Homework #1

(Due on Canvas by Sat, Sep. 7)

1. Gradient, divergence, and curl

Suppose $\vec{\boldsymbol{a}}$ and $\vec{\boldsymbol{b}}$ are constant vectors and $\vec{\boldsymbol{r}} = r\hat{\boldsymbol{r}}$ is a vector field, calculate the following expressions and simplify them to the fullest.

(1) $\boldsymbol{\nabla} \times (\vec{\boldsymbol{r}}/r)$

(2) $\boldsymbol{\nabla} \cdot (\vec{\boldsymbol{r}}/r)$

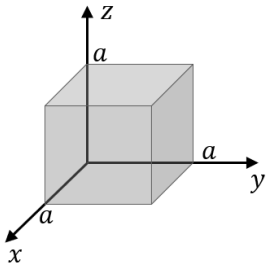
(3) $\boldsymbol{\nabla}(\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{r}})$

(4) $\boldsymbol{\nabla} \cdot [(\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{r}})\vec{\boldsymbol{b}}]$

(5) $\boldsymbol{\nabla} \times [(\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{r}})\vec{\boldsymbol{r}}]$

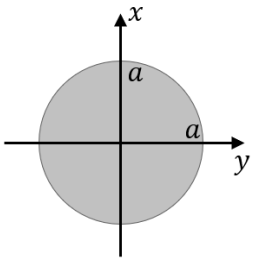
2. The divergence theorem

For vector field $\vec{v} = x^2\hat{\boldsymbol{x}} + y^2\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}$, verify the divergence theorem in the cubic region bounded by $0 \le x \le a$, $0 \le y \le a$, $0 \le z \le a$.



3. The Stokes' theorem

For vector field $\vec{v} = s^2\hat{\boldsymbol{\phi}} + \phi\hat{\boldsymbol{z}}$ expressed in the cylindrical coordinates, verify the Stokes' theorem in the circular region (with radius a) centered at the origin within the $z = 0$ plane.



4. Line charge distribution

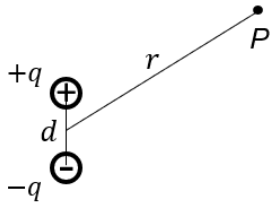
Calculate the electric field vector at distance z above the center of a circular loop of radius a that carries a uniform line charge λ.

5. Surface charge distribution

Consider a spherical surface of radius a carrying a uniform surface charge density σ, use (1) the Gauss's law and (2) the Coulomb's law to calculate the electric field vector at distance z (with $z > a$) from the center of the sphere. Compare the results.

6. Point charge distribution

An electric dipole consists of one positive charge $+q$ and one negative charge $-q$ separated by a distance d. Calculate the electric field vector at a distance r from the center of the dipole. Approximations can be adopted assuming $r \gg d$. Perform your work in the spherical coordinates.



7. Volume charge distribution

Two spheres, each of radius a and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap. Call the vector from the positive center to the negative center $\vec{\boldsymbol{d}}$. Show that the electric field vector in the region of overlap is constant, and find its expression.