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Markups in double auction markets

Wenjie Zhan[a,*], Daniel Friedman[b]

[a]*School of Management, Huazhong University of Science and Technology, and LEEPS Laboratory, University of California, Santa Cruz*
[b]*Department of Economics and LEEPS Lab, University of California, Santa Cruz*

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Abstract

We study the continuous double auction (CDA) market with simulated traders using three simple markup rules. Relative to analytic results obtained for markups in the simpler call market (CM), we find that uniform CDA markups have a more complex and non-monotonic impact on CDA market outcomes such as price, volume, and efficiency. The symmetric Nash equilibrium markup in the CDA is remarkably close to the most efficient markup in thick markets, which may partially explain the ''mysterious'' efficiency for CDA market.

JEL classification: D40; C72

Keywords: Markup; Simulations; Continuous double auction

1. Introduction

The continuous double auction (CDA) is the premier market format. It allows traders to make public-committed offers to buy and to sell, and allows traders to accept offers at any time during a trading period. Variants of CDA markets prevail in most modern financial exchanges and are featured options on B2B Internet sites. Numerous laboratory studies beginning with Smith (1962) show that CDA markets

*Corresponding author. Tel.: +86 27 62559779; fax: +86 27 87556437.
E-mail address: wjzhan@mail.hust.edu.cn (W. Zhan).

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are extraordinarily efficient even though each trader has little knowledge of the circumstances of the market. Smith (1982) calls this finding a major "scientific mystery."

Theoretical literature has made considerable progress but a full solution to the mystery remains elusive. The problem is that the CDA allows very complex strategies. Each trader observes a history of offers and transactions unfolding in real time, and must decide contingent on her own value or cost how much to offer and when (e.g., Friedman, 1984; Wilson, 1987; Easley and Ledyard, 1996).

Gode and Sunder (1993) develop automated agents to probe the inner workings of the CDA. They find that even non-strategic, random offers to buy (bids) and to sell (asks) produce highly efficient outcomes in CDA markets. The main requirements are that traders are persistent, and the asks involve only non-negative markups over the sellers' costs and the bids involve only non-negative markdowns from the buyers' values. CDA simulations more closely approximate human behavior when agents adjust beliefs and hence offers in response to previous transactions (Gjerstad and Dickhaut, 1998), or adjust offers directly (Cliff and Bruten, 1997).

The goal of the simulations we present below is not to approximate human behavior, but rather to gain insight into how traders' profit motive influences the performance of CDA market. We simplify strategies to a single dimension, called markup: each trader chooses some target profit rate m and uses it to determine his/her offers to buy or to sell. Thus a buyer's offer to buy (bid) is a linear function of his value, a seller's offer to sell (ask) is a linear function of her cost.

The effect of markup on market efficiency can be derived analytically in the simple call market (CM) format, as shown in Section 3. In the CDA format it seems impractical to derive analytic results, but computer simulations allow us to dissect market efficiency and other CDA market outcomes such as buyer and seller surplus, transaction volume, average transaction price, and price dispersion.

In particular, we seek answers to two questions:

(1) How do traders' markups influence CDA market outcomes? In particular, which markups are most efficient?
(2) Under various specifications of the game played in CDA markets, what are the Nash equilibrium (NE) markups, and how do they compare to the efficient markups?

The paper is organized as follows. Section 2 presents the CDA simulation design and defines the markup rules and the outcome variables. Section 3 derives analytic results for the simple CM format. Section 4 uses simulations to demonstrate how uniform markups influence CDA market outcomes.

Section 5 allows different traders to use different markups in the CDA. NE markups are found in a notional two-cartel game and in the full game with N buyers and N sellers. In general, one expects that NE will not be efficient, but it turns out that symmetric Nash equilibrium (SNE) markups in the thick CDA market are remarkably close to efficient markups. Thus the CDA seems to reconcile efficiency with profit seeking.

The last section summarizes, offers interpretations, and suggests future research. The appendix contains a flow chart of the C and MATLAB programs used in our CDA simulations, and also contains the derivation of the CM analytical results.

2. Simulation setup

2.1. CDA market rules

The participants or *traders* in the CDA market simulations are N automated buyers and N automated sellers. The baseline, called the *thick* market, is $N = 100$. In order to check robustness and to compare laboratory studies, we also examine a *medium* market with $N = 10$, and a *thin* market with $N = 4$.

The market is open for $T > 0$ periods, e.g., $T = 2500$ in the baseline. At the beginning of each period, every buyer i receives a valuation v_i, independently drawn from a distribution on (0, 200). The valuation applies to a single indivisible unit of the good. Likewise, each seller j has the capacity to produce a single indivisible unit of the good at a cost c_j independently drawn from a distribution on (0, 200). In the baseline, both buyers and sellers draw from the same uniform distribution.

The algorithm for the CDA market may be described as follows.

Initially in each period all buyers and sellers are active. In each cycle within a period, an active buyer or seller is selected randomly. A selected buyer submits a bid according to Eq. (1). Similarly, a selected seller submits an ask according to Eq. (2). A new bid goes into the bid queue that is sorted from the highest to the lowest. A new ask goes into the ask queue that is sorted from the lowest to the highest. The *best bid* is the highest in the bid queue, and the *best ask* is the lowest in the ask queue. Whenever the best bid is equal to or higher than the best ask, there is an immediate transaction at a price p that is equal to the best bid or the best ask, depending on which offer came first. A transaction removes the best buyer and the best seller from their queues, and they become inactive for the remainder of the period. The new best bid is the highest remaining in the bid queue, new best ask is the lowest remaining in the ask queue. Then a new cycle begins with another randomly selected buyer or seller. Cycles continue until no active seller will ask a price lower than any active buyer will bid. Then the period ends and market outcomes are recorded. Finally, outcomes are averaged over all periods.

See the Appendix for a flow chart for the C and MATLAB programs.

2.2. Outcome variables

When buyer i transacts at price p, his profit (or surplus) is $v_i\!-\!p$. Likewise, when seller j transacts at price p, her profit (or surplus) is $p\!-\!c_j$. Profit is 0 for buyers and sellers who do not transact in a given period. *Buyers' surplus* in a given period is the sum of profits earned by all buyers that period, and *sellers' surplus* is the corresponding sum of all individual seller profits. *Total surplus* is the sum of buyers' surplus and sellers' surplus. *Transaction volume* is the number of transactions in a

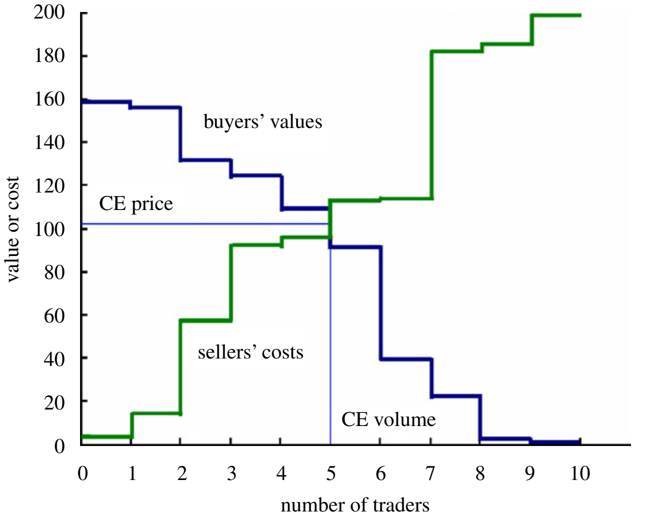

Fig. 1. Competitive equilibrium (CE) in a market with $N = 10$ buyers and sellers.

period. In a CDA market, different transactions may have different prices. Within each period we track the mean and standard deviation of prices. We report the *average price*, the mean transaction price averaged over all periods. The standard deviation is a measure of absolute price dispersion, but we focus mainly on (relative) *price dispersion*, defined within each period as the standard deviation of price divided by the CE price, i.e., the coefficient of variation, and averaged over all periods.

