PC3261: Classical Mechanics II
Assignment 2

1. [35 pts] A raindrop is falling through a mist collecting mass as it falls. Assume that the drop remains spherical and that the rate of accretion is proportional to the cross-sectional area of the drop multiplied by the speed of fall. You may assume uniform downward gravitational field of magnitude g, zero air resistance and the density ρ of the drop remains constant.

(a) Given that the initial radius of the drop is r_0, obtain a second-order *nonlinear* ordinary differential equation for $z(t)$ governing the dynamics of the raindrop as it is falling through the mist.

(b) Now, consider that the drop starts from rest when it is infinitesimally small, i.e. $r_0 \approx 0$, solve for $z(t)$, $\dot{z}(t)$ and $\ddot{z}(t)$.

2. [30 pts] A uniform chain of mass M and length L is held at rest hanging vertically downwards with its lower end touching a fixed horizontal table. The chain is then released.

(a) Assume that, before hitting the table, the chain falls freely under uniform gravity. Show that, while the chain is falling, the force that exerts on the table is always *three times* the weight of the chain actually lying on the table.

(b) Now, when all the chain has landed on the table, the loose end is pulled upwards with a constant force $Mg/3$. Find the height to which the chain will first rise. This time, assume that the force exerted on the chain by the table is *equal* to the weight of chain lying on the table.

3. [35 pts]

(a) Consider a system of point particles each with mass m_α. With respect to the origin of an inertial frame $\mathcal{O}$, the position vector of the particle is respectively given by $\mathbf{r}_\alpha(t)$. In the lecture, the time rate of change of total angular momentum of the system about the origin of the inertial frame is found to be

$$\dot{\mathbf{L}}(t) = \boldsymbol{\tau}^{\text{ext}}(t)\,,$$

where $\boldsymbol{\tau}^{\text{ext}}(t)$ is the total external torque about the origin of the inertial frame. Here, the internal forces among the particles are assumed to obey Newton's third law and central. The position vector of the center of mass of this system of particle is denoted by $\mathbf{R}_{\text{CM}}(t)$. And, we have shown during the lecture that the time rate of change of total angular momentum about the center-of-mass frame is given by

$$\dot{\mathbf{L}}_{\text{CM}}(t) = \boldsymbol{\tau}^{\text{ext}}_{\text{CM}}(t)\,,$$

where $\boldsymbol{\tau}^{\text{ext}}_{\text{CM}}(t)$ is the total external torque about the center-of-mass frame. Now, we consider a reference frame $\mathcal{O}'$ at which the position vector of its origin is $\mathbf{R}(t)$ with respect to the origin of $\mathcal{O}$. Denoting the total mass of the system by M, show that

$$\dot{\mathbf{L}}'(t) = \boldsymbol{\tau}'^{\text{ext}}(t) - M\left[\mathbf{R}_{\text{CM}}(t) - \mathbf{R}(t)\right] \times \frac{\mathrm{d}^2\mathbf{R}(t)}{\mathrm{d}t^2}\,.$$

(b) A uniform circular hoop, of mass M and radius R, is lying on a smooth horizontal table at which the hoop can slide freely. A bug of mass m can run on the hoop. The system is at rest when the bug starts to run. What is the angle turned through by the hoop when the bug has completed one lap of the hoop?

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