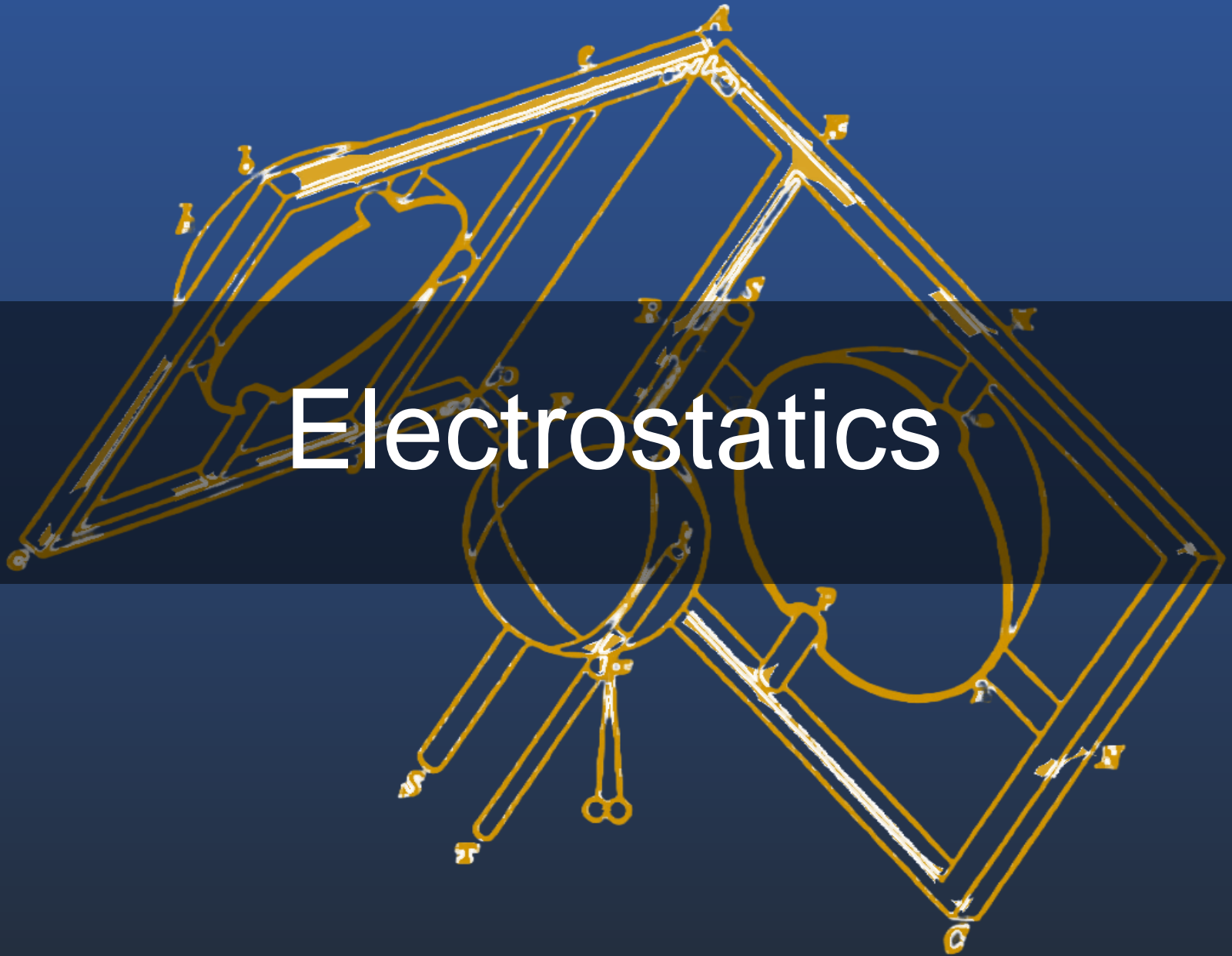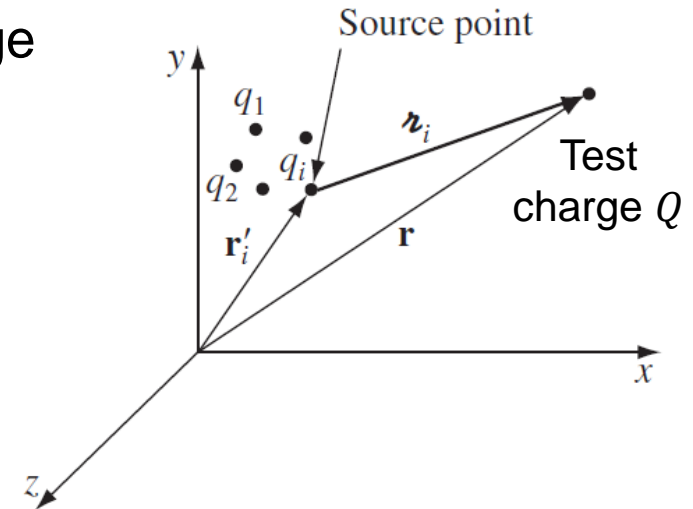# Electrostatics

Charge, electric field, and potential

Coulomb's law

- Force of n source charges on a test charge

  - Force from source charge q_i acting on test charge Q

    - Coulomb's law $F_i = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_i Q}{r_i^2}\widehat{\boldsymbol{r_i}}$

    - Permittivity $\varepsilon_0 = 8.85 \times 10^{12}$ C^2/(N m^2)

