

Student Name:

SIS ID (starts with letter "e"):

1. Current density, as a vector field, must satisfy the continuity equation.

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

In the magneto-static limit, argue that the current fluxes through the S_1 and S_2 surfaces must add up to zero for the object below.

In the magneto-static limit $\nabla \cdot \mathbf{J} = 0$

because $\frac{\partial \rho}{\partial t} = 0$

Choose the object as, Gaussian surface

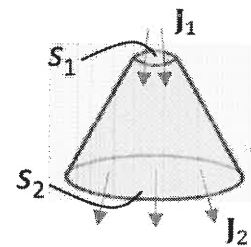
$$\nabla \cdot \mathbf{J} = 0 \Rightarrow \oint \mathbf{J} \cdot d\mathbf{a} = 0$$

$$\Rightarrow \int_{S_1} \mathbf{J} \cdot d\mathbf{a} + \int_{S_2} \mathbf{J} \cdot d\mathbf{a} + \int_{S_3} \mathbf{J} \cdot d\mathbf{a} = 0$$

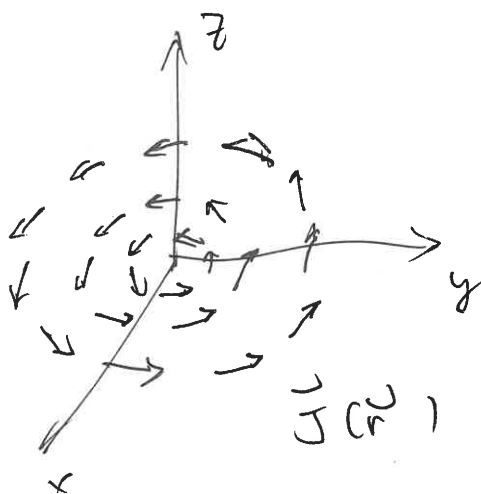
side surface of cone

0, because no current passes through there

$$\Rightarrow \int_{S_1} \mathbf{J} \cdot d\mathbf{a} + \int_{S_2} \mathbf{J} \cdot d\mathbf{a} = 0$$



2. Sketch a current density $\mathbf{J}(\mathbf{r})$ that has nonzero curl (hint: simpler to visualize on a 2D plane). Can you guess in what case this would happen?



$$\nabla \times \mathbf{J} \neq 0, \text{ from Ohm's law}$$

$$\nabla \times (\sigma \mathbf{E}) \neq 0, \text{ if } \sigma \text{ constant}$$

$$\Rightarrow \nabla \times \mathbf{E} \neq 0$$

This cannot happen in the static limit.

Some time-varying effect must occur.

We'll learn later that, from motional emf

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \longrightarrow \text{"eddy current"}$$