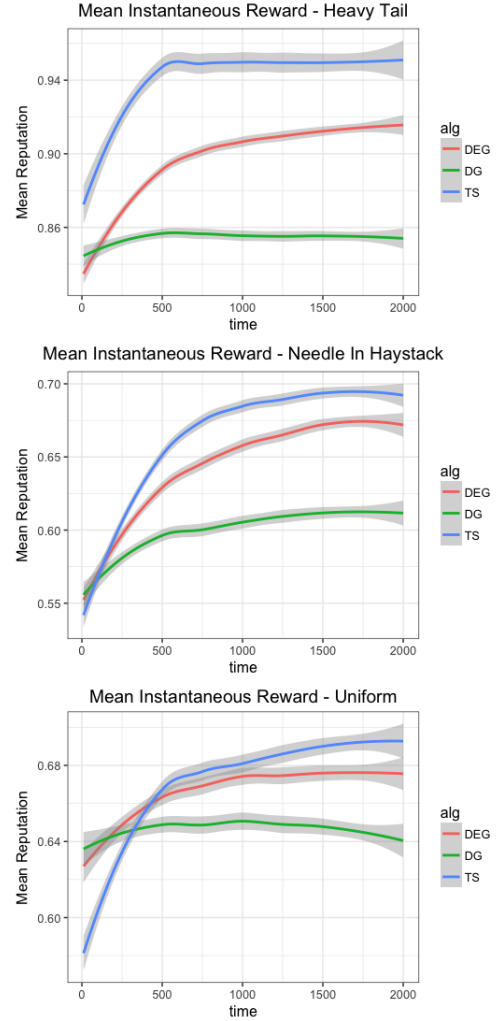
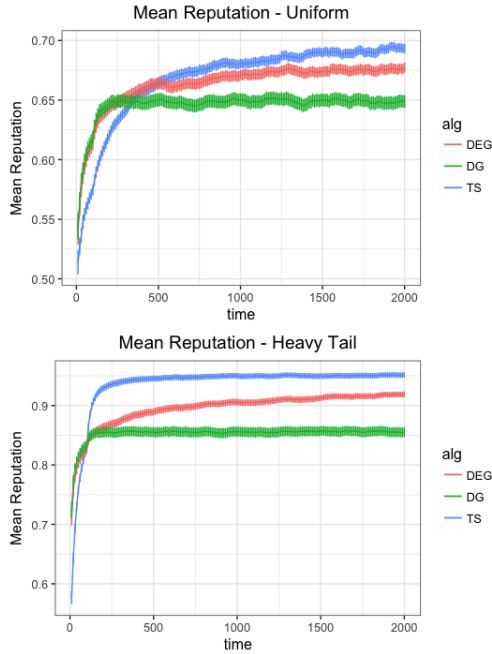


# Competing Bandits: The Perils of Exploration under Competition (Supplemental Material)

In this supplement, we provide plots and tables for our experiments, which were omitted from the main text due to page constraints. In all cases, the plots and tables here are in line with those in the main text, and lead to similar qualitative conclusions.

## A Plots for “Performance In Isolation”

We present additional plots for Section 3. First, we provide mean reputation trajectories for Uniform and Heavy Tail MAB instances. Second, we provide trajectories for instantaneous mean rewards, for all three MAB instances.<sup>1</sup> In all plots, the shaded area represents 95% confidence interval.



## B Permanent Duopoly

We present a missing detail for the permanent duopoly experiments from Section 4. Namely, we present full results for the Uniform instance: not only the market shares but also the EoG values. The results can be seen in Table 1. They are in the same format as those in the main text.

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<sup>1</sup>These trajectories are smoothed via a non-parametric regression. More concretely, we use this option in ggplot: [https://ggplot2.tidyverse.org/reference/geom\\_smooth.html](https://ggplot2.tidyverse.org/reference/geom_smooth.html).

	Uniform		
	$T_0 = 20$	$T_0 = 250$	$T_0 = 500$
TS vs DG	<b>0.46</b> $\pm 0.03$ EoG 230 (0)	<b>0.52</b> $\pm 0.02$ EoG 754 (800)	<b>0.6</b> $\pm 0.02$ EoG 910 (906.5)
TS vs DEG	<b>0.41</b> $\pm 0.03$ EoG 180 (0)	<b>0.51</b> $\pm 0.02$ EoG 910 (734)	<b>0.55</b> $\pm 0.02$ EoG 970 (987)
DG vs DEG	<b>0.51</b> $\pm 0.03$ EoG 470 (57.5)	<b>0.48</b> $\pm 0.02$ EoG 1000 (1088)	<b>0.45</b> $\pm 0.02$ EoG 1000 (1142)

Table 1: **Permanent duopoly**, for the Uniform MAB instance. Each cell describes a game between two algorithms, call them Alg1 vs. Alg2, for a particular value of the warm start  $T_0$ . Line 1 in the cell is the market share of Alg 1: the average (in bold) and the 95% confidence band. Line 2 specifies the “effective end of game” (EoG): the average and the median (in brackets). The time horizon is  $T = 2000$ .

