2024.3.7

CP violation in the weak decays of kaon mesons.

The neutral kaon mesons are produced by strong interaction, K^0, $\overline{K^0}$ $K^0 = d\bar{s}$, $\overline{K^0} = \bar{d}s$

But they decay as K_L, and K_S

Three features

(i) Oscillation $K^0 \rightleftharpoons \overline{K^0}$

(ii) kaon decay, two different lifetimes
τ_L, τ_S, decay as 2 pions, or 3 pions

(iii) CP is minutely violated.

K_S : $(|K^0\rangle - |\overline{K^0}\rangle)$, $CP = +1$ $\because$ $CP|K^0\rangle = -|\overline{K^0}\rangle$
$CP|\overline{K^0}\rangle = -|K^0\rangle$

K_L : $(|K^0\rangle + |\overline{K^0}\rangle)$, $CP = -1$

$CP\,|K^0\rangle = -|\overline{K^0}\rangle$, $CP\,|\overline{K^0}\rangle = -|K^0\rangle$

$CP\,|\pi^0\rangle = -|\pi^0\rangle$, 2 pions $CP = +1$
3 pions $CP = -1$

If CP is conserved

$K_S \to 2$ pions, $K_L \to 3$ pions

1964 experiment Cronin-Fitch

source. K^0 K_S → 57 ft ≈ 17.4 m away

$\bar{K}^0$ K_L Detectors.

produced

Only K_L remains.

expect to see only 3-pion decays

But found 45 2-pion decays among 22700 decays

If CP is conserved, K_L can only decay into 3 pions

This experiment indicates CP violation

only violated minutely

$$\frac{45}{22700} \sim 2 \times 10^{-3} \approx 0.2\%$$

$K_L \to 2\pi$ (rare decay)

Work out the mathematics of kaons

(i) Oscillation $K^0 \rightleftharpoons \overline{K^0}$

(ii) CP violation

(iii) Decay of kaons. K_L, K_S can decay with different life times τ_L, τ_S

The idea:

(i) start with K^0, $\overline{K^0}$ produced by strong interaction.
Treating as 2-state physical system, using QM to explain occuring of oscillation.
(similar to coupled pendulum problem in classical mechanics)

(ii) K^0, $\overline{K^0}$ are not eigenstates of CP.
construct CP eigenstates K_1, K_2

$$K_1 = \frac{1}{\sqrt{2}}(K^0 - \overline{K^0}), \qquad K_2 = \frac{1}{\sqrt{2}}(K^0 + \overline{K^0})$$

Note: K_1, K_2 just mathematical construct, actual physical particles are K^0, $\overline{K^0}$; K_L; K_S

Modify the states K_1, K_2 to get slightly CP broken states K_I, K_{II}

(iii) Introduce 'effective' Hamiltonian to explain decay. Change the actual Hamiltonian H to a non-Hermitian $\overline{H}$, $\overline{H} \neq \overline{H}^\dagger$

Use quantum mechanics to account for the oscillation. The kaon is a 2-state system

$\{K^0, \bar{K}^0\} \longrightarrow \{K_S, K_L\}$

produced by strong interaction

Decay by wk interaction

Assume CP is conserved, we have shown

Short lifetime $|K_S\rangle \sim |K^0\rangle - |\bar{K}^0\rangle \rightarrow CP = +1$

Long lifetime $|K_L\rangle \sim |K^0\rangle + |\bar{K}^0\rangle \longrightarrow CP = -1$

$CP|K^0\rangle \quad C(-)|K^0\rangle$
$= -C|K^0\rangle$
$= -|\bar{K}^0\rangle$

Schrödinger Equation

$$i\hbar \frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$$

$$H = H_{st} + H_{em} + H_{wk} = H_0 + H_{wk}$$

$$H_0 \equiv H_{st} + H_{em}$$

Discrete basis for n-state system

$$|\psi\rangle = \sum_{i=1}^{n} c_i |i\rangle$$

$i = 1, 2, \ldots n$

$$i\hbar \frac{\partial}{\partial t} c_i(t) = \sum_{j=1}^{n} H_{ij} c_j(t)$$

$i = 1, 2, \ldots n$

$$H_{ij} = \langle i|H|j\rangle$$

For 2-state system

$$i\hbar\, \partial_t c_1(t) = H_{11} c_1(t) + H_{12} c_2(t)$$

$$i\hbar\, \partial_t c_2(t) = H_{21} c_1(t) + H_{22} c_2(t)$$

notation $\partial_t c_1 \equiv \frac{\partial c_1}{\partial t}$

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

c_1, c_2 unknown

To solve this, choose a basis.

