Customer Behavior Modeling in Revenue Management and Auctions: A Review and New Research Opportunities

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Abstract

Customer behavior modeling has been gaining increasing attention in the operations management community. In this paper we review current models of customer behavior in the revenue management and auction literatures and suggest several interesting research directions in this area.

1 Introduction

It is common practice in operations management to characterize customer demand exogenously. For example, market size is often represented using a demand distribution (as in the newsvendor model), price sensitivity is almost always captured by a demand curve, and customer arrivals in revenue management and service settings are usually modeled using stochastic processes. The common feature among all these modeling approaches is that customers are passive: they do not engage in any decision-making processes and are simply governed by the demand profile specified at the outset. Yet, in our world of consumer sovereignty, all customers do, at some point, actively evaluate alternatives and make choices. This suggests that customers' decision processes (in determining, e.g., how much to pay, which product to buy, when to buy, etc.) deserves some attention. Herein, we shall use the term "customer behavior" to refer to the outcomes of these deliberation processes. For many practical problems, neglecting these decision processes on the demand side may have significant repercussions, since customer behavior in any market is intricately tied to firms' actions and the corresponding reactions from other customers. In our view, it is important to adopt a micro-perspective on such market interactions. This requires a high-resolution lens to zoom in on the incentives and decision processes of customers at the individual level.

There are two common market mechanisms to allocate supply to demand. The first is the price mechanism, in which goods are sold to customers at a particular posted price. Although the law of one price should prevail in perfectly competitive markets, casual inspection indicates a significant level of price dispersion in the real world.[1] When firms have the opportunity to price discriminate, it is natural to expect price variations to persist. In this environment, customers may react strategically to price fluctuations, and ignoring this behavioral aspect may lead to suboptimal pricing decisions. The second selling mechanism is auctions. There is a rich literature on this subject and it is well known that auction design has a significant impact on firms' revenues. Even the smallest modification in the rules of an auction can have critical ramifications on financial bottom-lines when individuals react to these rules. In the Internet age, there is more room for creativity in designing new auction formats, and the use of electronic auctions has become more widespread than ever. This generates new and exciting challenges in understanding the impact of customer behavior in these market mechanisms.

Industries have been paying careful attention to modeling customer behavior. The recent advent of customer relationship management systems has witnessed a wave of gathering and storing customer and transaction information and using the results to improve marketing, sales, and customer service (e.g., Siebel, e-piphany). In research communities, the significance of customer behavior modeling has become well-recognized as well. The literature on auction theory has a rich tradition in modeling seller and bidder behavior. Further, the relatively newer stream of research in dynamic pricing and revenue management is also paying increasing attention to modeling the behavior of individual customers. In these works, a wide variety of research methodology has been employed to study customer behavior. For instance, there has been mathematical analyses of game-

[1] Phillips (2005) list prices for a half-gallon of whole milk at different markets in a 16-block area of upper west side of Manhattan on a single day in May 2002. Prices vary by 44%. Furthermore, the price varied by more than \$0.40 even for two stores on the same block. Neither buyers nor sellers possess perfect information.

theoretic models, empirical examination of field data, as well as behavioral studies with laboratory experiments. In our opinion, this provides some testimony to the academic richness and practical applicability of customer behavior modeling.

In this paper, we will review some recent papers from the revenue management literature and auction literatures. The common theme of these papers is about how companies should design their selling mechanisms in order to maximize expected profits. Since customers will exhibit systematic responses to the selling mechanisms, firms are responsible for anticipating these responses when making their pricing and auctioning decisions.

In the first half of this paper, we review the recent literature on strategic customer behavior in revenue management. We classify the related papers in two groups. The first group of paper examines the effect of inter-temporal substitution by customers. That is, individuals may choose *when* to buy a particular product, in response to firms' dynamic pricing practices. In particular, when they anticipate price reductions, consumers may choose to wait for the sale. Additionally, other relevant issues include capacity rationing (i.e., manipulating product availability to influence purchase timing), valuation uncertainty, and consumer learning effects. These kinds of behavior imply that the dynamics of consumer demand depend directly on the seller's dynamic pricing strategies. We stress that this dependence is not captured by conventional models with exogenous demand arrival processes. Next, the second group of papers studies customer choice in multi-product revenue management settings. Here, the focus is on how individuals choose *which* product to buy. Along these lines, a common approach is to use discrete choice models to capture multi-product consumer demand. In addition, we also look at substitution and complementary effects across multiple products.

In the second half of this paper, we review literature on strategic bidder behavior in auctions. We classify the related papers in four groups. The first group of papers focuses on the findings related to the well-known revenue-equivalence result. The second group of papers studies the bidding behavior in multi-unit auctions, which offer more opportunities for strategic behavior compared to single-unit auctions. We then discuss in the third group of papers the recent developments in behavior modeling for online auctions. This is a new research area, and the common belief is that Internet bidders' behaviors are still not well studied. The last group of papers reveals the difficulties associated with applying theoretical and behavior auction results to real-life auctions. The main finding here is that many seemingly reasonable assumptions in the research papers may not hold in practice, so extra care has to be paid when designing and implementing real-life auctions.

2 Customer Behavior in Revenue Management

2.1 Inter-temporal Substitution and Strategic Customer Behavior

Inter-temporal substitution refers to the practice of delaying purchases to a future point in time. Until recently, the revenue management literature has almost completely neglected this issue. The standard modeling paradigm is to assume that demand arriving at each instance in time is either realized (leading to sales) or lost forever. There is no opportunity for demand to lie dormant in the market, in anticipation of future purchase opportunities. Recognizing that this assumption is unrealistic, recent work has begun to pay increasing attention to this issue.

We shall use the term "strategic customer behavior" to refer to this kind of inter-temporal substitution behavior. Although strategic behavior could potentially be a much broader concept, we shall fix our terminology to be consistent with recent papers in the literature. Additionally, an emerging convention is to use the term *strategic customers* to refer to those who may practice inter-temporal substitution, and to use the term *myopic customers* to refer to those who make a one-time purchase decision at their time of arrival.

Customer response to dynamic pricing The first step to understanding strategic customer behavior in revenue management is to develop models of how customers respond to firms' pricing strategies. There are several papers that aim in this direction. These papers start off assuming that the firm sets prices dynamically according to some established policy, and then investigates the optimal behavior of a single strategic customer in this setting. These models focus on Belobaba's (1989) expected marginal seat revenue (EMSR) policy and/or the optimal pricing policy characterized in Gallego and van Ryzin (1994), which shall be referred to as the GVR policy. Both of these policies have had important impact on practical implementations of revenue management systems, so it is important to understand rational consumer responses to these policies.

