Reinforcement Learning in the Double Auction: Literature Review

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1962: An Experimental Study of Competitive Market Behavior (Smith)

(1) Context and Motivation. This paper tests how market prices and quantities converge to competitive equilibrium in a classroom experimental market. The impetus is to see whether actual trading outcomes approximate classical supply-demand predictions under controlled conditions. By allowing buyers and sellers to trade repeatedly and observe each other's bids/asks, the author aims to shed light on market behavior when participants act strategically with limited information.

(2) Relevant Literature. Earlier experiments (notably Chamberlin's) investigated imperfect market settings but offered less clarity on processes that foster or hinder convergence to equilibrium. Smith's experiments build on these, adopting a multi-period design where private reservation values or costs remain constant over multiple trading rounds. This design attempts to capture salient features of organized exchange markets in a simplified laboratory setting.

(3) Main Hypothesis and Research Questions. - Does iterative bidding under partial information lead to prices near the competitive equilibrium predicted by standard supply-demand theory? - How quickly do market prices converge when participants learn from repeated rounds? - Do institutional factors (e.g., only sellers quoting prices) alter or delay convergence? - Is the Walrasian model or another dynamic adjustment rule more consistent with observed price paths?

(4) Experimental Design. Subjects split into buyers and sellers, each assigned a private reservation price (maximum willingness-to-pay for buyers or minimum willingness-to-sell for sellers). Each period allows public bids/asks; a contract is formed whenever a standing bid meets a standing ask. Buyers and sellers "drop out" for that period once they trade a single unit. This continues across multiple rounds with static supply and demand parameters, allowing subjects to gain experience. In some variations, only sellers post offers, or participants can trade multiple units, or supply/demand shifts mid-experiment.

(5) Methodology (Models). Several formal dynamic hypotheses are evaluated: - *Walrasian Adjustment*: Prices move according to excess demand at current quotes. - *Excess-Rent Adjustment*: Price changes relate to the combined "rents" (surplus) above/below the equilibrium level. - *Modified Versions*: Added terms allow potential biases from differences in buyer vs. seller surplus at equilibrium.

- **Result 1:** Most sessions showed strong convergence to competitive equilibrium, even with relatively small numbers of traders.
- **Result 2:** One-sided offers (only sellers quoting prices) led to weaker or unstable convergence, typically favoring buyers.
- **Result 3:** Some overshooting occurred immediately after large demand/supply shifts, revealing transient price volatility.
- **Result 4:** Excess-rent models often fit price dynamics better than simpler Walrasian formulations.
- **Result 5:** Allowing multiple trades per participant per round speeds learning and convergence.

(6) Main Takeaways. Repeated auction-style trading with public information and no collusion tends to yield outcomes close to standard supply-demand intersections. Institutions matter, though: if participants cannot both post bids and offers, the process can fail to reach or can significantly delay convergence. Empirically, "excess rent" explanations provide a compelling view of how large potential surpluses drive bargaining adjustments to equilibria.

(7) Future Directions. More refined experiments with real monetary incentives, dynamic cost/value updates, and different institutional rules can further clarify how well the "excess-rent" approach generalizes. Multi-unit and continuous-time designs might illuminate real-world complexities (e.g., speculation, inventory carryover). The link between individual bounded rationality and robust market-level convergence remains a core target for additional study.

1982: A Theory of Price Formation and Exchange in Oral Auctions (Wilson)

(1) Context and Motivation. Robert Wilson's *On Equilibria of Bid-Ask Markets* (Technical Report No. 452, 1984) tackles how decentralized market-mediated allocations can rapidly converge to competitive outcomes, particularly in oral double-auction (DA) markets. Building on Kenneth Arrow's demonstration that Walrasian models achieve efficient resource allocation under complete markets, Wilson focuses on oral auctions as a prime example of a market institution where buyers publicly bid and sellers publicly ask. Experimental evidence (e.g. Smith, Plott) shows that DA markets converge quickly to near-competitive prices, even with few traders and incomplete information. Wilson's aim is to extend insights from bilateral bargaining (e.g. Cramton, Rubinstein) into a multilateral setting to show how "endogenous impatience" among many buyers/sellers can yield near-competitive prices without exogenous discount rates.

(2) Relevant Literature. Classical competitive equilibrium theory (Arrow, Debreu) posits universal price-taking in complete markets, guaranteeing efficient allocations but leaving open how price formation transpires in real time. Empirical work by Smith (1962) and others established that in oral double auctions, with bidders publicly quoting prices, markets converge quickly and extract most of the available surplus. Simultaneously, game-theoretic research (Rubinstein on bilateral bargaining, Milgrom-Weber on auctions) explores price formation under incomplete information. Wilson's paper merges these threads: how can we reconcile bilateral-bargaining insights with the many-participant structure of double auctions to model the path to competitive outcomes?

(3) Main Hypothesis and Questions.

- **Hypothesis:** In DA markets, competitive pressure from multiple buyers and sellers endogenously induces "impatience" to trade, speeding convergence to near-Walrasian prices even without exogenous discounting.
- **Key Questions:**
    1. How do bilateral bargaining concepts (e.g. credible delay, signaling) generalize to multi-seller, multi-buyer DA environments?
    2. Do "sequential offers" in DA contexts mimic auction-like dynamics (English/Dutch) that approximate competitive equilibrium allocations?
    3. Under what conditions do untraded units or "failed" transactions remain negligible, so that most surplus is captured?

(4) Experimental Setting. While Wilson's paper is theoretical, it is motivated by laboratory double-auction experiments (Plott, Smith). Typically, participants are assigned roles as buyers (with induced redemption values) or sellers (with induced costs) for a single unit. Over a limited trading session, public bids and asks appear; any bid meeting or exceeding an ask (or vice versa) finalizes a trade at that price. Observed outcomes exhibit:

- Rapid convergence of average transaction prices toward the theoretical "intersection" of supply and demand.
- High efficiency, with most gains from trade realized.
- Bidding/asking strategies evolving over time, though the precise mechanism was not fully formalized prior to these models.

(5) Two Models (in Spirit). Wilson's analysis extends from bilateral bargaining to multilateral double auctions:

- **(a) Bilateral Bargaining Foundations:** Cramton (1984) showed that in a buyer-seller pair with private reservation values, discount factors create "delay" used as a signaling device. Wilson strips away exogenous discount rates, positing that competition itself acts as the "impatience" driver.
- **(b) Multi-Party DA Extension:** He proposes that each trader's risk of losing a trade to competitors replicates the role of discounting. Sellers/buyers make "serious" offers over time, with each posted quote revealing partial information and triggering updated beliefs by others. Eventually, each match (buyer-seller pair) trades at a price determined by rival competition, approximating the Walrasian outcome as the number of traders grows.

