Goal: proving $\int_V \nabla \times \vec{v}\, d\tau = -\oint_S \vec{v} \times d\vec{a}$

Divergence theorem $\int \nabla \cdot \vec{v}\, d\tau = \oint \vec{v} \cdot d\vec{a}$

substitute $\vec{v} \to \vec{v} \times \vec{c}$ where $\vec{c}$ is a constant

$$\int \nabla \cdot (\vec{v} \times \vec{c})\, d\tau = \oint (\vec{v} \times \vec{c}) \cdot d\vec{a}$$

LHS: product $\nabla \cdot (\vec{v} \times \vec{c}) = \vec{c} \cdot (\nabla \times \vec{v}) - \cancel{\vec{v} \cdot (\nabla \times \vec{c})} = \vec{c} \cdot (\nabla \times \vec{v})$

RHS: $(\vec{v} \times \vec{c}) \cdot d\vec{a} = d\vec{a} \cdot (\vec{v} \times \vec{c}) = \vec{c} \cdot (d\vec{a} \times \vec{v}) = -\vec{c} \cdot (\vec{v} \times d\vec{a})$

$$\Rightarrow \int \vec{c} \cdot (\nabla \times \vec{v})\, d\tau = -\oint \vec{c} \cdot (\vec{v} \times d\vec{a})$$

$$\Rightarrow \vec{c} \cdot \underline{\int \nabla \times \vec{v}\, d\tau} = -\vec{c} \cdot \underline{\oint \vec{v} \times d\vec{a}}$$ $\vec{c}$ arbitrary.

$$\Rightarrow \int \nabla \times \vec{v}\, d\tau = -\oint \vec{v} \times d\vec{a}$$