

2024.1.18

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(1)

Recap

concept of particle

Newton 1687 matter \rightarrow particle

particle is pointlike object with properties e.g., ^{mass,} position, momentum, energy.

Matter made out of particles.

Light made out of particle, corpuscles.

Maxwell 1860 - 1865

Matter not just particles, we need fields (wave), field = an extended

physical system, not pointlike

with field, unification { electricity
magnetism
light

Matter { particle
field.

2 physical
system

Hugo

1926 QM: dual nature of 'particle'
pointlike & field

One fundamental constituent of ⁽²⁾
particle; quantum particle $\begin{cases} \text{pointlike} \\ \text{wave like} \end{cases}$

QFT ~ 1950

fundamental constituent of matter

: quantum field,

local excitation of field \rightarrow particle

Nowadays \rightarrow strings (No exp.)

Matter (general) (4% of the content of the universe)

Matter \rightarrow particles with nonzero rest mass
(specific)

force field \rightarrow particle with zero rest mass

Classifications of particles (with 3 non-zero rest mass) :

Leptons

3 families $\begin{pmatrix} e \\ \nu_e \end{pmatrix} \dots$

hadrons

→ $\begin{cases} \text{baryons} \\ \text{mesons} \end{cases}$

→ quarks

3 families, e.g. $\begin{pmatrix} u \\ d \end{pmatrix}$

Hadrons : baryons made out of 3 quarks
mesons 2 (quark + antiquark)

Force fields : gravitational field : graviton

weak field : intermediate vector bosons
 Z, W^\pm

em field : photon

strong field : gluons

Discuss : how 3 quarks → baryons

how quark + antiquark → meson.

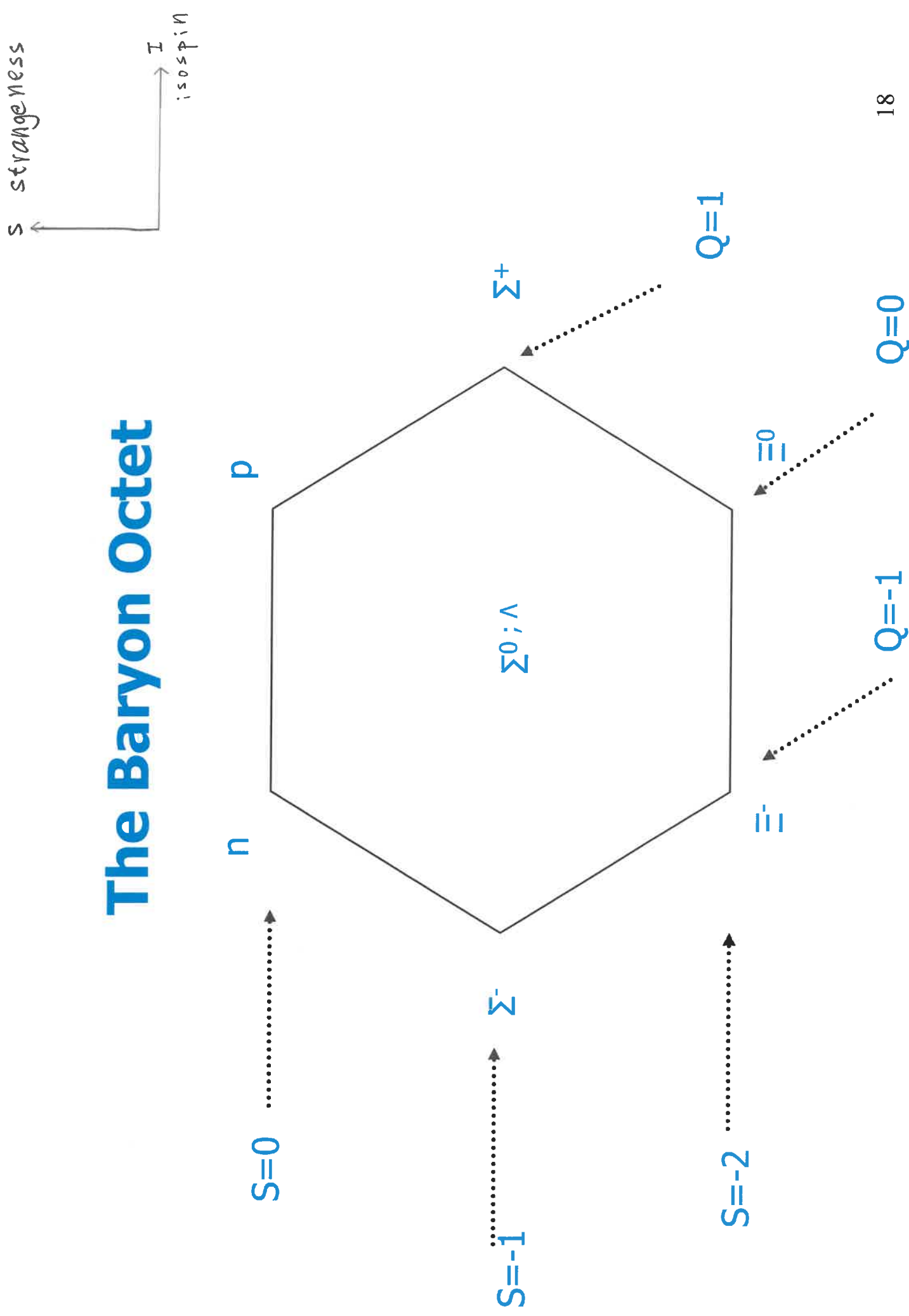
THE EIGHTFOLD WAY (1961)