(6) Main Findings. Although the paper focuses on constructing partial rather than complete equilibria, Wilson's informal and formal arguments conclude:

- **Endogenous impatience:** The fear that another agent will undercut a seller or outbid a buyer induces them to "move quickly," generating rapid price movements.
- **Dutch/English Auction Analogy:** After the first serious quote, the model effectively reduces to descending or ascending auctions. Continuation values closely track buyer/seller valuations, so final trade prices converge near the underlying cost/value gap.
- **Approximate Efficiency:** In large markets, the chance of missed trades (and unexploited surplus) is small, so overall outcomes mirror competitive equilibrium allocations. The equilibrium path ensures that the last-minute "cave-in" or acceptance is priced at or near one's true valuation/cost.
- **Challenges:** Some endgame scenarios are unresolved (e.g. if only one buyer or seller remains at the end). The partial nature of the equilibrium analysis leaves open certain off-equilibrium contingencies.

(7) Conclusion and Significance. Wilson's framework bridges fundamental theories of bilateral bargaining and multi-participant double auctions, suggesting that the "competitive pressure" in DA markets effectively replicates discounting-driven impatience. In so doing, he sheds light on how incomplete information can still produce near-Walrasian price convergence. Although incomplete in addressing endgames and certain out-of-equilibrium paths, the analysis is highly suggestive that oral double auctions embody a dynamic, self-enforcing mechanism for price formation. This lays groundwork for deeper, robust game-theoretic treatments explaining how, even under decentralized information, observed DA outcomes can be both high-efficiency and near-competitive in price.

1991: A Simple Testable Model of Price Formation in the CDA Market (Friedman)

(1) Context and Motivation. Friedman addresses the continuous double-auction (CDA) market mechanism—used in major financial and laboratory settings—and seeks a tractable, detail-rich model to predict how bid, ask, and acceptance behaviors evolve over a single trading period. Unlike fully strategic approaches (e.g., Wilson's equilibrium analysis), Friedman proposes a "Game Against Nature" assumption that simplifies agents' inference: each trader treats opposing bids/asks as exogenous random processes, ignoring potential feedback of one's own actions on others' future strategies. This assumption, plus Bayesian updating and expected-utility maximization, underpins his "simple testable model" meant as a benchmark for empirical comparisons.

(2) Main Hypothesis and Questions. - Traders neglect explicit strategic interdependence yet remain Bayesian decision-makers, updating beliefs about the "random" arrival of future offers. - *Central question:* Does this simplified "Game Against Nature" approach (with parametric assumptions) generate sufficiently accurate, testable predictions of real CDA price formation? - Sub-questions:

- Will risk-neutral buyers/sellers adopt *aggressive reservation-price strategies*, shaving/raising the best quote if profitable?
- Under repeated experimental sessions, does the model predict near-competitive allocations and rapid price convergence?

(3) Relevant Literature. Friedman places his model among:

- **Wilson (1987)**: Full rational sequential equilibrium with common knowledge, but high complexity.
- **Easley & Ledyard (1986)**: Ad-hoc behavioral rules, lacking explicit micro-foundations for agent updating/choice.
- **Friedman (1984)**: NCE (No-Congestion Equilibrium) and efficiency results but fewer specifics on path of price formation.
- **Empirical lab findings**: Show rapid convergence to near-CE outcomes, even with few traders and repeated sessions.

(4) Experimental Setup.

- **DA Mechanism:** Repeated one-period sessions, each T units long, with n buyers (endowed with valuations) and m sellers (with costs). Bids/asks evolve continuously; if a buyer (seller) accepts the current ask (bid), a transaction occurs and both exit.

- **Game Against Nature (BGAN) Assumptions:** - *BGAN1*: An agent's actions do not affect future "Nature-driven" offers from the other side. - *BGAN2*: Bayesian updating of unknown distribution of future offers. - *BGAN3*: Offers arrive as a (non-dogmatic) Poisson process with a known expected arrival rate.
- **Agent Utility:** Linear in trading profit, i.e. risk-neutral.
- **Objectives:** Provide explicit formulas for *reservation prices* $V(t)$, which guide acceptance or improvement of quotes.

(5) Main Findings.

- **Optimal Strategies:** Under the BGAN assumptions, each agent's best response is an *aggressive reservation price* rule:
  - e.g. a seller sets a dynamic $V_s(t)$ and accepts or tries to hold the ask if it exceeds $V_s(t)$.
- **Computable Equations:** For risk-neutral traders, Friedman derives differential equations for $V(t)$ in terms of a transform of the posterior distribution of potential bids/asks, giving a closed-form or numerically solvable expression for the reservation price path.
- **Near-Efficiency and Convergence:** In a single trading period with these "aggressive" strategies, outcomes are almost Pareto optimal. Under repeated sessions, the model plausibly converges to near-competitive prices, consistent with lab observations.
- **Further Refinements:** If traders were more sophisticated (e.g. recognizing their own offers might shift future behavior), or uncertain about multiple distribution parameters, the reservation price might shift but remain numerically close for most of t.

(6) Significance for Economic Change. Friedman's framework yields an *explicit path* for price formation in a non-tâtonnement, real-time DA setting without solving a complex game-theoretic equilibrium. The approach provides *testable*, parametric predictions about *who moves next*, at *what time*, with *which updated price*, making it ideal for comparing event-by-event with laboratory or real market data. Despite ignoring strategic feedback, the model recovers near-efficiency and emergent convergence, highlighting the possibility that robust market structures can overshadow agent-level strategic complexity.

(7) Conclusion and Future Research. Friedman proposes his "Game Against Nature" model as a tractable benchmark for CDA experiments, one that can be iterated to handle multi-unit or asset markets. He recommends empirical tests at multiple granularities (individual bids vs. sub-auctions vs. final outcomes) and suggests that if it explains lab data sufficiently, it could guide simpler parametric or extended risk/preference variants. Future work might incorporate partial feedback effects, co-evolving beliefs, or multi-round complexities to see whether strategic refinements significantly enhance predictive power beyond the "simple testable model."

1993: A Theory of Price Formation and Exchange in Oral Auctions (Easley-Ledyard)

(1) Context and Motivation. Easley and Ledyard explore how prices form in oral double-auction experiments, where a small group of buyers and sellers publicly quote bids and offers. Despite imperfect information, these markets often converge to near-competitive prices. Existing competitive supply-demand theory predicts final outcomes but does not explain the path to equilibrium. Their aim is to build a dynamic model of how bids, offers, and contract prices evolve over time.

(2) Relevant Literature. Previous work on stability often used a fictitious auctioneer adjusting prices in response to excess demand. However, laboratory double-auction markets (Smith, Williams) show rapid convergence even with few traders, raising questions about the decentralized mechanism leading to competitive outcomes. Easley and Ledyard contribute by proposing formal models to capture the observed price-formation process.

