Behavior of Trading Automata in a Computerized Double Auction Market

John Rust
Richard G. Palmer
John H. Miller

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John Rust, University of Wisconsin

Richard Palmer, Duke University

John H. Miller, Carnegie Mellon University

and

Santa Fe Institute

dat@sfi.santafe.edu

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Abstract: This paper reports the results of a series of tournaments held at the Santa Fe Institute beginning in March, 1990 in which computer programs played the roles of buyers and sellers in a simplified synchronized double auction market. We show that despite the decentralized nature of the trading process and traders' incomplete information about supply and demand, transaction price trajectories for a heterogeneous collection of computer programs typically converged to the competitive equilibrium, resulting in allocations that were nearly 100% efficient. We also show that a very simple trading strategy is a highly effective and robust performer in these markets. A simple rule-of-thumb was able to outperform more complex algorithms that used statistically-based predictions of future transaction prices, explicit optimizing principles, and sophisticated "learning algorithms".

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1. Introduction

This paper reports the results of a series of computerized double auction tournaments held at the Santa Fe Institute beginning in March 1990. The tournament consisted of over 30 computer programs (*automata traders*) playing the roles of buyers and sellers in a simplified syncrhonized double-auction (DA) market. The tournament was organized with several objectives in mind: 1) to get new insights on the form of effective trading strategies, 2) to compare the performance of automata traders and human traders, and 3) to create an artificial market to help us better understand the operation of the "invisible hand" in real world DA markets.

The remarkable efficiency properties of DA markets have been documented in numerous laboratory experiments using human subjects. By assigning subjects *tokens* with fixed *redemption values* and *token costs*, well-defined supply and demand curves can be constructed. The intersection of these curves defines the price and quantity at which neoclassical economic theory predicts trading will occur, the *competitive equilibrium* (CE) solution. The complication is that in most experimental markets each trader only knows only their own token values: no single trader has enough information to determine the market supply and demand curves in order to compute the CE. The nearly universal finding of more than two decades of human experiments is that despite the presence of incomplete information and the small number of traders, transaction prices and quantities quickly converge to CE. The resulting market allocations are highly efficient: traders are typically able to exploit close to 100% of the potential profits.

Although the textbook "supply equals demand" model may provide a good prediction of *closing* prices and quantities in DA markets, it fails to explain the dynamics by which this happens. A more sophisticated theory is required to show how the trading process aggregates traders' dispersed information, driving the market towards CE. The essence of the problem was clearly stated by Friederik Hayek nearly 50 years ago:

> "The problem is in no way solved if we can show that all the facts, *if* they were known to a single mind, would uniquely determine the solution; instead we must show how a solution is produced by the interactions of people each of whom possesses only partial knowledge. To assume that all the knowledge to be given to a single mind in the same manner in which we assume it to be given to us as the explaining economists is to assume the problem away and to disregard everything that is important and significant in the real world." (Hayek, 1945, p. 530)

Since the use of a computer tournament to gain insights into human trading behavior is somewhat unorthodox, section 2 briefly reviews current theories of DA markets. Although these theories have provided important insights into the nature of trading strategies and price formation, it is fair to say that none of them has provided a satisfactory resolution of "Hayek's problem". In particular, current theories assume a substantial degree of implicit coordination by requiring that traders have common knowledge of each other's strategies (in game-theoretic models), or by assuming that all traders use the same strategy (in learning models). Little is known theoretically about price formation in DA markets populated by heterogeneous traders with limited knowledge of their opponents. Although experimental studies have provided considerable empirical evidence on the nature trading *behavior* under these conditions, they have failed to cast light on trading *strategies* which are essentially unobservable.

In order to directly observe strategies, we sponsored a tournament in which entrants submitted trading programs playing the roles of buyers and sellers in a computerized DA market. To attract good programs we offered $10,000 in prizes, paid out in proportion to trading profits earned by entrants' programs over the course of the tournament. In return we obtained a heterogeneous collection of trading programs to populate a unique laboratory for studying decentralized price formation. Section 3 describes the rules of the tournament and the structure of our "synchronized DA", a modified version of the traditional continuous DA market designed to simplify the task of programming strategies and guarantee equal trading opportunities. Section 4 presents the results of the cash tournament held at the Santa Fe Institute in March 1990 and subsequent non-cash "scientific" and "evolutionary" tournaments held in 1991. We find that the top-ranked programs yield a fairly "realistic" working model of a DA market in the sense that their collective behavior is consistent with the key "stylized facts" of human experiments. We also find that a very simple trading strategy is a highly effective and robust performer in these markets. This strategy was able to outperform more complex algorithms that use statistically-based predictions of future transaction prices, explicit optimizing principles, or sophisticated "learning algorithms". The basic idea behind the approach can be described quite simply: *wait in the background and let others do the negotiating, but when bid and ask get sufficiently close, jump in and "steal the deal"*. However the results of our evolutionary tournaments show that when too many other traders try to imitate this strategy, market efficiency can fall precipitously due to negative information externalities. Specifically, if too many traders "wait in the background", little information is generated until just before the end of the trading period. This tends to produce "closing panics" as traders rush to unload their tokens in the final seconds of the trading period, resulting in failure to execute all potentially profitable transactions.

Long-run stability in the trading environment seems to require the presence of active bidders that provide a flow of information to "lubricate the market". However most of the active bidding strategies seem to be too impatient, exposing themselves to a high risk of mistakes and consequent exploitation by the background traders. Although a few of the more complex trading programs appear to be resistant to short-run exploitation, none of them appear strong enough to resist the parasitic effects of the background traders in the long-run. We show that a market dominated by background traders can be "quasi-stable" if a small but steady stream of short-lived "noise-traders" enters the market. The noise-traders provide a flow of information and source of new capital to keep the market running despite being nearly totally dominated by background traders. However since price volatility is very high in such a market, it is likely to present attractive opportunity for exploitation by new strategies. Section 5 concludes with some observations on how one might find such strategies.

2. Review of Previous Approaches to Analyzing DA Markets

Modern economic theory has attempted to explain the apparent disequilibrium behavior in DA markets as actually being the equilibrium outcome of a game of incomplete information. The "null hypothesis" is that observed trading behavior is a realization of a Bayesian-Nash equilibrium (BNE) of this game. Given the immensity of the strategy space (especially in continuous-time formulations), it has proven extremely difficult to characterize the equilibria of these games. The clearest characterizations have been obtained by Satterthwaite and Williams (1989a,b, chapter 4) for a class of static DA games known as the *k-Double Auction*. They have established that *truthtelling* is nearly a dominant strategy as the number of traders gets large. Unfortunately, this intuition does not appear to carry over to dynamic DA markets where truthful revelation is a very poor strategy: if a trader places a bid equal to her true redemption value and another trader accepts that bid, the bidder will clearly earn zero profit.

To the best of our knowledge the only characterization of equilibrium in a dynamic DA market is due to Wilson (1987). His equilibrium, described as a "waiting game Dutch Auction" (WGDA) by Cason and Friedman (chapter 8),

> "offers a concrete explanation of the mechanism by which the dispersed information about traders' valuations is manifested in the prices at which transactions are consummated. The mechanism, according to the present hypothesis, is multilateral sequential bargaining in which the traders are endogenously matched for transactions via a signalling process using delay as the primary signal." (p. 412)

Cason and Friedman (1991) have shown that beyond the prediction of high *ex post* trading efficiency, almost all of the other predictions of Wilson's model are inconsistent with the behavior of human traders in laboratory experiments. Wilson's model predicts that trade will occur in the efficient order,[1] whereas in human experiments the rank correlation between the order of transactions with the efficient order is typically much less than 100%. Indeed, Wilson's model predicts that in equilibrium all *ex post* efficiency losses will be due to unrealized intra-marginal trades, i.e. the market may trade too few tokens but never too many. However in experimental settings, a significant fraction of efficiency losses are due extra-marginal trades. This is due to the fact that in human experiments, buyers and sellers are not matched for transactions as predicted by Wilson's waiting game equilibrium: it is frequently the case that extra-marginal traders succeed in "bumping" intra-marginal traders. [2] Furthermore, bidding behavior seems to be poorly described as a sequence of Dutch auctions called exclusively by the current bidder or asker. In human experiments there is often stiff competition for the "right" to hold the the current bid or ask, which is frequently "stolen" by other more eager traders. Finally, Cason and Friedman show that in human experiments transaction price

[1] The efficient order is the trade sequence which maximizes surplus, i.e. the first trade occurs between the buyer with the highest redemption value and the seller with the lowest token cost, the second trade occurs between the buyer and seller with the next most valuable tokens, and so on.

[2] This suggests that differences in traders' "impatience" may also be a function of other factors we might call "aggressiveness" (or "stupidity"?) which may have no direct relation to the magnitude of their token values.

changes are significantly negatively autocorrelated. Wilson's model predicts zero autocorrelation in price changes since equilibrium transaction prices must follow a martingale to preclude intertemporal arbitrage.

Given enough freedom in the specification of traders' beliefs and risk aversion, Ledyard (1986) has shown that essentially any set of undominated strategy profiles can be "rationalized" as a BNE outcome of the DA trading game. Does this imply that game theory will eventually be able to provide an explanation for the experimental observations? We think there is a deeper reason why the game-theoretic approach may not yield a good model of human behavior. The problem is that the common knowledge underpinnings of game theoretic models presume an unreasonably high degree of implicit coordination amongst the traders, begging Hayek's question of how coordination is achieved in a decentralized market in the first place. Game theory also assumes that there is no *a priori* bound on traders' ability to compute their BNE strategies. However even traders with infinite, costless computing capacities may still decide to deviate from their BNE strategies if they believe that limitations of other traders force them to use sub-optimal strategies. Since traders can only observe behavior, they will never be certain of exactly which strategies their opponents are using. This learning problem is of such a high dimensionality relative to the limited number of observations available within typical trading periods that it may not pay to try to adopt a sophisticated Bayesian updating strategy. [3] Game-theoretic solutions can also be "non-robust" in the sense that they depend critically on particular common knowledge assumptions about the form of the probability distribution of traders' token values. Typically game theorists use the "independent private values model" where each trader's tokens are *IID* draws from a fixed distribution F. It is not known whether equilibria will be substantially different if token values are correlated across traders. To our knowledge, there is no convincing evidence from the experimental literature that presence or absence of common knowledge about the joint distribution of token values has a significant impact on trading outcomes. Indeed, subjects in most experimental studies are not given any information about how tokens are generated. In a game-theoretic model it would be impossible to even define the concept of equilibrium without such prior information.

In response to the difficulties of using game theory to analyze and explain experimental findings in dynamic DA markets, economists have begun to formulate explicit disequilibrium trading theories based on simple, yet plausible rules-of-thumb. Examples of this approach include Easley and Ledyard (chapter 3), Friedman (1991), and Garcia (1980). The results of these studies suggest that rationality is not a necessary condition for observing efficient outcomes and convergence to CE in DA markets. Gode and Sunder (1992, and chapter 6) provided a particularly striking demonstration of this result. They showed that markets populated by "Zero Intelligence" (ZI) strategies exhibit very high *ex post* efficiencies, and the corresponding price trajectories frequently converged to CE in the very first

[3] It has been shown (Freedman, 1963; Diaconis and Freedman, 1986) that Bayesian updating can be inconsistent in infinite-dimensional parameter spaces. In certain cases the prior distribution can completely overwhelm the data in the sense that the posterior will not converge to the "truth" even given an infinite number of observations. Intuitively, this will also be the case when the dimensionality of the object being learned is large relative to the number of observations.

trading period. A ZI seller with token cost C asks an amount $C + \tilde{U}$, where $\tilde{U}$ is uniformly distributed over its support S. Similarly a *ZI* buyer with redemption value R bids amount $R - \tilde{U}$. At each step t of trading, a ZI trader uses *IID* draws $\tilde{U}$ to construct its bids and asks, accepting the first profitable opposing bid or ask that comes along. Thus, ZI are traders are "minimally rational" in the sense that they do not attempt to optimize or learn from past observations, although they do avoid trading at a loss by always bidding below their redemption values or asking above their token costs. Gode and Sunder (1992) found that in markets populated entirely by ZI traders allocations were nearly 100% efficient outcomes and price trajectories typically converged to CE, provided the support of the set over which random draws $\tilde{U}$ are taken is not too large.[4] These findings strongly suggest that the nice properties of DA markets may have more to do with the properties of the institution itself than the rationality of traders *per se.*[5]

Although ZI traders may be collectively rational, there is a clear sense in which they are individually irrational. Specifically, we show that ZI traders, like *Truthtellers*, will be rapidly exterminated by even slightly more aggressive trading strategies. For this reason it seems doubtful that ZI traders will provide a good model of human trading behavior. Indeed, even though the price trajectories frequently converge to equilibrium, overall price volatility in DA markets populated by ZI traders is unrealistically high. Given the high transaction price volatility, it is somewhat surprising that ZI traders are even collectively rational. However if the support S of the distribution over which the ZI traders' bids and offers are randomized is too large, the probability increases that extra-marginal traders will succeed in "bumping" intra-marginal traders, resulting in significant efficiency losses. Truthtelling can be viewed as a limiting form of the ZI strategy as the support S converges to $\{0\}$. Since trade in a syncronized DA market populated entirely by truthtellers is necessarily fully efficient, it follows that ZI traders are also close to 100% efficient, at least when the upper support S is sufficiently small.[6]

One of the lessons from Axelrod's (1984) prisoner's dilemma tournament is that sometimes players can be "too smart for their own good" in the sense that sophisticated and self-interested behavior can be deterimental to achieving good cooperative outcomes. The question is whether this is true in DA markets as well. Is it the case that sophisticated optimizing strategies make individual traders better off, but reduce market efficiency? The problem is that "passive"

[4] The key restriction is to rule out placing bids or asks at a loss, which is true provide the support S is nonnegative. The nature of the trading rules also affect the results. ZI traders in continuous-time markets yield less efficient allocations than in continous-time markets than in the sychronized DA markets described in section 3, since they are able to place an unbounded number of bids and asks in the former. For details, see Gode and Sunder, chapter 6.

