Overview

April 9, 2018

Simulation Details

Considered $K = 3$, $Memory = 100$ (unless otherwise noted)
The Bandit priors that were considered:

- Uniform: Draw the mean rewards for the arms from [0.25, 0.75]
- "HeavyTail": We took the mean rewards to be randomly drawn from Beta($\alpha = 0.6, \beta = 0.6$). With this distribution it was likely to have arms that were at the extremes (close to 1 and close to 0) but also some of the arms with intermediate value means.
- Needle-in-haystack
    1. High - 2 arms with mean 0.50, 1 arm with mean 0.70 (+ 0.20)

Algorithms considered:

1. ThompsonSampling with priors of $Beta(1, 1)$ for every arm.
2. DynamicGreedy with priors of $Beta(1, 1)$ for every arm
3. Bayesian Dynamic ϵ-greedy with priors of $Beta(1, 1)$ for every arm and $\epsilon = 0.05$

Agent Algorithms considered:

1. HardMax
2. HardMaxWithRandom ($\epsilon = 0.05$)
3. SoftMax ($\alpha = 30$)

Memory Sizes

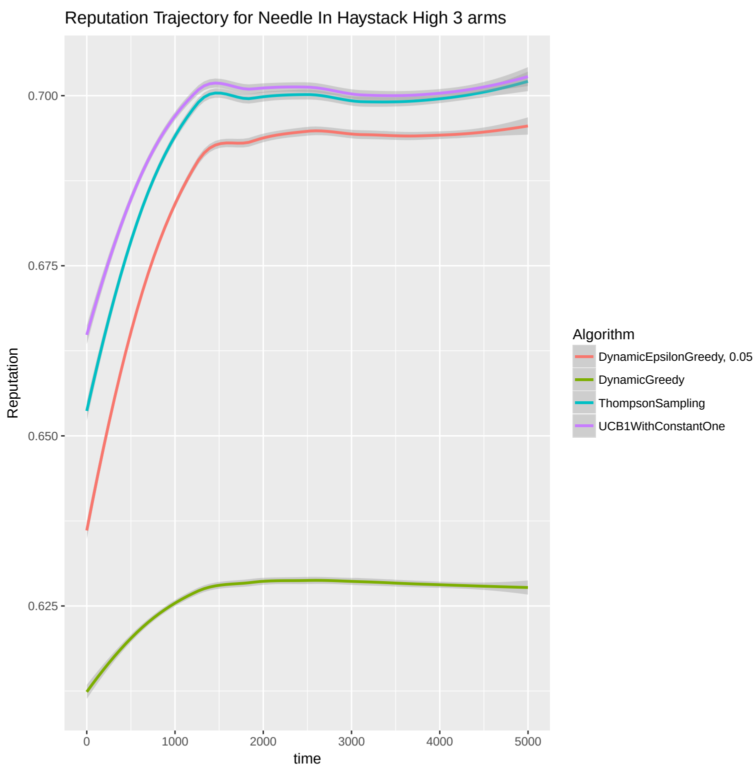
1. 100

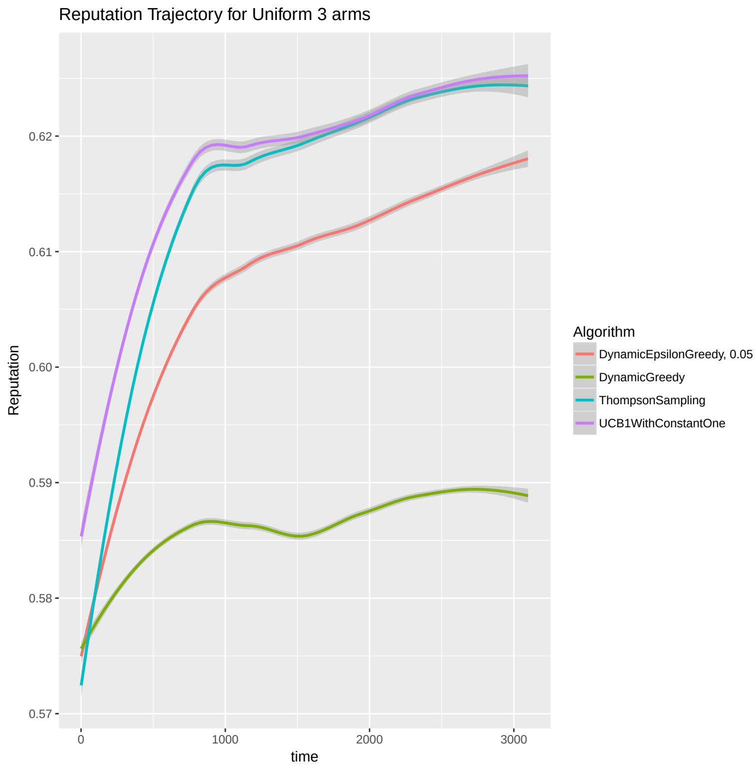
Simulation Procedure

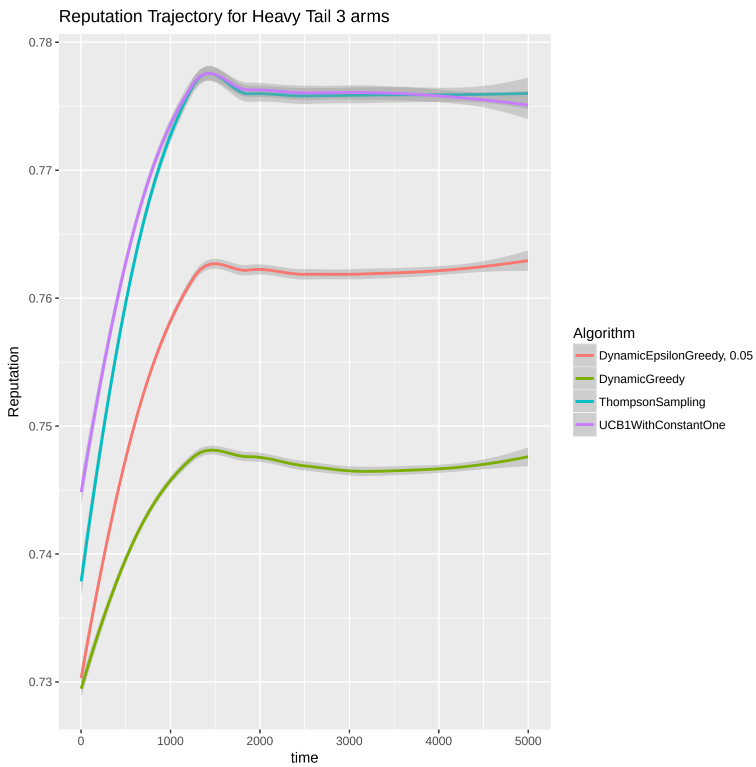
```
for Each prior p do
    Generate true distribution from p (except for needle-in-haystack, just use p itself)
    Generate T × K realizations for the arms
    for Each agent algorithm agentalg do
        for Each principal algorithm pair principalalg1, principalalg2 do
            for N simulations do
                Give the agents k observations from each principal
                Give principal 2 X free observations (the agents also get these observations)
                Run simulation for T periods
            end for
        end for
    end for
end for
```

Results

First, let's look at the preliminary simulation results on the instances we considered. What is plotted here is the reputation score for a memory size of 100 for each of the priors and each of the algorithms:







The first set of results we will consider are $X = 0$ and $k = 5$ so that there is simultaneous entry and only a small cold start. Recall to read the tables:

1. In bold is the average market share for principal 1 and with a 95% confidence interval