In one of the earliest papers on this topic, Anderson and Wilson (2003) consider strategic consumer response in a model in which protection limits are set according to Belobaba's (1989) EMSR rule. The firm has a fixed capacity, and sets a protection limit that caps the number of units sold to low-revenue demand arriving earlier. Leftover units are then sold to high-revenue demand arriving later. There is some probability that at the end of this procedure, some units will remain unsold and have to be discounted. The authors show how to calculate this probability. If this probability is sufficiently large, high-revenue demand may wait for last-minute discounts. The authors numerically investigate the revenue implications of this kind of strategic waiting behavior. In follow-up work, the authors also investigate how to set these booking limits optimally in the presence of strategic customers (see Wilson, Anderson and Kim, 2006).

In another paper, Zhou, Cho, Fan (2005) consider the setup in Gallego and van Ryzin (1994) and focus on the optimal purchasing strategy of a single strategic customer facing the GVR policy. The authors derive the threshold nature of the optimal purchasing policy. That is, customers should purchase immediately if the current price is below some threshold depending on their valuation and the current time. They also numerically study the case with multiple strategic customers, and find that strategic behavior may benefit the seller. Although strategic waiting may disable the seller from extracting the entire consumer surplus, the advantage now is that strategic customers facing too high a price are not immediately lost and they may be recovered later at lower prices.

Asvanunt and Kachani (2006) formulate the customer's purchasing problem as an optimal stopping problem, and derive optimality conditions for both the EMSR and GVR pricing policies. Consistent with earlier findings, they showed threshold properties for the customer's optimal purchasing policy. Furthermore, they also numerically investigate the impact of strategic behavior on firm revenues. They demonstrate that if the firm fails to account for strategic customer behavior, the loss in revenue can be substantial.

Dynamic pricing under strategic customer behavior With some understanding of how customers respond to well-known pricing policies, the natural next step is to study how firms should set their prices in the presence of strategic customers. After all, the EMSR and GVR policies were derived under the assumption of myopic customers, and it is no longer clear how optimal pricing policies should look like when customers behave strategically. Such investigations call for an equilibrium of sorts: customers should respond strategically to firms' pricing policies in a self-optimizing manner, and firms should also maximize revenue by taking these strategic responses into account.

One of the earliest papers to investigate optimal dynamic pricing under strategic customer behavior was by Aviv and Pazgal (2005). They analyze a model with a single price reduction at a fixed point in time T. In other words, a fixed "premium" price is charged prior to a fixed time T and a "discount" price p is charged after time T. There is a Poisson stream of strategic customers, with valuations drawn from some distribution. Customers arriving before time T may strategically wait to purchase at time T if doing so is beneficial, but customers arriving after T have no incentive to wait. The seller's problem is to choose the discount price p optimally. The authors consider two classes of discounting strategies: contingent (in which the magnitude of the discount may depend on remaining inventory) and fixed discounts (in which the discount price is announced at the start). For both cases, the authors show that the optimal purchasing strategy for customers before time T is a threshold strategy. Next, the authors study the seller's discount pricing problem for both contingent and fixed discounts, and they compare their results to the benchmark with myopic customers who do not wait. They demonstrate numerically that: (i) pre-commitment can benefit the seller when customers are strategic, (ii) strategic customer behavior leads to inventory levels having a more significant effect on the depth of discounts, and (iii) the losses resulting from neglecting strategic customer behavior can be substantial.

While the previous paper analyzes a single price reduction, Elmaghraby, Keskinocak and Gulcu (2004) extend this analysis by considering a finite number of price steps. However, the authors restrict attention to markdown pricing mechanisms. In their model, they assume that there is a deterministic, finite number of buyers, each with demand for multiple units. Buyers may have different valuations, which are publicly known (in the base model). The seller's problem is to choose a markdown mechanism in which prices decrease in steps according to a pre-announced schedule of prices. At each price step, buyers strategically bid for the number of units they are willing to buy at the current price. Despite the increased generality of their model, the authors show that a markdown mechanism with two price steps, i.e. a single price reduction, is optimal. (This parallels the results in Gallego and van Ryzin (1994), who also demonstrate the optimality of two prices in a deterministic approximation of their model.) Elmaghraby, Keskinocak and Gulcu analytically characterize the optimal prices when there are two customers, and demonstrate how to compute the optimal policy for multiple customers. They also consider the incomplete information problem, in which all valuations are privately known.

Su (2005) analyzes the dynamic pricing problem when there is a combination of strategic and myopic customers in the population. One of the main insights therein is to demonstrate that accounting for both types of customers is important because this has structural implications on the optimal pricing policy. Depending on the customer composition, optimal prices may be increasing, decreasing, or even non-monotone. To achieve these results, the author develops a model with customer heterogeneity along two dimensions: willingness to pay and willingness to wait. On one hand, customers may have high or low valuations. On the other hand, they may either be patient or impatient: patient customers are strategic and wait for discounts but impatient customers (in the extreme case) are myopic and make a one-time "buy-now-or-leave-forever" decision upon arrival. Unlike earlier models in revenue management (e.g. Belobaba, 1989, and Gallego and van Ryzin, 1994), which assume myopic customers, and unlike the demand models in economics and marketing (e.g. Conlisk, Gerstner and Sobel, 1984, and Xie and Shugan, 2001), which consist entirely of strategic customers, this model permits an arbitrary mixture of both. In this way, there are four customer segments (strategic-high-types, strategic-low-types, myopic-high-types, and myopic-low-types). However, an important restriction of this model is that each segment is assumed to arrive in a deterministic flow. Furthermore, instead of focusing on its adverse effects, this analysis highlights two ways in which strategic customer behavior may benefit the seller. First, strategic behavior implies that when prices are high initially, demand is not immediately lost and may still culminate in sales if prices are lowered eventually. (This effect is also stated by Zhou, Cho and Fan, 2005.) Second, under scarcity, strategic waiting generates competition for availability (at lower prices), which increases reservation prices and induces purchases (at higher prices).

Unlike other dynamic pricing setups, Gallien (2006) analyze a model in which the firm has a fixed inventory to sell over an infinite horizon, so there is no "end-of-season" deadline. Customers arrive according to a renewal process over an infinite horizon, and their valuations are independently drawn from some given distribution. Taking a mechanism design perspective, the author views each customer's type as his/her valuation and arrival time. Then, the seller's problem is to design a mechanism involving allocations to customers and transfer payments from them, subject to individual rationality constraints (since customers may not buy) and incentive compatibility constraints (since customer types are hidden information). Under some conditions, the author shows that the optimal mechanism is equivalent to a posted price mechanism $\{p_k\}_{k=1}^{K}$; that is, the posted price is p_k when there are k remaining units. Further, under the optimal mechanism, this sequence of prices increases with each sale, and customers either purchase upon arrival or never purchase. To a certain extent, this result rationalizes two common assumptions in revenue management models: first, the restriction to dynamic posted-price policies, and second, the assumption of "myopic" customers who either purchase immediately or not at all. The result that prices increase over time is consistent with earlier findings when customers do not wait strategically (see Arnold and Lippman, 2001, and Das Varma and Vettas, 2001). Some critics may argue that the infinite horizon assumption does not concur with many revenue management contexts; however, these insights should still apply in instances when the length of the time horizon is sufficiently large.

