NATIONAL UNIVERSITY OF SINGAPORE

PC4245 Particle Physics

(Semester II: AY 2021-22)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1. Write your Matric Number on the front cover page of each answer book.
2. This examination paper contains **4** questions and comprises **6** printed pages. Answer any **3** questions.
3. All questions carry equal marks.
4. Students should write the answers for each question on a new page.
5. This is a CLOSED BOOK examination.

1. (a) The kaon mesons are produced via strong interaction but decay by weak interaction. Describe briefly the Cronin-Fitch experiment on K^0 decay that evidenced CP violation.

Find the ratio of K_S (K short) and K_L (K long) in a beam of 10 GeV neutral kaons at a distance of 20 meters from where the beam is produced.

Note that the probability of getting $|K_L\rangle$ at time t is $P_L(t) = \frac{1}{2}e^{-\Gamma_L t/\hbar}$ and that of getting $|K_S\rangle$ at time t is $P_S(t) = \frac{1}{2}e^{-\Gamma_S t/\hbar}$, where $\Gamma_L = \frac{\hbar}{\tau_L}$, $\Gamma_S = \frac{\hbar}{\tau_S}$ and $\tau_L = 5.2 \times 10^{-8}$ sec, $\tau_S = 0.86 \times 10^{-10}$ sec .

(b) Consider the elastic scattering ($m_3 = m_1, m_4 = m_2$) in the lab frame (particle 2 at rest), $1+2 \rightarrow 3+4$.

Derive an expression of the differential cross section

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{\underset{\sim}{p_3}^2 S|M|^2}{m_2|\underset{\sim}{p_1}|\left|(E_1+m_2c^2)|\underset{\sim}{p_3}| - |\underset{\sim}{p_1}|E_3\cos\theta\right|}$$

in the usual notations.

Note: The following formula can be used

$$d\sigma = S\,|\mathcal{M}|^2\,\frac{\hbar^2}{4}\left[(\underline{p_1}\cdot\underline{p_2})^2 - (m_1m_2c^2)^2\right]^{-1/2} \frac{d^3\underset{\sim}{p_3}}{(2\pi)^3\,2\,p_3^0} \times \frac{d^3\underset{\sim}{p_4}}{(2\pi)^3\,2\,p_4^0}\,(2\pi)^4\,\delta^{(4)}(\underline{p_1} + \underline{p_2} - \underline{p_3} - \underline{p_4})$$

and for the lab frame (particle 2 at rest)

$$\sqrt{(\underline{p_1}\cdot\underline{p_2})^2 - (m_1m_2c^2)^2} = m_2\,|\underset{\sim}{p_1}|\,c]$$

2. (a) The spin/flavor wave function for a baryon Λ with spin up is given by

$$\left|\Lambda:\frac{1}{2}\frac{1}{2}\right\rangle = \frac{1}{2\sqrt{3}}[u(\uparrow)d(\downarrow)s(\uparrow) - u(\downarrow)d(\uparrow)s(\uparrow) + 6\text{ permutations}]$$

Calculate the magnetic dipole moment of Λ by considering only the first term of the wave function, namely,

$\frac{1}{2\sqrt{3}}\left(u(\uparrow)d(\downarrow)s(\uparrow)\right).$

Your answer should be expressed in terms of the quark magnetic dipole moments. In the usual notations, these are

$$\mu_u = \frac{2}{3}\frac{e\hbar}{2m_u c}, \quad \mu_d = -\frac{1}{3}\frac{e\hbar}{2m_d c}, \quad \mu_s = -\frac{1}{3}\frac{e\hbar}{2m_s c}.$$

Note: The magnetic dipole moment is related to spin by the relation

$\underset{\sim}{\mu} = \frac{q}{mc}\underset{\sim}{s}$.

The magnetic dipole moment of a baryon in the usual notations can be written as $\mu_B = \langle B\uparrow|(\underset{\sim}{\mu}1 + \underset{\sim}{\mu}2 + \underset{\sim}{\mu}3)_3\,|B\uparrow\rangle$

(b) The mass of a baryon in terms of the masses of its constituent quarks and the their spins, in the usual notations, is given by

$$M(\text{baryon}) = m_1 + m_2 + m_3 + A'\left[\frac{\underset{\sim}{s_1}\cdot\underset{\sim}{s_2}}{m_1 m_2} + \frac{\underset{\sim}{s_1}\cdot\underset{\sim}{s_3}}{m_1 m_3} + \frac{\underset{\sim}{s_2}\cdot\underset{\sim}{s_3}}{m_2 m_3}\right]$$

where $m_u = m_d = 363\text{ MeV}/c^2$, $m_s = 538\text{ MeV}/c^2$, and

$A' = \left(\frac{2m_u}{\hbar}\right)^2 50\text{ MeV/c}^2.$

(i) Prove that

$$\underset{\sim}{s_1}\cdot\underset{\sim}{s_2} + \underset{\sim}{s_1}\cdot\underset{\sim}{s_3} + \underset{\sim}{s_2}\cdot\underset{\sim}{s_3} = \begin{cases} -\frac{3}{4}\hbar^2, & \text{for } j = 1/2\text{ (octet)} \\ \frac{3}{4}\hbar^2, & \text{for } j = 3/2\text{ (decuplet)} \end{cases}$$

