Implementation of Primal-Dual Interior Point Methods Using MATLAB®

December 23, 2021

Overview

In this 2^{nd} report, there are three primal-dual interior point methods that are implemented using MATLAB®, which are:

- Central Path method of fixed step size α, and centering parameter σ.
- Central Path method of adaptive step size α, and centering parameter σ.
- Mehrotra Predictor-corrector method.

Where the results of these methods are shown in the following sections.

Problems to be Solved

All three implementations are used to solve three different problems, one with only *equality* constraints, another with only *inequality* constraints, and the latter has *mixed* constraints.

1^{st} Problem: Euqality-constraints Problem

The formulation of the first problem comes as follows:

$$\begin{aligned} min \;\; & z = x_1 + x_3 + 3x_4 \\ s.t. \;\; & 2x_1 + 2x_3 + 3x_4 = 10 \\ & -2x_2 - 2x_3 - 6x_4 = -6 \\ & x_i \geq 0, \; for \; i = 1, 2, 3, 4 \end{aligned}$$

2^{nd} Problem: Ineuqality-constraints Problem

The formulation of the first problem comes as follows:

$$\begin{aligned} min \;\; & z = 5x_1 + 2x_2 - 4x_3 \\ s.t. \;\; & 6x_1 + x_2 - 2x_3 \geq 5 \\ & x_1 + x_2 + x_3 \leq 4 \\ & 6x_1 + 4x_2 - 2x_3 \geq 10 \\ & x_i \geq 0, \; for \; i = 1, 2, 3 \end{aligned}$$

3^{rd} Problem: Another Inequality-constraint Problem

The formulation of the first problem comes as follows:

$$\begin{aligned} min \;\; & z = -30x_1 - 20x_2 \\ s.t. \;\; & 2x_1 + x_2 \leq 8 \\ & x_1 + 3x_2 \leq 8 \\ & \\ & x_i \geq 0, \; for \; i = 1, 2 \end{aligned}$$

Central Path Method

The function script submitted with this file, named ***Central_Path_IPM***, takes in:

- a matrix of constraints coefficients, A.
- Vectors of constants b and c.
- The types of the constraints, *Equal*, 0 for equality constraints, and 1 and -1 for greater-than-or-equal and less-than-or-equal constraints, respectively.
- User's method choice whether it is fixed- or adaptive-α method.
- The centering parameter σ.
- The fixed step size α, when the method is fixed central path.

and, returns x^*, λ^*, s^*, set of investigated x's and s's.

Mehrotra Predictor-corrector Method

The function script submitted with this file, named ***Mehrotra_IPM***, takes in:

- a matrix of constraints coefficients, A.
- Vectors of constants b and c.
- The types of the constraints, *Equal*, 0 for equality constraints, and 1 and -1 for greater-than-or-equal and less-than-or-equal constraints, respectively.

and, returns x^*, λ^*, s^*, set of investigated x's and s's and c, if updated.

1^{st} Problem Results

In this subsection, figures of 1^{st} problem objective function v.s. iterations, complementary condition and its central path. The fixed $\alpha = 0.45$ and the centering parameter $\sigma = 0.1$

Reduced objective function figures

In *Figure 1*, the plot of the objective function vs iterations for fixed-α central path method is shown, where the adaptive-α central path method is shown in *Figure 2*.

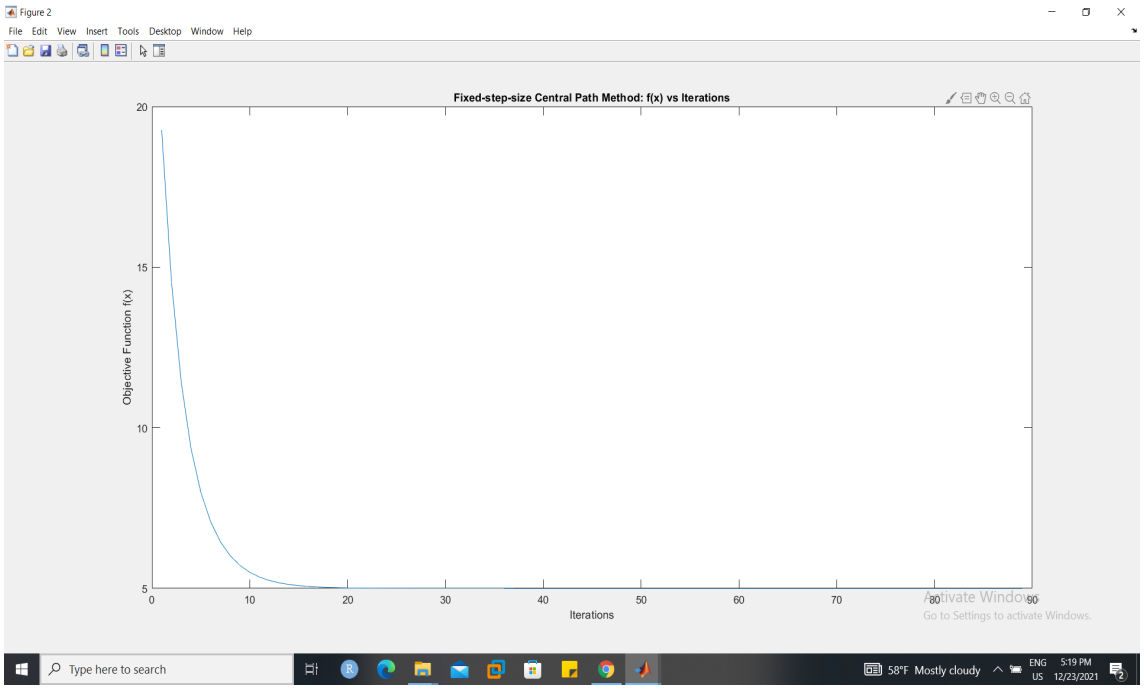

Figure 1: Fixed-α central path method result

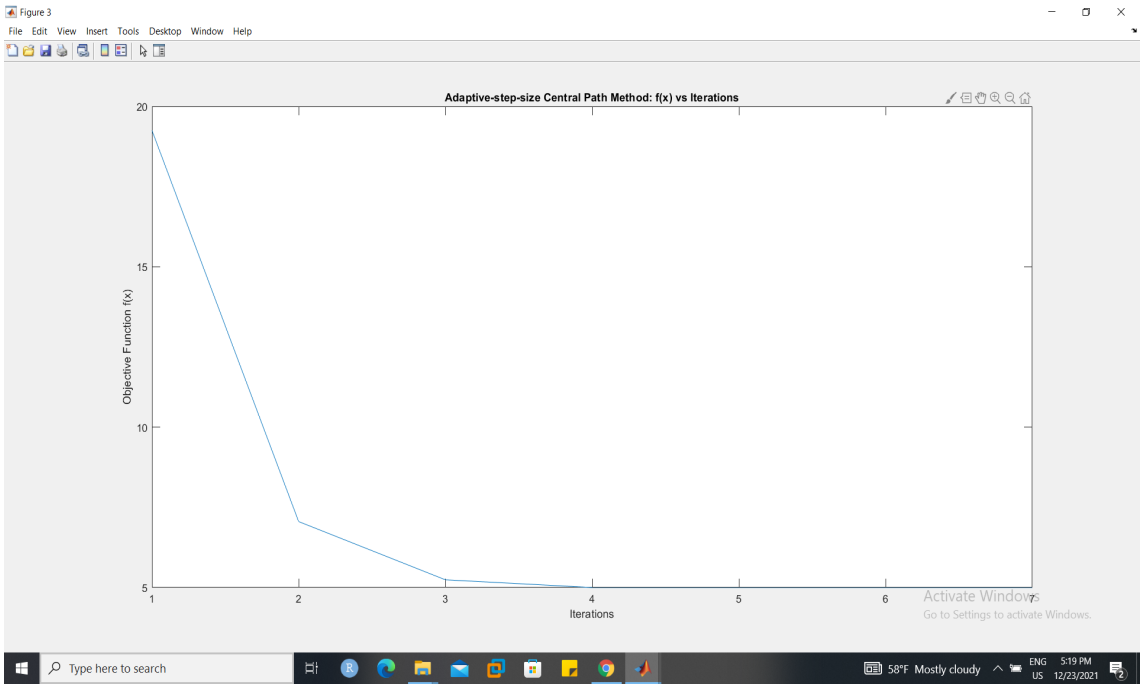

Figure 2: Adaptive-α central path method

in the following figure the Mehrotra method result is shown.