Efficiency is the key performance variable. It is defined as the total surplus as a percentage of the maximum feasible surplus (i.e., CE surplus), given the specific cost and value realizations that period. Fig. 1 illustrates the computation. The realized buyer values are sorted from highest to lowest and graphed as a stairstep demand curve, and the realized seller costs are sorted from lowest to highest and graphed as a supply curve. Their intersection defines the competitive equilibrium (CE) price and trading volume.[1] Since realized buyer values and seller costs are drawn randomly at the beginning of each period, the CE price and CE volume can vary across periods. Each period we record the ratio of transaction volume to CE volume and the ratio of average price to CE price, and we report the T-period averages.

In Fig. 1, CE volume is 5 and CE prices lie in the interval [95.3, 108.4]. As explained in every Principles of Economics text, the total surplus (buyers' surplus plus sellers' surplus) is maximized in CE and is represented by the area between the demand and supply curves to the left of their intersection. CE total surplus is 416.1 in Fig. 1. If the realized total surplus were 400 in a particular period, then efficiency

[1] With stairstep supply and demand curves, there can be a range of CE prices when the curves intersect on a vertical segment. Alternatively, when the intersection is on a horizontal segment, there can be a range of CE volumes. In such cases, we take the midpoint, but all choices of CE price and volume lead to the same CE total surplus and to the same efficiency.

would be $(400/416.1) \times 100\% \approx 96.1\%$. If the realized transaction volume were 4 that period, then the ratio of transaction volume to CE volume would be $4/5 = 0.8$. If the realized average price were 100 that period, then the ratio of average price to CE price would be $(100/((95.3+108.4)/2)) \approx 0.98$.

3. Markups and the CM

The baseline trader algorithms (the "standard markup") use a specific value $m \in [0,1]$ that determines bids b_i and asks a_j according to the formulas:

$$b_i = v_i(1-m), \tag{1}$$

$$a_j = c_j(1+m). \tag{2}$$

For example, if $m = 0.3$, then each buyer bids 30% below his own value and each seller asks 30% above her own cost. Thus, to the extent that $m > 0$, traders under-reveal their true willingness to transact, in order to improve the profitability of their transactions.

To understand the impact of markups, it is instructive to begin with the simple market format known as the CM. In the CM, each trader simultaneously makes a single bid or ask, and these offers are cleared at a uniform price that matches supply and demand.

Fig. 2 illustrates. The solid lines show true demand and supply in a very thick market, and the corresponding dashed lines show the demand and supply revealed in bids and asks generated by Eqs. (1)–(2) when $m = 0.3$. Point F indicates the resulting price and transaction volume under the CM market format. The figure suggests that, relative to CE (point E), transaction volume is reduced by about 30%, efficiency falls by about 10% due to the "deadweight loss" triangle ABE, and price also declines by about 10%.

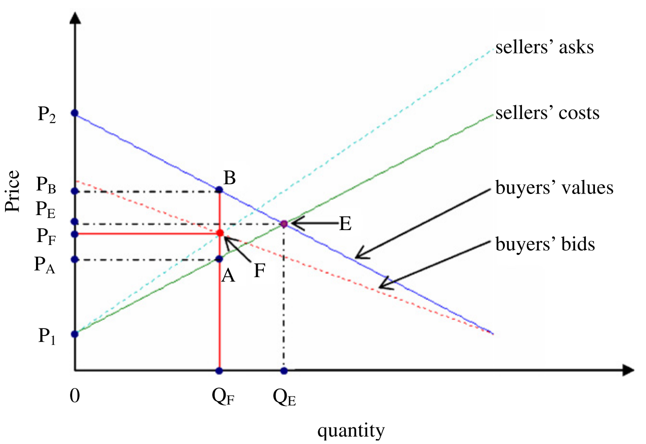

Fig. 2. True and revealed demand and supply when $m = 0.3$.

One obtains analytic results as follows. In our thick market with uniform distributions on (0,200), sampling variation is quite small. Neglecting sampling variation, inverse demand is $P^D = 200 - Q$ and inverse supply is $P^S = Q$ for $0 \leqslant Q \leqslant 200$. Equating these expressions for true demand and supply, we obtain CE price and quantity $P_E = Q_E = 100$. The CE sellers' surplus SS_E is the area of triangle $P_1EP_E = 100 * 100/2 = 5000$ and likewise CE buyers' surplus is $BS_E = 5000$. Thus CE total surplus is $TS_E = SS_E + BS_E = 10,000$.

When all buyers and all sellers choose markup $m \geqslant 0$, the revealed inverse demand becomes $(1-m)$ (200–Q) and revealed inverse supply becomes $(1+m)Q$. Equating revealed demand and supply, one obtains the following expressions for CM outcomes (F) relative to CE outcomes (E):

Volume $= Q_F/Q_E = 1-m$, price $= P_F/P_E = 1-m^2$, buyers' surplus $= BS_F/TS_E = 0.5+0.5m^2-m^3$, sellers' surplus $= SS_F/TS_E = 0.5-1.5m^2+m^3$ and efficiency $= TS_F/TS_E = 1-m^2$. See the Appendix for the derivations obtained in a somewhat more general setting, and see Fig. 3 for a graph of analytic functions of CM efficiency, CM buyers'/sellers' surplus. The graphs of analytic functions of CM volume and CM price are shown in Figs. 7 and 8, respectively. All the analytic functions are indistinguishable from simulation averages.

As one might expect, overall efficiency declines slowly at first and then more rapidly as m increases, and sellers' surplus also declines monotonically. Perhaps surprisingly, buyers' surplus increases slightly in m at first, and declines only when $m > 1/3$. The ratio of buyers' surplus to sellers' surplus increases in m over the entire range. The intuition for this "buyer bias" asymmetry can be seen in Fig. 2 and Eqs. (1)–(2). When m is near 1, then all bids are near 0, while asks are evenly spread between 0 and $(1+m)200 \approx 400$. Since transaction prices cannot exceed the highest

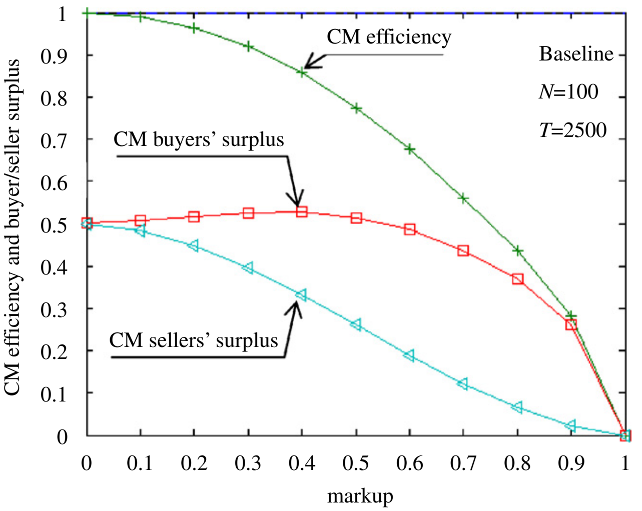

Fig. 3. Call market (CM) efficiency and buyers'/sellers' surplus in baseline.

bid, the prices are quite low. Hence the transactions that do occur are much more profitable for buyers than for sellers.

