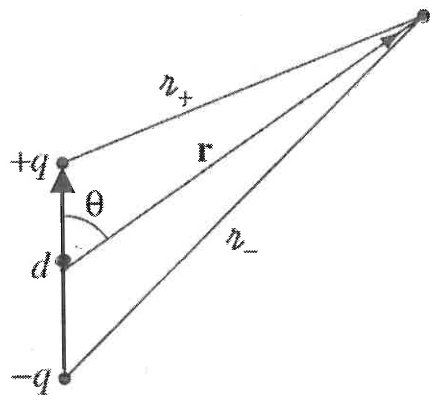
Dipole field at large r: $V_{\text{dip}}(\mathbf{r}) = \dfrac{1}{4\pi\epsilon_0}\dfrac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}$



$$r_\pm^2 = r^2 + \left(\frac{d}{2}\right)^2 \mp rd\cos\theta$$

$$= r^2\left(1 \mp \frac{d}{r}\cos\theta + \frac{d^2}{4r^2}\right)$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_+} - \frac{q}{r_-}\right)$$

$$\frac{1}{r_\pm} \approx \frac{1}{r}\left(1 \mp \frac{d}{r}\cos\theta\right)^{-1/2} \approx \frac{1}{r}\left(1 \pm \frac{1}{2}\frac{d}{r}\cos\theta\right)$$

$$\left[\ (1+\varepsilon)^\alpha \approx 1 + \alpha\varepsilon \quad \text{when } \varepsilon \to 0\ \right]$$

$$\Rightarrow \frac{1}{r_+} - \frac{1}{r_-} \approx \frac{d}{r^2}\cos\theta$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{qd\cos\theta}{r^2} \qquad |\vec{p}| = qd$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$