CIE 327 Course Project Report (Part I)

Rawan Eldalil
s-rawaneldalil@zewailcity.edu.eg

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Overview

In the Probability and Stochastic Processes (CIE 327) course, there is a significant focus on the stochastic processes and how to evaluate and calculate their associated statistics. For further emphasis on this topic, the project is all about implementing a program that calculates those statistics given the ensemble of the process. Using Python aided with Tkinter Graphical User Interface (GUI) tool, a program that asks the user to enter the number of sample functions associated with the process, and also those sample functions and other necessary inputs for statistics calculation to get the process-associated statistics is created.

How the program works

As soon as one runs the program, a window as shown in **Figure 1** that asks the user to enter the number of sample functions of the process, and as soon as one clicks the 'Enter' button after typing the number of sample functions, another label appears asks the user to enter the first sample function. And, that label

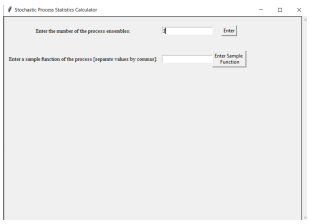

Figure 1: Entry of the number of sample functions of the process

is prompted every single time one clicks on the 'Enter Sample Function' button, as long as the number of sample functions entered is below the number entered by the user. Check **Figure 2** for illustration.

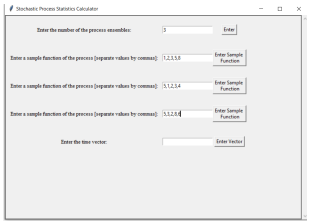

Figure 2: Entry of the sample functions.

After the user enters all the entered number of sample functions, a label appears to get the time vector entered by the user. Once the user gets the time vector in the entry and clicks on 'Enter Vector' button, the layout develops to one like in **Figure 3**, where the user is, then, asked to enter the number of the sample functions he/she wants to view. As soon as the user clicks on 'View Ensembles' button, the sample functions are plotted as shown in **Figure 4** and a set of buttons shown in **Figure 5** appears on the frame for different statistical calculations. Now, let's turn to each button in the set and see what it does. In

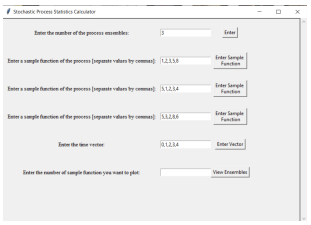

Figure 3: Time Vector Entry.

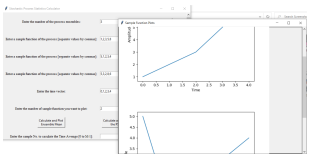

Figure 4: Sample Functions Plots.

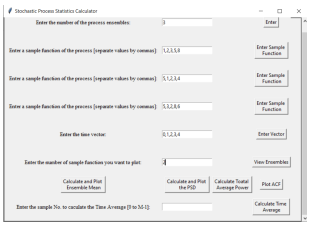

Figure 5: Button Set.

Figure 6, a three-dimensional plot of the Auto-Correlation Function (ACF) of the random process pops up once the user clicks on the 'Plot ACF' button. For the 'Calculate Total Average Power' button, once clicked,

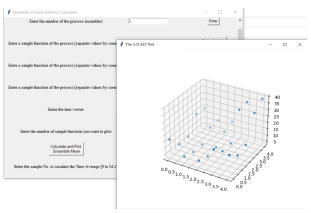

Figure 6: ACF 3D Plot.

a messagebox, like in **Figure 7**, pops up to show the total average power of the process. While clicking the

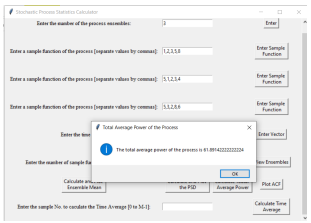

Figure 7: Time Average of the nth sample function.

'Calculate and Plot the PSD' button gets one a plot of the Power Spectral Density (PSD) of the process as shown in **Figure 8**. The 'Calculate and Plot Ensemble Mean' button in **Figure 9** plots the ensemble mean of the process once clicked. Notice in **Figure 4** that another label with an entry and a button next to it to guide the user to enter n of the n^{th} sample function for the time average calculation. Once the user clicks on the 'Calculate Time Average' button, a messagebox appears before the user with the time average result on it; this is illustrated in **Figure 10**. After the button 'Ok' on the messagebox is clicked, another label asks

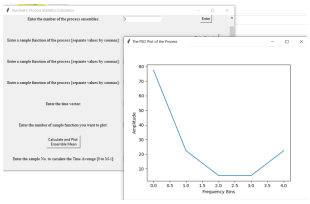

Figure 8: PSD Plot.

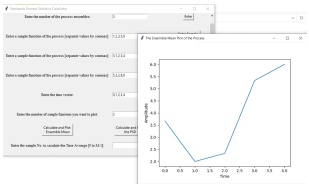

Figure 9: The Ensemble Mean Plot.

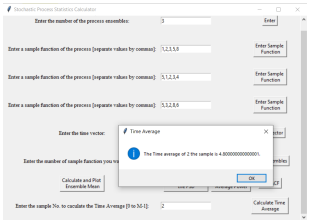

Figure 10: The Time Average Result

that user to enter i and j separated by a comma associated with the i^{th} and j^{th} samples of the process for the ACF calculation between the samples. As soon as the user submits i and j, a messagebox is prompted displaying the ACF result as shown in **Figure 11**. After the user clicks on 'Enter i and j' button, another entry field is displayed to enable the user to enter n of the n^{th} sample function for the time ACF calculation. Once the button 'Caculate Time ACF' is clicked, a plot of the time ACF against the time difference between the n^{th} sample function samples pops up as shown in **Figure 12**.

