$$\frac{5 \cdot 6.626 \cdot 10^{-34}}{4 \cdot 10^{-18} \cdot 9.109 \cdot 10^{-31} \cdot 2\pi} = 1.44714109 \times 10^{14}$$

I_sphere =2/5 mr^2
L = 2/5 mr^2 (v/r)
h/2 = 2/5 mvr
Hence,
v = 5h/4rm

→ v > c hence the electron cannot be treated like a classical solid sphere

PC4245 PARTICLE PHYSICS
HONOURS YEAR
Tutorial 2

1. Suppose we interpret the electron literally as a classical solid sphere of radius *r*, mass *m*, spinning with angular momentum $\frac{1}{2}\hbar$. What is the speed, *v*, of a point on its "equator"? Experimentally, it is known that *r* is less than 10^{-16}cm. What is the corresponding equatorial speed? What do you conclude from this?

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 4.8, page 154].

$S_i = \frac{\hbar}{2}\sigma_i$ for i=1,2,3

1. Find the eigenstates of the measurement operator, S_i, which will have a $\pm$ term
   - S_x has eigenstates $\chi_\pm = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm 1\end{pmatrix}$
2. A spinor (the state in question) can be represented via a combination of these eigenstates:

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \frac{a}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix} + \frac{b}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix}$$

3. Thus, for a state $\begin{pmatrix}\frac{1}{\sqrt{5}}\\ \frac{2}{\sqrt{5}}\end{pmatrix}$,

$$\begin{pmatrix}\frac{1}{\sqrt{5}}\\ \frac{2}{\sqrt{5}}\end{pmatrix} = \frac{a}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix} + \frac{b}{\sqrt{2}}\begin{pmatrix}1\\ -1\end{pmatrix}$$
$$\therefore \frac{1}{\sqrt{5}} = \frac{a+b}{\sqrt{2}} \text{ and } \frac{2}{\sqrt{5}} = \frac{a-b}{\sqrt{2}}$$
$$a = \frac{3}{\sqrt{10}} \text{ and } b = \frac{-1}{\sqrt{10}}$$
$$\text{Probability of measuring } +\frac{\hbar}{2} \text{ and } -\frac{\hbar}{2}:$$
$$|a|^2 = \frac{9}{10} \text{ and } |b|^2 = \frac{1}{10} \text{ respectively}$$

2. Suppose an electron is in the state $\begin{pmatrix}1/\sqrt{5}\\ 2/\sqrt{5}\end{pmatrix}$

   a. If you measured S_x, what values might you get, and what is the probability of each?

   b. If you measured S_y, what values might you get, and what is the probability of each?

   c. If you measured S_z, what values might you get, and what is the probability of each?

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 4.18, page 154].

3. a. Show that $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$. ("1" here really means 2 x 2 unit matrix; if no matrix is specified, the unit matrix is understood).

$$\sigma_1{}^2 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \mathbb{I}$$
$$\sigma_2{}^2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \mathbb{I}$$
$$\sigma_3{}^2 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \mathbb{I}$$

   b. Show that $\sigma_x\sigma_y = i\sigma_z$, $\sigma_y\sigma_z = i\sigma_x$, $\sigma_z\sigma_x = i\sigma_y$. These results are neatly summarized in the formula

$$\sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \quad \sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \quad \sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\sigma_i\sigma_j = \delta_{ij} + i\varepsilon_{ijk}\sigma_k$$

(summation over k implied), where δ_{ij} is the Kronecker delta:

$$\delta_{ij} = \begin{cases} 1, \text{ if } i = j \\ 0, \text{ otherwise} \end{cases}$$

and ε_{ijk} is the Levi-Civita symbol

$$\varepsilon_{ijk} \begin{cases} 1, \text{ if } ijk = 123, 231 \text{ or } 312 \\ -1, \text{ if } ijk = 132, 213 \text{ or } 321 \\ 0, \text{ otherwise} \end{cases}$$

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 4.19, page 155].