(3) Main Hypothesis and Questions. - How do buyers and sellers adjust bids/offers in response to observed prices and trades? - Can a simple aggregate dynamic (Model I) or a microfounded behavior-based dynamic (Model II) explain the convergence to competitive equilibrium? - What patterns of bidding, final contract quantities, and untraded units does each model predict?

(4) Experimental Setting. Subjects are divided into buyers and sellers, each assigned induced values (buyers) or costs (sellers). In repeated market sessions, participants post bids/offers; a binding trade occurs when a bid meets an offer. Only public information is bids, offers, and transaction prices. Over multiple trading days with the same schedules, prices typically stabilize near the equilibrium predicted by supply and demand intersections.

(5) Two Models.

- **Model I (Aggregate)**: - Posits that trades occur randomly among feasible pairs with prices in a "support set" determined by previous day's contracts. - Shows price convergence under weak assumptions but lacks explicit individual decision rules.
- **Model II (Behavior-Based)**: - Assumes each agent forms reservation prices (max willingness to pay or min willingness to accept) based on experience. - Bids and offers adjust over the day, and by day's end, traders "cave in" to near-equilibrium prices if they risk going unfilled. - Yields stronger predictions about the sequence of prices and which traders transact.

(6) Main Findings. - Both models predict eventual convergence to near-competitive prices. - Model II offers stricter predictions on intra-day price bounds and final untraded units, which match experimental data reasonably well but occasionally show violations (e.g., stubborn outliers). - Empirical checks on nine lab sessions show only a small fraction of transactions violate these theoretical bounds.

(7) Conclusion and Significance. These models provide insight into how individual bidding behavior, under limited information and learning, suffices to drive markets toward competitive outcomes. The simpler aggregate approach (Model I) explains why convergence can occur; the richer behavioral approach (Model II) clarifies how local bid/offer adjustments lead to stable prices. This work thus lays groundwork for a more complete theory of price formation in real-time trading markets.

1993: Allocative Efficiency of Markets with Zero-Intelligence Traders (Gode-Sunder)

(1) Context and Motivation. The paper explores how double auctions can achieve near-complete allocative efficiency even when traders lack rationality and profit-seeking motives. The authors replace human agents with simple "zero-intelligence" automata to observe whether market rules alone can ensure high efficiency.

(2) Relevant Literature. Previous studies (Becker, Smith) showed that rationality assumptions are often relaxed at the individual level, yet aggregate outcomes converge to equilibrium. Prior lab experiments indicated near 100% efficiency under double auctions with human participants, suggesting strong institutional effects.

(3) Main Hypothesis and Questions. - Hypothesis: In double auctions, the rules themselves largely determine efficiency, independent of individual cognition. - Does removing profit-maximizing behavior reduce efficiency significantly? - What role does the budget constraint play in driving convergence?

(4) Experimental Setup. The authors ran multiple double-auction sessions. They endowed buyers and sellers with redemption values and costs, respectively, then replaced human strategists with programs generating random bids/offers, sometimes constrained by their valuations.

(5) Methodology. - Two types of zero-intelligence traders: unconstrained (allowed to trade at a loss) vs. constrained (blocked from making loss-making trades). - Compare resulting price convergence and efficiency to human-trader benchmarks. - Measure efficiency as actual total surplus / maximum possible surplus.

- **Result 1:** Unconstrained zero-intelligence traders often made money-losing trades, lowering efficiency.
- **Result 2:** Constrained traders consistently reached near 100% efficiency, like humans.
- **Result 3:** Convergence to equilibrium price still occurred, even absent learning or memory.
- **Result 4:** Profit dispersion was higher for unconstrained and lower for humans.
- **Result 5:** Market rules ("budget constraint") largely drove the efficiency outcome.

(6) Main Takeaways. Imposing basic market discipline on non-rational automata suffices to achieve near-maximal surplus extraction. Individual profit-seeking is not necessary for high efficiency. However, rational behavior still matters for reducing profit variability across traders and stabilizing prices faster.

(7) Future Directions. Study mixed populations of humans and automata to isolate learning effects. Investigate other auction designs or complexity in zero-intelligence rules. Examine additional dimensions (information aggregation, speed of convergence) to enrich our understanding of market robustness.

1994: Computerized Double Auction Tournament (Rust, Palmer, Miller)

(1) Context and Motivation. Automata traders were developed to simulate behavior in a dynamic double auction market. The key motivation was to examine how decentralized trading converges to competitive outcomes despite limited information, heterogeneous strategies, and potential bidding mistakes.

(2) Relevant Literature. Prior studies on double auctions (Smith, Gode and Sunder, Friedman, Wilson) show that markets often converge to near-competitive equilibrium, even with simple or zero-intelligence traders. Game-theoretic efforts (Satterthwaite and Williams, Wilson) grapple with incomplete information and equilibrium existence but are less clear on real-time price formation. Experimental economics finds human traders also converge quickly, leaving open questions about individual learning vs. institutional forces.

(3) Hypothesis and Research Questions. The paper hypothesizes that institutional rules, not only individual rationality, drive price convergence. Key questions: - Do simple automata rules suffice for high efficiency? - Which strategic features distinguish best-performing traders? - How robust is efficiency under varied market structures?

(4) Experimental Setup. Multiple tournaments were run with over 30 automata, each receiving private token values and executing bids/asks. Various environments manipulated time steps, distributions of values, and market composition. Each simulation tracked market prices, orders, trades, and profits over thousands of rounds.

(5) Methodology. - Discretized double auction with alternating Bid/Ask and Buy/Sell steps. - "AURORA" trading rules (only current bid/ask holders can trade). - Wide parameter sweeps: different supply/demand curves, time constraints, token endowments. - Efficiency calculated as realized total profit / total potential surplus.

- **Result 1:** Prices typically converged near competitive equilibrium, even in the first trading periods.

- **Result 2:** Simple "wait-in-the-background" strategies (e.g., Kaplan) dominated, outperforming more complex predictive/adaptive algorithms.
- **Result 3:** Efficiency regularly exceeded 90%, showing resilience to incomplete information and heterogeneous trader types.
- **Result 4:** Despite convergence, a non-trivial share of trades were extra-marginal, mirroring human-lab findings.
- **Result 5:** In evolutionary settings, the top strategies eventually monopolized the market, yet introduced occasional "crashes" when everyone waited too long.

(6) Main Takeaways. Institutional features of double auctions (improvement rules, public bids/asks) funnel decentralized actions toward efficiency. Simple reactive rules can generate near-optimal outcomes, suggesting that complexity in agent design is not strictly necessary. However, universal "waiting" induces informational deadlock, showing negative externalities when learning is one-sided.