The asymmetry led us to consider two alternative markup specifications: exponential and shift. Exponential markups replace the factors $1 \pm m$ in (1)–(2) by $\exp(\pm m)$. This sort of markup is used in Cason and Friedman (1999) and it implies that even at $m = 1$ the bids are about 37% of true value, not 0%, while asks are about 270% of true cost, not 200%.[2] We also considered shift markups, in which $b_i = v_i - 100 \times m$ (truncated below at $b_i = 0$) and $a_j = c_j + 100 \times m$. We shall see when these alternatives produce results notably different from the baseline specification (1)–(2).

Before returning to the CDA market format, we should mention the sophisticated theoretical literature examining strategic behavior and efficiency in the CM. The markup is the *only* strategic variable in CM markets, and it turns out that rules of the form (1) and (2) are very close approximations of Bayesian Nash equilibria given uniformly distributed values and costs (e.g., Rustichini et al., 1994). As the number of buyers and sellers N increases, the equilibrium markup m approaches 0 and so efficiency is high. Indeed, CM markets are asymptotically efficient under quite general conditions (e.g., Cripps and Swinkels, 2006).

4. Uniform markups in the CDA

The complexity of the CDA seems to preclude analytic results parallel to those of the previous section. Therefore we use simulation methods to analyze the impact of markups on CDA market outcomes, and use the corresponding CM results as a benchmark. In this section we consider the case where all traders use the same markup m.

Recall that the efficiency is maximized in a CM at $m = 0$, where buyers and sellers fully reveal true values and costs and CE is achieved. One might think that the same would be true in the CDA market, but Fig. 4 shows otherwise. As m increases from 0 to 0.3, efficiency increases! The figure shows that sellers' surplus as well as buyers' surplus increases at first. Of course, as m increases further, eventually transaction volume dries up and efficiency declines.

Fig. 5 helps explain what is going on. Traders who transact in CE (viz., buyers with values v_i above the CE price p^* and sellers with cost c_j below p^*) are called intermarginal (IM); the other traders are called extramarginal (EM). In CE, by definition, all transactions are between IM buyers and IM sellers (T_1 in Fig. 5). When an IM buyer or seller fails to transact, there is a loss of total surplus equal to that trader's CE profit, viz., $v_i - p^* > 0$ for an IM buyer (T_4 in Fig. 5), and $p^* - c_j > 0$ for an IM seller (T_5). The sum of such losses is called *V-inefficiency* (VI). Formally,

$$\mathrm{VI} = \left\{ \sum_{i \in \mathrm{IMBN}} (v_i - p^*) + \sum_{j \in \mathrm{IMSN}} (p^* - c_j) \right\} \Bigg/ TS_{\mathrm{E}}, \tag{3}$$

[2] As a referee correctly comments, standard markups and exponential markups pick up different grid points in the same one-dimensional strategy space as m varies in the discrete set $\{0, 0.1, 0.2, \ldots, 0.9, 1.0\}$, while shift markups pick up grid points in a slightly different one-dimensional strategy space.

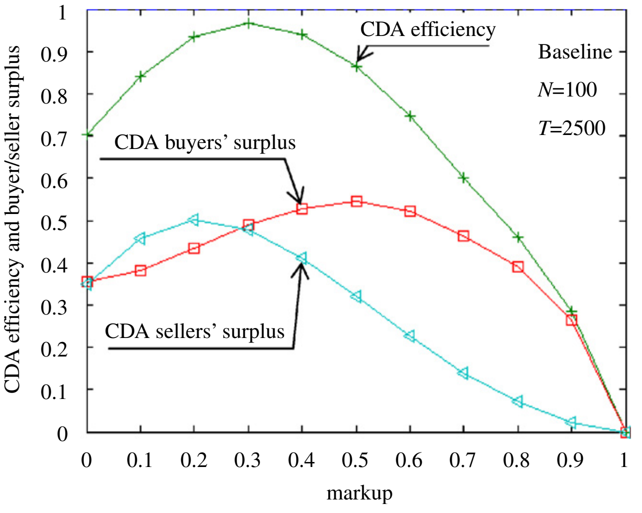

Fig. 4. CDA market efficiency and buyers/sellers surplus.

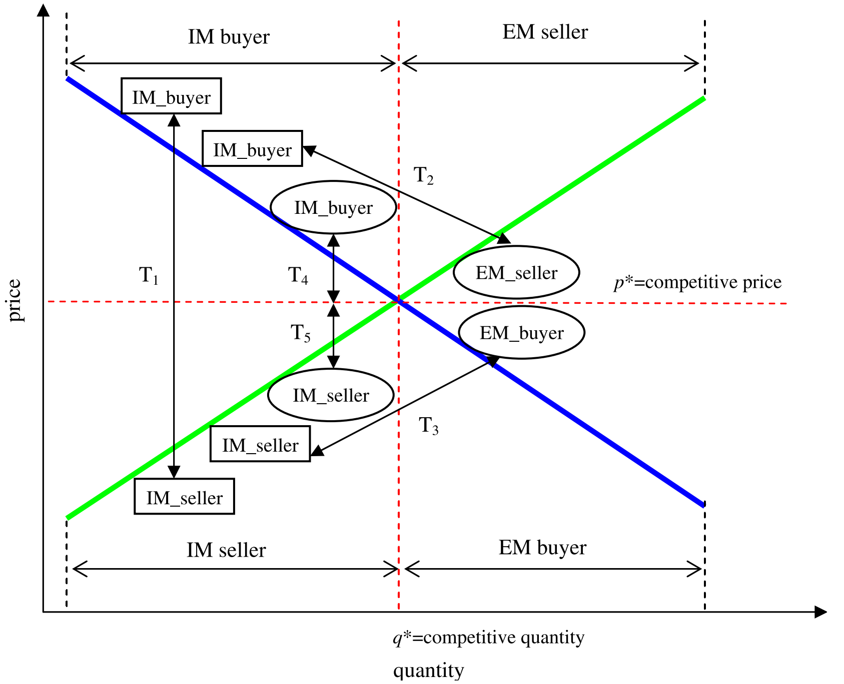

Fig. 5. Efficient and inefficient transactions in the CDA.

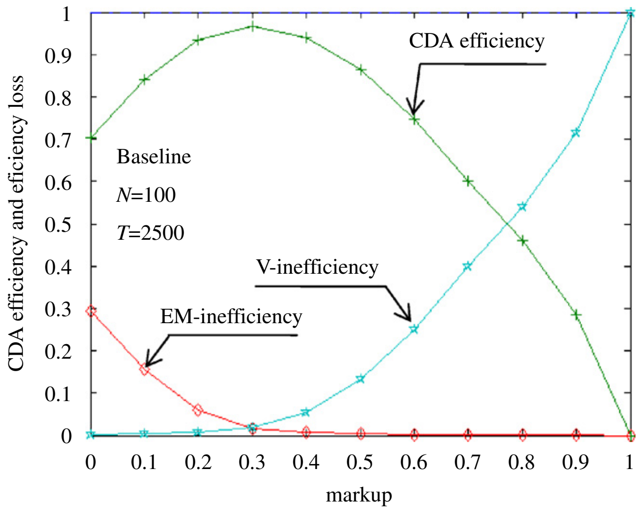

Fig. 6. Efficiency loss in CDA market.

where IMBN is the set of IM buyers who do not trade, IMSN is the set of IM sellers who do not trade, and TS_E is the CE total surplus. All losses in the CM with $m > 0$ are of this sort.

