Sequential Bargaining with Incomplete Information

Author(s): Drew Fudenberg and Jean Tirole

Source: *The Review of Economic Studies*, Apr., 1983, Vol. 50, No. 2 (Apr., 1983), pp. 221-247

Published by: Oxford University Press

Stable URL: https://www.jstor.org/stable/2297414

Review of Economic Studies (1983) L, 221–247
0034-6527/83/00160221$00.50

Sequential Bargaining with Incomplete Information

DREW FUDENBERG
University of California, Berkeley

and

JEAN TIROLE
CERAS, Ecole Nationale des Ponts et Chausées

This paper describes a simple two-person, two-period bargaining game, and solves it using the concept of perfect Bayesian equilibrium, in which the actions of each player convey information which is used by his opponent. The paper examines the effects of changes in bargaining costs, the size of the "contract zone", and the length of the bargaining process on such aspects of the solution as the probability of impasse and the likelihood of concessions. The combination of information transfer and the lack of pre-commitment embodied in perfectness yields many surprising results. Common perceptions about the effects of parameter changes on bargaining processes are suspect, and should be checked in the particular game being discussed.

INTRODUCTION

This paper analyses the simplest model of non-cooperative bargaining that captures both the fact that bargaining involves a succession of steps and that bargainers typically do not know the value to others of reaching an agreement. We focus particularly on the related issues of commitment and information transfer, which we explore using the concept of perfect Bayesian equilibrium. Such equilibria require that players update their beliefs about their opponents in accordance with Bayes rule, and are not misled by empty threats.

In our model, a seller has two chances to sell an indivisible object to a single buyer; both would prefer a trade today to the same trade tomorrow. The simplicity of the model allows us to completely characterize the set of equilibria. If the valuation of the seller is known, the perfect Bayesian equilibrium is generically unique, while if neither player knows the valuation of the other, there are multiple equilibria. In either case, the combination of information transfer and the lack of precommitment embodied in perfectness can yield surprising results. We find, for example, that the buyer may do better when he is more impatient, that increasing the size of the "contract zone" may decrease the probability of agreement, that prices can increase over time, and that increasing the number of periods can decrease efficiency.

Section 1 reviews the relevant literature. Section 2 introduces the model and our notion of equilibrium. Section 3 analyses the case of one-sided uncertainty, and Section 4 the case in which neither player knows the valuation of his opponent. Before proceeding, we define a bargaining process with gains from trade to be efficient if agreement is reached in the first period, weakly inefficient if agreement is reached at a later period, and strongly inefficient if agreement is not reached.

1. REVIEW OF THE LITERATURE

The literature on non-cooperative bargaining has until recently avoided combining the sequential and the incomplete information aspects.[1] (Some mileage has been obtained by focusing on each of the aspects in isolation.) The resolution of problems involving both aspects can be found in other fields of economics (Kreps–Wilson (1982*a*); Milgrom–Roberts (1982*a*), (1982*b*); Harris–Townsend (1981)).

Rubinstein (1982) considers the problem of how to divide a pie between two players. Players alternate making offers until an offer is accepted. The costs of bargaining are known; if they take the form of discounting or per-period costs there is in general a unique perfect equilibrium, which terminates in the first period. Thus the solution is in general efficient, as might be expected from the complete information aspect of the game. Binmore (1980) and McLennan (1981) generalize Rubinstein's model to bargaining over a utility set, and shows that as the period between offers collapses to zero, the solution converges to a generalized Nash bargaining solution, thus providing a non-cooperative justification based on "toughness" (i.e. costs of bargaining) for the cooperative Nash solution.

Chatterjee and Samuelson (1980) consider a one-period model with incomplete information. A buyer and a seller submit bids, and there is a trade if the seller's bid is less than the buyer's. It is clear that the incomplete information aspect of this auction-type procedure will in general lead to inefficiency, because agreement will not always be reached even when the valuation of the buyer exceeds that of the seller.

Crawford (1981) presents a two-period model in which commitment combined with incomplete information leads to a possibly inefficient outcome. Crawford's model does not discuss information transfers, because the players learn about their second-period costs only after making their first-period decisions. Moreover, while his equilibrium is perfect, given the game he studies, Crawford's model allows commitments based on a perhaps ad hoc "commitment technology".

Riley and Zeckhauser (1980) consider the sequential pricing decision of a seller who can commit himself to a sequence of offers, and show that a single take-it-or-leave-it price is superior to haggling. We shall see later that this result depends crucially on the assumption that the costs of continuing are the same for both players (see Sobel–Takahashi (1981) and Stokey (1979); in Riley–Zeckhauser (1980), these costs are zero).

Sobel and Takahashi (1981) consider a two-period model quite similar to ours. Their main focus is on the case of incomplete information about only the buyer's valuation, the seller's valuation being known. They show in particular that when the seller and the buyer have the same discount factor, the total surplus is maximized for a discount factor strictly between 0 and 1. They conclude with an interesting extension of their model to a potentially infinite horizon for a restricted class of distributions over the buyer's valuation.

2. THE MODEL

One seller and one buyer bargain over an indivisible object. At the start of the bargaining process, each player knows his own valuation (s for the seller, b for the buyer), but not the valuation of the other player. He has some probability distribution about it, which is common knowledge. Valuation should be taken in a broad sense, including the possibility of future transactions with other traders if disagreement occurs. Buyer and seller are assumed to be risk-neutral. In the first period, the seller makes an offer that the buyer accepts or rejects; if the buyer rejects the first offer, the seller makes a second offer in the following period. The payoffs are $[p, b-p]$ (respectively $[\delta_S p, \delta_B(b-p)]$) if agreement at price p is reached in the first (second) period, where $0 \leqq \delta_s, \delta_B \leqq 1$ are the discount factors of the seller and buyer respectively.[2] If agreement has not been reached after two periods, the game ends and the payoffs are $[\delta_S s, 0]$.[3]

We now turn to a formal definition of equilibrium. We must first specify the actions of a seller of type s. Let $p_1(s)$ be his first-period offer, and let $p_2(s, p_1)$ be his second-period offer if the buyer refuses the offer p_1 (in the case of mixed strategies, the notation is easily adapted). Note that we must define strategies for every subgame (in particular for every first-period offer). A buyer of type b accepts the offer p_1 with probability $r_1(p_1, b)$, and in the second period he accepts p_2 with probability $r_2(p_2, b)$. An equilibrium is a set of strategies (p_1, p_2, r_1, r_2) forming a "perfect Bayesian equilibrium": in each subgame no player can gain by deviating to another strategy given the strategies of the other players and given his information, the updating of priors being performed according to Bayes rule. As is usual the perfect equilibrium strategies are obtained by backward induction. This equilibrium concept is an instance of the sequential equilibrium of Kreps and Wilson (1982*b*), to which we refer the reader for a more general definition.

The buyer's second-period equilibrium strategy is easily computed: he accepts the offer p_2 if and only if p_2 does not exceed b; $r_2(p_2, b)=1$ if $p_2 \leqq b$ and $r_2(p_2, b)=0$ if $p_2 > b$. (Actually the buyer is indifferent between accepting and rejecting when $p_2 = b$, but see footnote 4.) The seller's second-period price maximizes his second-period profit given the buyer's second-period strategy, using the strategy $r_1(p_1, b)$ and p_1 to compute his posterior about the valuation of the buyer. Let us now consider the strategy of the buyer in the first period. He observes an offer p_1. Given the strategy $p_1(s)$, he is able to compute a posterior probability distribution for the valuation s of the seller. This in turn can be combined with $p_2(s, p_1)$ to infer a probability distribution for the second-period offer p_2. We simply require that buyer b's first-period action $r_1(p_1, b)$ maximizes his expected discounted payoff given this distribution. Lastly, we consider seller s's first-period action $p_1(s)$. We require it to maximize seller s's expected discounted payoff given the strategies $[r_1(p_1, b), r_2(p_2, b)]$ and his (potential) second-period action $p_2(s, p_1)$.

3. COMPLETE INFORMATION ABOUT THE SELLER, AND TWO TYPES OF BUYERS

We start with the simplest case where the valuation of the seller, s, is common knowledge. The incomplete information aspect of the bargaining process is here limited to the uncertainty the seller faces about the valuation of the buyer. We assume there are two types of potential buyers, with valuations $\underline{b}$ and $\bar{b}\,(\underline{b} < \bar{b})$. The seller has prior $(\frac{1}{2}, \frac{1}{2})$. We shall not restrict the set of offers, but it is clear that any offer will be between $\underline{b}$ and $\bar{b}$.

We consider only the case $s < \underline{b}$; the case $s > \underline{b}$ is trivial as the seller sets $p_1 = p_2 = \bar{b}$. We shall denote by $[\underline{r}_1(p_1), \underline{r}_2(p_2)]$ and $[\bar{r}_1(p_1), \bar{r}_2(p_2)]$ the strategies of buyer $\underline{b}$ and $\bar{b}$. Given the first-period strategies $\underline{r}_1(p_1)$ and $\bar{r}_1(p_1)$, in the case of rejection of will compute a posterior $[\underline{q}(p_1), \bar{q}(p_1) = 1 - \underline{q}(p_1)]$ by simple Bayesian updating (where $\underline{q}(p_1)$ is the updated probability of facing buyer $\underline{b}$). $p_2(p_1)$ will denote the seller's second-period offer if he charged p_1 in the first period. Since p_2 will actually take the values $\underline{b}$ and $\bar{b}$, we will describe it as a random variable with $\sigma_2(p_1) = \Pr[p_2(p_1) = \underline{b}]$.

We will prove that two types of equilibria for the two-period game may arise depending on the solution of the following one-period game.

One-period game. The seller makes an offer that the buyer accepts or rejects. If $\underline{b} > (\bar{b}+s)/2$, the seller announces $\underline{b}$ and the buyer accepts, whatever his type. In this case the seller will be called a "soft seller"; in general, "softness" would depend on the seller's prior and valuation as well as on the buyer's potential valuations. If $\underline{b} < (\bar{b}+s)/2$, the seller announces $\bar{b}$, and the buyer accepts if and only if he is of type $\bar{b}$; the seller is then called a "tough seller." For simplicity, throughout the paper we shall ignore borderline cases, such as $\underline{b} = (\bar{b}+s)/2$; our statements hold for almost all values of the parameters.

In the two-period game, the buyer's equilibrium second-period strategies are the same as in the one-period game: each buyer accepts all offers less than or equal to his valuation. Because the seller will never charge a price less than $\underline{b}$, we can take $\underline{r}_1(p_1)=1$ for $p_1 \leqq \underline{b}$, 0 otherwise. Thus, of the buyer's strategies we need only consider $\bar{r}_1(p_1)$.

In Figure 1 we introduce the diagrams we will use throughout to display equilibria. At the beginning of the first period, the seller chooses a first-period offer p_1. The set

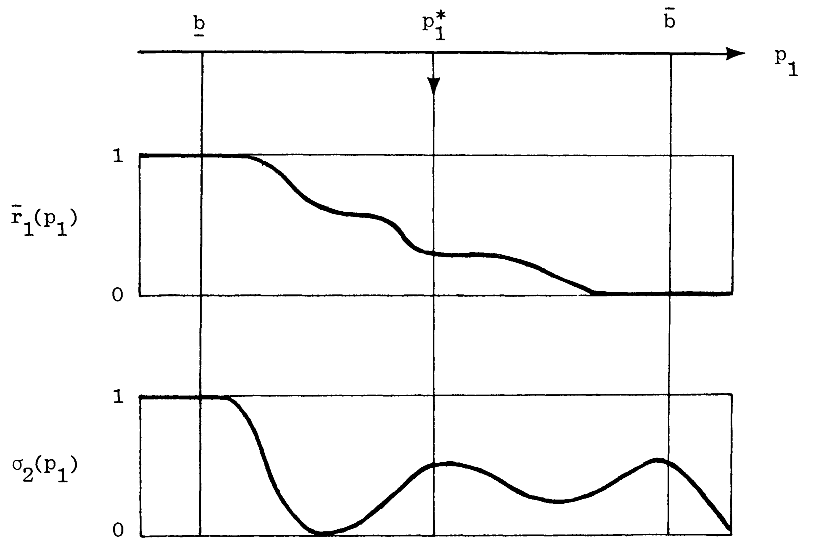

FIGURE 1

of first-period offers is drawn as a horizontal line. The probability that buyer $\bar{b}$ accepts p_1, $\bar{r}_1(p_1)$, is graphed between the first set of horizontal lines. (We recall that only $\bar{b}$'s strategy need be displayed.) In the event that p_1 is refused, the seller plays $\underline{b}$ ("soft") with probability $\sigma_2(p_1)$; this probability is graphed between the second set of horizontal lines. The arrows represent the first-period offers which may arise in equilibrium.

Next we introduce some definitions which will be useful in our characterization of equilibria. Let $\tilde{b}$ be the highest first-period price buyer $\bar{b}$ will pay when he expects that p_2 will be $\underline{b}$: $\tilde{b} \equiv (1-\delta_B)\bar{b}+\delta_B\underline{b}$. Let $\tilde{r}$ be the value of $\bar{r}_1$ which makes a tough seller just indifferent between playing soft and tough in the second period when $\underline{r}_1=0$: $\tilde{r} \equiv (\bar{b}+s-2\underline{b})/(\bar{b}-\underline{b})$. (One can check that $0<\tilde{r}<1$ when the seller is tough.)