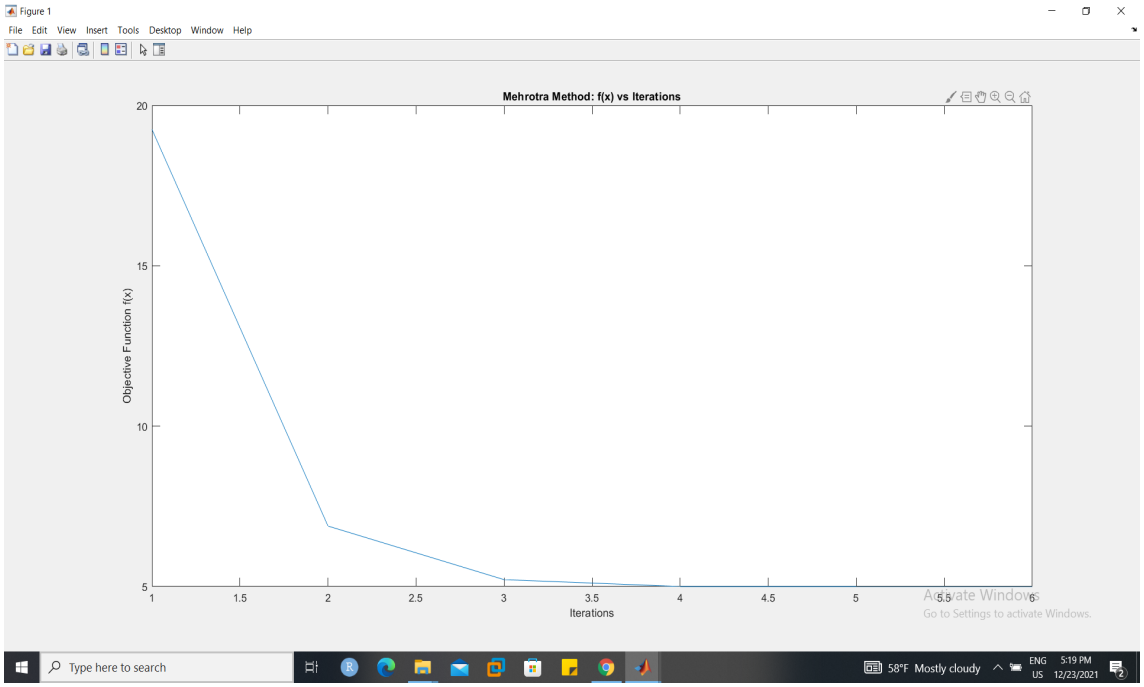

Figure 3: Mehrotra method result

Central path figures

In *Figure 4*, the plot of the x_1 a vs x_2 for fixed-α central path method is shown, where the adaptive-α central path method is shown in *Figure 5*.

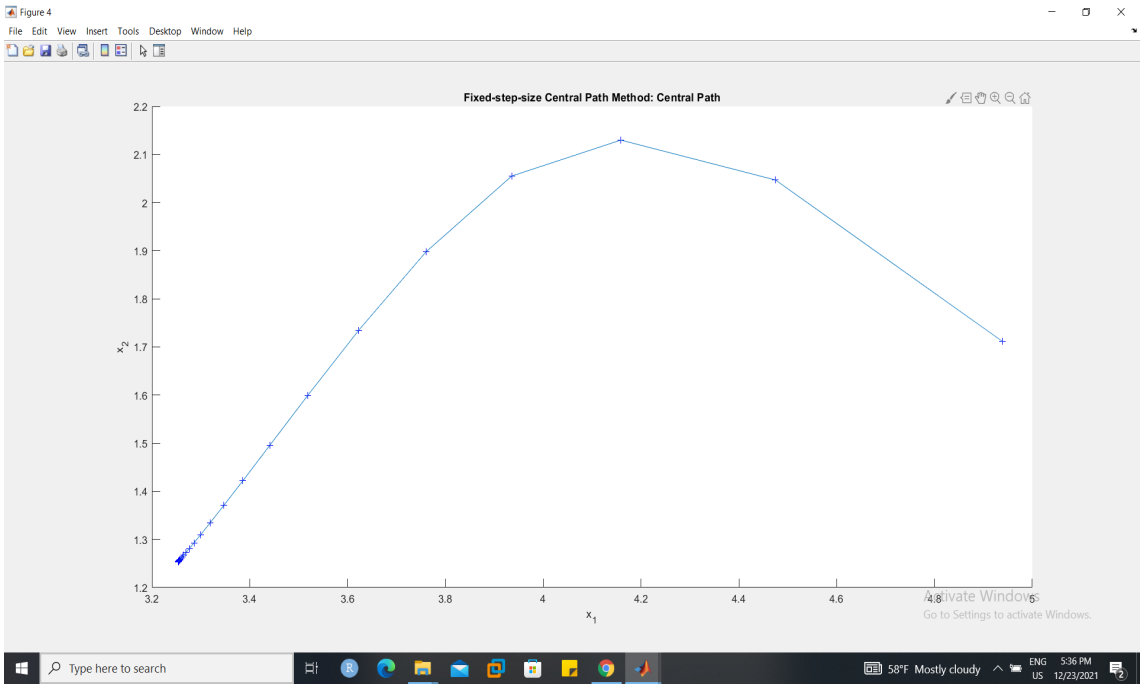

Figure 4: Fixed-α central path method result

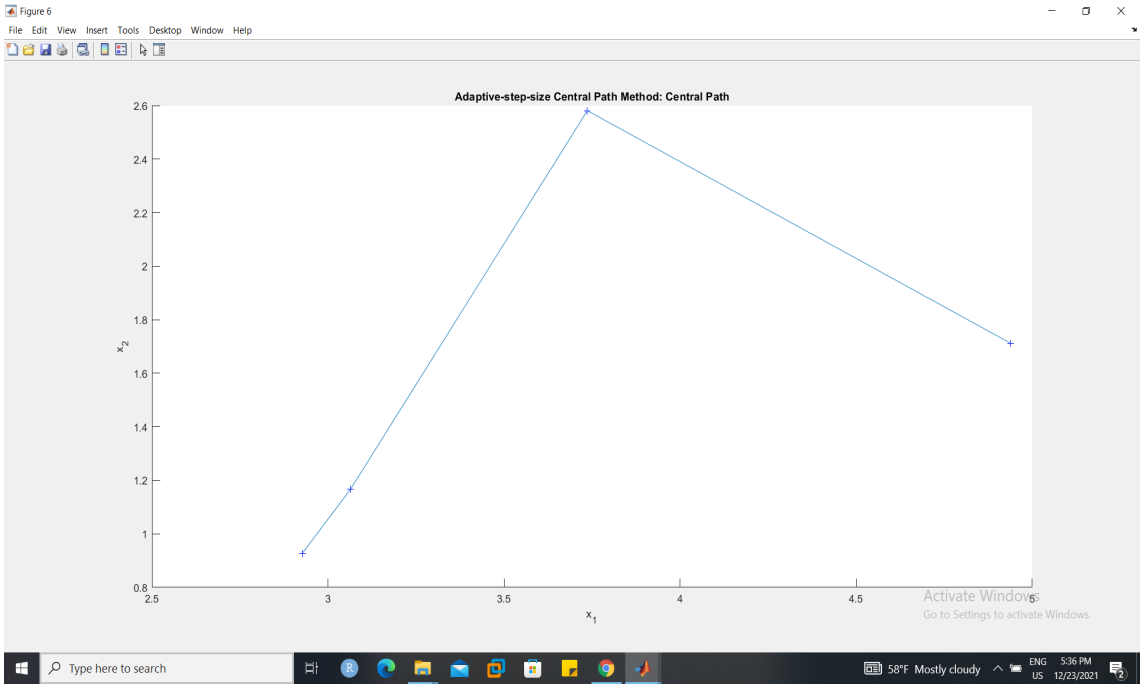

Figure 5: Adaptive-α central path method result

In *Figure 6*, the central path of the Mehrotra method is shown.

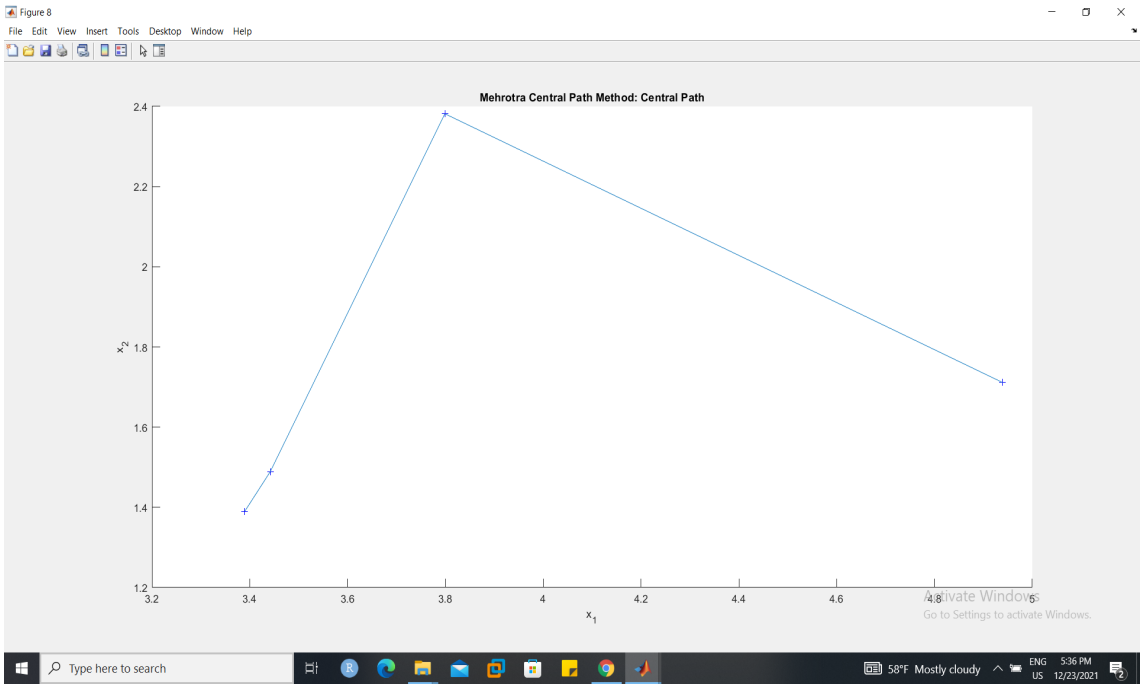

Figure 6: Mehrotra method result

Complementary condition figures

In *Figure 7*, the plot of the $x_1 s_1$ a vs $x_2 s_2$ for fixed-α central path method is shown, where the adaptive-α central path method is shown in *Figure 8*.