classify hadrons
(baryons, mesons)
according to the
multiplets (singlet,
octet, decuplet) of
 $SU(3)$ group, so
called unitary symmetry.
Extension of isospin scheme $SU(2)$ ¹⁷

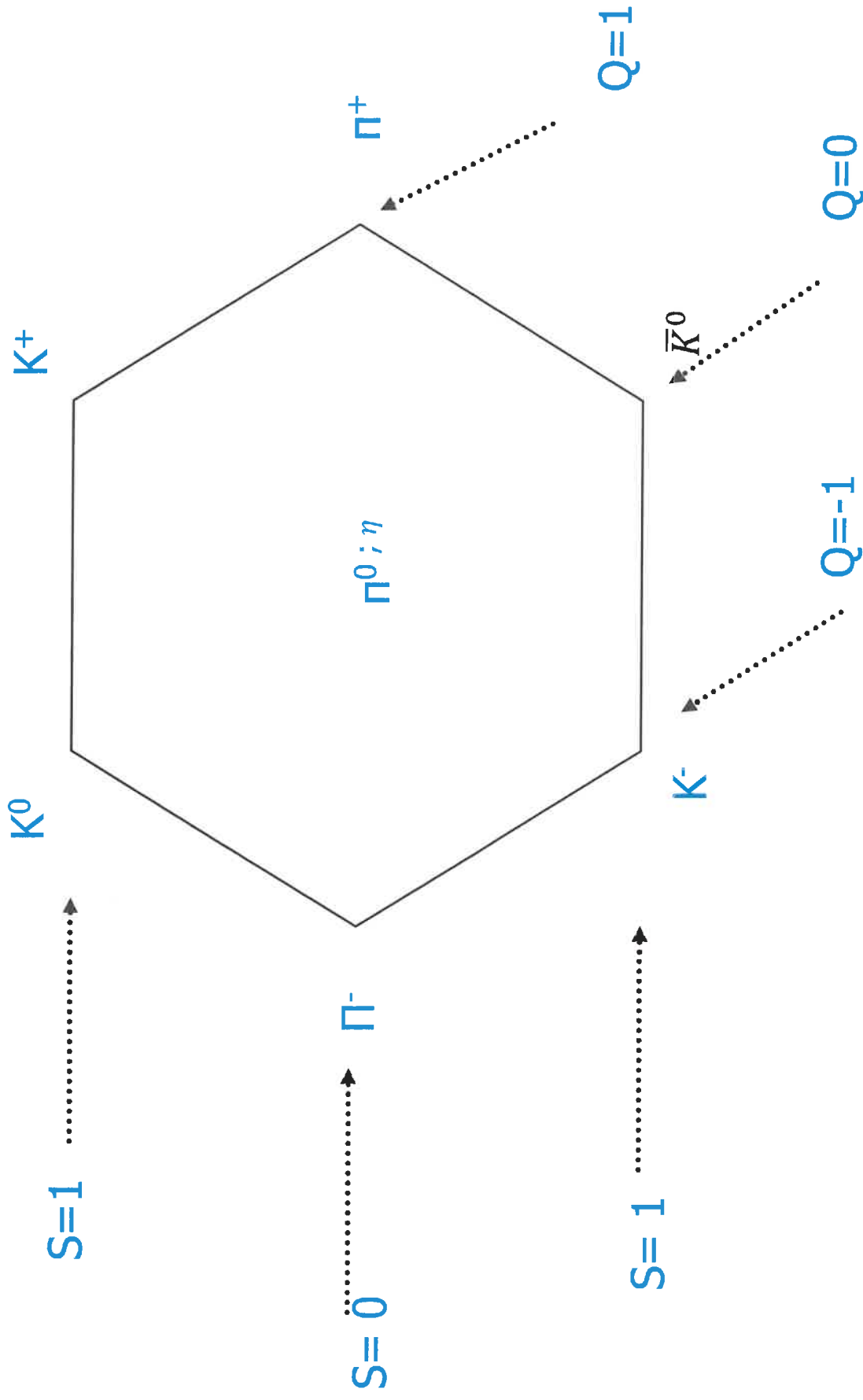
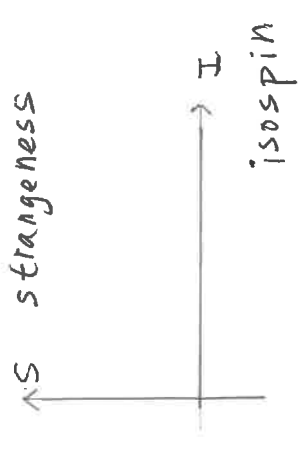
Classify hadrons (baryons and mesons) according to multiplets (singlets, octets, decuplets) of the unitary group $SU(3)$, so called unitary symmetry.

