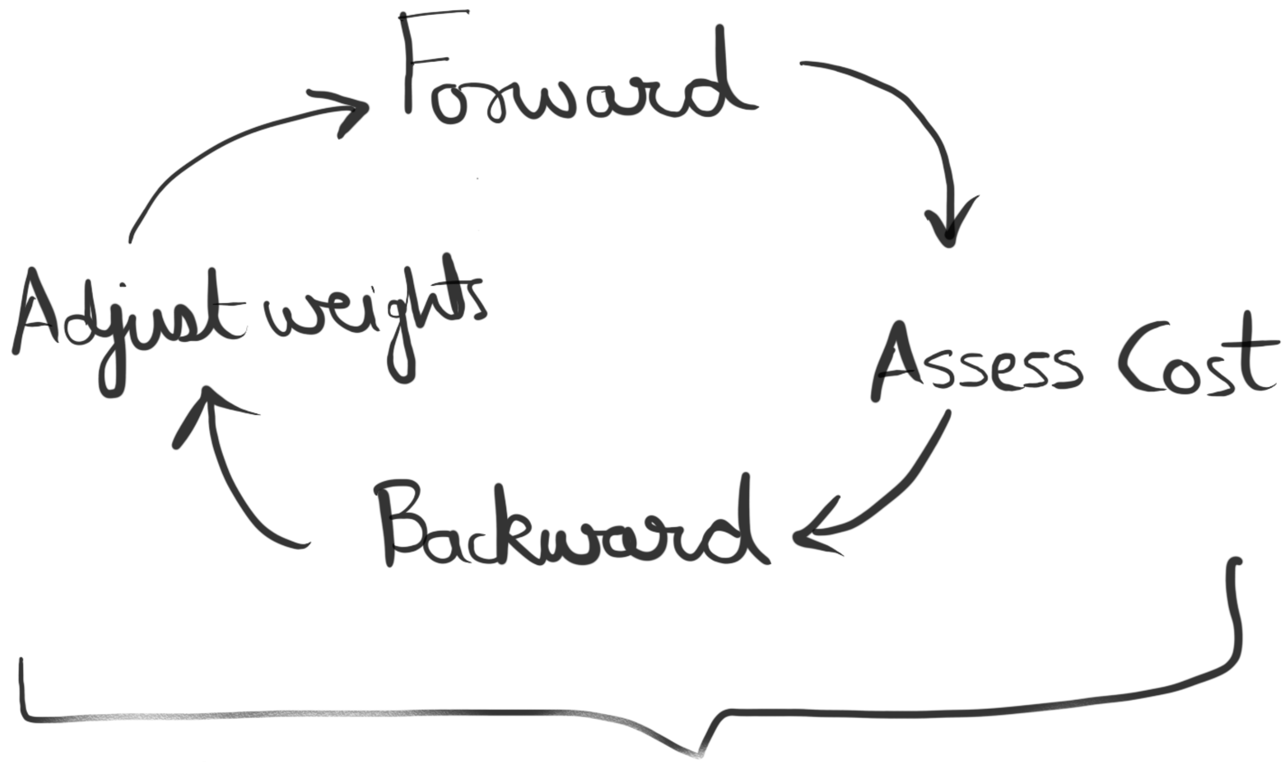
# Neural Networks:

Representation

Learning

Today's focus

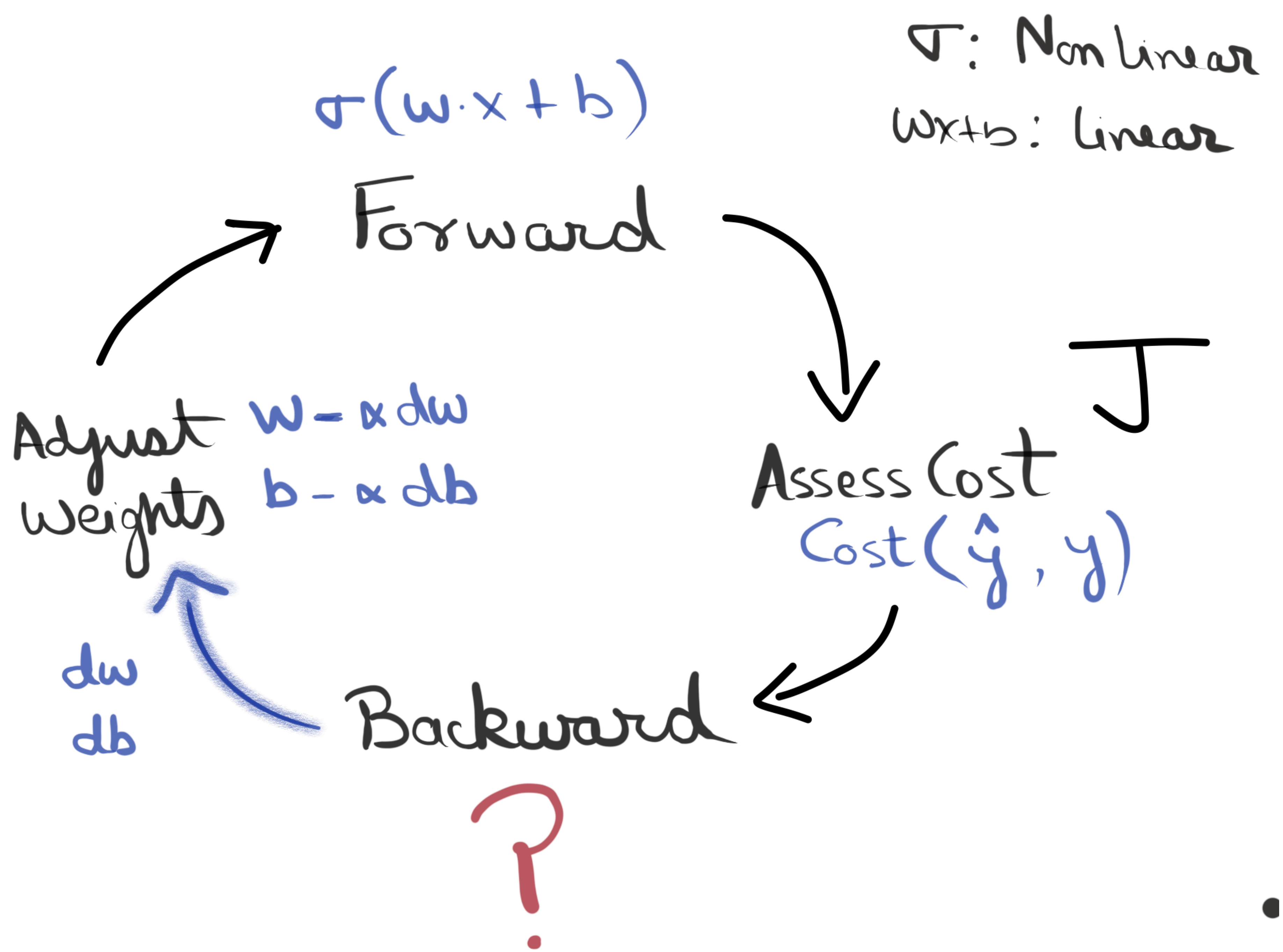
# Neural Network Learning

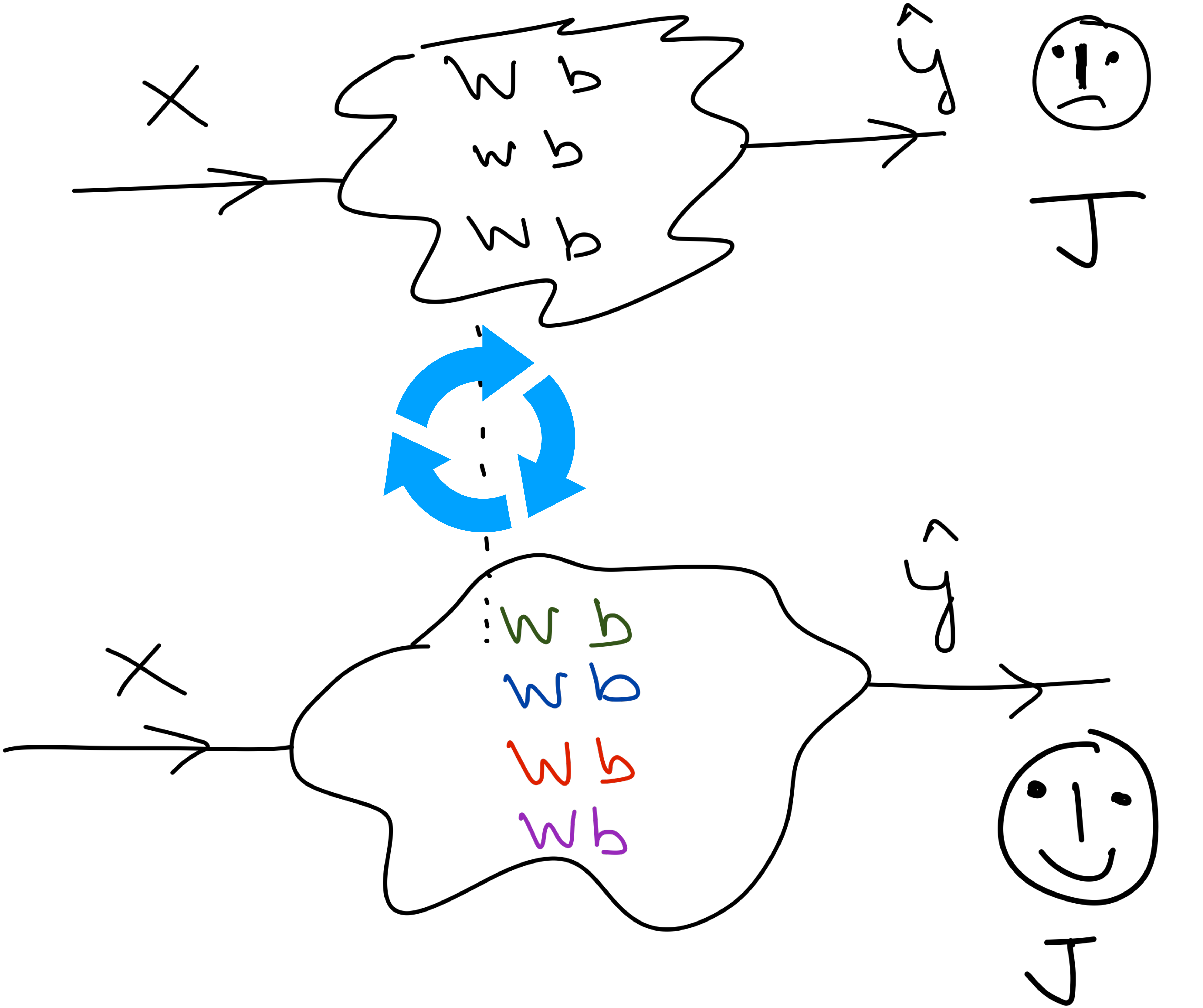



Just plain calculations:  +, - , x, / , functions