(7) Future Directions. - Explore hybrid adaptive strategies combining robust heuristics with limited learning. - Investigate how small noise-trader inflows influence price stability or volatility. - Assess whether adding more realistic timing uncertainties (network delays) affects outcomes. - Compare human vs. automata-based institutions in tandem designs. - Extend the analysis to continuous-time auctions or alternative market institutions.

1996: Price Formation in Double Auction Markets (Cason-Friedman)

(1) Context and Motivation. This paper investigates how prices form in continuous double auctions (DA), a critical setting where self-interested traders with private values and costs interact. The authors conduct controlled laboratory experiments to see how market prices converge when traders face uncertainty about others' valuations. This work aims to clarify how private information and strategic or non-strategic behavior shape equilibrium price discovery in real-time trading institutions.

(2) Relevant Literature. Prior DA experiments (Smith, Gode & Sunder, Wilson, Friedman) show near-competitive outcomes under diverse conditions, with efficient trading emerging quickly. However, open questions remain on the precise *process* of price formation. Theoretical proposals (Waiting Game, Bayesian models, zero-intelligence random bidding) differ on how time, risk, and competition drive bids/asks to equilibrium.

(3) Main Hypothesis and Research Questions. - Do DA markets still efficiently aggregate private information under random value and cost draws, yielding reliable price discovery? - How quickly do markets converge each period in the presence of changing fundamentals? - Does experience in repeated trading lead to predictable shifts in bidding/asking strategy? - Do strategic theories (e.g., Waiting Game, Bayesian Game) explain price formation better than simpler random-bidding models?

(4) Experimental Design. Subjects are randomly assigned roles as buyers or sellers, each with one unit per period. Valuations (buyers) and costs (sellers) are drawn anew each round from uniform distributions. Subjects post bids and asks continuously, accepting or rejecting counterpart quotes in real time, over multiple trading rounds. Various buyer-seller ratios are tested, and trader experience varies from novice to expert. Detailed log data of bids, asks, and transaction times enable fine-grained analysis of price dynamics.

(5) Methodology (Models). - *Waiting Game/Dutch Auction (WGDA)*: Strategic, sequential equilibrium. Traders wait, adjusting bids/asks slowly, with the fewer-side of the market extracting more surplus. - *Bayesian Game Against Nature (BGAN)*: Traders treat observed bids/asks as signals from a stationary data-generating process, updating reservation prices over time. Moderate positive autocorrelation in prices is predicted. - *Zero-Intelligence (ZI)*: Buyers and sellers randomly sample bids and asks within rational (no-loss) bounds. Yields near-competitive efficiency but often has negative price-change autocorrelation.

- **Result 1:** Despite random draws and incomplete information, DA trading achieves high efficiency ($\approx$ 85%–95%).
- **Result 2:** Efficiency improves with subject experience, but both extra-marginal trading and lost profitable trades persist.
- **Result 3:** Early transactions tend to involve high-surplus pairs, though out-of-order trades remain common.
- **Result 4:** Price-change autocorrelation is strongly negative for novices, moves toward zero or slightly positive for experts, casting doubt on strong martingale claims but also rejecting pure random-bid models.
- **Result 5:** In expert sessions, the smaller side of the market captures more surplus, partially confirming strategic theories like WGDA.

(6) Main Takeaways. Even with randomness, DAs maintain robust efficiency. Nevertheless, no single existing model (WGDA, BGAN, ZI) fully explains all aspects of price trajectories and bidding sequences. Zero-intelligence logic captures some early-stage price fluctuations; strategic waiting and learning become salient only after repeated practice. Hence price formation in these DA markets likely involves boundedly rational updates and partial strategic positioning.

(7) Future Directions. Building richer models that incorporate adaptive expectations, memory of past trades, or hybrid ZI-strategic bidding could yield better fits. Increasing market thickness (multiple units) and exploring variant auction rules can test how general these findings are. Moreover, calibrating model parameters (risk attitudes, waiting costs) against granular bid/ask path data may unify random-bid and game-theoretic approaches into a more realistic theory of double auction price formation.

1997: Zero is Not Enough: On the Lower Limit of Agent Intelligence for Continuous Double Auction Markets (Cliff-Bruten)

(1) Context and Motivation. Cliff and Bruten revisit Gode and Sunder's landmark finding that even "zero-intelligence" traders (who submit random bids/offers but avoid loss-making transactions) can achieve near-competitive outcomes in continuous double auction (CDA) markets. They argue that this is conditional on a symmetric (or balanced) supply and demand. In asymmetric markets, zero-intelligence-constrained (ZI-C) traders' transaction prices can deviate predictably from equilibrium, showing that zero intelligence is not always enough to guarantee convergence to the theoretical competitive price.

(2) Main Hypothesis and Questions. - Gode and Sunder claimed near-equilibrium prices derive more from the market structure than from individual rationality or learning. - Cliff and Bruten demonstrate that ZI-C performance depends strongly on whether supply and demand curves are roughly symmetric. - They introduce slightly more adaptive, "zero-intelligence-plus" (ZIP) traders to see if minimal adaptive mechanisms suffice to replicate human-like convergence under a broader range of market structures.

(3) Gode and Sunder's ZI-C vs. Cliff and Bruten's Analysis. - *ZI-C Agents*: Randomly pick bids/offers in a feasible range (bound by each agent's limit price), ensuring no loss-making deals. Gode and Sunder find they converge to equilibrium in some environments, primarily those with roughly symmetric supply/demand. - *Critique*: Cliff and Bruten show analytically and via simulation that when supply/demand is asymmetric (e.g., "flat" or "box" designs), mean transaction prices for ZI-C can deviate significantly from equilibrium. This "success" in earlier tests is somewhat an artifact of special parameter choices.

(4) Zero-Intelligence-Plus (ZIP) Traders. - ZIP traders maintain simple profit margins on top of cost (for sellers) or below value (for buyers). When market events (accepted or rejected bids/offers) suggest the margin is too low/high, it is adjusted via a simple Widrow-Hoff style learning rule. - Agents thus stochastically adapt their margins (learning rate, random shock, "momentum"), ensuring they adjust quotes in a direction suggested by recent market outcomes.

(5) Empirical Findings. - In symmetric markets, both ZI-C and ZIP yield near-equilibrium outcomes. - In asymmetric or "box" markets, ZI-C fails (average prices deviate substantially from equilibrium), whereas ZIP converges more robustly over multiple trading sessions. - ZIP also lowers profit dispersion more quickly, aligning more closely with experimental human data.

(6) Conclusion and Significance. While Gode and Sunder's ZI-C experiments showed that market structure strongly drives overall efficiency, Cliff and Bruten demonstrate that zero-intelligence alone does not guarantee convergence to competitive prices in all configurations. A minimal injection of adaptive behavior (ZIP) helps agents achieve human-like performance, even under asymmetry. This suggests a revised lower bound on agent sophistication for robust convergence in CDAs, and opens the door to further enhancements of learning rules and market complexity.