    - Separation vector $\boldsymbol{r}_i = \boldsymbol{r} - \boldsymbol{r}_i'$

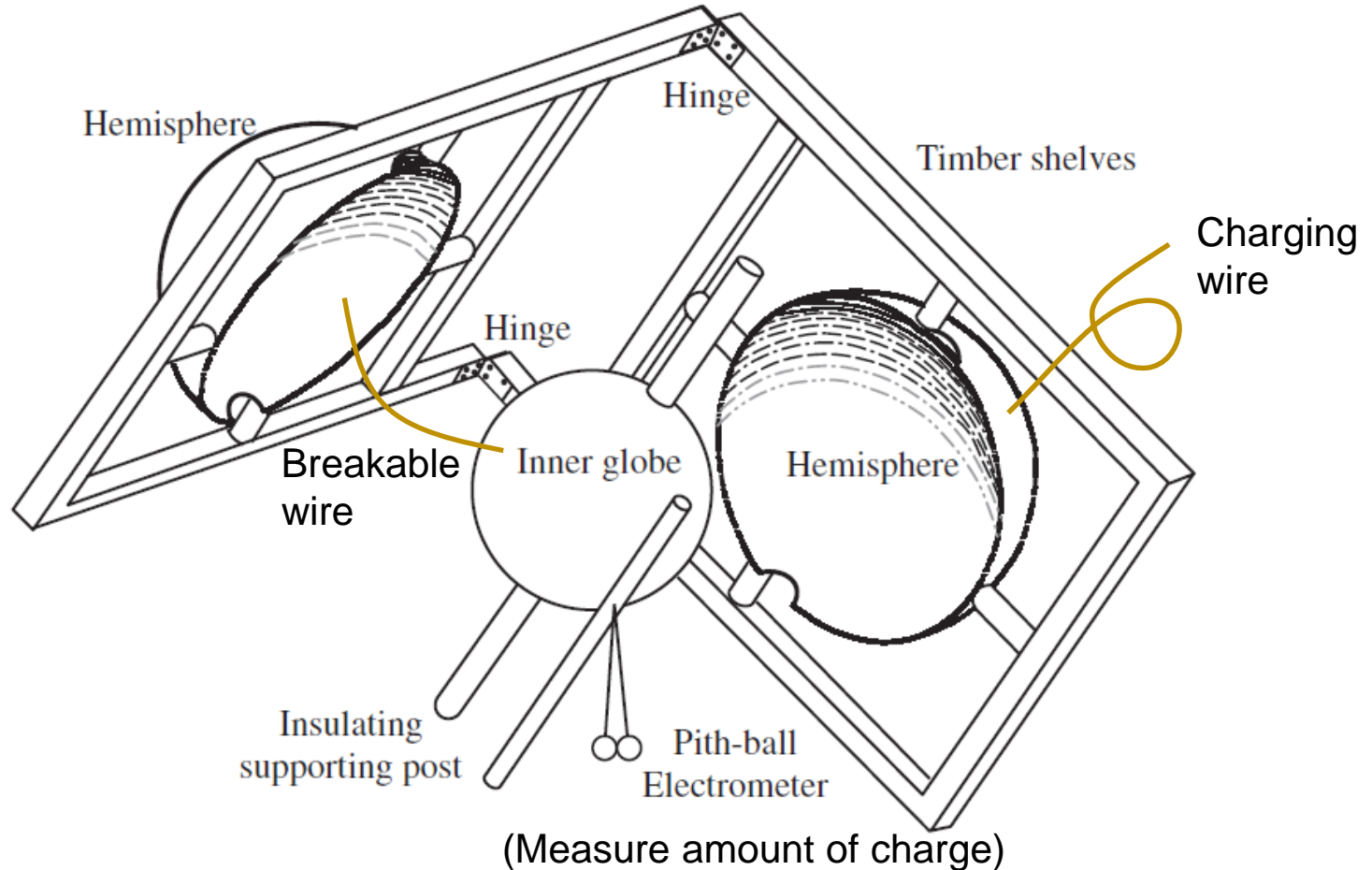   - Location of Q: $\boldsymbol{r}$, location of q_i: $\boldsymbol{r}_i'$

  - Principle of superposition

    - Total force acting on test charge $F = \sum\limits_{i=1}^{n} F_i$

    - Not a necessity, but an experimental fact

  * $\boldsymbol{\imath}$ in textbook is typed as $\boldsymbol{r}$ in our slides (Cursive "r")
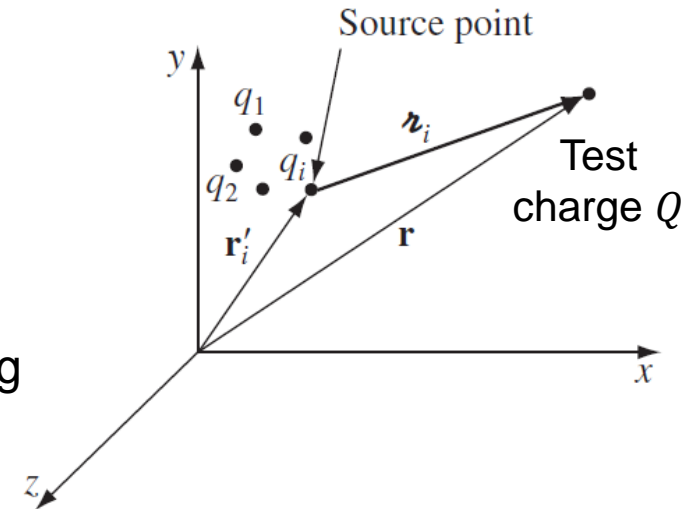
Coulomb's law

Cavendish's apparatus for determining $F \propto r^{-2}$ in Coulomb's law

Electric field induced by charge

- Relation of force and electric field

$$\mathbf{F} = Q\mathbf{E}$$

  o Electric field: force per unit charge

  o Real physical entity, as a vector field filling the space around charges

  o Negated theory of "ether"

- Electric field induced by discrete charges

$$\mathbf{E}(\mathbf{r}) \equiv \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{\imath_i^2} \hat{\boldsymbol{\imath}}_i$$

  o Separation vector $\boldsymbol{\imath}_i = \boldsymbol{r} - \boldsymbol{r}_i'$, contains $\boldsymbol{r}$

  o Principle of superposition also holds

Electric field induced by charge
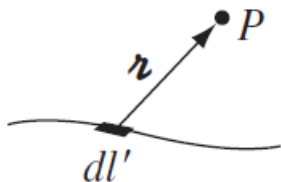
- Electric field induced by continuous charge distribution

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{1}{\imath^2} \hat{\boldsymbol{\imath}} \, dq$$

- ○ Add up contributions from infinitesimal charge elements dq

- ○ Three ways dq can be distributed

Line charge	**Surface charge**	**Volume charge**
$dq \to \lambda \, dl'$	$dq \to \sigma \, da'$	$dq \to \rho \, d\tau'$
$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\mathbf{r}')}{\imath^2} \hat{\boldsymbol{\imath}} \, dl'$	$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\mathbf{r}')}{\imath^2} \hat{\boldsymbol{\imath}} \, da'$	$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{\imath^2} \hat{\boldsymbol{\imath}} \, d\tau'$
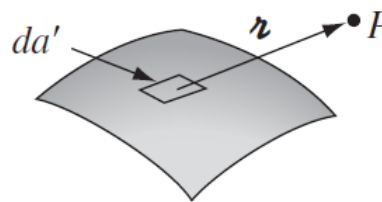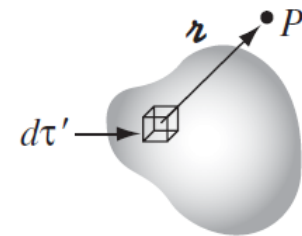
*λ, σ, ρ: charge per unit length, area, volume

Electric field induced by charge

- Electric field induced by continuous charge distribution

Example 2.2. Find the electric field a distance z above the midpoint of a straight line segment of length $2L$ that carries a uniform line charge λ (Fig. 2.6).