## C Temporary Monopoly

We present additional experiments on temporary monopoly from Section 4, across various MAB instances and various values of the incumbent advantage parameter  $X$ .

Each experiment is presented as a table with the same semantics as in the main text. Namely, each cell in the table describes the duopoly game between the entrant’s algorithm (the row) and the incumbent’s algorithm (the column). The cell specifies the entrant’s market share (fraction of rounds in which it was chosen) for the rounds in which he was present. We give the average (in bold) and the 95% confidence interval. NB: smaller average is better for the incumbent.

### Heavy-Tail MAB Instance

	TS	DEG	DG
TS	<b>0.054</b> $\pm 0.01$	<b>0.16</b> $\pm 0.02$	<b>0.18</b> $\pm 0.02$
DEG	<b>0.33</b> $\pm 0.03$	<b>0.31</b> $\pm 0.02$	<b>0.26</b> $\pm 0.02$
DG	<b>0.39</b> $\pm 0.03$	<b>0.41</b> $\pm 0.03$	<b>0.33</b> $\pm 0.02$

Table 2: Temporary Monopoly: Heavy Tail,  $X = 50$

	TS	DEG	DG
TS	<b>0.003</b> $\pm 0.003$	<b>0.083</b> $\pm 0.02$	<b>0.17</b> $\pm 0.02$
DEG	<b>0.045</b> $\pm 0.01$	<b>0.25</b> $\pm 0.02$	<b>0.23</b> $\pm 0.02$
DG	<b>0.12</b> $\pm 0.02$	<b>0.36</b> $\pm 0.03$	<b>0.3</b> $\pm 0.02$

Table 3: Temporary Monopoly: Heavy Tail,  $X = 200$

	TS	DEG	DG
TS	<b>0.0017</b> $\pm 0.002$	<b>0.059</b> $\pm 0.01$	<b>0.16</b> $\pm 0.02$
DEG	<b>0.029</b> $\pm 0.007$	<b>0.23</b> $\pm 0.02$	<b>0.23</b> $\pm 0.02$
DG	<b>0.097</b> $\pm 0.02$	<b>0.34</b> $\pm 0.03$	<b>0.29</b> $\pm 0.02$

Table 4: Temporary Monopoly: Heavy Tail,  $X = 300$

	TS	DEG	DG
TS	<b>0.002</b> $\pm 0.003$	<b>0.043</b> $\pm 0.01$	<b>0.16</b> $\pm 0.02$
DEG	<b>0.03</b> $\pm 0.007$	<b>0.21</b> $\pm 0.02$	<b>0.24</b> $\pm 0.02$
DG	<b>0.091</b> $\pm 0.01$	<b>0.32</b> $\pm 0.03$	<b>0.3</b> $\pm 0.02$

Table 5: Temporary Monopoly: Heavy Tail,  $X = 500$

### Needle-In-Haystack MAB Instance

	TS	DEG	DG
TS	<b>0.34</b> $\pm 0.03$	<b>0.4</b> $\pm 0.03$	<b>0.48</b> $\pm 0.03$
DEG	<b>0.22</b> $\pm 0.02$	<b>0.34</b> $\pm 0.03$	<b>0.42</b> $\pm 0.03$
DG	<b>0.18</b> $\pm 0.02$	<b>0.28</b> $\pm 0.02$	<b>0.37</b> $\pm 0.03$

Table 6: Temporary Monopoly: Needle In Haystack,  $X = 50$

	TS	DEG	DG
TS	<b>0.17</b> $\pm 0.02$	<b>0.31</b> $\pm 0.03$	<b>0.41</b> $\pm 0.03$
DEG	<b>0.13</b> $\pm 0.02$	<b>0.26</b> $\pm 0.02$	<b>0.36</b> $\pm 0.03$
DG	<b>0.093</b> $\pm 0.02$	<b>0.23</b> $\pm 0.02$	<b>0.33</b> $\pm 0.03$