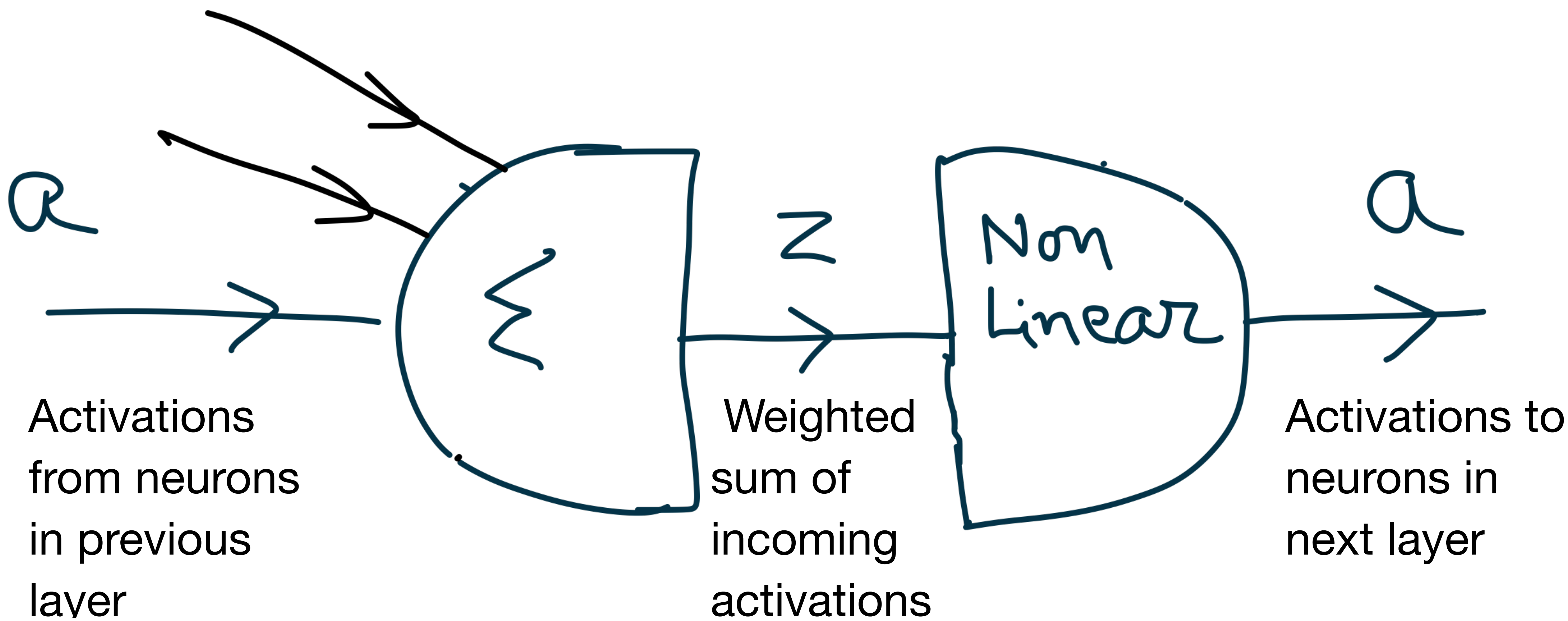
No Algorithmic Cleverness

No smart data structures


σ: Non linear
wx+b: linear
σ(w·x + b)
Forward
J
Assess Cost
Cost(ŷ, y)
Backward
?
dw
db
Adjust weights
w − α dw
b − α db