[5] We should also mention related work by Marimon, McGratten and Sargent (1990) who studied the behavior of a collection of Holland's (1975) *classifier systems* in a dynamic exchange economy. Although their economy is a more complicated dynamic market than the essentially static DA market studied here (in particular it can have multiple isolated equilibria), they also found that their artificial agents eventually converged to an equilibrium of the system.

[6] To the extent that humans sometimes make random mistakes, ZI traders may provide insight into why static sealed-bid DA markets are less efficient than dynamic DA markets. For example in the k-double auction, the level of inefficiency appears to be much more sensitive to the level of underbidding (as indexed by the size of the support S) than it is in a dynamic DA.

or "nice" strategies (such as Trutheller or ZI) that are consistent with high collective efficiency may be individually unsustainable in the presence of aggressive players. The issue turns on the potential gains from behaving aggressively versus behaving passively. We suspect that an analog of the Satterthwaite-Williams result carries over to dynamic DA markets: namely, that the gains from behaving strategically are negligible even in markets with very small numbers of traders. However in order to define the gains to behaving strategically we also have to define an appropriate notion of what it means to behave passively or "non-strategically". Identifying non-strategic behavior with truthtelling will not work in dynamic DA markets since truthtelling, like ZI, is easily exploitable. In a large and efficient market there is reasonably well-defined notion of *pricetaking* behavior: namely, placing all bids or asks at the market price and only accepting a bid or offer that is at least as good as the market price. In an efficient market it seems intuitively clear that pricetaking should be close to a dominant strategy. [7] However it is less clear exactly what it means to be a pricetaker in thin markets with small numbers of traders where initial transaction prices are highly volatile and potentially far from equilibrium, a situation that is typical of the first few periods of most experimental DA markets, as well as many markets observed in the field.

3. Structure of the DA Tournament

Most of the trading programs used in this study were submitted in response to advertisements for a "Double Auction Tournament" held at the Santa Fe Institute in March 1990. Cash prizes totalling $10,000 were offered to a maximum of 100 entrants in proportion to the trading profits earned by their programs over the course of the tournament. In addition to prize money and wide publicity, a substantial effort was devoted to make the programming and debugging of trading strategies as easy as possible. This included development of the *Santa Fe Token Exchange* (SFTE) which opens at the start of each hour for token trading over the worldwide Internet computer network. [8]

The computerized DA was implemented via a *message-passing protocol* which specifies the form of allowable messages that programs can send, such as bids, asks, and buy and sell orders. Entrants were provided with a simple "skeleton" trading program (written in C, Fortran, and Pascal) which handled all the message-passing housekeeping, allowing them to focus on the logic of their strategies, rather than on programming details. A central *monitor program* coordinates the trading process by communicating with all of the trading programs, executing their buy and sell orders and relaying their bids and asks to the other traders. Trading programs (which could also be interfaces to human

[7] Roberts and Postlewaite (1976) were the first to formally establish such a result in the context of a static complete-information Walrasian exchange economy. The results of SW (1989a,b) and RSW (1990) cited in section 2 can be interpreted as an extension of the Roberts-Postlewaite result to a non-Walrasian exchange economy with incomplete information.

[8] Many entrants reported that the SFTE was useful for refining their trading programs in advance of the actual tournament. We also distributed "free-ware" to allow entrants who did not have Internet access to set up their own local token exchanges.

traders) communicate only with the monitor and not directly with each other as illustrated in figure 3.1. It is important to note that the monitor program is only a clerk: it is not an "auctioneer" and has no market-clearing authority.[9]

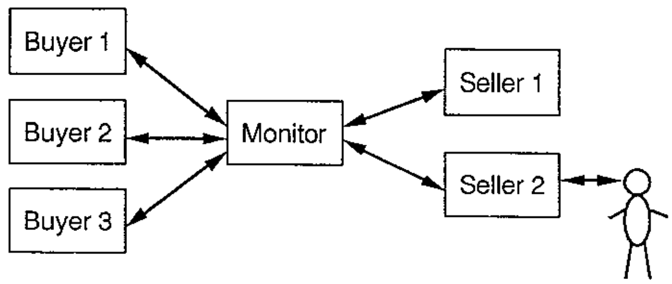

Figure 3.1: Interplayer Communication Via the Monitor

The structure of our computerized DA market is very similar to the continuous-time experimental DA markets described in section 2. The major differences are 1) time is discretized into alternating *Bid/Ask* (BA) and *Buy/Sell* (BS) steps, and 2) transactions are cleared according to "AURORA rules". The DA market opens with a BA step in which all traders are allowed to simultaneously post bids and asks. After the monitor informs the traders of each others' bids and asks, the holders of the *current bid* (highest outstanding bid) and *current ask* (lowest outstanding ask) enter into a BS step.[10] During the BS step, either player can accept the other player's bid or ask. If an acceptance occurs, a transaction is executed.[11] A *trading period* is simply a set of S alternating BA and BS steps.

The discretization of time was adopted to simplify the programming of trading strategies and improve the synchronization of communications between players and the monitor in a multiprocessing or network computing environment where delays may vary from player to player and moment to moment. In a continuous-time environment "faster" traders have an inherent advantage. This speed advantage may arise due to communication delays (e.g., simultaneous messages sent from a trader in Japan and Chicago may arrive at different times at a central computer in New York) or due to processing delays (e.g., machines may be able to recognize and respond to certain conditions faster than humans). By discretizing time and setting sufficiently wide response-time limits, we can effectively guarantee that all traders have equal trading opportunities. In the limit, our implementation of a discrete-time DA market is

[9] The monitor does enforce trading rules, and can impose upper and lower price limits. It also has the authority to censor illegal or late messages, although no cpu-time limits were imposed in the actual tournament.

[10] If a current bid (ask) does not exist, then all buyers (sellers) enter into the BS step.

[11] If both parties accept each other's offers, the monitor randomly chooses between the current bid and ask to determine the transaction price.

not restrictive since a continuous-time trading environment can be arbitrarily well approximated by a discrete-time environment with very many short trading intervals.[12]

The AURORA rules were inspired by similar rules used by the AURORA computerized trading system developed by the Chicago Board of Trade. AURORA Rules stipulate that only the holder of the current bid or current ask are allowed to trade. We adopted these rules as a substitute for *ad hoc* tie-breaking rules which are necessary in discrete-time trading environment when several traders are able to simultaneously accept an outstanding bid or ask. Presumably the Chicago Board of Trade had a very different motivation for considering the AURORA rules. [13] . In our case, we experimented with alternative sets of trading rules and found that the AURORA rules produced a more interesting DA game, as well as one that was perceived as "fairer" by participants. Strategically, our DA breaks into a series of two simultaneous move subgames: a multiplayer game to acquire the current bid or ask in a BA step, and a two-player game involving a binary accept/reject decision in a BS step. In early human experiments using random tie-breaking rather than AURORA rules, we found that traders often expressed frustration that trade execution seemed more a matter of luck than strategy due to the fact that other traders would repeatedly win random tie-breaks for the acceptance of an outstanding bid or ask. On the other hand, one might criticize the AURORA rules on the grounds that it makes it harder for traders to remain in the background since it forces them to "show their hand" by posting a winning bid or ask before being allowed to trade. We have found, however, that these rules do not place a significant constraint on background traders: they can still stay quietly in the background for most of the trading period, jumping in the moment they detect an attractive bid or ask. Indeed, this is precisely the strategy followed by the winner of the tournament.

An individual DA *game* is divided into one or more *rounds*, and each round is further divided into one or more *periods*. A single period of the DA game consists of a fixed number of alternating BA and BS steps as described above. The reason for structuring games to have multiple rounds and periods within rounds is to control players' abilities to learn about their opponents. Tokens and redemption values are fixed within each period of a given round, but are allowed to change between rounds. Thus DA games with many periods allow players to learn the value of each others' *tokens*, while DA games with many rounds allow players to learn about each others' *strategies.*

At the start of a DA game the monitor broadcasts *public information* to the traders, including the number of buyers and sellers and their identities, the number of rounds, periods and time steps, the number of tokens each agent will have, and the joint distribution F from which the traders' token values are drawn. Next, the monitor sends each trader a packet of *private information*, namely, their realized token values. Since public information is provided by

[12] The same point applies to price units in our DA market, which were rounded to the nearest integer.

[13] Since AURORA rules have the effect of making all transactions publicly observable, they may have been designed partly in response to trading abuses that were uncovered in the Chicago Exchanges in the late 1980's.

a simultaneous broadcast to all players, it serves as a means of ensuring that players have common knowledge about all relevant game parameters. The joint distribution F was communicated to players using a four digit `gametype` variable. Token values are represented by T_{jk} where j indexes the trader, and k indexes the token assigned to the trader. Tokens are randomly generated according to

$$T_{jk} = \begin{cases} A + B + C_k + D_{jk}, & \text{if } j \text{ is a } \textit{buyer}; \\ A \qquad\; + C_k + D_{jk}, & \text{if } j \text{ is a } \textit{seller}, \end{cases} \tag{3.1}$$

where[14] $A \sim U[0, R_1]$, $B \sim U[0, R_2]$, $C_k \sim U[0, R_3]$, and $D_{jk} \sim U[0, R_4]$. Notice that when $R_1 = R_2 = R_3 = 0$ we have the standard independent private-values model where tokens are independently uniformly distributed on the interval $[0, R_4]$. A `gametype` equal to 0 indicates an environment where redemption values were generated by an unspecified process.

The best way to understand what goes on in a DA market is to study the monitor output for the sample tournament game in Figure 3.2. The figure shows the first period of a DA game with two rounds and three periods per round. In this case there are four buyers and four sellers, and each trader is assigned four tokens. The implied supply and demand curves give a CE which is uniquely determined with a price of 691 at a quantity of 11. [15] The '+' next to each trader's token indicates an intra-marginal token, a '-' indicates an extra-marginal token, and an '=' denotes a token value equal to the CE price. The first BA step yields a current bid of 435 held by B1 (buyer 1) and a current ask of 1128 held by S2 (seller 2). Neither B1 nor S2 choose to accept the other's bid or ask at BS step 1, so the bidding begins again at BA step 2. The first transaction occurs in BS step 4 when both B3 and S1 simultaneously accept each other's offers. Here, a random tie-break results in S3 selling its first token (denoted by capital A) at B3's bid of 717. Immediately after the transaction the current bid and ask are set to zero and a new BA step starts up in period 5. The game continues this way until the final BS step is reached in step 25. The box at the end of the monitor output provides a summary of the period's trading activity: there were 10 transactions yielding a total profit of 604, which is 91% of the total surplus of 663. In this case the source of the inefficiency was due to two events. First, one intra-marginal trade was not consummated. Second, there were two extra-marginal trades made by B3 which displaced an equal number of intra-marginal trades. This example illustrates a typical feature of efficiency losses in dynamic DA markets, namely the coexistence of extra-marginal trades along with unconsummated intra-marginal trades. [16]

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[14] Each of the 4 digits of the `gametype` variable correspond to $\{R_1, \ldots, R_4\}$ according to the base-3 coding, $R_i = 3^{k(i)} - 1$ where $k(i)$ is the ith digit of `gametype`.

[15] The discrete nature of the market often implies that a non-unique equilibrium point emerges, in which case an equilibrium range of either prices or quantities results.

