PC3261: Classical Mechanics II
Assignment 7

1. [30 pts] A vaulter, carrying a pole with rest-length $10\,\mathrm{m}$, runs in the positive x-direction at speed $\beta = \sqrt{3}/2$. She enters a barn with rest-length $8\,\mathrm{m}$, which has its left door open and right one closed. At $t = 0$ in the barn's frame, the left door closes with the left end of the pole ($x' = 0$) just inside the door. At the same time, the right end of the pole is well inside the barn at $x = 5\,\mathrm{m}$. Denote the event $(t, x) = (0, 0)$ by A, and the event $(t, x) = (0, 5)$ by B. Thus, the barn doors are closed with the pole completely inside the barn. An instant later, the right door opens and the vaulter runs harmlessly out of the barn.

(a) Find t'_B in the vaulter's frame assuming that $t'_A = 0$.

(b) At time t'_B, what length of the pole is inside the barn according to the vaulter?

(c) Sketch the pole and the barn on a spacetime diagram, showing how the vaulter understands the sequence of these events.

2. [40 pts] A particle is moving along the x-axis. It is uniformly accelerated so that the magnitude of the acceleration measured in its *instantaneous* rest frame is always g, a constant.

(a) Defining four-acceleration as $\mathbf{a} \equiv \mathrm{d}\mathbf{u}/\mathrm{d}\tau$ where $\mathbf{u}$ is the four-velocity, show that four-acceleration is always orthogonal to four-velocity, i.e. $\mathbf{a} \cdot \mathbf{u} = 0$.

(b) Find $x(\tau)$ and $t(\tau)$ assuming $x(0) = x_0$, $t(0) = 0$ and $u^x(0) = 0$. Sketch the worldline of the particle on a spacetime diagram.

(c) Find the particle's three-velocity $\vec{\mathbf{v}}(t)$. Does the particle's speed ever exceed the speed of light? Explain.

3. [30 pts] Let S and S' be inertial frames of reference, with S' moving in the $+x$-direction with speed β relative to S. Consider a photon in S with wavelength λ_0, moving at an angle θ with respect to the $+x$-axis.

(a) What is the angle θ' with respect to the $+x'$-axis at which the photon moves with respect to a stationary observer in S'? And, what is the wavelength λ' of the photon measured by a stationary observer in S'?

(b) Even when the motion of the photon is perpendicular to the $+x$-axis ($\theta = \pi/2$) there is a shift of wavelength. This is called the *transverse Doppler effect* and arises due to time dilation. At what angle θ does the photon have to move so that there is *no* Doppler shift between frames S and S'?

(c) Instead of a photon, a bullet of mass m is fired in S with speed u at an angle θ with respect to the $+x$-axis. What is the angle θ' with respect to the $+x'$-axis at which the bullet moves with respect to a stationary observer in S'?

Last updated: November 9, 2023 10:09 am