$G \quad ML^3T^{-2}$
$c \quad LT^{-1}$
$\hbar \quad ML^2T^{-1}$

$l_p \to \sqrt{\frac{G\hbar}{c^3}}$

$t_p \to \sqrt{\frac{G\hbar}{c^5}}$

$m_p \to \sqrt{\frac{\hbar c}{G}}$

PC4245 PARTICLE PHYSICS
HONOURS YEAR
Tutorial 1

1. (a) From $c, \hbar,$ and G (Newton's constant of universal gravitation), construct a quantity ℓ_p with the dimension of length, a quantity t_p with the dimension of time, a quantity m_p with the dimension of mass. These are known as *Planck length*, the *Planck time* and *Planck mass*, respectively, after Max Planck, who first published then in 1899 – the year before the eponymous constant itself. Work out the actual numbers in meters, seconds, and kilograms. Also calculate the *Planck energy* ($E_p = m_p c^2$) in GeV. [These quantities set the scale at which quantum gravity is expect to be relevant.]

$$E_p = \sqrt{\frac{\hbar c^5}{G}}$$

(b) What is the gravitational analog to the fine structure constant? Fine the actual number, using (i) the mass of the electron, (ii) the Planck mass.

$\alpha = \frac{e^2}{4\pi\epsilon_0 r^2} \to \frac{e^2}{hc}$ comes from $F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$ by setting $q_1 = q_2 = e$

Similarly, $\alpha_G = \frac{m_p^2}{4\pi\epsilon_0 r^2} \to \frac{m_p^2}{hc}$ from $F_G = \frac{m_1 m_2}{4\pi\epsilon_0 r^2}$ by setting $m_1 = m_2 = m_p$

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 12.9, page 420].

2. What is the Gell-Mann-Nishijima formula? Can it be generalized?

$Q = I_3 + \frac{1}{2}(A + S)$

[$2Q = A + U + D + S + C + B + T$]

Note that $2I_3$ is replaced by $U + D$

U = upness $\quad D$ = downness

In MeV / c^2,
m_N = 938.565
m_Ξ = 1533
m_Λ = 1115.68
m_Σ = 1192.64

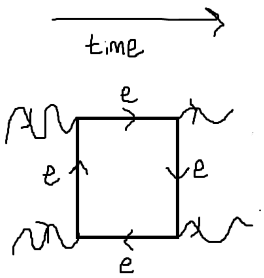
3. The *Gell-Mann/Okubo mass formula* relates the masses of members of the baryon octet (ignoring small differences between p and n; Σ^+, Σ^0, and Σ^-; and Ξ^0 and Ξ^-): $\quad 2(m_N + m_\Xi) = 3m_\Lambda + m_\Sigma$
Using this formula, together with the known masses of the *nucleon* N (use the average of p and n), Σ (again, use the average), and Ξ (ditto), "predict" the mass of the Λ. How close do you come to the observed value?
[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 1.4, page 56].
[*Answer:* m_Λ (observed) = 1116 MeV/c^2]

Calc: 1250 MeV / c^2
Actual: 1116 MeV / c^2

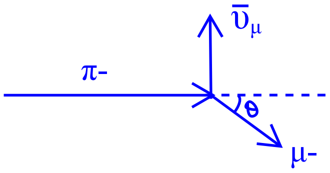
In MeV / c^2,
Δ = 1232
Σ* = 1385
Ξ* = 1533

4. The mass formula for decuplets is much simpler –equal spacing between the rows:
$$M_\Delta - M_{\Sigma^*} = M_{\Sigma^*} - M_{\Xi^*} = M_{\Xi^*} - M_\Omega$$
Using this formula (as Gell-Mann did) to predict the mass of the Ω^-. (Use the average of the first two spacing to estimated the third.) How close is your prediction to the observed value?
[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 1.6, page 57].
[*Answer:* M_Ω (observed) = 1672 MeV/ c^2]

Calc: 1683.5 MeV / c^2
Actual: 1672 MeV / c^2



5. Sketch the lowest-order Feynman diagram representing Delbruck scattering: $\gamma + \gamma \to \gamma + \gamma$. This process, the scattering of light by light, has no analog in classical electrodynamics.
[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 2.2, page 86].



$p_{\pi^-} = p_{\mu^-}\cos\theta$
$p_{\bar{\nu}_\mu} = p_{\mu^-}\sin\theta$
Thus, $\tan\theta = \frac{p_{\bar{\nu}_\mu}}{p_{\pi^-}}$

6. A pion traveling at speed v decays into a muon and neutrino, $\pi^- \to \mu^- + \bar{\nu}_\mu$. If the neutrino emerges at 90° to the original pion direction, at what angle does the μ come off?
[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 3.15, page 111].
[*Answer: tan* $\theta = (1 - m_\mu^2/m_\pi^2)/(2\beta\gamma^2)$]

7. Particle *A* (energy *E*) hits particle *B* (at rest), producing particles $C_1, C_2, \ldots$: $A + B \to C_1 + C_2 + \ldots + C_n$. Calculate the threshold (i.e., minimum *E*) for this reaction, in terms of the various particle masses.
[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 3.16, page 111].

[*Answer:* $E = \frac{M^2 - m_A^2 - m_B^2}{2m_B}c^2, \quad where\ M \equiv m_1 + m_2 + \cdots + m_n$]

8. Particle *A*, at rest, decays into particles *B* and *C* ($A \to B + C$).

   a. Find the energy of the outgoing particles, in terms of the various masses.

   [*Answer:* $E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A}c^2$]

   b. Find the magnitudes of the outgoing momenta.

$$\left(\begin{array}{l} Answer: \quad |\tilde{p}_B| = |\tilde{p}_C| = \frac{\sqrt{\lambda(m_A^2, m_B^2, m_C^2)}}{2m_A}c, \\ where\ \lambda\ is\ the\ so\text{-}called\ triangle\ function: \\ \quad \lambda(x,y,z) \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz. \end{array}\right)$$

   c. Note that λ factors: $\lambda(a^2, b^2 c^2) = (a + b + c)(a + b - c)(a - b + c)(a - b - c)$. Thus $|\tilde{p}_B|$ goes to zero when $m_A = m_B + m_C$, and runs imaginary if $m_A < (m_B + m_C)$. Explain. m_A = m_B + m_C is the threshold condition for the rxn to occur; B and C will have zero spatial momentum if the condition is true, and the rxn will not occur if RHS > LHS

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 3.19, page 112].

9. In reactions of the type $A + B \to A + C_1 + C_2 + \cdots$ (in which particle A scatters off particle B, producing $C_1, C_2, \ldots$), there is another inertial frame [besides the lab (B at rest) and the CM ($P_{TOT} = 0$)] which is sometimes useful. It is called the Breit, or "brick wall," frame, and it is the system in which A recoils with its momentum reversed ($\tilde{p}_{A\,after} = -\tilde{p}_{A\,before}$), as though it had bounced off a brick wall.

   Take the case of elastic scattering ($A + B \to A + B$); if particle A carries energy E, and scatters at an angle θ, in the CM, what is its energy in the Breit frame?

Find the velocity of the Breit frame (magnitude and direction) relative to the CM.

[This question is from the D J Griffiths, Introduction to Elementary Particles, 2nd Edition, Problem 3.24, page 112].