[16] The monitor output contains a number of other symbols. @ denotes a bid equal to the current bid, X and Y denotes a bid below the current bid and the minimum allowed price, respectively (both illegal), and Z denotes a bid above the maximum allowed price. $ denotes a bid above the current ask (and current bid), & denotes a bid above the current ask but not current bid, $\sim$ denotes a token traded at a loss, and ! denotes a crossing of the current bid and ask.

```
DA game 1  Fri May 24 02:14:05 1991
-------------------------------------
protocol:        5  monitor:      443  gametype:     6453
nrounds:         2  nperiods:       3  ntimes:         25
minprice:        1  maxprice:    2000  ntokens:         4
ran1:          728  ran2:          80  ran3:          242
ran4:           26  deadsteps:    100  timeout:        30
id  name            id  name
--  ------------    --  ----------
B1  silverbuffalo   S1  burchard
B2  staecker        S2  pricetaker
B3  perry           S3  breton
B4  anon2           S4  anderson
+------------------------------------------------------------------------------+
|                          Round  1, period 1                                  |
+-------+-------------------------+-------------------------+------------------+
| token |  B1    B2    B3    B4   |  S1    S2    S3    S4   |   Equilibrium    |
+-------+-------------------------+-------------------------+------------------+
|   a   | 754+  760+  761+  751+  | 651+  666+  646+  661+  |  691 to 691      |
|   b   | 722+  708+  717+  719+  | 661+  675+  659+  665+  |  av:  691.0      |
|   c   | 691=  705+  690-  702+  | 680+  683+  680+  693-  |  trades: 11      |
|   d   | 681-  678-  689-  691=  | 779-  776-  788-  774-  |                  |
+-------+-------------------------+-------------------------+------------------+
|t  step|  B1    B2    B3    B4   |  S1    S2    S3    S4   | cbid coff price  |
+-------+-------------------------+-------------------------+------------------+
|1  BA  | 435*  276   368   345   | 1550  1128* 1182  1999  |  435 1128        |
|     BS|                         |                         |  435 1128        |
|2  BA  |       440   574*        | 1127  1090   966* 1065  |  574  966        |
|     BS|                         |                         |  574  966        |
|3  BA  |  @    579   676*        |  803*  942   827   930  |  676  803        |
|     BS|                         |                         |  676  803        |
|4  BA  |  @    681   717*        |  652$  793   770   791  |  717  652 !      |
|     BS|             a>A         |  a>A                    |             717  |
|5  BA  |       216   606*  345   | 1075   713*  957  1999  |  606  713        |
|     BS|                         |                         |  606  713        |
|6  BA  |  @    611   635*        |  661*  712    X    703  |  635  661        |
|     BS|                         |                         |  635  661        |
|7  BA  |  @    640   642*        |              656*       |  642  656        |
|     BS|             b<B         |               <A        |             656  |
 . . .  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
|23 BA  |       557         651*  |        778   681   666* |  651  666        |
|     BS|                   c<C   |                     <B  |             666  |
|24 BA  |       232         550*  |  809         681* 1999  |  550  681        |
|     BS|                         |                         |  550  681        |
|25 BA  |       596         651*  |               "         |  651  681        |
|     BS|                         |                         |  651  681        |
+-------+-------------------------+-------------------------+-----Market-------+
|Trades |  1/2   2/3   4/2   3/3  |  3/3   3/3   2/3   2/2  |      20/21       |
|Profit |   27    48   125   117  |  141    62    45    39  |       604        |
|Eqlbrm |   94   100    96    99  |   81    49    88    56  |       663        |
|Effncy |  29%   48%  130%  118%  | 174%  127%   51%   70%  |       91%        |
+-------+-------------------------+-------------------------+------------------+
```

Figure 3.2: Sample Monitor Output

Parameter	Environment									
	BASE	BBBS	BSSS	EQL	LAD	PER	SHRT	SML	RAN	TOK
`gametype`	6453	6453	6453	0	0	6453	6453	6453	0007	6453
`minprice`	1	1	1	1	1	1	1	1	1	1
`maxprice`	2000	2000	2000	2000	2000	2000	2000	2000	3000	2000
`nbuyers`	4	6	2	4	4	4	4	2	4	4
`nsellers`	4	2	6	4	4	4	4	2	4	4
`ntokens`	4	4	4	4	4	4	4	4	4	1
`nrounds`	2	2	2	2	2	6	2	2	2	2
`nperiods`	3	3	3	3	3	1	3	3	3	3
`ntimes`	75	50	50	75	75	75	25	50	50	25
`games`	1624	1624	1624	1624	1624	1624	1624	3428	1624	1624
games/player	56	56	56	56	56	56	56	112	56	56
periods/player	336	336	336	336	336	336	336	336	336	336
conversion ratio ($\times 10^{-4}$)	6.11	8.95	9.73	3.48	3.57	6.97	7.04	6.32	1.04	20.6

Table 3.1: DA Trading Environments

Tournament entrants were told that their programs would be placed in an unspecified number of alternative *environments*. Each environment is a complete specification of all relevant parameters of the DA game listed in Table 3.1. Participants were told potential ranges for each of the parameters, but were not given any specific advance information about how the actual environments would be selected. The actual DA tournament consisted of playing a large number of DA games in ten separate environments presented in Table 3.1. Each of the ten environments were allocated $1,000 prize money, and separate conversion factors were calculated to translate token profits into dollar earnings. The conversion factor $c(i)$ for environment i is the ratio $1000/TS(i)$, where $TS(i)$ is the total surplus available in environment i. Due to the lack of 100% efficiency, actual dollar payments in the tournament amounted to $8,937. Overall, we ran a total of 2,233 games in the 10 separate environments, comprising 13,398 individual periods of play.

One can see from Table 3.1 that the tournament subjected programs to a wide range of trading conditions. The base case (BASE), was an environment similar to the one used in pre-tournament trials at the SFTE. Other environments include duopoly and duospony (BBBS and BSSS), a degenerate surplus distributions where all players receive the same token values shifted by a common random constant (EQL), an independent private values environment where each trader's token is an IID draw from a uniform distribution (RAN), a single-period environment that prevented players from learning from previous market outcomes (PER), a "high-pressure" environment where the traders' time allotment was very short (SHRT), and an environment where each trader was only assigned a single token (TOK). Our intention was to force programs to compete under a broad range of conditions in order to provide a rigorous and comprehensive test of their effectiveness.

To insure that tournament earnings were not due to a series of lucky token draws, we developed a sampling scheme that guaranteed that all trading programs had equal surplus endowments with probability 1.[17] Once a random set of token values was drawn according to the sampling scheme given in (3.1), trading programs were randomly selected to play in a set of N games (where $N = 30$ is the total number of entrants) subject to the constraints that no program played a copy of itself in the same game and all programs played all positions (B1, B2, S1, S2, etc.) an equal number of times. After this set of N games was completed, the scheme was repeated with a new set of token values. It follows that the differences in the trading profits earned by the traders can be ascribed to differences in their trading ability, since each program received the same endowment of tokens and encountered roughly the same collection of opponents in a large number of replications of the DA game.

4. Results of DA Tournaments

We received thirty programs for the first (cash) tournament held in March, 1990. Table 4.1 summarizes the entries, listed by the name of the participant(s) who submitted the program.[18] Of the thirty entries, fifteen were from economists, nine from computer scientists, three from mathematicians, and the remaining three were from an investment broker, a professor of marketing, and a joint entry from two cognitive scientists. Three of the entries were outgrowths of research papers describing formal models of DA trading behavior.[19] Several of the entries emerged from working groups which co-developed sets of strategies, in some cases pre-testing them in "local tournaments" using our double auction software. These groups include seven entries from the Economic Science Lab (ESL) at the University of Arizona, three from the University of Minnesota, and two each from the University of Colorado (Economics) and Carnegie Mellon University (Computer Science). The table also includes four entries from SFI including the ZI, Truthtelling and Pricetaking strategies discussed in section 2, as well as a "skeleton" strategy provided to entrants as a simple example of a working trading program. Due to potential conflict of interest none of the latter programs were entered in the cash tournament held in March 1990 although they were used as experimental controls in subsequent "scientific tournaments". All of the entrants programmed their strategies by replacing the bid/ask and buy/sell subroutines of the skeleton program with their own code. Although versions of the skeleton program were available in C, Fortran, and Pascal, almost all of the entries (26 out of 30) were programmed in C. Only two were written in Fortran, and two in Pascal.

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[17] In the case of two trading programs that were only programmed to play one side of the market, a *Skeleton* stand-in trader was substituted in the games they refused to play. There are slight variations in actual token endowments caused by the fact that one program occasionally "died" midway through a trading period, resulting in forfeiture of its potential surplus in the remaining periods of the game.

[18] In cases where trading programs were developed by teams of individuals and we were unable to determine the primary author, we substituted a program nickname supplied by the authors. We also received two anonymous entries.

[19] The Kennet-Friedman entry is based on Friedman's (1991) model of DA trading as a Bayesian game against nature (BGAN), and the Ledyard-Olson entry is based on Easley and Ledyard's model described in Chapter 3.

We found that the most useful way of comprehending the variety of strategies in Table 4.1 was to classify them along the following dimensions:

simple *vs.* complex
adaptive *vs.* non-adaptive
predictive *vs.* non-predictive
stochastic *vs.* non-stochastic
optimizing *vs.* non-optimizing.

Rust, Palmer and Miller (1992) describe how these categories are defined and provide a detailed analysis of individual trading programs. In general we found that although there was a substantial range in program complexity, the majority of the programs appeared to encode the entrant's "market intuition" using simple rules of thumb. Since these rules are "hardwired", most of the programs are also classified as non-adaptive. Exceptions include a neural network program submitted by cognitive scientists Dallaway and Harvey, and an "adaptive cellular curvefitter" submitted by mathematician Paul Burchard. Besides their private information about token values, most of the programs relied on only small number of public information variables, the current bid, ask, and elapsed time being the most important. Only 10 programs made use of the prior information about the distribution of of token values provided by the `gametype` variable. Although 20 programs made use of the number of buyers and sellers in the DA game, only two programs (Ledyard-Olson and Staecker) used this information in an explicit way, e.g. by including separate "monopoly subroutines". Most of the programs did not attempt to keep track of the behavior of individual opponents or make statistical predictions of future market quantities. An exception was the program of Mark Staecker, developed as part of a senior honors thesis at the University of Western Ontario, which predicted the next high bid, low ask and equilibrium price using market-level statistics from previous periods. Based on these predictions, Staecker's program decides if a transaction is likely to occur at the next BS step and if so, uses its predictions to place an "atractive" bid or ask in the next BA step.

The top-ranked program was submitted by economist Todd Kaplan of the University of Minnesota. It was one of the shortest programs submitted, and is classified as *simple, non-adaptive, non-predictive, non-stochastic, and non-optimizing*. The second-ranked program was submitted by computer scientist Mark Ringuette from Carnegie-Mellon University. Despite the fact that they were independently developed, both strategies are remarkably similar. The strategies can be described in one line as *wait in the background and let the others do the negotiating, but when bid and ask get sufficiently close, jump in and steal the deal.* These programs succeed in "stealing the deal" by bidding an amount greater than or equal to the *previous* current ask. Ringuette's program differs from Kaplan's by randomly overbidding the previous current ask. When time is running out or when a long time has elapsed since making its last trade, Ringuette's program defaults to a modified version of the Skeleton bidding strategy whereas Kaplan's program places a bid equal to the smaller of the current ask or its current token value. In practice this implies that Kaplan's program eventually defaults to Truthtelling mode when confronting patient opponents who delay making "serious" bids and asks.

4.1 Results of March 1990 Tournament

Table 4.2 presents the dollar payoffs earned by the eligible trading programs in the March 1990 tournament, broken down by environment. [20] The top program, Kaplan, earned a total of $408, $14 higher than the second place program of Ringuette. The gaps separating third, fourth and fifth place were $7.45, $10.51 and $8.63, respectively. While these differences in earnings may not seem economically significant, they are statistically very significant. Kaplan's earnings are over 2.5 standard deviations higher than Ringuette's second place earnings, and the gaps separating first from second, third, and fourth places are 3.8, 5.6 and 7.1 standard deviations, respectively. The average standard deviation in profits of $5.75 was only slightly higher than the $5.40 standard deviation in surplus allocations, calculated over 3,360 individual periods of play in the 10 environments. Recall that our procedure for generating tournament games guarantees that the token endowments of all traders are identical with probability 1. Given the large number of periods of play, an appeal to the law of large numbers allows us to be very confident that differences in traders' earnings reflect true differences in profitability rather than randomness due to player matchings and stochastic elements in the programs themselves.

The player rankings are also highly consistent across the 10 environments. The average Spearman rank correlation between overall tournament payoffs and payoffs in each of the 10 environments is 77%, ranging from a high of 95% in environment SML to a low of 72% in environment TOK. Kendall's W-statistic, which measures the degree of concordance in all the rankings, is highly significant at 79%, allowing us to easily reject the hypothesis that player rankings in different environments are independent. It is striking that Kaplan's program took first place in 7 out of 10 environments, coming in second place in environment EQL and third place in environment SHRT. The only place where Kaplan's program did not do well was the environment TOK where traders were endowed with only a single token. At the bottom end of the spectrum, the BGAN (Bayesian game against nature) program of Friedman and Kennet was consistently one of the worst performers in all ten environments. With earnings of $164.30 the BGAN is over 10 standard deviations below the earnings of the next highest competitor. [21]

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[20] The NN program of Dallaway and Harvey was disqualified from the March 1990 tournament because it consistently incurred large losses. The authors submitted a revised version for the subsequent scientific tournament which performed satisfactorily.

[21] BGAN may have suffered as a result of problems converting the program from PC Turbo Pascal to Sun Pascal. Although the converted program compiles without error, its numerical integration routines generate under- and overflow errors at runtime suggesting a possible incompatibility in its calls to certain functions. The low dollar ranking of the program of Bolcer should be disregarded since the program was only programmed to play the role of seller. In terms of dollar payoffs per game played, Bolcer's program is about equivalent in performance to the programs *Max* and *Terminator* which placed 19^{th} and 20^{th} in overall earnings.