Taking a different analytical approach, Xu and Hopp (2004) model dynamic pricing as a stochastic control problem in the spirit of Bitran and Caldentey (2003). The demand intensity $\lambda(t,p)$, which represents the rate at which sales occur, depends on both time and price. For concreteness, we focus this discussion on the exponential demand case with $\lambda(t,p) = e^{\alpha(t)p}$. The authors capture strategic behavior by endogenizing the time-inhomogeneity $\alpha(t)$ in the following way. Customers form rational expectations of prices by considering the seller's revenue maximization problem given $\alpha(t)$ and respond according to these expectations; customers' price sensitivities (which reflect their strategic response to prices) should then be consistent with $\alpha(t)$. The authors develop techniques to solve the control problem under this consistency condition.

In a series of papers, Levin, McGill, Nediak (2006a) assume that each customer at time t has a random valuation $B(t)$, so customer valuations are volatile and may stochastically vary over time. In their model, customers are (ex ante) identical. Customers have a discount factor and this is interpreted as their "degree of strategicity": when the discount factor is zero, the customer disregards future purchase opportunities and is thus myopic; at the other extreme, when there is no discounting, future purchases are as valuable as current purchases, so the customer is fully strategic. Given their realized valuation and the price in the current time period, customers choose a purchase probability. For the firm, the problem is to set prices dynamically, so that revenues are maximized under customers' response to these prices. The authors derive dynamic equilibrium conditions for this problem, show existence and uniqueness, and derive structural properties for two special cases: when all customers are myopic, and when there is sufficient inventory to satisfy all customers. The authors extend their analysis to the oligopolist setting in Levin, McGill, Nediak (2006b). Next, in Levin, McGill, Nediak (2006c), they incorporate demand learning when the seller is uncertain of some parameters of the demand process. Since there is imperfect information, customers are unable to predict future prices via rational expectations. Instead, consumers are assumed to believe that prices follow a certain stochastic process (although these beliefs may not be accurate) and choose purchase probabilities accordingly. In this setup, the authors develop simulation-based techniques to find optimal prices.

Apart from pricing, there are also several other strategic issues that influence strategic customer behavior. In a recent paper, Yin and Tang (2006) investigate the effect of inventory information on strategic customer behavior. They develop a model with two customer types and two exogenously determined price levels. Using this model, the authors consider two scenarios: the seller may either display all available units on the sales floor, or put them out one at a time. Interestingly, the authors show that when facing strategic customers, the seller is better off by displaying its inventory one unit at a time. Another related issue is reservation policies, which guarantee future availability and therefore have a direct impact on how strategic customers respond to firms' dynamic price schedules. Using a similar model, Elmaghraby, Lippman, Tang, and Yin (2006) compare two operating regimes. Under the "reservations" regime, customers unwilling to pay the full price for a product may place a reservation that will entitle them to the product at the discounted price if it remains unsold. Under the "no-reservations" regime, consumers who do not pay full price have to enter a lottery for the discounted product, i.e., there is no advantage to the early arrivals. The authors find situations in which offering reservations generates higher revenues, but consumers are worse off.

Most papers above assume that customers arrive evenly over the time horizon. Dasu and Tong (2006) consider a model in which all customers are present at the start, in the spirit of early papers by Stokey (1979, 1981) and Besanko and Winston (1990). Customers have privately known and i.i.d. valuations. Using this model, they analyze two pricing schemes: posted prices (in which the seller commits to a price path, cf. Stokey, 1979) and contingent prices (where the seller is free to set prices over time, cf. Stokey, 1981). They characterize buyers' optimal purchasing behavior in equilibrium for both cases. They also numerically compare the seller's revenue under posted and contingent prices, and find that neither scheme dominates the other. The authors also numerically study several other questions, such as the cost of ignoring strategic behavior, the effect of the capacity-to-demand ratio, and the role of providing inventory and sales information to customers.

Ahn, Gumus, and Kaminsky (2005) introduce a model with a different kind of inter-temporal demand. Unlike the papers above, demand here is not "strategic" in the sense that customers do not actively try to find the best deal. Rather, they remain in the market for a fixed number of time periods and purchase once the price is set below their valuation. In this way, demand faced in a particular period depends on prices over multiple periods in the past. This model is motivated by the observation that customers may wait for prices to fall within their budget; thus, they are not immediately lost when prices are too high. This persistence of demand is thus in line with the spirit of the papers reviewed above. With this setup, the authors analyze joint pricing and inventory decisions; this appears to be the first work to incorporate dynamic inventory decisions into a dynamic pricing model with strategic customer behavior. The authors identify structural properties, and also develop effective heuristics for some special cases.

Capacity rationing models In this emerging literature, there is another group of papers focusing on how firms can use pricing and rationing to extract maximum revenue. Since capacity is scarce, rationing is almost inevitable, so it is quite natural for firms to use this as a strategic tool in face of strategic customers. The common approach adopted by the group of papers below is to start with a two-period model; prices are higher in the first period but there may be limited availability in the second period. When capacity is observable, customers can perfectly anticipate the probability of availability in the second period. However, when capacity is unobservable, customers infer the fill rate through a learning process that converges after multiple repetitions of the underlying two-period model. Broadly speaking, all the models below investigate how rationing affects strategic demand by making customers more inclined to purchase earlier at higher prices.

Liu and van Ryzin (2005) study the effects of capacity rationing in a two-period model in which all customers are present at the start of the horizon. As mentioned above, customers who buy in the first period pay a premium and customers who (attempt to) buy in the second period may be rationed. Demand is deterministic and prices are fixed. Customers observe capacity levels, so fill rates in the second period can be perfectly anticipated. The authors find that the effectiveness of rationing depends on consumer risk preferences; in particular, it is never optimal to ration risk-neutral customers. They also extend their analysis to the oligopolist context and find that competition may hinder the profitability of capacity rationing. There is some critical number of firms beyond which rationing can not persist in equilibrium.

In another paper, Zhang and Cooper (2006) consider a similar two-period setup with deterministic demand and observable capacity. As in Liu and van Ryzin (2005), the authors verify the advantages of capacity rationing when prices are fixed. However, when they endogenize the firm's pricing decision, they find that rationing provides no additional benefit. This result suggests that the role of capacity rationing is to compensate for poor pricing decisions. ¿From another angle, this result also suggests that rationing and dynamic pricing serve as substitute control levers for the firm.

Next, we turn to the setting in which the capacity is not observed by customers. Gallego, Phillips, Sahin (2004) examine capacity rationing using a similar two-period setup. However, in their models, customers do not observe quantities and have to form expectations of product availability through interactions with the firm. The authors model this using multiple incarnations of the base model, with customers updating their expectations in each period via outcomes in the previous period, until this process converges to some equilibrium point. Consistent with earlier work, the conclusion is that rationing customers and disposing of excess units may be preferred over training customers to wait for the sale.

