1. $\nabla \cdot (f\vec{A}) = f(\nabla \cdot \vec{A}) + \vec{A} \cdot (\nabla f)$ Show proof

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

$$\nabla \cdot (f\vec{A})$$

$$= \frac{\partial}{\partial x}(fA_x) + \frac{\partial}{\partial y}(fA_y) + \frac{\partial}{\partial z}(fA_z)$$

$$= A_x\frac{\partial f}{\partial x} + f\frac{\partial A_x}{\partial x} + A_y\frac{\partial f}{\partial y} + f\frac{\partial A_y}{\partial y} + A_z\frac{\partial f}{\partial z} + f\frac{\partial A_z}{\partial z}$$

$$= f(\nabla \cdot \vec{A}) + \underbrace{(A_x\hat{x} + A_y\hat{y} + A_z\hat{z})}_{\vec{A}} \cdot \underbrace{\left(\hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}\right)}_{\nabla f}$$

$$= f(\nabla \cdot \vec{A}) + \vec{A} \cdot \nabla f$$

2. Prove $\nabla \times (\vec{A} \times \vec{B}) = (\vec{B} \cdot \nabla)\vec{A} - (\vec{A} \cdot \nabla)\vec{B} + \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A})$

BAC - CAB rule : $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

$$\nabla \times (\vec{A} \times \vec{B})$$

$$= \cancel{\vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A})}$$

* "∇" not like an ordinary vector in triple product

$x \to z \to y \to x$

$$\nabla \times (\vec{A} \times \vec{B})$$

$$= \underbrace{\hat{x}\left[\frac{\partial}{\partial y}(\vec{A} \times \vec{B})_z - \frac{\partial}{\partial z}(\vec{A} \times \vec{B})_y\right]}_{①} + \underbrace{\hat{y}[\quad]}_{②} + \underbrace{\hat{z}[\quad]}_{③}$$

$$① = \hat{x}\left[\frac{\partial}{\partial y}(A_x B_y - A_y B_x) - \frac{\partial}{\partial z}(A_z B_x - A_x B_z)\right]$$

$$= \hat{x}\left[B_y \frac{\partial A_x}{\partial y} + A_x \frac{\partial B_y}{\partial y} - B_x \frac{\partial A_y}{\partial y} - A_y \frac{\partial B_x}{\partial y} - B_x \frac{\partial A_z}{\partial z} - A_z \frac{\partial B_x}{\partial z} + B_z \frac{\partial A_x}{\partial z} + A_x \frac{\partial B_z}{\partial z}\right]$$

② $x \to y \quad y \to z \quad z \to x$ from ①

③ $y \to z \quad z \to x \quad x \to y$ from ②

Right hand side.

$(\vec{B}\cdot\nabla)\vec{A}$

$= (B_x\frac{\partial}{\partial x} + B_y\frac{\partial}{\partial y} + B_z\frac{\partial}{\partial z})(A_x\hat{x} + A_y\hat{y} + A_z\hat{z})$

$= (\underline{B_x\frac{\partial A_x}{\partial x}} + B_y\frac{\partial A_x}{\partial y} + B_z\frac{\partial A_x}{\partial z})\hat{x} + (\ldots\ldots)\hat{y} + (\ldots\ldots)\hat{z}$.

cancels with that in $-\vec{B}(\nabla\cdot\vec{A})$

terms in (1) $\Rightarrow (\vec{B}\cdot\nabla)\vec{A}$

terms in (1) $\Rightarrow \vec{A}(\nabla\cdot\vec{B})$

terms in (1) $\Rightarrow -\vec{B}(\nabla\cdot\vec{A})$

terms in (1) $\Rightarrow -(\vec{A}\cdot\nabla)\vec{B}$