Author/Nickname	Institution	F	L	C	A	S	P	O	CPU
Anon-1	Anonymous	CS	C	·	2	·	1	·	86
Anon-2	Anonymous	CS	C	·	3	·	1	·	89
Jacobson	Carnegie Mellon	CS	C	·	3	X	1	·	86
Ringuette	Carnegie Mellon	CS	C	·	2	X	1	·	85
Golden Buffalo	Colorado	E	C	·	2	X	2	·	88
Silver Buffalo	Colorado	E	C	·	2	X	2	·	88
Lin	Portland State	E	C	·	2	X	1	·	89
Perry	Portland State	M	C	·	3	X	2	·	88
Anderson	Minnesota	E	C	·	2	·	1	·	88
Breton	Minnesota	E	C	·	3	X	1	·	88
Bromiley	Minnesota	MK	F	·	2	·	1	·	90
Kaplan	Minnesota	E	C	·	3	·	1	·	86
Pricetaker	SFI	EM	C	·	2	X	1	·	88
Skeleton	SFI	EM	C	·	2	X	1	·	84
Truthteller	SFI	EM	C	·	1	·	1	·	84
ZI	SFI	EM	P	·	1	X	1	·	82
Exp	Arizona ESL	E	C	·	2	·	2	·	86
Free	Arizona ESL	E	C	·	2	·	2	·	87
Gamer	Arizona ESL	E	C	·	1	·	1	·	84
Max	Arizona ESL	E	C	X	2	·	2	X	157
Max-R	Arizona ESL	E	C	X	2	·	2	X	261
Slide	Arizona ESL	E	C	·	2	X	2	X	96
Terminator	Arizona ESL	E	C	·	2	·	2	·	89
Bolcer	UC Irvine	CS	P	·	2	·	2	·	86
Burchard	Princeton IAS	M	C	X	5	X	2	X	99
Dallaway/Harvey	Sussex	CS	C	X	5	·	1	X	91
Kennet/Friedman	Tulane/UCSC	E	P	X	2	·	2	X	181
Kindred	Duke	CS	C	·	2	X	1	·	89
Ledyard/Olson	Cal Tech	E	C	X	3	X	2	·	187
Leinweber	MJT Advisors	B	C	·	2	X	1	·	87
Lee	British Columbia	CS	C	·	2	X	1	·	88
Staecker	Western Ontario	CS	C	X	3	·	2	X	88
Utgoff	Massachusetts	CS	C	·	2	X	2	·	86
Wendroff/Rose	Los Alamos NL	M	F	·	3	·	2	·	88

Legend:

F Field—B = broker, CS = computer science, E = economics, M = math, physics, MK = marketing.

L Programming Language—C, F = Fortran, P = Pascal.

C Complex—X if program is "complex" as defined in 4.1.

A Adaptive—5-level ranking defined in section 4.2: 1=least adaptive, 5=most adaptive

S Stochastic—X if program makes use of random number generator.

O Optimizing—X if program uses an explicit optimization principle.

P Predictive—3-level ranking defined in section 4.4: 1= doesn't predict, 2=predicts market variables.

CPU Ratio of CPU time consumption to average for all programs.

Table 4.1: Taxonomy of DA Trading Programs

Trading program	Overall	BASE	BBBS	BSSS	LAD	EQL	PER	RAN	SHRT	SML	TOK
Kaplan	408	42	42	41	43	42	44	41	38	42	33
Ringuette	394	41	37	40	45	38	40	37	33	37	46
Staecker	387	41	39	37	43	36	41	35	35	38	42
Anon-2	376	33	36	40	40	39	33	34	39	36	46
Ledyard	367	34	38	37	38	36	36	35	35	37	41
Perry	366	34	41	36	40	36	32	38	35	35	40
Breton	360	35	33	37	36	37	35	37	35	37	40
Anderson	358	33	39	38	34	37	31	36	38	35	39
Anon-1	354	36	37	35	36	35	37	35	35	35	32
Burchard	344	34	35	34	34	34	35	39	36	34	28
Terminator	342	35	27	40	36	34	36	33	31	32	38
Golden Buffalo	340	34	35	35	35	35	32	37	34	30	33
Lee	337	33	34	32	35	35	35	37	35	36	27
Leinweber	333	34	34	32	39	36	32	36	35	34	22
Silver Buffalo	330	34	30	32	33	38	36	33	33	25	37
Slide	330	35	34	32	34	33	36	32	33	32	29
Jacobson	329	34	28	34	31	37	31	32	32	30	39
Bromiley	316	32	31	35	30	33	35	31	31	25	32
Max	299	31	32	33	30	28	32	25	22	32	34
Max-R	294	28	34	29	31	27	30	26	25	29	35
Utgoff	286	32	26	25	32	35	32	25	26	29	23
Kindred	271	30	24	33	32	34	31	30	27	25	4
Free	242	27	32	21	23	25	21	27	27	22	17
Gamer	230	25	20	22	24	24	25	21	25	21	22
Wendroff	228	25	20	21	25	24	23	26	25	15	25
Lin	224	21	20	23	22	25	17	25	25	19	27
Exp	210	16	17	23	26	26	20	11	14	23	34
Kennet	164	16	6	14	23	14	19	21	17	12	23
Bolcer	148	13	19	8	18	17	16	16	13	10	18
Total	$8967	897	881	898	945	928	903	893	870	847	905
Surplus	$10000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Table 4.2: Dollar Payoffs in March 1990 Double Auction Tournament

While we are very confident of our ability to distinguish the best and worst programs, we are much less confident about the relative rankings of the middle group of programs. One can see from Table 4.2 that after the large gaps separating the fourth and fifth place entries (Anon-2 and Ledyard-Olson), the differences in payoffs of the next group of programs are within 1 standard deviation of each other. The next significant difference in payoffs is a \$10 gap separating the 9^{th} place entry of Anon-1 from the 10^{th} place entry of Burchard. Even after 3360 periods, it's clear that we would need many more observations to be confident of the relative rankings of programs between 5^{th} and 9^{th} place. In general it's impossible to make any reliable performance distinctions if we can only observe traders over a small number of periods. The average dollar earnings of 9.4 cents per period of play is dominated by the per period standard deviation in profits of 10.0 cents. Most of the latter variation is attributable to the 9.2 cent standard deviation in surplus arising from traders' random token endowments. Thus, a computerized trading environment is virtually a necessity if one wants to reliably discriminate good traders from bad. It appears that it would be infeasible to make the same sorts of distinctions in markets with human traders given that it takes hundreds or even thousands of periods of play before one can be sure that differences in relative performance are statistically significant.

Total tournament payouts at the bottom of Table 4.2 provide a convenient a measure of trading efficiency, since conversion ratios from token profits to dollar payoffs were based on realized surplus rather than on realized profits. Thus, the total dollar payouts of $8,967 correspond to an 89.7% efficiency ratio. [22] Perhaps not surprisingly, efficiency was highest in the environment EQL where all traders in a given DA game have symmetric token endowments (shifted by a common random component). However, the learning problem in this environment is not trivial since traders were not given any prior information on the distribution of token values (i.e. `gametype` was set equal to 0), and therefore had no way of knowing that everyone else was assigned equal tokens.

It is probably also not a surprise that the least efficient environment was SML where there were only two buyers and two sellers. Human experiments reveal that the competitive properties of the DA market start to break down when there are so few traders. The SML environment is but a step away from the most extreme situation of bilateral bargaining, which is know to have inefficient outcomes owing to a high frequency of disagreement. The increased frequency of disagreement in the SML environment shows up in the distribution of trader's profits: even though surplus endowments are 0 only 5% of the time, traders walk away with 0 profits over 20% of the time. Other environments with relatively low efficiencies include SHRT (where there was a time constraint on trading), and BBBS and BSSS (duopoly and duopsony markets, respectively).

Overall efficiency levels appear to be somewhat lower than that observed in the later periods of experimental markets with human traders. We suspected that the low trading efficiencies of the bottom 10 programs were responsible for most of the aggregate inefficiencies, suggesting that running a tournament that excluded these programs would result in a more "realistic" and efficient market. Before doing so, we gave all entrants an opportunity to revise their programs in light of the results of the March, 1990 tournament. A second series of scientific tournaments were conducted in May, 1991 with seven revised entries. [23] The scientific tournament also included several new programs written by the authors, including *Skeleton*, *Pricetaker*, and *ZI*.

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[22] Payouts are net of profits earned by a *Skeleton* "stand-in" for the programs of Dallaway-Harvey and Bolcer which only played the roles of buyer and seller, respectively. If we were to include profits earned by *Skeleton*, aggregate market efficiency would be slightly higher.

[23] We received revised entries from Dallaway-Harvey, Ledyard-Olson, and Perry, and 4 revised entries from the Arizona ESL group, *Max*, *Max-R*, *Slide*, and *Terminator*.

4.2 Results of Scientific and Top 17 Tournaments

Overall player rankings in the scientific tournament were quite similar to the original tournament. Aggregate efficiency increased slightly from 90% to 92%, ranging from a low of 88% in SHRT to 95% in EQL. Once again the programs of Kaplan and Ringuette are the clear winners, with Kaplan's program trading at an overall efficiency level of 121%, significantly higher than Ringuette's efficiency of 116%. Staecker's program, trading at an efficiency level of 109%, comes in third place just as in the original tournament. Two independent copies of *Skeleton* placed 5 th and 8^{th} place, which is somewhat surprising given that *Skeleton* was provided to all participants in advance of the tournament and thus should have represented a fairly easy target to beat. [24] On the lower end of the scale the programs *Gamer*, *Exp*, and Lin—which were among the poorest performers in the original tournament—remained the poorest performers in the scientific tournament, trading at an average efficiency level of under 70%. *ZI* also performed quite poorly trading at an efficiency level of 75%, confirming our discussion in section 2. The Neural-Network trader of Dallaway and Harvey performed somewhat better than *ZI*, trading exclusively in the role of buyer at an efficiency level of 85%. *Pricetaker* turned out to be the median trader at rank 17 with an average efficiency level of 95%. This is somewhat below the 100% efficiency ranking we would have expected for our implementation of a naive pricetaking strategy.

In order to see whether the leading position of Kaplan and Ringuette is a result of general superiority or merely a relative superiority in their ability to exploit the lowest-ranked traders, we ran a third tournament consisting of the top 17 players from the scientific tournament. The results of the "Top 17" tournament are summarized in table 4.3. We see that once again, Kaplan and Ringuette remain the leaders even after elimination of the lowest-ranked traders. Ringuette does slightly better than Kaplan in terms of overall earnings, although the difference is not statistically significant.[25] Kaplan's program comes in first place in 5 of the 10 environments whereas Ringuette's program is first in 4 environments, and the two programs are tied for first place in environment SML. The fact that Kaplan and Ringuette were able to maintain their high efficiency ratios in this tighter, more competitive market suggests that they are in fact generally superior to all of the other programs.

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[24] Two copies of *Skeleton* were included in the scientific tournament as a further check on the statistical reliability of our rankings. The relative performance of the two copies is rather close in all environments except TOK, where the two copies traded at efficiency levels of 41% and 84%, respectively.

[25] Note that overall earnings in the scientific tournament were computed by summing *token* profits as opposed to dollar earnings in the March 1990 tournament. This different implicit weighting scheme may account for the change in overall rankings of Kaplan and Ringuette.

Trading program	Overall	BASE	BBBS	BSSS	LAD	EQL	PER	RAN	SHRT	SML	TOK
Ringuette	117	121	103	116	120	113	119	120	91	126	130
Kaplan	114	110	109	122	103	120	129	111	118	126	116
Anon-1	108	109	110	110	111	108	103	106	113	117	109
Anderson	105	100	104	102	115	106	104	103	102	107	101
Staecker	101	111	103	107	99	103	108	96	97	117	126
Burchard	101	94	92	92	91	107	84	112	95	81	84
Perry	98	103	104	100	97	106	106	95	87	91	97
Anon-2	97	99	102	98	108	95	93	93	88	107	124
Lee	96	90	88	81	96	105	90	101	88	90	88
Ledyard/Olson	96	90	90	89	98	104	94	97	90	96	103
Golden Buffalo	94	90	94	88	101	91	91	97	81	80	118
Breton	92	102	94	91	94	92	98	91	81	94	105
Leinweber	90	86	83	88	87	89	87	98	85	67	33
Skeleton	89	91	89	78	89	100	84	93	93	62	39
Jacobson	89	97	91	92	96	80	87	87	88	94	104
Silver Buffalo	82	87	84	74	87	91	81	79	80	71	102
Pricetaker	71	81	81	95	77	68	87	57	87	98	96
Market	97	98	95	96	98	99	97	96	92	96	98

Table 4.3: Trading Efficiencies in the "Top 17" Tournament

The only program that improved significantly in the Top 17 tournament was Anderson, which moved from 12^{th} place to 4^{th} place. The trading efficiencies of the remaining programs generally significantly declined, especially Staecker, Lee, *Skeleton*, Breton, and *Pricetaker*. However despite these declines, average market efficiency increased to 97%. The latter efficiency levels are as good as, if not superior to, efficiency levels observed in comparable human experiments. Indeed we found that when we participated as human traders in the Top 17 market, it was difficult to consistently trade at higher than 100% efficiency, whereas we found it relatively easy to consistently trade at higher than 100% efficiency in markets that included the lowest ranked trading programs. This suggests that the Top 17 market may serve as a good working model of a "competitive market" such as observed in experimental markets with human traders.