Subsequently, Ovchinnikov and Milner (2005) also consider a similar model with multiple repeated instances of a two-period model. In each instance $i = 1, \ldots, n$, the seller determines the number of units x_i to put on sale in the second period. However, unlike Gallego, Phillips, Sahin (2004), the authors do not model customers' wait-or-buy decision in terms of individual utilities. Instead, they model aggregate waiting behavior in the following way: customers' propensities to wait in each instance is updated in a manner that depends on the number of units that were available on sale in the previous model instance. The authors develop techniques to solve for the optimal x_i^* that maximizes total revenue over all instances of the underlying two-period model.

Su and Zhang (2006) develop a framework that incorporates both approaches outlined above. When quantity is observable (as in Liu and van Ryzin, 2005), the seller has essentially committed to some fill rate; when quantity is unobservable (as in Gallego, Phillips, Sahin, 2004), there is no commitment and an equilibrium is derived based on the rational expectations approach pioneered by Muth (1961). The authors demonstrate that the ability to commit to some capacity level can enhance the firm's revenue. They also investigate approaches to sustain such commitments in a credible (subgame-perfect) manner. Such approaches involve contractual arrangements within a vertical channel (e.g. between the seller and its supplier). In this way, this paper sheds light on the effect of strategic customer behavior on supply chain performance.

Valuation uncertainty So far, we have looked at strategic customer behavior of a particular form; customers strategically delay purchases with hopes of getting a better deal in the future. However, there may be other reasons for delaying purchase. In particular, when customers are uncertain about their valuation for the product, it may be wise to wait until more information is available. For example, in the context of airlines, travelers who are not certain about their plans may choose to wait. This situation lends itself naturally to advance purchase discounts, in which customers who buy early are compensated for bearing risk (c.f. Xie and Shugan, 2001, and Dana, 1998). In the revenue management literature, there are a few papers that examine customer behavior in the presence of valuation uncertainty, as outlined below.

Gallego and Sahin (2006) extend this line of research by considering selling call options to customers who face uncertainty in their valuations. A call option (x, p) is sold at price p and it entitles the holder to claim a unit of capacity at the strike price x after realizing his or her valuation. There are two commonly observed special cases of such call options: $(0, p_1)$ options are non-refundable prices paid before valuations are realized, and $(p_2, 0)$ options are spot prices paid after valuations are realized (or equivalently, they can be seen as fully refundable prices). The authors find that selling capacity options can improve revenues significantly over advance purchase discounts.

Most of the papers in the literature assume that customers' valuations are uncertain in an independent fashion. For example, whether Traveler A values a seat on an airplane does not affect Traveler B's valuation. However, in practice, stochastic customer valuations may be correlated. For example, the valuation for a snow-shovelling contract depends on actual but uncertain levels of snowfall. Yu, Kapuscinski, and Ahn (2005) treat this case in a model of advance selling. They also consider the case with stochastic but independent customer valuations.

In another paper, Koenigsberg, Muller, and Vicassim (2006) develop a similar two-period, two-class model; the market size and composition is fixed but consumers face uncertainty in their own valuations. The authors find that optimal prices should increase over the two periods, thus resembling advance purchase discounts. The authors additionally investigate the feasibility of offering a last-minute discount at the end of period-two (so this becomes a three-period model); they conclude that this is beneficial only when customers are uncertain about whether such deals will be offered.

2.2 Customer Behavior and Multi-Product Revenue Management

Revenue management models have mainly focused on the case with identical units of the same product. In practice, firms often sell multiple products that exhibit demand dependencies. Then, how should the firm make revenue management decisions? The answer to this question, naturally, depends on the nature of these demand dependencies. In our review below, we consider two broad settings. First, demand dependencies across products may simply be driven by customer choice; that is, when customers choose one out of a set of products. Second, demand dependencies may arise due to substitution or complementarity effects across products; for instance, customers who purchase a product may purchase another related product.

Choice from a set of products Fundamentally, customer choice can be modeled using a discrete choice framework. This may be a general choice model, or may also be specialized to more commonly used models such as the multinomial logit model. In any case, the customer's choice depends critically on the set of available products. Therefore, there are two related questions: which set of options to make available, and how to price each of these options.

This issue of customer choice was first investigated by Talluri and van Ryzin (2004), who study a revenue management problem under a discrete choice model of customer behavior. There are n fare products, each associated with an exogenous revenue. At each point in time, the firm chooses to offer a subset of these fare products. Given the subset of offered products, customers choose an option (which may also be a no-purchase option) according to some discrete choice model, such as logit choice (see Anderson et al., 1992, or Ben-Akiva and Lerman, 1985). The authors show that there is an ordered family of efficient subsets $S_1, \ldots, S_m$, such that the optimal policy is to open one of these subsets S_k at any time, with the optimal index k increasing in remaining capacity and decreasing in remaining time. They show that the policy can be implemented using nested protection levels if and only if the sequence of efficient subsets is nested, i.e. $S_1 \subseteq \cdots \subseteq S_m$. The authors also provide conditions under which nesting by fare order is optimal. Finally, they develop an estimation procedure based on the Expectation-Maximization (EM) method that jointly estimate arrival rates and choice model parameters, when no-purchase outcomes are censored.

In a subsequent paper, van Ryzin and Liu (2004) extend this analysis to the network setting. Each product consists of a fare class and an itinerary, which may use up resources on multiple legs of the network. The dynamic program of finding the optimal offer sets becomes computationally intractable. Similar to Gallego, Iyengar, Phillips and Dubey (2004), the authors adopt a deterministic approximation by reinterpreting the purchase probability as the deterministic sale of a fixed quantity (smaller than one unit) of the product. Under this interpretation, the revenue management problem can be formulated as a linear program, and the authors show that its solution is asymptotically optimal as demand and capacity are scaled up. Using the LP formulation and the asymptotic results, the authors extend the notion of efficient subsets, introduced by Talluri and van Ryzin (2004), to the network setting. Finally, the authors also discuss implementation heuristics to convert their static LP solution into dynamic control policies.

Zhang and Adelman (2006) further extend this network revenue management setup by adopting an approximate dynamic programming approach. The authors approximate the value function for the problem using an affine function of the state vector. They show that this approximation yields tighter bounds on the value function, compared to the choice-based linear programming approach of van Ryzin and Liu (2004) and Gallego, Iyengar, Phillips and Dubey (2004). Like these previous authors, Zhang and Adelman (2006) also explore computational techniques such as decompositionbased heuristics and column generation ideas, and apply these ideas to solve the problem for a multinomial logit choice model.

There is another group of papers by Zhang and Cooper (2005a, 2005b), who examine the issue of customer choice from a somewhat different angle. While the papers reviewed above look at customer choice between different fare products drawing from a common pool of resource (possibly in a networked fashion), Zhang and Cooper consider customer choice over parallel flights, each drawing from separate inventories.