But another sort of loss is possible in the CDA: an EM trader may transact, creating an overall loss equal to her (negative) profit at CE price p^*. For example, when EM buyer i trades with an IM seller (T_3 in Fig. 5), there is a loss of total surplus equal to $p^* - v_i > 0$. Likewise, when EM seller j trades with an IM buyer (T_2), the total surplus decreases by $c_j - p^* > 0$. The sum of such losses in a market period is called the *EM-inefficiency* (EMI). Thus

$$\mathrm{EMI} = \left\{ \sum_{i \in \mathrm{EMBT}} (p^* - v_i) + \sum_{j \in \mathrm{EMST}} (c_j - p^*) \right\} \Big/ TS_E, \tag{4}$$

where EMBT is the set of EM buyers who trade, and EMST is the set of EM sellers who trade. Unwinding the definitions, one can verify that

$$\text{CDA efficiency} = 1 - \mathrm{EMI} - \mathrm{VI}. \tag{5}$$

That is, the realized CDA surplus plus the losses due to EMI and VI sum to the CE total surplus.[3]

Fig. 6 shows that EMI is maximized at $m = 0$ in a thick CDA market. It decreases fairly quickly towards 0 as m increases. For the same reasons as in the CM market, V-inefficiency rises at an increasing rate in m: with larger markups, fewer asks fall below bids and transaction volume goes to zero as m increases to 1. The figure shows

[3]Two slightly different conventions for decomposing efficiency losses can be found in Rust et al. (1993) and Cason and Friedman (1996).

that the sum of EM- and V-inefficiency is minimized at $m = 0.3$, so by (5) efficiency is maximized at that point.

Fig. 7 bolsters this explanation by dissecting transaction volume. Normalizing CE volume to 1.0, the figure shows that CM volume (which automatically excludes EM traders) declines steadily as markup m increases according to the equation CM volume $= 1-m$ (see Section 3 and the Appendix). CDA volume also decreases, but starts out substantially higher.

Fig. 7 decomposes the CDA transaction volume into three components:

$$\text{CDA volume} = Q_1 + Q_2 + Q_3, \tag{6}$$

where Q_1 is the normalized number of transactions between IM buyers and IM sellers (T_1 in Fig. 5), Q_2 is the normalized number of IM buyers trading with EM sellers (T_2), and Q_3 is the normalized number of EM buyers trading with IM sellers (T_3). Together Q_2 and Q_3 account for EM inefficiency. (They have no counterpart in CM, and they are the source of CDA volume in excess of CE volume.) Fig. 7 also shows Q_4, the normalized number of IM buyers failing to transact (T_4 in Fig. 5), and Q_5, the normalized number of IM sellers failing to transact (T_5). Together Q_4 and Q_5 account for V-inefficiency. Over the range $m \in (0,0.3)$, efficiency-enhancing volume Q_1 increases and efficiency-impairing volume Q_2 and Q_3 decrease quickly, while the sources of V-inefficiency, Q_4 and Q_5 increase slightly. However, over the range $m \in (0.3,1)$, Q_1 decreases, Q_2 and Q_3 decrease slightly while Q_4 and Q_5 increase quickly as markup m increases.

Fig. 8 examines the average transaction price, normalizing CE price to 1. The (uniform) CM price decreases gradually from 1 to 0 as $1-m^2$, as noted in Section 3. The CDA average transaction price increases slightly at first, and then decreases

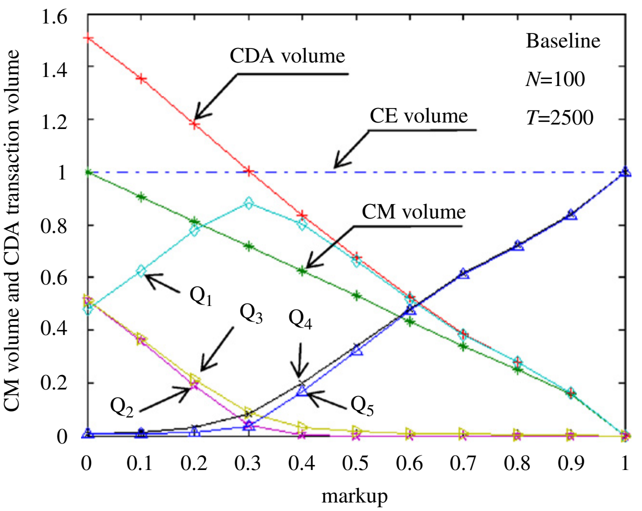

Fig. 7. Effect of markup on transaction volume.

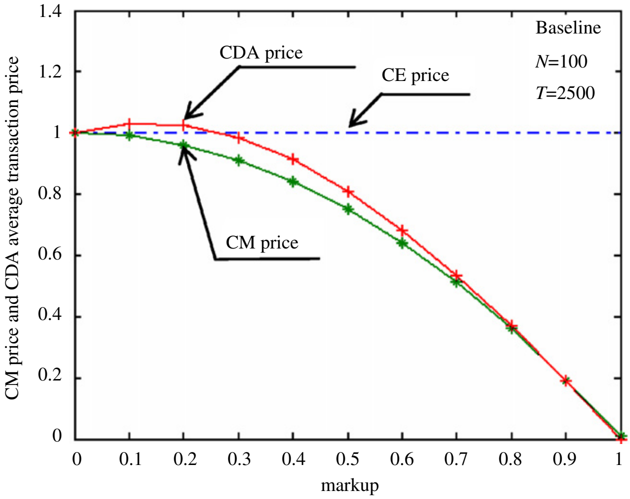

Fig. 8. Effect of markup on average transaction price.

gradually with $m \in [0.1, 1]$.[4] With exponential markup, the CDA average transaction price is similar to Fig. 8 except that: (1) the increase in average transaction price occurs over the range $m \in (0,0.2)$; (2) when $m = 1$, the average transaction price is 0.66, not 0. With shift markups, the CDA average transaction price remains very close to the CE price over the entire range of m.

Finally, Fig. 9 shows price dispersion in the baseline CDA market. It decreases in m due to the narrowing range of bids. In medium ($N = 10$) and thin ($N = 4$) markets, price dispersion declines in a roughly parallel fashion from a slightly higher value at $m = 0$. With the exponential and shift markup rules, the price dispersion also decreases as markup m increases. By way of comparison, the price dispersion is 0.17, 0.26, and 0.31, respectively in thick, medium, and thin markets for the main simulation algorithm (called ZI-C) in Gode and Sunder (1993).[5]

The other results are also fairly robust to variations from the baseline simulation. Efficiency is maximized in a thin CDA market at $m = 0.1$, and in a medium market at $m = 0.2$. Efficiency is maximized with exponential markups and with shift

[4] When the transaction price determination rule changes from the first offer (either the best bid or the best ask) to the average of the best bid and the best ask, the slight initial increase in average price disappears. An intuitive explanation for the increase is that as sellers' markup m increases, low bids are less likely to be accepted. Although the overall distribution of bids decreases slightly for small positive markups, the distribution of asks shifts up so as to select more strongly those bids towards the upper end of the distribution.

