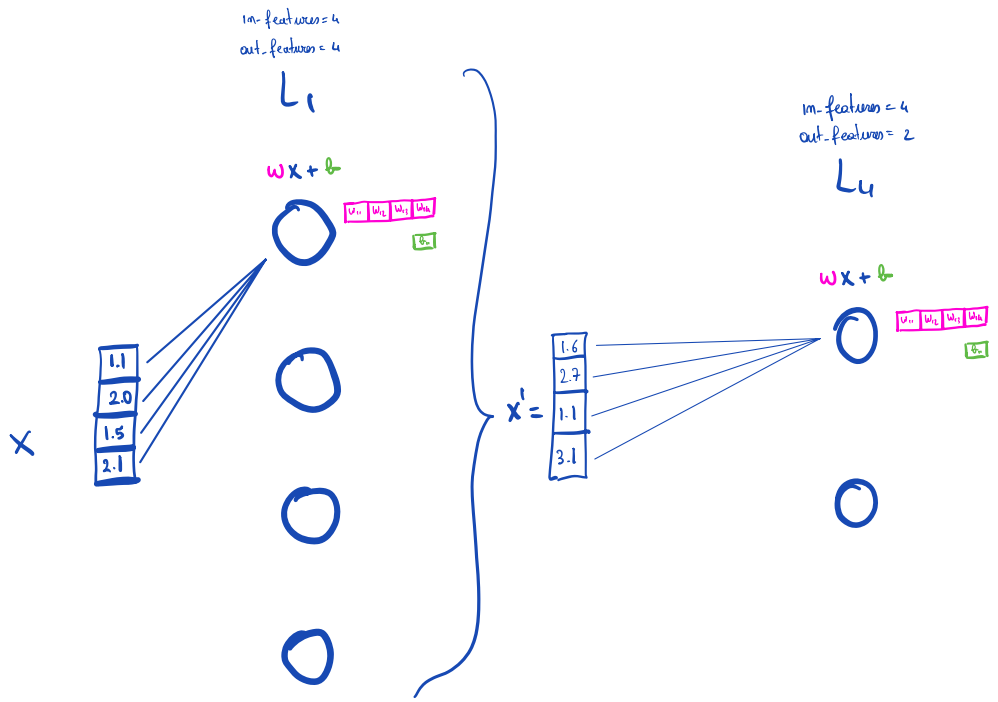
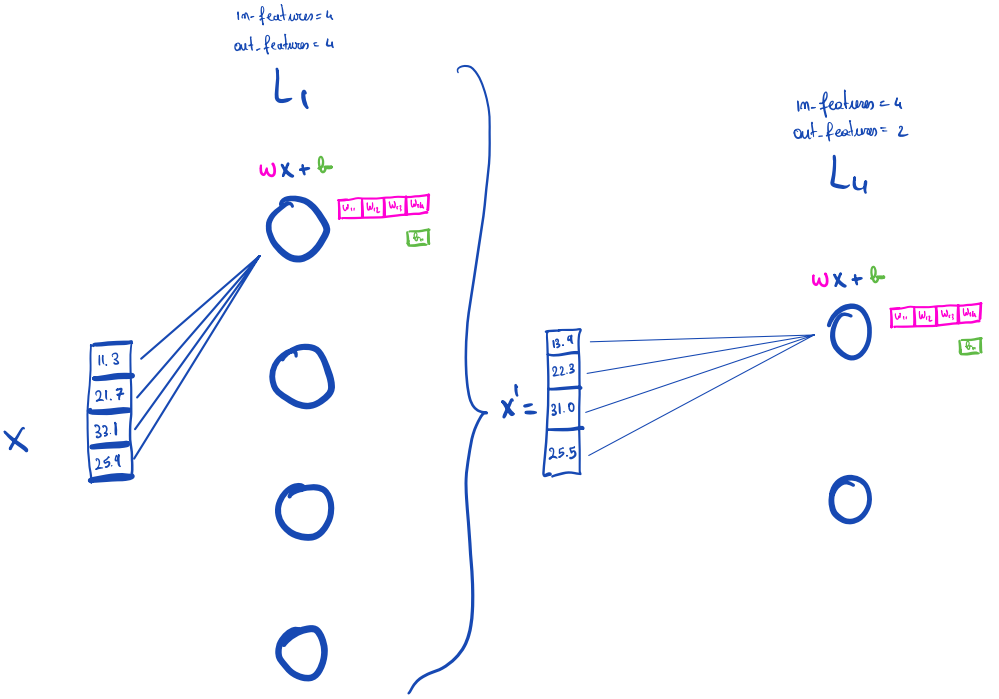
Normalization 101



Problem: covariate shift



Why is it bad?

