song recommendation : Velmurugan

Quiz winner

Agenda

1. Introduction to prefix sum
2. Questions on prefix sum

Question

Given N array elements and Q queries. For each query calculate sum of all elements in range [L, R]

Note $L <= R$

$1 <= N , Q <= 10^5$

example arr =

-3	6	2	4	5	2	8	-9	3	1
0	1	2	3	4	5	6	7	8	9

queries

L	R	ans
4	8	9
3	7	10
1	3	12
7	7	-9
0	4	14

Brute force logic

For q queries find sum for range L, R

Code

```
querysum ( q , arr , n) {
    for (i = 0 ; i< q ; i++ ) {
        // L, R input
        sum=0;
        for (j = L, j<=R ; j++)
            sum += arr[j] ;
    }   print (sum)
}
```

T.C = $O(n * q)$

S.C = $O(1)$

$10^5 * 10^5 = 10^{10}$

Given Indian cricket team score for first 10 overs of batting. After every over the score is given below

Overs	1	2	3	4	5	6	7	8	9	10
Score board	2	8	14	29	31	49	65	79	88	97

Quiz 1

Totals runs scored in 7th over :

total end of 7th over - total at end of 6th over

65 - 49 = 16 runs

Quiz 2

How many runs were scored from 6th to 10th over?

total at end of 10th over - total at end of 5th over

97 - 31 = 66

Quiz 3

How many runs were scored in just 10th over ?

total score at end of 10th over - total at (10-1) over

97 - 88 = 9

Quiz 4

How many runs were scored from 3rd to 6th over?

score [6] - score [3-1] = 49-8 = 41

Quiz 5

How many runs were scored from 4th to 9th over?

score [9] - score [4-1] = 88 - 14 = 74

General Formula [ith to jth over]

score [j] - score [i-1] // constant calculation

Observation

1. Scoreboard has cummulative score. so it can calculate range sum in O(1) operation

---

How to calculate value for prefix sum?

Definition

ps [i] - sum of individual elements from 0 to i

arr =

2	5	-1	7	1

Initialize

ps [0] = arr [0] = 2

ps [1] = arr [0] + arr [1] = 2+5

ps [2] = arr [0] + arr [1] + arr [2] = 2+5+(-1)

ps =

2	7	6	13	14

Quiz 6

Calculate the prefix sum array of following array

arr =

10	32	6	12	20	1

⟶

10	42	48	60	80	81

psum for arr

Brute Force

```
ps = int [n]
for (i=0;  i< n ;  i++) {
        sum=0 ;
        for ( j= 0 ;  j <= i ; j++)        sum+= arr [i]
        psum [i]  = sum ;
}
```

Observation & optimization prefix sum array

$ps[0] = arr[0]$

$ps[1] = \cancel{arr[0]} + arr[1] = ps[0] + arr[1]$

$ps[2] = arr[0] + arr[1] + arr[2] = ps[1] + arr[2]$

$ps[3] = a[0] + a[1] + a[2] + a[3] = ps[2] + arr[3]$

⋮

$$\boxed{ps[i] = ps[i-1] + arr[i]}$$

pseudo code

```
ps[N] ;   ps[0] = arr[0] ;
for (i=1; i<=n; i++)
    ps[i] = ps[i-1] + arr[i]
```

T.c = O(n)

S.c = O(n)

How to answer the queries

	0	1	2	3	4	5	6	7	8	9
arr =	-3	6	2	4	5	2	8	-9	3	1
psum =	-3	3	5	9	14	16	24	15	18	19

queries

L	R	ans
4	8	18 - 9 = 9
3	7	15 - 5 = 10
1	3	9 - (-3) = 12
7	7	15 - 24 = -9
0	4	14

ps[8] - ps[4-1]

ps[7] - ps[3-1]

ps[3] - ps[1-1]

ps[7] - ps[7-1]

ps[4] - ~~ps[0-1]~~ 0 = ps[4]

Generalize equation to find sum

sum[L, R]	if	L ≠ 0	ps[R] - ps[L-1]
	if	L = 0	ps[R]

Question 1 Optimized code

T.C = O(n+q)
S.C = O(n)

```
def    query sum (q, arr, n){
        ps[N];  ps[0] = arr[0];
         for (i=1; i<n; i++)                      O(n)
              ps[i] = ps[i-1] + arr[i]            O(n)

        for (i=0; i<q; i++){
             L,R // input
             if (l==0){                           T.C = O(Q)
                  print(ps[R]);
             }
             else {
                  print(ps[R] - ps[l-1]);
             }
        }
}
```

$10^5 + 10^5 = 2 \times 10^5$

Can we modify the array ?

	0	1	2	3	4	5	6	7	8	9
arr =	-3	6	2	4	5	2	8	-9	3	1

↓ modify

arr =	-3	3	5	9						

Advantage:

No additional space

Disadvantage:

You lose the array

pseudo code

```
for (i=1; i<n; i++)
    arr[i] = arr[i-1] + arr[i]
```

Question 2

Given an array of size N and Q queries with start and end index. For every query, return the sum of all even indexed elements from L to R

Example

arr =	2	3	1	6	4	5
	0	1	2	3	4	5

query

L	R	ans
1	3	1 (arr[2])
2	5	1+4 = 5
0	4	7
3	3	0

Brute Force

For every query, get L, R and iterate to get even index sum

T.C = $O(n * q)$

Observation

Should we calculate the prefix sum of entire array ?