Table 7: Temporary Monopoly: Needle In Haystack,  $X = 200$

	TS	DEG	DG
TS	<b>0.1</b> $\pm 0.02$	<b>0.28</b> $\pm 0.03$	<b>0.39</b> $\pm 0.03$
DEG	<b>0.089</b> $\pm 0.02$	<b>0.23</b> $\pm 0.02$	<b>0.36</b> $\pm 0.03$
DG	<b>0.05</b> $\pm 0.01$	<b>0.21</b> $\pm 0.02$	<b>0.33</b> $\pm 0.03$

Table 8: Temporary Monopoly: Needle In Haystack,  $X = 300$

	TS	DEG	DG
TS	<b>0.053</b> $\pm 0.01$	<b>0.23</b> $\pm 0.02$	<b>0.37</b> $\pm 0.03$
DEG	<b>0.051</b> $\pm 0.01$	<b>0.2</b> $\pm 0.02$	<b>0.33</b> $\pm 0.03$
DG	<b>0.031</b> $\pm 0.009$	<b>0.18</b> $\pm 0.02$	<b>0.31</b> $\pm 0.02$

Table 9: Temporary Monopoly: Needle In Haystack,  $X = 500$

### Uniform MAB Instance

	TS	DEG	DG
TS	<b>0.27</b> $\pm 0.03$	<b>0.21</b> $\pm 0.02$	<b>0.26</b> $\pm 0.02$
DEG	<b>0.39</b> $\pm 0.03$	<b>0.3</b> $\pm 0.03$	<b>0.34</b> $\pm 0.03$
DG	<b>0.39</b> $\pm 0.03$	<b>0.31</b> $\pm 0.02$	<b>0.33</b> $\pm 0.02$

Table 10: Temporary Monopoly: Uniform,  $X = 50$

	TS	DEG	DG
TS	<b>0.12</b> $\pm 0.02$	<b>0.16</b> $\pm 0.02$	<b>0.2</b> $\pm 0.02$
DEG	<b>0.25</b> $\pm 0.02$	<b>0.24</b> $\pm 0.02$	<b>0.29</b> $\pm 0.02$
DG	<b>0.23</b> $\pm 0.02$	<b>0.24</b> $\pm 0.02$	<b>0.29</b> $\pm 0.02$

Table 11: Temporary Monopoly: Uniform,  $X = 200$

	TS	DEG	DG
TS	<b>0.094</b> $\pm 0.02$	<b>0.15</b> $\pm 0.02$	<b>0.2</b> $\pm 0.02$
DEG	<b>0.2</b> $\pm 0.02$	<b>0.23</b> $\pm 0.02$	<b>0.29</b> $\pm 0.02$
DG	<b>0.21</b> $\pm 0.02$	<b>0.23</b> $\pm 0.02$	<b>0.29</b> $\pm 0.02$

Table 12: Temporary Monopoly: Uniform,  $X = 300$

	TS	DEG	DG
TS	<b>0.061</b> $\pm 0.01$	<b>0.12</b> $\pm 0.02$	<b>0.2</b> $\pm 0.02$
DEG	<b>0.17</b> $\pm 0.02$	<b>0.21</b> $\pm 0.02$	<b>0.29</b> $\pm 0.02$
DG	<b>0.18</b> $\pm 0.02$	<b>0.22</b> $\pm 0.02$	<b>0.29</b> $\pm 0.02$

Table 13: Temporary Monopoly: Uniform,  $X = 500$

## D Reputation vs. Data Advantage

This section presents all experiments on data vs. reputation advantage (Section 5).

Each experiment is presented as a table with the same semantics as in the main text. Namely, each cell in the table describes the duopoly game between the entrant’s algorithm (the **row**) and the incumbent’s algorithm (the **column**). The cell specifies the entrant’s market share for the rounds in which hit was present: the average (in bold) and the 95% confidence interval. NB: smaller average is better for the incumbent.

	TS	DEG	DG
TS	<b>0.0096</b> $\pm 0.006$	<b>0.11</b> $\pm 0.02$	<b>0.18</b> $\pm 0.02$
DEG	<b>0.073</b> $\pm 0.01$	<b>0.29</b> $\pm 0.02$	<b>0.25</b> $\pm 0.02$
DG	<b>0.15</b> $\pm 0.02$	<b>0.39</b> $\pm 0.03$	<b>0.33</b> $\pm 0.02$

