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Exercise 1.1. Find the particles's velocity and acceleration vectors. What are the magnitude and direction of the particle's acceleration?

Solution: Starting with the position vector of the particle undergoing uniform circular motion,

$$\mathbf{r} = R\cos\omega t \hat{\mathbf{e}}_x + R\sin\omega t \hat{\mathbf{e}}_y. \tag{1}$$

From the definition of the velocity vector as the rate of change of the position vector w.r.t. time,

$$\mathbf{v} \equiv \frac{d\mathbf{r}}{dt} = R(-\omega \sin \omega t \hat{\mathbf{e}}_x + \omega \cos \omega t \hat{\mathbf{e}}_y)$$

$$= R\omega(-\sin \omega t \hat{\mathbf{e}}_x + \cos \omega t \hat{\mathbf{e}}_y).$$
(2)

Similarly, from the definition of the acceleration vector as the rate of change of the velocity vector w.r.t. time,

$$\mathbf{a} \equiv \frac{d\mathbf{v}}{dt} = R\omega(-\omega\cos\omega t\hat{\mathbf{e}}_x - \omega\sin\omega t\hat{\mathbf{e}}_y)$$

$$= -R\omega^2(\cos\omega t\hat{\mathbf{e}}_x + \sin\omega t\hat{\mathbf{e}}_y),$$
(3)

which yields a magnitude,

$$a = \sqrt{\mathbf{a} \cdot \mathbf{a}} = \sqrt{R^2 \omega^4 \cos^2 \omega t} \underbrace{(\hat{\mathbf{e}}_x \cdot \hat{\mathbf{e}}_x)}_{1} + R^2 \omega^4 \sin^2 \omega t} \underbrace{(\hat{\mathbf{e}}_y \cdot \hat{\mathbf{e}}_y)}_{1}$$

$$= R\omega^2,$$
(4)

and a direction,

$$\hat{\mathbf{a}} = \frac{\mathbf{a}}{a} = -\cos\omega t \hat{\mathbf{e}}_x - \sin\omega t \hat{\mathbf{e}}_y,\tag{5}$$

pointing radially inwards towards the centre of the circular path.