Finally, let $\pi(\tilde{b})=\frac{1}{2}\tilde{b}+\frac{1}{2}\delta_S\underline{b}$ be the seller's profit when $p_1=\tilde{b}$ (as we know $\bar{b}$ accepts $\tilde{b}$); and let $\pi(\bar{b},\tilde{r})$ be the seller's profit when $p_1=\bar{b}$ and $\bar{r}_1(\bar{b})=\tilde{r}$.

Proposition 1. *When the buyer has complete information about the seller, there exists a unique perfect Bayesian equilibrium. If the seller is soft, he always plays $\underline{b}$ in the second period* ($\sigma_2=1$), *and p_1 is either $\underline{b}$ or $\tilde{b}$. If the seller is tough, and* $\max(\pi(\underline{b}),\pi(\tilde{b}))>\pi(\bar{b},\tilde{r})$, *the outcome is the same as with a soft seller. If $\pi(\bar{b},\tilde{r})>\max(\pi(\underline{b}),\pi(\tilde{b}))$, then $p_1=\bar{b}$, $\bar{r}_1(\bar{b})=\tilde{r}$, and $\sigma_2(\bar{b})=0$.*

Proof. We first remark that in any period, buyer $\underline{b}$ refuses any offer higher than $\underline{b}$, and that any offer (less than or) equal to $\underline{b}$ is accepted by any buyer, as the seller never charges less than $\underline{b}$ in the second period. Now assume the seller is soft. Then if the first period offer p_1 has been rejected, the seller's posterior is such that $\underline{q}(p_1) \geqq \frac{1}{2}$.

This in turn implies that the seller plays soft in the second period:

$$\underline{b} > \tfrac{1}{2}\bar{b} + \tfrac{1}{2}s \Rightarrow \underline{b} > \bar{q}(p_1)\bar{b} + \underline{q}(p_1)s.$$

Buyer $\bar{b}$, anticipating the seller's second-period offer of $\underline{b}$, will accept the first-period offer if and only if

$$\bar{b} - p_1 \geqq \delta_B(\bar{b} - \underline{b}) \Leftrightarrow p_1 \leqq \delta_B \underline{b} + (1-\delta_B)\bar{b} = \tilde{b}.$$

Thus the seller won't consider first-period offers above $\tilde{b}$. We can conclude that there are two first-period offers which can arise in equilibrium, $\underline{b}$ and $\tilde{b}$. (Offers in between do not increase the probability of agreement, and thus are dominated by $\tilde{b}$ from the point of view of the seller.) The expected payoffs for the seller are:

$$\begin{cases} \pi(\underline{b}) = \underline{b} \\ \pi(\tilde{b}) = \tfrac{1}{2}\tilde{b} + \tfrac{1}{2}\delta_S \underline{b}. \end{cases}$$

It is easy to check that there are parameter values which make either payoff larger, and so either equilibrium can occur.

Now assume the seller is tough. Consider the strategy of buyer $\bar{b}$ in the first period. First, regardless of the seller's strategy in the second period, he will be willing to accept any offer less than or equal to $\tilde{b} = \delta_B \underline{b} + (1-\delta_B)\bar{b}$. Consider an offer p_1 exceeding $\tilde{b}$. If buyer $\bar{b}$ played $\bar{r}_1(p_1) > \bar{r}$, the seller would play soft in the second period, and it would pay for buyer $\bar{b}$ to refuse $(\bar{r}_1(p_1) = 0)$, a contradiction. Suppose now that buyer $\bar{b}$ played $\bar{r}_1(p_1) < \bar{r}$. Then the seller would be tough in the second period, and buyer $\bar{b}$ would be better off accepting p_1 (since $p_1 > \tilde{b}$), except maybe if $p_1 = \bar{b}$. Thus for $\tilde{b} < p_1 < \bar{b}$, the only possible equilibrium strategy for buyer $\bar{b}$ in the first period is the mixed strategy: $\bar{r}(p_1) = \bar{r}$. But in order to be willing to play a mixed strategy, buyer $\bar{b}$ must be indifferent between accepting and refusing the first-period offer p_1. Remember now that $\bar{r}_1(p_1) = \bar{r}$ means the seller is indifferent between playing tough and soft in the second period. If $\sigma_2(p_1)$ is the probability of playing soft in the second period, buyer $\bar{b}$ is indifferent in the first period if and only if:

$$\bar{b} - p_1 = \sigma_2(p_1)\delta_B(\bar{b} - \underline{b}) \Leftrightarrow \sigma_2(p_1) = \frac{\bar{b} - p_1}{\delta_B(\bar{b} - \underline{b})}.$$

Notice that we must have $\bar{r}_1(b) = \bar{r}$ as well, because the seller can always approximate the corresponding payoff arbitrarily closely by playing $p_1 = \bar{b} - \varepsilon$, which is accepted with probability $\bar{r}_1$.

Thus we have again shown that there exist unique perfect Bayesian equilibrium strategies. They are represented in Figure 3.

From looking at buyer $\bar{b}$'s first-period strategy, we see that the best strategy for the seller in the first period is to choose one of three offers $(\underline{b}, \tilde{b}, \bar{b})$, as $\bar{r}_1(p_1)$ is constant over $[\underline{b}, \tilde{b}]$ and over $(\tilde{b}, \bar{b}]$. The respective payoffs for the seller are:

$$\begin{cases} \pi(\underline{b}) = \underline{b} \\ \pi(\tilde{b}) = \tfrac{1}{2}\tilde{b} + \tfrac{1}{2}\delta_S \underline{b} = \tfrac{1}{2}(\delta_B \underline{b} + (1-\delta_B)\bar{b}) + \tfrac{1}{2}\delta_S \underline{b} \\ \pi(\bar{b}, \bar{r}) = \tfrac{1}{2}\bar{r}\bar{b} + \left(\dfrac{2-\bar{r}}{2}\right)\delta_S \underline{b} = \dfrac{\bar{b} + s - 2\underline{b}}{2(\bar{b} - \underline{b})}\bar{b} + \dfrac{\bar{b} - s}{2(\bar{b} - \underline{b})}\delta_S \underline{b}. \end{cases}$$

(The last payoff is computed observing that the seller is indifferent between playing tough and soft in the second period.) One can check that any of these payoffs may be the largest, depending on the parameters.[5] ||

Even in such a simple model, we can identify some important features of the bargaining process. First we examine the case of a soft seller, displayed in Figure 2.

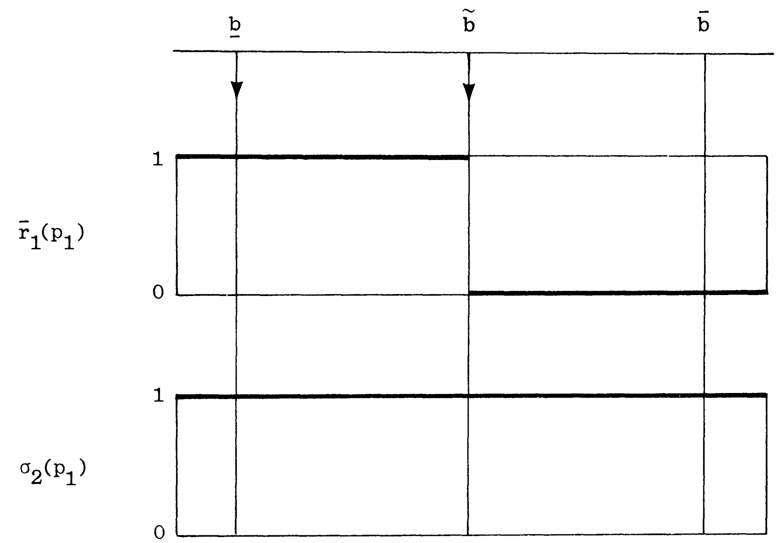

FIGURE 2
One soft seller

1. There is an adverse selection phenomenon in that the seller's prior will dictate his behaviour. In our computations the prior about the valuation of the buyer was $(\frac{1}{2}, \frac{1}{2})$. If the prior were different, the same argument would go through except for the computation of $\pi(\tilde{b})$. If $\delta_S < 1$ and the seller expects with a high probability that the buyer has valuation $\underline{b}$, then he will play soft in the first period ($p_1^* = \underline{b}$); on the contrary, if he is fairly convinced that he is facing buyer $\bar{b}$, he will charge $p_1^* = \tilde{b}$. This reflects the intuition that in general a buyer is better off if the seller believes he is not very eager to buy.

2. This case shows that agreement need not occur in the first period, even in a situation where agreement would always be reached in a one-period bargaining process. Thus extending the horizon may decrease the efficiency of the bargaining process (here the one-period efficient process becomes a two-period weakly inefficient process).

3. Concessions may (and in general will) arise in perfect equilibrium even if buyers are risk-neutral (take the case where $p_1^* = \tilde{b}$). This condition would still hold if the seller could commit himself to a sequence of offers at the start of the bargaining process (it is easy to see that if $\pi(\tilde{b}) > \pi(\underline{b})$, the best strategy for a seller who can commit himself is to announce ($p_1 = \tilde{b}, p_2 = \underline{b}$), which leads to the same outcome as the perfect equilibrium). One can check that if the buyer and the seller have identical bargaining costs ($\delta_B = \delta_S$), the best strategy for a seller who can commit himself is to announce ($p_1 = \underline{b}, p_2 = \underline{b}$). (It can also be checked that when the seller is tough and can commit himself, he sticks to a fixed price, $\underline{b}$ or $\bar{b}$.) The non-haggling result of Stokey (1979) and Riley–Zeckhauser (1980) holds with identical costs of bargaining, but may not hold when the buyer is more "impatient" than the seller.

4. A lower discount factor of the buyer increases $\tilde{b}$, and makes this offer more attractive to the seller. Thus apart from the direct effect, an increase in the rate of time preference of the buyer is detrimental to him in that it shrinks the range of parameters for which the seller plays soft in the first period of the two-period process. We shall see later that an increase in time preference need not make a player worse off.

5. The seller may be made better off when one period is added to the bargaining process, even if he discounts the future. If the values of the parameters are such that $\pi(\tilde{b}) > \pi(\underline{b})$ (as for $\delta_S = 1$), the seller is made better off by the addition of a second

period which allows him to extract some of the surplus of the high valuation buyer without losing the chance to sell to the low valuation buyer.

We now turn to the case of a tough seller, displayed in Figure 3. What are the new features exhibited by this equilibrium?

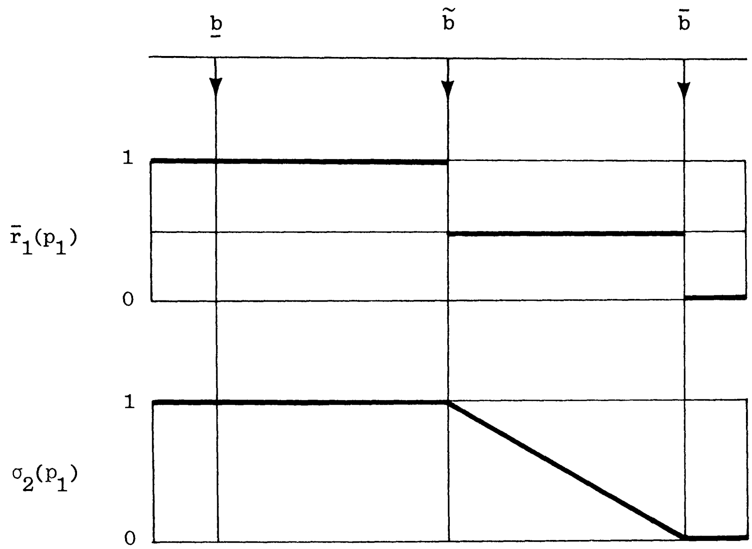

FIGURE 3
One tough seller

6. The outcome may be strongly inefficient. If the parameters are such that the seller chooses $\bar{b}$ in the first period, with probability $\frac{1}{2}$ agreement won't be reached. Contrary to the case of a soft seller, where introducing a second period may only lead to a less efficient outcome (although the equilibrium is at most weakly inefficient), introducing a second period in the case of a tough seller may decrease or increase efficiency. The strongly inefficient outcome becomes weakly inefficient or efficient if the equilibrium offer is $\underline{b}$ or $\tilde{b}$: it becomes "even more inefficient" if the equilibrium offer is $\bar{b}$ in that the offer $\bar{b}$ is now refused with some probability by buyer $\bar{b}$. Hence increasing the horizon may have very different effects.

7. Surprisingly, a decrease in the buyer's discount factor may increase the buyer's payoff, even if one takes into account the direct effect on welfare. To see this, choose values of the parameters such that $\pi(\bar{b}, \bar{r}) > \pi(\tilde{b}) > \pi(\underline{b})$ and $(\pi(\bar{b}, \bar{r}) - \pi(\tilde{b}))$ is "small" (it is easy to check that this is feasible). The seller's first-period offer is $\bar{b}$, and buyer $\bar{b}$ has a zero payoff. Now decrease δ_B so that $\pi(\tilde{b}) > \pi(\bar{b}, \bar{r})$ (remember that $\pi(\bar{b}, \bar{r})$ does not depend on δ_B). The offer $\tilde{b}$ becomes more attractive to the seller, and the payoff for the buyer $\bar{b}$ now is $(\bar{b} - \tilde{b}) > 0$. What is the intuition behind this somewhat counterintuitive result? Having a low discount factor makes buyer $\bar{b}$ willing to accept high first-period offers, thus increasing the relative attractiveness for the seller of seeking a "compromise" with buyer $\bar{b}$ compared to taking a tough stance at the start. The crux of the problem is that buyer $\bar{b}$ *cannot* commit himself to accept high offers if his discount factor is high, and thus cannot prevent the seller from asking $\bar{b}$. Buyer $\bar{b}$ would like to offer to accept offers greater than $\tilde{b}$, but if the seller believed him buyer $\bar{b}$ would want to pretend to be buyer $\underline{b}$, refuse the offer and face offer $\underline{b}$ in the second period.

