Algorithmic Competition, with Humans

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SEA Meetings
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[1]Federal Trade Commission. This work represents my views alone and not those of the Commission, its Commissioners, or the United States Government.

Algorithmic (automated) pricing

For many (most?) firms, pricing partially *automated*

Does this increase markups above competitive levels?

- Yes, no, maybe, depending on model

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Algorithms may:

- Provide *commitment* to irrational pricing off-path
- Improve *prediction* so pricing on-path is ex post rational

This paper

Study **algorithmic competition with managerial override**

1. Firms design algorithms, mapping rivals' price to own
2. (or 2+) Algorithms run *or* firms choose prices manually

Choosing manually is *costly* but *useful* if algorithms are "failing"

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I show this is:

(Tractable):	analytic solution exists
(Instructive):	highlights roles of prediction and commitment
(Falsifiable):	significantly refines equilibrium predictions
(Sufficient):	explains patterns in real pricing data...
(Necessary*):	...in ways existing models cannot

Literature

The game, in general

Two-stages, symmetric differentiated duopoly, demand $q(p_i, p_{-i})$

Stage 1: Firms simultaneously set algorithms σ_i

Stage 2: Firms simultaneously defer to algorithm *or* choose price

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- Action set is $\{ \underbrace{\mathbb{R}^+}_{\substack{\text{Override,}\\ \text{set a price}}}, \underbrace{\sigma}_{\substack{\text{Defer to}\\ \text{algorithm}}} \}$
- One-shot pricing
- Overriding may come at cost c; no marginal costs

Pricing stage details

Algorithms in place, $\sigma = (\sigma_1, \sigma_2)$ define subgame.

If rival chooses a *price* p_{-i}:

$$a_i = \text{arg max}_{a \in \{\mathbb{R}^+, \sigma\}} \; p_i q_i(p_i, p_{-i}) - c * 1[a_i \in \mathbb{R}^+]$$

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If rival chooses its *algorithm* σ_{-i}:

$$a_i = \arg\max_{a \in \{\mathbb{R}^+, \sigma\}} p_i q_i(p_i, x_{-i} + z_{-i} p_i) - c * 1[a_i \in \mathbb{R}^+]$$

In Pictures Pricing Stage Lemma

Informal summary of theoretical results

If $c = \infty$, *any* price between Bertrand and collusive is possible.

Details

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If $c = 0$, $\exists$ equilibrium in which:

- Algorithms *match* changes in rival price but *undercut* levels
- Only one firm, chosen at random, overrides
- Undercutting designed so:

$$\sigma_i(p^{BR}(\sigma_i)) = p^{BR}\big(p^{BR}(\sigma_i)\big)$$

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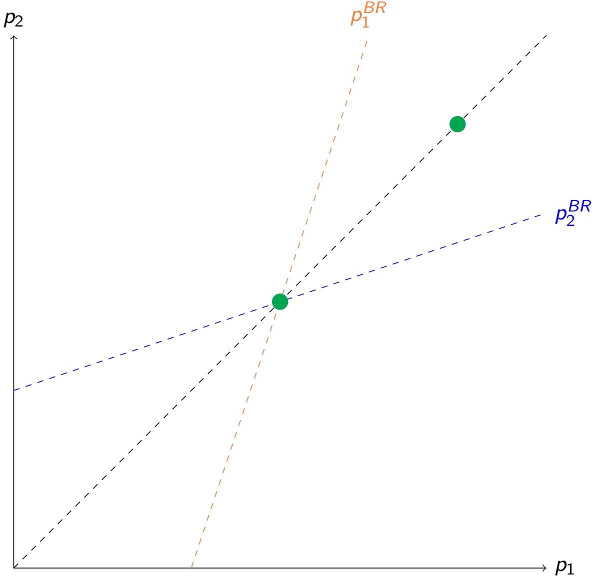
"Collusion by algorithm" sometimes also an equilibrium