Big change in input of a layer ⇒ Big change in output of a layer ⇒ Big change in loss ⇒

⇒ Big change in gradient ⇒ Big change in the weights of the network

⇒ Network learns slowly!

Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift

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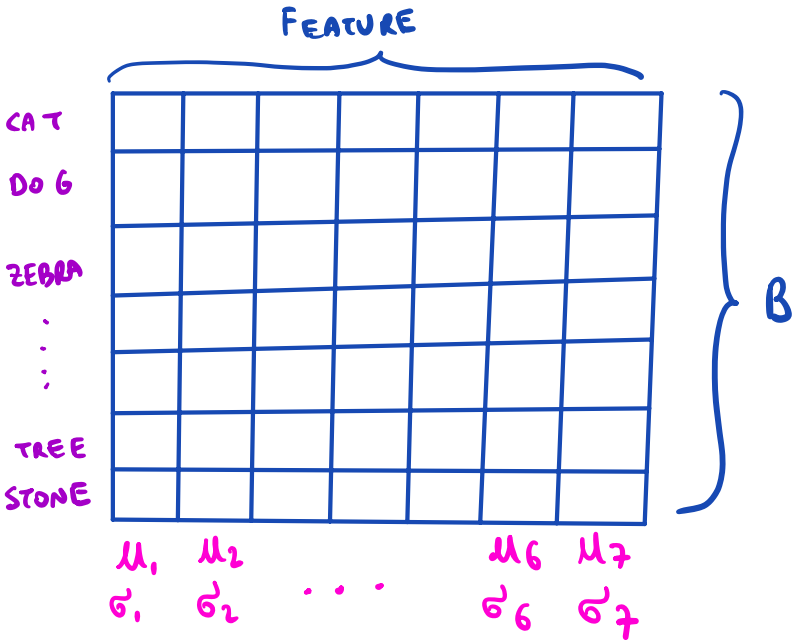
Abstract

Training Deep Neural Networks is complicated by the fact that the distribution of each layer's inputs changes during training, as the parameters of the previous layers change. This slows down the training by requiring lower learning rates and careful parameter initialization, and makes it notoriously hard to train models with saturating nonlinearities. We refer to this phenomenon as *internal covariate shift*, and address the problem by normalizing layer inputs. Our method draws its strength from making normal-

3 Normalization via Mini-Batch Statistics

Since the full whitening of each layer's inputs is costly and not everywhere differentiable, we make two necessary simplifications. The first is that instead of whitening the features in layer inputs and outputs jointly, we will normalize each scalar feature independently, by making it have the mean of zero and the variance of 1. For a layer with d-dimensional input $\mathrm{x} = (x^{(1)} \ldots x^{(d)})$, we will normalize each dimension

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$



we introduce, for each activation $x^{(k)}$, a pair of parameters $\gamma^{(k)}, \beta^{(k)}$, which scale and shift the normalized value:

$$y^{(k)} = \gamma^{(k)}\widehat{x}^{(k)} + \beta^{(k)}.$$

These parameters are learned along with the original model parameters, and restore the representation power of the network. Indeed, by setting $\gamma^{(k)} = \sqrt{\mathrm{Var}[x^{(k)}]}$ and $\beta^{(k)} = \mathrm{E}[x^{(k)}]$, we could recover the original activations, if that were the optimal thing to do.

In the batch setting where each training step is based on the entire training set, we would use the whole set to normalize activations. However, this is impractical when using stochastic optimization. Therefore, we make the second simplification: since we use mini-batches in stochastic gradient training, *each mini-batch produces estimates of the mean and variance* of each activation. This way, the

The problem with Batch Norm is that each statistic depends on what other items are in the batch. To get good results, we must use a big batch size

Layer Normalization

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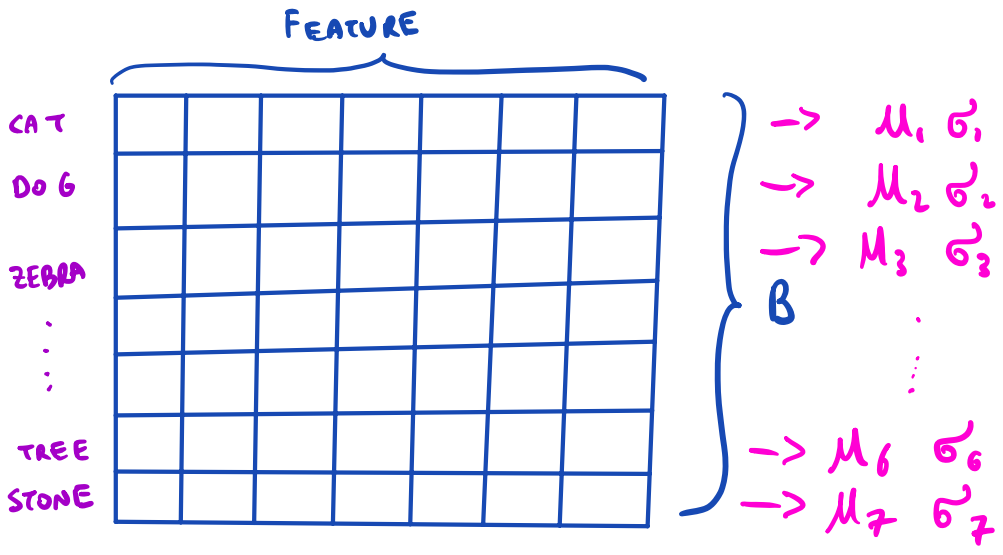
3 Layer normalization