In their first paper, Zhang and Cooper (2005a) consider the "block demand" model in which "blocks" of customers arrive sequentially, with class-j demand arriving in period j. They extend the classical single-flight setup to multiple parallel flights and model customer choice behavior across flights. The model of consumer choice depends on a preference mapping as well as inventory availability over all flights. The problem is to find optimal dynamic booking limits for future classes of customers on each flight. The authors derive upper and lower bounds for the value function in their model. They also provide simulation-based techniques to solve the dynamic program, and propose heuristics based on a linear programming approximation.

Next, in Zhang and Cooper (2005b), the authors continue their line of research, but consider pricing decisions instead of availability decisions. Time is discrete and at most one customer arrives in each period. In each time period, the firm chooses a price (out of a discrete set of allowable prices) for each flight. Given this vector of prices, the customer's choice is given by some choice probabilities, which depend on the prices offered. The authors develop bounds for the value function of this dynamic pricing problem. They use these bounds to derive value-approximation and policy-approximation heuristics, and numerically show that these heuristics are, in most cases, near optimal and superior to other heuristics based on pooling ideas.

There are also several papers that look at customer choice in revenue management systems, applied to a health care setting. In this case, customers refer to patients, who choose between capacity sources. Gupta and Wang (2005) formulate a revenue management problem of a clinic that sees both same-day patients as well as regular patients. The clinic has a fixed number of appointment slots over a single day. Regular patients request appointment slots before-hand, and same-day patients arrive at the start of the day. The clinic wishes to balance the needs of the patients who book in advance and the patients who require a same-day appointment. Regular patients choose appointment slots according to a discrete choice model. Thus, the clinic's problem is to decide which appointment requests to accept in order to maximize revenue. In a similar setup, Green, Savin, Wang (2006) investigate how to design the outpatient appointment schedule, but they focus on the dynamic scheduling of different classes of patients into service.

Substitution and complementarity across products Now, we turn to substitution and complementarity effects across different products. In general, these effects can be captured using multi-dimensional demand functions (mapping prices into demands for each product). Such a preference-based approach is versatile and can be applied to rather general substitution and complementarity patterns across different products. Beyond this approach, there are also other related scenarios that fall under this category. For instance, consider the following situation. A customer who has purchased a particular product may also be willing to purchase a related product, especially if a discount is offered. Such practices are called cross-selling or up-selling, and are quite common in practice. The questions here would be how to choose the accompanying product, and what price to charge for the bundle. As another example, consider the following situation. When the product requested by a customer is sold out, the customer may still be willing to accept a substitute. In such situations, should these substitution offers be made, and how should they be priced? With this framework in mind, we now turn to review the related papers.

Maglaras and Meissner (2006) consider a multi-product revenue management problem with multi-dimensional demand functions that map prices (for each product) into demand rates (for each product). There is a common resource, and different products deplete the resource at different rates. With this model, the authors formulate a dynamic pricing problem and a capacity allocation model, and show that both formulations can be reduced to a common framework in which the firm controls the aggregate rate at which the resource is depleted. For a deterministic (fluid) formulation of the model, the authors characterize the optimal controls in closed-form and suggest several static and dynamic heuristics. These heuristics are also shown to be asymptotically optimal.

In another paper, Cooper, Homem-de-Mello, Kleywegt (2006) considers the fact that customers may choose between different fare products (focussing on the airline context). In particular, the availability of low-fare tickets will reduce sales for high-fare tickets. However, the focus of this paper is different because the central question here is: what happens when revenue managers fail to recognize this kind of demand dependency? The authors show that in this case, a spiral-down effect may occur in the following manner. The revenue manager estimates demand based on observations of past sales, without considering any dependency between the high-fare and low-fare products. Thus, as more low-fare tickets are made available, low-fare sales will increase and high-fare sales will decrease, leading to decreased estimates of high-fare demand and lower choices of protection levels for high-fare tickets. As this estimation and optimization procedure iteratively continues, high-fare sales, protection levels, and revenues follow a downward spiral. Here, the modeling setup involves the basic two-class model, in which an optimal protection limit is set according to Littlewood's rule. The authors do not explicitly model the choice behavior of customers, but instead focus on modeling the adaptive procedure of estimation and optimization. They identify conditions under which spiralling down occurs.

Next, we turn to demand substitution across different "classes" of products. For instance, in the rental car business, customers who request a "mid-size" car but find it unavailable may be willing to accept an upgrade to a "full-size" car. Shumsky and Zhang (2004) consider this situation and formulate a dynamic capacity allocation problem with upgrading. In their model, there are n customer classes and n products. Time is discrete. The customer arrivals $(d_1^t, \ldots, d_n^t)$ in each period t is given by a joint distribution function. Each customer may receive a product corresponding to his/her class or be upgraded by at most one class. Prices are fixed, and there may be usage costs for satisfied demand and penalty costs for unsatisfied demand. The firm allocates units to arrival customers, and may also ration demand. Therefore, demand substitution is driven by the firm's actions rather than through customer choice. The authors show that the optimal allocation policy involves a greedy allocation (fulfill as much demand as possible without upgrading) followed by upgrading capped by a dynamic protection limit for each product. They also consider a simplified special case with two products and two time periods, and analyze the optimal initial capacity decision in this case.

Finally, we turn to cross-selling. This refers to the practice of offering each customer a choice of the requested product or a package containing the requested product as well as another product. Such practices are becoming increasingly common in the e-commerce setting. Netessine, Savin, Xiao (2006) introduce an investigation of cross-selling practices into the revenue management literature. In the model, the firm manages a set of products, faces stochastic customer arrivals, and makes dynamic cross-selling decisions based on current inventory levels of each product. There are m products and m classes of customers, each requesting one of the products i at a fixed price p_i. Each class-i customer's reservation price for a bundle with products i and j is described by the distribution F_{ij}. In this setting, firms have to select the complementary product to offer as well as the optimal price for the packaged offer; this leads to a combinatorial optimization problem super-imposed onto the dynamic pricing problem. The authors focus on two practical settings:

with and without an opportunity to replenish inventory in the event of stockouts. The authors refer to these as the Emergency Replenishment Model and the Lost Sales Model respectively. For the former, the authors demonstrate that the problem can be decomposed into a separate problem for each product. For both models the authors provide bundling and pricing heuristics, and test their effectiveness numerically.

In a subsequent paper, Aydin and Ziya (2006) consider a similar practice and refer to it as upselling. In the model, there are two products, a regular product and a promotional product. The regular product is assumed to be always available and its price is exogenously fixed. The promotional product has a fixed inventory and has to be sold by a certain deadline. The firm first chooses the price for the promotional product. Customers arrive stochastically and may purchase either product. If the customer purchases the regular product, the firm may additionally decide whether or not to make an upsell offer and if so, how much discount to give. In this sense, the authors jointly consider the problem of dynamically pricing as well as upselling the promotional product. The authors also consider the upselling problem when the price of the promotional product is static and set at the start. Under dynamic pricing, the authors find that the firm's upselling decision does not depend on inventory level and remaining time. The authors also find that the benefit of upselling is greater under static pricing compared to dynamic pricing.