Technical details | If $c > 0$

The theory, in pictures



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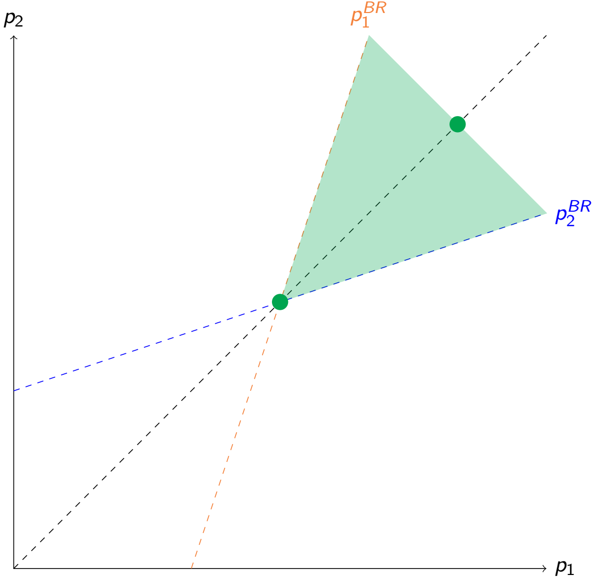

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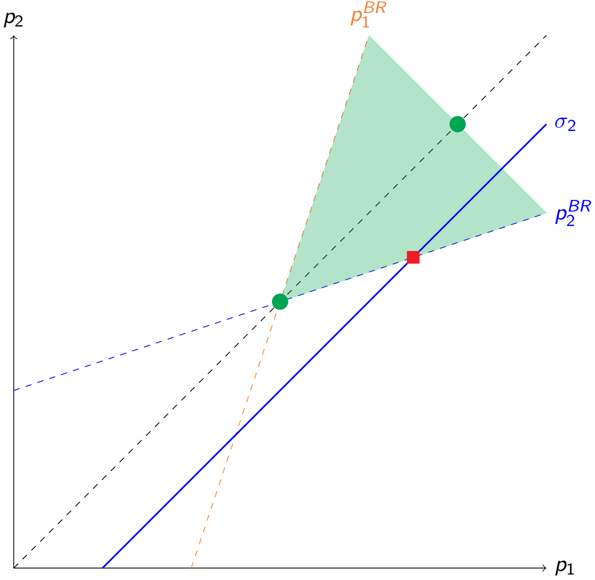

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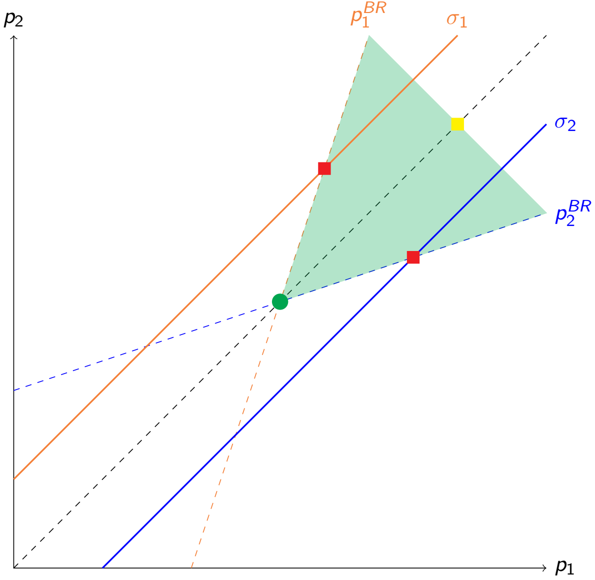

Commitment vs. Prediction

Suppose state of nature θ drawn between algorithm-setting and pricing stages:

$$q_i = q(p_i, p_{-i}, \theta)$$

or

$$\pi_i = q(p_i, p_{-i})(p - \theta)$$

If $c = 0$ and algorithms cannot condition on θ, then prices generically Bertrand.

Prediction vs. commitment, a synthesis

	Full Commitment	No Commitment
Full Prediction	"Anything goes"	One-sided override or collusion
No Prediction	"Anything goes"	Generically Bertrand

In sustaining supracompetitive prices, prediction and commitment are *substitutes*: one or the other is good enough