4.3 Aggregate Behavior of Computerized Traders: Some "Stylized Facts"

A more detailed analysis of tournament results reveals that the top-ranked trading programs do in fact yield a fairly realistic working model of a a DA market in the sense that their collective behavior is consistent with the following stylized facts of human DA markets: 1) convergence to CE, 2) high *ex post* efficiency levels, 3) reductions in transaction price volatility and efficiency losses in successive trading periods reflective of apparent "learning" effects,

4) existence of extra-marginal as well as intra-marginal efficiency losses, 5) low rank correlations between the realized order of transactions and the "efficient" order, and 6) negatively autocorrelated transaction price changes. Even in markets that include the less efficient lower-ranked traders, transaction price trajectories appear to be very similar to those observed in human markets. Indeed, we typically find that transaction prices and quantities converge close to the CE in the very first trading period.

Figure 4.1 shows a typical outcome, game BASE012 of the scientific tournament. The figure plots the induced supply and demand curves and the transaction price trajectories in each of the three trading periods. All three trajectories converged to the CE, generating nearly 100% efficient outcomes. In this case the market traded at 100% efficiency in the first trading period, compared to 98% in the second and third periods. Despite the high *ex post* efficiency, the rank correlation coefficient between the order of the buyer's and seller's trades and the "efficient order" is very low, corresponding to what we observe in human experiments. For example the rank correlation for buyers and sellers in period 1 is 60% and 48%, respectively, falling to just 10% in period 3. The information in the right border of figure 4.1 shows the times and traders involved in each transaction made in period 3. For example the 1^{st} token was traded in BS step 3 when buyer B3 accepted the offer of seller S4 (at a price of 429), and the 8^{th} token was traded in BS step 51 when buyer B2 accepted the offer of seller S1 (at a price of 436). If trades we made in the efficient order as predicted by Wilson's WGDA theory described in section 2, then the first token should have been traded by B2 and S4 and the 8^{th} token should have been traded by B2 and S2. The automata traders frequently trade extra-marginal tokens, something that is commonly observed in human experiments but is also ruled out by game theoretic models such as Wilson's WGDA. For example in figure 4.1 we see that in period 3 buyer B4 succeeded in buying 3 tokens, "bumping" buyer B1 who only succeeded in buying 1 token (in an efficient allocation, each buyer and seller trade their two most valuable tokens). As a result, the traders failed to exploit 27 units of potential surplus in period 3.[26]

For comparison, figure 4.2 presents typical price trajectories in a market with 100% ZI traders. The nature of the ZI strategy implies that transaction price sequences in successive periods are *IID*, so these traders cannot exhibit the type of learning behavior that is characteristic of human DA markets. It is evident that overall transaction price volatility is significantly higher for ZI traders in all periods. The zig-zag pattern of transaction price trajectories implies serial correlation coefficients in the neighborhood of -50% predicted by Cason and Friedman (chapter 6). Finally, notice that ZI traders are very impatient: they complete all their transactions in the first third of the trading period.

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[26] The 4-way decomposition of lost surplus, EM, IM, BS, and SS on the right hand border of figure 4.1 is explained in section 4.4 and the appendix. The left border of figure 4.1 presents other statistics on the trading process, broken out by period. These include the correlation coefficient of transaction price changes (C), *ex post* efficiency (E), time of last transaction (T), Spearman rank correlations of the order of the transactions with the efficient order for buyers (B) and sellers (S), and the maximum and average absolute percentage deviation of transaction prices from the midpoint equilibrium price (A), (M).

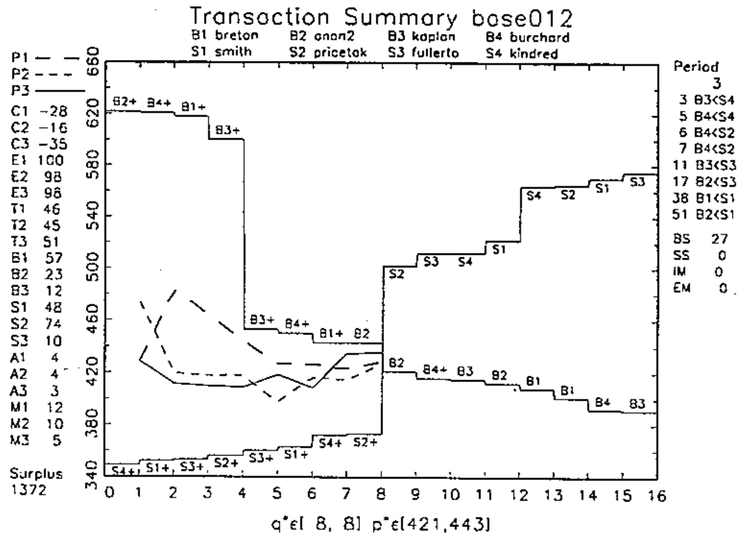

Figure 4.1: Price Trajectories in DA Game BASE012

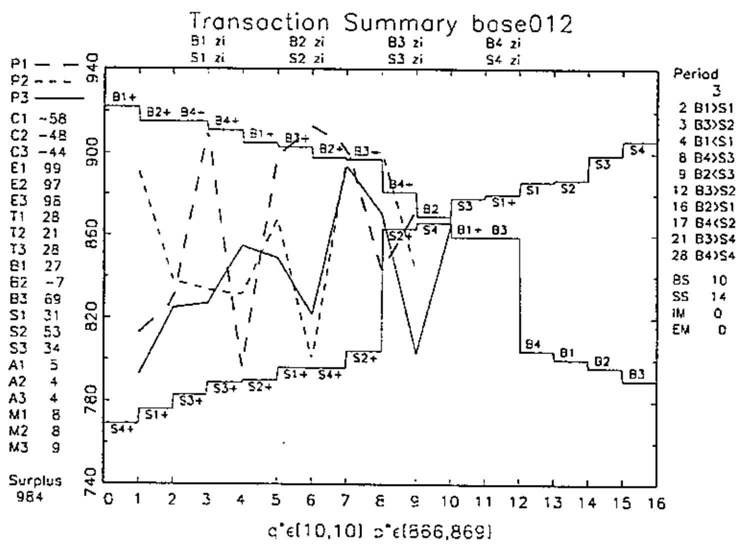

Figure 4.2: Price Trajectories for ZI Traders

The behavior displayed in figures 4.1 appears qualitatively similar to outcomes of human experiments. To demonstrate that these examples are not atypical, table 4.4 presents an array of trading statistics averaged over all tournament environments. In order to highlight learning effects, we break out the statistics by trading period. To help put these statistics in perspective, we also present results for a tournament with 100% ZI traders. We can see from table 4.4 that the Top-17 traders show significant interperiod learning effects: trading efficiency increases, the fraction of price trajectories hitting the CE target increases, and price variability decreases (whether measured by the coefficient of variation or by the percentage deviation from the midpoint equilibrium price). The only statistics that show no systematic improvement in successive periods are the rank correlation coefficients of buyers' and sellers' transaction sequences *vis a vis* the efficient trading order and the serial correlation coefficient for transaction price changes. Both of these statistics are significantly different from the predictions of the WGDA theory (100% and 0% respectively), but are roughly consistent with what we observe in human experiments (30-40%, and -25%, respectively). [27] Efficiency levels are also roughly comparable, although they actually appear to be slightly higher than what we observe in human DA markets.

Learning effects are much less pronounced when we include the lowest ranked trading programs, and of course are completely absent in markets with 100% ZI traders. Notice that while ZI traders attain the highest efficiency levels, price volatility (measured by any of the statistics in the second panel of table 4.4) is significantly higher than the Top-17 traders. ZI traders are noticeably less patient, exchanging only 6% of their tokens in the second half of the trading period compared to over 33% for the Top-17. The statistics on the "hit rates" show that despite the high efficiency of ZI markets, less than 20% of all price trajectories actually converge to equilibrium, compared to nearly 27% in the third period in comparable markets with Top-17 traders. If we widen the target slightly and count any price trajectory that is within 5% of the equilibrium price interval, then the hit rate increases to over 60% for the Top-17 traders compared to just over 50% for the ZI traders. Hit rates for all traders in the scientific tournament are slightly lower than the ZI traders. The other major difference between ZI and the Top-17 traders is that the rank order correlations of the trading sequences are significantly higher, and the serial correlation coefficient of transaction price changes are significantly more negative.

We are presently collecting detailed data sets that will allow us to go beyond the simple stylized facts outlined above and conduct more precise statistical comparisons of the behavior of human and computer traders. Our conjecture is that humans will display much more dramatic interperiod learning effects than the Top-17 programs. It is also probable that we will find significant differences in the stochastic properties of price trajectories in later periods of the game, as well as differences in the timing of of bids, asks, and transactions.

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[27] See Friedman and Cason, chapter 8. They suggest that serial correlation coefficients may be closer to 0 in experiments with more experienced subjects, but the available data are inconclusive.

Statistic (standard deviation)	Top 17 Traders 2327 Games			All Traders 4274 Games			ZI Traders 2295 Games		
	P1	P2	P3	P1	P2	P3	P1	P2	P3
BRANK	47.8	46.4	46.0	48.5	46.9	46.5	57.6	57.4	56.6
	(.9)	(.9)	(.9)	(.6)	(.6)	(.6)	(.9)	(.9)	(.9)
SRANK	48.0	47.5	47.2	48.9	48.8	48.5	53.1	52.2	52.5
	(.9)	(.9)	(.9)	(.6)	(.6)	(.6)	(.9)	(.9)	(.9)
CORRCHG	-26.9	-24.0	-23.9	-29.9	-27.9	-27.2	-48.1	-46.6	-47.4
	(1.1)	(1.1)	(1.1)	(.8)	(.8)	(.8)	(.9)	(1.0)	(.9)
COEFVAR	8.1	6.5	6.2	10.7	8.7	8.6	12.7	12.7	12.7
	(.1)	(.2)	(.2)	(.2)	(.1)	(.1)	(.2)	(.2)	(.2)
DEV, MAX	19.1	15.4	14.6	23.4	19.8	19.4	27.1	27.0	27.2
	(.4)	(.3)	(.3)	(.3)	(.3)	(.3)	(.5)	(.5)	(.5)
DEV, AVERAGE	.42	.15	.07	-.71	-1.24	-1.20	3.23	3.46	3.17
	(.26)	(.22)	(.21)	(.20)	(.18)	(.17)	(.22)	(.22)	(.22)
DEV, LAST	.21	.23	0.02	-.03	-.68	-.59	.73	1.15	.60
	(.20)	(.19)	(.18)	(.19)	(.18)	(.20)	(.24)	(.24)	(.24)
ABS-DEV, AVERAGE	9.9	8.0	7.7	11.4	9.9	9.7	11.9	12.0	11.9
	(.2)	(.2)	(.2)	(.2)	(.1)	(.1)	(.2)	(.2)	(.2)
ABS-DEV, LAST	5.0	4.6	4.4	6.6	6.5	6.5	6.7	6.8	6.9
	(.2)	(.2)	(.2)	(.2)	(.2)	(.2)	(.2)	(.2)	(.2)
HIT RATE	23.5	26.3	27.2	18.5	18.7	18.9	19.0	18.0	19.7
	(.9)	(.9)	(.9)	(.6)	(.6)	(.6)	(.8)	(.8)	(.8)
HIT RATE 2	48.8	52.3	53.2	38.6	39.2	39.4	36.9	36.6	37.6
	(1.0)	(1.0)	(1.0)	(.7)	(.7)	(.7)	(1.0)	(1.0)	(1.0)
HIT RATE 5	57.9	62.4	62.6	48.2	49.4	48.6	52.3	51.4	51.2
	(1.0)	(1.0)	(1.0)	(.8)	(.8)	(.8)	(1.0)	(1.0)	(1.0)
HIT RATE 10	82.6	86.2	86.8	77.0	76.7	76.8	82.4	81.1	81.2
	(.8)	(.7)	(.7)	(.6)	(.6)	(.6)	(.8)	(.8)	(.8)
PCT2ND	40.2	34.3	33.4	27.6	26.1	25.7	5.7	5.7	5.9
	(.5)	(.4)	(.4)	(.3)	(.3)	(.3)	(.2)	(.2)	(.2)
EFF, AV	94.8	96.7	96.9	90.6	91.4	91.5	97.0	97.0	96.9
	(.2)	(.1)	(.1)	(.5)	(.3)	(.3)	(.1)	(.1)	(.1)
EFF, TOTAL	95.6	96.8	97.0	92.3	92.1	91.8	97.9	97.8	97.9

Legend:

BRANK	Rank correlation of buyers' transactions with efficient order
SRANK	Rank correlation of sellers' transactions with efficient order
CORRCHG	Correlation coefficient of transaction price changes
COEFVAR	Coefficient of variation of transaction prices
DEV,MAX	Maximum % deviation from midpoint equilibrium price
DEV,AV	Average % deviation from midpoint equilibrium price
DEV,LAST	% deviation of last transaction price from midpoint eq. price
ABS-DEV,AV	Average absolute % deviation from midpoint equilibrium price
ABS-DEV,LAST	Absolute % deviation of last transaction price from midpoint eq. price
HIT RATE	% price trajectories where $q_l \in [\underline{q}, \overline{q}]$ and $p_l \in [\underline{p}, \overline{p}]$
HIT RATE 2	% price trajectories where $q_l \in [\underline{q}, \overline{q}]$ and $p_l \in [.98\underline{p}, 1.02\overline{p}]$
HIT RATE 5	% price trajectories where $q_l \in [\underline{q}, \overline{q}]$ and $p_l \in [.95\underline{p}, 1.05\overline{p}]$
HIT RATE 10	% price trajectories where $q_l \in [\underline{q} - 1, \overline{q} + 1]$ and $p_l \in [.9\underline{p}, 1.1\overline{p}]$
PCT2ND	% of trades occuring in 2nd half of trading period
EFF,AV	Average of Profits / Surplus for individual games
EFF,TOTAL	Total Profits / Total Surplus

Where: p_l is last transaction price,
$[\underline{p}, \overline{p}]$ is equilibrium price interval,
$[\underline{q}, \overline{q}]$ is equilibrium quantity interval.