4. a. Show that $e^{i\pi\sigma_z/2} = i\sigma_z$.

$$\text{Using } \exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots,$$
$$\exp(i\pi\frac{\sigma_z}{2}) \approx 1 + \frac{i\pi}{2}\sigma_z + (\frac{i\pi}{2}\sigma_z)^2\frac{1}{2!} + (\frac{i\pi}{2}\sigma_z)^3\frac{1}{3!}$$
$$\text{Using } \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots \text{ and } \cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots$$
$$= \cos\frac{\pi}{2} + i\sigma_z(sin\frac{\pi}{2})$$
$$= i\sigma_z$$

b. Find the matrix U representing a rotation by 180° about the y axis, that it coverts “spin up”, into “spin down”, as we would expect.

Rotation about the y-axis above at angle of π:

$$U_{\hat{y}} = \exp\left(-\frac{i\theta}{\hbar}S_y\right) = \exp\left(-\frac{i\pi}{2}\sigma_y\right) = \begin{pmatrix} 0 & ie^{-\frac{i\pi}{2}} \\ -ie^{\frac{i\pi}{2}} & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & ie^{-\frac{i\pi}{2}} \\ -ie^{\frac{i\pi}{2}} & 0 \end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\ -ie^{\frac{i\pi}{2}}\end{pmatrix} = \begin{pmatrix}0\\ -i(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})\end{pmatrix} = \begin{pmatrix}0\\ -i(0+i)\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$

c. More generally, show that

$$U(\theta) = exp(-i\underset{\sim}{\theta}\cdot\underset{\sim}{\sigma}/2) = cos\frac{\theta}{2} - i(\overset{\sim}{\hat{\theta}}\cdot\underset{\sim}{\sigma})\,sin\frac{\theta}{2}$$

$$U_{\hat{n}} = \exp\left(-\frac{i\theta}{\hbar}S_{\hat{n}}\right) = \exp\left(-\frac{i\theta}{2}\underset{\sim}{\sigma}\cdot\hat{n}\right)$$

Expand using Taylor Series to get answer

where θ is the magnitude of $\underset{\sim}{\theta}$, and $\overset{\sim}{\hat{\theta}} = \underset{\sim}{\theta}/\theta$

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 4.21, page 155].

Use this:

THE BARYON DECUPLET

qqq	Q	S	Baryon
uuu	2	0	Δ^{++}
uud	1	0	Δ^{+}
udd	0	0	Δ^{0}
ddd	−1	0	Δ^{-}
uus	1	−1	Σ^{*+}
uds	0	−1	Σ^{*0}
dds	−1	−1	Σ^{*-}
uss	0	−2	Ξ^{*0}
dss	−1	−2	Ξ^{*-}
sss	−1	−3	Ω^{-}

I = n_up + n_down
= 1

I_3 = 1/2(n_up - n_down + n_antidown - n_anti-up)
= 1/2(1-1)
= 0

5. The Σ^{*0} can decay into $\Sigma^+ + \pi^-$, $\Sigma^0 + \pi^0$, or $\Sigma^- + \pi^+$. Suppose you observed 100 such disintegrations, how many would you expect to see of each type ?

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 4.32, page 156].

Using the notation $|I_1\ I_2\ m_{I_1}\ m_{I_2}\rangle \to |I_1\ m_{I_1}\rangle\,|I_2\ m_{I_2}\rangle$ and $|I_1\ I_2\ I\ m\rangle \to |I\ m\rangle$ alongside the Clebsch-Gordon coeffs,

▸ Table

$$\Sigma^+\pi^- : |1,1\rangle\,|1,-1\rangle = \frac{1}{\sqrt{6}}|2,0\rangle + \frac{1}{\sqrt{2}}|1,0\rangle + \frac{1}{\sqrt{3}}|0,0\rangle$$
$$\Sigma^0\pi^0 : |1,0\rangle\,|1,0\rangle = \sqrt{\frac{2}{3}}|2,0\rangle - \frac{1}{\sqrt{3}}|0,0\rangle$$
$$\Sigma^-\pi^+ : |1,-1\rangle\,|1,1\rangle = \frac{1}{\sqrt{6}}|2,0\rangle - \frac{1}{\sqrt{2}}|1,0\rangle + \frac{1}{\sqrt{3}}|0,0\rangle$$