Table 14: Data Advantage: Heavy Tail,  $X = 200$

	TS	DEG	DG
TS	<b>0.021</b> $\pm 0.009$	<b>0.16</b> $\pm 0.02$	<b>0.21</b> $\pm 0.02$
DEG	<b>0.26</b> $\pm 0.03$	<b>0.3</b> $\pm 0.02$	<b>0.26</b> $\pm 0.02$
DG	<b>0.34</b> $\pm 0.03$	<b>0.4</b> $\pm 0.03$	<b>0.33</b> $\pm 0.02$

Table 15: Reputation Advantage: Heavy Tail,  $X = 200$

	TS	DEG	DG
TS	<b>0.25</b> $\pm 0.03$	<b>0.36</b> $\pm 0.03$	<b>0.45</b> $\pm 0.03$
DEG	<b>0.21</b> $\pm 0.02$	<b>0.32</b> $\pm 0.03$	<b>0.41</b> $\pm 0.03$
DG	<b>0.18</b> $\pm 0.02$	<b>0.29</b> $\pm 0.03$	<b>0.4</b> $\pm 0.03$

Table 16: Data Advantage: Needle In Haystack,  $X = 200$

	TS	DEG	DG
TS	<b>0.35</b> $\pm 0.03$	<b>0.43</b> $\pm 0.03$	<b>0.52</b> $\pm 0.03$
DEG	<b>0.26</b> $\pm 0.03$	<b>0.36</b> $\pm 0.03$	<b>0.43</b> $\pm 0.03$
DG	<b>0.19</b> $\pm 0.02$	<b>0.3</b> $\pm 0.02$	<b>0.36</b> $\pm 0.02$

Table 17: Reputation Advantage: Needle In Haystack,  $X = 200$

	TS	DEG	DG
TS	<b>0.27</b> $\pm 0.03$	<b>0.23</b> $\pm 0.02$	<b>0.27</b> $\pm 0.02$
DEG	<b>0.4</b> $\pm 0.03$	<b>0.3</b> $\pm 0.02$	<b>0.32</b> $\pm 0.02$
DG	<b>0.36</b> $\pm 0.03$	<b>0.29</b> $\pm 0.02$	<b>0.3</b> $\pm 0.02$

Table 18: Reputation Advantage: Uniform,  $X = 200$

	TS	DEG	DG
TS	<b>0.2</b> $\pm 0.02$	<b>0.22</b> $\pm 0.02$	<b>0.27</b> $\pm 0.03$
DEG	<b>0.33</b> $\pm 0.03$	<b>0.32</b> $\pm 0.03$	<b>0.35</b> $\pm 0.03$
DG	<b>0.32</b> $\pm 0.03$	<b>0.31</b> $\pm 0.03$	<b>0.35</b> $\pm 0.03$

Table 19: Data Advantage: Uniform,  $X = 200$

	TS	DEG	DG
TS	<b>0.0017</b> $\pm 0.002$	<b>0.06</b> $\pm 0.01$	<b>0.18</b> $\pm 0.02$
DEG	<b>0.04</b> $\pm 0.009$	<b>0.24</b> $\pm 0.02$	<b>0.25</b> $\pm 0.02$
DG	<b>0.12</b> $\pm 0.02$	<b>0.35</b> $\pm 0.03$	<b>0.33</b> $\pm 0.02$

Table 20: Data Advantage: Heavy Tail,  $X = 500$

	TS	DEG	DG
TS	<b>0.022</b> $\pm 0.009$	<b>0.13</b> $\pm 0.02$	<b>0.21</b> $\pm 0.02$
DEG	<b>0.26</b> $\pm 0.03$	<b>0.29</b> $\pm 0.02$	<b>0.28</b> $\pm 0.02$
DG	<b>0.33</b> $\pm 0.03$	<b>0.39</b> $\pm 0.03$	<b>0.34</b> $\pm 0.02$

Table 21: Reputation Advantage: Heavy Tail,  $X = 500$

	TS	DEG	DG
TS	<b>0.098</b> $\pm 0.02$	<b>0.27</b> $\pm 0.03$	<b>0.41</b> $\pm 0.03$
DEG	<b>0.093</b> $\pm 0.02$	<b>0.24</b> $\pm 0.02$	<b>0.38</b> $\pm 0.03$
DG	<b>0.064</b> $\pm 0.01$	<b>0.22</b> $\pm 0.02$	<b>0.37</b> $\pm 0.03$

Table 22: Data Advantage: Needle In Haystack,  $X = 500$

	TS	DEG	DG
TS	<b>0.29</b> $\pm 0.03$	<b>0.44</b> $\pm 0.03$	<b>0.52</b> $\pm 0.03$
DEG	<b>0.19</b> $\pm 0.02$	<b>0.35</b> $\pm 0.03$	<b>0.42</b> $\pm 0.03$
DG	<b>0.15</b> $\pm 0.02$	<b>0.27</b> $\pm 0.02$	<b>0.35</b> $\pm 0.02$