A basis can be $(K^0, \bar{K}^0)$ or (K_S, K_L)

We choose

$$|K^0\rangle = |1\rangle \quad , \quad |\bar{K}^0\rangle = |2\rangle .$$

Hence

$$H_{11} = \langle 1|H|1\rangle = \langle K^0|H|K^0\rangle$$

$$= \langle K^0|H_0 + H_{wk}|K^0\rangle$$

$$= \langle K^0|H_0|K^0\rangle + \langle K^0|H_{wk}|K^0\rangle$$

Assume $|K^0\rangle$, $|\bar{K}^0\rangle$ are eigenstates of H_0

$= H_{st} + H_{em}$

$$H_0|K^0\rangle = E_0|K^0\rangle$$

$$H_0|\bar{K}^0\rangle = E_0|\bar{K}^0\rangle$$

$$\therefore \quad H_{11} = E_0\langle K^0|K^0\rangle + \langle K^0|H_{wk}|K^0\rangle$$

$$= E_0 + \underbrace{\langle K^0|H_{wk}|K^0\rangle}_{\text{small}}$$

$$\simeq E_0$$

$\rightarrow \quad H_{22} = E_0$

$$H_{12} = \langle k^0 | H | \bar{k}^0 \rangle$$

$$= \langle k^0 | H_0 + H_{wk} | \bar{k}^0 \rangle$$

$$= \langle k^0 | H_0 | \bar{k}^0 \rangle + \langle k^0 | H_{wk} | \bar{k}^0 \rangle$$

$$= E_0 \langle k^0 | \bar{k}^0 \rangle + \langle k^0 | H_{wk} | \bar{k}^0 \rangle$$

$$= \langle k^0 | H_{wk} | \bar{k}^0 \rangle = -A$$

$$\rightarrow \quad i\hbar \frac{\partial}{\partial t} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\rightarrow \quad i\hbar \frac{\partial}{\partial t} c_1 = E_0 c_1 - A c_2$$

$$i\hbar \frac{\partial}{\partial t} c_2 = -A c_1 + E_0 c_2$$

unknown c_1, c_2 $\qquad \partial_t \equiv \frac{\partial}{\partial t}$

Adding, $i\hbar \partial_t (c_1 + c_2) = (E_0 - A) c_1 + (E_0 - A) c_2$

$$= (E_0 - A)(c_1 + c_2)$$

subtracting $i\hbar \partial_t (c_1 - c_2) = (E_0 + A)(c_1 - c_2)$

solution obvious

$$c_1 + c_2 = a\, e^{-\frac{i}{\hbar}(E_0 - A)t}$$

a = arbitrary constant

$$c_1 - c_2 = b\, e^{-\frac{i}{\hbar}(E_0 + A)t}$$

$\rightarrow$ $c_1 =$ HW

$c_2 =$

$\therefore$ $|\psi\rangle = c_1 |K^0\rangle + c_2 |\bar{K}^0\rangle$

$|1\rangle = |K^0\rangle$

$|2\rangle = |\bar{K}^0\rangle$

$=$

$$|\psi\rangle = c_1 |K^0\rangle + c_2 |\bar{K}^0\rangle$$

$$= \frac{1}{2}\left(a e^{-iE_2 t/\hbar} + b e^{-iE_1 t/\hbar}\right) |K^0\rangle$$

$E_2 = E_0 - A$

$E_1 = E_0 + A$

$$+ \frac{1}{2}\left(a e^{-iE_2 t/\hbar} - b e^{-iE_1 t/\hbar}\right) |\bar{K}^0\rangle \qquad \text{(HW)}$$

3 different situations

(i) At time $t=0$, let $|\psi\rangle = |K^0\rangle$

$$a = b = 1$$

So at time $t \geq 0$

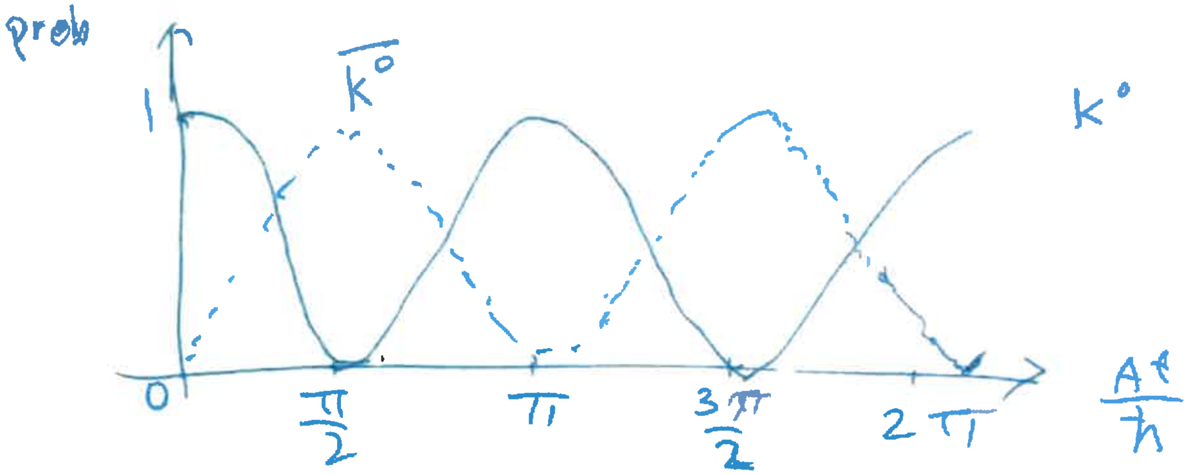
$$|\psi\rangle = \frac{1}{2} e^{-iE_0 t/\hbar}\left(e^{iAt/\hbar} + e^{-iAt/\hbar}\right) |K^0\rangle$$