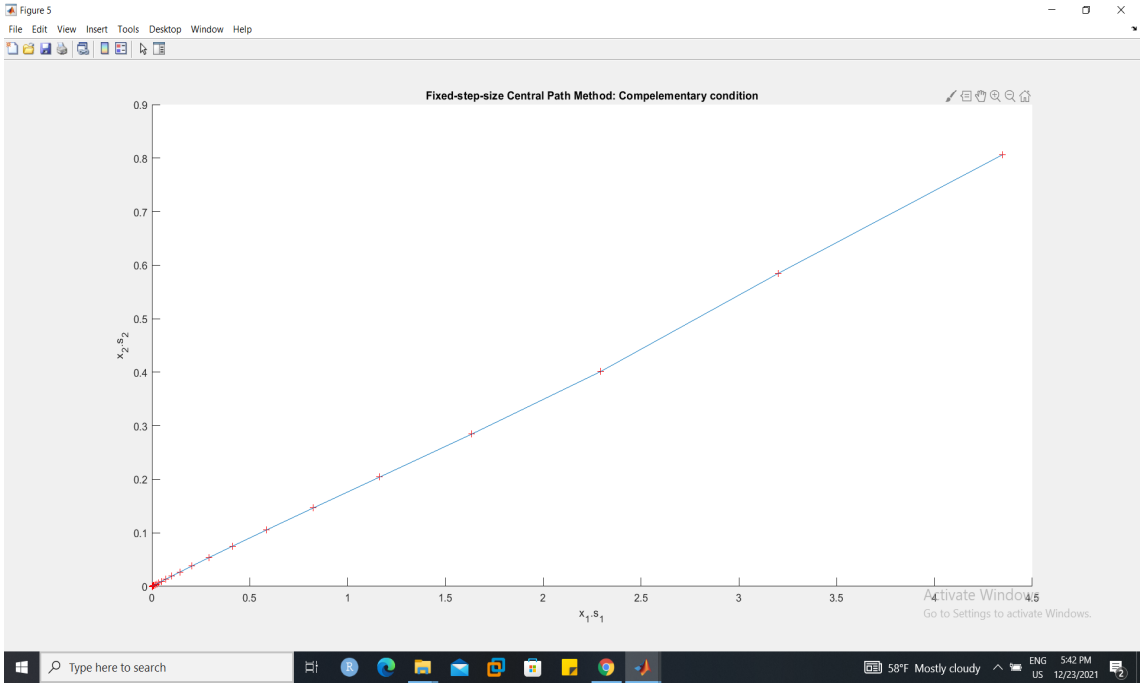

Figure 7: Fixed-α central path method result

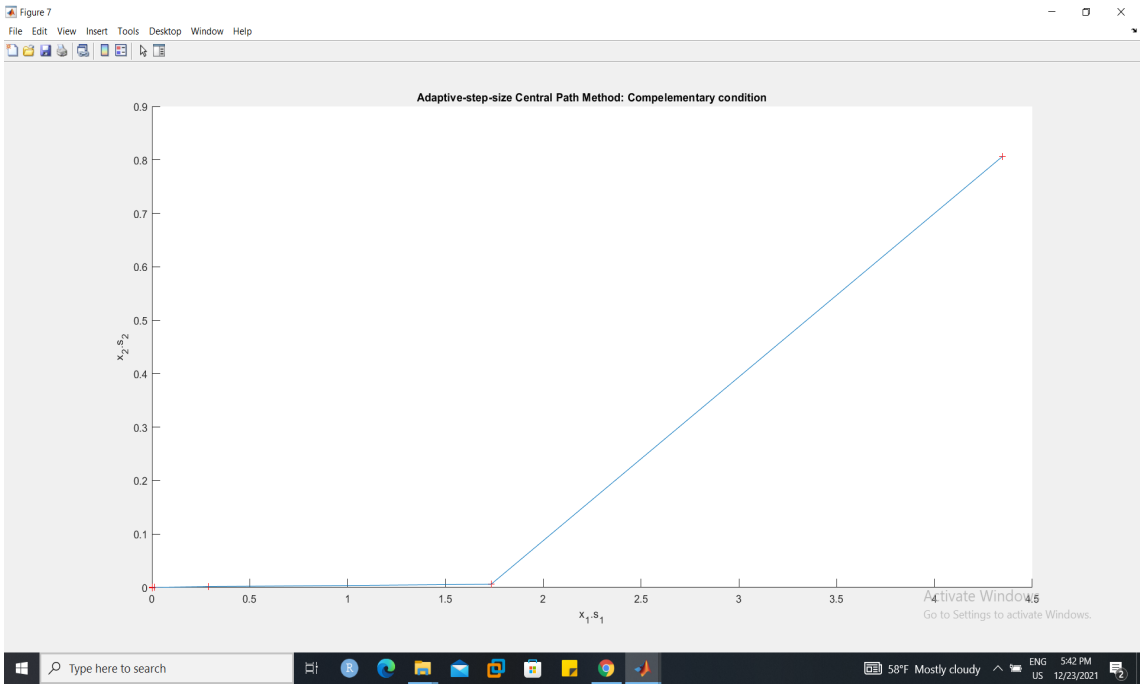

Figure 8: Adaptive-α central path method result

In *Figure 9*, the central path of the Mehrotra method is shown.

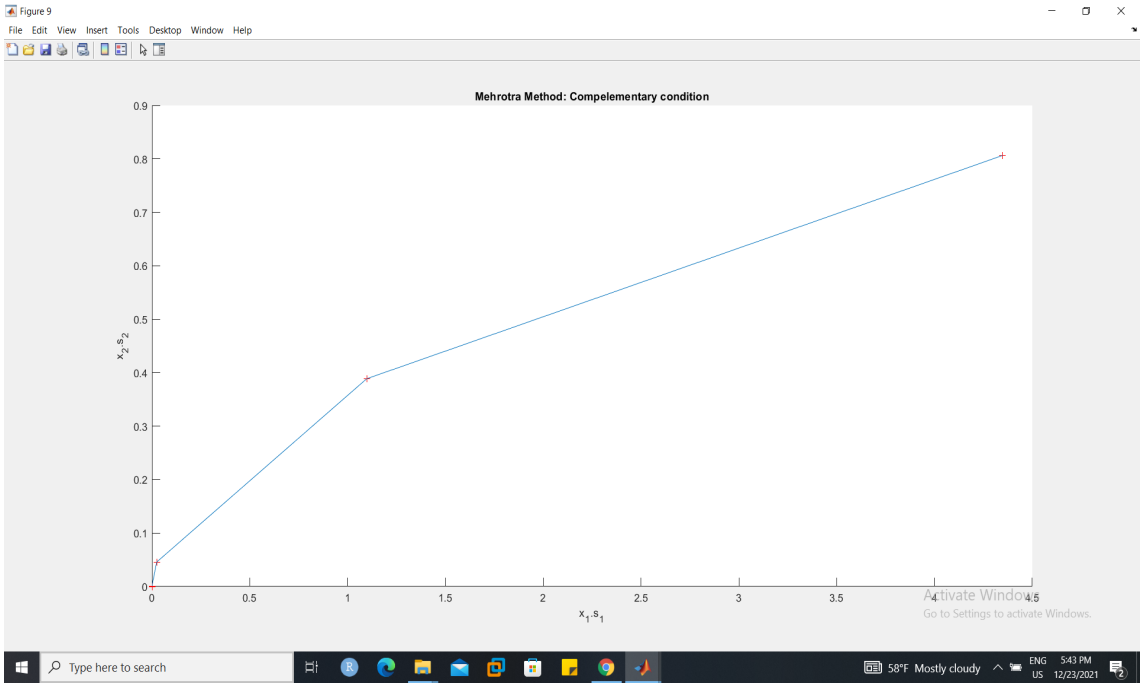

Figure 9: Mehrotra method result

2^{nd} Problem Results

In this subsection, figures of 2^{nd} problem objective function v.s. iterations, complementary condition and its central path. The fixed $\alpha = 0.45$ and the centering parameter $\sigma = 0.1$

Reduced objective function figures

In *Figure 10*, the plot of the objective function vs iterations for fixed-α central path method is shown, where the adaptive-α central path method is shown in *Figure 11*.