2. The var term is the variance of the market shares
3. The "share" line means the % of simulations that ended up with one principal getting more than 90% of the market.

Simultaneous Entry

Question of Interest: What algorithms should win under different agent response models?
Conjecture: HardMaxWithRandom and SoftMax should eventually lead to the better algorithm winning as long as, on the instance we are considering there is a gap in the long-term reward of the better algorithms.
Why? Looking at the preliminary calibration plots, since there is some randomness in the choice rule of the agents, each principal should always be getting some free observations (our parameters such as the α in SoftMax and ϵ in HardMaxWithRandom tune how many expected observations a principal should get for a fixed T). Let's fix attention on the case of ThompsonSampling vs Bayesian/Dynamic Greedy. Looking at each of the preliminary plots we see that ThompsonSampling beats Dynamic / Bayesian Greedy for sufficiently large T. Thus, if we calibrate the time horizon long enough so that the principal playing Thompson Sampling should, in expectation, get sufficiently many samples to be on the part of the curve where learning is almost done then it should be able to accumulate a higher reputation score (by having better arm selection than DynamicGreedy) and eventually get more of the market.

Conjecture: Under HardMax, the early rounds should dictate almost everything since we suppose the algorithms start with little initial information and thus the difference between the algorithms should not make much of a difference. Since the rewards are randomly drawn, lucky early round draws may dictate the course of the game. Thus a conjecture is that the mean market share will be 50/50.

The first conjecture seems to be confirmed in the simulations. Using the calibration from above ($\epsilon = 0.05$ for HMR and $\alpha = 30$ for SoftMax). However, the HardMax results require some more validation. First, we look at the results for low T (1000) but high N (1200). Then we extend the time horizon, but look at lower N.

