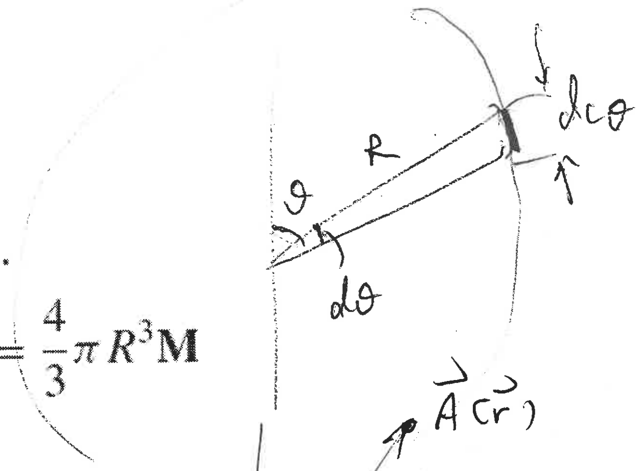
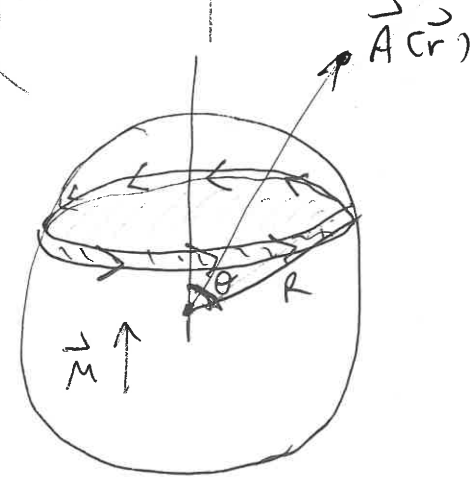
Example 6.1. Find the magnetic field of a uniformly magnetized sphere.



- Field outside the sphere same as what would be for a dipole $\mathbf{m} = \frac{4}{3}\pi R^3 \mathbf{M}$



uniform $\vec{M}$ $\quad \vec{J}_b = 0 \qquad \vec{K}_b = \vec{M} \times \hat{n} = M_0\, \hat{z} \times \hat{r} = M \sin\theta\, \hat{\phi}$

Assume point of interest $|\vec{r}| \gg R$

$d\vec{m} \parallel \hat{z}, \quad \vec{A} \propto d\vec{m} \times \hat{r} \propto \hat{z} \times \hat{r} \propto \hat{\phi}$

$$d\vec{m} = \vec{a}\, dI = \vec{a} \cdot K_b \cdot d\ell_\theta = \hat{z}\left[\pi (R \sin\theta)^2\right](M_0 \sin\theta)(R\, d\theta)$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{d\vec{m} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{\hat{z} \times \hat{r}}{r^2} \int_0^\pi \pi R^3 M \sin^3\theta\, d\theta$$

$$= \frac{\mu_0}{4} \frac{\hat{z} \times \hat{r}}{r^2} M R^3 \underline{\int_0^\pi \sin^3\theta\, d\theta}$$

$$= \frac{\mu_0}{4\pi}\left(M \frac{4}{3}\pi R^3\right) \frac{\hat{z} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{\vec{m}_s \times \hat{r}}{r^2}$$

$$\vec{m}_s = \hat{z} M \cdot \frac{4}{3}\pi R^3$$

$$-\int_0^\pi \sin^2\theta\, d(\cos\theta)$$

$$= -\int_{\theta=0}^{\theta=\pi} (1 - \cos^2\theta)\, d(\cos\theta) \qquad u \equiv \cos\theta$$

$$= \int_1^{-1} (u^2 - 1)\, du = \frac{4}{3}$$