C2.exmp2.2

○ Integration sometimes can get formidable, need to device new tools to simplify problems.
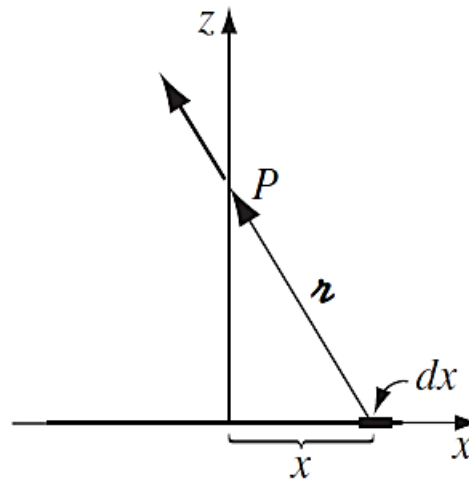
Gauss's law

- Electric field lines

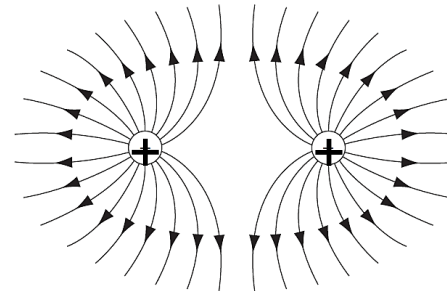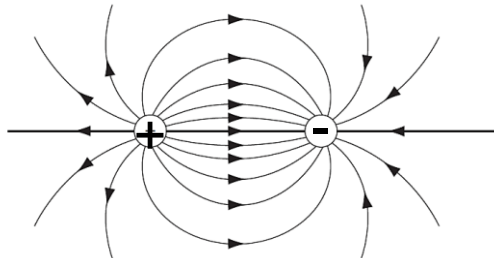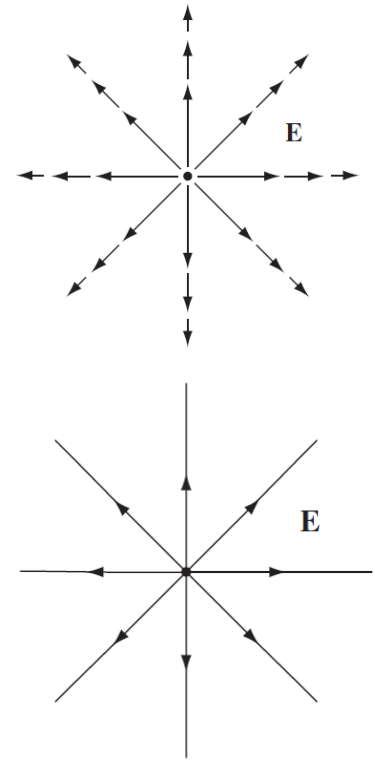 - Source charge q at the origin

    $$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

  - Draw vector field – field falls off like $1/r^2$

  - Connect up the arrows – electric field lines
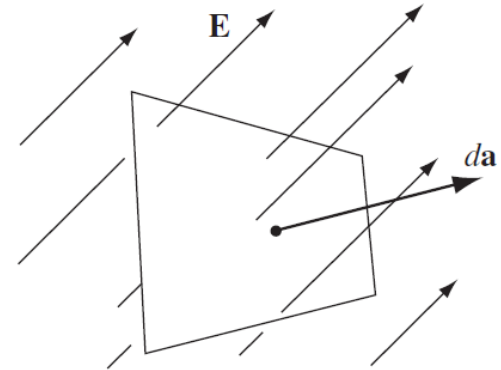
    - Direction of line indicates field direction

    - Density of line indicates field magnitude

  - Field lines begin from positive charges and end on negative ones

Gauss's law

- **Electric field flux** $\quad \Phi_E \equiv \int_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{a}$

  o A measure of the number of field lines passing through an area

- **Gauss's law**

  o The flux through any closed surface is a measure of the total charge inside

$$\oint \mathbf{E} \cdot d\mathbf{a} = \int \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\hat{\mathbf{r}}\right) \cdot (\underbrace{r^2 \sin\theta \, d\theta \, d\phi \, \hat{\mathbf{r}}}_{d\mathbf{a}}) = \frac{1}{\epsilon_0}q$$

  Spherical surface
  of radius r

  o The surface integral can be any shape, not necessarily spherical

  o Multiple charges $\quad \oint \mathbf{E} \cdot d\mathbf{a} = \sum_{i=1}^{n}\left(\oint \mathbf{E}_i \cdot d\mathbf{a}\right) = \sum_{i=1}^{n}\left(\frac{1}{\epsilon_0}q_i\right)$

  $$\Rightarrow \boxed{\oint \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0}Q_{\text{enc}}}$$ (Q_{enc}: total charge enclosed in the integrated surface)

Gauss's law

- Gauss's law

  - Gauss's law in the differential form

$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}} \quad \text{(integral form)}$$

Divergence theorem

Consider volume distribution

$$\oint_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{a} = \int_{\mathcal{V}} (\nabla \cdot \mathbf{E}) \, d\tau$$

$$Q_{\text{enc}} = \int_{\mathcal{V}} \rho \, d\tau$$

$$\Longrightarrow \quad \int_{\mathcal{V}} (\nabla \cdot \mathbf{E}) \, d\tau = \int_{\mathcal{V}} \left(\frac{\rho}{\epsilon_0} \right) d\tau$$
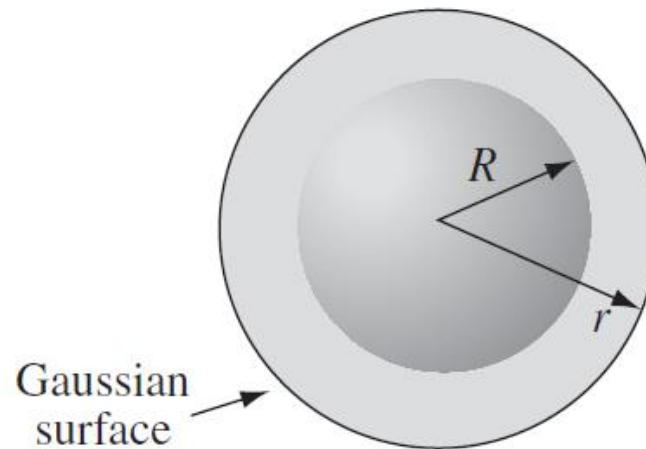
$$\Longrightarrow \quad \boxed{\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho} \quad \text{(differential form)}$$

  - Differential form more compact, but integral form easier to use

  - Use of Gauss's law to calculate electric field

    - Need (1) Gauss's law in integral form and (2) symmetry arguments

Gauss's law

- Application of Gauss's law

Example 2.3. Find the field outside a uniformly charged solid sphere of radius R and total charge q.