1998: Price Formation in Double Auctions (Gjerstad & Dickhaut)

(1) Context and Motivation. Gjerstad and Dickhaut propose a model of trader behavior in the double-auction (DA) institution, focusing on how simple belief formation and myopic surplus maximization can rapidly drive prices toward competitive equilibrium. Past experiments (Smith, Plott) show DAs converge quickly even under imperfect information. Earlier theories (Easley & Ledyard, Friedman, Gode & Sunder, Wilson) had either no explicit belief formation or relied on specific assumptions about bidding rules. Gjerstad and Dickhaut's goal is to formulate how traders form probabilities ("beliefs") that their bids/offers will be accepted and then act to maximize expected surplus, thus providing a decentralized mechanism of price discovery consistent with lab data.

(2) Main Hypothesis and Questions. - **Hypothesis:** In continuous double auctions, traders can form simple, data-driven beliefs about the likelihood of each quote being accepted, and use these to pick bids/offers maximizing expected payoff. - **Key questions:**

- How do we formalize belief formation solely from observed market activity (frequencies of bids/offers accepted/rejected)?
- Can such beliefs plus a myopic best-response rule consistently yield near-competitive prices and allocations?
- Does the model adapt quickly to shifts in supply/demand, matching experimental results?

(3) Comparing with Prior Models (e.g. Gode & Sunder). - Gode and Sunder's zero-intelligence-constrained (ZI-C) traders randomly choose quotes but avoid unprofitable transactions, achieving efficiency under certain supply-demand conditions but lacking any explicit learning/belief mechanism. - *Critique:* Gjerstad and Dickhaut find that while ZI-C can produce high efficiency, it often fails to explain real-time convergence or the path of prices under broad market variations. - *Their Advance:* By endogenizing traders' beliefs (via empirical frequencies of accepted/rejected quotes) and linking these to a standard expected-surplus maximization rule, they show a plausible path of within-day price evolution consistent with lab data.

(4) The Proposed "Belief + Myopic Surplus Max" Model.

- **Belief Formation:** - Sellers compute $\hat{p}(a)$ = probability that an ask a will be taken, inferred from the fraction of historical asks near a that were accepted vs. rejected, plus interpolation assumptions. - Buyers compute $\hat{q}(b)$ similarly for a bid b. - Adjust for "spread reduction" rules in the DA (new asks must undercut old ones, etc.) to yield zero probability outside feasible bounds.

- **Strategy:** - At any moment, a seller picks an ask a^* maximizing $\hat{p}(a) \times (\text{price} - \text{cost})$. - A buyer picks a bid b^* maximizing $\hat{q}(b) \times (\text{valuation} - \text{price})$. - Timing of who moves next is tied to exponential waiting-time functions keyed to potential surplus (intuitively, higher surplus means more "impatience" to move).

(5) Empirical/Simulation Findings. - **Convergence to Equilibrium Price:** Simulations in a symmetric 4-buyer/4-seller environment quickly stabilize near the theoretical equilibrium price, matching or exceeding the efficiency observed in human-subject experiments. - **Comparison to ZI-C:** In the same environment, ZI-C fails to converge; it remains scattered around equilibrium. Gjerstad and Dickhaut's beliefs-based model yields smaller price deviations and faster stability. - **Shifting Supply/Demand:** When the supply and demand curves are shifted mid-experiment, the model's beliefs adjust so that transaction prices move to the new equilibrium in only 1–2 trading periods. - **Behavioral Anomaly:** The model predicts that trades initiated by sellers might *lower* average prices than those initiated by buyers, contrasting with some experimental evidence. This suggests real human traders might have more nuanced heuristics.

(6) Conclusion and Significance. Gjerstad and Dickhaut provide a distinct mechanism for DA price formation relying on endogenously learned acceptance probabilities. Their model not only explains high efficiency but also illuminates the time path by which quotes converge near competitive levels, adapting even under changing conditions. While further refinements are needed—especially concerning certain systematic differences in who initiates trades and how final transaction prices relate to initiators—the results strongly support that relatively simple, decentralized belief formation and myopic profit-seeking suffice to replicate key features of double-auction outcomes. This work thereby enriches our understanding of how real-time bidding dynamics yield near-competitive results with limited information.

2001: High-Performance Bidding Agents for the Continuous Double Auction (Tesauro & Das)

(1) Context and Motivation. Tesauro and Das tackle the design of high-performing bidding agents for real-time continuous double auctions (CDAs), an institution widely used in financial markets (e.g. equities, commodities). Previous research, including the Santa Fe Double Auction Tournament (Rust et al. 1992), demonstrated that simple "sniping" strategies could outperform many bidding rules but lacked adaptive capabilities. Meanwhile, early adaptive bidders (e.g. Cliff's ZIP and Gjerstad & Dickhaut's GD) showed promise in simulations but had not been extensively tested in real-time, multi-unit, asynchronous CDA environments. Tesauro and Das aim to enhance these strategies for multi-unit trading and assess their performance under realistic market rules, including modifications to ZIP and GD tailored to persistent orders and real-time constraints.

(2) Main Hypothesis and Questions. - *Hypothesis:* By modifying ZIP and GD for multiple units and real-time order persistence, agents can achieve superior surplus (profits) and faster/robust convergence to near-competitive outcomes compared to simpler or non-adaptive strategies. - *Key questions:*

- How do ZIP and GD need to be altered to handle multiple-unit holdings and real-time market feedback (stochastic asynchronous trade arrivals, persistent open orders)?
- Do these modifications surpass known benchmarks, e.g. Kaplan's sniping approach and Zero-Intelligence (ZI) random bidding?
- In head-to-head tests and in mixed populations, which strategies yield higher efficiency and surplus?

(3) Modified ZIP and GD Strategies.

- **Multi-unit ZIP:** - Original ZIP adjusts a single profit margin m based on trading outcomes. - Tesauro and Das introduce a margin array (one margin per unit) and scale margins for higher/lower-value units. - In real-time CDA, an agent must also re-quote if no trades occur for a while ("beating" best outstanding quotes).
- **MGD (Modified Gjerstad-Dickhaut):** - Original GD forms beliefs from recent accepted/rejected orders about the probability that a new quote at price p will transact. - Key enhancements: reset probability boundaries based on previous period's highest/lowest trade, handle multi-unit differently (submit quotes for least valuable untraded unit), and reduce volatility.

(4) Experimental Setup. - Discrete-time simulator approximates real-time as "random activation" of agents each step. Each agent has multiple units with cost/valuation drawn from uniform $[100, 200]$, typically leading to about half the units being profitable at equilibrium. - Several market rules tested: e.g. NYSE-style improvement rule (new bid must exceed old best bid, new ask must undercut old best ask), persistent open orders (expire only upon trade or period end). - Agents also deployed in MAGENTA, a genuine real-time multi-agent platform, though fewer runs are feasible there.

