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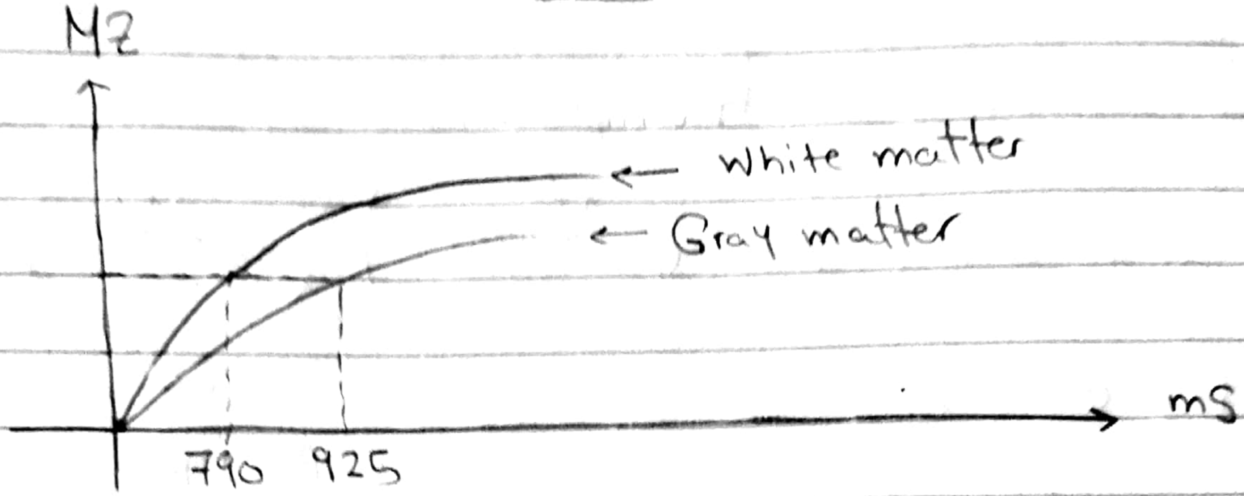
Ass. 1
MRI
Sheet 2

1



2 Saturation (long TR & Short TE)

$$S = \rho \left(1 - \cancel{e^{-\frac{TR}{T_1}}}\right)\left(e^{-\frac{TE}{T_2}}\right)$$

at long TR, exponential tend to zero

$$\therefore S = \rho \left(e^{-TE/T_2}\right)$$

$$\therefore -\ln\left(\frac{S}{\rho}\right) = \frac{TE}{T_2}$$

$$\therefore T_2 = \frac{-TE}{\ln(S/\rho)}$$

③ ① White Matter

$$S_1 = \rho_1 \left(1 - e^{-\frac{TR}{T_1}}\right)\left(e^{-\frac{TE}{T_2}}\right)$$

$$= 70 \left(1 - e^{-\frac{TR}{0.76/10^3}}\right)\left(e^{-TE/61}\right)$$

Gray Matter

$$S_2 = 85 \left(1 - e^{-TR/1.09/10^{-3}}\right)\left(e^{-TE/61}\right)$$

by equaling $S_1 = S_2$

$$\therefore \quad 70\left(1 - e^{-TR/0.76/10^3}\right) = 85\left(1 - e^{-TR/10/10^3}\right.$$

$$\frac{1 - e^{-\frac{TR}{0.76/10^3}}}{1 - e^{-\frac{TR}{1.09/10^3}}} = \frac{85}{70} \quad \{\text{by Calculator}\}$$

$$\therefore \; TR = 1.028 \text{ Sec.}$$

to get TE

$$90\left(1 - e^{-\frac{1.028}{1.08}}\right)\left(e^{-TE/100*10^3}\right) = 125\left(1 - e^{-\frac{1.028}{2.15}}\right)\left(e^{-TE/109*10^3}\right)$$

$$\therefore \; TE = 0.183 \text{ Sec.}$$

(ii) $T_i = ?$ to get signal $= 90\%$

→ CSF is the tissue greatest value of Time $T_1 = 20\ S$

$\therefore S = \cancel{M_0}\left(1 - 2e^{-\frac{T_i}{T_1}}\right) = 0.9\cancel{M_0}$

$\therefore 1 - 2e^{-\frac{T_i}{T_1}} = 0.9$

$\therefore T_i = 59.9$ Sec.