Repeated version

Most settings w/ algorithmic pricing involve *repeated* interaction

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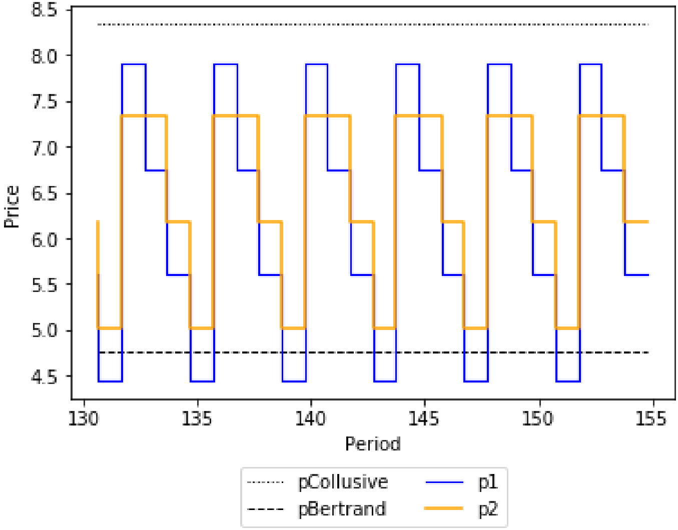
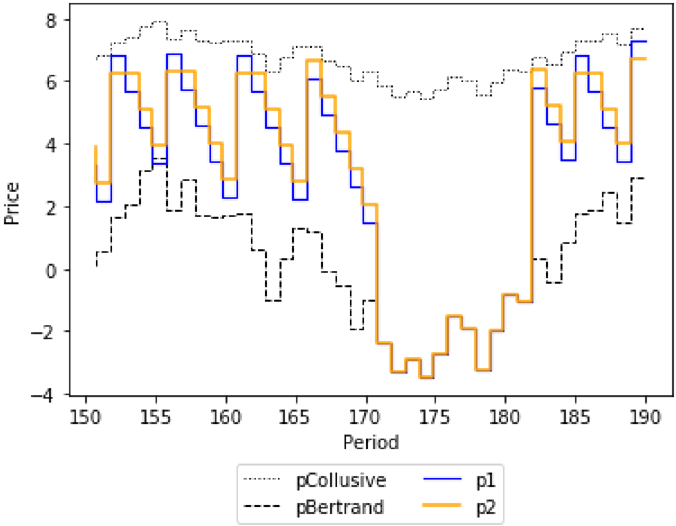
Simulation Details

Edgeworth Cycles



Bertrand Reversion



From theory to data

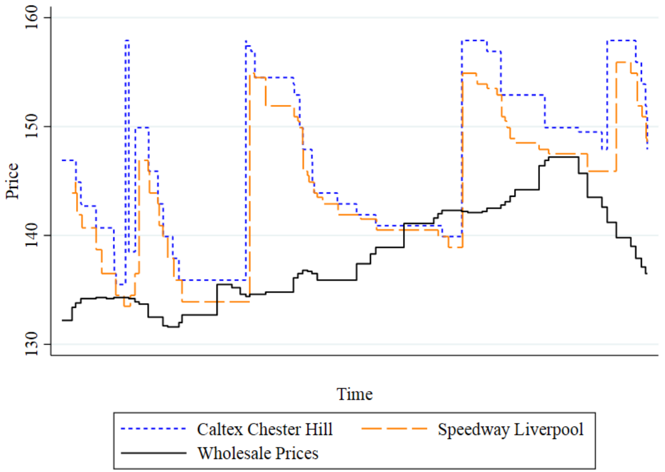
Key model predictions:

- Existence of Edgeworth cycles Here
- Price decreases should seem automated
  - Decreases should be uniform in size
  - Price "matching" should happen quickly Here
- Price increases should resemble override
  - Prices should better reflect marginal costs after increases Here
  - Increases should be timed for when opportunity cost of undercutting low Here
- Cost volatility may lead to "freefall" pricing
  - Unusually large price decreases only when marginal costs volatile Here
  - Less so for increases Here

Conclusion

Edgeworth cycles in real life

FuelWatch: timestamped price changes for every gas station in New South Wales, 2018-present



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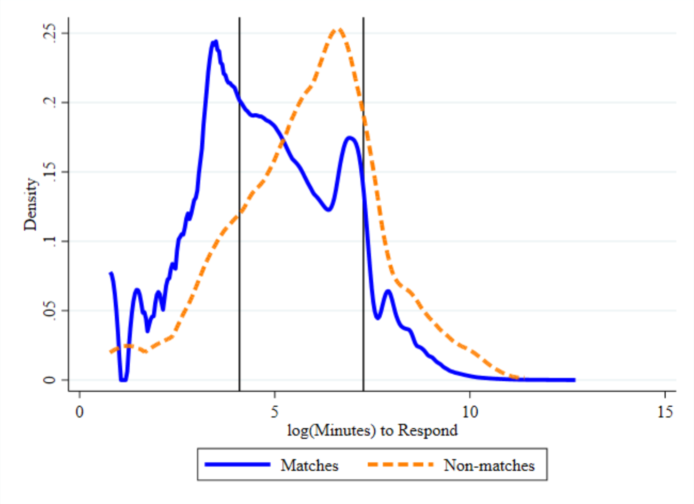
Speed of responses



Note: vertical lines at 1 hour and 1 day

Details Back

Only resets are strategic

What predicts p_{it}?

