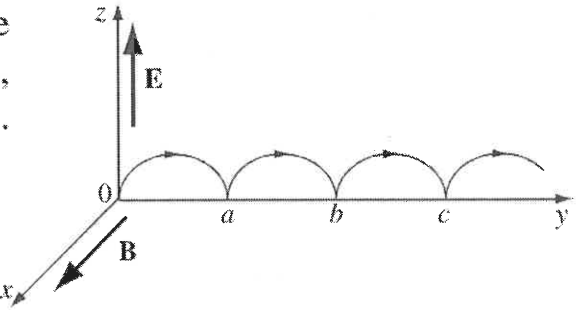
Example 5.2. Cycloid Motion. A more exotic trajectory occurs if we include a uniform electric field, at right angles to the magnetic one. Suppose, for instance, that **B** points in the x-direction, and **E** in the z-direction, as shown in Fig. 5.7. A positive charge is released from the origin; what path will it follow?



$\vec F$ in the yz plane $\quad V_x(0) = 0 . \Rightarrow \vec r(t) = (0, y, z)$

$\vec v = (0, V_y, V_z) = (0, \dot y, \dot z) \qquad \vec E = (0, 0, E) \quad \vec B(B, 0, 0)$

$\vec F = Q(\vec E + \vec v \times \vec B) = Q[E\hat z + V_z B\hat y - V_y B\hat z] = Q(E\hat z + B\dot z\hat y - B\dot y\hat z)$

$= m\vec a = m(\ddot y\,\hat y + \ddot z\,\hat z)$

$\Rightarrow QB\dot z = m\ddot y, \quad QE - QB\dot y = m\ddot z$, where $\omega \equiv \frac{QB}{m}$ = "cyclotron frequency"

$\Rightarrow \ddot y = \omega\dot z, \quad \ddot z = \omega\left(\frac{E}{B} - \dot y\right) \longrightarrow \begin{cases} y(t) = C_1\cos\omega t + C_2\sin\omega t + \frac{E}{B}t + C_3 \\ z(t) = C_2\cos\omega t - C_1\sin\omega t + C_4 \end{cases}$

$y(0) = z(0) = 0, \quad \dot y(0) = \dot z(0) = 0$ initial conditions $\Rightarrow C_1, C_2, C_3, C_4$

$\Rightarrow y(t) = \frac{E}{\omega B}(\omega t - \sin\omega t) \qquad z(t) = \frac{E}{\omega B}(1 - \cos\omega t)$