Results for t= 1000 Needle In Haystack High Memory= 100

	TS vs DEG	TS vs DG	DG vs DEG	TS vs TS	DEG vs DEG	DG vs DG
HM	**0.49** +/- 0.03 Var: 0.24 Share: 96 %	**0.51** +/- 0.03 Var: 0.24 Share: 94 %	**0.46** +/- 0.03 Var: 0.23 Share: 92 %	**0.51** +/- 0.03 Var: 0.24 Share: 97 %	**0.5** +/- 0.03 Var: 0.24 Share: 94 %	**0.5** +/- 0.03 Var: 0.23 Share: 89 %
HMR	**0.51** +/- 0.02 Var: 0.18 Share: 76 %	**0.52** +/- 0.02 Var: 0.18 Share: 72 %	**0.48** +/- 0.02 Var: 0.17 Share: 69 %	**0.51** +/- 0.02 Var: 0.19 Share: 80 %	**0.5** +/- 0.02 Var: 0.18 Share: 74 %	**0.52** +/- 0.02 Var: 0.16 Share: 66 %
SM	**0.52** +/- 0.02 Var: 0.071 Share: 4.3 %	**0.53** +/- 0.02 Var: 0.079 Share: 4.8 %	**0.47** +/- 0.02 Var: 0.079 Share: 7 %	**0.51** +/- 0.01 Var: 0.063 Share: 3.5 %	**0.5** +/- 0.02 Var: 0.078 Share: 6.7 %	**0.5** +/- 0.02 Var: 0.083 Share: 7.2 %

Results for t= 1000 Heavy Tail Memory= 100

	TS vs DEG	TS vs DG	DG vs DEG	TS vs TS	DEG vs DEG	DG vs DG
HM	**0.42** +/- 0.03 Var: 0.24 Share: 97 %	**0.41** +/- 0.03 Var: 0.23 Share: 97 %	**0.55** +/- 0.03 Var: 0.23 Share: 91 %	**0.49** +/- 0.03 Var: 0.25 Share: 98 %	**0.52** +/- 0.03 Var: 0.23 Share: 90 %	**0.5** +/- 0.03 Var: 0.22 Share: 88 %
HMR	**0.43** +/- 0.02 Var: 0.19 Share: 79 %	**0.43** +/- 0.02 Var: 0.18 Share: 79 %	**0.54** +/- 0.02 Var: 0.16 Share: 64 %	**0.5** +/- 0.03 Var: 0.2 Share: 86 %	**0.52** +/- 0.02 Var: 0.16 Share: 66 %	**0.5** +/- 0.02 Var: 0.15 Share: 61 %
SM	**0.54** +/- 0.01 Var: 0.051 Share: 3.3 %	**0.49** +/- 0.01 Var: 0.056 Share: 3.4 %	**0.53** +/- 0.01 Var: 0.046 Share: 3.7 %	**0.5** +/- 0.01 Var: 0.047 Share: 1.5 %	**0.51** +/- 0.01 Var: 0.044 Share: 2.7 %	**0.5** +/- 0.01 Var: 0.044 Share: 3.8 %

Results for t= 1000 Uniform Memory= 100

	TS vs DEG	TS vs DG	DG vs DEG	TS vs TS	DEG vs DEG	DG vs DG
HM	**0.5** +/- 0.03 Var: 0.23 Share: 93 %	**0.52** +/- 0.03 Var: 0.23 Share: 92 %	**0.52** +/- 0.03 Var: 0.23 Share: 90 %	**0.5** +/- 0.03 Var: 0.23 Share: 94 %	**0.5** +/- 0.03 Var: 0.23 Share: 90 %	**0.49** +/- 0.03 Var: 0.22 Share: 88 %
HMR	**0.49** +/- 0.02 Var: 0.16 Share: 65 %	**0.5** +/- 0.02 Var: 0.16 Share: 64 %	**0.51** +/- 0.02 Var: 0.15 Share: 59 %	**0.5** +/- 0.02 Var: 0.17 Share: 70 %	**0.5** +/- 0.02 Var: 0.16 Share: 62 %	**0.49** +/- 0.02 Var: 0.15 Share: 55 %
SM	**0.5** +/- 0.01 Var: 0.056 Share: 3.8 %	**0.5** +/- 0.01 Var: 0.06 Share: 2.3 %	**0.48** +/- 0.01 Var: 0.058 Share: 2.6 %	**0.49** +/- 0.01 Var: 0.054 Share: 2 %	**0.5** +/- 0.01 Var: 0.056 Share: 2.9 %	**0.49** +/- 0.01 Var: 0.058 Share: 3.3 %

