PC3261: Classical Mechanics II
Assignment 3

1. [25 pts] A uniform sphere has mass M and radius a. The shere is released from rest on a plane that is inclined at an angle β to the horizontal and for which the coefficient of static friction is μ_s.

(a) Suppose the sphere rolls without slipping. Determine the acceleration of the CM and the required frictional force in terms of M, g and β. Deduce that there is a critical angle of inclination β_c below which there is pure rolling motion and above which there is slipping.

(b) Suppose $\beta > \beta_c$ so that that slipping occurs. Determine the acceleration of the CM and the angular acceleration. Deduce a relation between the CM velocity and the angular velocity.

(c) Show that the mechanical energy of the sphere is constant during rolling but decreases quadratically with time during slipping.

2. [25 pts] A spool is constructed from two identical cylinders of radius a connected by a cylindrical axle of radius $b < a$. The mass of the spool is M and the moment of inertia of the spool about its axis is I. The spool is placed at rest on a horizontal plane and a constant external force F, making an angle β with the horizontal, is applied by pulling on an inextensible massless string wound around the axle. Determine the possible responses of the spool that involve pure rolling for $0 \leq \beta \leq \pi/2$. Also, calculate the maximum CM acceleration in pure rolling.

3. [25 pts] A uniform rope of length ℓ and mass M is on a table with a segment of length x_0 hanging over the edge. The rope is released from rest.

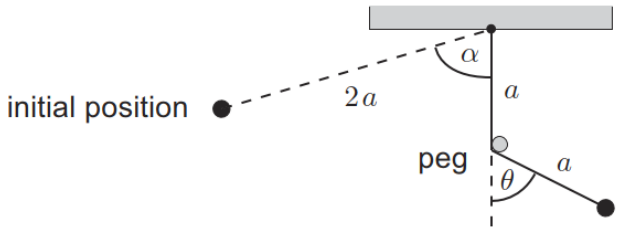
(a) Suppose the table is smooth, find the time t_s taken for the rope to slide off the table from Newton's second law.

(b) Solve the same problem by applying work-energy theorem, and compare the result with **(a)**.

(c) Suppose the motion is subjected to friction, with a coefficient of sliding friction μ between the rope and table. Find the time taken for the rope to slide off the table.

(d) Calculate the total loss in mechanical energy of the rope at time t_s in terms of μ, x_0/ℓ and $Mg\ell$.

4. [25 pts] A heavy ball is attached to a fixed point O by a light inextensible string of length $2a$. The ball is drawn back until the string makes an acute angle α with the vertical and is then released. A thin peg is fixed a distance a vertically below O in the path of the string as shown. In a game, the contestant chooses the value of α and wins a prize if the ball strikes the peg. Ignoring frictions, find the winning value of α.



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