8. Finally we analyse the effect of adding a second period in terms of the welfare of the players. By the welfare of buyer $\underline{b}$ or $\bar{b}$, we naturally mean his welfare if "nature

chooses him". Whether the seller is tough or soft, at least one of the potential players (seller, two buyers) is not made worse off when a second period is added to the bargaining process. If the seller is soft, he charges $\underline{b}$ in the one-period process; and as he is free to charge $\underline{b}$, which is accepted, in the first period of the two-period process, he cannot be made worse off. If the seller is tough, he plays $\bar{b}$ in the one-period game, so that no buyer has a positive surplus; thus the buyer cannot be worse off in the two-period process (and buyer $\bar{b}$ may be made better off). The tough seller may be worse off in the cases where he chooses $\underline{b}$ or $\bar{b}$, and may be worse off or better off in the cases where he chooses $\tilde{b}$.

4. INCOMPLETE INFORMATION ON BOTH SIDES

By assuming that the buyer was completely informed about the valuation of the seller, we have so far neglected the transfer of information through the seller's offer. We now study bargaining under incomplete information for both the seller and the buyer. To keep things simple, we stick with the same two-potential-buyers ($\underline{b}$ and $\bar{b}$) framework, and we assume that there are two types of potential sellers, with valuations $\underline{s}$ and $\bar{s}(\underline{s}<\bar{s})$. The buyer has a prior $(\frac{1}{2},\frac{1}{2})$ over the type of the seller, and this probability distribution is common knowledge. We shall consider the case where there is always a potential gain from trade $(\bar{s}<\underline{b})$.

The buyer now cares about the identity of the seller, since the two types of sellers will in general make different second-period offers. Bayes' rule dictates how the buyer should update when he observes an offer which has a positive probability.[6] For an offer which in equilibrium is made by only one type of seller, the buyer's posterior should concentrate all the weight on that type. For an offer made by both types, the posterior depends on the weights given that offer by each type. Lastly, we shall need to specify how the buyer's prior is revised if the observed first-period offer is not made by either seller in equilibrium. Bayes's rule then places no restriction on the buyer's posterior. The leeway we have in choosing these conjectures leads to a multiplicity of equilibria for some parameters.[7] One might, therefore, want to impose some additional restrictions on the posteriors. We will do so only after completely characterizing the perfect equilibria which can be supported by *some* Bayesian conjectures, because it is difficult to say what restrictions are reasonable. (See Section 6 and Kreps–Wilson (1982*b*).)

A first-period offer p_1 which both sellers play with positive probability will be called a common offer. We adopt the convention that if two offers p_1 and p_1' are such that $\bar{r}_1(p_1)=\bar{r}_1(p_1')=0$, the offers are actually the same, since they are both refused in the first period, and they both lead to the same posterior $(\frac{1}{2},\frac{1}{2})$ about the valuation of the buyer.

Proposition 2. *In equilibrium there is at most one common offer.*

The proof of this proposition is somewhat tedious, and can be found in the Appendix, but the idea is simple. Assume that in equilibrium both sellers are indifferent between two offers, $p_1<p_1'$. Maximization by the seller implies that, in general, the probability of acceptance by buyer $\bar{b}$ will be higher at p_1 than at p_1'. This means that the seller is more likely to face buyer $\underline{b}$ in the second stage when he has charged p_1 in the first stage. This in turn confers a relative disadvantage at p_1 to seller $\underline{s}$, since he is more concerned than seller $\bar{s}$ by the possibility of disagreement in the second period.

If there is *no* common offer, the equilibrium will be called separating, because the first-period offer identifies the seller. If there is a common offer, but at least one seller sometimes makes a different first-period offer, the equilibrium will be called semi-separating. Finally, if the sellers always make the same first-period offer, the equilibrium will be called pooling, because the first-period offer never reveals the identity of the seller.

With two potential sellers, there are three cases: (A) both sellers are soft ($\underline{s}<\bar{s}<2\underline{b}-\bar{b}$); (B) $\underline{s}$ is soft, $\bar{s}$ is tough ($\underline{s}<2\underline{b}-\bar{b}<\bar{s}$); or (C) both sellers are tough ($2\underline{b}-\bar{b}<\underline{s}<\bar{s}$).

Proposition 3. *With two potential soft sellers, the equilibrium is unique and is a pooling equilibrium. Furthermore, it is identical to the one-soft-seller equilibrium.*

The proof, which parallels that of Proposition 1, is omitted. The equilibrium can be supported by many different conjectures. For example, the buyer may have "passive conjectures": no matter what the first-period offer, the buyer does not change his prior.

We will now examine case (B), $\underline{s}$ soft and $\bar{s}$ tough, in some detail (the case of two tough sellers is quite similar; we discuss it briefly later on). In the interest of brevity, many of the proofs have been placed in the Appendix.

Lemma 3. *Seller* $\underline{s}$: (i) *always plays soft* ($\underline{b}$) *in the second period.* (ii) *from* (i), *would never charge* p_1 *such that* $\bar{r}_1(p_1)=0$.

Lemma 3 states that: (i) the soft seller cannot become tough in the second period, since in equilibrium any offer which is accepted by the low valuation buyer is also accepted by the high valuation buyer; (ii) therefore the soft seller will never charge a price which is sure to be refused.

We now characterize in succession pooling, separating and semi-separating equilibria.

Pooling equilibrium

Assume that the offer p_1 is the outcome of a pooling equilibrium. Observe that the buyer's posterior after observing p_1 is $(\frac{1}{2},\frac{1}{2})$.

Lemma 4. $p_1 \leqq \mathring{b} \equiv \bar{b}-\frac{1}{2}\delta_B(\bar{b}-\underline{b})$.

Lemma 4 puts an upper bound on what a pooling equilibrium offer can be. Since for a pooling equilibrium offer, there is some probability, namely $\frac{1}{2}$, that the seller is soft, this limit has to be lower than $\bar{b}$, which would be the most buyer $\bar{b}$ would be willing to accept if he knew with certainty that the seller were tough. For a similar reason, this upper bound exceeds $\tilde{b}$.

Define:

$$\bar{r}=\frac{\bar{b}+\bar{s}-2\underline{b}}{\bar{b}-\underline{b}}$$

(value of $\bar{r}_1$, which makes $\bar{s}$ indifferent between $\underline{b}$ and $\bar{b}$ in the second period)

$$\pi(\underline{b})=\underline{b}$$

(payoff to any seller when $p_1=\underline{b}$)

$$\pi(\tilde{b})=\frac{\tilde{b}}{2}+\delta_S\frac{\underline{b}}{2}$$

(payoff of any seller when $p_1=\tilde{b}$)

$$\pi(\mathring{b},\bar{r})=\frac{\bar{r}}{2}\mathring{b}+\frac{2-\bar{r}}{\bar{r}}\delta_S\underline{b}$$

(payoff of any seller when $p_1=\mathring{b}$ and $\bar{r}_1\,(\mathring{b})=\bar{r}$)

$$\pi(\bar{b},0)=\delta_S\frac{(\bar{b}+\bar{s})}{2}$$

(payoff of $\bar{s}$ when $p_1=\bar{b}$ and $\bar{r}_1(\bar{b})=0$. Recall from Lemma 3 that $\underline{s}$ would never charge a price that was sure to be refused).

Proposition 4. *Pooling equilibria are characterized as follows, depending on which of the four payoffs* $\{\pi(\underline{b}), \pi(\tilde{b}), \pi(\mathring{b}, \bar{r}), \pi(\bar{b}, 0)\}$ *is largest.*

(i) *If* $\pi(\underline{b})$ *is largest, a pooling equilibrium exists and the path is unique, with* $p_1=\underline{b}$, $\bar{r}_1(\underline{b})=1$, *and* $\bar{\sigma}_2(\underline{b})=1$;

(ii) *If* $\pi(\tilde{b})$ *is largest, a pooling equilibrium exists and the path is unique, with* $p_1=\tilde{b}$, $\bar{r}_1(\tilde{b})=1$, *and* $\bar{\sigma}_2(\tilde{b})=1$;

(iii) *If* $\pi(\mathring{b}, \bar{r})$ *is largest, any* p_1 *in an interval* $[b', \mathring{b}]$, *with* $\tilde{b}<b'<\mathring{b}$, *can arise in a pooling equilibrium as can* $\underline{b}$ *or* $\tilde{b}$;

(iv) *If* $\pi(\bar{b}, 0)$ *is largest, no pooling equilibrium exists. Each case arises for a non-negligible set of parameters.*

Proof. Lemmas 5, 6, and 7 in the Appendix show that if a pooling equilibrium exists, it will either be at $p_1=\tilde{b}$ or $p_1=\underline{b}$, with $\bar{r}_1(p_1)=1$; at $p_1\in[\tilde{b}, \mathring{b}]$, with $\bar{r}_1(p_1)=\bar{r}$; or at $\mathring{b}$, with $\bar{r}_1(p_1)\leqq\bar{r}$. These lemmas thus establish that any pooling equilibrium will be one of the four types above. The equilibrium payoffs for the sellers of the various possible pooling equilibria are given above, followed by the payoff to seller $\bar{s}$ of deviating from a pooling equilibrium to charge $\bar{b}$ in both periods.

First, we assume that $\pi(\underline{b})$ or $\pi(\tilde{b})$ is largest, and construct a pooling equilibrium at $\underline{b}$ or $\tilde{b}$ respectively. We use "passive conjectures": the buyer learns nothing

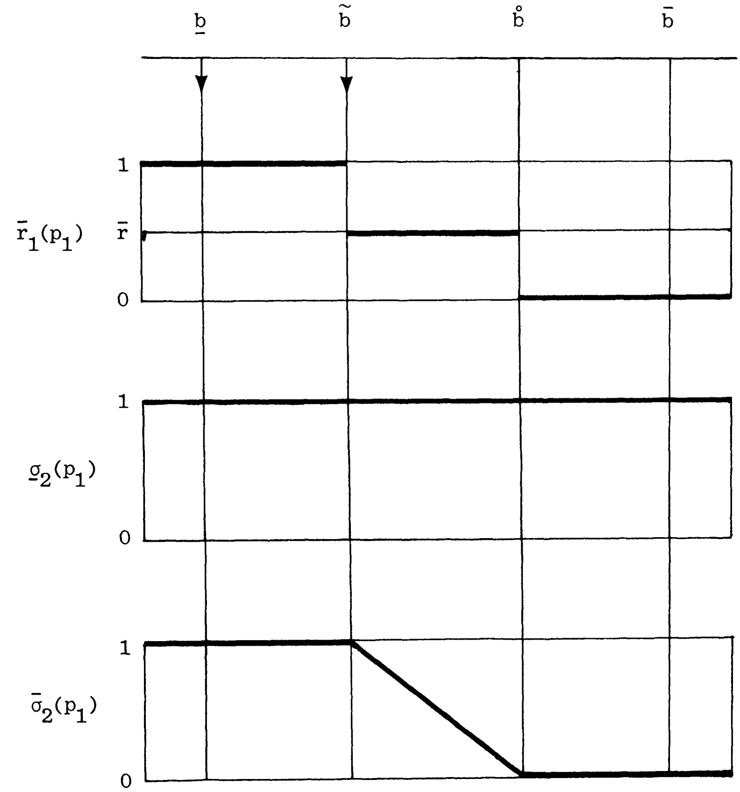

FIGURE 4

One soft seller, one tough seller. Pooling equilibrium from zero probability events. Combined with posterior given by Bayesian updating at the pooling equilibrium, the buyer's posterior will be $(\frac{1}{2}, \frac{1}{2})$ whatever the first-period offer. The equilibrium strategies are displayed in Figure 4.

Seller $\bar{s}$'s randomized second period strategy, when his first offer is between $\tilde{b}$ and $\mathring{b}$, has to make buyer $\bar{b}$ indifferent between accepting and refusing in the first period. Since buyer $\bar{b}$ has conjectures $(\frac{1}{2}, \frac{1}{2})$ about the valuation of the seller, $\bar{\sigma}_2(p_1)$ must satisfy:

$$\bar{b}-p_1=\delta_B(\bar{b}-\underline{b})(\tfrac{1}{2}+\tfrac{1}{2}\sigma_2(p_1)) \Leftrightarrow \bar{\sigma}_2(p_1)=\frac{2(\bar{b}-p_1)}{\delta_B(\bar{b}-\underline{b})}-1.$$

Thus $\bar{\sigma}_2(p_1)$ is linearly decreasing in p_1 and takes value 1 at $\tilde{b}$ and value 0 at $\mathring{b}$. Given that $\pi(\bar{b}, 0) < \max(\pi(\underline{b}), \pi(\tilde{b}))$, seller $\bar{s}$ won't be tempted to charge more than $\mathring{b}$ in the first period. Since both sellers have the same preferences over the offers less than or equal to $\mathring{b}$,[8] there will be a pooling equilibrium at $\underline{b}$ or $\tilde{b}$, depending on which is largest.