Table 4.4: Summary Statistics for Successive Trading Periods

4.4 Analysis of DA Efficiency Losses

One of the stylized facts of human DA markets is that a major fraction of efficiency losses are due to trades of extra-marginal tokens. In order to quantify the magnitude of these losses, it's useful to distinguish between four types of inefficiencies that can occur in DA markets:

IM: value of lost surplus of non-traded intra-marginal tokens (i.e. those that lie to the left of the equilibrium quantity, q^*) when the actual number of trades q is less than q^* (0 otherwise),

EM: value of lost surplus due to trade of extra-marginal tokens (i.e. those that lie to the right of q^* on the supply and demand curves) when the actual number of trades q is greater than q^* (0 otherwise),

BS: value of lost surplus due to trades of extra-marginal buyers' tokens that displaced potential trades of an equal number of buyers' intra-marginal tokens,

SS: value of lost surplus due to trades of extra-marginal seller's tokens that displaced potential trades of an equal number of seller's intra-marginal tokens.

Table 4.5 presents an "inefficiency audit" that summarizes this 4–way decomposition of efficiency losses. The last column of each section of table presents the ratio of total profits to total surplus in each period of play, and the remaining columns present a percentage breakdown of lost surplus due to each of the 4 sources, EM, IM, SS, and BS. In general the audit reveals that total extra-marginal efficiency losses—the sum of EM, SS, and BS—constituted a substantial fraction of total efficiency losses in all environments except TOK. As noted above, this finding is consistent with the results of human experiments that show that efficiency losses are frequently a result of trading too many tokens rather than too few tokens. Extra-marginal efficiency losses were identically zero only in the EQL and TOK environments. In EQL this result is to be expected given the nature of the token distribution which resulted supply and demand curves with large steps, each four units wide, a unique equilibrium price, and a four unit range of market clearing quantities. In the single token TOK environment, if an EM efficiency loss occurs it is more likely that some trader has taken a loss on a transaction. We can see that in the scientific tournament EM losses in the TOK environment are non-zero, reflecting the fact that some traders (principally the programs *Free* and Kindred) were trading at a loss. Once these lower ranked programs were removed in the Top 17 tournament, EM efficiency losses were virtually eliminated.

Overall, Table 4.5 shows that the largest single source of inefficiency is IM, indicating that generally too few rather than too many tokens were traded. This effect is most noticeable in environments TOK and SHRT. Such a result is to be expected for the SHRT environment due to the small number of trading steps. However the large value of IM in the TOK environment is surprising, given that one would expect it would be much easier to trade a single token instead of four.[28] Intra-marginal efficiency losses were also large in the duopsony and duopoly environments BSSS and BBBS. In BBBS this may possibly reflect sellers' attempts at "collusion" in order to restrict output in an attempt to share joint monopoly profits. Note however that the other large source of efficiency losses is due to excessive competition on the long side of the market. Thus, in environment BSSS where there are 3 sellers for each buyer, the large value of the SS efficiency losses indicate that the six sellers became engaged in "price wars" as they competed to sell their tokens to the two buyers.

Table 4.5 show that the level of intra-marginal efficiency losses are typically significantly lower in the Top 17 tournament, reflecting the fact that the majority of the lower-ranked trading programs did poorly as a consequence of failing to trade all of their potentially profitable tokens. On the other hand, the relatively higher levels of extra-marginal efficiency losses in the Top 17 tournament provide another indication that this is indeed a relatively more competitive and aggressive market.

4.5 Results of the "Evolutionary Tournament"

A limitation of the previous tournaments is that trading programs were not allowed to play against themselves. In order to provide a simple model of of imitation and growth processes we decided to conduct an "evolutionary tournament" based on ideas from evolutionary biology (Axelrod 1984, Maynard Smith, 1992).[29] The idea is that in real world markets the best traders will attempt to "clone" their strategies in order to gather a larger market share. These processes will tend to lead to expansion in the number of traders using effective trading rules and declines in the number using poor trading strategies. However the changing market composition may also present opportunities: some trading rules (such as Anderson's) may actually perform better in a tighter, more competitive market.

In an evolutionary tournament each trading program is assigned a measure of "fitness", and an initial population of programs evolves over time according to a specified set of "replicator dynamics". Specifically, let the fitness level of program i in game t be given by its *capital stock* $K_i(t)$. A trader's fitness evolves over time according to the law of motion:

$$K_i(t) = K_i(t-1) + \Pi_i(t) - S_i(t) \tag{4.1}$$

where $\Pi_i(t)$ is trader i's profit in game t, and $S_i(t)$ is the surplus (i.e. token value) assigned to trader i in game t. Thus, our measure of fitness corresponds to trading efficiency: fitness increases when trading efficiency exceeds 100%

[28] This result may indicate possible programming problems if the majority of entrants developed their programs on the assumption that traders would normally be endowed with four tokens. Although tournament rules explicitly noted the possibility of single token environments, all pre-tournament games run on SFTE involved four token environments. Entrants may have assumed that single token environments would constitute a negligible share of tournament profits and focused their attention on the four token case.

[29] Actually, from a strict biological perspective it would be more accurate to call it an "ecological tournament" since the set of species (trading programs) is fixed and only their relative proportions are allowed to change over time.

Case		Scientific Tournament					Top 17 Tournament				
Enviroment	Period	EM	IM	SS	BS	Total	EM	IM	SS	BS	Total
BASE	1	10.1	35.8	17.3	36.8	93.7	35.8	33.7	13.2	17.3	97.0
	2	9.6	43.0	12.4	34.9	93.4	41.0	31.0	12.2	15.9	97.8
	3	10.2	41.4	13.9	34.5	93.4	49.0	19.4	14.4	17.2	98.2
BBBS	1	4.2	45.2	2.1	48.5	91.2	10.0	53.1	4.8	32.1	93.7
	2	3.2	48.7	2.1	45.9	90.3	13.7	43.7	4.1	38.5	95.9
	3	3.2	47.4	2.0	47.4	90.6	13.8	34.2	4.9	47.1	95.9
BSSS	1	0.8	55.0	42.0	2.2	91.3	6.8	36.5	50.7	6.0	94.2
	2	0.7	51.0	46.1	2.2	92.2	9.6	13.4	70.4	6.6	96.0
	3	0.3	57.0	41.3	1.5	91.6	11.1	16.0	67.5	5.4	96.0
EQL	1	0.0	82.9	5.7	11.5	94.8	0.0	53.1	26.0	20.8	98.3
	2	0.0	89.7	3.2	7.1	94.9	0.0	50.0	24.3	25.7	98.8
	3	0.0	89.3	4.2	6.5	95.1	0.0	50.8	19.7	29.5	98.9
LAD	1	8.8	56.6	13.8	20.8	94.0	21.0	26.2	28.6	24.2	97.8
	2	6.4	53.4	18.6	21.6	93.9	23.4	17.2	26.2	33.2	98.3
	3	6.2	53.9	16.8	23.1	93.7	29.5	14.3	25.5	30.7	98.4
RAN	1	21.0	30.8	16.9	31.3	91.8	44.2	13.3	20.5	22.0	95.6
	2	14.6	36.7	11.4	37.3	91.5	49.9	13.2	14.9	22.1	96.4
	3	11.6	38.5	10.6	39.3	90.8	52.6	15.9	12.4	19.1	96.6
SML	1	7.8	72.7	9.1	10.4	91.1	8.3	69.2	16.9	5.51	93.0
	2	7.9	72.5	7.6	11.9	91.3	15.9	50.0	24.6	9.5	96.3
	3	6.8	74.0	6.7	12.4	90.8	16.3	43.7	23.5	16.5	97.1
TOK	1	11.7	85.5	1.4	1.3	95.0	0.0	95.7	2.3	2.0	97.5
	2	2.9	95.6	0.5	1.0	93.2	0.0	94.2	5.8	0.0	99.1
	3	2.3	96.3	0.5	0.9	94.0	0.0	95.9	2.7	1.4	98.2
SHRT	1	3.2	62.7	11.4	22.6	88.3	4.5	70.5	15.5	9.5	89.3
	2	3.0	62.6	10.6	23.8	88.1	7.5	68.0	12.8	11.7	92.9
	3	2.5	64.5	10.3	22.8	87.6	6.4	67.1	15.3	11.2	93.4

Table 4.5: Analysis of Efficiency Losses

and declines otherwise. A real world interpretation is that traders are buying and selling shares of stock. In game t the trader purchases shares of stock at an initial cost of $S_i(t)$, closing out his account at the end of the day by selling off his shares for $\Pi_i(t)$. The competitive hypothesis that price and quantity converge to CE can be re-interpreted as the "efficient markets hypothesis" that the expected value of end-of-day holdings $\Pi_i(t)$ equals the initial purchase price $S_i(t)$. However if markets are not completely efficient, then superior traders should be able to make positive expected profits, tending to increase their capital stocks over time.

Our "replicator dynamic" posits that the fraction $p_j(t)$ of traders using strategy j in game t is proportional to the relative capital share of the type j traders:

$$p_j(t) = K_j(t) \Big/ \sum_{i=1}^{I} K_i(t) \tag{4.2}$$

These dynamics reflect what one might call the "Dean Witter Philosophy": namely, if Dean Witter manages $x\%$ of the stock of investment capital, then on any given day we would expect that approximately $x\%$ of the traders in the market will be employed by Dean Witter.

We begin the evolutionary tournament by endowing all traders with equal initial capital shares, $K_i(0) = \overline{K}$, $i = 1, \ldots, I$. In a DA market with L buyers and M sellers, we begin the evolution by taking L *IID* draws (with replacement) from the population of buyers and M *IID* draws from the population of sellers to form the market using the multinomial distribution (4.2). Given a randomly selected set of traders we play a DA game from the BASE environment with a randomly selected set of tokens. After trading is complete the capital stocks of the L buyers and M sellers are updated according to (4.1), new selection probabilities are computed according to (4.2), and a new set of players is drawn for game $t = 1$. It is also easy to construct a market where a constant fraction of noise-traders, say, $p\%$, enters the market in each game t. For each of the L buyers and M sellers in the DA market, we draw *IID* Bernoulli random variables with parameter p. If the outcome of the m^{th} Bernoulli random variable B_m is 0, then the m^{th} seller is selected from the pool of I "permanent" traders according to (4.2), otherwise if $B_m = 1$ then the m^{th} seller is selected at random from a fixed set of noise-trader programs (and similarly for buyers).[30]

We have not proved any results about the limiting behavior of the evolutionary tournament such as whether or not the limiting set of traders form a *stable set* along the lines of Maynard Smith's notion of "evolutionary stable strategies" (ESS). Indeed, it is not even clear that one can define the precise conditions under which the evolution of capital stocks constitutes an ergodic stochastic process. It is easy to see that in the case of a closed market with no inflow of noise-traders the aggregate capital stock is indeed an ergodic stochastic process: any market with non-zero efficiency losses must eventually hit an absorbing state of zero capital with probability 1. However, our computer simulations suggest that the I-dimensional stochastic process of capital shares is non-ergodic. Indeed, our computer simulations indicate that the long-run outcome of competition among our fixed set of trading programs is unstable, i.e. the stable set is empty.

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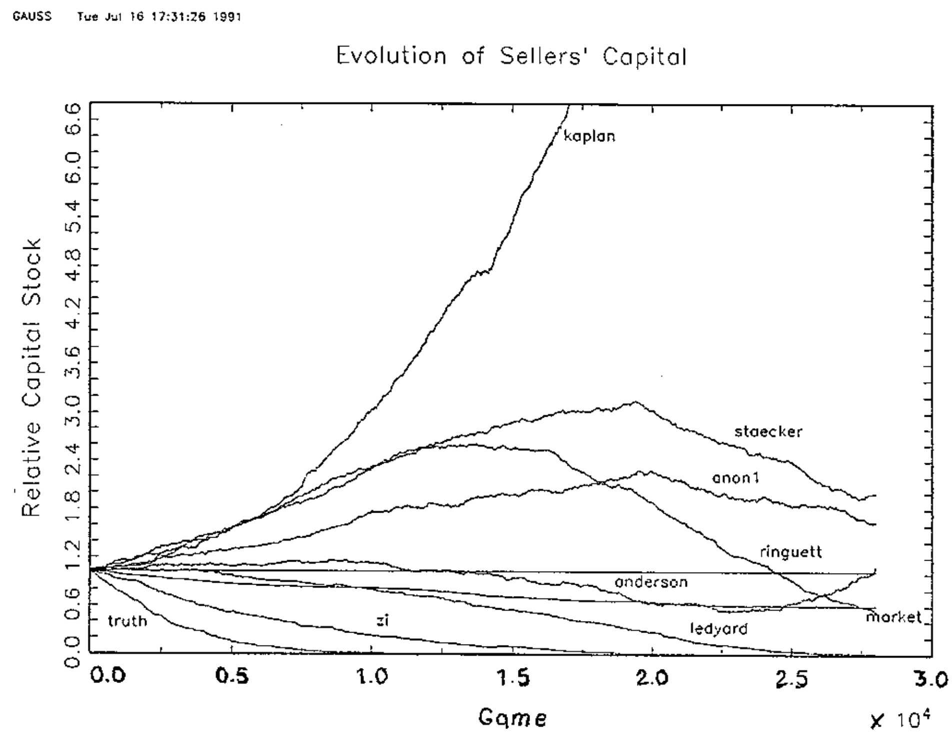
[30] There are two ways to handle the profits (or losses) earned by noise-traders. One way is to update capital stocks of the noise-traders according to (4.2), treating them as a subset of the set of permanent traders. This effectively provides a lower bound of p/N on any trader's participation probability, where N is the number of noise-traders. The other way is not to update the capital stocks for the programs which were selected as noise-traders. This latter method may still indirectly increase the capital stocks of the permanent traders to the extent that they are able to systematically extract surplus from the noise-traders.

