p is defined as,

$$\begin{aligned}p &\sim \frac{206265}{7876} = 26.189\,06'' \\ &\approx 26.189''\end{aligned} \tag{3}$$

(c) and

$$s = \frac{\Delta\theta_{\varepsilon \text{ Boo}}}{p} = \frac{2.9}{26.18906} \approx 0.1107\,\text{mm}$$
$$\approx 0.11\,\text{mm} \tag{4}$$

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Question 3. Suppose that a radio telescope receiver has a bandwidth of 50 MHz centered at 1.430 GHz. (1 GHz = 1000 MHz). Assume that, rather than being a perfect detector over the entire bandwidth, the receiver's frequency dependence is triangular, meaning that the sensitivity of the detector is 0% at the edges of the band and 100% at its center.

(a) Assume that the radio dish is a 100% efficient reflector over the receiver's bandwidth and has a diameter of 100 m. Assume also that the source NGC 2558 (a spiral galaxy with an apparent visual magnitude of 13.8) has a constant spectral flux density of $S = 2.5\,\text{mJy}$ over the detector bandwidth. Calculate the total power measured at the receiver.

(b) Estimate the power emitted at the source in this frequency range if $d = 100\,\text{Mpc}$. Assume that the source emits the signal isotropically.

Notes: $1\,\text{Mpc} = 3.086 \times 10^{22}\text{m}$

Hints:

- *The integral of a triangular function is the area of a triangle.*
- *Does the power emitted at the source depend on f_ν?*

Solution: (a) Since the receiver has a triangular frequency dependence,

$$P = S_0\left[\pi\left(\frac{D}{2}\right)^2\right]\frac{\Delta f}{2} = \left(2.5 \times 10^{-29}\right)\left[\pi\left(\frac{100}{2}\right)^2\right]\frac{50 \times 10^6}{2} \approx 4.909 \times 10^{-18}\text{W}$$
$$\approx 4.9 \times 10^{-18}\text{W}. \tag{5}$$

(b) Using the inverse square law,

$$P_{\text{source}} = \frac{P_{\text{receiver}}}{\pi\left(\frac{D}{2}\right)^2}\left(4\pi d^2\right) = \frac{16 P_{\text{receiver}} d^2}{D^2} = \frac{16\left(4.909 \times 10^{-18}\right)\left(100 \times 3.0857 \times 10^{22}\right)^2}{100^2}$$
$$\approx 7.479 \times 10^{28}\text{W}$$
$$\approx 7.5 \times 10^{28}\text{W}. \tag{6}$$

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Question 4. Consider an achromatic doublet. Let R_1 and R_2 be the radii of curvature of the converging lens and $-R_2$ and R_3 be the radii the curvature of the diverging lens.

(a) Express the focal length for red light (f_{r}) in terms of R_1, R_2, R_3, and the refractive indices of the crown and flint glasses.

(b) Express the focal length for blue light (f_{b}) in terms of R_1, R_2, R_3, and the refractive indices of the crown and flint glasses.

(c) Let $f_{\mathrm{r}} = f_{\mathrm{b}}$. Find an equation relating R_1, R_2, R_3. Simplify the equation.

Solution: (a) For the crown (converging) lens made of crown glass with surfaces of radii R_1 and R_2, it has a focal length $f_{\mathrm{c,\ r}}$ of,

$$\frac{1}{f_{\mathrm{c,\ r}}} = (n_{\mathrm{c,\ r}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{7}$$

For the flint (diverging) lens with surfaces of radii $-R_2$ and R_3, it has a focal length $f_{\mathrm{f,\ r}}$ of,

$$\frac{1}{f_{\mathrm{f,\ r}}} = (n_{\mathrm{f,\ r}} - 1)\left(\frac{1}{-R_2} - \frac{1}{R_3}\right) = (n_{\mathrm{f,\ r}} - 1)\left(-\frac{1}{R_2} - \frac{1}{R_3}\right). \tag{8}$$

Thus, for red light

$$\begin{aligned}\frac{1}{f_{\mathrm{r}}} &= \frac{1}{f_{\mathrm{c,\ r}}} + \frac{1}{f_{\mathrm{f,\ r}}} \\ &= (n_{\mathrm{c,\ r}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_{\mathrm{f,\ r}} - 1)\left(-\frac{1}{R_2} - \frac{1}{R_3}\right).\end{aligned} \tag{9}$$

(b) and for blue light,

$$\frac{1}{f_{\mathrm{b}}} = (n_{\mathrm{c,\ b}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_{\mathrm{f,\ b}} - 1)\left(-\frac{1}{R_2} - \frac{1}{R_3}\right). \tag{10}$$

(c) Let $f_{\mathrm{r}} = f_{\mathrm{b}}$,

$$\begin{aligned}&(n_{\mathrm{c,\ r}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_{\mathrm{f,\ r}} - 1)\left(-\frac{1}{R_2} - \frac{1}{R_3}\right) \\ &= (n_{\mathrm{c,\ b}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_{\mathrm{f,\ b}} - 1)\left(-\frac{1}{R_2} - \frac{1}{R_3}\right)\end{aligned} \tag{11}$$

$$\implies \quad [(n_{\mathrm{c,\ r}} - 1) - (n_{\mathrm{c,\ b}} - 1)]\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = [(n_{\mathrm{f,\ b}} - 1) - (n_{\mathrm{f,\ r}} - 1)]\left(-\frac{1}{R_2} - \frac{1}{R_3}\right) \tag{12}$$

$$\implies \quad (n_{\mathrm{c,\ r}} - n_{\mathrm{c,\ b}})\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n_{\mathrm{f,\ b}} - n_{\mathrm{f,\ r}})\left(-\frac{1}{R_2} - \frac{1}{R_3}\right) \tag{13}$$

$$\therefore \quad (n_{\mathrm{c,\ r}} - n_{\mathrm{c,\ b}})\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n_{\mathrm{f,\ r}} - n_{\mathrm{f,\ b}})\left(\frac{1}{R_2} + \frac{1}{R_3}\right). \tag{14}$$

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