(5) Results.

- **Homogeneous Populations:** - Both ZIP and GD (especially the MGD variant) achieve near-maximal efficiency ($> 99\%$ of theoretical surplus). - Kaplan's sniping yields slightly lower efficiency and more volatile prices, while Zero-Intelligence (ZI) lags significantly.

- **Mixed One-in-Many Tests:** - A single agent of one type vs. an otherwise homogeneous population of another. - Results show MGD and ZIP can profitably "invade" populations of weaker strategies; sniping does well only if it remains relatively rare (otherwise it collapses).
- **Balanced-Group Tests:** - Two strategies with equal numbers of agents, matched in pairs with identical limit prices. - MGD consistently outperforms the original GD, ZIP, Kaplan, and ZI, suggesting it is the strongest known real-time bidder for multi-unit CDA among those tested.

(6) Conclusion and Significance. Tesauro and Das demonstrate that carefully adapted versions of ZIP and GD—especially MGD—achieve superior performance in real-time CDA markets where agents hold multiple units and must manage persistent orders. In both homogeneous and heterogeneous agent populations, MGD generally dominates simpler or less adaptive strategies (ZI, Kaplan-sniping). These findings underscore that (i) minimal-intelligence agents (ZI) are not enough for top-tier performance, and (ii) sniping is beaten by adaptive, belief-based bidders when multiple agents can snipe simultaneously. In sum, MGD sets a new benchmark for high-efficiency, high-surplus bidding in CDAs and suggests further refinements for adaptive agent design under realistic market conditions.

2007: Markup Strategies

(1) Context and Motivation. Zhan and Friedman tackle a longstanding question about why continuous double auction (CDA) markets exhibit high efficiency. They focus on the role of "markups," i.e., systematic markups or markdowns that traders add to their true cost or value. Earlier literature (e.g. Gode & Sunder's "zero-intelligence" approach) suggested even random bids can yield near-competitive outcomes, but Zhan & Friedman test how simple profit-seeking markup rules affect the CDA's aggregate performance. They aim to clarify how uniform markups shift prices, volumes, and efficiency in simulated CDA environments, and to investigate whether there exist Nash equilibria in markup choice.

(2) Main Hypothesis and Questions.

- Does a uniform markup rule among traders monotonically reduce market efficiency, or can there be some optimal markup that *enhances* efficiency relative to zero markup?
- Are there "Nash equilibrium" markup choices that individual traders might adopt, and if so, do they coincide with the most efficient markup (thereby explaining the CDA's "mysteriously" high efficiency)?

(3) Relevant Literature. The authors build on:

- **Gode & Sunder (1993)**: Even "non-strategic" zero-intelligence CDA traders produce high efficiency, but do not systematically explore profit-driven markups.

- **Gjerstad-Dickhaut (1998)** and **Cliff-Bruten (1997)**: More sophisticated adaptive or belief-based CDA strategies.
- **Call-market analyses** (Rustichini et al. 1994, Cripps & Swinkels 2006) show that uniform markup in a single-shot clearing can reduce volume and efficiency, but do not directly apply to the multi-offer CDA.

(4) Experimental Setting. They simulate a standard CDA with N buyers and N sellers, each having a single unit with randomly drawn private values/costs from uniform $(0, 200)$. The market runs over T discrete periods. Traders use simple rules that fix a single markup m: a buyer bids $b_i = v_i(1-m)$ and a seller asks $a_j = c_j(1+m)$ (the "standard markup" rule). Variants include "exponential" and "shift" markup functions. Key outcomes are measured: efficiency (surplus / max possible surplus), transaction volume, average transaction price, price dispersion, and buyer/seller surplus shares.

(5) Main Findings.

- **Optimal Uniform Markup**: Unlike the single-period call market (where $m = 0$ is the unique best choice), in a multi-step CDA an intermediate markup around $m \approx 0.1$ to 0.3 can *increase* realized efficiency (by reducing wasted extramarginal trades).
- **Nash Equilibria in Markup Choice**: The authors examine both a "two-cartel" game (all buyers share one markup, all sellers share another) and a full $2N$-player game (each trader chooses individually). In the two-cartel case, the equilibrium markup pair is typically $(m_b \approx 0.6, m_s \approx 0.5)$. But in the full game, *symmetric* Nash equilibrium choices cluster near the *efficient* markup.
- **Explanation for High CDA Efficiency**: The interplay between greed (larger markup can yield higher per-transaction profit) and fear (excessive markup risks no transaction) results in an equilibrium that is surprisingly close to maximizing total surplus.

(6) Significance for Economic Change. Their simulations help resolve Smith's "mystery" of why CDA is so efficient, suggesting that simple strategic markups can self-organize to near-maximal efficiency. The "fear–greed" trade-off means traders spontaneously limit their markups to avoid missing profitable deals. This insight refines prior non-strategic or single-period analyses, showing how repeated interactions and partial revelation produce near-competitive outcomes.

(7) Conclusion and Future Research. Zhan and Friedman show that, in simulated CDA markets, a modest uniform markup can yield the best overall efficiency, and that the symmetric Nash equilibrium markup is indeed near this efficiency-maximizing level. Future work can relax the assumption of uniform markup or single-dimensional strategies (e.g. time-based offers, or contingent markups depending on realized value/cost) to see if this near-coincidence of equilibrium and efficiency persists. Additionally, real-world continuous auctions might display more timing strategies (like sniping) or multi-unit complexities, all further opportunities for exploring how simple profit-motivated heuristics can sustain highly efficient outcomes.

2010: The Agent-Based Double Auction Markets: 15 Years On (Chen & Tai)

(1) Context and Motivation. Chen and Tai revisit the legacy of Andrews and Prager (1994), who introduced GP-driven (Genetic Programming) traders in double-auction markets, highlighting how "novelty-discovering" agents can spur continual change. Standard economic models, they argue, often lack such internal sources of dynamism, whereas GP-based autonomous bidders actively explore and exploit new tactics. Their study extends previous agent-based double-auction (DA) setups—especially Gode & Sunder's zero-intelligence model and the Santa Fe Double Auction Tournaments—by letting GP agents repeatedly evolve strategies against a variety of hand-coded opponents.

(2) Main Hypothesis and Questions. - *Central Claim:* If at least one market participant is an autonomous GP agent in a DA environment, it can eventually outperform non-autonomous strategies (human-written or simpler adaptive ones), because it continually hunts for new exploitable patterns. - *Key questions:*

- Do GP-based traders truly surpass a wide range of known DA strategies (e.g., Kaplan's sniping, Gjerstad-Dickhaut's belief-based approach) over sufficient trading rounds?
- How does the GP population size and learning time affect the emergent performance?
- Does the DA mechanism itself accommodate (or hamper) continuous novelty emergence and indefinite cycles of strategic change?