Gaussian surface

C2.exmp2.3

○ The field outside the sphere is exactly the same as it would have been if all the charge had been concentrated at the center

Divergence of electric field

- Directly calculate divergence
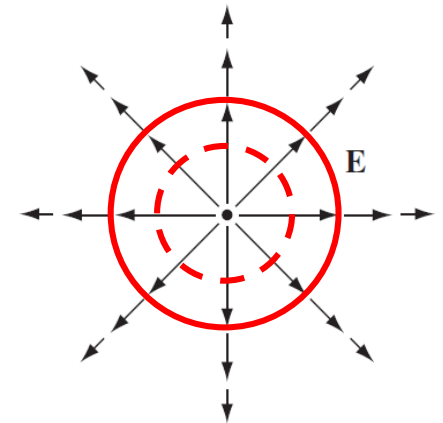
  o According to Coulomb's law

  $$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_{\text{all space}} \frac{\hat{\imath}}{\imath^2} \rho(\mathbf{r}')\, d\tau'$$

  $$\nabla \cdot \mathbf{E} = \frac{1}{4\pi\epsilon_0} \int \boxed{\nabla \cdot \left(\frac{\hat{\imath}}{\imath^2} \right)} \rho(\mathbf{r}')\, d\tau'$$

  $$\nabla \cdot \left(\frac{\hat{\boldsymbol{r}}}{r^2} \right) = \nabla \cdot \left(\frac{\hat{\boldsymbol{r}}}{r^2} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{1}{r^2} \right) = 0$$

  o The derivation above is correct anywhere but the origin ($r = 0$), where the divergence should go to infinity

    ▪ Consider special case of point charge and Gauss's law with varying volume to integrate

> **?** This seems to contradict the Gauss's law, what went wrong
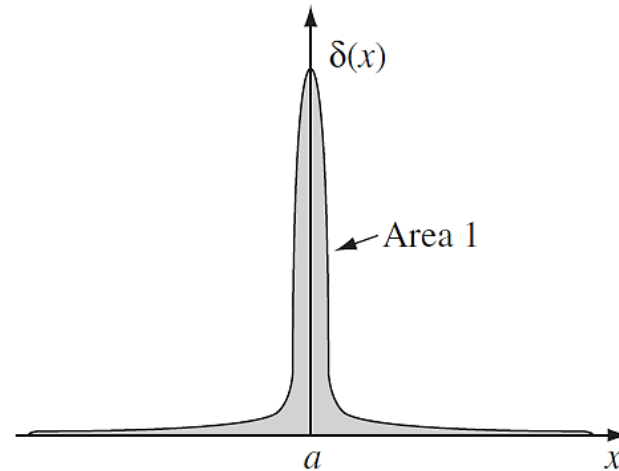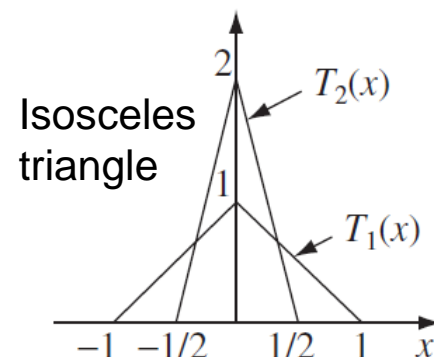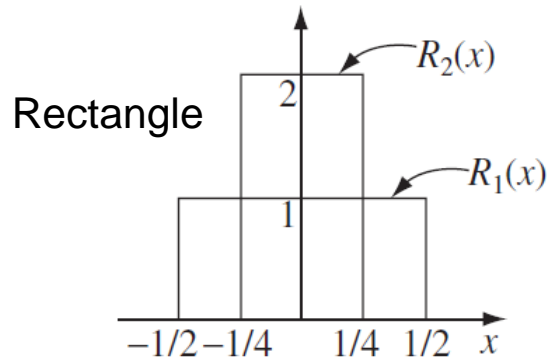
Divergence of electric field

- Delta function

  o Infinitely high, infinitesimally narrow

  o 1D Delta function

  $$\delta(x) = \begin{cases} 0, & \text{if } x \neq 0 \\ \infty, & \text{if } x = 0 \end{cases}$$

  with $\displaystyle\int_{-\infty}^{\infty} \delta(x)\, dx = 1$

  ■ Can be understood as the limit of a sequence of functions

  Rectangle

  Isosceles triangle

Divergence of electric field

- Delta function

  o 1D Delta function

    ▪ When in an integral, "picks out" the value of a function

    Since $\delta(x)$ anywhere 0 but at $x = 0$

    [$f(x)$ being an ordinary function not going to infinity]

    $$f(x)\delta(x) = f(0)\delta(x)$$

    $$\Rightarrow \int_{-\infty}^{\infty} f(x)\delta(x)\,dx = f(0)\int_{-\infty}^{\infty} \delta(x)\,dx = f(0)$$

    And, one can shift $\delta(x)$ to $\delta(x-a)$ to pick out another one

    $$f(x)\delta(x-a) = f(a)\delta(x-a)$$

    $$\Rightarrow \boxed{\int_{-\infty}^{\infty} f(x)\delta(x-a)\,dx = f(a)}$$

    ▪ A frequently used expression $\delta(kx) = \dfrac{1}{|k|}\delta(x)$

Divergence of electric field

- Delta function

  - 3D Delta function $\boxed{\delta^3(\mathbf{r}) = \delta(x)\,\delta(y)\,\delta(z)}$

  with $\displaystyle\int_{\text{all space}} \delta^3(\mathbf{r})\,d\tau = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(x)\,\delta(y)\,\delta(z)\,dx\,dy\,dz = 1$

  - Picks out a function value $\displaystyle\int_{\text{all space}} f(\mathbf{r})\delta^3(\mathbf{r}-\mathbf{a})\,d\tau = f(\mathbf{a})$

- Back to calculating divergence of electric field

$$\nabla\cdot\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int \nabla\cdot\left(\frac{\hat{\boldsymbol{\imath}}}{\imath^2}\right)\rho(\mathbf{r}')\,d\tau'$$

$$\nabla\cdot\left(\frac{\hat{\boldsymbol{\imath}}}{\imath^2}\right) = 4\pi\delta^3(\boldsymbol{\imath})$$

$$\nabla\cdot\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int 4\pi\delta^3(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')\,d\tau' = \frac{1}{\epsilon_0}\rho(\mathbf{r})$$
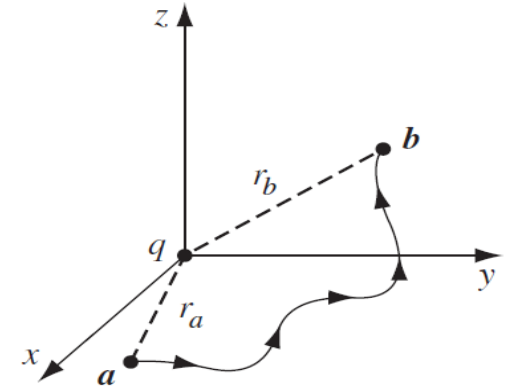
Gauss's law recovered

Curl of electric field

- Calculate curl for point charge at origin

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

$$d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$$

$$\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E}\cdot d\mathbf{l} = \frac{1}{4\pi\epsilon_0}\int_{\mathbf{a}}^{\mathbf{b}}\frac{q}{r^2}dr = \frac{-1}{4\pi\epsilon_0}\frac{q}{r}\Big|_{r_a}^{r_b} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right)$$