This scheme is an extension of the isospin classification, $SU(2)$.
E.g. proton and neutron form an isodoublet.

The Baryon Octet



The Meson Octet



$SU(3)$ Octet and Nonet

An $SU(3)$ octet consists of 2 isodoublets, 1 isotriplet, and 1 isosinglet. These isomultiplets refer to $SU(2)$.

An nonet consists of an $SU(3)$ octet and an $SU(3)$ singlet.

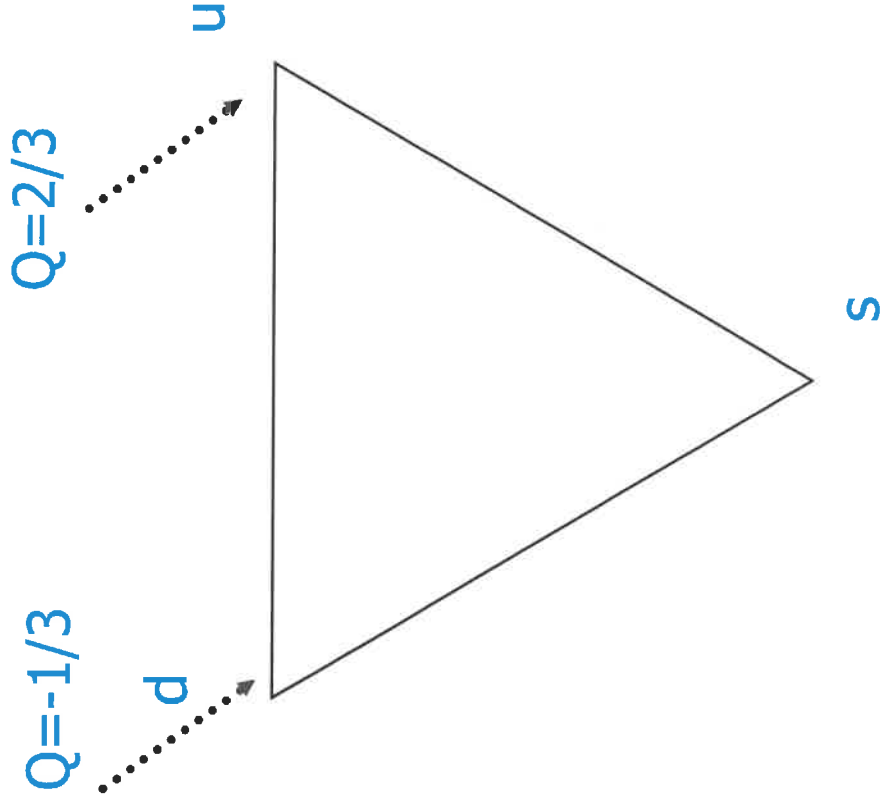
An nonet is a $SU(3)$ reducible representation, and is equivalent to an irreducible $SU(3)$ octet representation and an irreducible $SU(3)$ singlet representation.