(3) Relevant Literature. They connect to prior computational DA research:

- **Gode & Sunder (1993)** showed zero-intelligence (ZI) agents can yield near-competitive outcomes, implying market structure can dominate agent rationality.
- **Santa Fe Double Auction Tournaments** found certain "sniping" (Kaplan) or simple rule-based codes (Ringuette) surprisingly strong. But these were human-coded, not evolving.
- **Andrews & Prager (1994)** first introduced genetic programming for DA strategy but only made one trader autonomous.
- **AIE-DA (Chen et al.)** extends GP to multi-population learning, showing GP replicates known lab results (e.g. rapid price convergence).

Chen and Tai position their work as the next step—testing whether GP-based "novelty discoverers" consistently outdo non-evolving DA strategies.

(4) Experimental Setup.

- **Market Rules:** A discrete double auction with 4 buyers, 4 sellers, each holding 4 commodity units per trading day. The DA uses an Aurora-like matching where at most one deal occurs per time step, at the midpoint of best bid/ask.
- **Opponent Strategies:** They sample from a large set including Kaplan (sniper), Ringuette (background undercut), Gjerstad-Dickhaut (expected-profit maximizing), ZI or ZIP variants, etc.
- **GP Trader Details:** - Population sizes vary (5, 20, 30, . . . , 100). - Terminals/function sets derived from successful human-coded rules. - Every N days, GP evolves via standard genetic operators (tournament selection, crossover, mutation).
- **Tournament Format:** Each run randomly picks 8 strategies (4 buyers, 4 sellers) without repetition. Only 0 or 1 GP trader is included, so no co-evolution among multiple GP players.
- **Key Metrics:** Individual Efficiency (ratio of realized profits to the best possible surplus) and profit-volatility (std. deviation).

(5) Main Findings.

- **GP Outperforms Hand-Coded Strategies:** Over several thousand trading days, GP's average individual efficiency surpasses even top human-coded algorithms (Kaplan, GD, etc.).
- **Learning Curve:** Smaller populations (e.g. size 5) require longer to beat the competition, while larger populations (size 100) converge more rapidly on strong bidding policies.
- **Profit Variation Trade-off:** GP yields high average gains, sometimes with higher volatility; it can occupy extremes of the "profit vs. stability" frontier.
- **Implication for Constant Change:** Because GP-based agents seek "novel" strategies, they can force ongoing adaptation from other (non-evolving) players, illustrating indefinite cycles of emergent behavior.

(6) Significance for Economic Change. Chen and Tai highlight that injecting even a single GP-driven "novelty discoverer" into DA markets disrupts static equilibria: such an agent perpetually exploits hidden inefficiencies in any fixed or non-autonomous strategy. This finding underscores how simple, static, or even moderately adaptive code can be systematically beaten, illuminating a critical mechanism of indefinite endogenous change reminiscent of Marshall's "biological economy." They argue future work might combine multiple co-evolving GP traders or integrate real human players to study how adaptive intelligence fosters unending transformations in market dynamics.

(7) Conclusion and Future Research. Concluding that "novelties-discovering" GP agents can eventually prevail over widely varied DA strategies, the authors propose further experiments pitting multiple GP traders or mixing them with human subjects. They foresee that real-time adaptivity can disrupt stable market equilibria, launching ever-renewing evolution of trading tactics. This underscores the potential for agent-based models to capture intrinsic, ongoing economic change, a hallmark of real market systems.

2011: IBM's Study on ZIP and GDX with Humans

(1) Context and Motivation. De Luca and Cliff revisit and extend the seminal work by IBM (Das et al. 2001) in which human traders and algorithmic ("robot") traders competed in a Continuous Double Auction (CDA). While IBM's earlier study found that certain strategies (notably ZIP and an Extended Gjerstad-Dickhaut, EGD) could outperform human participants, no one had since replicated these findings. Motivated by the decreased cost of computing resources and a desire to test more advanced trading algorithms, the authors develop the OpEx "lab-in-a-box" platform to run controlled CDA experiments. They aim to (a) replicate the original human-vs.-ZIP experiments and (b) test, for the first time, how humans fare against the GDX algorithm (an enhanced version of Gjerstad-Dickhaut).

(2) Main Hypothesis and Questions.

- *Core hypothesis*: Algorithmic trading strategies, in particular GDX (Tesauro & Bredin's 2002 extension of GD), can consistently outperform human traders in a CDA environment, much like ZIP and EGD did in previous IBM work.
- Are GDX agents more efficient and profitable than both human traders *and* the simpler ZIP agents?
- Does the more sophisticated pricing logic of GDX (using Dynamic Programming) translate to higher per-agent surplus, and at what cost to overall market efficiency or profit dispersion?

(3) Relevant Literature. They cite:

- **IBM's 2001 study (Das et al.)**: Found that both EGD (a GD extension) and ZIP outperformed humans, capturing the majority of surplus.
- **Smith's classical experiments (1962)** showing fast convergence to equilibrium in CDA with human participants.
- **GDX** from Tesauro & Bredin (2002) as a strategic sequential version of GD with dynamic optimization.
- **ZIP** from Cliff & Bruten (1997), widely used as a baseline for adaptive bidding strategies.

(4) Experimental Setup.

- **OpEx Platform**: A new open-source experimental framework ("lab-in-a-box") with netbooks as trader terminals and an exchange server that implements CDA matching. They replicate real-time trading sessions where human participants place orders via a GUI, while agent participants send programmatic orders.
- **Human vs. Agent Design**: Typically 6 human traders vs. 6 agent traders, each subgroup having 3 buyers and 3 sellers. The limit prices (costs for sellers, valuations for buyers) are arranged so that a known "competitive equilibrium" price exists. Market shocks periodically shift this equilibrium.
- **Timing Rules**: Each "round" is a 3-minute window (or sometimes discrete-event simulation mode) during which agents or humans can update bids/offers. Ten rounds make a full experiment.

(5) Main Findings.

- **Human vs. ZIP or GDX:** Across several new experiments, both algorithms indeed outperform human participants in terms of individual surplus on average, corroborating the IBM result that adaptive agents can exploit human traders' systematic inefficiencies.
- **ZIP vs. GDX:**
  - GDX (the dynamic-programming version of GD) *generally yields higher efficiency for the agents themselves* than does ZIP in agent–human matches. GDX is more profitable to the robot group.
  - However, the *overall market* runs more smoothly (higher overall efficiency, lower profit dispersion) with ZIP than with GDX. GDX's aggressiveness can drive more volatility or more uneven outcomes.
- **Agent–Agent Tests:** GDX beats ZIP in purely algorithmic matches if it has enough computational advantage or time. In real-time setups (where quick reaction can matter), ZIP can occasionally match or exceed GDX's performance because ZIP is simpler and faster to post an advantageous quote.