- For any closed loop ($r_a = r_b$) $\quad \oint \mathbf{E}\cdot d\mathbf{l} = 0$

$\Longrightarrow$ $\boxed{\nabla \times \mathbf{E} = 0}$ due to Stoke's theorem

Stoke's theorem

$$\int_{\mathcal{S}} (\nabla \times \mathbf{v})\cdot d\mathbf{a} = \oint_{\mathcal{P}} \mathbf{v}\cdot d\mathbf{l}$$

- Any static charge distribution

$$\nabla \times \mathbf{E} = \nabla \times (\mathbf{E}_1 + \mathbf{E}_2 + \ldots) = (\nabla \times \mathbf{E}_1) + (\nabla \times \mathbf{E}_2) + \ldots = 0$$

Electric potential

- Vector field **E** cannot take arbitrary form

  - Crucial constraint: $\nabla \times \mathbf{E} = \mathbf{0}$

$$\frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x} \qquad \frac{\partial E_z}{\partial y} = \frac{\partial E_y}{\partial z} \qquad \frac{\partial E_x}{\partial z} = \frac{\partial E_z}{\partial x}$$

  ? Any chance the vector field can be described more easily?

- Electric potential: $\boxed{V(\mathbf{r}) \equiv -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}}$
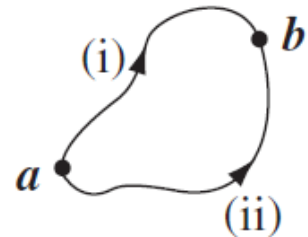
  - Unit: joules per coulomb

  - $\mathcal{O}$: a reference point (usually taken as infinity)

  - Integral does not depend on path

    - $\nabla \times \mathbf{E} = \mathbf{0}$

    - $\oint \mathbf{E} \cdot d\mathbf{l} = 0$

    - $\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l}$ is path independent

    Equivalent statements

Electric potential

- Electric potential: $V(\mathbf{r}) \equiv -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}$

  o Potential difference between two points is more meaningful

  $$V(\mathbf{b}) - V(\mathbf{a}) = -\int_{\mathcal{O}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l} + \int_{\mathcal{O}}^{\mathbf{a}} \mathbf{E} \cdot d\mathbf{l}$$

  $$= -\int_{\mathcal{O}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l} - \int_{\mathbf{a}}^{\mathcal{O}} \mathbf{E} \cdot d\mathbf{l} = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l}$$

  On the other hand, the theorem for gradient gives

  $$V(\mathbf{b}) - V(\mathbf{a}) = \int_{\mathbf{a}}^{\mathbf{b}} (\nabla V) \cdot d\mathbf{l} \implies \boxed{\mathbf{E} = -\nabla V}$$

  o Scalar field V gives full information of vector field $\mathbf{E}$

  o Can be off by a constant if choosing a different reference point

  $$V'(\mathbf{r}) = -\int_{\mathcal{O}'}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l} = -\int_{\mathcal{O}'}^{\mathcal{O}} \mathbf{E} \cdot d\mathbf{l} - \int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l} = K + V(\mathbf{r})$$
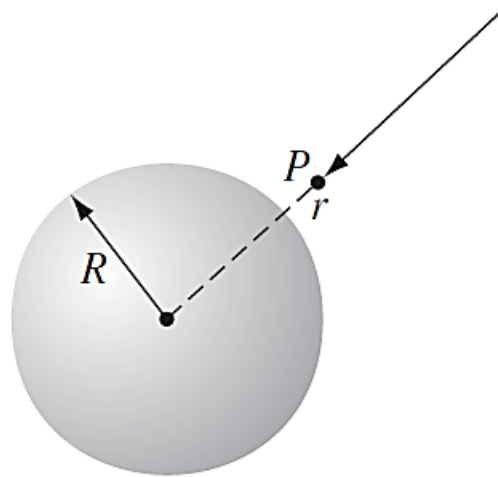
  new

Electric potential

- Application of electric potential

Example. Find the potential of a point charge q at origin

Example 2.7. Find the potential inside and outside a spherical shell of radius R (Fig. 2.31) that carries a uniform surface charge. Set the reference point at infinity.

Electric potential

- Poisson's equation of potential

    o Poisson's equation $\boxed{\nabla^2 V = -\dfrac{\rho}{\epsilon_0}}$

    $$\nabla \cdot \mathbf{E} = \nabla \cdot (-\nabla V) = -\nabla^2 V = \frac{\rho}{\epsilon_0}$$

    o In regions with no charge, Laplace's equation $\boxed{\nabla^2 V = 0}$

    o Curl of a gradient always zero $\quad \nabla \times \mathbf{E} = \nabla \times (-\nabla V) = \mathbf{0}$

- Potential of a localized charge distribution

    o Pick infinity as the reference point $\quad \mathcal{O} = \infty$

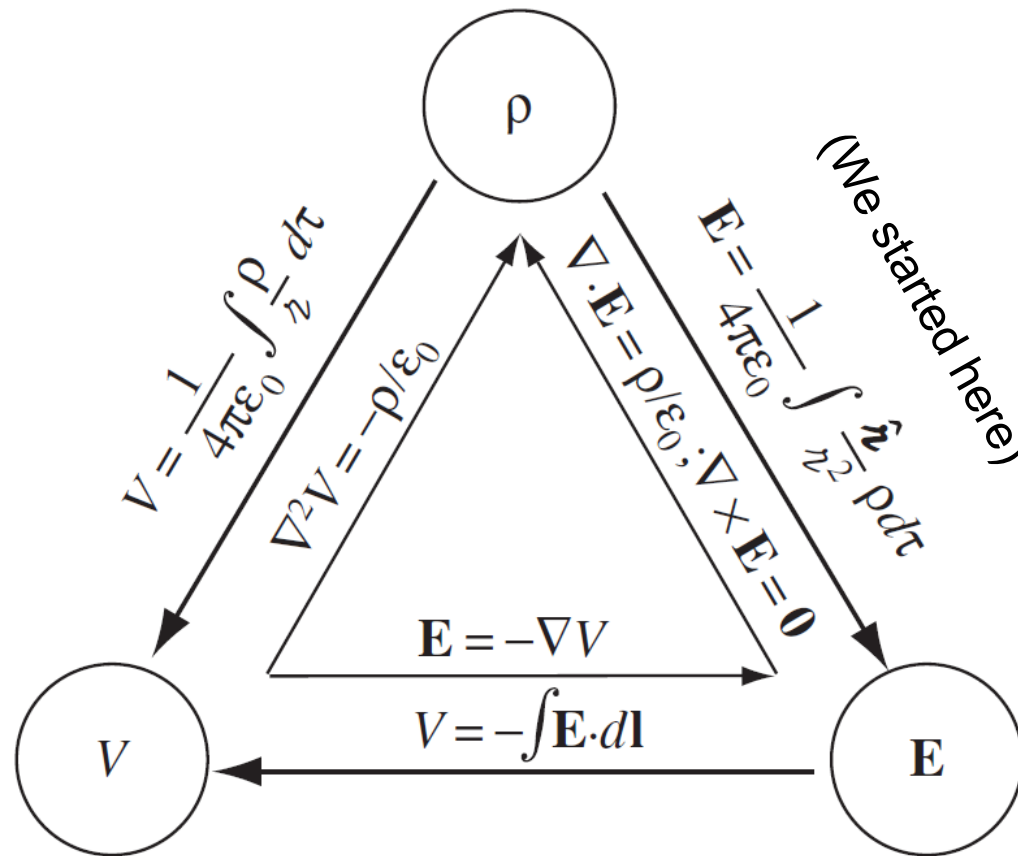   o Principle of superposition holds $\quad V = V_1 + V_2 + \dots$