Define jump as

1. $p_{it} > p_{i,t-dt}$, i.e., i increases price
2. $p_{i,t-dt} \leq p_{i,t-2dt}$ $\forall i$, i.e., not a match of a rival's increase

	Partial Correlation	
Variable	Non-Jumps	Jumps
Rival Price	0.73	0.41
Wholesale Price	0.18	0.36
Traffic Volume	0.02	0.04

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Resets timed strategically

Probit regressions to predict 1[Jump occurs at t]:

Variable	(1) Coef (Std. Err)	(2) Coef (Std. Err)	(3) Coef (Std. Err)
Wholesale Cost	0.280***	0.281***	0.301***
	(0.017)	(0.017)	(0.016)
Rival Price	0.023**	0.023**	0.009
	(0.010)	(0.010)	(0.011)
Lagged Own Price	−0.235***	−0.236***	−0.238***
	(0.012)	(0.012)	(0.01)
Traffic Volume	-	-	−0.056***
	-	-	(0.005)
N	728,032	728,032	475,190
Pr[Y=1]	0.071	0.071	0.072

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Freefall pricing

Construct volatility measures using (1) historical rack price volatility and (2) OVX index

Variable	(1) Coef (Std. Err)	(2) Coef (Std. Err)	(3) Coef (Std. Err)	(4) Coef (Std. Err)	(5) Coef (Std. Err)
ΔWholesale	-	-0.030^{***}	-0.043^{***}	-0.008	-0.016
	-	(0.010)	(0.010)	(0.010)	(0.011)
Volatility	0.078^{***}	0.071^{***}	-	0.073^{***}	-
	(0.013)	(0.013)	-	(0.015)	-
OVX	-	-	0.054^{***}	-	0.069^{***}
	-	-	(0.013)	-	(0.022)
N	412,622	412,538	412,538	412,538	412,538
Fixed Effects	-	-	-	Monthly	Monthly

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Conclusions

Algorithmic competition with managerial override is:

(Tractable):	analytic solution exists
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Implications:

1. Algorithms can do damage! But must consider (1) extent and ease of human involvement, (2) predictive abilities of algorithms
2. Instead of puzzling over Edgeworth cycles, back out an algorithm/human combo that generates it

Thank you!

Questions or comments?
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https://mleisten.github.io

Related Literature

Algorithms as commitment: Brown and Mackay (2021), Salcedo (2015)

Algorithms as prediction: Miklos-Thal and Tucker (2019), O'Connor and Wilson (2019)

Algorithms as learning: Asker et al. (2021), Assad et al. (2020), Johnson et al. (2020), Calvano et al. (2020), Klein (2019)

Forebears in conduct: Klemperer and Meyer (1989), Rubinstein and Abreu (1988), Maskin and Tirole (1988), Salop (1986)

Gasoline: Assad et al. (2020), Byrne and DeRoos (2019), Clark and Houde (2013), Wang (2009), Hosken et al. (2008), Noel (2007)

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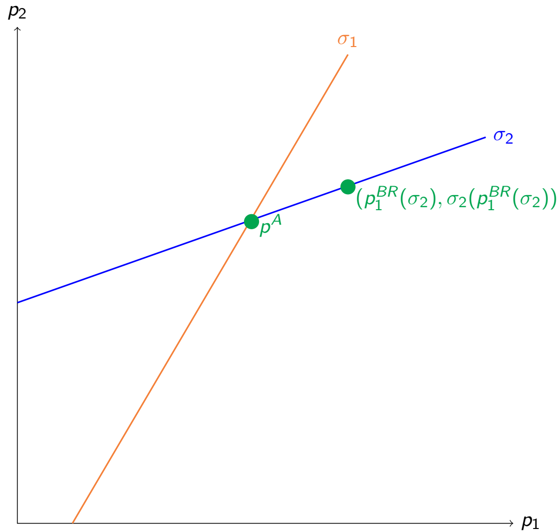
Pricing, Illustrated