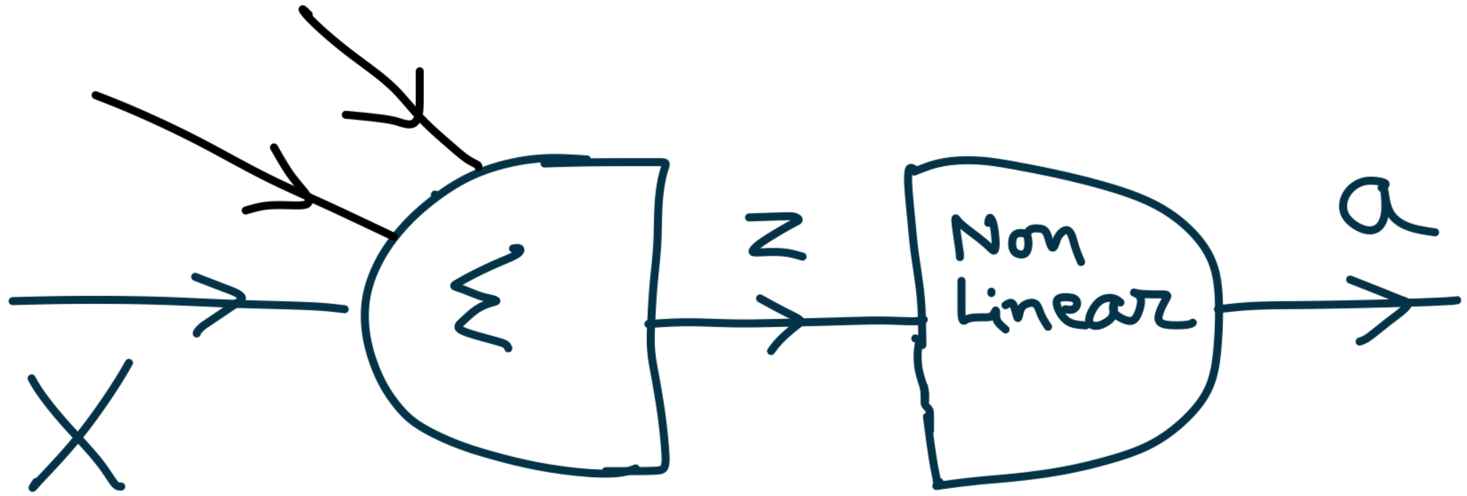

x
W b
w b
W b
$\hat{y}$
J
x
W b
W b
W b
W b
$\hat{y}$
J

# An Artificial Neuron

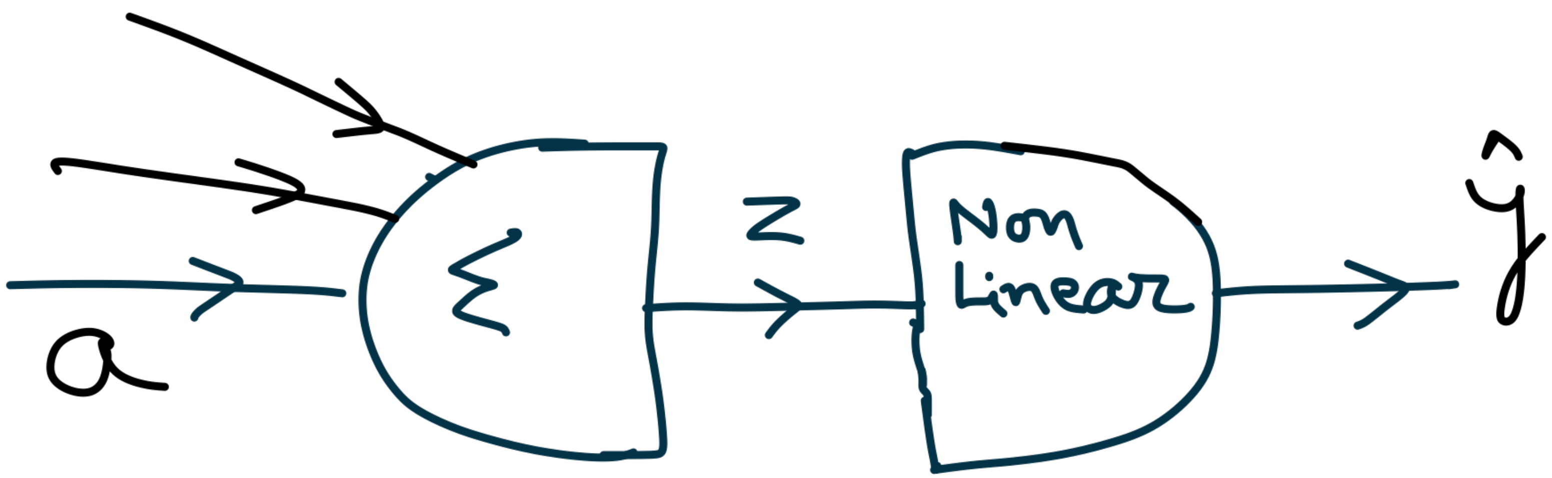


Incoming activations are program inputs

$a^{[0]}$




Layer 1 Neurons

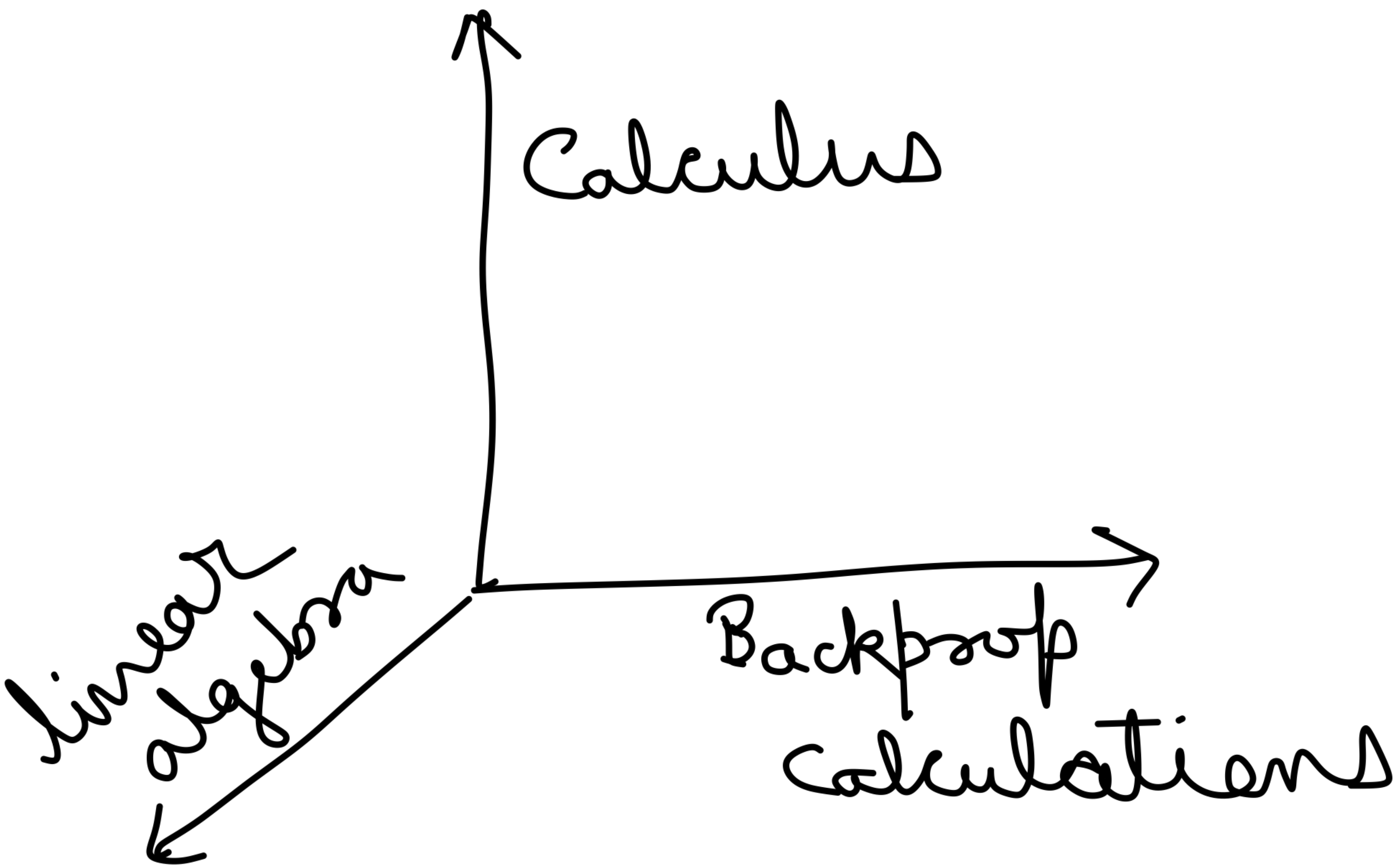



Outgoing activation is program output

$a^{[L]}$

Layer L Neurons

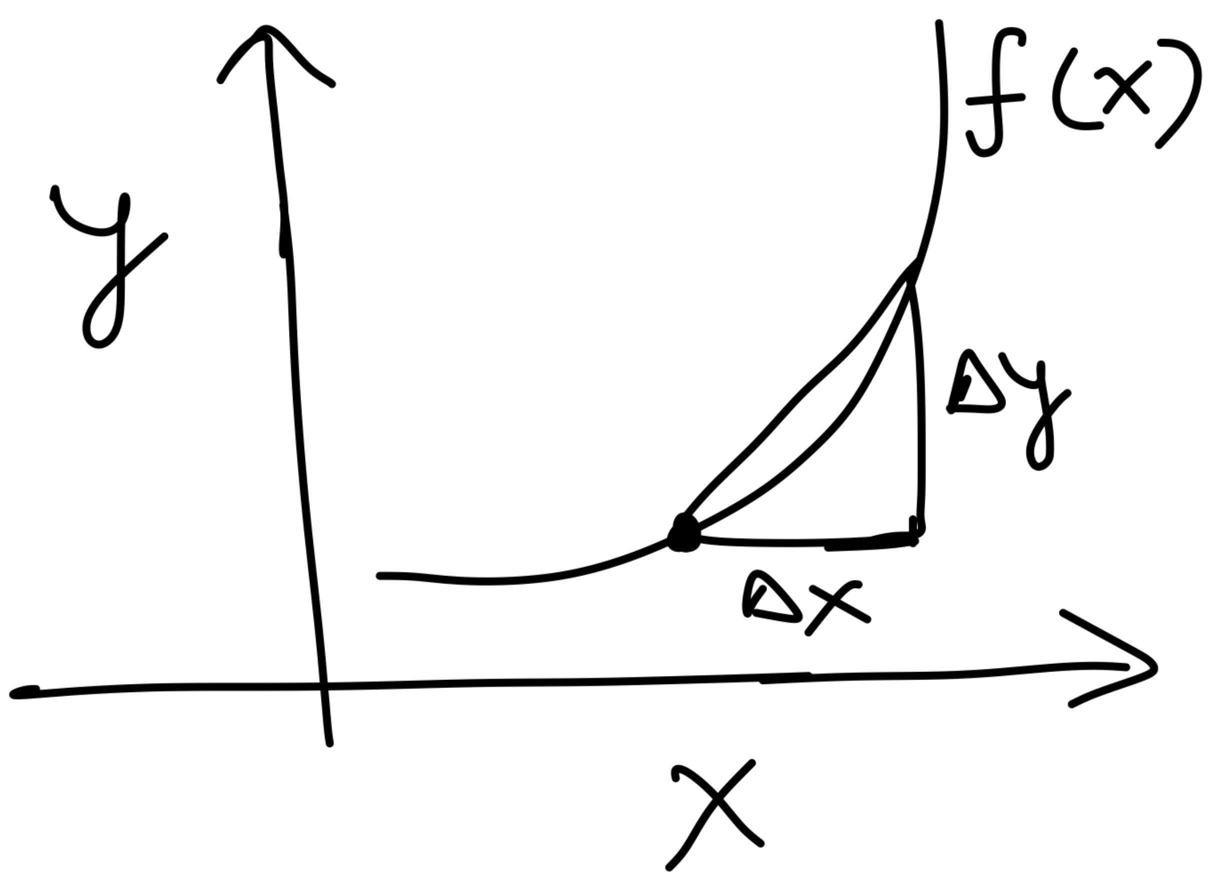
# Understanding Backpropagation



# Derivative of a Function $y = f(x)$

*Slope, Gradient, Rate of change*, $f'(x)$