3 Auctions

In this section we review some classic results from auction theory describing seller and bidder behavior. We will also discuss the new Internet auctions that are typically dynamic with more complicated bidding strategies.

3.1 The Impact of Bidding Behavior on the Revenue Equivalence Theorem

Most classic auction models assume independent and private valuations of bidders. (See Vickrey 1961, Milgrom and Weber 1982, the recent survey by Klemperer 1999, and earlier surveys by McAfee and McMillan 1987, Milgrom 1989.)

The well-known revenue equivalence result states that if all bidders are risk-neutral and have independent private values for the auctioned items, then all four of the standard single unit auctions have the same expected sales price (or seller's revenue). The four standard single unit auctions are the English auction, the Dutch auction, first-price sealed-bid auction, and the second-price sealed-bid auction. In particular, the first-price sealed bid and Dutch auctions are "strongly equivalent," or "strategically equivalent." [2] The English and second-price sealed-bid auctions are strategically equivalent when bidders item values for the auctioned item are private and independently drawn from the same probability distribution. In that case, the optimal strategy in both auctions is to bid up to or stay in until the value.

There are several tests on the revenue equivalence results between the first-price and Dutch auctions, and between the second-price and English auctions. In terms of the strategic equivalence between the Dutch auction and the first price sealed-bid auction, we have observed contradicting experimental results. Cox, Roberson and Smith (1982)'s experiments suggest that the prices in first-price sealed-bid auctions are higher than Dutch auction prices. Smith (1991) suggest two possible explanations: 1) the utility depends not only on the monetary outcome but also on the "suspense of waiting" in the Dutch auction; and 2) bidders may underestimate the increased risk associated with waiting in the Dutch auction. On the other hand, Lucking-Reily(1999) reports that the prices in first-price sealed-bid auctions are lower than Dutch auction prices in his experiments.

One possible explanation of this contradicting results may be the speeds of the Dutch clocks used in different auctions. A fast auction clock may result in less revenues if a slower clock was used instead (Kwasnica and Katok, 2003).

For the English auction and the second price sealed-bid auction, most of the experimental results show that the second-price and English auctions are not strategic equivalent. The prices in second-price auctions are significantly higher than the prices in English auctions (e.g., Kagel, Harstad and Levin, 1987; Kagel and Levin 1993, and and Harstad, 2000), except studies by Coppinger, Smith, and Titus (1980) and Cox, Roberson and Smith (1982), where bidding above valuations are not allowed.

One of the possible explanations of overbidding in second-price auctions is due to subjects "illusion that bidding in excess of value improves the probability of winning with no real cost to the bidder, as the second-high-bid price is paid." (Kagel et al, 1987). However, Garratt, Walker, and Wooders (2004) show that the bidding strategies for bidders with or without experience can be different. Experienced bidders show no greater tendency to overbid than to underbid.

[2]two games are strategically equivalent if they have the same normal form except for duplicate strategies. In our setting, for every strategy in the first-price auction, we can also determines a strategy in the Dutch auction which results in the same outcomes.

3.2 Behavior Modeling in Multi-unit Auctions

Multi-unit demand auctions offer more opportunities for strategic bidder behavior. Depending on whether every winning bidder pays the same price or not, we can have two classes of auctions: A uniform price auctions in which every winning bidder pays the same price, or a discriminatory auction in which winning bidders pay amounts depending on the amount bids.

For uniform price auctions, researchers have found two very different types of behavior: demand reduction and over bidding. Demand reduction occurs if bidders have non-increasing demand for homogeneous goods. This is because there is an incentive to reduce demand on some units in an effort to win other units at more favorable prices (see, for example, Ausubel and Cramton, 1996 and Engelbrecht-Wiggans and Kahn, 1998). Over bidding occurs if there are complementarities between items. The value of a package of items may exceed the sum of its parts so there are incentives for bidders to bid above the value they place on any individual item (see, for example, Krishna and Rosenthal, 1996). Kagel and Levin (2001) point out that both types of incentives appear in the recent Federal Communications Commission (FCC) spectrum auctions: In the nationwide narrow-band auction bidders appear to have had non-increasing demands, while in the broadband auction there appear to have been complementarities among items.

Feldman and Reinhart (1995) discuss the differences between uniform-price and discriminatory-price formats in terms of bidding behavior and seller revenue. Their analysis shows that auction participants shade their bids under a discriminatory-price format. They also apply their model to data from the International Monetary Fund (IMF) gold auctions run in 1976-80. During this period, IMF sold one-fifth of its gold stock at 45 sealed-bid auctions to create a fund to assist developing countries. In general, in sealed-bid auctions awards are made either at the price that was bid (discriminatory-price format) or at a single, market-clearing price (uniform-price format). Their results suggest the superior revenue-generating properties of uniform-price over discriminatory-price auctions.

Kagel and Levin (2001) compare bidding in the uniform price auctions with a dynamic Vickrey/Ausubel auction (Ausubel, 1997). Theoretically, the Vickrey auction promotes full efficiency since it eliminates any incentive for demand reduction. Experiments have shown that Vickrey auction can raise greater expected revenue than the uniform price auction (Maskin and Riley, 1989; Ausubel and Cramton, 1996). Kagel and Levin (2001) show experimentally that the dynamic Vickrey auction eliminates the demand reduction found in the uniform price auctions, thereby improving economic efficiency. However, it raises less average revenue than in uniform price sealed bid auctions.

3.3 Behavior Modeling in Internet Auctions

With recent advances in information technology and the growth of electronic commerce over the Internet, online auctions have expanded rapidly with millions of transactions occurring every day. As a result, Internet auctions have attracted the attention of academic researchers.

Although there are some studies on the effect of auction formats (Lucking-Reiley 1999), and the last-minute bidding phenomenon (Roth and Ockenfels 2002), Internet bidders' behaviors are still not well-studied. Online bidders' behavior, together with strategies employed by software agents widely used in Internet auctions, are sophisticated and can be very different from the types of behavior observed in traditional auctions. Dholakia and Soltysinski (2001) report evidence of herd behavior bias: bidders will often be influenced by the behavior of other bidders when choosing items to bid on. This behavior occurs because of the lack of key information about an item. Online bidders can neither fully assess the trustworthiness of an online seller nor thoroughly evaluate an item (as they could at an off-line auction or in a retail environment), so they perceive existing bids to be evidence of an item's quality, making it worthy of their own bid. Kamins, Dreze and Folkes (2004) find that when the seller specifies a high external reference price (a reserve price), the final amount bid is greater than when the seller specify a low external reference price (a minimum bid amount). This finding is consistent with research in the behavioral price literature, which shows that as the level of an advertised reference price increases in a comparative price advertisement, consumers purchase intentions increase (see, for example, Blair and Landon 1981). See Park and Bradlow (2005) for more reviews. Park and Bradlow (2005) also propose a dynamic parametric stochastic model of bidding behavior that pays attention to bidding dynamics. Using a database of notebook auctions from one of the largest Internet auction sites in Korea, they demonstrate that their dynamic model captures the key behavioral aspects of bidding behavior, which include: whether an auction will have a bid at all, (if so) who has bid, when they have bid and how much they have bid over the entire sequence of auction bids. Furthermore, they provide a useful tool for managers at auction sites to conduct their customer relationship management. Specifically, the managers can use their tool to evaluate the "goodness" (whether) of the listed auction items and the "goodness" (who, when, and how much to bid) of the potential bidders in their Internet auctions.

An interesting paper that studies the strategic bidding behavior in sponsored search auctions is by Edelman and Ostrovsky (2005). They estimate that Overture's revenue from sponsored search (between June 15, 2002 and June 14, 2003) could have been more than 60% higher if it had been able to prevent the strategic behavior of the bidders. They also find that this type of behavior remains present on both Google and Overture nowadays: When two or more advertisers activated autobidders, their bids tended to form a distinctive sawtooth pattern of gradual rises in price followed by sudden drops. This outcome is caused by the first-price auction, which is naturally unstable, in the sense that if bids can be adjusted frequently, bidders will not state their true valuations, but will keep revising their bids in response to other bidders' behavior. As a result, a sawtooth bidding pattern might appear and it reduces market efficiency. Better auction designs could reduce this strategic behavior and raise search engines revenue, as well as increase the overall efficiency of the market.

Feng, Shen, Zhan (2006) study auctions for a set of ranked items where each buyer has unit demand. This setting has promising applications in areas such as keyword auctions in the search engine advertising industry, the sale of quality ranked raw materials, etc. An auction mechanism suitable for this setting is the simultaneous pooled auction (SPA), where each bidder simultaneously submits a single bid and is allocated an object based on the rank of his bid among all the bids. However, one severe problem inherent in the SPA is that some bidders may incur ex post losses; that is, they pay more than what they value the received objects. The loss can impact the bidding amounts from bidders. They propose a tailored VCG mechanism that generates the same expected revenue as the SPA does, while bidders do not incur any ex post loss.

Both eBay and Amazon supply bidders with simple software agents, so participants in Internet markets can be human bidders bidding in person, or artificial agents employed by bidders. Thus, the interactions between human bidders and agents may impact the performance of market rules. Gonzalez, Hasker, and Sickles (2004) study bidding behavior in eBay computer monitor auctions. Their analysis reject the use of Jump Bidding (Avery 1998) or "Snipe or War" bidding (Roth and Ockenfels, 2000) even though they find that over 11% of the bids in their data sets were submitted in the last minute. If sniping becomes even more widespread on eBay than they are today, eBay will be gradually transformed into a sealed bid second price auction (Ockenfels and Roth, 2002). Ockenfels and Roth (2002) also discuss the manner in which late bids are caused both by sophisticated, strategic reasoning and by irrationality and inexperience, the interaction of late bidding and incremental bidding, and the relation between market design and artificial agent design.

3.4 Testing Theorems: Practical Auction Design

It is widely recognized that the appropriate choice of auction format is a matter of great practical concern. The ultimate goal of any auction design theory and experiment is to make sure that it will perform reasonably well in practice, that is, achieve high efficiency or collect high revenue. We have reviewed many papers on auction theory and the experiments performed to test these models. Applying them to real life can have totally different outcomes.

The FCC held its first radio spectrum auction in July 1994. The simultaneous multiple-round auction format, which is an ascending bid auction in which all licenses were offered simultaneously, was used this this auction. Cramton (1995) describes the auction rules and how bidders prepared for the auction. He also posed several questions for auction theory, including the impact of jump bidding and joint bidding / collusion. He also discusses how to adjusting the bid increment in response to bidding activity. In the end, the government collected $617 million for ten licenses, and the auction was viewed as a huge success. It serves as an excellent example of applying economic theory to practical problems of allocating scarce resources.

Cramton (1997) further analyzes six spectrum auctions conducted by the FCC from July 1994 to May 1996. All these auctions use simultaneous multiple-round auctions. Bidders can successfully formed efficient aggregations of licenses, although the actual bidding behavior differed substantially in the auctions. He believes that the extent of bidder competition and price uncertainty play an important role in determining behavior. In several of the auctions, bidding credits and installment payments also play major roles in bidding behavior.

"Jump bidding" behavior has been observed in FCC auctions. Jump bidding means that a bidder increases his bid higher than the minimum required increment, and raise his own standing high bid. Such bidding behavior is deemed irrational by standard auction theory, but it occurred in all of the first three FCC auctions, especially in the first two narrow-band license offerings.

The 1990s had also witnessed failures of several US auctions because of the collusion among the bidders. They utilized the final digits of their bids to let others know the lot identification numbers and even phone numbers. Some of the companies were imposed forfeitures by the FCC. [36]

An important factor that determines whether an auction will be successful or not is the degree of bidder collusion. There is only limited research on collusion in the literature. Robinson (1985) suggests that collusion may be easier to achieve in a second-price auction than in a first-price auction. In a second-price auction, the designated winner can bid a very high amount while all the other bidders bid zero. Note that no other bidder has any incentive to deviate from this agreement. However, in a first-price auction, ideally the designated winner should bid a very small amount, while all the others bid zero. But all the others may have incentive to cheat on the agreement. McAfee and McMillan (1992) show that for private value auctions, it is possible for the cartel to designate the winner and fairly (incentive-compatibly) divide the gains by making appropriate side payments. Hendricks and Porter (1989) analyze what environment and mechanisms may facilitate collusion and study the methods of detecting collusion.

The English auction is the most widely used auction format in practice. However, since it is easy to find out who is bidding and how much, English auction enables the formation of collusive arrangements in real time comparing with sealed-bid auctions. The English auction enables collusion without prior agreements. The outer continental shelf petroleum lease bids are publicized by the Government, thus it is very easy for the bidders to collude. (Cox, Isaac and Smith, 1983).

The important thing to realize here is that auction design has to be context dependent. Good auctions pay attention to the specific details of the situation, and have to relax many "reasonable" assumptions made in the theory papers and experiment papers. Special attention has to be paid to potential collusive activity and entry-deterring behavior.

The 3G mobile-phone auctions in Europe conducted at the turn of the century illustrate the above point perfectly. Although the auction items are very similar in each of the nine auctions. The difference in auction design, in some cases only subtle differences, can result in very different results: Switzerland got 20 euros per capita while the United Kingdom received 650 euros per capita. Klemperer (2004) analyzes the differences in auction designs from different countries and the resulting bidder strategies, why some auctions can facilitate collusion between firms and failed to attract entrants. He also points out that the sequencing of the auctions was crucial.

We also want to point out that the failure of many online marketplaces are related to bad auction design. Online marketplaces have been built and run for commodities in the hope of achieving better market efficiency. However, many of them failed due to various reasons. Bad auction design and excessive squeezing of sellers' profits which eventually lead to less and less sellers are two important reasons for many market places to fail. Shen and Zhang (2006) survey over 350 B2B e-marketplaces in an effort to discover what is the current state of the industry, what caused it to be so, and what we can do for the future half a decade after its zenith. The foundation of the study was based on B2B e-marketplaces and relevant information that was collected from the Internet.