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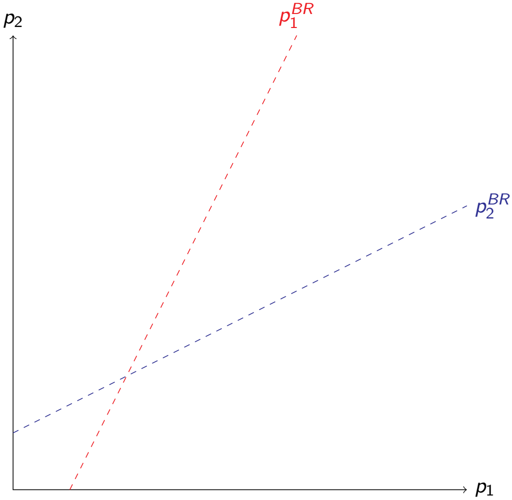

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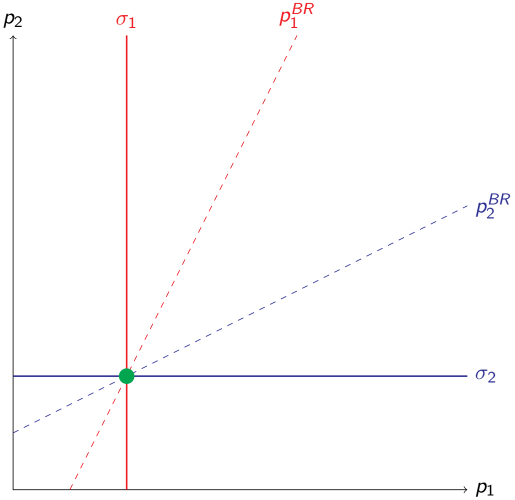

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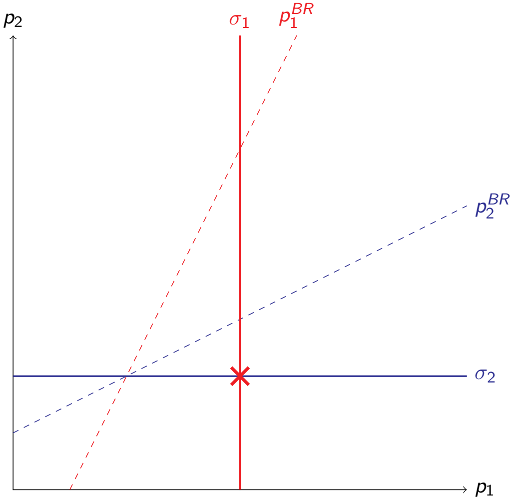

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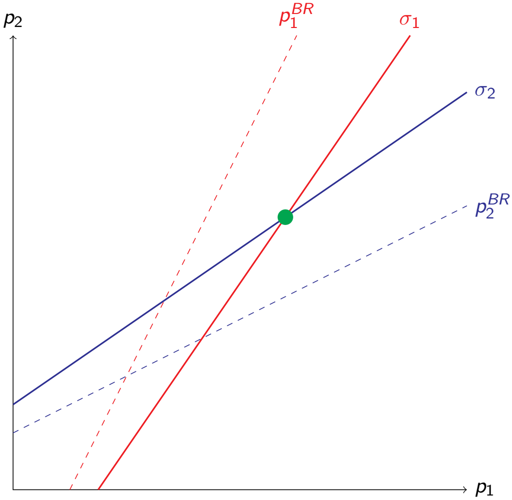
Upward sloping algorithms soften competition, sustain supracompetitive prices



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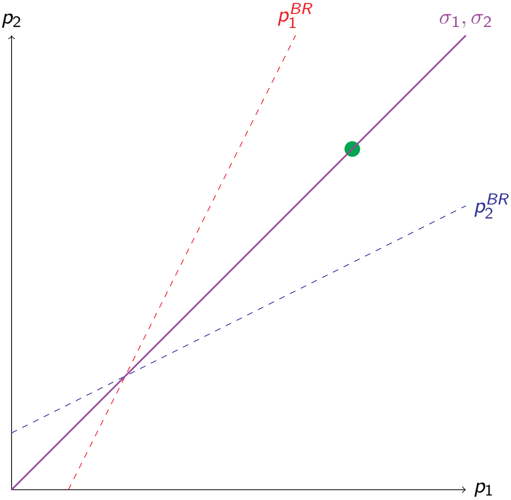
Price matching algorithms yield collusive prices (as in Salop (1986))