[5] Price dispersion in the ZI-C model is lower than that in uniform markup = 0.3 for two reasons. The first reason is that ZI-C markups exceed 0.3 on average; e.g., it can be shown that buyer markups average 0.5 under standard conventions. The second reason is that ZI-C sellers with lower costs have higher markups on average (and analogously for buyers under other conventions). As noted in the concluding discussion, price dispersion is lower when markups are decreasing functions of a trader's CE surplus.

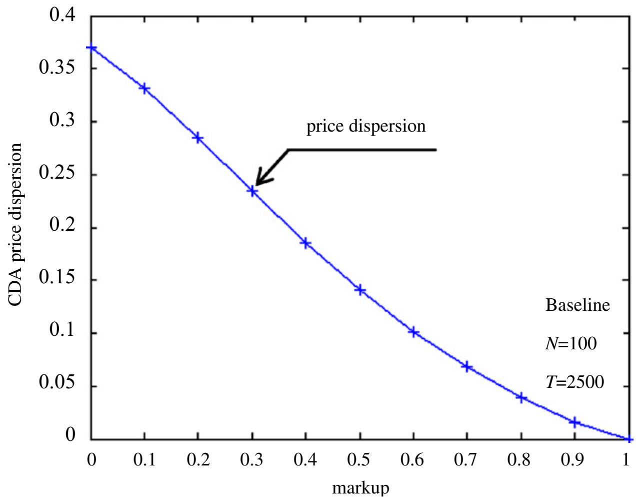

Fig. 9. Effect of markup on CDA price dispersion.

markups at the same values of m as in standard markups for all three markets (thick, medium and thin). The buyer bias disappears with shift markups but is only slightly attenuated with exponential markups.

5. Nash equilibria in CDA games

5.1. Two-cartel game

Suppose that $m = m_b$ for all buyers i in Eq. (1) while $m = m_s$ all sellers j in Eq. (2). A fanciful interpretation is that all buyers belong to a cartel, and all sellers belong to a second cartel, and the members of each cartel agree on a common markup. Formally, the first player B chooses m_b from the set $\{0.0, 0.1, 0.2, \ldots, 0.9, 1.0\}$ and the second player S simultaneously chooses m_s from the same set. The payoffs are respectively buyers' surplus and sellers' surplus, computed numerically by averaging across periods and normalized by the CE surplus each period. Note that the two-player game is competitive in the sense that larger m_s reduces B's payoff at each m_b, and larger m_b reduces S's payoff at each m_s.

Does this two-player game have a NE in pure strategies? Fig. 10 shows that it is a dominant strategy in the baseline market for the buyer cartel to set $m_b = 0.6$, because that maximizes profit in a CDA market for each seller cartel's choice m_s.

In turn, Fig. 11 shows that the best response for the seller cartel to the buyer cartel's dominant strategy is $m_s = 0.5$. As a result, we conclude that the unique NE for the baseline two-player game is ($m_b{}^* = 0.6$, $m_s{}^* = 0.5$).

The same two-player game for the medium market ($N = 10$) has the unique NE ($m_b{}^* = 0.5$, $m_s{}^* = 0.6$). In the thin market ($N = 4$), the NE moves down to

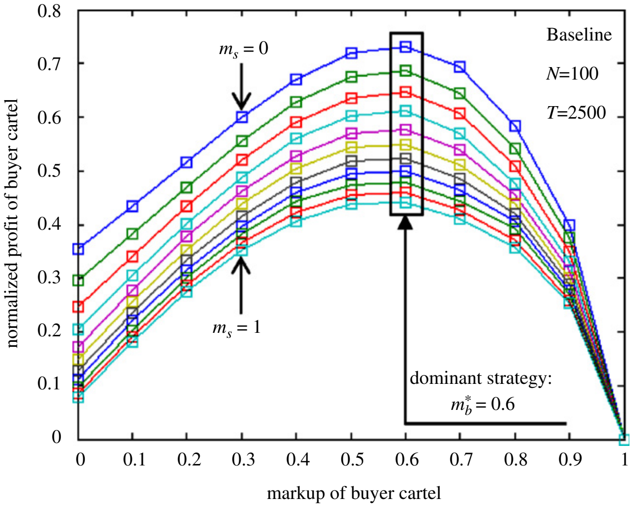

Fig. 10. Dominant strategy for the buyer's cartel.

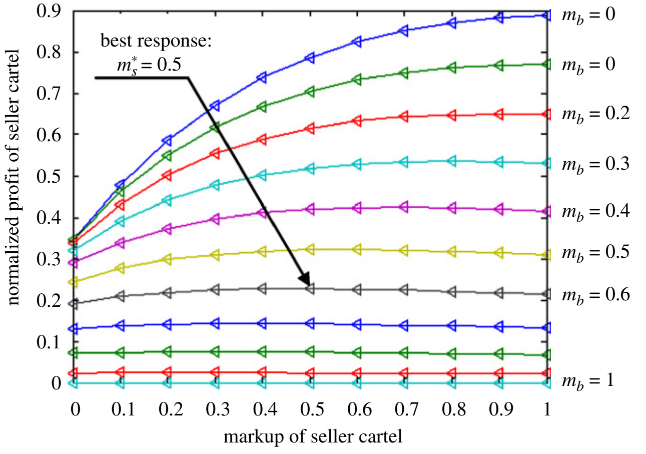

Fig. 11. Best response for the seller's cartel.

($m_b^* = 0.4$, $m_s^* = 0.5$). With exponential markups, the NE markup of this two-cartel game is ($m_b^* = 0.8$, $m_s^* = 0.4$) in a thick market, ($m_b^* = 0.7$, $m_s^* = 0.4$) in a medium market, and ($m_b^* = 0.5$, $m_s^* = 0.4$) in a thin market; adjusting for the different grid points, the Nash equilibria are fairly close to their standard markup counterparts. With shift markups, the Nash equilibria become ($m_b^* = 0.6$, $m_s^* = 0.6$) in a thick market, ($m_b^* = 0.5$, $m_s^* = 0.5$) in a medium market, and ($m_b^* = 0.4$, $m_s^* = 0.4$) in a thin market.

5.2. The 2N-player game

Thus more relevant CDA game allows each of the N buyers and each of the N sellers simultaneously to choose his/her own markup from the same set $\{0.0, 0.1, 0.2, \ldots, 0.9, 1.0\}$. In SNE,[6] all buyers choose the same m_{b} and all sellers choose the same m_s, and no single buyer or seller can increase profit by deviating to a different markup. Do such equilibria exist?

To find out, we check for profitable deviations. First, for buyers we numerically compute the payoffs to a single deviator buyer choosing various markup values m_i when all other $(N-1)$ regular buyers choose some particular m_{b} and all N sellers choose some particular m_{s}. The deviator buyer's profit depends sensitively on his value v_i. Numerical explorations confirm that averaging profit over the distribution of values (uniform over (0, 200)) can be closely approximated by a simple average of profit for values $\{190, 180, 170, \ldots, 10\}$. The deviator buyer's profit is normalized by $1/N$ of CE surplus period by period and averaged over all periods. The deviator buyer's best response m_i^* is defined as the markup that makes the largest profit. For each value of m_{b} and m_{s}, we check for the symmetry condition ($m_i^* = m_{\mathrm{b}}$, m_{s}). Similarly, we consider a single deviator seller choosing various markup values m_j when all other $(N-1)$ regular sellers choose some particular m_{s} and all N buyers choose some particular m_{b}. We compute the deviator's payoff by averaging realized profit over costs $\{10, 30, 40, \ldots, 190\}$, and denote the best response by m_j^*. For each m_{b} and m_{s}, we check for the symmetry condition (m_{b}, $m_j^* = m_s$). When both symmetry conditions hold ($m_i^* = m_{\mathrm{b}}$, $m_j^* = m_{\mathrm{s}}$), we have SNE of our $2N$-player game.