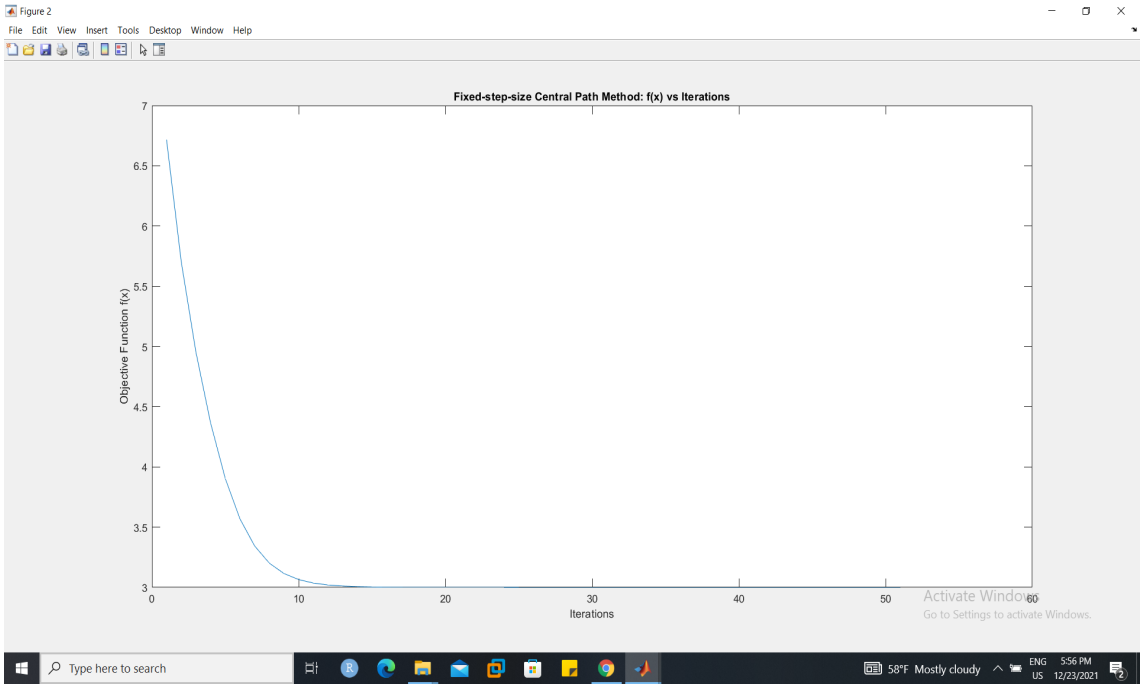

Figure 10: Fixed-α central path method result

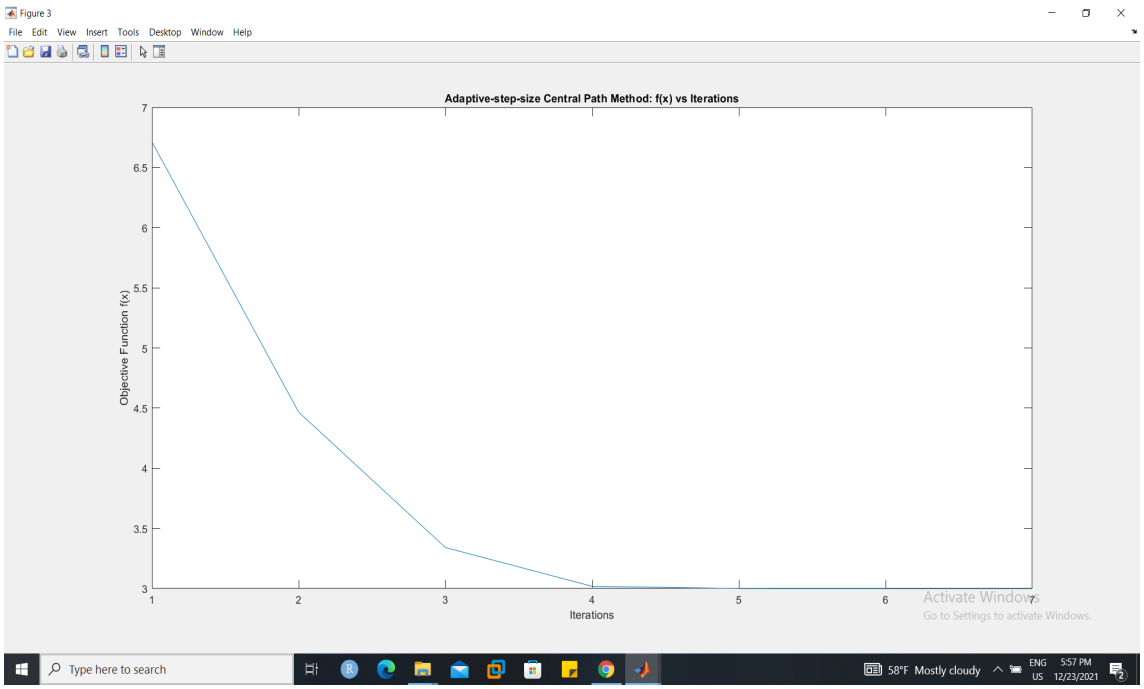

Figure 11: Adaptive-α central path method

in the following figure the Mehrotra method result is shown.

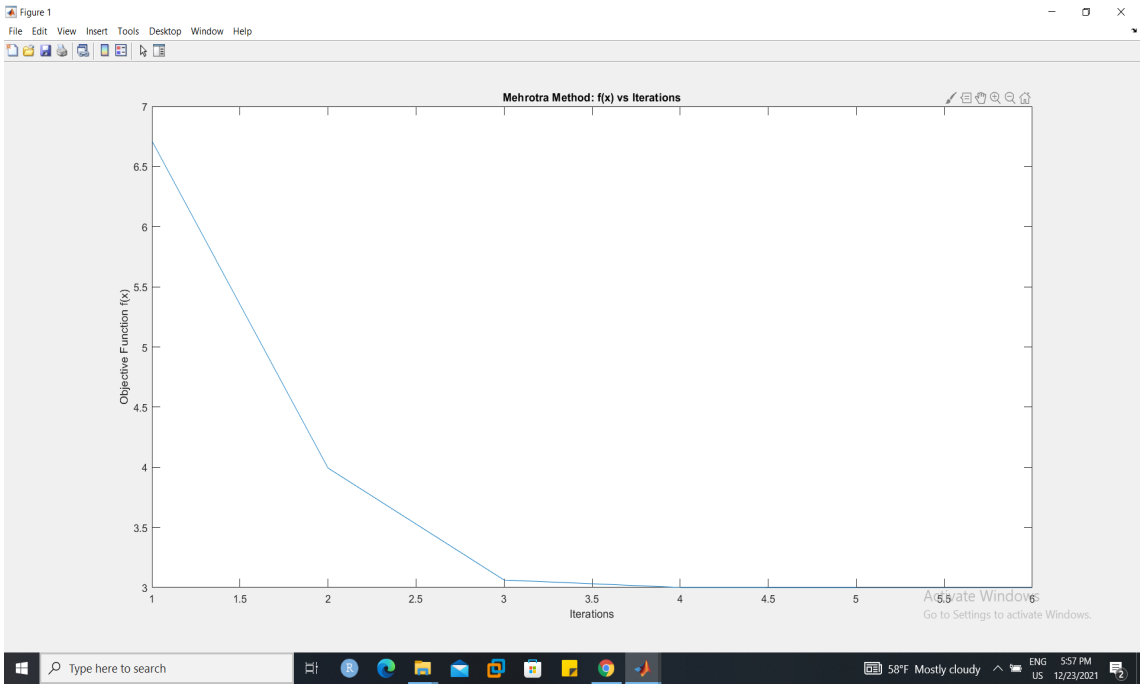

Figure 12: Mehrotra method result

Central path figures

In *Figure 13*, the plot of the x_1 a vs x_2 for fixed-α central path method is shown, where the adaptive-α central path method is shown in *Figure 14*.