The Quark Model (1964)

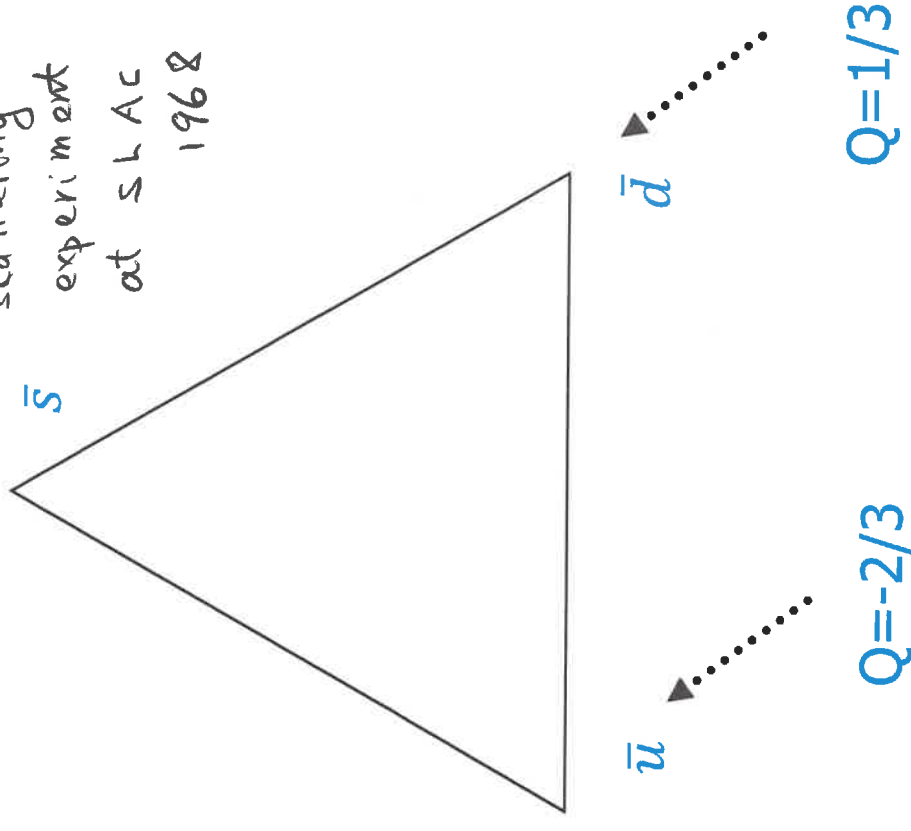
Quarks indirectly
detected in the
deep inelastic e^-p

$S=1$

scattering
experiment
at SLAC
1968



$S=0$



1.4 Theoretical Framework

1.4.1 Quantum field theories

To every elementary particle, we associate a field operator $\psi(\underline{x})$, $x^0 = ct$, $\underline{x} = (x^1, x^2, x^3)$, $\psi(\underline{x})$ acts on state vectors of a Hilbert space. The field operator $\psi(\underline{x})$ obeys equation of motion. For free particles, equations of motion are known. Usually can obtain equation of motion from action S

$$S = \int d^4x \mathcal{L} \quad \mathcal{L} = \text{Lagrangian density}.$$

For particles in interaction, interaction terms are usually derived from a symmetry principle, called principle of local gauge invariance.