Fig. 12 shows the results for a thick ($N = 100$) CDA market averaged over 2500 periods when $m_{\mathrm{s}} = 0.3$. Note that when the regular buyers' margin is $m_{\mathrm{b}} = 0.4$, the deviator buyer's best response is $m_i^* = 0.4$, satisfying the first symmetry condition.

Fig. 13 shows the deviator seller's average profit with different markup when all the buyers' markups are 0.4 and regular sellers' markups are 0.3. The best response for the deviator seller is also $m_j^* = 0.3$, so the second symmetry condition is satisfied. Thus $\{m_i^* = 0.4, m_j^* = 0.3\}_{i,j = 1,2,\ldots,N}$ is an SNE for the $2N$-player game. Thorough exploration of other values for m_{b} and m_{s} confirms that this SNE is unique.

For medium and thin markets, the unique SNE markup turns out to be $\{m_i^* = 0.3,\ m_j^* = 0.3\}_{i,j = 1,2,\ldots,N}$. With an exponential markup, the epsilon SNE markup is $\{m_i^* = 0.5,\ m_j^* = 0.3\}_{i,j = 1,2,\ldots,N}$ in the thick market.[7] The unique SNE markups are $\{m_i^* = 0.4,\ m_j^* = 0.3\}_{i,j = 1,2,\ldots,N}$ in the medium market, and $\{m_i^* = 0.3,\ m_j^* = 0.2\}_{i,j = 1,2,\ldots,N}$ in the thin market. With a shift markup, epsilon SNE markups are $\{m_i^* = 0.4,\ m_j^* = 0.4\}_{i,j = 1,2,\ldots,N}$ in the

[6] SNE strategies are symmetric contingent on trader type; i.e., all traders use the same mapping from type to markup. All buyers choose the same strategy, as do all sellers, but the common buyer markup (i.e., markdown) in Eq (1) may or may not be the same as the seller markup in Eq (2).

[7] In this case, the deviator seller earns 0.3412, 0.3519, 0.3508, 0.3443, and 0.3358 respectively for $m_{\mathrm{s}} = 0.1$, 0.2, 0.3, 0.4, and 0.5. The deviator buyer maximizes profit at $m_i = m_{\mathrm{b}} = 0.5$. Thus $m_j = 0.3$ is part of an epsilon NE for epsilon = (0.3519–0.3508) = 0.0011.

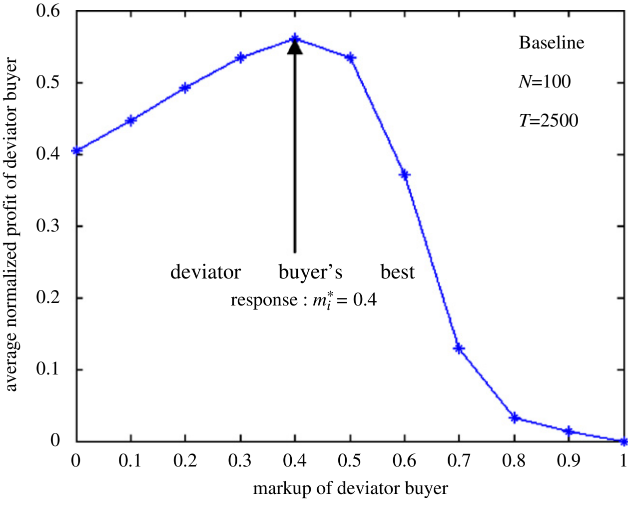

Fig. 12. Deviator buyer's average normalized profit with ($m_b = 0.4$, $m_s = 0.3$).

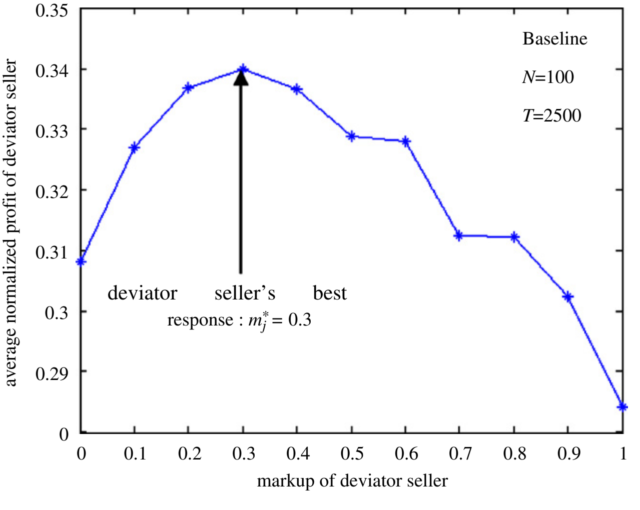

Fig. 13. Deviator seller's average normalized profit with ($m_b = 0.4$, $m_s = 0.3$).

thick market,[8] and $\{m_i^* = 0.4,\ m_j^* = 0.4\}_{i,j=1,2,\ldots,N}$ in the medium market.[9] The unique SNE markup is $\{m_i^* = 0.3,\ m_j^* = 0.3\}_{i,j=1,2,\ldots,N}$ in the thin market.

[8]In this case, the deviator seller earns 0.3681, 0.3949, 0.4201, 0.4301, 0.4313, and 0.4203 respectively for $m_s = 0.1$, 0.2, 0.3, 0.4, 0.5, and 0.6. Thus $m_j = 0.4$ is part of an epsilon NE for epsilon = (0.4313–0.4301) = 0.0012.

[9]In this case, the deviator buyer earns 0.39480, 0.42529, 0.43723, 0.43679, and 0.42225 respectively for $m_b = 0.1$, 0.2, 0.3, 0.4, and 0.5, while the deviator seller earns 0.34911, 0.37510, 0.38846, 0.38746, and 0.37221 respectively for $m_s = 0.1$, 0.2, 0.3, 0.4, and 0.5. Thus $m_j = 0.4$ is part of an epsilon NE for epsilon = (0.38846–0.38746) = 0.0100.

Table 1
Summary of market efficiency

Density	Markup	CM	CDA			
		$m = 0$ (%)	$m = 0$ (%)	$m =$ NE (2-cartel) (%)	$m =$ SNE (2N-player) (%)	$m =$ most efficient (%)
Thick	Standard	100	70.4	77.5	95.4	96.6
	Exponential	100	70.1	82.5	95.0[a]	96.7
	Shift	100	70.1	82.9	97.1[a]	97.4
Medium	Standard	100	77.2	70.0	87.9	89.9
	Exponential	100	77.0	72.8	86.1	90.2
	Shift	100	77.3	77.6	86.9[a]	92.3
Thin	Standard	100	82.3	65.8	77.4	86.7
	Exponential	100	82.1	67.2	81.6	86.7
	Shift	100	82.1	71.3	80.6	88.0

Notes: Data are averaged over $T = 2500$, 5000, and 25,000 periods, respectively in thick, medium, and thin markets.
[a]Epsilon SNE.