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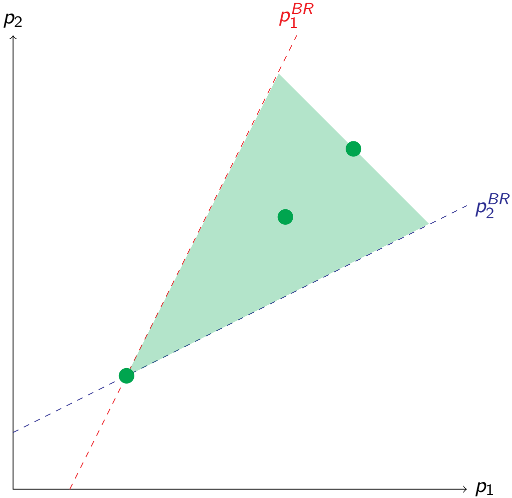
Really, "anything goes"



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Pricing stage game

Proposition: One of these must be true:

1. Algorithms are sustained in an equilibrium. Prices are p^A.
2. One firm overrides its algorithm in an equilibrium. Prices are $(p^*(\sigma_{-i}), \sigma_{-i}(p_i^*(\sigma_{-i})))$.
3. The unique equilibrium is Bertrand pricing.

Note: Bertrand may still exist if (1) or (2) is true.

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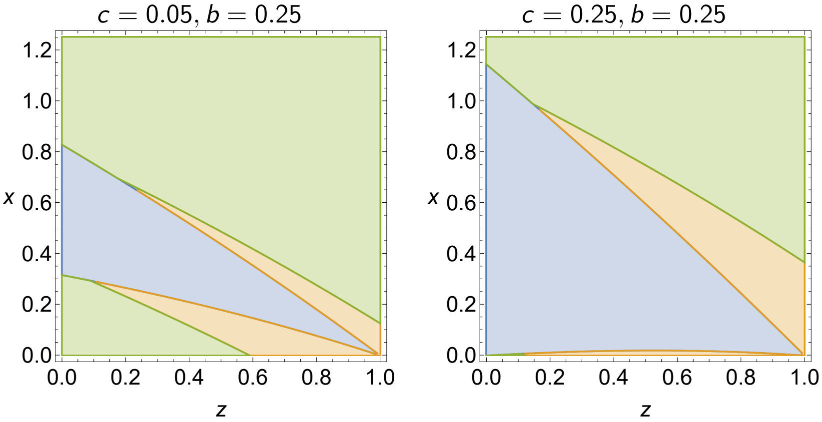
Pricing stage: example

Linear demand: $q_i = 1 - p_i + bp_{-i}$



Algorithmic, One-sided override, Bertrand

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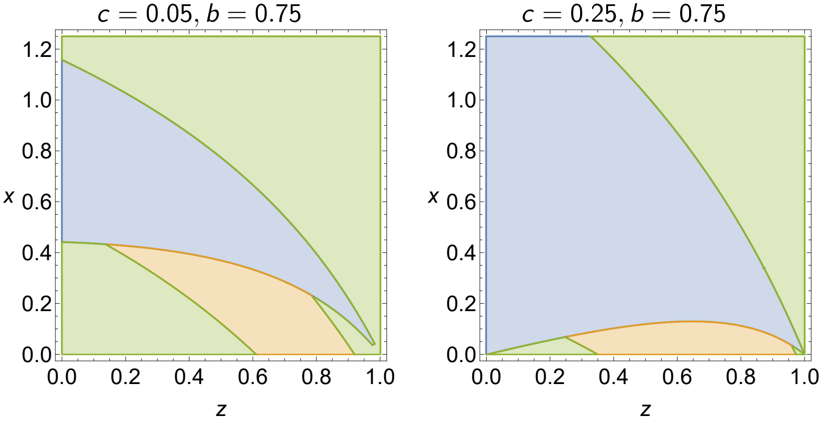

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Public randomization

Four types of equilibrium: "Algorithmic", "Only Firm 1 Overrides", "Only Firm 2 Overrides", "Bertrand"

Existence profile $\mathrm{r}(\sigma) \in \{0,1\}^4$, with entry $r_j = 1$ if equilibrium of type j exists when algorithms are σ

Assumptions:

- Probability of equilibrium type j is measurable w.r.t. $\mathrm{r}(\sigma)$.
- If $\exists$ a non-Bertrand equilibrium, Bertrand is never played.
- All non-Bertrand equilibria are played with positive probability.