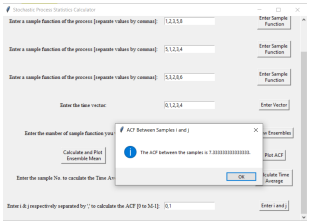

Figure 11: The ACF between the i^{nth} and j^{nth} Samples of the Process

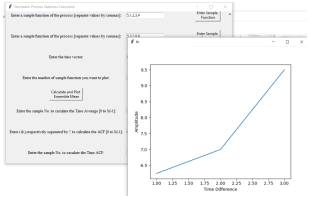

Figure 12: The Time ACF of the n^{th} Sample Function

Cosine Random Process Statistics Calculation

The random process info

The random process is generated in the 'Generate Cosines' cell; it consists of 7 cosine sample functions. The angular frequency (i.e. Omega-c) is 2π, and the amplitude of the sinusoid is equal to 5. The time vector entered to the program is 1000-sample array ranging from -5 to 5. The thetas of the sample functions of process are:

- The first sample function has a value of theta equals to $\pi/7$
- The second sample function has a value of theta equals to $\pi/6$
- The third sample function has a value of theta equals to $\pi/5$
- The forth sample function has a value of theta equals to $\pi/4$
- The fifth sample function has a value of theta equals to $\pi/3$
- The sixth sample function has a value of theta equals to $\pi/2$
- The seventh sample function has a value of theta equals to π

Results

The Ensemble Mean Plot

The ensemble mean plot of the cosine random process shown in **Figure 13** is sinusoidal; it is because the number of the sample functions is small, and not all possible values of theta are included as in the case of continuous form of the process that has the value of the ensemble mean equals to zero.

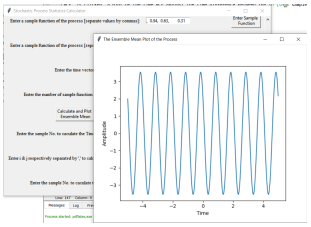

Figure 13: The ensemble mean plot of the process

The ACF 3-D Plot

The large number of the ACF points in the 3-D space shown in **Figure 14** makes it somehow difficult for one to distinguish the shape of the function, yet one observes a pattern keeps repeating on the edges of the silhouette of the plot which looks like a sinusoid.

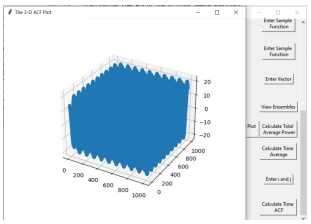

Figure 14: The 3D ACF Plot

The PSD Plot

The PSD plot in **Figure 15** show two peaks (or impulses) at two distinctive frequencies. Actually, it should have a peak at one frequency which is 2π; the other one is just repeated because of how the FFT function works.

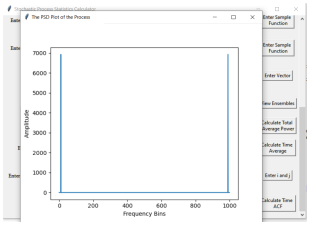

Figure 15: The PSD plot of the process

The Time Average of the 6^{th} Sample

The time average of the 6^{th} sample function is shown in **Figure 16**, and it approximately equals to $-1.3489 * 10^{-19}$.

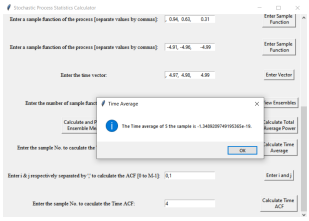

Figure 16: The Time Average of the 6^{th} Sample

The ACF between the 1^{st} and 6^{th} Samples of the Process

The ACF between the 1^{st} and 6^{th} Samples of the process is shown in **Figure 17**, and it approximately equals to 11.013.

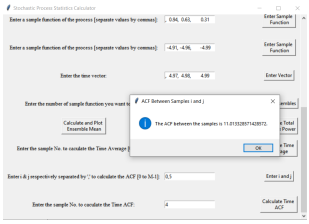

Figure 17: The ACF between the 1^{st} and 6^{th} Samples of the process

The Time ACF of the 5^{th} Sample Function of the Process

The time ACF of the 5^{th} sample function of the process is shown in **Figure 18**, and it is a sinusoid.

The Relation between Statistical Mean and Time Mean

The average of the sample functions' time mean values is almost equal to the average of the ensemble mean of the process. Check the 'The relation between the statistical mean and time mean' cells in the notebook.

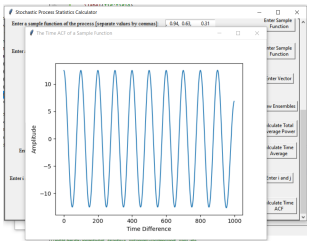

Figure 18: The Time ACF of the 5^{th} Sample Function of the Process

The Relation between Statistical ACF and Time ACF

Due to the difficulty of observing the exact shape of the statistical ACF, one can't really tell if there is a direct relation between them.