$$\frac{dy}{dx}, \quad \frac{\partial y}{\partial x_1}, \quad \underbrace{\frac{\Delta y}{\Delta x}, \quad \frac{f(x+\Delta x) - f(x)}{\Delta x}}_{\text{as } \Delta x \to 0}$$



Common Derivatives:

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\frac{d}{dx} k = 0$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \log x = 1/x$$

# Chain Rule of Derivatives

$$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$$

$$\frac{d}{dx} \log(x^2) = \frac{1}{x^2} \cdot (2x)$$

$$\frac{d}{dx} e^{x^3} = e^{x^3} \cdot 3x^2$$

# Partial Derivatives

when does $\frac{dy}{dx}$ becomes $\frac{\partial y}{\partial x}$

when $y$ is a function of more than one variable. e.g.

$$y = f(x_1, x_2) = x_1^2 + 2x_2^2 + 3x_1 x_2$$

$$\cancel{\frac{dy}{dx_1}} \quad \frac{\partial y}{\partial x_1} = 2x_1 + 3x_2$$ } treat $x_2$ as constant

$$\cancel{\frac{dy}{dx_2}} \quad \frac{\partial y}{\partial x_2} = 4x_2 + 3x_1$$ } treat $x_1$ as constant

# What, Why and How

What are we after ?

$$\boxed{\frac{\partial J}{\partial w}, \frac{\partial J}{\partial b}}$$

J is Cost$(\hat{y}, y)$

Why?

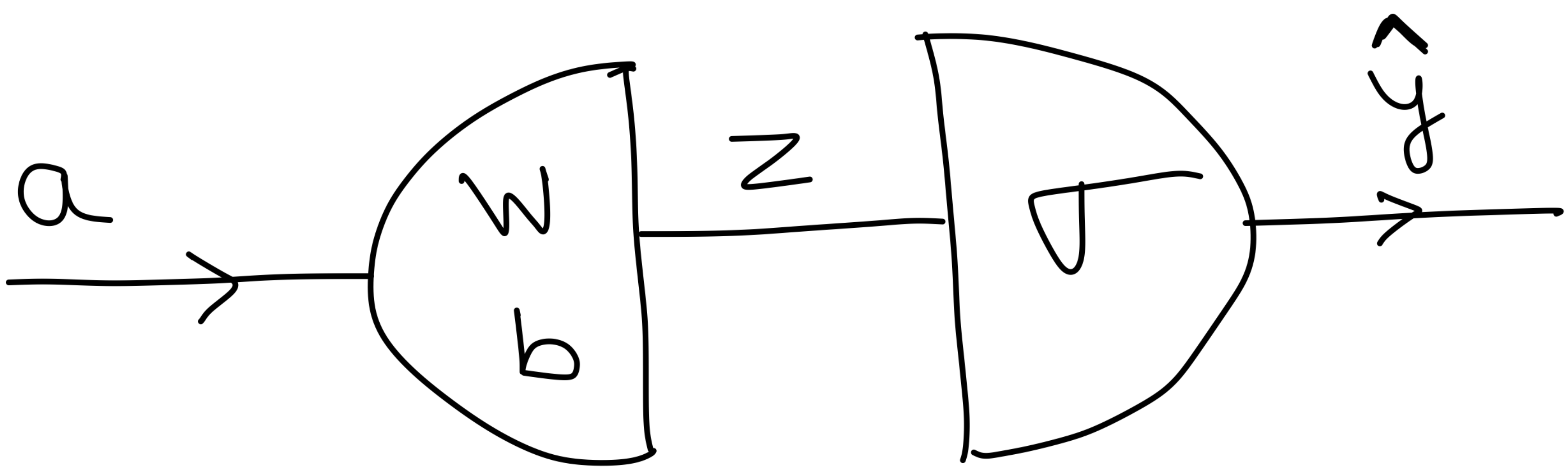
Gradient Descent says the best strategy to update weights is:

$$w := w - \alpha \frac{\partial J}{\partial w}$$

$$b := b - \alpha \frac{\partial J}{\partial b}$$

How to get $\dfrac{\partial J}{\partial w}$, $\dfrac{\partial J}{\partial b}$

We need to start from output layer




σ : sigmoid function

W, b are not directly connected with cost, there is J, Z in between

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Recap: Forward

$$Z = W \cdot a + b$$

$$\hat{y} = \text{sigmoid}(Z)$$

$$J = \text{Cost}(\hat{y}, y)$$

Cost function for our problem

$$J(y,\hat{y}) = -\frac{1}{N}\sum_{1}^{N} y\log\hat{y} + (1-y)\log(1-\hat{y})$$

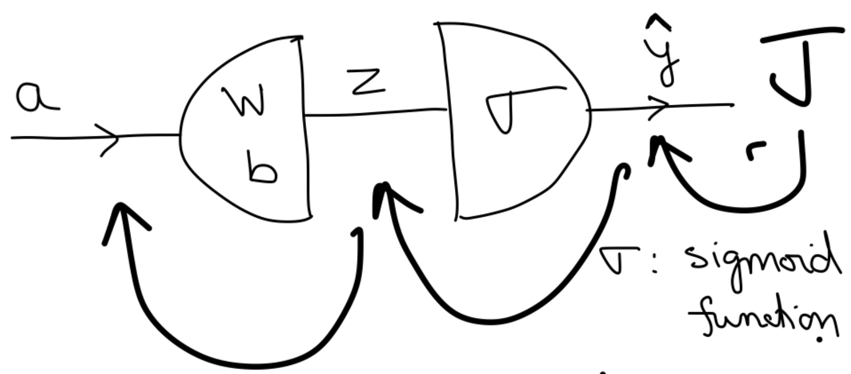
Average loss across all training samples

Cost is high when $y$ differs too much from $\hat{y}$ and vice-versa

# Going Backward (last layer)

J is a function of $\hat{y}$ is a function of z is a function of w, b, a

$J(\hat{y}(z(w, b, a)))$ - - - - -




$$\frac{\partial J}{\partial z} = \frac{\partial \hat{y}}{\partial z} \frac{\partial J}{\partial \hat{y}}$$

$$\frac{\partial J}{\partial z} = \frac{\partial \, \text{Sigmoid}(z)}{\partial z} \cdot \frac{\partial J(y, \hat{y})}{\partial \hat{y}}$$

Going Backward (last layer)

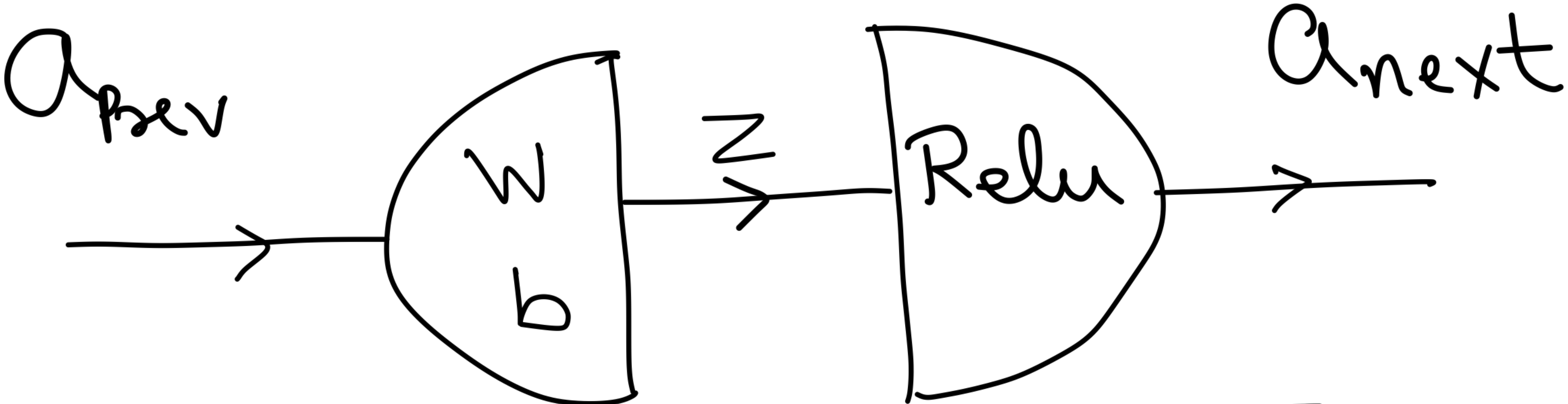
$$\frac{\partial J}{\partial w} = \frac{\partial Z}{\partial w} \cdot \frac{\partial J}{\partial Z}$$