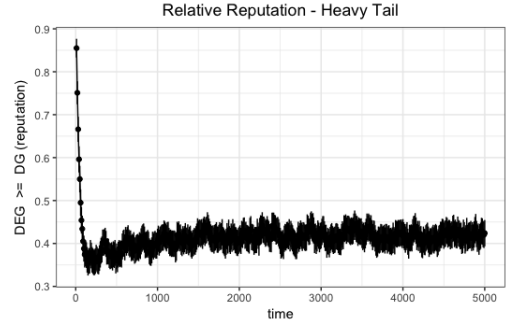
Table 23: Reputation Advantage: Needle In Haystack,  $X = 500$

	TS	DEG	DG
TS	<b>0.14</b> $\pm 0.02$	<b>0.18</b> $\pm 0.02$	<b>0.26</b> $\pm 0.03$
DEG	<b>0.26</b> $\pm 0.02$	<b>0.26</b> $\pm 0.02$	<b>0.34</b> $\pm 0.03$
DG	<b>0.25</b> $\pm 0.02$	<b>0.27</b> $\pm 0.02$	<b>0.34</b> $\pm 0.03$

Table 24: Data Advantage: Uniform,  $X = 500$

	TS	DEG	DG
TS	<b>0.24</b> $\pm 0.02$	<b>0.2</b> $\pm 0.02$	<b>0.26</b> $\pm 0.02$
DEG	<b>0.37</b> $\pm 0.03$	<b>0.29</b> $\pm 0.02$	<b>0.31</b> $\pm 0.02$
DG	<b>0.35</b> $\pm 0.03$	<b>0.27</b> $\pm 0.02$	<b>0.3</b> $\pm 0.02$

Table 25: Reputation Advantage: Uniform,  $X = 500$



## E Mean Reputation vs. Relative Reputation

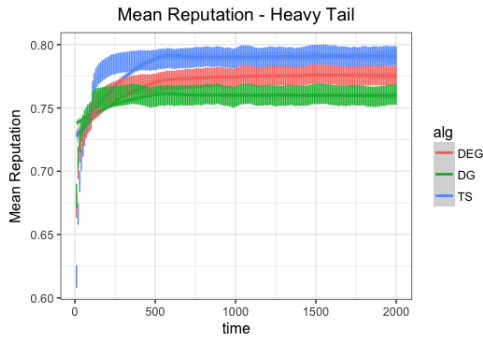
We present the experiments omitted from Section 6. Namely, experiments on the Heavy-Tail MAB instance with  $K = 3$  arms, both for “performance in isolation” and the permanent duopoly game. We find that  $\text{DEG} > \text{DG}$  according to the mean reputation trajectory but that  $\text{DG} > \text{DEG}$  according to the relative reputation trajectory *and* in the competition game. As discussed in Section 6, the same results also hold for  $K = 10$  for the warm starts that we consider.

The result of the permanent duopoly experiment for this instance is shown in Table 26.

	Heavy Tail		
	$T_0 = 20$	$T_0 = 250$	$T_0 = 500$
TS vs. DG	<b>0.4</b> $\pm 0.02$ EoG 770 (0)	<b>0.59</b> $\pm 0.01$ EoG 2700 (2979.5)	<b>0.6</b> $\pm 0.01$ EoG 2700 (3018)
TS vs. DEG	<b>0.46</b> $\pm 0.02$ EoG 830 (0)	<b>0.73</b> $\pm 0.01$ EoG 2500 (2576.5)	<b>0.72</b> $\pm 0.01$ EoG 2700 (2862)
DG vs. DEG	<b>0.61</b> $\pm 0.01$ EoG 1400 (556)	<b>0.61</b> $\pm 0.01$ EoG 2400 (2538.5)	<b>0.6</b> $\pm 0.01$ EoG 2400 (2587.5)

Table 26: Duopoly Experiment: Heavy-Tail,  $K = 3$ ,  $T = 5000$ . Each cell describes a game between two algorithms, call them Alg1 vs. Alg2, for a particular value of the warm start  $T_0$ . Line 1 in the cell is the market share of Alg 1: the average (in bold) and the 95% confidence band. Line 2 specifies the “effective end of game” (EoG): the average and the median (in brackets).

The mean reputation trajectories for algorithms’ performance in isolation:



Finally, the relative reputation trajectory of DEG vs. DG: