Algorithmic Learning in Double Auctions

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Introduction

- In the 1990s, a series of computerized tournaments found that sophisticated learning algorithms could not beat simple trading rules in repeated double auctions (Rust et al., 1992).
- In the last 30 years, there has been tremendous progress in the ability of self-learning algorithms to overcome complex games like Chess, Go, and Atari.
- These developments come from:
  - Processing data from millions of games through self-play.
  - Flexible non-linear approximation of the "long run" value from actions at any possible state.
- Can self-play and flexible representation enable learning algorithms to discover strategies that do better than simple trading strategies?

Research Questions

- Can Deep Q-learning Networks and Genetic Algorithms learn to identify and beat simple sniping rules (e.g. Kaplan trading strategy, Zero Intelligence Traders) in noisy trading environments by training through sufficient self-play?
- What kind of information is necessary to allow this to happen? How do changes in market rules lead to the emergence of different strategies? Which market rules fecilitate learning and which do not?

Theoretical Literature

- Samuelson (1980) studies static games with a buyer bidding to a seller with an unknown reservation price. Bids rise in buyer-valuation and beliefs about seller-valuation. Uncertainty about sellers' valuation transfers gains from trade from buyer to seller and permits trade in situations that, under certainty, would not be possible. Risk aversion leads to conservative bidding i.e. truth-telling.
- Sobel and Takahashi (1983) study dynamic games and show that when buyers are unable to commit to certain prices, information gleaned from failed offers is vital. Offers that are not accepted help buyers and sellers update their beliefs about each others' reservation prices, and inform strategies in the next period.
- Sattettbwaite and Williams (1989) find that in static double auctions, as the number of traders increases, truth-telling becomes the nearly dominant strategy, as no trader is able to manipulate the market price with falsehood. With smaller number of traders, small amount of "bid shading" is optimal.
- Wilson (1987): Characterises a BN equilibrium for a single period

Experimental Literature

- Easley and Ledyard (1986) suggest that experimental data with humans rejects Walrasian, Marshallian, and Bayesian Nash (BN) theories of price adjustment. They propose a decision-making heuristic that uses “reservation prices” and “expectation formation from history” that better matches the data.
- Rust et al. (1992): ”Kaplan” Sniping strategy: that waits for bid-ask differences to close without revealing any information, and then steals the deal if profitable to do so. Kaplan beats complex algorithms like using neural networks and genetic algorithms that process a large amount of information. This “background” strategy, although dominates others, is not evolutionarily stable and needs the presense of “active” strategies that generate information.
- Tesauro et al. (2001): finds strategies to beat the Kaplan strategy in the continuous double auction.

Terminology

There are many closely related market mechanisms:

- Double Auction - traders (buyers/sellers) message the bid/ask offer, and decide whether to buy/sell.
- Single Auction - Buyers post bids in single or multiple rounds, and the seller chooses a winner and a payment amount from the bids.
- Posted Price - Sellers (buyers) announce ask (bid) prices and then buyers (sellers) accept or reject.

We will focus on the double auction.

Typology of Auctions

Auction Type	Examples
Single-dimensional vs multi-dimensional	Auction based on price vs one based on price, date, quality
One-sided or multi-sided	Art auction vs Call market (buyers and sellers)
Open-cry or sealed-bid	Bids (winning or otherwise) are revealed or they are not
First-price, second-price or k-th price	Winner pays their bid, the second-highest bid or the k-th highest bid
Single-unit or multi-unit	Auction for one barrel of wine vs for X barrels of wine in one go
Single-item or multi-item / combinatorial	Single item vs Bundles of products (e.g. 10 barrels of wine, 1 box of fish, etc.)

Double Auction

- Two-sided auction where buyers place **bids** and sellers place **asks**.
- Types:
  - Periodic - bids and asks are recieved for a fixed duration, quantity demanded and supplied for each price is computed, and market clearing price is determined. e.g. NYSE Call Market
  - Continuous - the market does not close, but the auctioneer immediately matches bids and asks as many as it can in a continuous fashion. e.g. Comodity trading at Chicago
- These are most commonly used in stock markets where buyers and sellers try to sell blocks of shares (multi-unit auctions).
- Firms looking to raise money can go public and float Initial Public Offerings (IPOs).

Market Clearing

- At any time the prevailing bids and asks can be tallied up to find the quantity demanded and quantity supplied at any given price.
- A range of prices may clear the market, in the figure it is 20-20$.

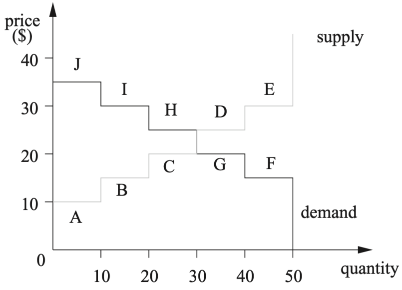

Fig. 2. Illustrative supply and demand curves for a double auction.

Why Double Auctions?

The main benefit of double auctions is that they economize on information and ensure markets clear.

If an auctioneer wanted to clear this market, she would have to compute the demand and supply curves from everybody's reservation price and choose the market price that would clear the market. This involves processing and knowing a lot of facts.

But double auctions have shown that even with extremely limited information and just a small number of traders, prices quickly converge to market clearing levels.

Research Context

- We want to approximate the New York Stock Exchange (NYSE) as closely as possible.
  - Equity Exchange
  - Continuous Double Auction
  - Open-outcry
  - Multiple stocks
  - Daily 930 to 1600
  - Agents:
    - Market Makers: track bid-ask for upto 100s of stocks, determine openeing/closing auctions, freezes, and market prices. They make profits from small differences in bid-ask prices. They create a market when nobody wants to buy/sell. Their purposes is to shrink the bid/ask spread and provide liquidity to the market.
    - Brokers: make bid-asks for multiple stocks.

Appendix

Samuelson (1980)

- When buyers have to offer, they "shade" their bids below reservation price. Thus if the reservation price is lower, bids are lower.
- If the buyer **believes** that sellers reservation prices have decreased, then bids would be lower.
- If there was no uncertainty about reservation prices, the offerer would get all the gains from trade. With uncertainty, the offerer makes at least as large a profit as the recipient. Thus, recipient of offer gains from uncertainty, while offerer loses out.
- Given the valuation distributions, the expected group profit (buyer + seller) may differ given who is to make the offer. For instance, if the seller knew the buyer's reservation perfectly and not vice-versa, and if seller had to make the offer, then trade would only take place when buyer's reservation exceeded the sellers. But if there was uncertainty, (inefficient) trade would happen even if that was not the case. Thus the agent with more information should be making the offer in order to maximize social efficiency.
- Risk aversion makes agents more and more truthful.

Easley and Ledyard (1986)- I

- Experiments with human subjects in bid-ask markets show that the actual price does not overlap exactly with market clearing prices.
- However, after a few repetitions, we see that prices converge to near the market-clearing price (theoretically predicted).
- Thus, the data rejects the "Walrasian tatonnement auctioneer", where an imaginary auctioneer announces hypothetical prices and clears the market.
- The transaction data also reject the “Marshallian” process where the buyer with the highest valuation purchases from a seller with the lowest valuation (because a wide range of prices are possible).

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- However, after a few repetitions, we see that prices converge to near the market-clearing price (theoretically predicted).
- Thus, the data rejects the "Walrasian tatonnement auctioneer", where an imaginary auctioneer announces hypothetical prices and clears the market. This is also one possible Nash equilibrium.
- The transaction data also reject the "Marshallian" process where the buyer with the highest valuation purchases from a seller with the lowest valuation (because a wide range of prices is possible). The correlation between buyer rank and seller rank is close to 0 (roughly random).

Easley and Ledyard (1986)- II

- It is hard to test Bayesian Nash Equilibrium models. Firstly, because if agents are risk averse, then anything is possible. Secondly, it is hard to pin down "common knowledge". Thirdly, the games are repeated, which is not an assumption in Wilson's (1987) model. However, Wilson's model does predict the same order of transactions as Marshallian, and thus is rejected.
- Agent must decide what to do (bid/ask), when to do, and whether to do (accept/reject).
- Each agent knows: rules of the auction, own valuation, sequence/timing/amount/identity/status of past offers.
- Behavioural model:
    - Reservation prices (unobserved) - these are not the same as own-valuation and can change as the auction proceeds. These refer to cut-off values, beyond which offers are accepted.
    - Trades occur between buyers with highest reservation price and the seller with the lowest reservation price.
    - If buyer completes a trade, but has overplayed (relative to historical data), she would adjust reservation prices.

Easley and Ledyard (1986)- III

- The following behavioral model has the following implications:
    - Minimum and maximum contract prices (on any day) will constrain the competitive price, and this gap is shrinking.
    - Minimum prices rise during excess demand, and maximum prices fall during excess supply.
    - Number of units traded on any day, would be close to competitive levels, plus or minus one unit.
    - Sellers with a valuation below minimum prices and buyers with a valuation above maximum price will trade first.
- Thus this behavioral model of "learning" performs better than Walrasian, Marshallian or Game theoretic models.

Rust et al 1992 - I

- Time discretized in to bid/ask and buy/sell steps.
- BA step: Monitor registers all bid/asks and highest outstanding bid ("current bid") and lowest outstanding ask ("current ask").
- BS step: Current bid and ask traders have to decide to accept/reject. If they accept then transaction goes through.
- One game has many rounds, and each round have couple of such trading periods (BA/BS). Token values / reservation prices are fixed in each round.
- Public information (Parameters of game): no. of buyers and identities, transaction records, bid/ask records, time steps and rounds, no. of tokens, and distribution of token values.
- Private information: token value at start of round.
- Buyers get slightly higher valued tokens than sellers, so they want to buy.

Rust et al 1992 - II

- Results:
  - Simple programs did better than complex ones on average: "wait in the background, and when bid-ask get close, jump in and steal the deal".
    - Kaplan's buyer: when the current ask and current bid are close (10% in percentage terms), the bid would be exactly equal to the current ask (if the bid if profitable at that value).
    - Ringuette's buyer: waits until current bid is close to current ask minus a profit margin. This value is a fraction of the range of token values for that round. And then randomly overbids.
  - Adaptable programs that used information from previous steps, rounds, games did not perform well. This included Genetic algorithms and neural networks.
  - Information about distribution of token values, number of periods and rounds, and number of sellers and buyers did not help programs. Timesteps and total tokens used were indeed important. The current bid and ask were used by all. Thus what matters is: (token value, current bid, current ask, timesteps left, tokens left).
  - The most successful programs did not behave strategically - track identities and histories of opponents.

Rust et al 1992 - III

- Programs did not make individual predictions about opponents. Few made aggregate market predictions. In general, programs did not try to predict the future.
- Programs did add noise to their own actions to prevent recognition by others. For instance, a Kaplan could be identified easily while Ringuette would not.
- Kaplan did the best and Bayesian Game Against Nature did the worst.
- Buyers get slightly higher valued tokens than sellers, so they want to buy.