Table 1 summarizes market efficiency for various markups. Full revelation ($m = 0$) produces efficiency shortfalls relative to CE of about 30%, 23%, and 18%, respectively in the thick, medium, and thin CDA markets. NE in the 2-cartel game suffers efficiency shortfalls of about 17–34%. By contrast, in thick markets, the SNE markups in the full 2N-player game deliver efficiency amazingly close to maximal. The efficiency shortfall is less than 4% in all three cases. In medium and thin markets, the efficiency shortfalls of the 2N-player game widen somewhat, but still are considerably less than those in full revelation or in the 2-cartel game.

In simulations not reported here, we also checked efficient and NE markups when the buyer values and seller costs were drawn from non-uniform distributions. With buyer values and seller costs drawn from the same truncated normal distributions centered at 100, the markups (efficient, NE, and SNE) tend to be smaller than those reported above. When the buyer distribution had a much larger mean than the seller distribution, the markups tended to be closer to those just analyzed. Efficiency at the SNE markup of the 2N-player game remains close to the CDA maximum.

6. Discussion

Our simulations of CDA markets lead to the following main conclusions.

(1) As markup increases, EMI (due to transactions involving EM traders) decreases while V-inefficiency (due to missing intramarginal trades) increases. Their sum is minimized, and hence efficiency is maximized, at a 30% markup in thick markets with $N = 100$ buyers and 100 sellers. In medium ($N = 10$) markets,

efficiencies are maximized at a 20% markup, while in thin ($N = 4$) markets, efficiencies are maximized at a 10% markup, and decline slowly as m increases slightly. These results hold when all traders use the same value of m under each of the three markup rules.

(2) The SNE markup of the $2N$-player CDA game is at or near the efficient markup for both buyers and for sellers in thick, medium, and thin markets with all three markup rules. Consequently, there is very little shortfall from maximal CDA efficiency.

All simulation evidence to date points to the same general conclusion: the SNE markups in CDA $2N$-player games are surprisingly close to the efficient markups, especially in thick markets. A possible interpretation is that the CDA market format is so efficient for a subtle reason. In increasing the markup, traders face a tradeoff between greed (larger profit if there is a transaction) and fear (a greater probability of failing to transact). This tradeoff in SNE seems to parallel the tradeoff between EMI and V-inefficiency in the CDA. Of course, the parallel fails in the CM and perhaps in other market formats.

In the baseline simulation as well as in the variants considered in this paper, each trader had to choose a markup that applied to any possible value or cost. That is, the trader has to commit to his markup before learning his value or cost that period. In future work, it might be worth considering non-linear bid and ask functions, i.e., markups contingent on the realized value or cost. The strategy set of non-linear functions is far more complex, but it allows simulated traders to act more like human traders. Indeed, the results of Friedman and Ostroy (1995) suggest that traders will prefer to have markup be an increasing function of value in CH as well as in CDA markets, and such strategies should reduce CDA price dispersion. It would be interesting to see whether NE behavior in this richer setting still leads to efficiency.

Another important strategic dimension in the CDA is the timing of bids and asks. Rust et al. (1993, 1994) find that a ‘‘sniping strategy’’ (waiting until the last second to make a serious offer is the winner in their market tournament, which suggests that timing is also an important dimension. Gjerstad (2006) finds that slight differences between buyers and sellers in the pace of offers lead to small differences in mean price and efficiency but to relatively large differences in the ratio of buyers' surplus to sellers' surplus. The timing of bids and asks can be investigated by having each trader gradually move her bid towards the value prescribed in the markup rule, with the adjustment speed either constant, or dependent on the time remaining and/or her CE surplus and/or observed behavior. Again, one could investigate whether the additional strategic dimensions break the empirical connection between NE and efficiency in the CDA market.

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Appendix A. Flow chart for the CDA market

See Fig. A1 for further details.

Appendix B. Derivation of CM results

As in Fig. 2, let the true demand function be

$$P^{\mathrm{D}} = \alpha_2 - \beta q \tag{A.1}$$

and the true inverse supply function be

$$P^{\mathrm{S}} = \alpha_1 + \beta q. \tag{A.2}$$

Thus both functions have the same slope parameter $\beta > 0$. Suppose that $\alpha_2 >> \alpha_1 > 0$, i.e., the highest buyer value α_2 far exceeds the lowest seller costs α_1. Applying the markup rule in Eqs. (1)–(2), the revealed demand function is

$$R^{\mathrm{D}} = (1-m)\alpha_2 - (1-m)\beta q \tag{A.3}$$

and the revealed inverse supply function is

$$R^{\mathrm{D}} = (1+m)\alpha_1 + (1+m)\beta q, \tag{A.4}$$

where $m \in [0,1]$ is the uniform markup value.

Equating P^{D} and P^{S} we obtain the CE quantity

$$Q_{\mathrm{E}} = \frac{\alpha_2 - \alpha_1}{2\beta} \tag{A.5}$$

and inserting Q_{E} into (A.1) or (A.2), we obtain CE price

$$P_{\mathrm{E}} = \frac{\alpha_1 + \alpha_2}{2}. \tag{A.6}$$

At each value of m, equate R^{D} and R^{S} to obtain the CM quantity

$$Q_{\mathrm{F}} = \frac{(1-m)\alpha_2 - (1+m)\alpha_1}{2\beta} \tag{A.7}$$

and insert Q_{F} into (A.3) or (A.4) to obtain the CM price

$$P_{\mathrm{F}} = \frac{(1-m^2)(\alpha_1 + \alpha_2)}{2}. \tag{A.8}$$

From (A.5) and (A.7) we obtain

$$\frac{Q_{\mathrm{F}}}{Q_{\mathrm{E}}} = 1 - \left(\frac{\alpha_1 + \alpha_2}{\alpha_2 - \alpha_1}\right) m. \tag{A.9}$$