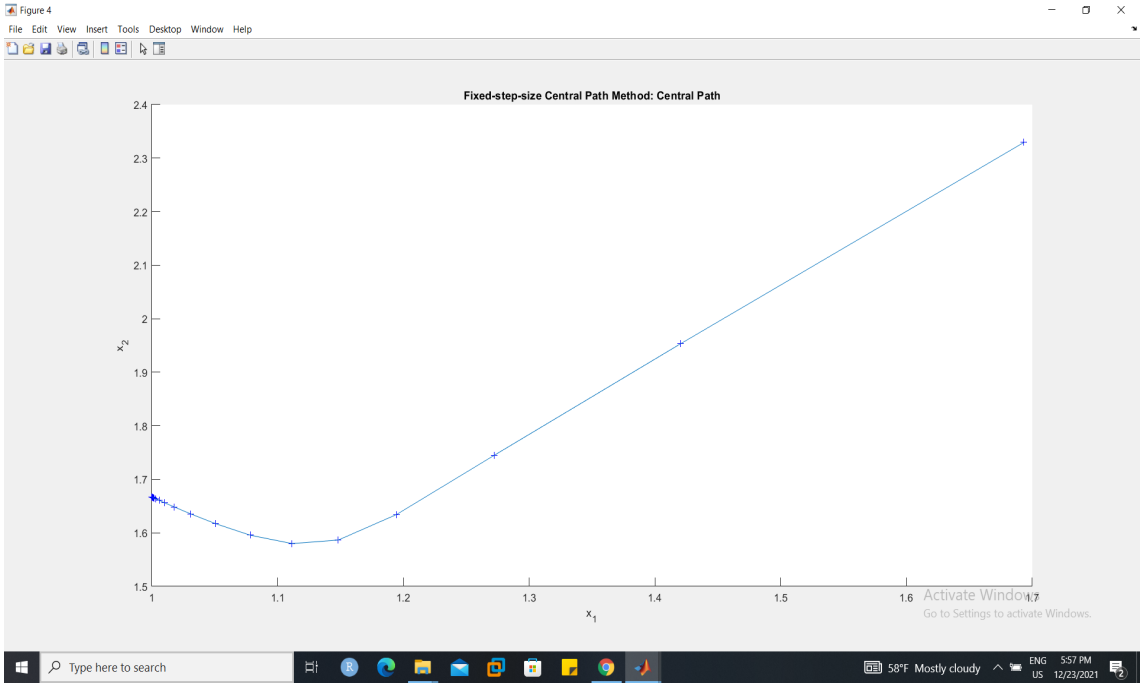

Figure 13: Fixed-α central path method result

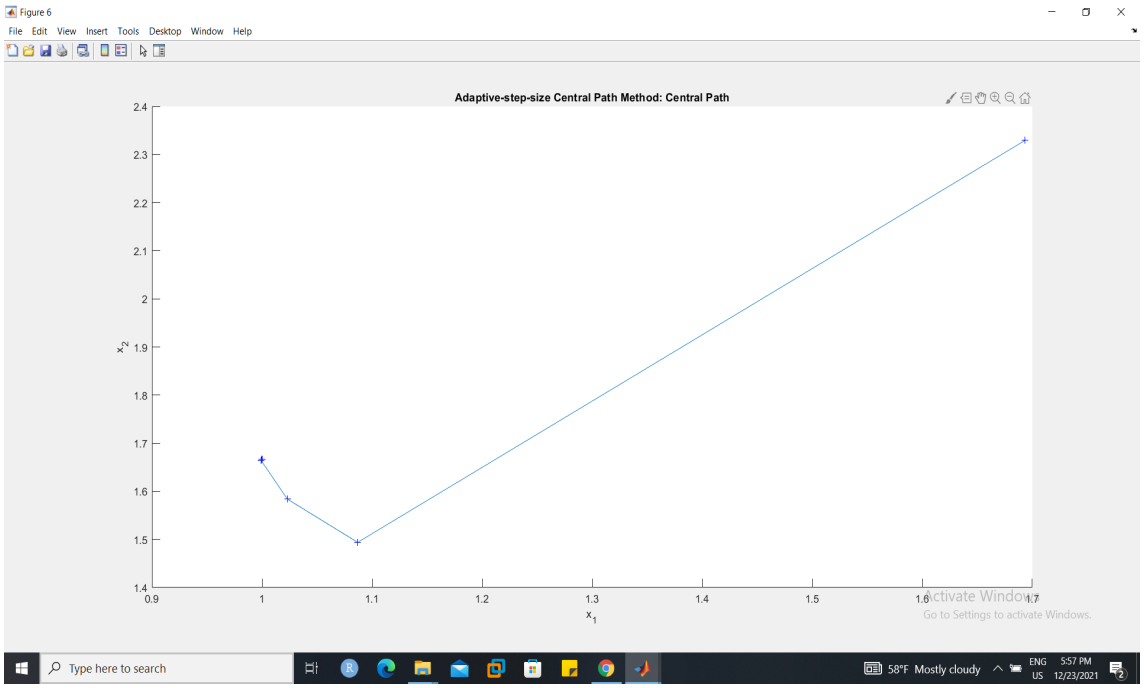

Figure 14: Adaptive-α central path method result

In *Figure 15*, the central path of the Mehrotra method is shown.

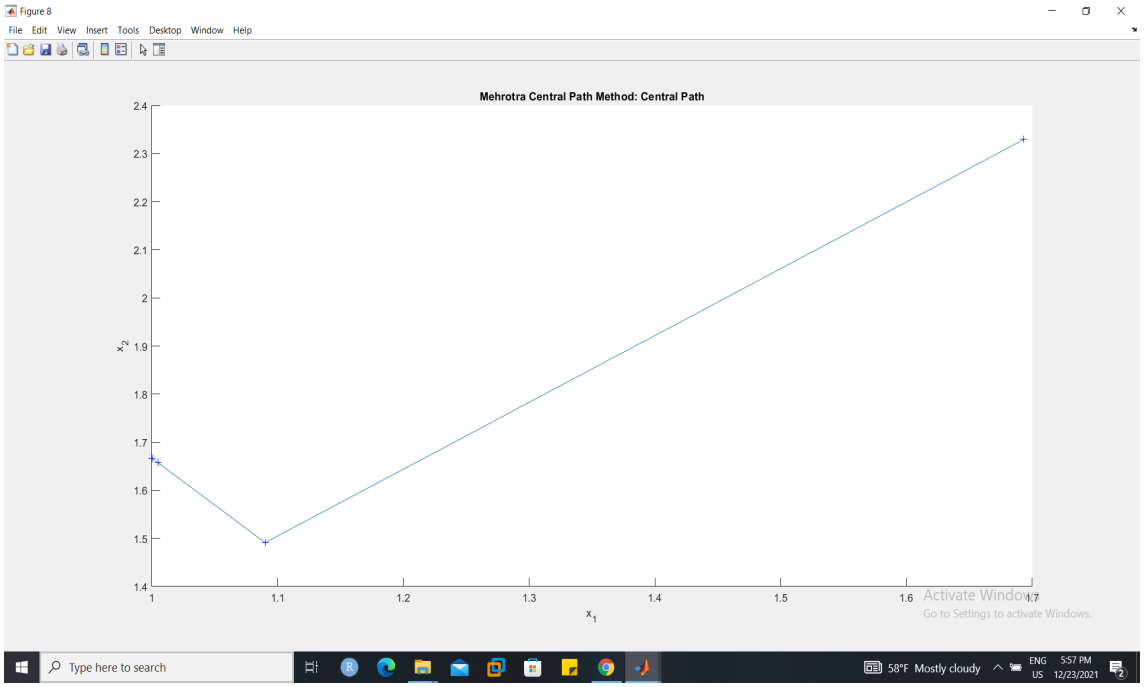

Figure 15: Mehrotra method result

Complementary condition figures

In *Figure 16*, the plot of the $x_1 s_1$ a vs $x_2 s_2$ for fixed-α central path method is shown, where the adaptive-α central path method is shown in *Figure 17*.