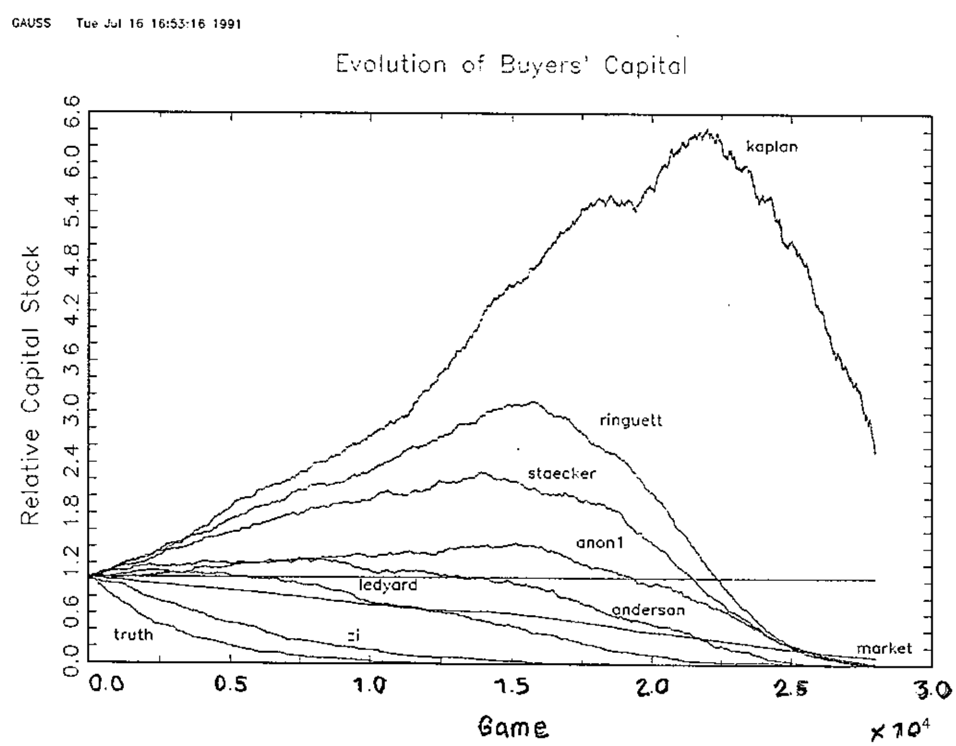
Figures 4.3 and 4.4 clearly illustrate this result in the case of an evolutionary tournament conducted under the BASE environment. We evolved capital stocks for buyers and sellers separately to test whether there are any asymmetries in the traders' performance. All 66 traders (33 buyers and 33 sellers) were equally endowed with an initial capital stock of 80,000. We stopped the tournament after 28,000 games when the buyers' capital stock had dwindled to less than 8% of its original value, as can be seen from the line labelled "market" in figure 4.3. The striking feature about both figures is that after about 5,000 games Kaplan's program emerges as the clear leader, dominating the both the buyers' and sellers' market after 20,000 games. The programs of Ringuette and Staecker have the second and third largest capital shares, but notice that after 20,000 games their influence begins to diminish, losing out to the dominant competition of the Kaplan traders. This effect is especially pronounced among the buyers, and after 22,000 games the capital stocks of Ringuette and Staecker have been reduced to less than their initial allocations, allowing Kaplan to achieve near total domination of the market. However this is precisely when Kaplan's program began to head into a precipitous decline, losing more than half of its capital stock in the succeeding 6,000 games.

The reason for Kaplan's fall is clear: the success of a "wait in the background" strategy depends on being in a market populated with active bidders. If all traders attempt to try to wait in the background, little information will be generated as each trader waits for the others to make the first move. By the end of the period there will be a trading panic as all traders attempt to unload their tokens. Thus, there is a much higher likelihood that the period will expire with unexploited surplus left on the table. This leads to a sharp fall in aggregate efficiency, precipitating the market "crash" that is evident in figure 4.4.

It follows that it cannot be collectively stable for all traders to adopt a "wait in the background" strategy: doing so creates a serious *information externality* that prevents the market from converging to CE. However in Kaplan's case the negative impact of the information externality is actually not the primary reason for his precipitous decline. The primary reason is that his program switches into "truthtelling mode" once it succeeds in monopolizing the market. Specifically, if a long time has elapsed since the last trade, or if time remaining in the trading period is running out, Kaplan's program places a bid equal to the minimum of the current ask and $T-1$ where T is the value of the next untraded token. In practice, the current ask is likely to exceed $T-1$ after a long period of inactivity (otherwise the program would have already jumped in), implying that Kaplan's program effectively becomes a truthteller by bidding $T-1$. Long periods of inactivity—a sign of impasse between buyers and sellers—are quite likely occur when at least one side of the market is dominated by background traders like Kaplan or Ringuette. It follows that once these programs succeed in monopolizing the market Kaplan's program necessarily switches into truthtelling mode.

Although truthtelling is a very bad strategy in a market populated by even slightly smarter strategies (as is clearly evident from the fact that *truthteller* provides the lower envelope for the capital trajectories in figures 4.3 and 4.4), truthtelling can be collectively stable if all traders adopt it since a market with 100% truthtellers is necessarily 100% efficient. Indeed, that is what we found when we ran a tournament with 100% Kaplan traders in the BASE





Figures 4.3 and 4.4: Results of the "Evolutionary Tournament"

environment. This suggests that Kaplan's two-stage wait-in-the-background/truthtelling strategy might actually be a clever way to gain and maintain a monopoly position. [31] However closer examination of figures 4.3 and 4.4 reveal the inherent danger of this approach. By game 22,000 Kaplan's program had virtually monopolized the buyers' market, but it still had not completely monopolized the sellers' market. [32] Thus, Kaplan's buyers tended to get locked into a waiting game with each other, causing them to switch into truthtelling mode resulting in a net windfall to the sellers (who were still predominantly in background mode). This scenario is clearly reflected in the fact that the steady decline in seller's capital decelerated after game 22,000, so that by game 28,000 sellers still had nearly 60% of their initial capital stock. Since Kaplan's program dominated the seller's market it gained the most, multiplying its initial capital stock over 12 times to 984,735.

Given the overall similarity in the programs of Kaplan and Ringuette, why does Kaplan dominate in the long run? Initially both Kaplan and Ringuette succeed in exploiting the other programs, as can be seen by their relatively equal capital shares in the first 5,000 games of the evolutionary tournament. However, once Kaplan and Ringuette start to co-dominate the market, Ringuette's efficiency levels fall from nearly 120% to well below 100% allowing Kaplan to take the lead. It is not easy to identity the precise cause of this outcome, although parts of the reasons have been discussed in our comparison of Kaplan and Ringuette in Rust, Palmer and Miller (1992). It appears that the principal reason for Ringuette's decline is that his program is more impatient to trade than Kaplan's, switching from the role of a background trader to an active bidder much sooner than Kaplan does. On the other hand, when Ringuette's program encounters a period of inactivity, it invokes a much more aggressive active bidding strategy than Kaplan, namely a slightly modified version of *Skeleton*. However since *Skeleton* consistently loses out to Kaplan when the latter is in "background mode", it should not be surprising that Ringuette's program also loses given that it typically switches to "skeleton mode" long before Kaplan's program has switched to truthtelling mode. Specifically, Ringuette's program switches to skeleton mode whenever the number of elapsed steps since the last trade exceeds the smaller of 12, or 60% of the remaining steps in the game. [33] In comparison Kaplan's program switches to truthtelling mode whenever the number of time steps since the last transaction exceeds one half of the number of remaining steps, or whenever 5 steps have elapsed since the last transaction and the number of steps since his program last traded exceeds two thirds of the remaining steps. Thus, at the beginning of a DA game dominated by Kaplan and Ringuette, Ringuette's program

[31] It is unlikely that Kaplan designed his program with this idea in mind since the rules for awarding cash prizes in the March, 1990 tournament are inconsistent with the use of the kinds of evolutionary tournaments which we subsequently designed.

[32] By game 28,000 the top sellers were Kaplan (64%), Staecker (10%), Anon-1 (8%), Perry (7%), Anderson (6%), and Ringuette (3%). In contrast, Kaplan controlled nearly 98% of the buyers' capital.

[33] Ringuette apparently also intended to return a bid from skeleton whenever there were fewer than 1/8 of the total number of steps remaining in the game, however due to an apparent programming error, this option is never invoked.

will switch to skeleton mode after BS step 12 whereas Kaplan's program will still be playing the role of a background trader.

In biological terms, our evolutionary tournaments have suggested the following conclusions: 1) Kaplan's strategy appears to be able to successfully invade a population of non-Kaplan's (including Ringuette), 2) a collection of 100% Kaplan strategies is not collectively stable. These conclusions suggest that the outcome of "closed" evolutionary tournaments (i.e. ones that exclude subsequent entry of noise-traders) may be characterized by cycles of "booms" and "crashes" in the populations of Kaplan's. In a boom period, Kaplan's program invades and overtakes a population of non-Kaplan's. However a crash begins once Kaplan's program attains a near monopoly, dying off on account of negative information externalities which cause it to switch into truthtelling mode. In the latter case Kaplan's relative capital share will shrink until a sufficient number of active bidders are present in the market to enable Kaplan's program to remain in background sufficiently frequently to counterbalance the losses incurred in truthtelling mode. We conducted evolutionary tournaments with noise-traders and found that only a small fraction of active bidders—5 or 10 percent—is necessary to achieve stability in capital shares. However there is little evidence that any of the other programs can successfully invade a population of Kaplan's that are operating in background mode. Otherwise we would have observed growth in their capital shares as Kaplan's program started to monopolize the market. Except for a slight upturn in Anderson's capital share in figure 4.4, there is no evidence that this is happening. [34] By remaining in the background, Kaplan's program is able to capitalize on the mistakes of the active bidders, lifting its efficiency well above 100% and ensuring its growth while pushing the active bidders' efficiency well below 100% and ensuring their decline. In the absence of an active bidding strategy that can successfully invade (or at least coexist) with a population of Kaplan's, there would appear to be no mechanism to stabilize the resulting cycles of booms and crashes. However given the passive nature of a "wait in the background" strategy, it is difficult to see how one could exploit it. Since this strategy is essentially parasitic, we might pose the key open question in biological terms: are there strategies and environments that are resistant to invasion by Kaplanites?

5. Conclusions

In this paper we have studied the behavior of a collection of computer programs playing the roles of buyers and sellers in a discretized version of a dynamic double auction market. One of our objectives was to use this market in an attempt to understand the operation of the "invisible hand". We found that despite the decentralized nature of the trading process and traders' incomplete information about supply and demand, the transaction price trajectories of a heterogeneous collection of computer programs typically converged to the competitive equilibrium,

[34] The upturn in Anderson's share of sellers' capital appears to coincide with Kaplan's monopolization of the buyers' market. Thus, the upturn seems more likely to reflect Anderson's ability to capture some of windfall gains provided by Kaplan's buyers when they entered truthtelling mode than Anderson's ability to successfully invade a population of Kaplan's.

resulting in allocations that were nearly 100% efficient. Our findings complement and extend previous theoretical and experimental insights by Easley and Ledyard, Gode and Sunder, and Wilson, taking us one step closer to resolving Hayek's problem; namely to "show how a solution is produced by the interactions of people each of whom possesses only partial knowledge." Specifically, the fact that convergence occurs in markets where traders use simple rules-of-thumb suggests that it is the DA *institution*, rather than the rationality of the traders *per se*, that is responsible for the emergence of competitive outcomes. The DA trading rules, particularly the "New York Rules" governing the improvement of standing bids and offers, appear to act as a "funnel" that guides the uncoordinated actions of a heterogeneous collection of decision rules towards the CE. Other institutional features, including the discrete vs. continuous nature of trading process, do not appear to play a significant role in generating competitive outcomes. In particular, our imposition of the "AURORA Rules" restricting which traders are eligible to accept the standing bid or ask does not appear to impose a significant constraint on trading opportunities or prevent the market from converging to CE.

