Student Name:

SIS ID (starts with letter "e"):

1. For vector field $\boldsymbol{v} = x^2 z\,\hat{\mathbf{x}} + y^2 x\,\hat{\mathbf{y}} + (y + 2z)\,\hat{\mathbf{z}}$, calculate the divergence and curl of $\boldsymbol{v}$.

$$\nabla \cdot \vec{v} = \frac{\partial}{\partial x}(x^2 z) + \frac{\partial}{\partial y}(y^2 x) + \frac{\partial}{\partial z}(y + 2z)$$

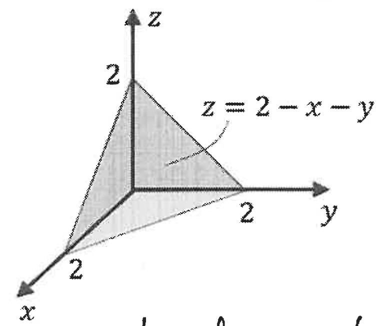
$$= 2xz + 2yx + 2$$

$$\nabla \times \vec{v} = \hat{x}\left[\frac{\partial}{\partial y}(y+2z) - \frac{\partial}{\partial z}(y^2 x)\right] + \hat{y}\left[\frac{\partial}{\partial z}(x^2 z) - \frac{\partial}{\partial x}(y+2z)\right]$$

$$+ \hat{z}\left[\frac{\partial}{\partial x}(y^2 x) - \frac{\partial}{\partial y}(x^2 z)\right] = \hat{x} + x^2\hat{y} + y^2\hat{z}$$

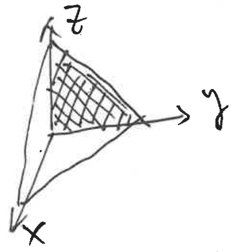
2. Consider the scalar field $T = xyz$, calculate the volume integral of T within the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $z = 2 - x - y$.

$$\int_a^b \int_{p(z)}^{q(z)} \int_{r(y,z)}^{s(y,z)} T(x, y, z)\, dx\, dy\, dz$$



- For $\int_a^b dz$, $\underline{a = 0,\ b = 2}$ by examining the boundaries of the tetrahedron

- For $\int_{p(z)}^{q(z)} dy$, need to consider bounds of y that are z-dependent

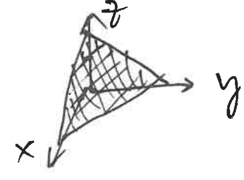


Consider the triangle in the z-y plane,

within it, $y \in [0, 2-z]$, so $\underline{p(z) = 0,\ q(z) = 2-z}$

- For $\int_{r(y,z)}^{s(y,z)} dx$ Consider bounds of x that are y- & z-dependent

Consider the triangle oriented along (1,1,1)



within it, $x \in [0, 2-z-y]$, so $\underline{r(y,z) = 0}$, $\underline{s(y,z) = 2-z-y}$