

# 1 Relations for the Reflection Quotient

Represent the  $(3, n)$ -Torus knot as the closure of the braid word  $(\sigma_1 \sigma_2)^n$ . Consider the generating set  $S = \{a, b, c\}$  where  $a, b, c$  are as in Figure 1.

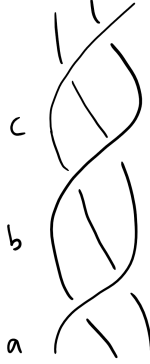


Figure 1: The generators of the reflection quotient

**Proposition 1.1.** *The generating relations for a  $(3, n)$ -Torus link are*

$T(3, 3k)$	$T(3, 3k + 1)$	$T(3, 3k + 2)$
$(cba)^{k-1}cbabc(abc)^{k-1}a$	$(cba)^{k-1}cbababc(abc)^{k-1}a$	$(cba)^k c(abc)^k a$
$(cba)^{k-1}cbc(abc)^{k-1}aba$	$(cba)^k bcb(abc)^k aba$	$(cba)^k bab(abc)^k aba$

*Proof.* Use the twist relations from Figure 2. □

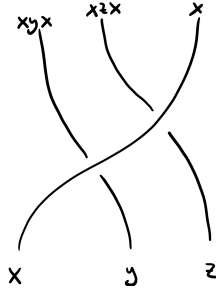


Figure 2: The relations induced by a twist

**Example 1.2.** For the  $(3, 4)$ -Torus knot, the above relations are  $cbababca$  and  $cbabcbabcbaba$ , so  $|ab| = |bc| = 3$  and  $|ac| = 2$  yields a Coxeter quotient.

**Conjecture 1.3.** *The  $(n, n+1)$ -Torus Knot has a Coxeter quotient isomorphic to  $S_n$ . More generally, so does the  $(n, m(n+1))$ -Torus Knot for all  $m \geq 1$ .*