$$\frac{\partial J}{\partial b} = \frac{\partial Z}{\partial b} \cdot \frac{\partial J}{\partial Z}$$

$$\boxed{\frac{\partial J}{\partial a} = \frac{\partial Z}{\partial a} \cdot \frac{\partial J}{\partial Z}}$$

Need this to continue going backwards

# Going Backward (middle layers) $i^{th}$ Layer




forward:

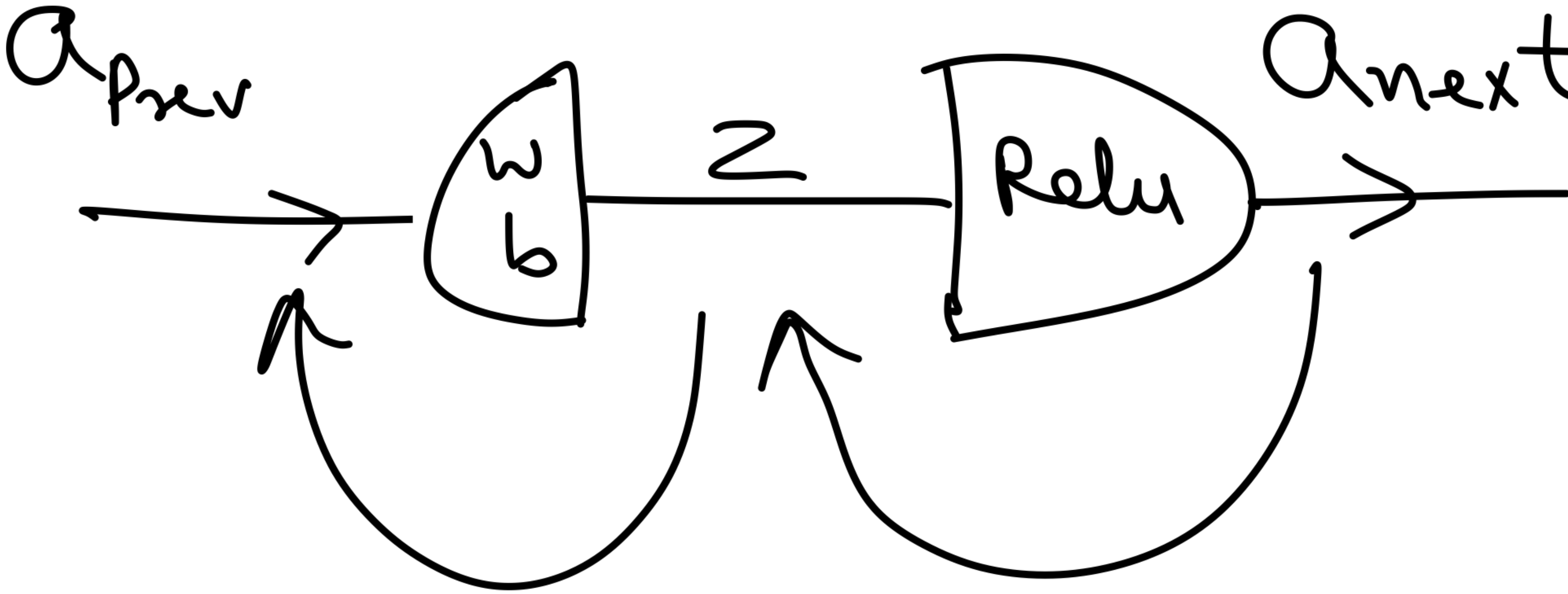
$$a_{next} = Relu(z)$$

$$z = W a_{prev} + b$$

$$\underbrace{\frac{\partial J}{\partial a_{next}}}$$

We know this.
Handed to us by $i+1^{th}$ Layer

# Going Backward (middle layers)




$$\frac{\partial J}{\partial z} = \frac{\partial a_{next}}{\partial z} \cdot \frac{\partial J}{\partial a_{next}}$$

$$\frac{\partial J}{\partial w} = \frac{\partial z}{\partial w} \cdot \frac{\partial J}{\partial z}$$

# Going Backward (middle layers)

$$\frac{\partial J}{\partial b} = \frac{\partial Z}{\partial b} \frac{\partial J}{\partial Z}$$

$$\boxed{\frac{\partial J}{\partial a_{prev}} = \frac{\partial Z}{\partial a_{prev}} \cdot \frac{\partial J}{\partial Z}}$$

Need this to continue going backward