Moreover, because buyer $\bar{b}$ accepts all first-period offers less than or equal to $\tilde{b}$ no matter what his conjectures, $\pi(\underline{b})$ and $\pi(\tilde{b})$ are the payoffs to making those offers in *any* equilibrium. This fact, combined with Lemmas 5, 6, and 7 which showed that the equilibrium payoff in any pooling equilibrium is less than or equal to $\max(\pi(\underline{b}), \pi(\tilde{b}), \pi(\mathring{b}, \bar{r}))$, implies that if $\pi(\underline{b})$ or $\pi(\tilde{b})$ is the largest, the pooling equilibrium path is unique.

Assume now that $\pi(\mathring{b}, \bar{r}) > \max(\pi(\underline{b}), \pi(\tilde{b}), \pi(\bar{b}, 0))$. Define b' in $(\tilde{b}, \mathring{b})$ by: $\pi(b', \bar{r}) = \max(\pi(\underline{b}), \pi(\tilde{b}), \pi(\bar{b}, 0))$. We claim that any $p_1^* \in [b', \mathring{b}]$ is a pooling

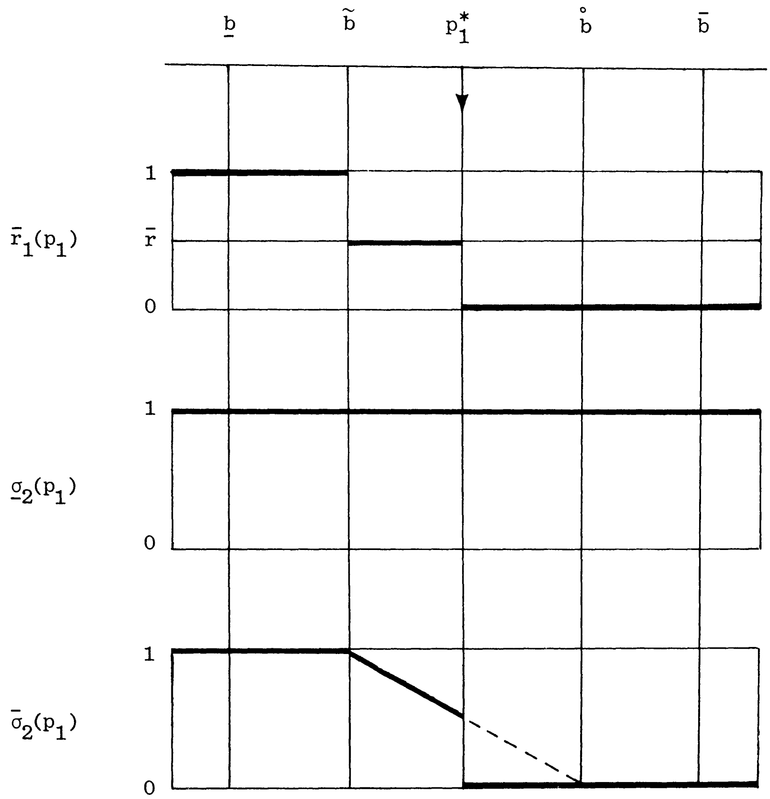

FIGURE 5

One soft seller, one tough seller. Pooling equilibrium

equilibrium with appropriate conjectures. We shall choose conjectures $(\frac{1}{2}, \frac{1}{2})$ up to p_1^* included, and $(1, 0)$ after; this means that for $p_1 > p_1^*$, the buyer believes with certainty that he faces seller $\underline{s}$. With these conjectures, there exists a unique tuple of equilibrium strategies, represented in Figure 5.

As before, $\bar{\sigma}_2(p_1)$ is given by, between $\tilde{b}$ and p_1^*:

$$\bar{\sigma}_2(p_1) = \frac{2(\bar{b} - p_1)}{\delta_B(\bar{b} - \underline{b})} - 1.$$

Note that to obtain this kind of pooling equilibrium, we introduced a discontinuity in the conjectures at p_1^*. This was not necessary. All that matters is that the probability that the buyer attaches to facing seller $\underline{s}$ increases "quickly enough" after p_1^*; this condition is not satisfied by passive conjectures. If one requires the conjectures to be "non-increasing", then the equilibria with first-period offers p_1^* strictly less than $\mathring{b}$ disappear.[9]

Assume that $\pi(\mathring{b}, \bar{r}) > \max(\pi(\underline{b}), \pi(\tilde{b})) > \pi(\bar{b}, 0)$. We claim that either $\underline{b}$ or $\tilde{b}$, depending on the largest of the two numbers $\pi(\underline{b})$ and $\pi(\tilde{b})$, is a pooling equilibrium with appropriate conjectures. Namely take any conjecture up to $\tilde{b}$ (we know that conjectures do not matter for $p_1 \leqq \tilde{b}$), and $(1, 0)$ for $p_1 > \tilde{b}$. The buyer, when he observes $p_1 > \tilde{b}$, is convinced that he faces seller $\underline{s}$. One can show that the unique equilibrium strategies associated with those conjectures are represented by Figure 6. If one requires the conjecture to be non-increasing, then it is easy to check that there is no pooling equilibrium at $\underline{b}$ or $\tilde{b}$.[10]

Finally, if $\pi(\bar{b}, 0)$ is the largest, no pooling equilibrium exists, because seller $\bar{s}$ prefers to deviate and charge $\bar{b}$ in the first period.

This study, combined with the lemmas, and the observation that each of the four numbers $\pi(\underline{b})$, $\pi(\tilde{b})$, $\pi(\mathring{b}, \bar{r})$, and $\pi(\bar{b}, 0)$ is the largest over a non-negligible set of parameters, completes the proof of Proposition 4. ‖

A few interesting things can be said about these pooling equilibria.

1. Consider the case where $\pi(\mathring{b}, \bar{r}) > \max(\pi(\underline{b}), \pi(\tilde{b}), \pi(\bar{b}, 0))$. Then $\mathring{b}$ is a pooling equilibrium. At the pooling equilibrium $\mathring{b}$, the soft seller is better off than when the buyer knows he is a soft seller [in that case, his profit is $\max(\pi(\underline{b}), \pi(\tilde{b}))$]. This is not very surprising; roughly speaking, the soft seller "free rides" on the tough seller, who has a credible threat of playing tough in the second period.

What about the tough seller? Remember that with complete information about his valuation, he charges $\underline{b}$, $\tilde{b}$ or $\bar{b}$ depending on the largest of $\pi(\underline{b})$, $\pi(\tilde{b})$ and $\pi(\bar{b}, \bar{r})$, and that $(\pi(\bar{b}, \bar{r}) > \pi(\mathring{b}, \bar{r}))$. Thus in the case being considered, the tough seller is worse off in the pooling equilibrium $\mathring{b}$ than in the case where the buyer knows his valuation. Note that with incomplete information about the seller, a tough seller who charges $\bar{b}$ in a two-period game with complete information about his valuation might charge $\underline{b}$ or $\tilde{b}$, instead of $\mathring{b}$ or $\bar{b}$.

The previous two observations are very intuitive, and may remind the reader of adverse selection models, such as the insurance market (Rothschild–Stiglitz [1976], Wilson [1977]) where high risk agents may benefit from the presence of low risk agents, whereas the latter may be hurt by the presence of the former.

Note also that if buyer $\bar{b}$ refuses $\mathring{b}$ [which he does with probability $(1 - \bar{r})$], the soft seller makes a concession $(p_2 = \underline{b})$ whereas the tough seller plays even tougher $(p_2 = \bar{b})$. Thus concessions are not a general rule with incomplete information about the seller.

2. When traders have no cost of bargaining $(\delta_S = \delta_B = 1)$, there is no pooling equilibrium.[11] This again is intuitive since the absence of bargaining costs makes the two-period game look like a one-period game, where, we know, the two sellers will behave differently.

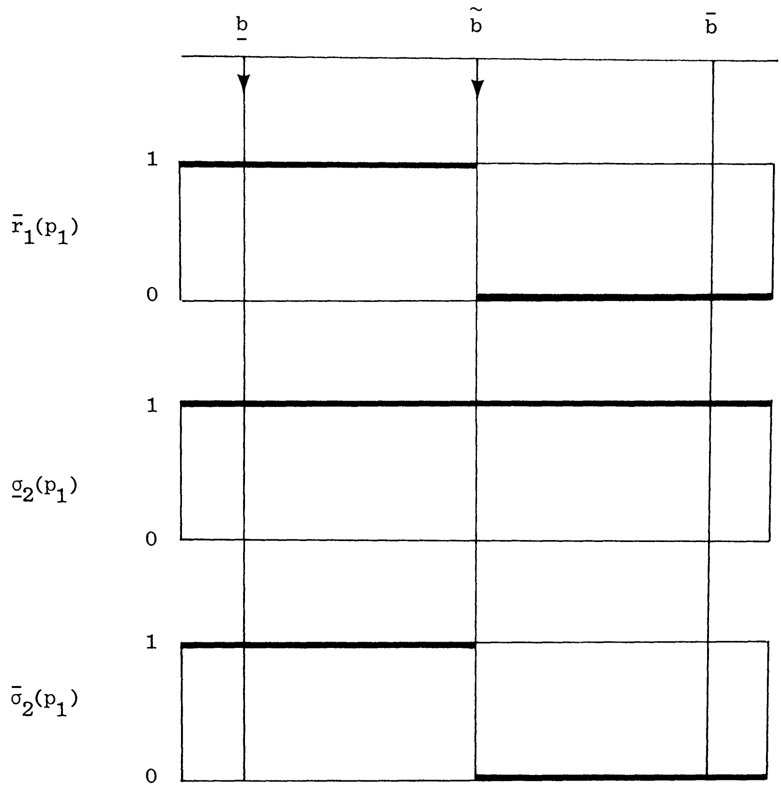

FIGURE 6

One soft seller, one tough seller. Pooling equilibrium

3. It is often presumed in the applied literature on bargaining that "enlarging the contract zone" increases the probability of agreement. We can see that this is not always true. To show this, consider again a set of parameters such that:

$$\pi(\overset{\circ}{b}, \bar{r}) > \max[\pi(\underline{b}), \pi(\tilde{b}), \pi(\bar{b}, 0)]$$

and the corresponding pooling equilibrium at $\overset{\circ}{b}$. Decrease $\bar{s}$ slightly, it enlarges the contract zone. But $\bar{r}$ decreases, and thus first-period disagreement occurs more frequently.

Separating and semi-separating equilibria

We now characterize separating and semi-separating equilibria. We begin with a proposition which will be useful for both.

Proposition 5. *In equilibrium, the only offer which can be made by seller $\underline{s}$ only is $\underline{b}$ or $\tilde{b}$, the only offer which can be made by seller $\bar{s}$ only is $\bar{b}$. If $\bar{b}$ is made by $\bar{s}$ only, then $\bar{r}_1(\bar{b}) < \bar{r}$.*

This proposition follows from Lemmas 8, 9, and 10 in the Appendix.

Separating equilibrium

Proposition 6. *A separating equilibrium exists if and only if the sellers would choose different offers were their valuations known. The equilibrium path is unique up to the choice of $\bar{r}_1(\bar{b})$: in the first period $\underline{s}$ charges $\underline{b}$ or $\tilde{b}$, and $\bar{s}$ charges $\bar{b}$.*

Proof. With complete information seller $\bar{s}$ chooses $\bar{b}$ rather than $\underline{b}$ or $\tilde{b}$ if and only if

$$\pi(\bar{b}, \bar{r}) > \max(\pi(\underline{b}), \pi(\tilde{b})). \tag{1}$$

First we show that inequality (1) is a necessary condition for existence.

$$\text{Call:} \quad \pi[\bar{b}, \bar{r}_1(\bar{b}), \underline{s}] = \tfrac{1}{2}\bar{r}_1(\bar{b})\bar{b} + \tfrac{1}{2}[2 - \bar{r}_1(\bar{b})]\delta_S \underline{b},$$

(seller $\underline{s}$'s payoff when charging $\bar{b}$).

$$\pi[\bar{b}, \bar{r}_1(\bar{b}), \bar{s}] = \tfrac{1}{2}\bar{r}_1(\bar{b})\bar{b} + \tfrac{1}{2}[1 - \bar{r}_1(\bar{b})]\delta_S \bar{b} + \tfrac{1}{2}\delta_S \bar{s}$$

(seller $\bar{s}$'s payoff when charging $\bar{b}$ if $\bar{r}_1(\bar{b}) \leqq \bar{r}$).

By definition of $\bar{r}$:

$$\pi(\bar{b}, \bar{r}, \underline{s}) = \pi(\bar{b}, \bar{r}, \bar{s}) = \pi(\bar{b}, \bar{r})$$

and for

$$0 < \bar{r}_1(\bar{b}) < \bar{r}: \pi(\bar{b}, \bar{r}_1(\bar{b}), s) < \pi(\bar{b}, \bar{r}_1(\bar{b}), \bar{s}).$$

Seller $\underline{s}$ must prefer announcing $\underline{b}$ or $\tilde{b}$ to announcing $\bar{b}$:

$$\max(\pi(\underline{b}), \pi(\tilde{b})) > \pi(\bar{b}, \bar{r}_1(\bar{b}), \underline{s}). \tag{2}$$

Seller $\bar{s}$ must prefer announcing $\bar{b}$ to announcing $\underline{b}$ or $\tilde{b}$:

$$\max(\pi(\underline{b}), \pi(\tilde{b})) < \pi(\bar{b}, \bar{r}_1(\bar{b}), \bar{s}). \tag{3}$$

Thus we have to find $\bar{r}_1(\bar{b})$ satisfying inequalities (2) and (3). This problem is represented in Figure 7.