Our second objective was to compare the behavior of human and automata traders. Overall, we found that the top-ranked trading programs appear to yield a "realistic" working model of a DA market in the sense that their collective behavior is consistent with the key "stylized facts" observed in human DA experiments: 1) convergence to CE, 2) high *ex post* efficiency, 3) reductions in transaction price volatility and efficiency losses in successive trading periods reflective of apparent "learning" effects, 4) existence of extra-marginal as well as intra-marginal efficiency losses, 5) low rank correlations between the realized order of transactions and the "efficient" order, and 6) negatively autocorrelated transaction price changes. More detailed statistical comparisons of human and computer traders will have to await the completion of matching human experiments to be conducted in our discretized DA market. The fact that our collection of program traders seem to behave similarly to human traders may not seem surprising if programmers are merely encoding their "market intuition" into their computer programs. However, given the complexity of the DA environment and the sophistication of human intelligence, it is not obvious that human behavior in these markets can be captured by a few simple decision rules.

Our final objective was to characterize the form of effective trading strategies in DA markets. We studied a collection of over 30 computer programs ranging in complexity from simple rules-of-thumb to sophisticated adaptive/learning procedures employing some of the latest ideas from the literature on artificial intelligence and cognitive science. In order to evaluate the programs, we conducted an extensive series of computer tournaments involving hundreds of thousands of individual DA games, covering a wide range of trading environments and compositions of trading partners. To our surprise, a single program emerged as the clear winner in nearly all of the tournaments and trading environments. The winning program, submitted by economist Todd Kaplan of the University of Minnesota, was one of the simplest programs that we studied, and can be characterized as *non-adaptive, non-predictive, non-stochastic,* and *non-optimizing*. The basic idea behind the program is to *wait in the background to let others do the* *negotiating, but when bid and ask get sufficiently close, jump in and "steal the deal"*. The program makes no use of prior information about the joint distribution of token values, and relies on only a few key variables such as its privately assigned token values, the current bid and ask, its number of remaining tokens, and the time remaining in the current period. The fact that one can design an effective trading program relying on only a few sufficient statistics confirms Hayek's observation about "the remarkable economy of knowledge that is required in order to take the right action in a competitive market".

It appears that the success of Kaplan's strategy is due to the fact that in an efficient market, if the current bid and ask are close, then it is likely to be the case that either 1) bid and ask are close to the equilibrium price interval, or 2) the current bid or ask are close as a result of a mistake in which one of the holders' failed to place their bid or ask at a sufficiently favorable price. Kaplan's program attempts to "steal the deal" by placing a bid equal to the previous asking price, but only if it can make a profit at that price. As a result Kaplan's program tends to earn at least a normal profit if case 1) holds, and a super-normal profit if another trader has made a mistake. Since the decision of how much to bid is much more difficult than the binary buy/sell decision, it is not surprising that mistakes in bidding are a primary source of poor trading performance. By staying out of the bidding game, Kaplan's program is able to avoid making bidding mistakes on its own account while capitalizing on bidding mistakes of others.

Another reason for the relatively poor performance of the complex, adaptive, optimizing, and predictive strategies is the inherent difficulty of making accurate inferences in a noisy marketplace given only a limited number of observations on one's opponents. The randomness in traders' token endowments is the dominant source of uncertainty in any particular DA game. The additional variation in profits induced by mistakes or stochastic elements in the trading strategies is insignificant in comparison. As a result, one needs a very large number of observations on trading outcomes to be able to reliably distinguish good traders from bad. It follows that it is virtually impossible to try to recognize and exploit the individual idiosyncrasies of one's individual trading partners unless one is interacting with the same group over a very long horizon. The low signal/noise ratio of realized trading profits combined with the high dimensionality of the space of possible trading histories and trading environments implies that programs based on general learning principles (such as neural networks and genetic algorithms) require many thousands of DA training games before they are able to trade even semi-effectively. [35] Nearly all of the top-ranked programs were based on a fixed set of intuitive rules-of-thumb that encoded the programmer's prior knowledge of trading process. This finding suggests that our hopes of using computerized agents endowed with general principles of artificial intelligence to evaluate alternative institutional designs may be too ambitious.

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[35] To quote from the entry by Dallaway and Harvey: "Given that we are doing the equivalent of evolving monkeys that can type Hamlet, we think the monkeys have reached the stage where they recognize that they should not eat the typewriter. If we could have a 4 billion year time extension before handing in the entry, we are completely confident of winning."

Given the simplicity, robustness, and effectiveness of the "wait in the background" strategy, it seems likely other traders would attempt to imitate it, leading to growth in the relative numbers of these sorts of background traders. On the other hand less profitable traders should gradually exit the market due to competitive pressures. In order to study the long-run equilibrium of such market we conducted an "evolutionary tournament" in which the fraction of each type of trader was proportional to its share of the total capital stock. The capital of each trader was updated after each DA game, increasing or decreasing by the difference between realized profits and the trader's surplus allocation in the game. Thus, the capital stocks and relative numbers of each type of trader grew or shrank depending on whether it traded at greater than or less than 100% efficiency. Starting from equal initial capital endowments, the background traders succeeded in exploiting and driving out the active bidders, nearly monopolizing the market. However the background traders create a negative "information externality" by waiting for their opponents to make the first move. If all traders do this, little information will be generated and the market would be unable to function efficiently. In order to avoid such a deadlock, Kaplan's program defaults to a "truthtelling" bidding strategy if a sufficiently long time has elapsed since a trade has occurred. Although a collection of 100% truthtellers is necessarily 100% efficient, it can be easily exploited by even slightly more sophisticated strategies.

The long-run stability of this market depends on whether it is open or closed to new entrants. A closed market tends to be unstable, exhibiting cycles of booms and crashes in the population of background traders. In a boom period the background traders invade and overtake a population of active bidders. A crash begins when the background traders achieve a near monopoly, because the negative information externalities that they create cause Kaplan's program to switch into truthtelling mode. Only in a knife-edge case where Kaplan's traders are able to simultaneously monopolize both sides of the market is a stable equilibrium achieved with 100% truthtelling. But typically the background traders will succeed in monopolizing one side of the market before the other, resulting in its precipitous decline as a result of systematic exploitation by the background traders on the other side of the market.

However in an open market, active bidding by a steady flow of short-lived noise-traders succeeds in stabilizing the pattern of booms and crashes in the number of Kaplan traders. Only a small fraction of noise-traders, comprising less than 10% of the market, is necessary to keep Kaplan's traders in background mode sufficiently frequently to counterbalance the losses they incur in truthtelling mode. Kaplan's traders make up at least 90% of the market in the long-run since the growth of a competitive fringe causes Kaplan's traders to shift into background mode, exploiting and eventually halting the growth of the fringe.

Although the noise-traders facilitate long-run stability in market shares, the limiting market is still quite unstable. In particular, transaction price volatility is unrealistically high—a consequence of the fact that the Kaplan traders are frequently in truthtelling mode. It is unlikely that this situation could persist in the presence of truly adaptive traders, since they would eventually discover best-replies that exploit the fact that Kaplan's program eventually switches into truthtelling mode. Since the trading programs submitted to our initial DA tournament were designed to do well in a sequence of short-run encounters with heterogeneous opponents rather than in long-run interactions with homogeneous opponents, it is not surprising that none were successful in exploiting this particular idiosyncrasy.

The open question is whether there exist strategies that are capable of dominating Kaplan's "wait in the background" strategy over a non-trivial range of environments. If we were to run another DA tournament, it seems likely that entrants would attempt to beat Kaplan by developing more sophisticated delay and "endgame" strategies rather than reverting to truthtelling mode after a fixed amount of time. Thus, even though Wilson's WGDA equilibrium appears to be inconsistent with the behavior of humans and computer programs, we view our results as confirming his insight on the importance of delay as a key ingredient of an effective trading strategy. The main complicating factor is that traders generally don't have any prior knowledge about the strategies used by their opponents, and it may be very difficult to learn those strategies unless one is interacting with the same group for a very long period of time. If this is the case, then simple rules-of-thumb such as Kaplan's may enable one to capture the key features of an effective strategy in a anonymous market consisting of short-run encounters with heterogeneous (and impatient) opponents, whereas more complicated adaptive/learning procedures may do better in a market where one repeatedly trades over a long period of time with a fixed set of opponents. In future work we plan to investigate whether one can develop hybrid rules that graft adaptive/learning procedures onto simple, effective rules-of-thumb, using the rule-of-thumb as a fall-back, but creating the possibility that it might be improved in light of trading experience. [36] Our hope is to characterize strategies that are undominated over a broad range of environments and consistent with long-run market stability.

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[36] For example, if a learning rule were "grafted" onto Kaplan's strategy, it might be smart enough to recognize that other traders were taking advantage of its truthtelling mode, adjusting its delay and bidding rules to insure its long-run survival.

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Appendix: Explanation of the 4-Way Breakdown of Efficiency Losses

Efficiency losses in any DA game can be decomposed into the following four categories:

IM: value of lost surplus of non-traded intra-marginal tokens (i.e. those that lie to the left of the equilibrium quantity, q^*) when the actual number of trades q is less than q^* (or 0 otherwise),

EM: value of lost surplus due to trade of extra-marginal tokens (i.e. those that lie to the right of q^* on the supply and demand curves) when the actual number of trades q is greater than q^* (or 0 otherwise),

BS: value of lost surplus due to trades of extra-marginal buyers' tokens that displaced potential trades of an equal number of buyers' intra-marginal tokens,

SS: value of lost surplus due to trades of extra-marginal seller's tokens that displaced potential trades of an equal number of sellers' intra-marginal tokens.

In order to define these quantities, we first define IMB and IMS as the sum of the values of all intra-marginal tokens *not traded* by buyers and sellers, respectively, during the period. Let EMB and EMS denote the sum of the values of all extra-marginal tokens *traded* by buyers and sellers during the period. Then we have:

$$\text{LOST SURPLUS} = \text{SURPLUS} - \text{PROFIT} = \text{IMB} - \text{EMB} + \text{EMS} - \text{IMS} \tag{1}$$

Let NIMB, NIMS denote the number of intra-marginal tokens that failed to be traded by buyers and sellers, respectively. Let NEMB and NEMS denote the number of extra-marginal tokens traded for the two respective sides of the market. Then we also have the identity

$$q - q^* = \text{NEMB} - \text{NIMB} = \text{NEMS} - \text{NIMS} \tag{2}$$

Clearly when $q = q^*$ there are no *net* intra-marginal or extra-marginal trades, so that EM=IM=0. Then the total amount of lost surplus can then be unambiguously divided into the two categories BS $=$ IMB $-$ EMB and SS $=$ EMS $-$ IMS. However if $q > q^*$, then we face the problem of how to allocate lost surplus due to trades of extra-marginal tokens among the three categories EM, BS, and SS, and if $q < q^*$ we face a similar problem of allocating lost surplus among the categories IM, BS, and SS. Define NBS, NSS, NEM, NIM as follows:

$$\begin{aligned}
\text{NEM} &= max(q - q^*, 0)\\
\text{NIM} &= max(q^* - q, 0)\\
\text{NBS} &= min(\text{NIMB}, \text{NEMB})\\
\text{NSS} &= min(\text{NIMS}, \text{NEMS})
\end{aligned} \tag{3}$$

Then it is easy to see that the following identities hold:

$$\begin{aligned}\text{NIMB} &= \text{NIM} + \text{NBS}\\ \text{NEMB} &= \text{NEM} + \text{NBS}\\ \text{NIMS} &= \text{NIM} + \text{NSS}\\ \text{NEMS} &= \text{NEM} + \text{NSS}\end{aligned} \tag{4}$$

Definition (3) provides an unambiguous way of decomposing the total number of inefficient trades into the 4 categories BS, SS, EM, and IM. There is no unambiguous way of deciding of decomposing the value of lost surplus, however. For example in the DA game illustrated in figure 3.2 buyer B2 traded two extra-marginal tokens and there are 3 intra-marginal tokens that B1 and B2 failed to trade, and 1 intra-marginal token that S3 failed to trade. Thus, NEM=0, NIM=1, NBS=2, and NSS=0, which implies that EM=0 and SS=0. To compute the value of BS and IM, we need to determine which of the 3 intra-marginal buyer's tokens were "bumped". We assume that any one of these tokens is equally likely to have been bumped, and thus we value each "bumped" buyer's token at IMB/3 and compute BS as the value of B3's two extra-marginal tokens less 2IMS/3. IM is computed as IMB/3 less the value of S3's untraded token. More generally EM, IM, BS and SS can be defined as follows:

$$\begin{aligned}\text{EM} &= \text{NEM}\,(\overline{\text{EMS}} - \overline{\text{EMB}})\\ \text{IM} &= \text{NIM}\,(\overline{\text{IMB}} - \overline{\text{IMS}})\\ \text{BS} &= \text{NBS}\,(\overline{\text{IMB}} - \overline{\text{EMB}})\\ \text{SS} &= \text{NSS}\,(\overline{\text{EMS}} - \overline{\text{IMS}})\end{aligned} \tag{5}$$

where $\overline{\text{EMS}} \equiv \text{EMS}/\,max(\text{NEMS}, 1)$ is the average value of extra-marginal sellers' tokens, and $\overline{\text{IMS}}$, $\overline{\text{EMB}}$, and $\overline{\text{IMB}}$ are defined similarly. Using identities (1) through (4) it is easy to verify that the definitions of EM, IM, BS and SS insure that they are always non-negative, and that the following identity holds:

$$\text{SURPLUS LOST} = \text{EM} + \text{IM} + \text{BS} + \text{SS}. \tag{6}$$