(ii) If the three quarks have the same mass m, show that for an octet baryon,

$$M(\text{baryon}) = 3m - \frac{3}{4}\frac{\hbar^2}{m^2} A'$$

and for a decuplet baryon

$$M(\text{baryon}) = 3m + \frac{3}{4}\frac{\hbar^2}{m^2} A' \;.$$

(vii) If two of the three quarks have the same mass, say $m_1 = m_2$, show that for an octet baryon,

$$M(\text{baryon}) = 2m_1 + m_3 + A'\left[\frac{\underset{\sim}{s}_1\cdot\underset{\sim}{s}_2}{{m_1}^2} - \frac{\underset{\sim}{s}_1\cdot\underset{\sim}{s}_2+\frac{3}{4}\hbar^2}{m_1 m_3}\right]$$

and for a decuplet baryon,

$$M(\text{baryon}) = 2m_1 + m_3 + A'\left[\frac{\underset{\sim}{s}_1\cdot\underset{\sim}{s}_2}{{m_1}^2} - \frac{\underset{\sim}{s}_1\cdot\underset{\sim}{s}_2-\frac{3}{4}\hbar^2}{m_1 m_3}\right] \;.$$

Hence or otherwise, obtain the mass of the octet baryon Λ

$$M(\Lambda) = 2m_u + m_s - A'\,\frac{3}{4}\frac{\hbar^2}{{m_u}^2}\,.$$

Note: The baryon Λ is an isosinglet,

$$\Lambda = \frac{1}{\sqrt{12}}\,[2(usd - dsu) + uds - sdu - dus + sud]$$

3. (a) Show that the helicity operator $s(\underset{\sim}{p}) \equiv \underset{\sim}{\Sigma}\cdot\underset{\sim}{p}/|\underset{\sim}{p}|$ commutes with the Hamiltonian of a Dirac particle, $H = c\underset{\sim}{\alpha}\cdot\underset{\sim}{p} + \beta mc^2$. Here $\underset{\sim}{\Sigma}$ is the spin operator of the Dirac particle.

Explain qualitatively why the helicity of a particle is, in general, not an invariant.

Note: $\underset{\sim}{\Sigma} = \begin{pmatrix}\underset{\sim}{\sigma} & 0\\ 0 & \underset{\sim}{\sigma}\end{pmatrix}$, $\quad \underset{\sim}{\alpha} = \begin{pmatrix}0 & \underset{\sim}{\sigma}\\ \underset{\sim}{\sigma} & 0\end{pmatrix}$, $\quad \beta = \begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix}$, $\quad I = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

and $\underset{\sim}{\sigma}$ are the Pauli matrices $\quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

(b) For a massive fermion, show that handedness is not a good quantum number. That is, show that γ^5 does not commute with H.

Note: $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$

(c) Describe briefly two experimental evidences that suggest each quark flavor must come in three color varieties.

4. (a) Draw the lowest-order Feynman diagrams for the electron-electron scattering

$$e^- + e^- \rightarrow e^- + e^-.$$

Using the Feynman rules for quantum electrodynamics, obtain the scattering amplitude $\mathcal{M}$ for the above process.

Note: For vertex, $ig\gamma^\mu$; for propagators, $\frac{-ig^{\mu\nu}}{q^2}$, $\frac{i}{q_\mu\gamma^\mu - mc}$

(b) Consider the electron-electron scattering at very high energy so that the mass of the electron can be ignored (i.e., set m = 0).

Define the spin-averaged quantity $\langle|\mathcal{M}|^2\rangle$.

The scattering amplitude $\mathcal{M}$ can be written as $\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2$.

Using the Casimir trick, show that

(i) $\quad < |\mathcal{M}_1|^2 > = \frac{g^4}{4(p_1 - p_3)^4} Tr(\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_3) \cdot Tr(\gamma_\mu \not{p}_2 \gamma_\nu \not{p}_4),$

(ii) $\quad < |\mathcal{M}_1 \, \mathcal{M}_2^*| > = \frac{-g^4}{16(p_1 \cdot p_3)(p_1 \cdot p_4)} Tr(\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_4 \gamma_\mu \not{p}_2 \gamma_\nu \not{p}_3).$

Hence or otherwise obtain an expression of $\langle|\mathcal{M}|^2\rangle$.

Note: (I) for massless particles the conservation of momentum ($\underline{p}_1 + \underline{p}_2 = \underline{p}_3 + \underline{p}_4$) implies that

$$\underline{p}_1 \cdot \underline{p}_2 = \underline{p}_3 \cdot \underline{p}_4, \quad \underline{p}_1 \cdot \underline{p}_3 = \underline{p}_2 \cdot \underline{p}_4, \quad \underline{p}_1 \cdot \underline{p}_4 = \underline{p}_2 \cdot \underline{p}_3 \,.$$

(II) $\sum_s u^{(s)}(\underline{p})\bar{u}^{(s)}(\underline{p}) = \not{p} + mc. \quad \not{p} \equiv \gamma^\mu p_\mu$

(OCH)

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