Notice that $\max(\pi(\underline{b}), \pi(\tilde{b})) > \pi(\bar{b}, 0, \underline{s}) = \delta_S \underline{b}$.

Thus one can satisfy inequalities (2) and (3) for some choice of $\bar{r}_1(\bar{b})$ (actually for a set of such values) if and only if:

$$\max(\pi(\underline{b}), \pi(\tilde{b})) < \pi(\bar{b}, \bar{r})$$

Next we assume that (1) is satisfied. We have to find strategies and conjectures which form a perfect Bayesian equilibrium. Let $(s(p_1), 1 - s(p_1))$ be the buyer's posterior

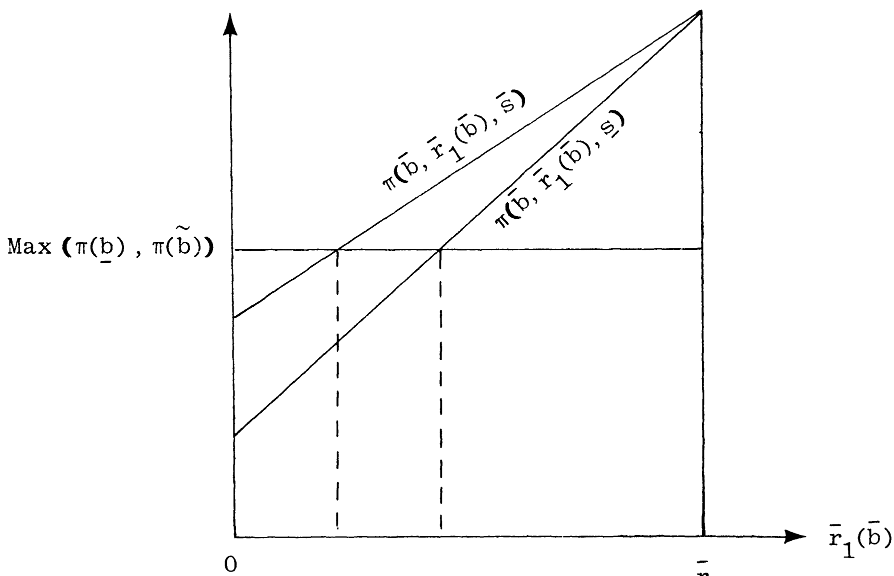

FIGURE 7

about the seller's valuation ($s(p_1)=1$ means that the buyer is convinced that the seller has valuation $\underline{s}$).

Let

$$s(p_1)=\begin{cases} 1 & p_1 \leqq \tilde{b} = \bar{b} - \delta_B(\bar{b}-\underline{b}) \\ \dfrac{\bar{b}-p_1}{\delta_B(\bar{b}-\underline{b})} & \tilde{b} \leqq p_1 \leqq \bar{b} \\ 0 & p_1 \geqq \bar{b}. \end{cases} \tag{4}$$

Note that these conjectures are continuous and "non-increasing".

Let $\bar{r}_1(\bar{b})$ be a probability satisfying the two inequalities (2) and (3). Then the strategies displayed in Figure 8 form a perfect equilibrium relative to the conjectures described in (4):

It is clear that there are other ways of constructing conjectures and strategies supporting the separating equilibrium. Nevertheless in terms of outcomes and payoffs, the separating equilibrium is unique up to the choice of $\bar{r}_1(\bar{b})$ in the eligible open set, i.e. up to the choice of a payoff for seller $\bar{s}$. ||

Remark 1. Seller $\underline{s}$ is not affected by the fact that the buyer does not know his valuation if the equilibrium is a separating equilibrium. On the contrary, even in a separating equilibrium, seller $\bar{s}$ is made worse off compared to the case of complete information about the seller. (To see this, remember that $\bar{r}_1(\bar{b})<\bar{r}$.) Thus there is an adverse selection effect even in a separating equilibrium (as in the insurance model).

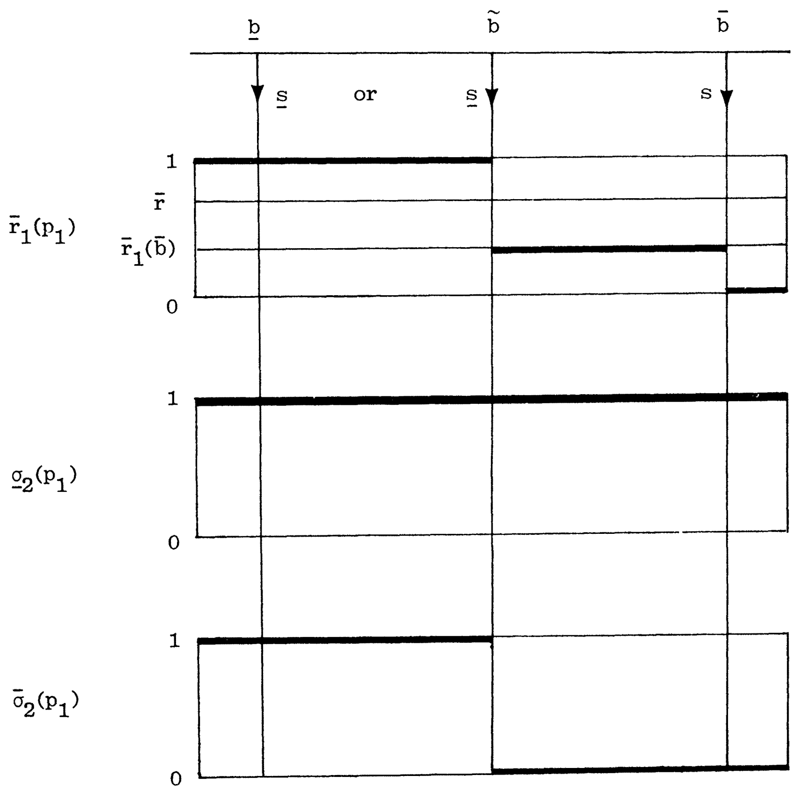

FIGURE 8

One soft seller, one tough seller. Separating equilibrium

Remark 2. An increase in any of the discount factors is favourable to the existence of a separating equilibrium. An increase in the seller's discount factor increases the attractiveness for the tough seller of $\bar{b}$ relative to $\tilde{b}$ and $\underline{b}$; an increase in the buyer's discount factor decreases the seller's payoff when playing $\tilde{b}$.

A decrease in the seller's discount factor can only increase the efficiency of the separating equilibrium, for two reasons: first, the soft seller may switch from $\tilde{b}$ to $\underline{b}$, since $\tilde{b}$ becomes less attractive; second, he is less attracted by $\bar{b}$ so that the set of eligible values for $\bar{r}_1(\bar{b})$ shifts to the right. A decrease in the buyer's discount factor increases the attractiveness of $\tilde{b}$ relative to $\underline{b}$, and thus can only lead to a less efficient outcome.

Existence of a semi-separating equilibrium

From Propositions 2 and 5 a semi-separating equilibrium can be of three types:

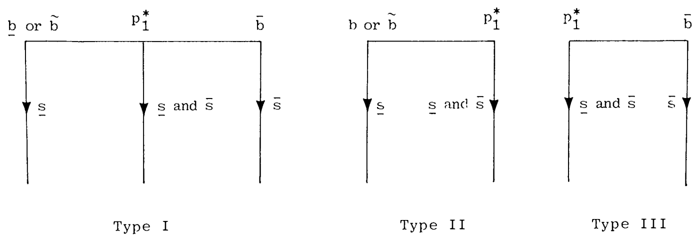

FIGURE 9

The analysis of the different types of semi-separating equilibria is somewhat lengthy, and is performed in the Appendix. The results are actually very simple, and are summarized in Proposition 7 (which encompasses Propositions 11, 12, 13 of the Appendix).

Proposition 7. *A necessary and sufficient condition of existence of a semi-separating equilibrium of type I or II is that* $\pi(\bar{b}, \bar{r}) > \max(\pi(\underline{b}), \pi(\tilde{b}))$, *i.e. a semi-separating equilibrium of type I or II exists if and only if a separating equilibrium exists. This condition is necessary, but not sufficient, for the existence of an equilibrium of type III. Lastly, the common offer* p_1^* *exceeds (is lower than)* $\mathring{b}$ *in a semi-separating equilibrium of type II (of type III).*

Summary of the existence results

We recall the payoffs which play a role in the existence problem:

$$\begin{cases} \pi(\underline{b}) = \underline{b} \\ \pi(\tilde{b}) = \tfrac{1}{2}[\bar{b} - \delta_B(\bar{b} - \underline{b})] + \dfrac{\delta_S}{2}\underline{b} = \tfrac{1}{2}\tilde{b} + \dfrac{\delta_S}{2}\underline{b} \\ \pi(\mathring{b}, \bar{r}) = \tfrac{1}{2}\bar{r}\mathring{b} + \delta_S\left(\dfrac{2-\bar{r}}{2}\right)\underline{b} \quad \text{where } \mathring{b} = \bar{b} - \dfrac{\delta_B}{2}(\bar{b} - \underline{b}) \quad \text{and } \bar{r} = \dfrac{\bar{b} + \bar{s} - 2\underline{b}}{\bar{b} - \underline{b}} \\ \pi(\bar{b}, \bar{r}) = \tfrac{1}{2}\bar{r}\bar{b} + \delta_S\left(\dfrac{2-\bar{r}}{2}\right)\underline{b} \\ \pi(\bar{b}, 0) = \delta_S\dfrac{\bar{b} + \bar{s}}{2} \end{cases}$$

It is clear that $\pi(\bar{b}, \bar{r}) > \pi(\bar{b}, 0)$ and $\pi(\bar{b}, \bar{r}) > \pi(\mathring{b}, \bar{r})$.

We distinguish two cases depending on the relative sizes of $\pi(\bar{b}, 0)$ and $\pi(\mathring{b}, \bar{r})$. The existence of pooling, separating and semi-separating equilibria depends on the comparison of max $(\pi(\underline{b}), \pi(\tilde{b}))$ and $\pi(\bar{b}, \bar{r})$, $\pi(\bar{b}, 0)$, $\pi(\mathring{b}, \bar{r})$, which latter three values can be in two different orders: $\pi(\bar{b}, \bar{r}) > \pi(\bar{b}, 0) > \pi(\mathring{b}, \bar{r})$ or $\pi(\bar{b}, \bar{r}) > \pi(\mathring{b}, \bar{r}) > \pi(\bar{b}, 0)$. The values of max $(\pi(\underline{b}), \pi(\tilde{b}))$ for which the various equilibria exist are displayed for these two cases in figures 10 and 11 respectively.

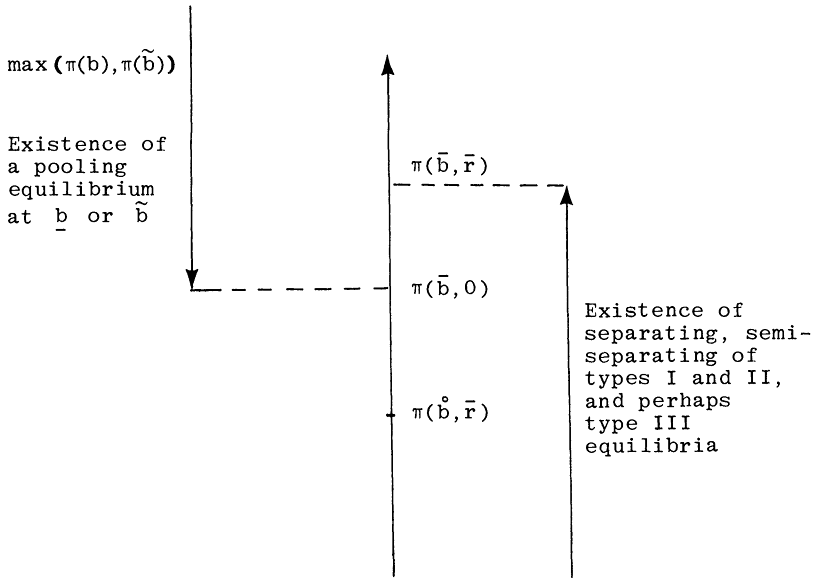

FIGURE 10

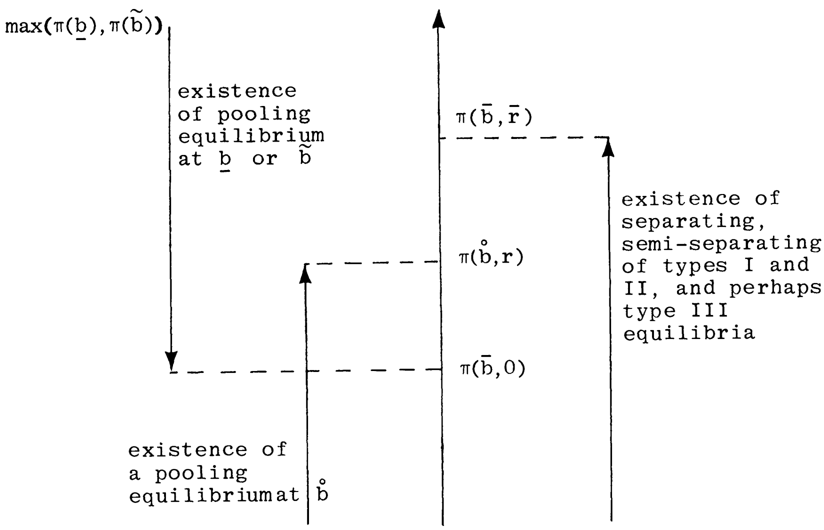

FIGURE 11

(C) When both sellers are tough the analysis and the results are quite similar to that just performed. All the pooling equilibria of proposition 4 can occur, but "less often". A pooling equilibrium can also occur at $p_1 = \bar{b}$, if each seller would have played $\bar{b}$ with complete information. Separating equilibria again only exist when the sellers would have played differently with complete information; as the sellers are now more similar, this condition is harder to satisfy.