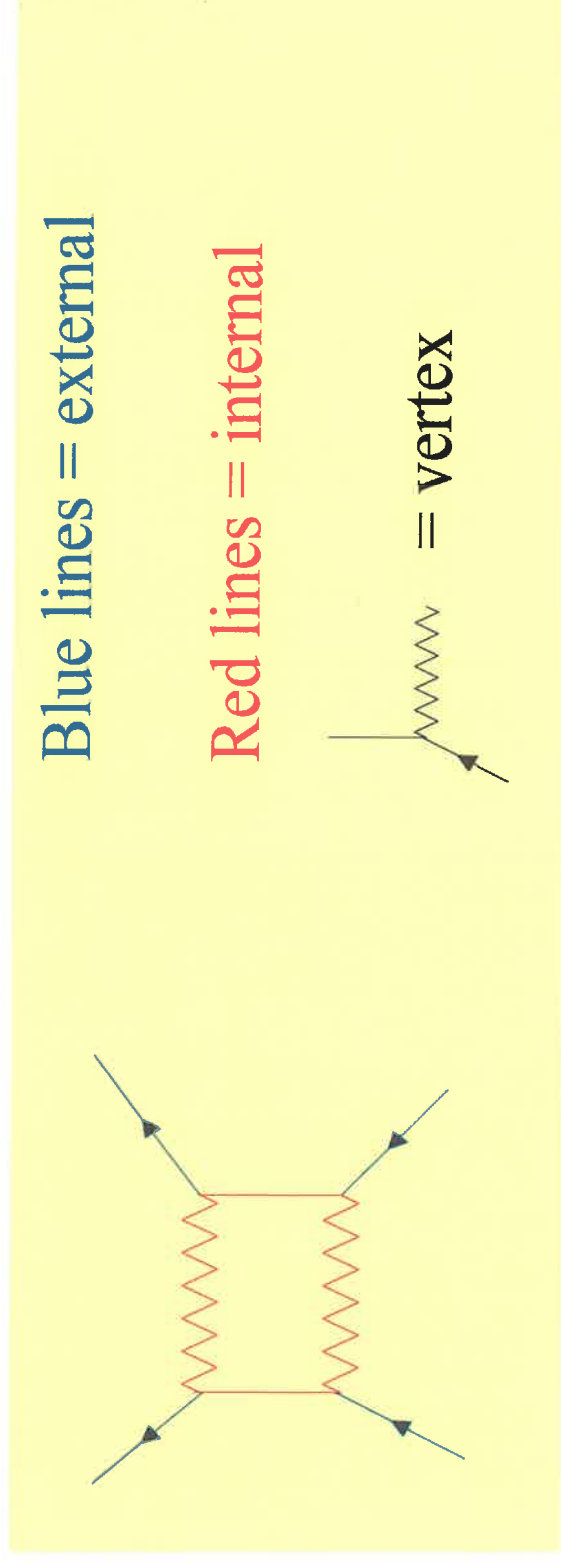
Two types of interaction terms:

$\bar{\psi}(\underline{x})\psi(\underline{x})\varphi(\underline{x})$	Yukawa
$\bar{\psi}(\underline{x})\gamma^\mu\psi(\underline{x})A_\mu(\underline{x})$	Gauge field theories

In quantum theory, $\exp(-iS)$ determines the physics, $S = \text{action}$.

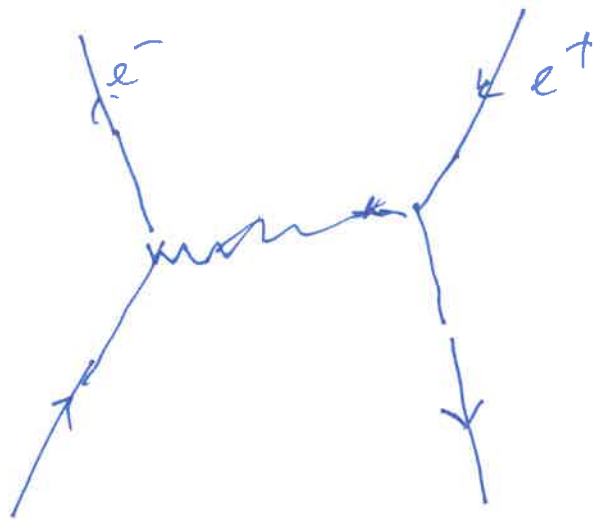
1.4.2 Feynman diagram

1. A Feynman diagram consists of external lines (lines which enter or leave the diagram) and internal lines (lines start and end in the diagram). External lines represent physical particles (observable). Internal lines represent virtual particles (A virtual particle is just like a physical particle except its mass can assume any value i.e. not on mass-shell). Vertices represent interactions. 4-momentum p^μ must be conserved at each vertex; in fact all conservation laws.
e.g.