$$+ \frac{1}{2} e^{-iE_0 t/\hbar}\left(e^{iAt/\hbar} - e^{-iAt/\hbar}\right) |\bar{K}^0\rangle$$

$$= e^{-iE_0 t/\hbar}\left[\cos\frac{At}{\hbar} |K^0\rangle + i \sin\frac{At}{\hbar} |\bar{K}^0\rangle\right]$$

This explains oscillation $\because$ at $t=0$, $|\psi\rangle = |K^0\rangle$, at $t = \frac{\pi}{2}\frac{\hbar}{A}$, $|\psi\rangle = |\bar{K}^0\rangle$

period of oscillation = HW



So far, oscillation $K^0 \rightleftharpoons \overline{K^0}$ is explained and is due to off-diagonal elements $H_{12} = H_{21} = -A \neq 0$

Next consider

(ii) Assume $b = 0$ (arbitrary constant), the state

$$|\psi\rangle = \frac{a}{2} e^{-iE_2 t/\hbar} |K^0\rangle + \frac{a}{2} e^{-iE_2 t/\hbar} |\overline{K^0}\rangle$$

$$= \frac{a}{2} e^{-iE_2 t/\hbar} (|K^0\rangle + |\overline{K^0}\rangle)$$

$\langle\psi|\psi\rangle = 1$, $\quad \frac{|a|^2}{4}(1 + 1) = \frac{|a|^2}{2}$

$$\rightarrow \quad a = \sqrt{2}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}} e^{-iE_2 t/\hbar} (|K^0\rangle + |\overline{K^0}\rangle)$$

$$= e^{-iE_2 t/\hbar} |K_2\rangle, \quad |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K^0}\rangle)$$

So the $|K_2\rangle$ is an eigenstate of the Hamiltonian H with eigenvalue $E_2 = E_0 - A$

i.e. $H|K_2\rangle = E_2|K_2\rangle$

proof:
substitute $|\psi\rangle = e^{-iE_2t/\hbar}|K_2\rangle$ into

the S.E.

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$$

$$LHS = e^{\frac{-iE_2t}{\hbar}}\cdot E_2|K_2\rangle$$

$$RHS = H|\psi\rangle = H e^{-iE_2t/\hbar}|K_2\rangle$$

$$= e^{-iE_2t/\hbar} H|K_2\rangle$$

$LHS = RHS \longrightarrow$ $H|K_2\rangle = E_2|K_2\rangle$

Note $|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$ is also an eigenstate of CP,

$$CP|K_2\rangle = \frac{1}{\sqrt{2}}(CP|K^0\rangle + CP|\bar{K}^0\rangle)$$

$$= -|K_2\rangle \qquad \text{(HW)}$$

i.e. $|K_2\rangle$ is a common eigenstate of H and CP.

Can we identify $|K_2\rangle$ as $|K_L\rangle$?

(iii) $a = 0$. Find $|\psi\rangle =$ HW

$$\frac{1}{2}b e^{-iE_1t/\hbar}(|K^0\rangle - |\bar{K}^0\rangle)$$

put $b = \sqrt{2}$ $\quad = e^{-iE_1t/\hbar}|K_1\rangle \quad |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$

$$\rightarrow \quad |K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

$$H\,|K_1\rangle = E_1\,|K_1\rangle\,, \qquad E_1 = E_0 + A$$

so $|K_1\rangle$ is a common eigenstate of CP and the Hamiltonian, i.e.

$$[CP,\ H] = 0$$

for the state $|K_1\rangle$ and $|K_2\rangle$

Proof: show $[CP, H]\,|K_1\rangle = 0$, $[CP, H]\,|K_2\rangle = 0$

Hence $[CP, H]\,|\psi\rangle = 0$, where $|\psi\rangle = c_1|K_1\rangle + c_2|K_2\rangle$

At this stage, we cannot identify

$$|K_1\rangle \sim |K_S\rangle, \qquad |K_2\rangle \sim |K_L\rangle$$

because K_1 conserves CP but not K_S; similarly K_2 cannot be identified as K_L.

Next to construct a model that accounts for CP violation; one way is to write

$$|K_S\rangle = \frac{\left(|K_1\rangle + \varepsilon\,|K_2\rangle\right)}{\sqrt{1+|\varepsilon|^2}} \qquad |\varepsilon| \text{ small parameter}$$