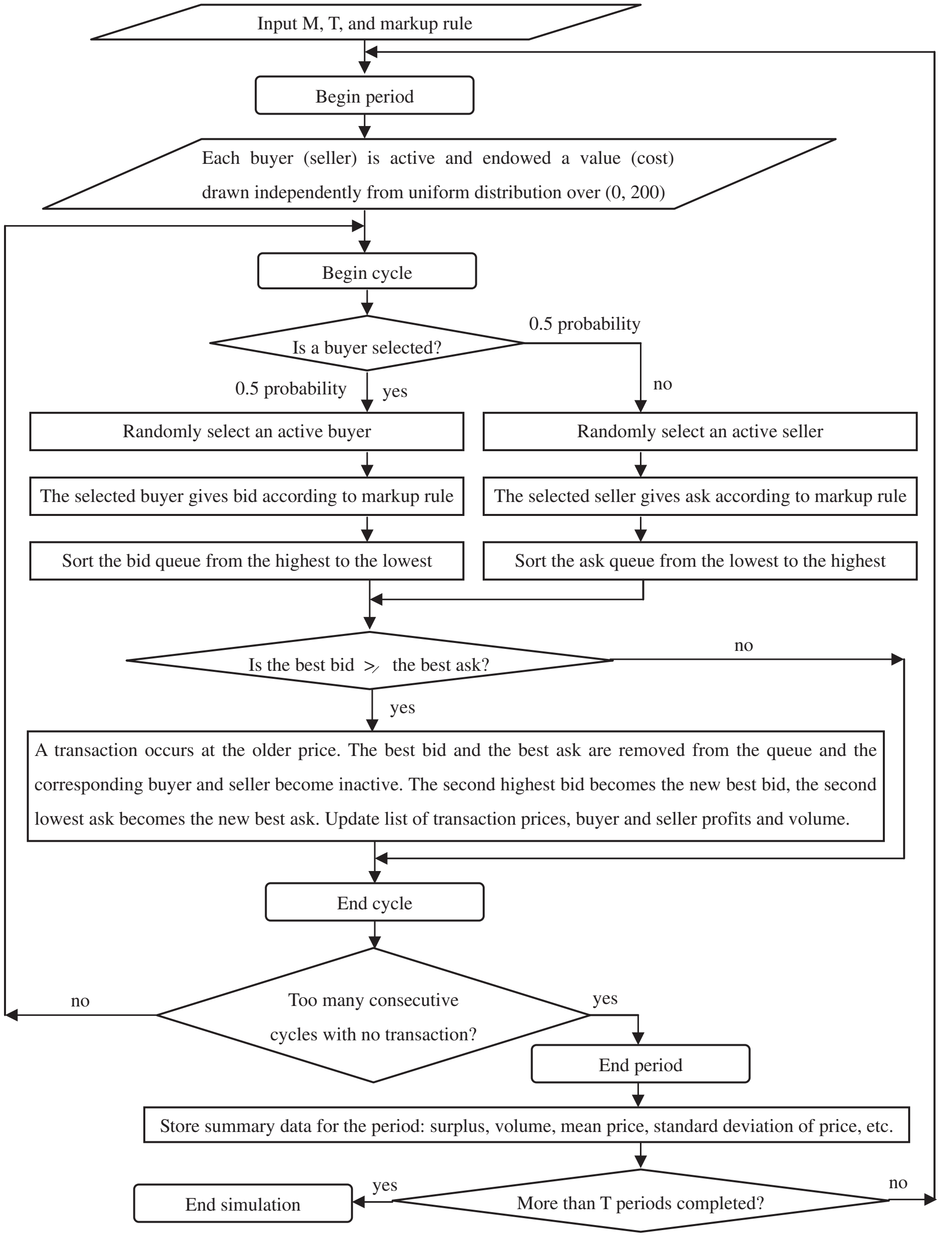

Fig. A1. Flow chart for the continuous double auction market.

Thus normalized CM volume is maximal when $m = 0$, and it decreases linearly in m. From (A.6) and (A.8) we obtain

$$\frac{P_{\mathrm{F}}}{P_{\mathrm{E}}} = 1 - m^2. \tag{A.10}$$

Thus the normalized CE price is maximal when $m = 0$ and decreases quadratically in m, independently of α_1 and α_2.

Evaluating (A.1) and (A.2) at $q = 0$, we get:

$$P_2 = \alpha_2 \tag{A.11}$$

and

$$P_1 = \alpha_1. \tag{A.12}$$

From (A.5), (A.11), and (A.12), the CE surplus is

$$S_{\mathrm{EP_2P_1}} = \frac{|P_2 - P_1|Q_{\mathrm{E}}}{2} = \frac{(\alpha_2 - \alpha_1)^2}{4\beta}. \tag{A.13}$$

Insert Q_{F} into (A.1) to obtain

$$P_{\mathrm{B}} = \frac{(1+m)(\alpha_1 + \alpha_2)}{2} \tag{A.14}$$

and insert Q_{F} into (A.2) to obtain

$$P_{\mathrm{A}} = \frac{(1-m)(\alpha_1 + \alpha_2)}{2}. \tag{A.15}$$

By (A.5), (A.7), (A.14), and (A.15), the deadweight loss at given m is

$$S_{\mathrm{EAB}} = \frac{|P_{\mathrm{B}} - P_{\mathrm{A}}|\,|Q_{\mathrm{E}} - Q_{\mathrm{F}}|}{2} = \frac{m^2(\alpha_1 + \alpha_2)^2}{4\beta}. \tag{A.16}$$

CM efficiency is

$$\frac{S_{\mathrm{EP_2P_1}} - S_{\mathrm{EAB}}}{S_{\mathrm{EP_2P_1}}} = 1 - \left(\frac{\alpha_1 + \alpha_2}{\alpha_2 - \alpha_1}\right)^2 m^2. \tag{A.17}$$

Thus CM efficiency is the maximal at $m = 0$ and decreases quadratically in m.

By (A.7), (A.8), (A.11), and (A.14), the buyers' surplus at given m is

$$\begin{aligned} S_{\mathrm{BFP_FP_2}} &= \frac{[(P_{\mathrm{B}} - P_{\mathrm{F}}) + (P_2 - P_{\mathrm{F}})]Q_{\mathrm{F}}}{2} \\ &= \frac{[(2m^2 + m - 1)\alpha_1 + (2m^2 + m + 1)\alpha_2]}{8\beta}[(1-m)\alpha_2 - (1+m)\alpha_1] \\ &= \frac{-2(\alpha_1 + \alpha_2)^2 m^3 + (\alpha_2^2 - 3\alpha_1^2 - 2\alpha_1\alpha_2)m^2 v + (\alpha_1 - \alpha_2)^2}{8\beta}. \end{aligned} \tag{A.18}$$

By (A.7), (A.8), (A.12), and (A.15), the sellers' surplus at given m is

$$\begin{aligned} S_{\mathrm{AFP_FP_1}} &= \frac{[(P_{\mathrm{F}} - P_{\mathrm{A}}) + (P_{\mathrm{F}} - P_1)]Q_{\mathrm{F}}}{2} \\ &= \frac{[(-2m^2 + m - 1)\alpha_1 + (-2m^2 + m + 1)\alpha_2]}{8\beta}[(1-m)\alpha_2 - (1+m)\alpha_1] \\ &= \frac{2(\alpha_1 + \alpha_2)^2 m^3 + (\alpha_1^2 - 3\alpha_2^2 - 2\alpha_1\alpha_2)m^2 + (\alpha_1 - \alpha_2)^2}{8\beta}. \end{aligned} \tag{A.19}$$

Normalized buyers' surplus is

$$\frac{S_{\mathrm{BFP_FP_2}}}{S_{\mathrm{EP_2P_1}}} = \frac{1}{2} + \frac{(\alpha_2^2 - 3\alpha_1^2 - 2\alpha_1\alpha_2)}{2(\alpha_2 - \alpha_1)^2}m^2 - \left(\frac{\alpha_1 + \alpha_2}{\alpha_2 - \alpha_1}\right)^2 m^3. \tag{A.20}$$

Thus normalized buyers' surplus is a cubic function of m. Similarly, normalized sellers' surplus is

$$\frac{S_{\mathrm{AFP_FP_1}}}{S_{\mathrm{EP_2P_1}}} = \frac{1}{2} + \frac{(\alpha_1^2 - 3\alpha_2^2 - 2\alpha_1\alpha_2)}{2(\alpha_2 - \alpha_1)^2}m^2 + \left(\frac{\alpha_1 + \alpha_2}{\alpha_2 - \alpha_1}\right)^2 m^3. \tag{A.21}$$

Finally, note that (A.20) and (A.21) sum to (A.17), i.e., CM efficiency is the sum of buyers' and sellers' surpluses.

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