2. The diagram is symbolic, the lines do not represent particle trajectories.

$$e^- e^+ \rightarrow e^- e^+$$



$$t; m_x$$

$$\downarrow$$

$$q$$

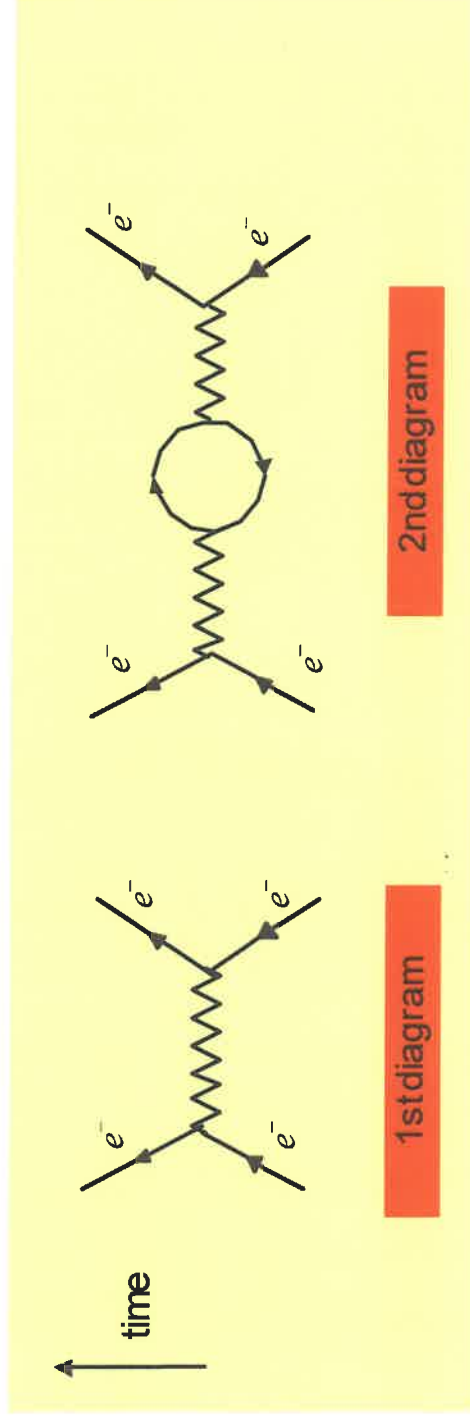
m photon

3. Each Feynman diagram stands for a complex number (scattering amplitude) which can be computed from Feynman's rules. The sum total of all Feynman diagrams with the same external lines represents a physical process.

There are infinitely many Feynman diagrams for a particular physical process. Each vertex in the diagram introduces a factor $\sqrt{\alpha}$ (α coupling constant).

For QED $\alpha_e = \frac{1}{137}$, thus higher order diagrams with many vertices will contribute less to the process.

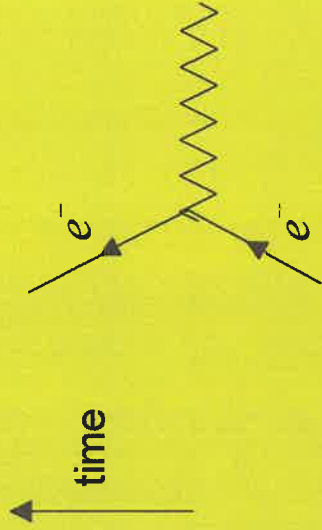
e.g. Electron-electron scattering $e^- e^- \rightarrow e^- e^-$



The 2nd diagram (1-loop) contributes less than the first diagram (tree).

4. At each vertex, the energy- momentum p^μ must be conserved.

e.g. $e^- \rightarrow e^- + \gamma$ violates energy conservation

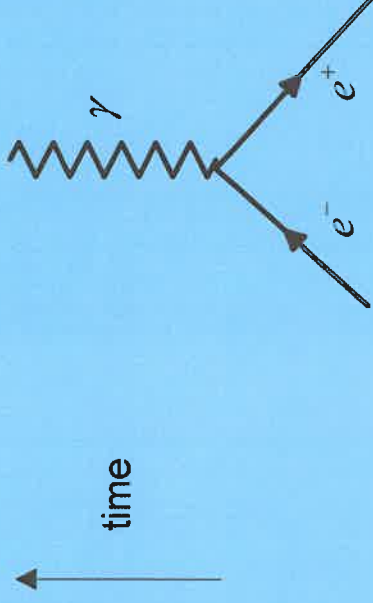


In **cm** frame, the e^- is initially at rest
The energy of the emitted electron and photon is