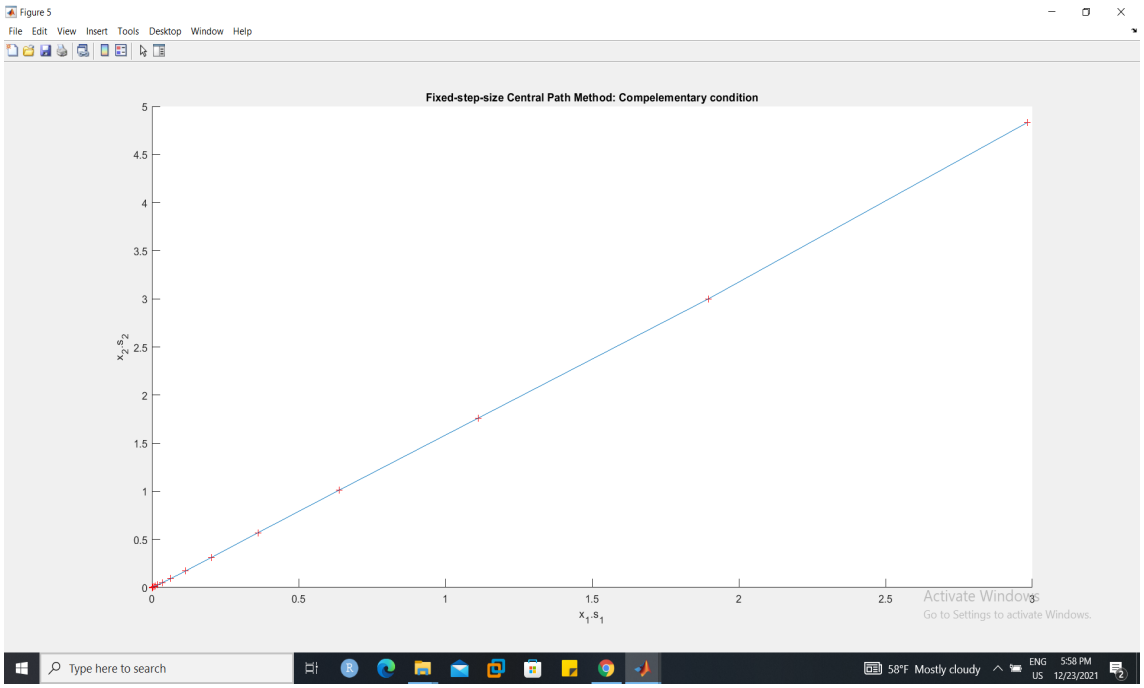

Figure 16: Fixed-α central path method result

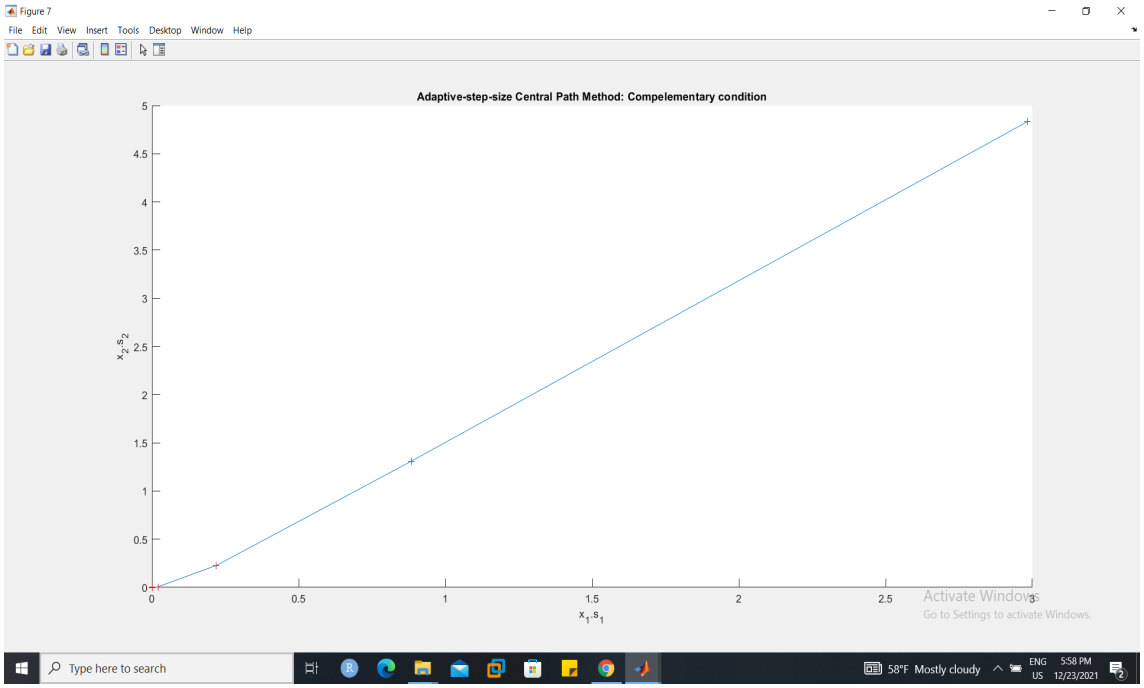

Figure 17: Adaptive-α central path method result

In *Figure 18*, the central path of the Mehrotra method is shown.

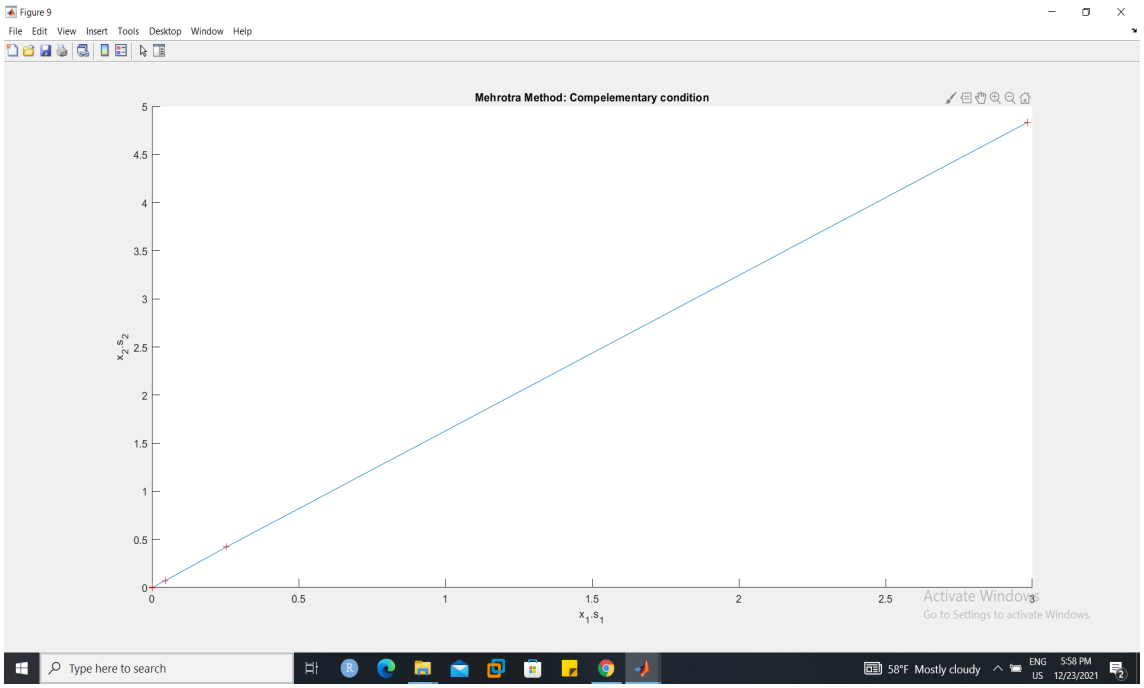

Figure 18: Mehrotra method result

3^{rd} Problem Results

In this subsection, figures of 3^{rd} problem objective function v.s. iterations, complementary condition and its central path. The fixed $\alpha = 0.45$ and the centering parameter $\sigma = 0.1$

Reduced objective function figures

In *Figure 19*, the plot of the objective function vs iterations for fixed-α central path method is shown, where the adaptive-α central path method is shown in *Figure 20*.

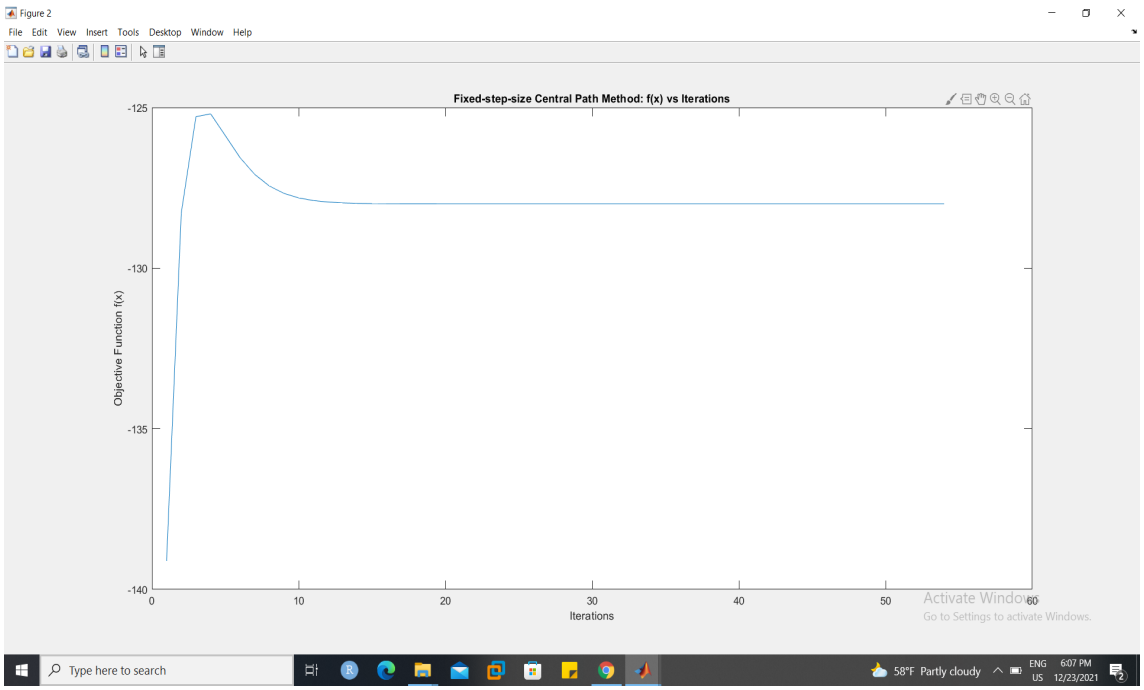

Figure 19: Fixed-α central path method result

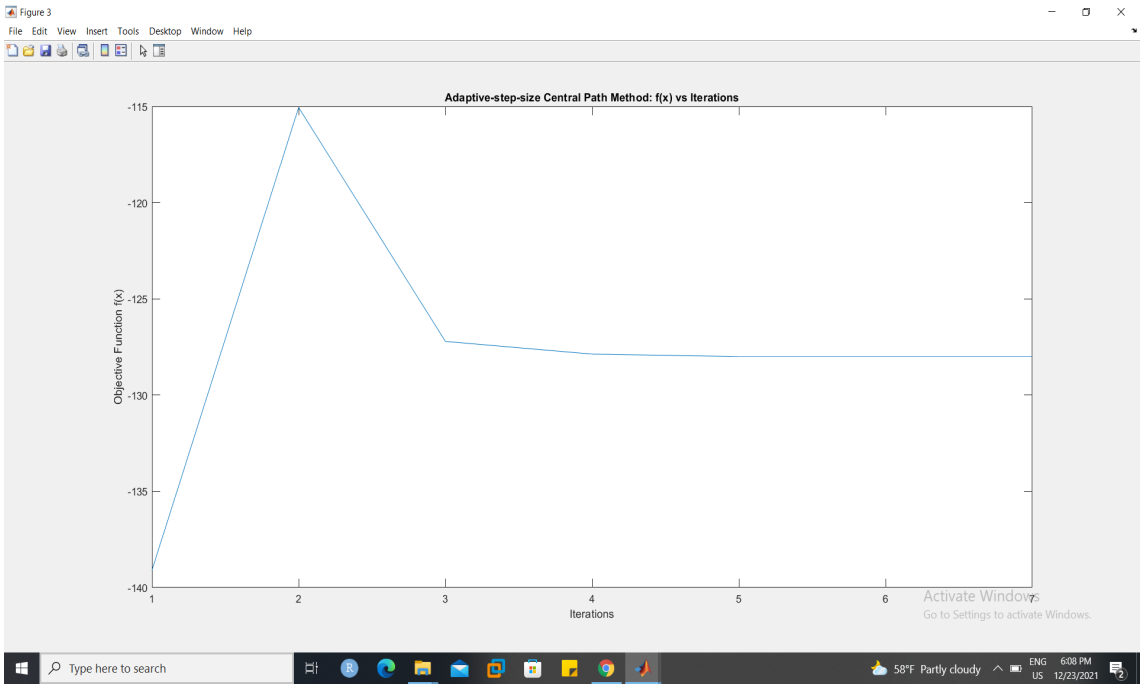

Figure 20: Adaptive-α central path method

in the following figure the Mehrotra method result is shown.

