PC4245 PARTICLE PHYSICS
HONOURS YEAR
Tutorial 4

1. Construct the normalized spinors $u^{(+)}$ and $u^{(-)}$ representing an electron of momentum $\underset{\sim}{p}$ with helicity ± 1 . That is find the *u's* that satisfy the Dirac equation with positive energy p°, and are eigenspinors of the helicity operator $(\underset{\sim}{\Sigma} \cdot \underset{\sim}{p} \,/\, |\underset{\sim}{p}|)$ with eigenvalues ± 1.

   Solutions: $$u^{(\pm)} = \sqrt{p^\circ + mc}\begin{pmatrix} W^{(\pm)} \\ \frac{\pm|\underset{\sim}{p}|}{p^\circ + mc} W^{(\pm)} \end{pmatrix},$$

   $$W^{(\pm)} = \frac{1}{\sqrt{2|\underset{\sim}{p}|(|\underset{\sim}{p}| \pm p^3)}}\begin{pmatrix} p^3 \pm |\underset{\sim}{p}| \\ p^1 + ip^2 \end{pmatrix}$$

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 7.7, page 268].

2. [The purpose of this problem is to demonstrate that particles described by the Dirac equation carry "intrinsic" angular momentum $(\underset{\sim}{S})$ in addition to their orbital angular momentum $(\underset{\sim}{L})$, neither of which is separately conserved, although their sum is. It should be attempted only if you are reasonably familiar with quantum mechanics.]

   a) Construct the Hamiltonian, H, for the Dirac equation. [Hint: Solve equation (7.19) for $p^\circ c$ Solution: $H = c\gamma^\circ(\underset{\sim}{\gamma} \cdot \underset{\sim}{p} + mc)$, where $\underset{\sim}{p} \equiv (\hbar/i)\nabla$ is the momentum operator.]

   b) Find the commutator of H with the orbital angular momentum $\underset{\sim}{L} \equiv \underset{\sim}{x} \wedge \underset{\sim}{p}$.
   [Solution:$[H, \underset{\sim}{L}] = -i\hbar c\gamma^\circ(\underset{\sim}{\gamma} \wedge \underset{\sim}{p})$]
   Since $[H, \underset{\sim}{L}]$ is not zero, $\underset{\sim}{L}$ by itself is not conserved. Evidently there is some other form of angular momentum lurking here. Introduce the "spin angular momentum," $\underset{\sim}{S}$, defined by the equation $\underset{\sim}{S} \equiv (\hbar/2)\underset{\sim}{\Sigma}$.

   c) Find the commutator of H with the spin angular momentum, $\underset{\sim}{S} \equiv (\hbar/2)\underset{\sim}{\Sigma}$.
   [Solution:$[H, \underset{\sim}{S}] = i\hbar c\gamma^\circ(\underset{\sim}{\gamma} \wedge \underset{\sim}{p})$]

   It follows that the total angular momentum, $\underset{\sim}{J} = \underset{\sim}{L} + \underset{\sim}{S}$, is conserved.

   d) Show that every bispinor is an eigenstate of $\underset{\sim}{S}^2$, with eigenvalue $\hbar^2 s(s+1)$, and find *s*. What, then, is the spin of a particle described by the Dirac equation?

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 7.8, page 268].

3. The charge conjugation operator C takes a Dirac spinor ψ into the 'charge conjugate' spinor ψ_C, given by

$$\psi_C = i\gamma^2\psi^*$$

where γ^2 is the Dirac third gamma matrix.
Find the charge conjugates of $u^{(1)}$ and $u^{(2)}$, and compare them with $v^{(1)}$ and $v^{(2)}$.

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 7.9, page 269].

4. Evaluate the amplitude M for electron-muon scattering in the CM system,

$$M = -\frac{g_e^2}{\left(\underline{p}_1 - \underline{p}_3\right)^2}[\bar{u}^{(s_3)}(\underline{p}_3)\gamma^\mu u^{(s_1)}(\underline{p}_1)][\bar{u}^{(s_4)}(\underline{p}_4)\gamma_\mu u^{(s_2)}(\underline{p}_2)]$$

assuming the e^- and μ approach one another along the z-axis, repel, and return back along the z-axis. Assume the initial and final particles all have helicity +1.

$$[Answer : M = -2g_e^2]$$

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 7.26, page 270].

(p1-p3)^2 = (p1)^2 + (p3)^2 - 2 p1 • p3
= (E1/c)^2 + (E3/c)^2 - (p1_curl)^2 - (p3_curl)^2 - 2[(E1E3/c^2) - (p1_curl•p3_curl)]
(p1_curl•p3_curl) = |p1_curl||p3_curl|cos180 = |p1_curl||p3_curl|

= (E1/c)^2 + (E3/c)^2 - (p1_curl)^2 - (p3_curl)^2 - 2[(E1E3/c^2) - |p1_curl||p3_curl|]