(6) Significance for Economic Change. These experiments reinforce that algorithmic traders can reliably surpass human participants' surplus capture in a CDA, even under lab conditions that allow direct competition. They also confirm that more sophisticated agent algorithms (like GDX) can command even higher share of surplus—though they may introduce somewhat lower *overall* market efficiency and higher price volatility. Such insights matter both for understanding how real-world electronic markets might evolve under greater algorithmic competition, and for the design of future "human-safe" or "human-friendly" markets.

(7) Conclusion and Future Research. De Luca and Cliff successfully replicate IBM's earlier finding that GD-like algorithms outperform humans in CDA markets, and they show GDX edges out ZIP. They also note that GDX's improvements sometimes come at the cost of less stable or less equitable market outcomes. Future work might involve mixing professionals vs. amateurs, adjusting agents' time-lags, or creating more complex multi-unit or multi-asset settings. Ultimately, they suggest that such low-cost "lab-in-a-box" setups can expand empirical study of human–agent interplay under many different market designs, illuminating how algorithms can systematically exploit human traders' behaviors.

2017: Chen's Literature Review

(1) Context and Motivation. Chen surveys how agent-based computational economics (ACE) has evolved through four major "origins" or traditions, each of which influenced how agents in ACE models are conceived. These four sources—the markets origin, the cellular-automata origin, the tournament origin, and the experimental-economics origin—have fostered increasingly diverse and interdisciplinary notions of "agents." The paper draws out how different intellectual inquiries (ranging from simple emergent systems to human-like cognitive modeling) shape the landscape of agents in ACE. Chen's overarching goal is to show how ACE has steadily integrated new conceptual layers—random or zero-intelligence agents, adaptive autonomous agents, and eventually cognitively calibrated or culturally sensitive agents—to reflect complex economic and social phenomena.

(2) Main Hypotheses and Questions.

- *Historical Scope:* How did ACE emerge from distinct research traditions (e.g. cellular automata in computational science, evolutionary tournaments in game theory) and what distinct agent paradigms did each tradition champion?
- *Agent Taxonomy:* In what ways have ACE agent designs progressed from simple "programmed" or "zero-intelligence" agents to more autonomous, adaptive, and human-like agents (with cognitive, psychological, or cultural traits)?
- *Interdisciplinary Integration:* How do advances in cognitive psychology, machine learning, and economics converge in shaping agent-based modeling of real-world markets and interactions?

(3) Relevant Literature. Chen's review connects to:

- **Cellular Automata** tradition: Schelling's segregation model, Sakoda's checkerboard model, and Albin's early mapping of checkerboard to "cellular automata" for social processes.
- **Tournament-based Research:** Axelrod's Iterated Prisoner's Dilemma contests and the Santa Fe Double Auction tournaments (Rust et al.), which led to explorations of autonomous agent evolution.

- **Experimental Economics:** Arifovic's calibrations of learning agents to replicate macro-lab experiments (e.g. cobweb, overlapping generations), and others linking human data to agent designs (reinforcement learning, EWA, etc.).
- **Agent Paradigms in ACE:** Zero-intelligence (Gode & Sunder), near-ZI variants (Cliff-Bruten), heterogeneous/cognitive agents (Camerer, Ho, Stahl), and psychologically or culturally enriched agents.

(4) Approach and Thematic Outline. Chen structures the survey around four "gates" into ACE:

- **Markets Origin:** The impetus to replace or refine Walrasian/aggregative models with explicit decentralized market processes.
- **Cellular Automata Origin:** Emergence of simple, local, rule-based agents (like Schelling's or Sakoda's grids) that display complex global patterns.
- **Tournament Origin:** Studies like Axelrod's IPD competitions or Santa Fe's Double Auction contests, leading to human-written but also "open" submissions from autonomous algorithms (GAs, GPs).
- **Experimental-Economics Origin:** Linking artificial agents to replicate or explain data from human-subject experiments (calibrated reinforcement learning, belief learning, etc.).

Within each origin, Chen highlights the associated agent paradigms (simple "programmed" vs. adaptive "autonomous" vs. cognitively "human-like").

(5) Main Findings.

- **Key Agent Paradigms:**
  1. *Simple Programmed Agents* (from cellular automata tradition): random or zero-intelligence traders can surprisingly yield near-competitive outcomes; "KISS principle" and emergent complexity from minimal rules.
  2. *Autonomous Agents* (tournament tradition): genetic algorithms and genetic programming let agents discover and adapt strategies on their own, shifting from closed to "open" evolutionary tournaments.
  3. *Human-like Agents* (experimental-economics tradition): calibrated or fitted learning models (reinforcement, EWA, level-k) replicate real lab data; new frontiers add working-memory capacity, personality traits (Big Five), cultural dimensions, etc.
- **Independence vs. Overlaps:** Chen clarifies that "simple," "random," and "low-intelligence" agents do not necessarily coincide: each has distinct formal underpinnings (algorithmic simplicity, entropy-based randomness, limited cognition).

- **Calibration and Complexity:** As agent designs incorporate more behavioral or cognitive detail, they become more powerful but also less tractable—leading to the "invisibility" of learned strategies. The tension between simplicity and descriptive realism endures.

(6) Significance for Economic Change. These agent varieties address two major deficits in conventional economics: the missing "real" market process (displacing Walras's auctioneer) and the missing "real" human complexity (bounded cognition, heterogeneous psychology). By evolving from cellular automata to cognitive, psychologically rich agents, ACE aims to capture the ongoing novelty and adaptation in social systems—offering more robust models of dynamic, out-of-equilibrium processes. This methodology is arguably the next step in bridging "fully decentralized" processes with behavioral realism, further illuminating how emergent macrostructures (market prices, social norms) arise from micro-level agent interaction.

(7) Conclusion and Future Research. Chen concludes that ACE research is continuing to broaden agent design. Future directions likely include:

- *Stronger Interdisciplinary Linkages:* Incorporating insights from cognitive neuroscience, personality theory, cross-cultural psychology, etc., to refine agent decision rules.
- *Exploratory Modeling and Policy:* Varieties of agents can systematically test how different cognitive or interactive assumptions affect macro outcomes, assisting robust policy design.
- *Realistic Microstructure:* Incorporating real-world data on trading behavior, neural correlates, or personality measures to calibrate agents in more complex simulations.

Overall, "varieties of ACE agents" highlight how flexible and wide-ranging agent-based methods can be, and how future cross-pollination with behavioral, cognitive, and cultural sciences can deepen economists' understanding of emergent economic phenomena.