Scattering amplitude, $\mathcal{M}$ for $\Sigma^+\pi^-$ $(j=1)$ is equal to

$$\mathcal{M}_a = \langle\text{out}|\text{in}\rangle = \left(\frac{1}{\sqrt{6}}\langle 2,0| + \frac{1}{\sqrt{2}}\langle 1,0| + \frac{1}{\sqrt{3}}\langle 0,0|\right)|1,0\rangle = \frac{1}{\sqrt{2}}\mathcal{M}_1$$

$$\mathcal{M}_b = \langle\text{out}|\text{in}\rangle = \left(\sqrt{\frac{2}{3}}\langle 2,0| - \frac{1}{\sqrt{3}}\langle 0,0|\right)|1,0\rangle = 0$$

$$\mathcal{M}_c = \langle\text{out}|\text{in}\rangle = \left(\frac{1}{\sqrt{6}}\langle 2,0| - \frac{1}{\sqrt{2}}\langle 1,0| + \frac{1}{\sqrt{3}}\langle 0,0|\right)|1,0\rangle = \frac{1}{\sqrt{2}}\mathcal{M}_1$$

$$\sigma_a : \sigma_b : \sigma_c = |\mathcal{M}_a|^2 : |\mathcal{M}_b|^2 : |\mathcal{M}_c|^2 = \frac{1}{2} : 0 : \frac{1}{2}$$

THE MESON NONET

$q\bar{q}$	Q	S	Meson
$u\bar{u}$	0	0	π^0
$u\bar{d}$	1	0	π^+
$d\bar{u}$	−1	0	π^-
$d\bar{d}$	0	0	η
$u\bar{s}$	1	1	K^+
$d\bar{s}$	0	1	K^0
$s\bar{u}$	−1	−1	K^-
$s\bar{d}$	0	−1	$\bar{K}^0$
$s\bar{s}$	0	0	??

6. Consider pion-nucleon scattering, $\pi N \to \pi N$. There are six elastic processes:

(a) $\pi^+ + p \to \pi^+ + p$ (b) $\pi^0 + p \to \pi^0 + p$
(c) $\pi^- + p \to \pi^- + p$ (d) $\pi^+ + n \to \pi^+ + n$
(e) $\pi^0 + n \to \pi^0 + n$ (f) $\pi^- + n \to \pi^- + n$

and four charge-exchange processes:

(g) $\pi^+ + n \to \pi^0 + p$ (h) $\pi^0 + p \to \pi^+ + n$
(i) $\pi^0 + n \to \pi^- + p$ (j) $\pi^- + p \to \pi^0 + n$

Since the pion carries $I = 1$, and the nucleon $I = \frac{1}{2}$, the total isospin can be $\frac{3}{2}$ or $\frac{1}{2}$. So there are just two distinct amplitudes here: M_3, for $I = \frac{3}{2}$, and M_1, for $I = \frac{1}{2}$. From the Clebsch-Gordan tables we find the following decompositions:

