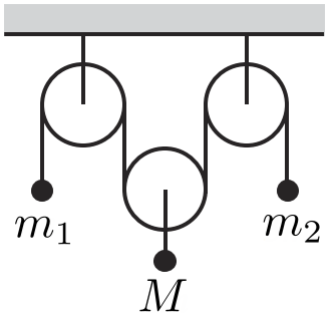
PC3261: Classical Mechanics II
Assignment 4

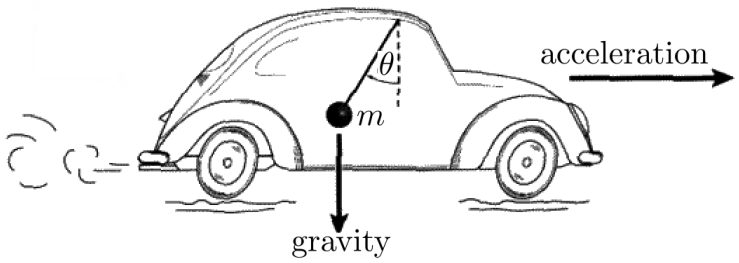
1. [20 pts] Consider a double Atwood machine as shown below. The center pulley is free to move vertically and it has a mass M. The string connecting the three masses is massless and inextensible. Masses m_1 and m_2 hang on the left and right respectively from the fixed pulleys. The acceleration of gravity is g. All three pulleys are frictionless so that the string slides freely over them.



(a) Obtain the conditions for static equilibrium from the principle of virtual work.

(b) Solve for the accelerations of all three masses from the d'Alembert's principle.

2. [20 pts] A pendulum with a massless string of length ℓ and mass m is attached to a moving car. The car is accelerated uniformly at a along a horizontal track starting with an initial horizontal speed v_0. The angle between the mass m and the vertical is denoted as θ.



(a) Obtain the equation of motion for $\theta(t)$ from the d'Alembert's principle.

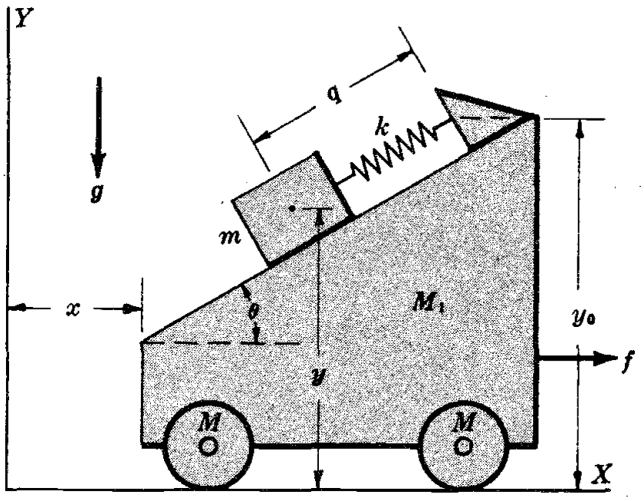
(b) Determine the angle, θ_{eq}, when the mass remains at rest in stable equilibrium. Give an expression for the tangent of this angle, $\tan\theta_{\text{eq}}$.

(c) Set $\theta(t) = \theta_{\text{eq}} + \epsilon(t)$, that is, measure the motion with respect to the equilibrium position. Obtain the equation of motion for $\epsilon(t)$ for small oscillations around θ_{eq} and obtain the angular frequency of these small oscillation.

3. [20 pts] A uniform solid ball has a few turns of light string wound around it. The end of the string is held steady and the ball is allowed to fall under gravity. Using d'Alembert with Lagrange multiplier, find the acceleration of the ball and the tension in the string.

4. [20 pts] A particle slides on a smooth inclined plane whose inclination θ in increasing at a constant rate Ω. If $\theta(0) = 0$ at which the particle starts from rest from a distance $x(0) = x_0$ from the bottom of the inclined. Obtain the equation of equation of motion from the Lagrange's equation and solve for $x(t)$.

5. [20 pts] A block of mass m is free to slide along the inclined plane on the cart under the action of gravity and the spring. The body of the cart has mass M_1. Each wheel hass mass mass M, radius r and moment of inertia I about its axle. A constant force f is exerted on the cart. Denote q_0 as the value of q when the spring is unstretched.



(a) Using x and y as generalized coordinates, obtain the equations of motion from the d'Alembert principle.

(b) Using x and q as generalized coordinates, obtain the equations of motion from the Lagrange's equation.

Last updated: October 3, 2023 10:34 pm