    o Discrete charges $\quad V(\mathbf{r}) = \dfrac{1}{4\pi\epsilon_0} \displaystyle\sum_{i=1}^{n} \dfrac{q_i}{r_i}$

    $$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{r^2} \hat{\imath}\, d\tau'$$

    Can check

    o Continuous charge $\quad V(\mathbf{r}) = \dfrac{1}{4\pi\epsilon_0} \displaystyle\int \dfrac{\rho(\mathbf{r}')}{r}\, d\tau'$

Charge, electric field, and potential

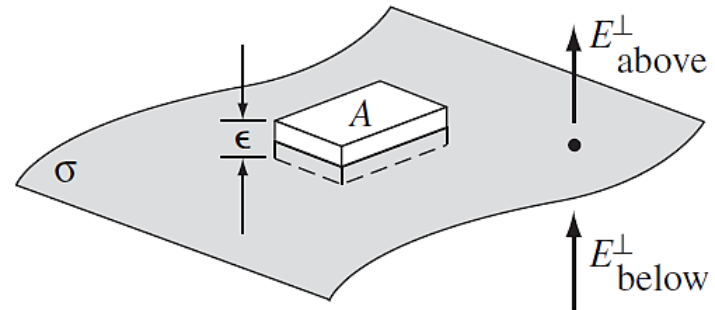Differential equations need boundary conditions to solve

Boundary conditions

- Boundary conditions of **E** across a 2D charged surface

  - Normal component of **E**

    "Gaussian pillbox" with $\varepsilon \to 0$

    $$\oint_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}} = \frac{1}{\epsilon_0} \sigma A$$

    $$\Longrightarrow \boxed{E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{1}{\epsilon_0} \sigma}$$
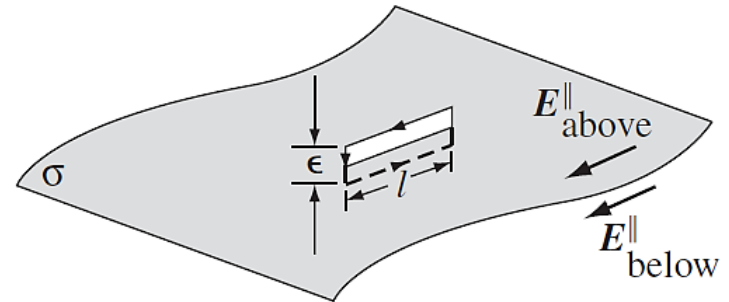
  - Tangential component of **E**

    Thin loop with $\varepsilon \to 0$

    $$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

    $$\Longrightarrow \boxed{\mathbf{E}^{\parallel}_{\text{above}} = \mathbf{E}^{\parallel}_{\text{below}}}$$

  - Summarizing above $\quad \mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \dfrac{\sigma}{\epsilon_0} \hat{\mathbf{n}}$
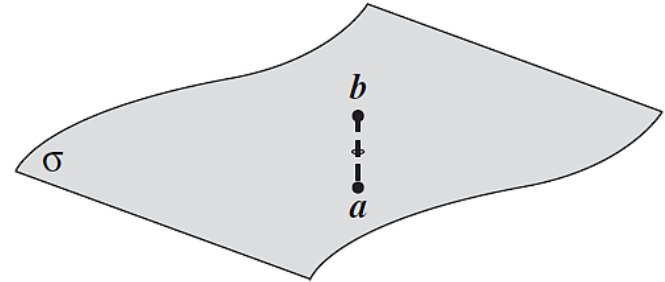
Boundary conditions

- Boundary conditions of V across a 2D charged surface

  o Potential is continuous (across any boundary)

  $$V_{\text{above}} - V_{\text{below}} = -\int_a^b \mathbf{E} \cdot d\mathbf{l}$$

  Path length → 0

  $$V_{\text{above}} = V_{\text{below}}$$

  o Gradient of potential is discontinuous

  $$\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}$$

  $$\mathbf{E} = -\boldsymbol{\nabla} V$$

  $$\boldsymbol{\nabla} V_{\text{above}} - \boldsymbol{\nabla} V_{\text{below}} = -\frac{1}{\epsilon_0}\sigma\,\hat{\mathbf{n}}$$

  $$\frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = -\frac{1}{\epsilon_0}\sigma$$
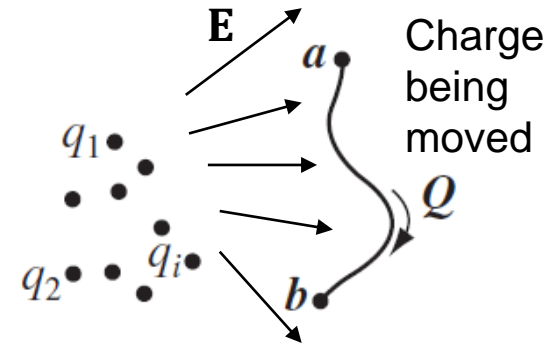
  where we define normal derivative of V

  $$\frac{\partial V}{\partial n} = \boldsymbol{\nabla} V \cdot \hat{\mathbf{n}}$$

Energy in electrostatics

- Work done to move a charge

    o Integrate force over distance

    $$W = \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{F} \cdot d\mathbf{l} = -Q \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l}$$

    $$= Q[V(\mathbf{b}) - V(\mathbf{a})]$$

    E
    a
    Charge being moved
    q_1
    Q
    q_2 q_i
    b

    o Electrostatic force is conservative (path independent)

    o Can confirm the unit of electric potential

    o Work for bringing from infinitely far to r

    $$W = Q[V(\mathbf{r}) - V(\infty)]$$

    $$W = QV(\mathbf{r}) \quad \text{with the potential reference point set to infinity}$$
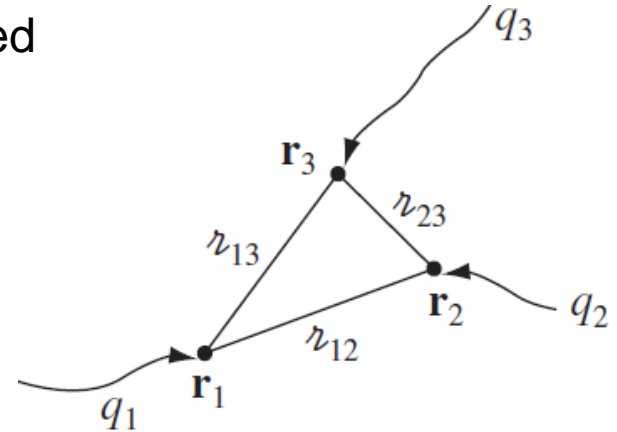
Energy in electrostatics