(iii) For adipose tissue

range for $T_1 = \frac{0.2+0.75}{2}$
$= 0.475$ ~~Se~~

When $TR = 100\ ms$

$S_1 = \cancel{P}\left(1 - e^{-\frac{0.1}{0.475}}\right) =$

When $TR = 500\ ms$

$S_2 = \cancel{P}\left(1 - e^{-\frac{0.5}{0.475}}\right) =$

$\therefore \frac{S_1}{S_2} = 0.2916$

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④ ⓘ $\boxed{M_{0A} = M_{0B}}$

$\rightarrow M_{xyA}(t) = M_{0A}\left(e^{-\frac{TE}{T_{2A}}}\right)\left(1 - e^{-\frac{TR}{T_{1A}}}\right)$

$\rightarrow M_{xyB}(t) = M_{0B}\left(e^{-\frac{TE}{T_{2B}}}\right)\left(1 - e^{-\frac{TR}{T_{1B}}}\right)$

$\therefore \Delta S_{xy}(t) = M_{xy\,A(t)} - M_{xy\,B(t)}$

to get max value { diff & = zero }

$$\frac{dS_{xy}(t)}{dTE} = 0$$

$$\therefore \cancel{M_{0A}}\left(1 - e^{-\frac{TR}{T_1A}}\right)\left(\frac{-1}{T_2A}\right)e^{-\frac{TE}{T_2A}}$$

$$= \cancel{M_{0B}}\left(1 - e^{-\frac{TR}{T_1B}}\right)\left(\frac{-1}{T_2B}\right)e^{-\frac{TR}{T_1B}}$$

$$\therefore \frac{T_2B\left(1 - e^{-\frac{TR}{T_1A}}\right)}{T_2A\left(1 - e^{-\frac{TR}{T_1B}}\right)} = \frac{e^{-\frac{TE}{T_2B}}}{e^{-\frac{TE}{T_2A}}}$$

$$\ln\left(\quad\right) = -\frac{TE}{T_2B} + \frac{TE}{T_2A}$$

$\left(-\frac{1}{T_1B} + \frac{1}{T_2A}\right)$

$$\therefore TE = \ln\left(\quad\right) * \frac{1}{-\frac{1}{T_2B} + \frac{1}{T_2A}}$$

(ii) $M_{zA(t)} = M_{oA}\left(1-e^{-\frac{TR}{T_1A}}\right)$

$M_{zB(t)} = M_{oB}\left(1-e^{-\frac{TR}{T_1B}}\right)$

$\Delta S_z(t) = M_{oB}\left(1-e^{-\frac{TR}{T_1B}}\right) - M_{oA}\left(1-e^{-\frac{TR}{T_1A}}\right)$

to get Max value { diff. & = 0 }

$\frac{dS_z(t)}{TR} = 0$

$\therefore \cancel{M_{oB}}\left(\frac{-1}{T_1B}\right) e^{-\frac{TR}{T_1B}} = \cancel{M_{oA}}\left(\frac{-1}{T_1A}\right) e^{-\frac{TR}{T_1A}}$

$\therefore \ln\left(\frac{T_1A}{T_1B}\right) = -\frac{TR}{T_1A} + \frac{TR}{T_1B}$

$\therefore TR = \ln\left(\frac{T_1A}{T_1B}\right) * \left(\frac{1}{\frac{-1}{T_1A} + \frac{1}{T_1B}}\right)$

(iii) $B_o = 1T$, A → white Matter, B → Gray Matter

Avera

$T_1 = 0.92$ Sec.	$T_2 = 80.5$ ms	A
$T_1 = 1.62$ Sec.	$T_2 = 85$ ms	B

$\therefore TR = \ln\left(\frac{0.92}{1.62}\right) * \left(\frac{1}{\frac{-1}{0.92} + \frac{1}{1.62}}\right)$

$= 1.205$ sec.

$\therefore TE = \ln\left(\frac{85*10^{-3}\left(1-e^{\frac{-1.205}{0.92}}\right)}{80.5*10^{-3}\left(1-e^{\frac{-1.205}{1.62}}\right)}\right) * \frac{1}{\frac{-1}{85*10^{-3}} + \frac{1}{80.5*10^{-3}}}$

$= 2.234$ sec.