$(\gamma m_e c^2 + \hbar \omega) > m_e c^2$ (energy of e^- at rest)

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}, \quad \beta = \frac{v}{c}$$

$e^- + e^+ \rightarrow \gamma$ violates conservation of momentum 3-momentum



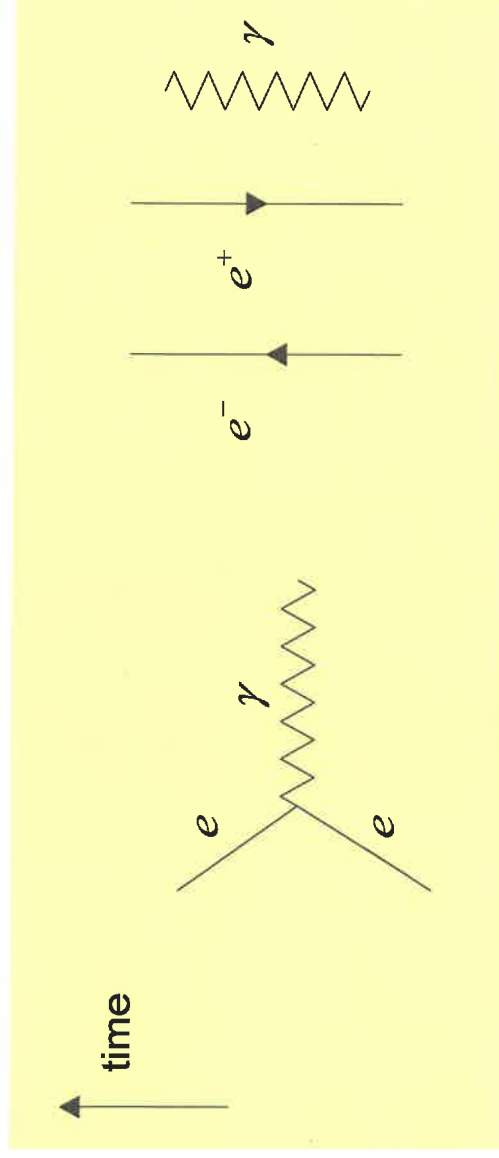
In **cm** frame total momentum of e^- and e^+ (positron) = 0, but total momentum after annihilation = momentum of γ (photon) $\neq 0$.

5. Each virtual particle (internal line) is represented by the “propagator” (a function describes the propagation of the virtual particle). The virtual particles are responsible for the description of force fields through which interacting particles affect on another.

(a) QED

$$\begin{aligned} \text{Coupling constant} \quad \alpha_e &= \frac{q_e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137} \\ q_e &= 1.602 \times 10^{-19} \text{ Coul}, \quad \hbar = 1.055 \times 10^{-34} \text{ Joule-Sec} \\ c &= 2.998 \times 10^8 \text{ m/s}, \quad \frac{1}{4\pi\epsilon_0} = 8.9875 \times 10^9 \end{aligned}$$

All **em** phenomena are ultimately reducible to following elementary process (primitive vertex)

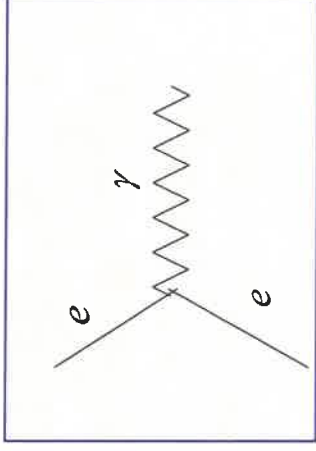


$$\begin{aligned} L &= \bar{\psi}\gamma^\mu D_\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + m\bar{\psi}\psi \\ &= \bar{\psi}\gamma^\mu\partial_\mu\psi - ie\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + m\bar{\psi}\psi \end{aligned}$$

$$\begin{aligned} \text{Interaction vertex} \quad \bar{\psi}\gamma^\mu\psi A_\mu &= j^\mu A_\mu \\ \text{and} \quad F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \end{aligned}$$

All **em** processes can be described by patching together two or more of the primitive vertices.

Note: The primitive QED vertex



by itself does not represent a possible physical process as it violates the conservation of energy.

Some examples of electromagnetic interaction

1. Møller Scattering $e^- e^- \rightarrow e^- e^-$