4 Future Research

We believe that modeling customer behavior is becoming one of the most important research areas in operations management. Understanding consumer decision-making and characterizing their responses to firms' decisions is an important ingredient in making good operational decisions. Although we have seen a growing number of papers devoted to this area, there are still several important research directions that we think deserve more attention from the research community.

4.1 Auctions in OM Settings

Recently we have seen an increasing interest in auction research within the operations management community (e.g., Vulcano, van Ryzin, and Maglaras (2002) Chen et al. (2005), Chu and Shen (2006a,b) and their reviews). We believe it is important to consider bidders' behaviors in this line of research. For instance, Vulcano, van Ryzin and Maglaras (2002) analyze a dynamic auction in which a seller with C units to sell faces a sequence of buyers separated into T time periods. They assume the buyers' valuations for a single unit are private and independent. There are a random number of buyers in each period. They prove that dynamic variants of the first-price and second-price auction mechanisms maximize the sellers expected revenue. Their main result is to show that under certain circumstances, the optimal auctions significantly outperform the traditional revenue management mechanism and a simple auction heuristic. The traditional revenue management mechanism use list prices in each period together with capacity controls, and the simple auction heuristic simply allocates units to each period and runs a sequence of standard, multi-unit auctions with fixed reserve prices in each period.

We feel that it is important to consider customer behavior in this problem. Customers are generally heterogeneous, and they may have very different valuations for the item. Some forms of the strategic behavior discussed in this paper may well apply to this problem setting. For instance, different customers may choose different time periods to participate in the auction. Furthermore, identical distributions for customer valuations may not be reasonable. The distributions may need to be adjusted as high-value bidders win and leave the auctions. Thus, the subsequent distributions should shift to left. With this in mind, some bidders may choose to wait and participate in the later rounds of auctions.

The number of units offered in each round of auctions, and whether the customers know this information or not, can also have an impact on bidding strategy. Uncertain about how many units are left in the remaining auctions, customers may bid more aggressively compared to the situation in which bidders know the quantity available in each round. A related question facing the seller is whether he should randomize the capacity available in each round.

For multiple round auctions, bidders typically can learn from past auctions. It will be interesting to understand how this affects their behavior and the seller's revenue.

We believe studies that can characterize bidder's behavior in the auction settings above, and in general for revenue management problems, are important and may reveal interesting managerial insights.

4.2 Bounded Rationality

In this paper, we have argued that it is important to understand and correctly model customer behavior in many operations management problems. We have seen how incorporating customer behavior may generate new insights or identify important issues that may have been overlooked before. Many theoretical studies reviewed above are based upon normative models of customer behavior. That is, given a particular decision model, customers are assumed to make optimal decisions (e.g., purchasing, bidding) that maximize their utility. Yet, it is conceivable that individuals, especially at the consumer level, are subject to psychological biases and cognitive limitations. For example, given limited information-processing resources, customers may use simple heuristics to make complex decisions. There may be intrinsic psychological tendencies to distort decisions toward particular alternatives. We shall collectively refer to this set of behavioral phenomena as *bounded rationality*. While these issues have been well-studied in related disciplines, such as psychology, behavioral economics, and marketing, they have not yet been embraced in operations management. Nevertheless, we feel that bounded rationality has tremendous potential in generating new research questions, providing additional tools, and bringing a fresh perspective on "mainstream" operations management research.

There are a couple of papers that have initiated our enquiry along these lines. Popescu and Wu (2006) study the effect of loss aversion on dynamic pricing strategies. Loss aversion refers to the systematic tendency for individuals to perceive losses, relative to some reference point, as being more significant than gains of the same objective magnitude. This effect was introduced by Kahneman and Tversky (1979) in their ground-breaking work on prospect theory. In the context of dynamic pricing, when a firm interacts repeatedly with the same customer pool, loss aversion and reference dependence implies that demand would be sensitive to the firm's pricing history. In particular, compared to the situation when previous prices were low, consumers are more likely to buy when previous prices are high, even when the current price is fixed at the same level. This suggests that when firms set prices over time, they have to take into account consumer memory of past prices. To capture this effect, the authors use a reference dependent demand function $D(p, r)$ that is decreasing in the price p and increasing in the reference price r, which is assumed to be an exponentially weighted sum of past prices. They formulate and solve the dynamic programming problem. They find that optimal prices converge to a constant steady state price when consumers are loss averse (but when consumers are loss-seeking, the optimal policy involves price cycles). Finally, the authors show that when managers ignore such reference effects, firms will price too low and lose revenue.

Customers who delay purchases may do so based on strategic considerations (e.g, with the hopes of securing “good deals”), but there may also be behavioral causes. In other words, consumers sometimes wait even when it is optimal (from an objective perspective) to buy immediately. We often see procrastinators waiting until the last minute in a wide range of situations (e.g., holiday shopping, preventive health care, maintenance services, etc). In a recent paper, Su (2006) considers this kind of behavior and refers to it as consumer inertia. This behavior is modeled using an additional utility premium that is required to trigger purchases, i.e., the consumer chooses to buy now if and only if $U \geq U' + \Gamma$, where U is the utility from buying now and U' is the utility from waiting. (A standard model of rational decision-making would have $\Gamma = 0$.) Interestingly, this model of inertia is consistent with several well-established behavioral regularities, such as loss aversion (e.g., travelers with valuation uncertainty will face a loss if they purchase a non-refundable plane ticket that turns out to be unsuitable), subjective over/under-weighting of the probability of future price changes and future availability, and hyperbolic discounting (in the presence of immediate transaction costs).

Apart from the examples above, there are many other behavioral issues that deserve attention in the revenue management literature. Many studies on customer behavior implicitly assume that individuals respond to firms' practices in an optimal manner. These consumers have access to multitudes of information and unlimited processing capabilities that make optimal decision-making feasible. Yet, we know that this is hardly realistic. Do consumers monitor prices continuously over time? Are they able to respond immediately when prices fall to a certain threshold level? Can they calculate these “optimal” decision thresholds in the first place? Do they know how to submit optimal bids? Is it reasonable to assume that all consumers understand the “game” that they are playing amongst themselves? These are all questions that cast doubt on standard analytical models that, in the absence of well-accepted alternatives, are predominant in research studying rational customer responses to firm strategies. In our opinion, the time is ripe to begin looking for alternative models that incorporate bounded rationality. What are the implications when consumers resort to simplifying decision-making heuristics, such as the availability and representativeness heuristics (see Tversky and Kahneman, 1974)? What if consumers satisfice instead of optimize (see Simon, 1982)? What if consumers face constraints on cognitive ability, memory, and processing power (see the monograph by Rubinstein, 1998)? How do behavioral considerations affect the way consumers interact strategically with one another and with the seller (see Camerer, 2003)? It would be interesting to understand the effects of these limitations on revenue management practice.

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