NATIONAL UNIVERSITY OF SINGAPORE

PC4245 Particle Physics

(Semester II: AY 2022-23)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1. Write your Matric Number on the front cover page of each answer book.
2. This examination paper contains **4** questions and comprises **5** printed pages. Answer any **3** questions.
3. All questions carry equal marks.
4. Students should write the answers for each question on a new page.
5. This is a CLOSED BOOK examination.

1. (a)(i) Briefly outline the C S Wu cobalt 60 experiment that demonstrated parity is broken in the weak decay. Explain why human heart on the left cannot be used to demonstrate parity is not conserved.

(ii) Suppose you wanted to inform someone on a distant planet that humans have their hearts on the left side. How could you communicate this unambiguously without sending an actual "handed" object?

(b) If the Hamiltonian of an elementary particle is invariant under space inversion, and if the state is nondegenerate, show that there can be no electric dipole moment in that state.

(c) Consider the weak decay of a charged pion into a muon and neutrino

$$\pi^+ \to \mu^+ + \nu_\mu$$

Sketch the mirror reflection of the decay process and hence show that the charged pion decay violates the parity conservation.

Sketch the decay process under a combination of space inversion and charge conjugation and hence show that the decay process conserves the CP.

2. (i) The charge conjugation operator C takes a Dirac spinor ψ into the 'charge conjugate' spinor $\psi_c = C\,\overline{\psi}^T$, where $\overline{\psi}^T$ is the transpose of the Dirac adjoint of ψ and $C = i\,\gamma^2\gamma^0$.

Show that $\psi_c = i\,\gamma^2\,\psi^*$, where γ^μ is the Dirac gamma matrix.

The Dirac equation for a fermion interacting minimally with an electromagnetic field $A_\mu(x)$ is

$$\gamma^\mu\,(i\hbar\partial_\mu - qA_\mu)\psi = mc\,\psi.$$

Show explicitly that ψ_c satisfies

$$\gamma^\mu \left(i\hbar\partial_\mu + qA_\mu\right)\psi_c = mc\,\psi_c.$$

Note that the transpose of γ^μ is γ^{μ^T}, and $\gamma^{\mu^T} = -\,C^{-1}\gamma^\mu\, C$

(ii) The free electron solution of the Dirac equation is given by

$$\psi(\underline{x}) = e^{-i\underline{p}\cdot\underline{x}/\hbar}\, u^{(s)}(\underline{p})\,,$$

$$u^{(s)}\left(\underline{p}\right) = \sqrt{p^0 + mc}\;\begin{pmatrix} w^s \\ \dfrac{\underset{\sim}{\sigma}\cdot\underset{\sim}{p}}{p^0 + mc}\, w^s \end{pmatrix}$$

where w^s is a spinor, and $s = 1, 2$.

Find the charge conjugates of $u^{(s)}$, $s = 1, 2$ and compare them with

the positron bispinor $v^{(s)}$.

Here $v^{(s)}$ refers to the free positron solution

$$\psi(\underline{x}) = e^{+i\underline{p}\cdot\underline{x}/\hbar}\, v^{(s)}(\underline{p})$$

$$v^{(s)}\left(\underline{p}\right) = \sqrt{p^0 + mc}\;\begin{pmatrix} \dfrac{\underset{\sim}{\sigma}\cdot\underset{\sim}{p}}{p^0 + mc}\, w^s \\ w^s \end{pmatrix}$$

Note: $\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$, $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$,

$\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

2. **(a)** The Fermi golden rule for a particle (mass m_1) decay into n particles can be written as

$$d\Gamma = \frac{S}{2\hbar m_1}|\mathcal{M}|^2 \frac{d^3\underset{\sim}{p_2}}{(2\pi)^3 2p_2^0}\,\frac{d^3\underset{\sim}{p_3}}{(2\pi)^3 2p_3^0}\cdots\frac{d^3\underset{\sim}{p_n}}{(2\pi)^3 2p_n^0}\,.$$

$\cdot\, (2\pi)^4 \delta^{(4)}(\underline{p}_1 - \underline{p}_2 - \underline{p}_3 - \cdots - \underline{p}_n)$

Consider the decay of a pion into two photons, $\pi^o \to \gamma + \gamma$. If the amplitude for the process is $\mathcal{M}(\underset{\sim}{p_2}, \underset{\sim}{p_3})$, show that the decay rate can be written as

$$\Gamma = \frac{1}{32\pi\hbar m_1}|\mathcal{M}|^2$$

Here $\underset{\sim}{p_2}$ and $\underset{\sim}{p_3}$ are respectively the 3- momenta of the two photons. What are the values of $\left|\underset{\sim}{p_2}\right|$ and $\left|\underset{\sim}{p_3}\right|$?

(b) Find the lowest order amplitude $\mathcal{M}$ for the electron-muon scattering. Calculate the spin-averaged quantity $\langle|\mathcal{M}|^2\rangle$.

The following formulas can be used without proof:

(1) $Tr[\gamma^\mu(\not{p}_1 + m_1 c)\gamma^v(\not{p}_3 + m_3 c)]$
$= 4[p_1^\mu p_3^v + p_3^\mu p_1^v + g^{\mu v}(m_1 m_3 c^2 - (\underline{p}_1 \cdot \underline{p}_3))]$

(2) $\sum_s u^{(s)}(\underline{p})\ \bar{u}^{(s)}(\underline{p}) = \not{p} + mc$

Note: For vertex: $ig\gamma^\mu$; for propagators: $\frac{-ig^{\mu v}}{\underline{q}^2}$, $\frac{i}{q_\mu\gamma^\mu - mc}$

4(i) Draw a one-loop Feynman diagram (vacuum polarization) for the electron-muon scattering $e^- + \mu^- \to e^- + \mu^-$.

Derive the scattering amplitude $\mathcal{M}$ for the one-loop diagram of the above process, using the Feynman rules for quantum electrodynamics.

Note: Vertex: $ig\gamma^\mu$; propagators: $\frac{-ig^{\mu v}}{\underline{q}^2}$, $\frac{i}{q_\mu\gamma^\mu - mc}$

([illegible]) Using renormalization procedure, show that the scattering amplitude for the above process up to and including the one-loop diagram for vacuum polarization is given by

$$\mathcal{M} = -\frac{g_R^2(t)}{(\underline{p}_1-\underline{p}_3)^2}[\bar{u}^{(s_3)}(\underline{p}_3)\gamma^\mu u^{(s_1)}(\underline{p}_1)][\bar{u}^{(s_4)}(\underline{p}_4)\gamma_\mu u^{(s_2)}(\underline{p}_2)]\,.$$

Here $g_R(t)$ is the renormalized coupling constant, $t = (\underline{p}_1 - \underline{p}_3)^2$ is the momentum transfer square.

Explain the difference between regularization and renormalization.

The following can be assumed without proof,

$$I^{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} Tr[\gamma^\mu \frac{1}{\not{p}_1-\not{p}_3-\not{k}-mc}\gamma^\nu \frac{1}{\not{k}-mc}]$$

$$= \frac{ig^{\mu\nu}\, t}{12\pi^2}\left(ln(\frac{M^2}{m^2}) - f(\frac{-t}{m^2c^2})\right)$$

where M is the cut-off and $f(\frac{-t}{m^2c^2})$ a finite function in the variable t.

(OCH)

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