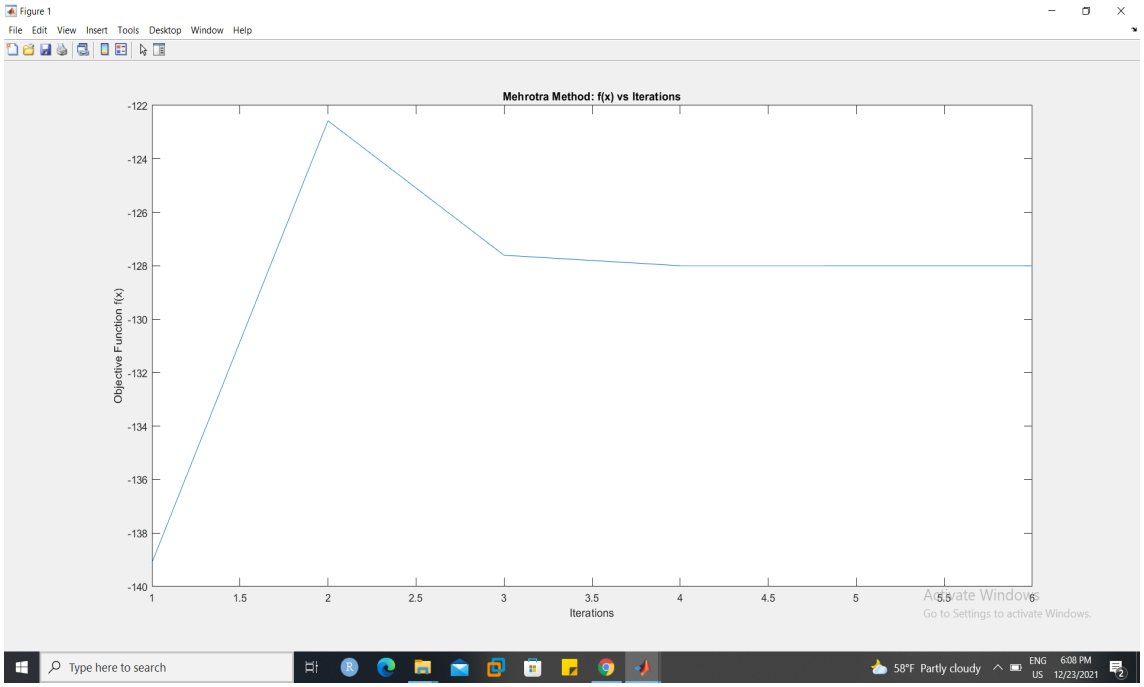

Figure 21: Mehrotra method result

Central path figures

In *Figure 22*, the plot of the x_1 a vs x_2 for fixed-α central path method is shown, where the adaptive-α central path method is shown in *Figure 23*.

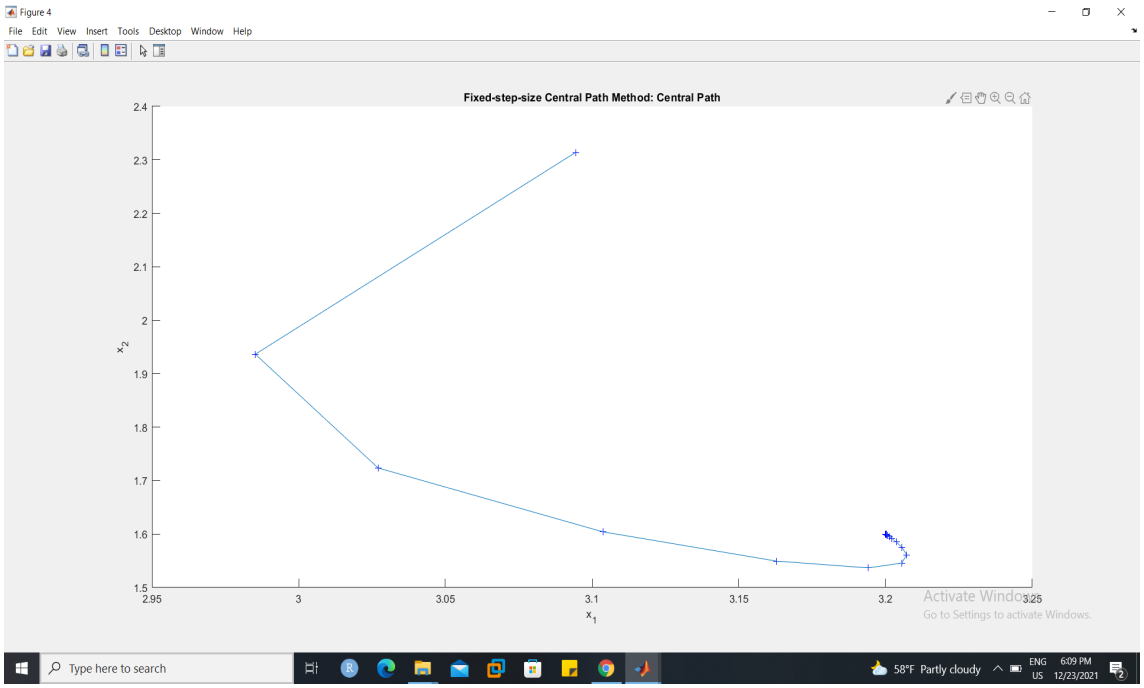

Figure 22: Fixed-α central path method result

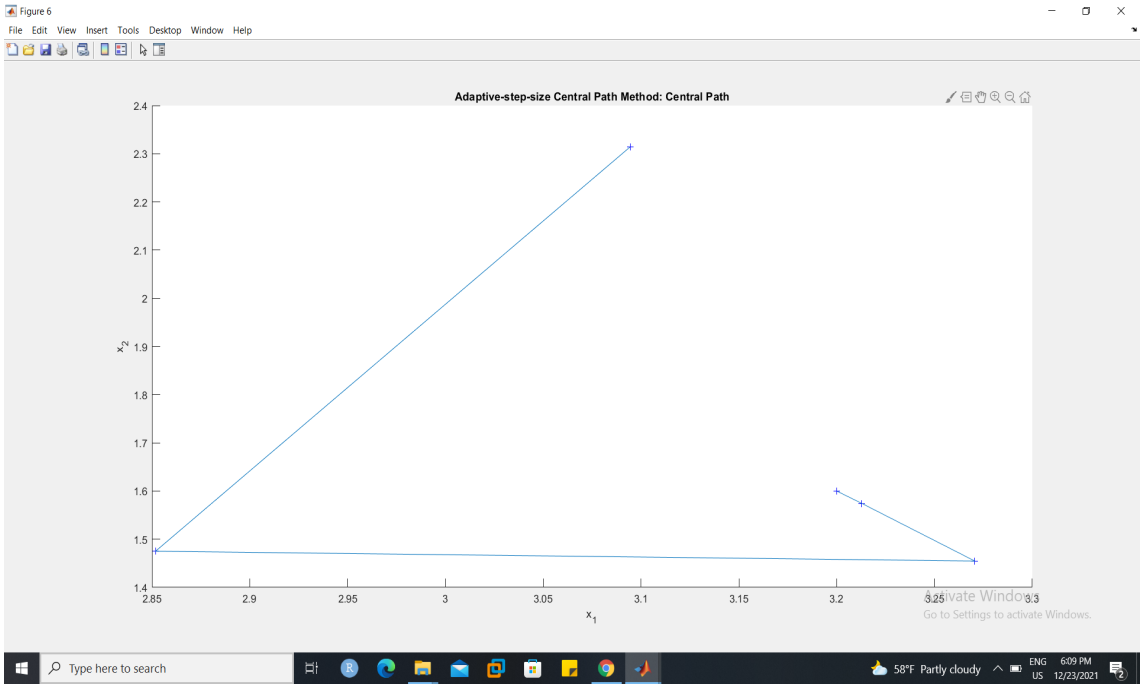

Figure 23: Adaptive-α central path method result

In *Figure 24*, the central path of the Mehrotra method is shown.