Define $\underline{r}$ to be the value of $\bar{r}_1(\cdot)$ which makes $\underline{s}$ indifferent in the second period. $\pi(\bar{b}, \underline{r}, \bar{s})$ and $\pi(\bar{b}, \underline{r}, \underline{s})$ are the returns to $\bar{s}$ and $\underline{s}$ respectively of $p_1 = \bar{b}$ when $\bar{r}_1(\bar{b}) = \underline{r}$.

Proposition 8. *With two tough sellers the following conditions are necessary and sufficient for the existence of the associated pooling equilibria. No other pooling equilibria exist. Consider the five numbers* $\pi(\underline{b})$, $\pi(\tilde{b})$, $\pi(\mathring{b}, \bar{r})$, $\pi(\bar{b}, \underline{r}, \bar{s})$, *and* $\pi(\bar{b}, \underline{r}, \underline{s})$.

(a) *If* $\pi(\underline{b})$ *is the largest, there exists a pooling equilibrium with* $p_1 = \underline{b}$.

(b) *If* $\pi(\tilde{b})$ *is the largest, there exists a pooling equilibrium with* $p_1 = \tilde{b}$.

(c) *If* $\pi(\mathring{b}, \bar{r})$ *is the largest, any* p_1 *in an interval* $[b', \mathring{b}]$ *with* $\tilde{b} < b' < \mathring{b}$ *can arise in a pooling equilibrium.*

(d) *If* $\pi(\bar{b}, \underline{r}, \underline{s}) > \max(\pi(\underline{b}), \pi(\tilde{b}))$, *there exists a pooling equilibrium with* $p_1 = \bar{b}$. *Note that the above cases are* not *disjoint. There exists* no *pooling equilibrium if*

$$\pi(\bar{b}, \underline{r}, \bar{s}) > \max(\pi(\underline{b}), \pi(\tilde{b}), \pi(\mathring{b}, \bar{r})) > \pi(\bar{b}, \underline{r}, \underline{s}).$$

Proposition 9. *With two tough sellers a separating equilibrium exists if and only if the two sellers would choose different first-period prices when the seller's valuation is known, that is if and only if:*

$$\pi(\bar{b}, \bar{r}, \bar{s}) > \max(\pi(\underline{b}), \pi(\tilde{b})) > \pi(\bar{b}, \underline{r}, \underline{s}).$$

Proposition 10. *With two tough sellers, the possible semi-separating equilibria take the same three forms as in Figure 9. Moreover, the relationship between the conditions for separating and semi-separating equilibria are the same as in Proposition 7. Types I and II exist if and only if there exists a separating equilibrium; this condition is necessary but not sufficient for the existence of a semi-separating equilibrium of Type III.*

Proofs of the above three propositions are available from the authors upon request.

5. ALTERNATING OFFERS

The model discussed to date gives the seller quite a bit of power, making both the first- and second-period offer. We have also considered a model with the buyer making the second-period offer. We considered the case in which the buyer has incomplete information about the valuation of the seller, which can take two values, but the valuation of the buyer is common knowledge. We again completely characterized the equilibrium set. (The derivations are available from the authors upon request.)

The main difference is that no strictly separating equilibrium can exist with alternating offers (a price is strictly separating if one of the two potential sellers strictly prefers the offer(s) he makes to this price). A strictly separating equilibrium with alternating offers cannot be strongly inefficient since in equilibrium the buyer knows the valuation of the seller in the second period; and if there is no strong inefficiency, the sellers cannot be separated, since they have the same payoff at each equilibrium offer.

6. THE PROBLEM OF THE CHOICE OF CONJECTURES

In this section, we briefly discuss the arbitrariness of the conjectures. The only constraint we have imposed on them is that they coincide with the outcome of the Bayesian updating whenever the latter is feasible. One might want to impose some more restrictions on the shape of those conjectures. One restriction we mentioned in Section 4 was that the conjectures be non-increasing in p_1. This in some sense says that the monotonicity property which usually characterizes the set of equilibrium offers also holds for offers which do not arise in equilibrium. Another restriction would be to require that the conjectures be continuous in p_1, so that small differences in actions do not produce large differences in the information conveyed. Most of the conjectures we used were continuous. Myerson (1978)'s concept of "proper equilibrium" requires that conjectures be generated by small "trembles" about the optimal strategies, where the likelihood of a tremble depends on its loss of payoff. This interesting approach is not tractable in our model. Another method is to impose restrictions directly on the set of allowable equilibria, with the restrictions on the conjectures left implicit.

Fortunately, our model was sufficiently simple to allow the analysis to proceed without additional restrictions. We therefore avoided them, except to note which equilibria are removed by monoticity constraints.

7. CONCLUSION

This paper has considered the problem of sequential two-person bargaining when the traders have incomplete information about each other. With incomplete information, any offer or rejection of an offer must be regarded as a signal. In a sequential bargaining process this signal can be used to update the prior about the other trader's private information, and thus to adjust subsequent actions. We solved the case in which the private information is the valuation of the trader, and where the bargaining process has two periods. The latter institutional assumption is of course very restrictive, and it would be desirable to develop a solution to the same problem when the length of the bargaining process is *a priori* unlimited, in precisely the same way that Rubinstein solved the two-person bargaining process with complete information (see Sobel–Takahashi (1981) for a start on this extension).

We also assumed that each trader had only two possible valuations for the object. In an earlier version of this paper we solved the case in which the seller's valuation was common knowledge while the buyer's valuation was uniformly distributed. Crampton (1982) has extended that mode to uniform distributions over each trader's valuation.

Our simple model allowed us to consider the effect of the parameters of the bargaining game (valuation, discount factors) on the strategic behaviour of the players, and the efficiency of the equilibrium. In particular we found that some very intuitive results cannot be taken for granted. First, a decrease in the buyer's discount factor may make him better off in spite of the fact that, being more impatient, he becomes more vulnerable to a high demand. We explained this phenomenon by the impossibility of commitment to take given actions (here to accept a "compromising offer") which is required by the concept of perfect equilibrium. Second, increasing the contract zone (e.g., by making the seller more eager to sell) may increase the possibility of disagreement. Third, if the buyer has incomplete information about the seller, the seller may charge a higher price in the second period than in the first; the buyer may nevertheless refuse such a first-period offer since there is the possibility that the seller is soft and will charge less in the second period. Fourth, increasing the number of periods may have surprising welfare effects: it can decrease efficiency even when the one-period game has an inefficient solution (see Section 4, case where the seller is tough). The list could be extended. The moral is that general assertions about the effects of parameter changes on the bargaining process are suspect. They should be tested in the particular bargaining game being discussed.

APPENDIX

Proofs of Proposition 2, of the Lemmas, and characterization of semi-separating equilibria

Proof of Proposition 2.

The expected second-period profit for a seller with valuation s and posterior $(\underline{q}(p_1), \bar{q}(p_1))$, and playing strategy $\sigma_2(p_1)$ is:

$$\pi_2(p_1) = \sigma_2(p_1)\underline{b} + [1-\sigma_2(p_1)][\underline{q}(p_1)s + \bar{q}(p_1)\bar{b}]. \tag{5}$$

We also know that $\underline{q}(p_1) \geqq \frac{1}{2}$.

Consider two offers p_1 and p'_1 $(p_1 < p'_1)$ such that both sellers are indifferent between p_1 and p'_1, given the equilibrium strategies. For any first-period offer, both sellers have the same first-period expected profits, and update their prior about the buyer in the same way, since the buyer's first-period behaviour is allowed to depend only on his information (p_1), and not on the type of the seller, which he does not know. Thus, with obvious notation:

$$\bar{\pi}_2(p_1) - \bar{\pi}_2(p'_1) = \underline{\pi}_2(p_1) - \underline{\pi}_2(p'_1). \tag{6}$$

In the proof we shall use the following lemmas:

Lemma 1. *Either* $\bar{q}(p_1) < \bar{q}(p'_1)$ *or* $\bar{q}(p_1) = \bar{q}(p'_1) = \frac{1}{2}$.

Proof. Assume $\bar{q}(p_1) \geqq \bar{q}(p'_1)$: This would mean that, when asking p'_1 rather than p_1, the seller (whatever this type) does not reduce the probability of acceptance by buyer $\bar{b}$. Thus except if buyer $\bar{b}$ refuses p_1 and p'_1 with probability one,

$$\bar{r}_1(p_1) = \bar{r}_1(p'_1) = 0$$

or

$$\bar{q}(p_1) = \bar{q}(p'_1) = \tfrac{1}{2},$$

the seller would not be indifferent between the two offers. ||

Lemma 2. *Buyer* $\bar{b}$ *accepts any offer* p_1 *less than or equal to* $\tilde{b}$ $(\bar{r}_1(p_1) = 1$ *for* $p_1 \leqq \tilde{b})$.

Proof. The best offer that the buyer can possibly face in the second period is $\underline{b}$; even if this is the case, due to discounting, it pays for buyer $\bar{b}$ to accept any p_1 less than $\tilde{b}$. For $p_1 = \tilde{b}$, buyer $\bar{b}$ is indifferent between accepting and refusing, if the second offer actually is $\underline{b}$. There is no loss of generality involved in assuming that $\bar{r}_1(\tilde{b}) = 1$. If $\bar{r}_1(\tilde{b}) < \bar{r}$, the seller can always charge $(\tilde{b} - \varepsilon)$ for ε sufficiently small. ||

Proof of Proposition 2. We have seen that we can assume:

$$\underline{q}(p_1) > \underline{q}(p'_1) \geqq \tfrac{1}{2}.$$

Hence seller $\underline{s}$, compared to seller $\bar{s}$, is made relatively worse off when changing from p'_1 to p_1 as far as the second-period opportunities are concerned (disagreement is more costly for seller $\underline{s}$, so that, *ceteris paribus*, he is more affected than seller $\bar{s}$ by an increase in the probability of facing buyer $\underline{b}$). Using (5) and (6), it is easy to see that the equality in equation (6) can be maintained only if, in equilibrium: $\underline{\sigma}_2(p_1) = \bar{\sigma}_2(p_1) = \underline{\sigma}_2(p'_1) = \bar{\sigma}_2(p'_1) = 1$, that is, only if both sellers' best strategy in the second period is to play soft in both cases. This in turn implies that $p_1 < \tilde{b}$ and $p'_1 \leqq \tilde{b}$, and thus $p_1 = \underline{b}$ and $p'_1 = \tilde{b}$. The last stage of the proof consists in noticing that the equality $\pi(\underline{b}) = \pi(\tilde{b})$ in general will not hold; more technically, one can easily show that the set of parameters for which $(\pi(\underline{b})=\pi(\tilde{b}))$ holds is of measure zero in the space of parameters. ||

Proofs of the Lemmas mentioned in the text.

Proof of Lemma 3. We already know that: $\forall p_1 : \underline{q}(p_1) \geqq \frac{1}{2}$. Since seller $\underline{s}$ already plays soft in the second period if $\underline{q}=\frac{1}{2}$, *a fortiori*, he will play soft if the probability of disagreement when playing tough is higher. Therefore any offer such that $\bar{r}_1(p_1)=0$ is dominated by the offer $\tilde{b}$ from the point of view of the seller $\underline{s}$. ||

Proof of Lemma 4. According to Bayes rule, the posterior of the buyer about the valuation of the seller is the same as the priors in a pooling equilibrium: $(\frac{1}{2}, \frac{1}{2})$. Thus from Lemma 3(i) the buyer has a probability of at least $\frac{1}{2}$ of facing a soft offer in the second period, if he refuses p_1 in the first period. Thus a necessary condition for the buyer to accept p_1 (whatever his type) is:

$$b-p_1 \geqq \tfrac{1}{2}\delta_B(b-\underline{b}).$$

Thus if $p_1 > \mathring{b}$, the buyer, whatever his type, refuses with probability one. From Lemma 3(ii), seller $\underline{s}$ does not charge p_1 in the first period, a contradiction. ||

Lemma 5. *Either $\bar{r}_1(p_1) \geqq \bar{r}$ and/or $p_1 = \mathring{b}$.*

Proof. If $\bar{r}_1(p_1) < \bar{r}$, seller $\bar{s}$ plays tough in the second period: $\bar{\sigma}_2(p_1)=0$; since $\underline{\sigma}_2(p_1)=1$, buyer $\underline{b}$ accepts if $p_1 < \mathring{b}$, a contradiction. If $p_1 > \mathring{b}$, $\bar{r}_1(p_1)=0$, which contradicts Lemma 3(ii). Thus if $\bar{r}_1(p_1) < \bar{r}$, $p_1 = \mathring{b}$. ||

Lemma 6. *The only possible pooling equilibria with $\bar{r}_1(p_1) > \bar{r}$ are $p_1=\underline{b}$ and $p_1=\tilde{b}$ [and $\bar{r}_1(\underline{b})=\bar{r}_1(\tilde{b})=1$].*

Proof. If $\bar{r}_1(p_1) > \bar{r}$, $\bar{\sigma}_2(p_1)=\underline{\sigma}_2(p_1)=1$, and thus buyer $\bar{b}$ accepts p_1 if and only if p_1 is less than $\tilde{b}$. ||

Lemma 7. *p_1 such that $\tilde{b} < p_1 < \mathring{b}$ can be a pooling equilibrium only if $\bar{r}_1(p_1)=\bar{r}$.*

(This follows from Lemmas 5 and 6.)