- Energy of a point charge configuration

  - Equals to the work required to bring charges together from infinity

    - First charge q_1 to $\mathbf{r_1}$, no work required

    - q_2 to $\mathbf{r_2}$ $\quad W_2 = \dfrac{1}{4\pi\epsilon_0} q_2 \left(\dfrac{q_1}{\imath_{12}} \right)$

    - q_3 to $\mathbf{r_3}$ $\quad W_3 = \dfrac{1}{4\pi\epsilon_0} q_3 \left(\dfrac{q_1}{\imath_{13}} + \dfrac{q_2}{\imath_{23}} \right)$

    - $W = W_1 + W_2 + W_3$

  - Total work (energy) for n charges

$$W = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \sum_{j>i}^{n} \frac{q_i q_j}{\imath_{ij}} = \frac{1}{8\pi\epsilon_0} \sum_{i=1}^{n} \sum_{j\neq i}^{n} \frac{q_i q_j}{\imath_{ij}} = \frac{1}{2} \sum_{i=1}^{n} q_i \underbrace{\left(\sum_{j\neq i}^{n} \frac{1}{4\pi\epsilon_0} \frac{q_j}{\imath_{ij}} \right)}_{V(\mathbf{r}_i)}$$

Count once for each pair

$$\boxed{W = \frac{1}{2} \sum_{i=1}^{n} q_i V(\mathbf{r}_i)}$$

Potential q_i feels due to all **other** charges

Energy in electrostatics

- Energy of a continuous charge distribution

  ○ Generalize point charge equation to

$$W = \frac{1}{2} \int \rho V \, d\tau$$

with V: actual potential, without excluding the charge of interest

$$\rho = \epsilon_0 \boldsymbol{\nabla} \cdot \mathbf{E}$$

$$W = \frac{\epsilon_0}{2} \int (\boldsymbol{\nabla} \cdot \mathbf{E}) V \, d\tau$$

Integrate by parts $\int_{\mathcal{V}} f(\boldsymbol{\nabla} \cdot \mathbf{A}) \, d\tau = -\int_{\mathcal{V}} \mathbf{A} \cdot (\boldsymbol{\nabla} f) \, d\tau + \oint_{\mathcal{S}} f\mathbf{A} \cdot d\mathbf{a}$

$$W = \frac{\epsilon_0}{2} \left[-\int \mathbf{E} \cdot (\boldsymbol{\nabla} V) \, d\tau + \oint V\mathbf{E} \cdot d\mathbf{a} \right] = \frac{\epsilon_0}{2} \left(\int_{\mathcal{V}} E^2 \, d\tau + \oint_{\mathcal{S}} V\mathbf{E} \cdot d\mathbf{a} \right)$$

Vanishes
when $\mathcal{V} \to \infty$

$$W = \frac{\epsilon_0}{2} \int E^2 \, d\tau$$

all space

  ○ <u>Cannot</u> be directly compared to equation of point charge, see textbook
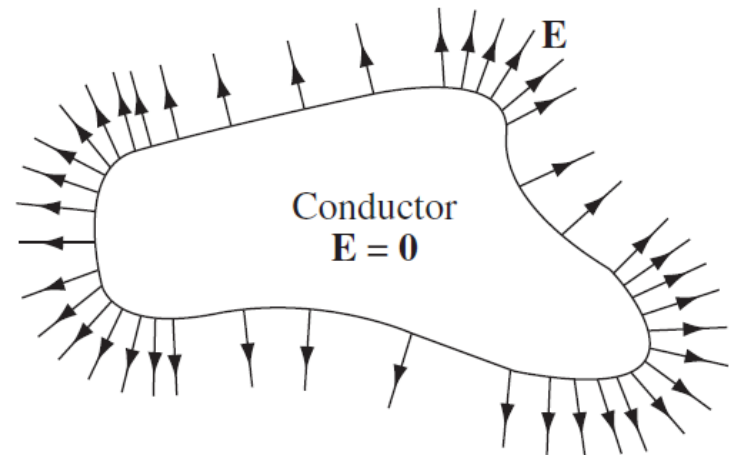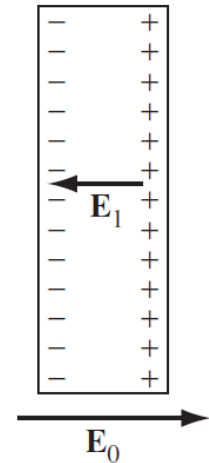
Conductor electrostatics

- Conductors

  - Free electrons – solid-state metals and doped semiconductors

  - Free ions – Electrolyte, salt water, lithium ion battery

  - Unlimited supply of free charges, which are free to move

- Electrostatics of perfect conductors

  - $\mathbf{E} = 0$ inside a conductor

    - If not, charge will flow to induce a new surface charge distribution that exactly cancels the internal field

  - $\rho = 0$ (net charge volume density) inside a conductor

    - Because $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$

  - A conductor is an equipotential

    - For any two points, $V(\mathbf{b}) - V(\mathbf{a}) = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l} = 0$

Conductor electrostatics

- Electrostatics of perfect conductors

  o Any net charge only resides on the surface (minimizes energy)

  o Surface net charges serves to cancel the internal field

  o $\mathbf{E}$ is always perpendicular to the surface, just outside the conductor

    ▪ Recall boundary conditions

    $$E_{\text{above}}^{\perp} - E_{\text{below}}^{\perp} = \frac{1}{\epsilon_0}\sigma$$

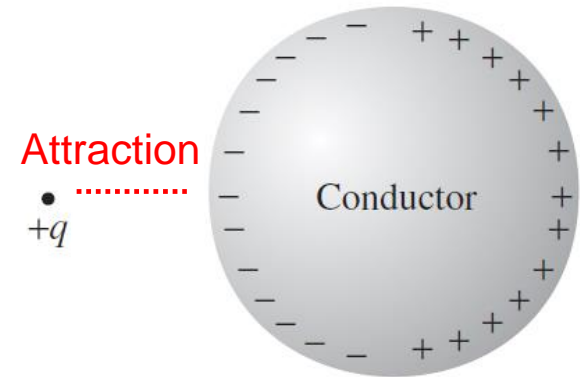 $$\mathbf{E}_{\text{above}}^{\parallel} = \mathbf{E}_{\text{below}}^{\parallel} = 0$$