We now consider the layer normalization method which is designed to overcome the drawbacks of batch normalization.

Notice that changes in the output of one layer will tend to cause highly correlated changes in the summed inputs to the next layer, especially with ReLU units whose outputs can change by a lot. This suggests the "covariate shift" problem can be reduced by fixing the mean and the variance of the summed inputs within each layer. We, thus, compute the layer normalization statistics over all the hidden units in the same layer as follows:

$$\mu^l = \frac{1}{H}\sum_{i=1}^{H} a_i^l \qquad \sigma^l = \sqrt{\frac{1}{H}\sum_{i=1}^{H}\left(a_i^l - \mu^l\right)^2} \tag{3}$$

where H denotes the number of hidden units in a layer. The difference between Eq. (2) and Eq. (3) is that under layer normalization, all the hidden units in a layer share the same normalization terms μ and σ, but different training cases have different normalization terms. Unlike batch normalization, layer normaliztion does not impose any constraint on the size of a mini-batch and it can be used in the pure online regime with batch size 1.



Root Mean Square Layer Normalization

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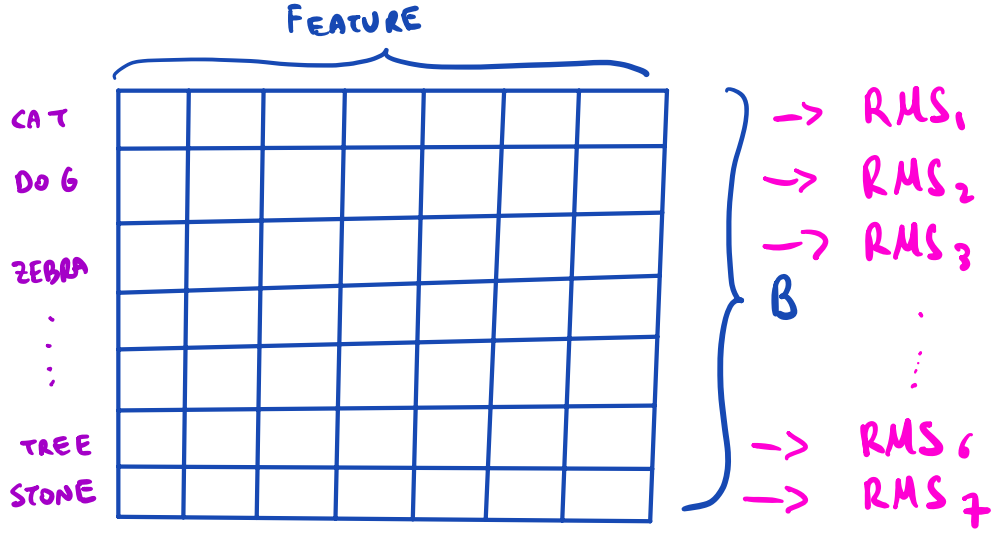
4 RMSNorm

A well-known explanation of the success of LayerNorm is its re-centering and re-scaling invariance property. The former enables the model to be insensitive to shift noises on both inputs and weights, and the latter keeps the output representations intact when both inputs and weights are randomly scaled. In this paper, we hypothesize that the re-scaling invariance is the reason for success of LayerNorm, rather than re-centering invariance.

We propose RMSNorm which only focuses on re-scaling invariance and regularizes the summed inputs simply according to the root mean square (RMS) statistic:

$$\bar{a}_i = \frac{a_i}{\text{RMS}(\mathbf{a})} g_i, \quad \text{where} \ \ \text{RMS}(\mathbf{a}) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} a_i^2}. \tag{4}$$

Intuitively, RMSNorm simplifies LayerNorm by totally removing the mean statistic in Eq. (3) at the cost of sacrificing the invariance that mean normalization affords. When the mean of summed inputs is zero, RMSNorm is exactly equal to LayerNorm. Although RMSNorm does not re-center



In this case each item is treated independently.