Lemma 8. *If the offer p_1 is made by seller $\underline{s}$ only, $p_1=\underline{b}$ or $p_1=\tilde{b}$.*

Proof. The buyer is convinced that the seller is $\underline{s}$, and thus will play $\underline{b}$ in the second period. If $p_1 > \tilde{b}$, buyer $\bar{b}$ refuses the offer. But by Lemma 3(ii), seller $\underline{s}$ would not charge p_1, which will be refused anyway. Thus p_1 can be only one of the two values $\underline{b}$ and $\tilde{b}$. ||

Lemma 9. *If the offer p_1 is made by seller $\bar{s}$ only, then $\bar{r}_1(p_1) < \bar{r}$.*

Proof. If $\bar{r}_1(p_1) \geqq \bar{r}$, seller $\bar{s}$ might as well play soft. Thus both sellers would have the same payoff at p_1; this means that both sellers would be indifferent between p_1 and seller $\underline{s}$'s offer ($\underline{b}$ or $\tilde{b}$ from Lemma 8). In the proof of Proposition 2, we have seen that this may be possible only if $\pi(\underline{b})=\pi(\tilde{b})$, which is generically untrue. (If $\pi(\underline{b})=\pi(\tilde{b})$, it may be possible to have a separating equilibrium, one seller charging $\underline{b}$ and the other $\tilde{b}$.) ||

Lemma 10. *If the offer p_1 is made by seller $\bar{s}$ only, $p_1=\bar{b}$.*

Proof. By Lemma 9: $\bar{r}_1(p_1)<\bar{r}$. This implies that seller $\bar{s}$ plays tough in the second period, $\bar{\sigma}_2(p_1)=0$. But remember that buyer $\bar{b}$, when observing p_1 is convinced that the seller has valuation $\bar{s}$; thus he is willing to accept any offer p_1 less than $\bar{b}(\bar{r}_1(p_1)=1$ if $p_1<\bar{b})$, a contradiction. ‖

Existence of Semi-Separating Equilibria.

Define

$$\pi(p_1, \bar{r}_1(p_1), \underline{s})=\tfrac{1}{2}\bar{r}_1(p_1)p_1+\frac{\delta_S}{2}(2-\bar{r}_1(p_1))\underline{b}$$

and

$$\pi(p_1, \bar{r}_1(p_1), \bar{s})=\tfrac{1}{2}\bar{r}_1(p_1)p_1+\frac{\delta_S}{2}(1-\bar{r}_1(p_1))\bar{b}+\frac{\delta_S}{2}\bar{s}.$$

These are the expected profits of the soft and tough sellers when they make an offer p_1 which is accepted with probability $\bar{r}_1(p_1)\leqq\bar{r}$ by buyer $\bar{b}$.

We first consider the existence of a semi-separating equilibrium of Type I.

Semi-separating equilibria of type I

A necessary condition for the existence of an equilibrium of this type is the existence of three numbers $(p_1^*, \bar{r}_1(p_1^*), \bar{r}_1(\bar{b}))$ such that

$$\tilde{b}<p_1^*<\bar{b} \tag{7}$$

$$0<\bar{r}_1(p_1^*)<\bar{r} \tag{8}$$

$$\max(\pi(\underline{b}), \pi(\tilde{b}))=\pi(p_1^*, \bar{r}_1(p_1^*), \underline{s}) \tag{9}$$

$$\pi(\bar{b}, \bar{r}_1(\bar{b}), \bar{s})=\pi(p_1^*, \bar{r}_1(p_1^*), \bar{s}). \tag{10}$$

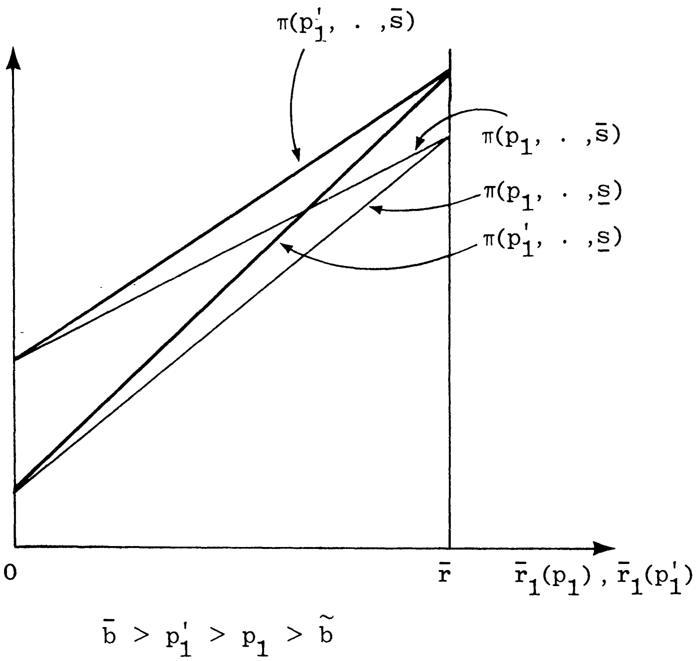

FIGURE 12

Note that (10) implies that $\bar{r}_1(p_1^*) > \bar{r}_1(\bar{b})$ so that seller $\underline{s}$ would not be tempted to charge $\bar{b}$ in the first period. Moreover, (8) and (9) imply that seller $\bar{s}$ would not want to charge $\underline{b}$ or $\tilde{b}$ (since his payoff would be equal to the one he would have by charging p_1^* in the first period and playing suboptimally $\underline{b}$ in the second period).

For each $p_1 \in (\tilde{b}, \bar{b})$, one can draw the graph of $\pi(p_1, \cdot, \underline{s})$ and $\pi(p_1, \cdot, \bar{s})$ between 0 and $\bar{r}$, as in Figure 12.

From (9) and (10), a necessary condition for the existence of a semi-separating equilibrium of type I is:

$$\pi(\bar{b}, \bar{r}) = \pi(\bar{b}, \bar{r}, \bar{s}) > \max(\pi(\underline{b}), \pi(\tilde{b})). \tag{11}$$

We now show that if (11) is satisfied, there exists a semi-separating equilibrium of type I; in other words, the existence condition is the same as for a separating equilibrium.

The existence proof is sketched by means of a figure. First represent $\pi(\bar{b}, \bar{r}, \bar{s})$ and $\max(\pi(\underline{b}), \pi(\tilde{b}))$ (see Figure 13). Choose p_1^* arbitrarily close to $\bar{b}$ (but still less than $\bar{b}$). Draw $\pi(p_1^*, \cdot, \underline{s})$ and $\pi(p_1^*, \cdot, \bar{s})$ [we know that $\pi(p_1^*, 0, \underline{s}) < \max(\pi(\underline{b}), \pi(\tilde{b}))$]. The intersection of $\pi(p_1^*, \cdot, \underline{s})$ and $\max(\pi(\underline{b}), \pi(\tilde{b}))$ defines a probability $\bar{r}_1(p_1^*)$ such that $0 < \bar{r}_1(p_1^*) < \bar{r}$. Note that $\bar{r}_1(p_1^*)$ is bounded away from zero because $\bar{r}_1(p_1^*)$ is for any choice of p_1^* higher than the value given by the intersection of $\pi(\bar{b}, \cdot, \underline{s})$ and $\max(\pi(\underline{b}), \pi(\tilde{b}))$.

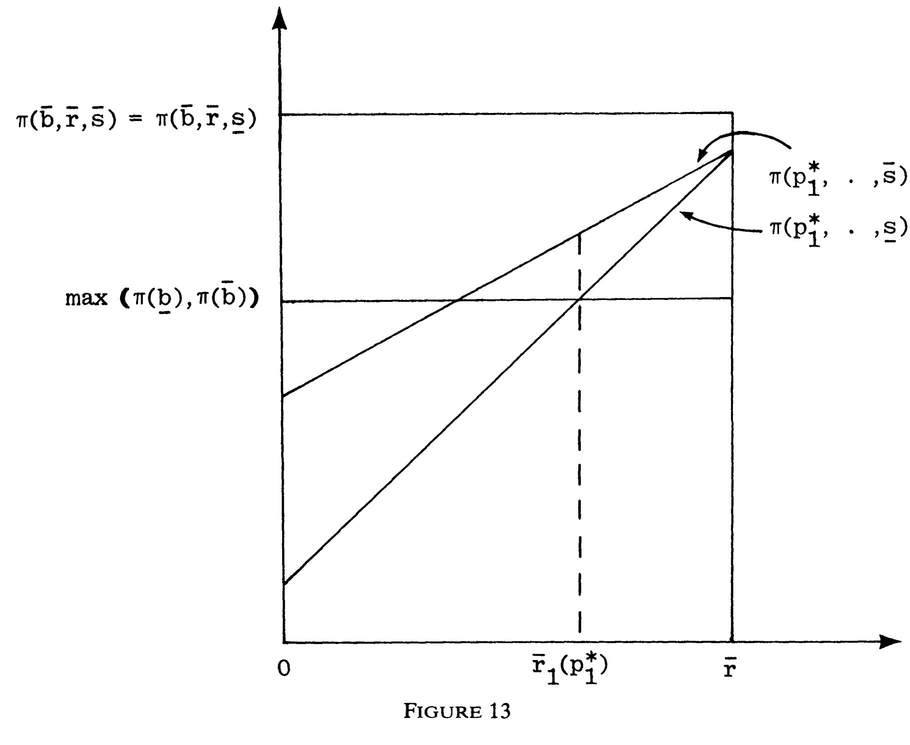

FIGURE 13

The last step consists in finding a $\bar{r}_1(\bar{b})$ satisfying equation (10); if we have been careful to choose p_1^* close enough to $\bar{b}$,

$$\pi(\bar{b}, 0, \bar{s}) < \pi(p_1^*, \bar{r}_1(p_1^*), \bar{s}) < \pi(\bar{b}, \bar{r}, \bar{s})$$

then by the intermediate value theorem, it is possible to find $\bar{r}_1(\bar{b})$ satisfying (10).

What we have shown until now is that, if (and only if) (1) holds, one can find $(p_1^*, \bar{r}_1(p_1), \bar{r}_1(\bar{b}))$ satisfying the necessary conditions. We still have to prove that we can actually construct conjectures and strategies giving a semi-separating equilibrium of Type I.

The conjectures we choose are the same as the ones described in (4). Seller $\underline{s}$ randomizes between ($\underline{b}$ or $\tilde{b}$) (depending on the larger of $\pi(\underline{b})$ and $\pi(\tilde{b})$) and p_1^*; seller $\bar{s}$ randomizes between p_1^* and $\bar{b}$; the random strategies are chosen such that the posterior of the buyer at p_1^* computed by simple Bayesian updating from those randomized strategies coincides with $s(p_1^*)$ (there is an infinity of ways of doing this). The equilibrium strategies corresponding to the conjectures are illustrated in Figure 14.

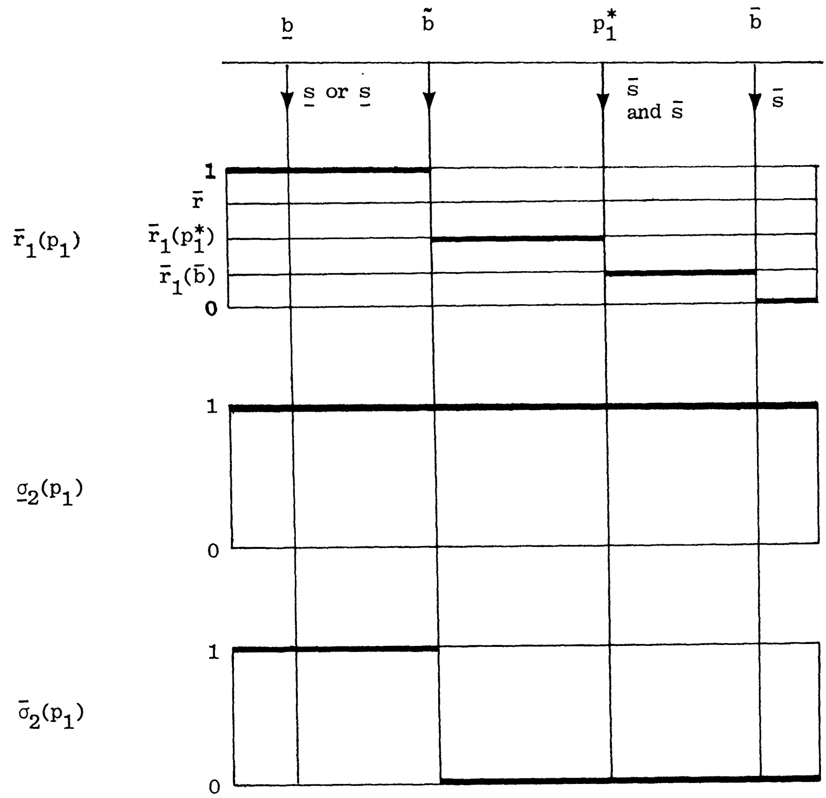

FIGURE 14

One soft seller, one tough seller. Semi-separating equilibrium

Which offers p_1^* can arise as the common offer of a semi-separating equilibrium of Type I depends on $\pi(\bar{b}, 0, \bar{s})$. If $\pi(\bar{b}, 0, \bar{s}) < \max(\pi(\underline{b}), \pi(\tilde{b}))$, the smallest p_1^* will be given by the equation $\max(\pi(\underline{b}), \pi(\tilde{b})) = \pi(p_1^*, \bar{r}, \underline{s})$. If not, it will be given by the solution of the system [(9), (10)] where $\bar{r}_1(\bar{b}) = 0$. [This is then a system of two equations with two unknowns: p_1^* and $\bar{r}_1(p_1^*)$.]