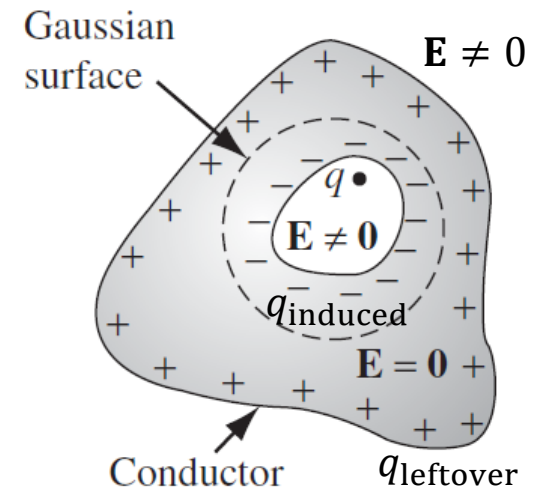# Conductor electrostatics

- Induced charges

  o Charge placed outside a metal

    - Induced charge serves to cancel field inside conductor

    - Net force of attraction

  o Charge in the cavity of a hollow metal

    - Inside the cavity: $\mathbf{E} \neq 0$

    - Induced charge $q_{\text{induced}} = -q$ at inner wall

    - Inside the conductor: $\mathbf{E} = 0$

    - Leftover charge $q_{\text{induced}} = q$ at outer wall

    - Outside the conductor: $\mathbf{E} \neq 0$

Conductor electrostatics

- Induced charges

  o Faraday cage

    ▪ If no charge is placed in the cavity of a hollow conductor, $\mathbf{E} = 0$ in the cavity regardless of the outside conditions
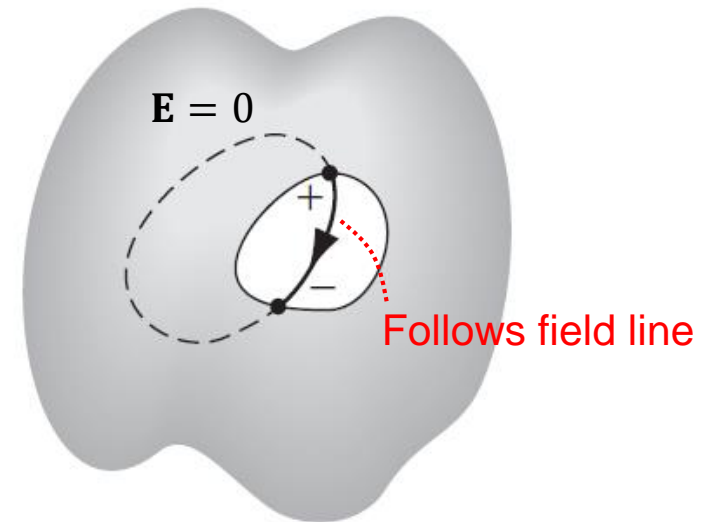
    If not, can construct a loop of integration, whose trajectory in the cavity follows the field line

    ➡ $\oint \mathbf{E} \cdot d\mathbf{l} \neq 0$

    ➡ Contradicts $\boldsymbol{\nabla} \times \mathbf{E} = 0$

    ➡ $\mathbf{E} = 0$

    $\mathbf{E} = 0$

    Follows field line

    ▪ Protects sensitive apparatus inside the cavity by shielding out external electric fields

Conductor electrostatics

- Induced charges

Example 2.10. An uncharged spherical conductor centered at the origin has a cavity of some weird shape carved out of it (Fig. 2.46). Somewhere within the cavity is a charge q. *Question:* What is the field outside the sphere?
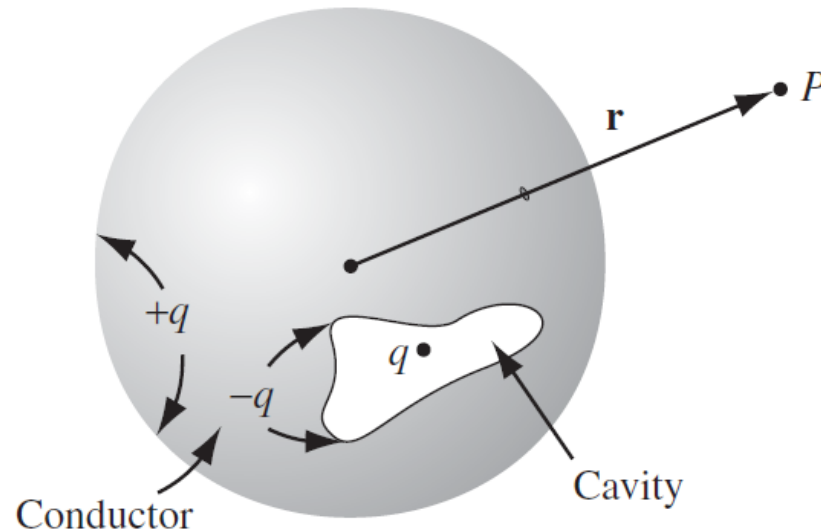
Conductor electrostatics

- Surface charge and force on a conductor

  - Boundary conditions

$$\left\{ \begin{array}{l} \mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \dfrac{\sigma}{\epsilon_0}\hat{\mathbf{n}} \\[2mm] \dfrac{\partial V_{\text{above}}}{\partial n} - \dfrac{\partial V_{\text{below}}}{\partial n} = -\dfrac{1}{\epsilon_0}\sigma \end{array} \right.$$

  On the surface of a perfect conductor $\longrightarrow$

$$\left\{ \begin{array}{l} \mathbf{E} = \dfrac{\sigma}{\epsilon_0}\hat{\mathbf{n}} \\[2mm] \sigma = -\epsilon_0\dfrac{\partial V}{\partial n} \end{array} \right.$$

  - Force (per unit area) exerted on the conductor

    - Can prove (textbook p.104) : for any surface across which is discontinuous, force needs to be calculated by

$$\mathbf{f} = \sigma \mathbf{E}_{\text{average}} = \frac{1}{2}\sigma(\mathbf{E}_{\text{above}} + \mathbf{E}_{\text{below}})$$

    - For conductors $\mathbf{f} = \dfrac{1}{2\epsilon_0}\sigma^2\hat{\mathbf{n}}$

Conductor electrostatics

- Capacitors

  o We can define a potential difference between two conductors, without specifying locations of the integral

  $$V = V_+ - V_- = - \int_{(-)}^{(+)} \mathbf{E} \cdot d\mathbf{l}$$

  +Q   any path   -Q

  o Although $\mathbf{E}$ is geometry dependent, we know $\mathbf{E} \propto Q$, and $V \propto Q$

  o Can define ratio as capacitance $C \equiv \dfrac{Q}{V}$

    - A purely geometrical quantity, determined by shapes, sizes, and separation of the two conductors

    - Unit: farads (F), or Coulomb per volt

    - Always positive