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What if $c > 0$?

Generally difficult to characterize. Less ambitious question: is collusion by algorithm possible?

Equivalent to: Can firm i do better than collusion by inducing rival to override their algorithm, as in the $c = 0$ case?

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Rule of thumb / conjecture: Scope for collusion by algorithm decreasing, then increasing, in c.

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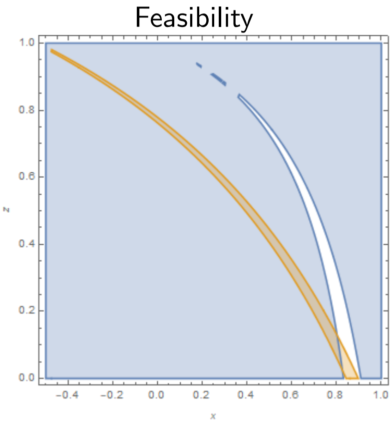
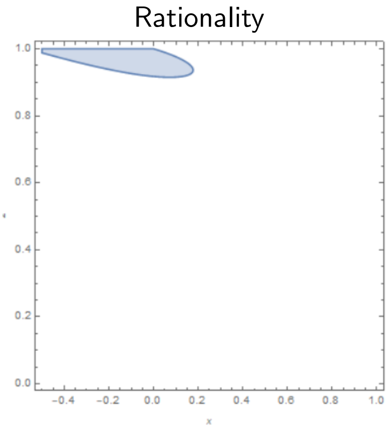
Example: $q_i = 1 - p_i + .5p_{-i}$





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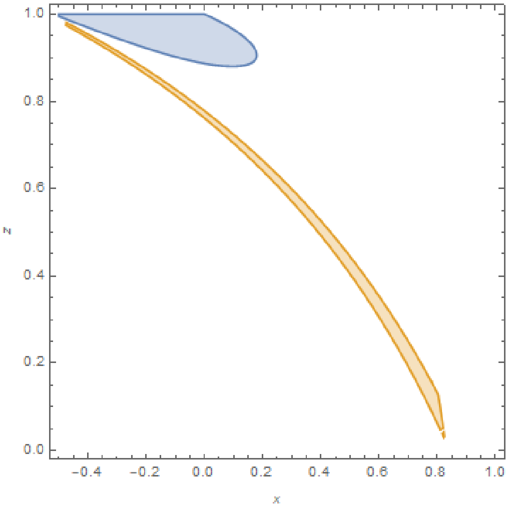

Rationality, Feasibility

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Searching for "equilibrium"

Simulate to find equilibrium in algorithms $\{x_i\}$:

1. Start with $x_{-i} = 0$, set grid X
2. For each gridpoint $x \in X$:
    - Simulate i average payoffs over time, setting $x_i = x$.
    - Set x_{-i} equal to i's best gridpoint
    - Iterate to convergence

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Speed of responses

i changes price at t, previous rival price change at $t - dt$.

Compute distance between i's latest price change and last rival's price change:

$$M_{it} = \left|(p_{it} - p_{i,t-dt}) - (p_{-i,t-dt} - p_{-i,t-2dt})\right|$$

Plot distribution of dt when $M_{it} = 0$ versus not

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Placebo test: "freerise"

Variable	(1) Coef (Std. Err)	(2) Coef (Std. Err)	(3) Coef (Std. Err)	(4) Coef (Std. Err)	(5) Coef (Std. Err)
ΔWholesale	-	0.049**	0.046*	0.045**	0.045**
	-	(0.024)	(0.025)	(0.021)	(0.022)
Volatility	-0.022	-0.013	-	0.032	-
	(0.021)	(0.021)	-	(0.025)	-
OVX	-	-	-0.046^{**}	-	0.049
	-	-	(0.022)	-	(0.040)
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