$$\left.\begin{aligned}
\pi^+ + p &: |11\rangle\left|\tfrac{1}{2}\tfrac{1}{2}\right\rangle = \left|\tfrac{3}{2}\tfrac{3}{2}\right\rangle \\
\pi^0 + p &: |10\rangle\left|\tfrac{1}{2}\tfrac{1}{2}\right\rangle = \sqrt{2/3}\left|\tfrac{3}{2}\tfrac{1}{2}\right\rangle - (1/\sqrt{3})\left|\tfrac{1}{2}\tfrac{1}{2}\right\rangle \\
\pi^- + p &: |1-1\rangle\left|\tfrac{1}{2}\tfrac{1}{2}\right\rangle = (1/\sqrt{3})\left|\tfrac{3}{2}-\tfrac{1}{2}\right\rangle - \sqrt{2/3}\left|\tfrac{1}{2}-\tfrac{1}{2}\right\rangle \\
\pi^+ + n &: |11\rangle\left|\tfrac{1}{2}-\tfrac{1}{2}\right\rangle = (1/\sqrt{3})\left|\tfrac{3}{2}\tfrac{1}{2}\right\rangle + \sqrt{2/3}\left|\tfrac{1}{2}\tfrac{1}{2}\right\rangle \\
\pi^0 + n &: |10\rangle\left|\tfrac{1}{2}-\tfrac{1}{2}\right\rangle = \sqrt{2/3}\left|\tfrac{3}{2}-\tfrac{1}{2}\right\rangle + (1/\sqrt{3})\left|\tfrac{1}{2}-\tfrac{1}{2}\right\rangle \\
\pi^- + n &: |1-1\rangle\left|\tfrac{1}{2}-\tfrac{1}{2}\right\rangle = \left|\tfrac{3}{2}-\tfrac{3}{2}\right\rangle
\end{aligned}\right\}$$

Reactions (a) and (f) are pure $I = \frac{3}{2}$:

$$M_a = M_f = M_3 \tag{1}$$

Consider pion-nucleon scattering, $\pi N \to \pi N$. There are six elastic processes:

(a) $\pi^+ + p \to \pi^+ + p$ (b) $\pi^0 + p \to \pi^0 + p$
(c) $\pi^- + p \to \pi^- + p$ (d) $\pi^+ + n \to \pi^+ + n$
(e) $\pi^0 + n \to \pi^0 + n$ (f) $\pi^- + n \to \pi^- + n$

and four charge-exchange processes:

(g) $\pi^+ + n \to \pi^0 + p$ (h) $\pi^0 + p \to \pi^+ + n$
(i) $\pi^0 + n \to \pi^- + p$ (j) $\pi^- + p \to \pi^0 + n$

The others are all mixtures: for example

$$M_c = \frac{1}{3}M_3 + \frac{2}{3}M_1, \qquad M_j = (\sqrt{2}/3)\, M_3 - (\sqrt{2}/3)\, M_1 \tag{2}$$

The cross sections, then, stand in the ratio

$$\sigma_a : \sigma_c : \sigma_j = 9|M_3|2 : \quad |M_3 + 2M_1|2 : \quad 2|M_3 - M_1|2 \tag{3}$$

At a *CM* energy of 1232 MeV there occurs a famous and dramatic bump in pion-nucleon scattering, first discovered by Fermi in 1951; here the pion and nucleon join to form a short-lived “resonance” state— the*Δ*. We know the*Δ* carries $I = \frac{3}{2}$, so we expect that at this energy $M_3 » M_1$, and hence

$$\sigma_a : \sigma_c : \sigma_j = 9 : 1 : 2 \tag{4}$$

(i) Referring to equations (1) and (2), work out all the πN scattering amplitudes, M_a through M_j, in terms of M_1 and M_3.

(ii) Generalize equation (3) to include all 10 cross sections.

(iii) In the same way, generalize equation (4).

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 4.28, page 156].

7. For two hadrons to interconvert, $A \rightleftharpoons B$, it is necessary that they have the same mass (which in practice means that they must be antiparticle of each other), the same charge and the same baryon number. In the Standard Model, with the usual three generations show that A and B would have to be neutral mesons, and identify their possible quark contents. What, then, are the candidate mesons? Why doesn’t neutron mix with antineutron, in the same way as K^0 and $\overline{K^0}$ mix to produce K_1 and K_2?
Why don’t we see the missing the strange vector mesons K^{0*} and $\overline{K^{0*}}$?

u-d <-> ud-
s-d <-> sd-
c-s <-> cs- etc.

Neutron and antineutron have different baryon number (+1 udd vs -1 u-d-d-) so cannot be antiparticle of each other.

Neutron strange vector mesons decay readily via strong int. and have no time to interconvert

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 4.38, page 157].