Homework #2

(Due on Canvas by Wed, Sep. 25)

1. Parallel-plate capacitor

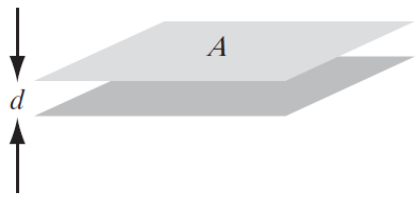
(1) Two infinite parallel conducting planes carry equal but opposite uniform surface charge densities $\pm\sigma$. Apply the Gauss's law to find the electric fields in all charge-free regions in space.

(2) A parallel-plate capacitor is comprised of two conducting planes of area A held at a small gap distance d apart.

(i) Write a few statements to argue why the finding in (1) about the electric field inside the gap would be approximately valid when d is small.

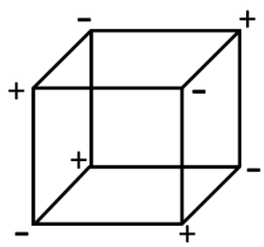
(ii) Assuming the two plates are carrying opposite charges $\pm Q$, calculate the difference of electric potential between them.

(iii) Calculate the capacitance.



2. Energy of a point charge distribution

Suppose four $+q$ and four $-q$ charges take the configuration shown in the graph below, where they are located at the eight vertices of a cube with edge length a. Calculate the electrostatic energy stored in this charge configuration.



3. Energy of a continuous charge distribution

Consider a solid sphere with radius R carrying a non-uniform volume charge density $\rho = kr$ (for $r \leq R$), where k is a constant and r is the radial coordinate.

(1) Calculate the electric potential $V(\vec{\boldsymbol{r}})$ inside and outside the sphere.

(2) Calculate the electrostatic energy stored in the charged sphere.

4. Conductor electrostatics

Consider the textbook example 3.2 that we have worked out in class (Note: C2.exmp3.2.pdf). Assume we have full knowledge over the image charge magnitude (q') and its position (b), calculate the following:

(1) Induced surface charge density distribution on the conductor.

(2) Total induced charge, by integrating the surface charge density over the conductor surface.

(3) Work required to bring charge q from infinity to this configuration.

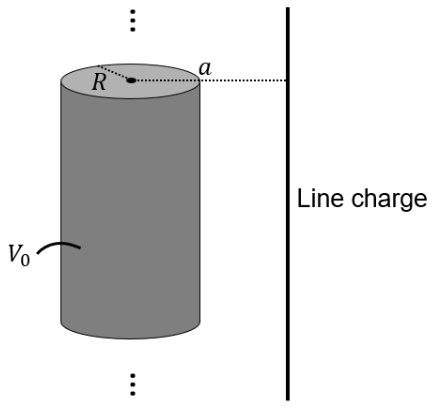
5. The method of images

A straight wire with line charge density λ is placed parallel to, and at a distance (a) away from the axis of an infinitely-long solid conducting cylinder (of radius R) held at a fixed voltage V_0 (reference point for potential is at infinity).

(1) Apply the method of images. What would be the position of the image charge? (Hint: the image charge is another line charge with density $-\lambda$).

(2) Calculate the electric potential at any point outside the cylinder. Express your results in cylindrical coordinates where the z axis is aligned to the axis of the cylinder.

(3) The induced surface charge density distribution on the cylinder.



6. Separation of variables

A rectangular pipe, running parallel to the z-axis (from $-\infty$ to $+\infty$), has three grounded metal sides, at $y = 0$, $y = a$, and $x = 0$. The fourth side, at $x = b$ is maintained at a specified potential $V_0(y)$.

(1) Develop a general formula for the potential inside the pipe.

(2) Find the potential explicitly, for the case $V_0(y) = V_0$ (a constant).