No , calculate prefix sum for even index values

Hint :

	0	1	2	3	4	5
arr =	2	~~1~~ 0	11	~~6~~ 0	0	~~7~~ 0
$psum_e$ =	2	2	13	13	13	13

Step 1 : ps[0] = arr[0]

Step 2 : if i is odd

ps[i] = ps[i-1]

else : ps[i] = ps[i-1] + arr[i]

Quiz 7

Construct the prefix sum for even index elements for the given array

	0	1	2	3	4
arr =	2	4	3	1	5

0	1	2	3	4
2	2	5	5	10

ps_e

L	R	sum
1	4	8
0	4	10

ps[4] - ps[0] = 10 - 2

ps[4] = 10

pseudo code

```
void    sum Of Even Index (arr, Q, n){
    ps[N] ; ps[0] = arr[0];
    for (i=1; i<n; i++) {
        if (i % 2 ==0)
            ps[i] = ps[i-1] + arr[i]
        else
            ps[i] = ps[i-1]
    }

    for (i=0; i<q; i++) {
        //L, R input
        if (L==0)
            print (ps[R])
        else
            print (ps[R] - ps[L-1])
    }
}
```

T.C = O(n+q)

S.C = O(n)

Question 2

Extension Sum of all odd indexed elements

```
if (i % 2 ==0)    ps[i] = ps[i-1]
else    ps[i] = ps[i-1] + arr[i]
```

Question 3

Given an array of size N, count the number of special index in the array.

Note: Special index are those removing which sum of even indexed element is equal to sum of odd indexed elements.

arr =

4	3	2	7	6	-2
0	1	2	3	4	5

explanation

i	array after removing					S_e	S_o	
0	3	2	7	6	-2	8	8 ✓	+1
1	4	2	7	6	-2	9	8 ✗	
2	4	3	7	6	-2	9	9 ✓	+1
3	4	3	2	6	-2	4	9 ✗	
4	4	3	2	7	-2	4	10 ✗	
	0	1	2	3	4			

Quiz 8

What will be the sum of elements at ODD index in the resulting array after removing element at index 2 ?

arr =

4	1	3	7	10
0	1	2	3	4

4	1	7	10
0	1	2	3

1+10 = 11

Quiz 9

sum of elements at odd index after removing index 3 ?

to remove

arr =

0	1	2	3	4	5	6	7	8	9
2	3	1	4	0	-1	2	-2	10	8

2	3	1	0	-1	2	-2	10	8

Observation

Before index 3

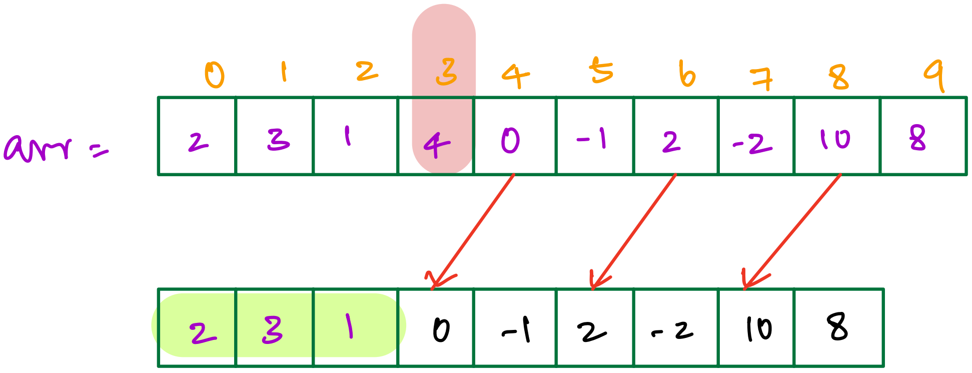
S_o remains S_o = 3

After index 3

S_o becomes S_e = -1 - 2 + 8

Quiz 10

sum of elements at even index after removing index 3?



arr =

0	1	2	3	4	5	6	7	8	9
2	3	1	4	0	-1	2	-2	10	8

2	3	1	0	-1	2	-2	10	8

$$Se = pse[3] - \cancel{pse[0-1]} + ps_0[9] - ps_0[3]$$

$$\cancel{2} + \cancel{1} + (\cancel{-1}) + (\cancel{-2}) + 8 = 8$$

Observation

sum of odd index elements after removing 3

= sum of odd till [0, 2] +

sum of even from [4, 9]

sum of even index elements after removing 3

= sum of even till [0, 2] +

sum of odd from [4, 9]

Step 1 : Calculate pso and pse

	0	1	2	3	4	5	6	7	8	9
arr =	2	3	1	4	0	-1	2	-2	10	8
Psum e =	2	2	3	3	3	3	5	5	15	15
psum o =	0	3	3	7	7	6	6	4	4	12

after removing index 3

even sum	odd sum
se [0, 2] + so [4, 9]	so [0, 2] + se [4, 9]
3 + 5 = 8	3 + 12 = 15

Step 2:

iterate through each value of i, check if se == so to count special index

i = 0

$$Se = Pso[1, n-1] = Pso[n-1] - Pso[0]$$

$$So = Pse[1, n-1] = Pse[n-1] - Pse[0]$$

else

$$Se = Pse[0, i-1] + Pso[i+1, n-1]$$

$$So = Pso[0, i-1] + Pse[i+1, n-1]$$

pseudo code

```
int    count Special index ( arr, n) {
        // calculate   Pse
        // calculate   Pso


        // count  special  index
        for (i=0;  i< n;  i++) {
                int  se , so ;
                if  (i==0) {
                        se =  Pso [n-1] - Pso [i];
                        so =  Pse [n-1] - Pse [i];
                }
                else {
                        se =  Pse [i-1] +  Pso [n-1] - Pso [i]
                                // se  till  i-1  +  so [i+1, n]
                        so =  Pso [i-1] + Pse [n-1] - Pse [i];
                                // so  till  i-1  +  se [i+1, n]
                }
                if  (so == se) {
                        count ++;
                }
        }

        return count;

}
```

Next class

Carry Forward

Subarray