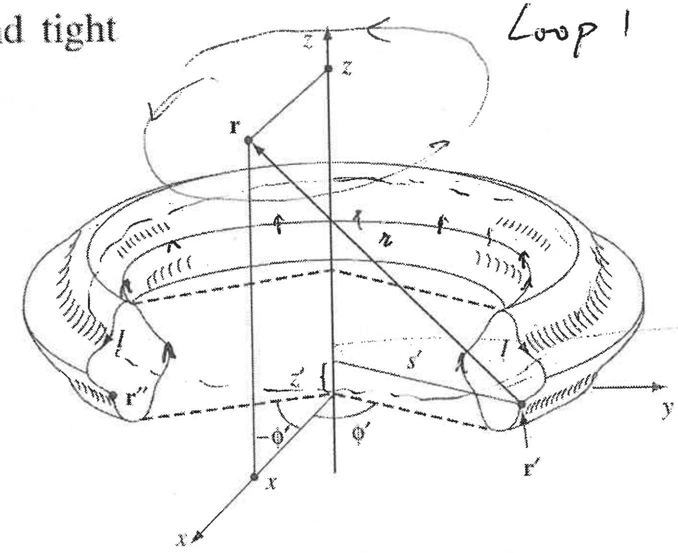
Example 5.10. A toroidal coil consists of a circular ring, or "donut," around which a long wire is wrapped (Fig. 5.38). The winding is uniform and tight enough so that each turn can be considered a plane closed loop.



Field at $\vec{r} = (x, 0, z)$ due to current element at $\vec{r}\,'$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{\vec{I}\times\vec{\mathscr{r}}}{\mathscr{r}^3}\,dl'$$

$$\vec{r}\,' = (s'\cos\phi', s'\sin\phi', z')$$

$$\vec{\mathscr{r}} = (x - s'\cos\phi', -s'\sin\phi', z - z')$$

$$\vec{I} = I_s\hat{s} + I_z\hat{z} = (I_s\cos\phi', I_s\sin\phi', I_z)$$

$$d\vec{B} \propto \vec{I}\times\vec{\mathscr{r}} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ I_s\cos\phi' & I_s\sin\phi' & I_z \\ x - s'\cos\phi' & -s'\sin\phi', & z - z' \end{vmatrix}$$

dB_x & $dB_z \propto \sin\phi'$, for any $\vec{r}\,'$, there exists $\vec{r}\,''$ at $-\phi'$ that cancels out dB_x & dB_z at $\vec{r}\,'$

$\vec{B} \parallel \hat{y} \Rightarrow \vec{B} \parallel \hat{\phi}$ everywhere in space

$$B\cdot 2\pi s = \begin{cases} \mu_0 I\cdot N & (\text{inside}) \\ 0 & (\text{outside}) \end{cases}$$

$$\vec{B} = \begin{cases} \dfrac{\mu_0 I N}{2\pi s}\hat{\phi} & (\text{inside}) \\ 0 & (\text{outside}) \end{cases}$$