Proposition 11. *A semi-separating equilibrium of Type I exists if and only if condition* (1): $\pi(\bar{b}, \bar{r}) > \max(\pi(\underline{b}), \pi(\tilde{b}))$ *is fulfilled, that is, if and only if a separating equilibrium exists.*

Semi-separating equilibrium of type II

A necessary condition for the existence of a semi-separating equilibrium of Type II is the existence of (p_1^*, $\bar{r}_1(p_1^*)$, $\bar{r}_1(\bar{b})$) satisfying (7), (8), (9), (10) except that (10) is now an inequality:

$$\pi(\bar{b}, \bar{r}(\bar{b}), \bar{s}) \leqq \pi(p_1^*, \bar{r}_1(p_1^*), \bar{s}). \tag{10'}$$

It is then clear that the existence condition is the same as for a semi-separating equilibrium of Type I. Indeed there is now one more constraint, coming from the fact that the Bayesian updating at p_1^* must give a posterior probability of facing seller $\bar{s}$ bigger than $\frac{1}{2}$ (before this posterior probability was arbitrary between 0 and 1). Notice that this implies that: $(\bar{r}_1(p_1^*) < \bar{r} \Rightarrow \bar{\sigma}_2(p_1^*) = 0)$, and thus that $p_1^* > \mathring{b}$. The previous proof is still valid with this constraint on p_1^* since we showed existence by taking p_1^* arbitrarily close to $\mathring{b}$. We can keep the conjectures given by (4), since they are such that $s(p_1) < \frac{1}{2}$ for $p_1 > \mathring{b}$. Hence we can find a random strategy for seller $\underline{s}$ between ($\underline{b}$ or $\tilde{b}$) and p_1^* such that the posterior of the buyer is $s(p_1^*)$ (where p_1^* is as in the proof of Proposition 9). The equilibrium strategies associated with these conjectures are given in Figure 15.

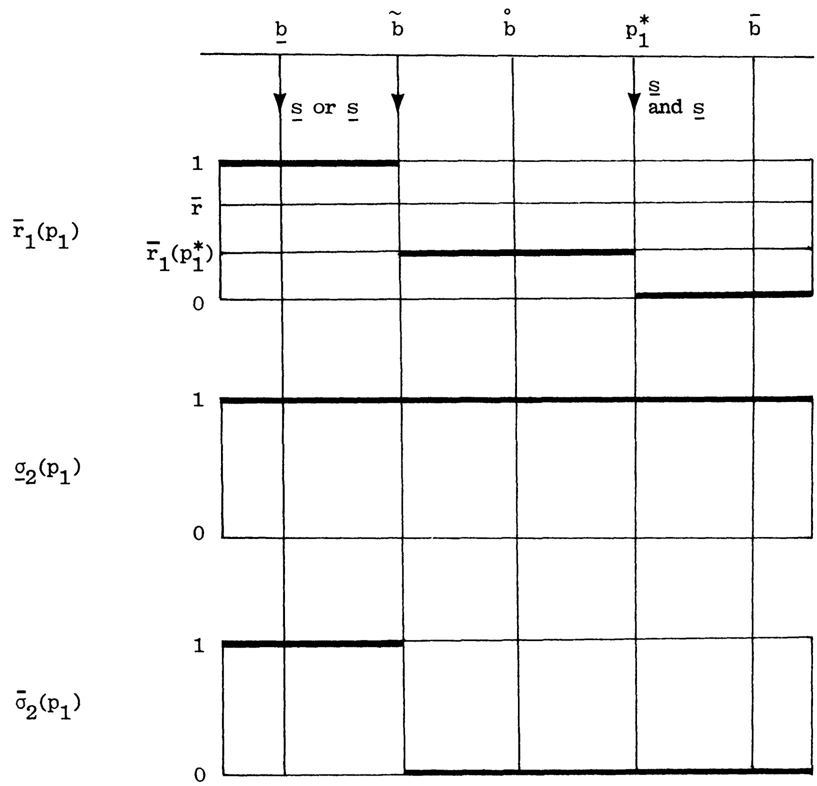

FIGURE 15

One soft seller, one tough seller. Semi-separating equilibrium

Proposition 12. *A semi-separating equilibrium of Type II exists if and only if condition* (1): $\pi(\bar{b}, \bar{r}) > \max(\pi(\underline{b}), \pi(\tilde{b}))$ *is fulfilled.*

Semi-separating equilibria of type III

A necessary condition for the existence of a semi-separating equilibrium of Type III is the existence of $(p_1^*, \bar{r}_1(p_1^*), \bar{r}_1(\bar{b}))$ satisfying (7), (8), (9), (10) except that (9) is now replaced by an inequality:

$$\max(\pi(\underline{b}), \pi(\tilde{b})) \leqq \pi(p_1^*, \bar{r}_1(p_1^*), \underline{s}). \tag{9'}$$

Once again this leads to the same necessary condition (1) for the existence of such an equilibrium. But remember that we have a constraint on the buyer's posterior. This posterior must put a higher weight on seller $\underline{s}$ than on seller $\bar{s}$. This in turn implies that $p_1^* < \mathring{b}$ (otherwise buyer $\bar{b}$ would refuse p_1^* with certainty which contradicts Lemma 3).

We already notice that the proof of existence used in the two previous cases does not extend. Actually, a semi-separating equilibrium of Type III may fail to exist even if condition (1) is fulfilled: If $\max(\pi(\underline{b}), \pi(\tilde{b})) \geqq \pi(\mathring{b}, \bar{r}, \underline{s})$, we will not be able to find $(p_1^*, \bar{r}_1(p_1^*))$ satisfying (8), (9′) and the condition $\tilde{b} < p_1^* < \mathring{b}$.

Proposition 13. *A necessary condition for the existence of semi-separating equilibrium of Type III is that condition* (1) *be fulfilled. Condition* (1) *is not sufficient: An equilibrium of this type may fail to exist even if* (1) *holds.*

First version received April 1981; *final version accepted September* 1982 (*Eds.*)

We would like to thank Franklin Fisher, David Levine, Eric Maskin, Paul Ruud, Joel Sobel and the two referees for helpful comments. Research support from the NSF and Sloan Foundation Grant No. 89729 is gratefully acknowledged.

NOTES

1. For cooperative bargaining and the axiomatic approach, see e.g. Roth (1979).

2. We might have assumed that costs of bargaining were fixed costs per period C_S, C_B; but we focused on the discounting case because, for example, given that buyers are free not to play, the only equilibrium with fixed costs and a single valuation for the seller is a trivial one in which the seller's first-period offer is accepted by all buyers who choose to play. We thank Eric Maskin for reminding us of the individual-rationality constraint.

3. Thus we assume that there is no flow benefit for the seller. This is certainly the case if the seller is an intermediary (shopkeeper, agent, or dealer) or if the good can not be "consumed" or used by the owner during the bargaining process. The alternative, which could be more appropriate for goods such as land, is to assume that the seller derives utility from the consumption of the object while bargaining.

4. If the buyer with valuation b refuses $p_2 = b$ with some probability the seller could shade his offer a bit so that it is accepted with probability one. In the case of a discrete distribution of buyer valuations, it will be in the interest of the seller to do so, while with a continuum of buyers $r_2(b, b)$ is unimportant as each buyer has measure zero.

5. If the buyer and seller do not discount the future, $(\delta_S = \delta_B = 1)$, then $\pi(\bar{b}, \bar{r}) > \pi(\tilde{b}) = \pi(\underline{b})$. If the seller is myopic $(\delta_S = 0)$, then he announces $\bar{b}$ or $\tilde{b}$ as $2\underline{b} \gtrless \tilde{b} = (1-\delta_B)\bar{b} + \delta_B\underline{b}$.

6. Kreps–Wilson (1982*a*) and Milgrom–Roberts (1982*a*) encountered the same problem in their incomplete information solutions to Selten's chain store paradox (see also Milgrom–Roberts' (1982*b*) paper on limit pricing).

7. We shall sometimes use the word "conjecture" for "posterior" in general; Bayesian equilibrium requires that these conjectures be derived from Bayes rule, when applicable.

8. Whenever seller $\bar{s}$ randomizes in the second period, one may as well assume as far as his payoff is concerned, that he plays $\underline{b}$ in the second period. More generally whenever $\bar{r}_1(p_1) > \bar{r}$, the two sellers expect the same payoff from charging p_1 in the first period.

9. Consider an offer $t \in (p_1^*, \mathring{b})$. If $\bar{r}_1(t) < \bar{r}$, the tough seller plays tough in the second period, and, since $t < \mathring{b}$ and $s(t) \leqq \frac{1}{2}$, buyer $\bar{b}$ accepts. Thus $\bar{r}_1(t) \geqq \bar{r}$, and p_1^* cannot be an equilibrium.

10. Assume $\bar{r}_1(\mathring{b}) < \bar{r}$. Then the tough seller plays tough in the second period, and since $s(\mathring{b}) \leqq \frac{1}{2}$, buyer $\bar{b}$ accepts the offer $\mathring{b}$. Thus $\bar{r}_1(\mathring{b}) \geqq \bar{r}$, and the seller prefers to offer $\mathring{b}$ in the first period rather than $\underline{b}$ or $\tilde{b}$. We thank Joel Sobel for pointing out an error in our previous treatment of this case.

11. $\pi(\underline{b}) = \underline{b} = \pi(\tilde{b}) < \pi(\mathring{b}, \bar{r}) = [\underline{b} + \frac{1}{2}(\bar{b} + \bar{s})] < \pi(\bar{b}, 0) = (\bar{b} + \bar{s})/2$.

REFERENCES

BINMORE, K. (1980), "Nash Bargaining Theory: II" (London School of Economics I.C.E.R.D.).

CHATTERJEE, K. and SAMUELSON, W. (1980), "The Simple Economics of Bargaining" (mimeo, Pennsylvania State University).

CRAMPTON, P. (1982), "Bargaining with Incomplete Information: A Two-Period Model with a Continuum of Buyers and Sellers" (Stanford University Graduate School of Business R.P. 652).

CRAWFORD, V. (1981), "A Theory of Disagreement in Bargaining", *Econometrica*, **50**, 607–638.

FARBER, H. and KATZ, H. (1979), "Interest Arbitration, Outcomes, and the Incentives to Bargain", *Industrial and Labor Relations Review*, **33**, 55–63.

HARRIS, M. and TOWNSEND, R. (1981), "Resource Allocation under Asymmetric Information", *Econometrica*, **49**, 33–64.

HARSANYI, J. (1967–68), "Games with Incomplete Information Played by Bayesian Players, I–III", *Management Science*, **14**, 159–183, 320–334, 486–501.

KREPS, D. and WILSON, R. (1982*a*), "Reputation and Imperfect Information", *Journal of Economic Theory*, **27**, 253–279.

KREPS, D. and WILSON, R. (1982*b*), "Sequential Equilibria", *Econometrica*, **50**, 863–894.

McLENNAN, A. (1981), "A General Non-Cooperative Theory of Bargaining" (University of Toronto, mimeo).

MILGROM, P. and ROBERTS, J. (1982*a*), "Predation, Reputation and Entry Deterrence", *Journal of Economic Theory*, **27**, 280–317.

MILGROM, P. and ROBERTS, J. (1982*b*), "Limit Pricing and Entry under Incomplete Information", *Econometrica*, **50**, 443–460.

MYERSON, R. (1978), "Refinements of the Nash Equilibrium Concept", *International Journal of Game Theory*, **7**, 73–80.

RILEY, J. and ZECKHAUSER, R. (1980), "Optimal Selling Strategies: When to Haggle, When to Hold Firm" (D.P. No. 88D, Kennedy School, Harvard).

ROTH, A. (1979) *Axiomatic Models of Bargaining* (Berlin: Springer Verlag).

ROTHSCHILD, M. and STIGLITZ, J. (1976), "Equilibrium in Competitive Insurance Markets: An Essay in the Economics of Imperfect Information", *Quarterly Journal of Economics*, **90**, 629–649.

RUBINSTEIN, A. (1982), "Perfect Equilibrium in a Bargaining Model", *Econometrica*, **50**, 97–110.

SELTEN, R. (1965), "Spieltheoretische Behandlung eines Oligopolmodells mit Nachfrageträgheit", *Zeitschrift für die gesamte Staatswissenschaft*, **12**, 301–324.

SOBEL, J. and TAKAHASHI, I. (1981), "A Multi-Stage Model of Bargaining" (D.P. 80-25*, University of California at San Diego).

STAHL, I. (1972) *Bargaining Theory* (The Economic Research Institute, Stockholm.)

STOKEY, N. (1979), "Intertemporal Price Discrimination", *Quarterly Journal of Economics*, **93**, 355–371.

WILSON, C. (1977), "A Model of Insurance Markets with Incomplete Information", *Journal of Economic Theory*, **16**, 167–207.