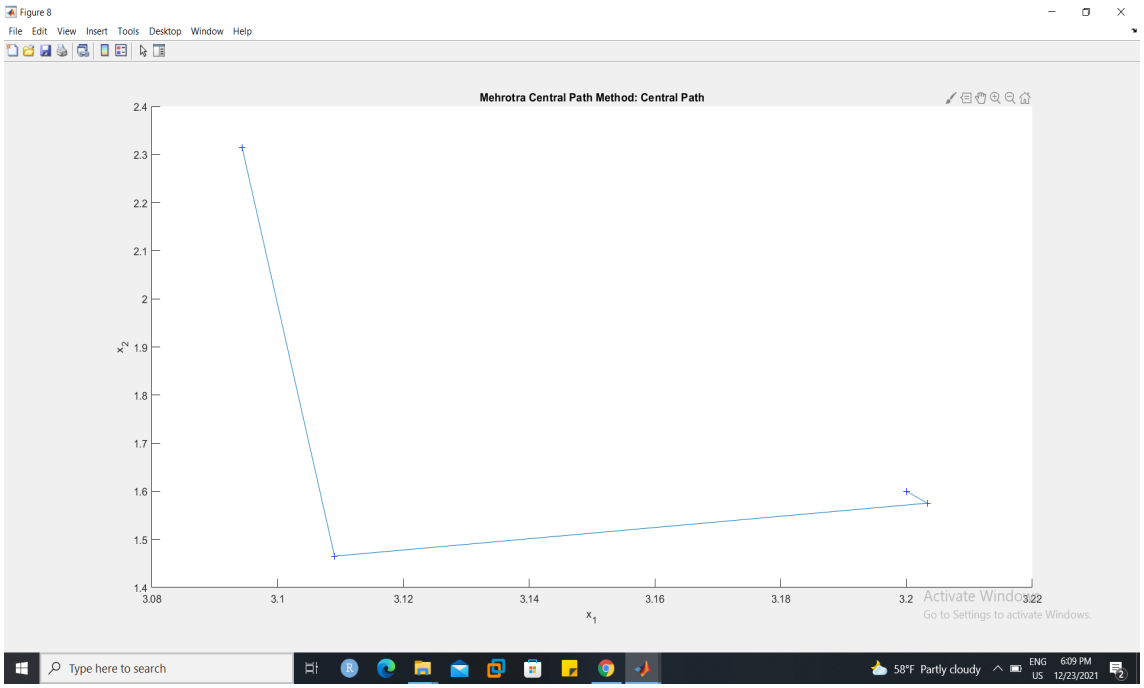

Figure 24: Mehrotra method result

Complementary condition figures

In *Figure 25*, the plot of the $x_1 s_1$ a vs $x_2 s_2$ for fixed-α central path method is shown, where the adaptive-α central path method is shown in *Figure 26*.

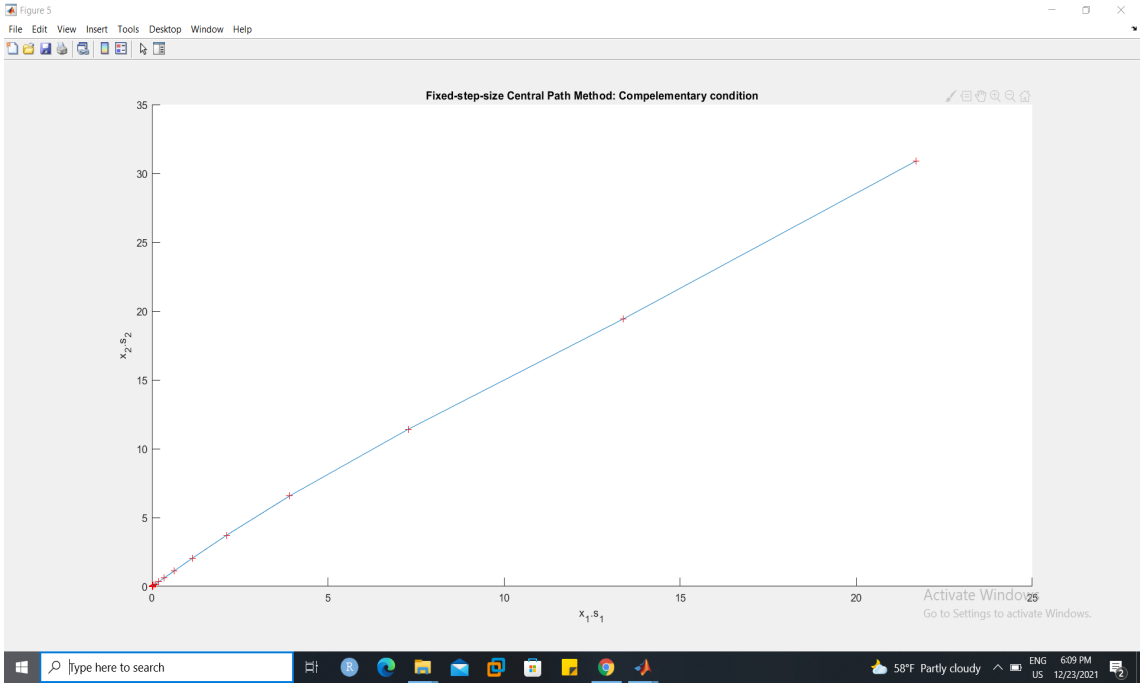

Figure 25: Fixed-α central path method result

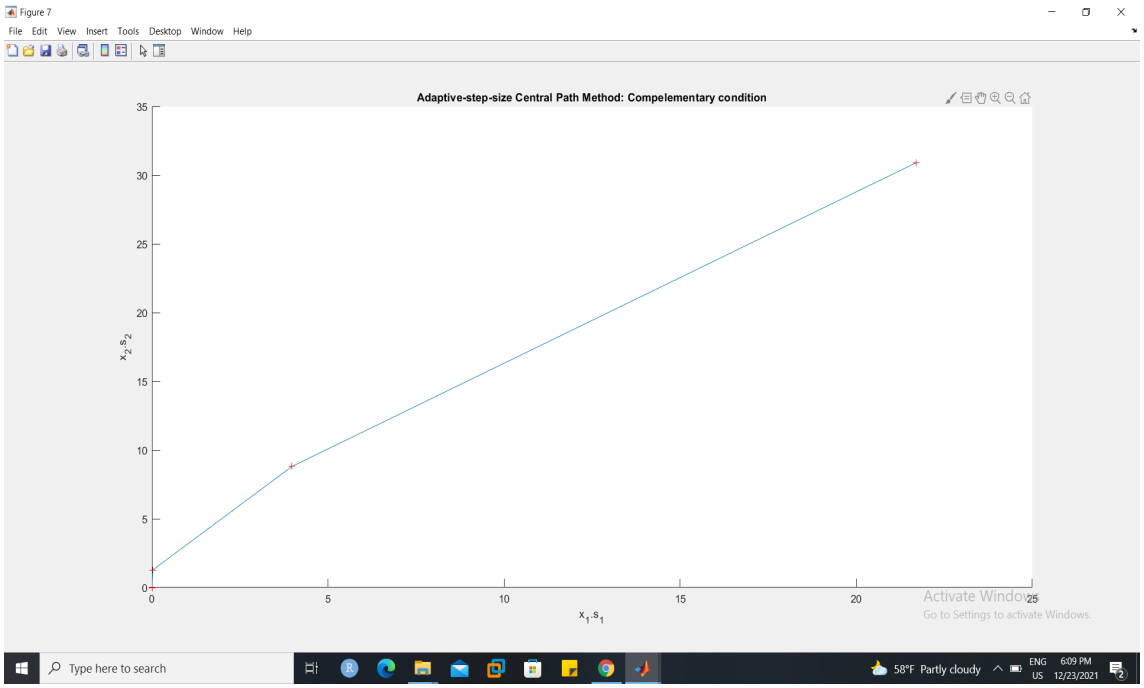

Figure 26: Adaptive-α central path method result

In *Figure 27*, the central path of the Mehrotra method is shown.

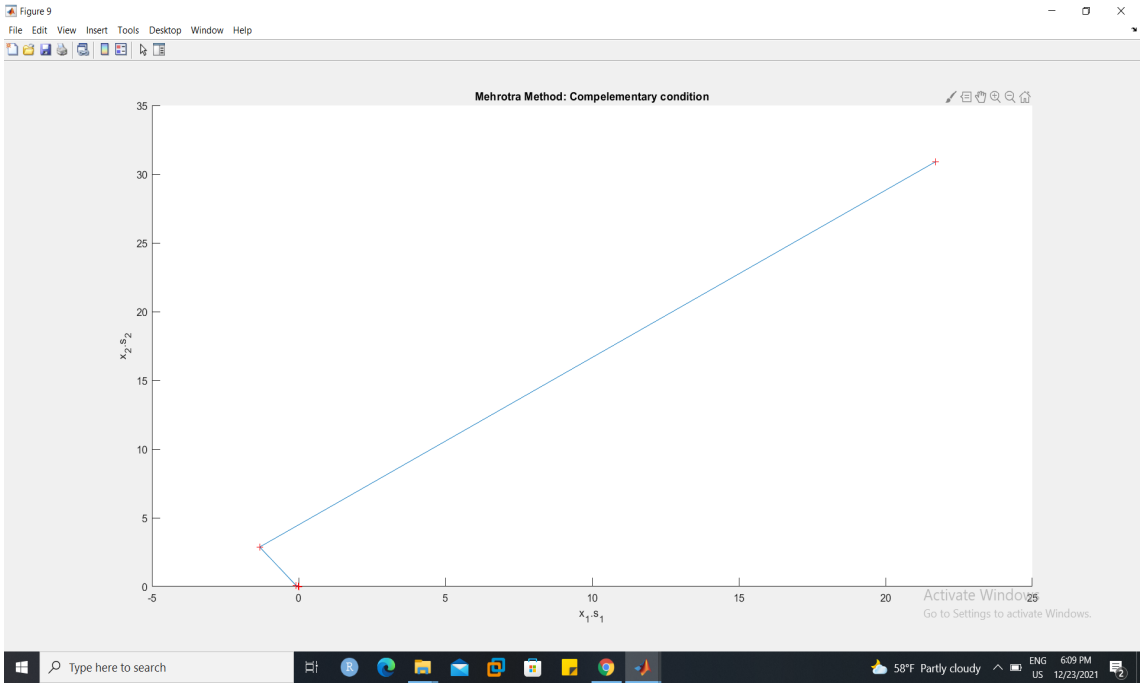

Figure 27: Mehrotra method result