PC3261: Classical Mechanics
Assignment 5

1. [20 pts] A spring pendulum consists of a point mass m attached to one end of a massless spring with spring constant k. The other end of the spring is tied to a fixed pivot point. When no weight is loaded on the spring, its length is ℓ_0. Assume that the motion of the system is confined to a vertical plane. With suitable generalized coordinate(s), derive the equation(s) of motion. Solve the equation(s) of motion in the approximation of small angular and radial displacements from equilibrium.

2. [20 pts] Consider a one-dimensional mechanical system described by the following Lagrangian function:

$$\mathcal{L}(q(t), \dot{q}(t), t) = \mathrm{e}^{\gamma t}\left[\frac{1}{2}\, m\dot{q}^2(t) - \frac{1}{2}\, kq^2(t)\right],$$

where γ, m, k are positive constants.

(a) Obtain the Euler-Lagrange equation of motion. Is there any constant of motion? How would you describe the motion?

(b) Now, perform a point transformation to another generalized coordinate given by

$$Q = Q(q(t), t) = \exp\left(\frac{\gamma t}{2}\right) q(t)\,.$$

Obtain the Lagrangian function and Euler-Lagrange equation of motion in terms of the new generalized coordinate. Is there any constant of motion? How would you describe the relationship between the two solutions given by these two Lagrange functions?

3. [30 pts] A particle of mass m is constrained to slide without friction on a rotating hoop of radius a whose axis of rotation is through a vertical diameter. The hoop is rotating at a constant angular speed ω.

(a) With suitable generalized coordinate(s), write down the Lagrangian function of the system. Identify any constant of motion that may exist.

(b) Locate the positions of equilibrium positions of the particle for $\omega < \omega_c$ and $\omega > \omega_c$ where $\omega_c \equiv \sqrt{g/a}$. Which of these equilibrium positions are stable and unstable?

(c) For stable equilibrium, perform a small perturbation about the equilibrium position θ_0: $\theta(t) = \theta_0 + \epsilon(t)$ where $|\epsilon(t)| \ll \theta_0$, determine the oscillation frequency of small amplitude oscillation about the equilibrium point.

4. [30 pts] A particle of mass m is acted on by a force whose potential energy is $U(r)$.

(a) Obtain the Lagrangian function in a spherical coordinate system which is rotating with *constant* angular speed Ω about the z-axis.

(b) Show that your Lagrangian function has the same form as in a fixed coordinate system with the addition of a velocity-dependent potential energy function U' which gives the centrifugal and Coriolis forces.

(c) Calculate from the velocity-dependent potential U' the components of the centrifugal and Coriolis forces in the radial and azimuthal directions.

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