Results for t= 2000 Heavy Tail Memory= 100

	TS vs BEG	TS vs BG	BG vs BEG	TS vs TS	BEG vs BEG	BG vs BG
EU	**0.42** +/- 0.06 Var: 0.23 Share: 93 %	**0.45** +/- 0.06 Var: 0.24 Share: 96 %	**0.57** +/- 0.06 Var: 0.23 Share: 92 %			
Noisy EU	**0.47** +/- 0.05 Var: 0.16 Share: 64 %	**0.44** +/- 0.05 Var: 0.17 Share: 70 %	**0.54** +/- 0.05 Var: 0.14 Share: 52 %			
Logit	**0.57** +/- 0.02 Var: 0.035 Share: 3.2 %	**0.51** +/- 0.03 Var: 0.042 Share: 2.8 %	**0.53** +/- 0.03 Var: 0.053 Share: 4.4 %			

Results for t= 15000 Heavy Tail Memory= 100

	TS vs BEG	TS vs BG	BG vs BEG	TS vs TS	BEG vs BEG	BG vs BG
EU	**0.43** +/- 0.06 Var: 0.23 Share: 95 %	**0.47** +/- 0.06 Var: 0.24 Share: 95 %	**0.56** +/- 0.06 Var: 0.22 Share: 89 %			
Noisy EU	**0.67** +/- 0.02 Var: 0.034 Share: 14 %	**0.55** +/- 0.03 Var: 0.053 Share: 16 %	**0.6** +/- 0.04 Var: 0.082 Share: 29 %			
Logit	**0.6** +/- 0.01 Var: 0.0066 Share: 0 %	**0.56** +/- 0.02 Var: 0.022 Share: 0.4 %	**0.52** +/- 0.03 Var: 0.041 Share: 2.4 %			

Results for t= 2000 Uniform Memory= 100

	TS vs BEG	TS vs BG	BG vs BEG	TS vs TS	BEG vs BEG	BG vs BG
EU	**0.5** +/- 0.06 Var: 0.23 Share: 93 %	**0.45** +/- 0.06 Var: 0.24 Share: 95 %	**0.5** +/- 0.06 Var: 0.23 Share: 90 %			
Noisy EU	**0.48** +/- 0.05 Var: 0.14 Share: 51 %	**0.51** +/- 0.05 Var: 0.14 Share: 52 %	**0.5** +/- 0.04 Var: 0.12 Share: 40 %			
Logit	**0.53** +/- 0.03 Var: 0.048 Share: 2 %	**0.52** +/- 0.03 Var: 0.054 Share: 2.4 %	**0.49** +/- 0.03 Var: 0.056 Share: 2.8 %			

Results for t= 15000 Uniform Memory= 100

	TS vs BEG	TS vs BG	BG vs BEG	TS vs TS	BEG vs BEG	BG vs BG
EU	**0.52** +/- 0.06 Var: 0.22 Share: 88 %	**0.47** +/- 0.06 Var: 0.23 Share: 90 %	**0.47** +/- 0.06 Var: 0.23 Share: 89 %			
Noisy EU	**0.56** +/- 0.02 Var: 0.036 Share: 7.2 %	**0.59** +/- 0.03 Var: 0.055 Share: 18 %	**0.48** +/- 0.03 Var: 0.067 Share: 19 %			
Logit	**0.55** +/- 0.01 Var: 0.0088 Share: 0 %	**0.57** +/- 0.02 Var: 0.026 Share: 0.8 %	**0.45** +/- 0.02 Var: 0.035 Share: 0.8 %			

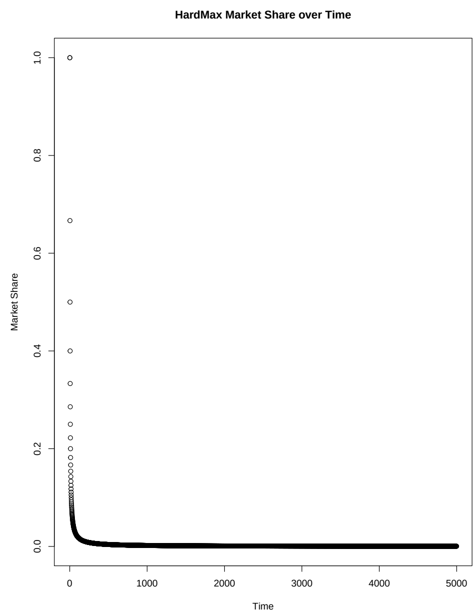
Results for t= 2000 Needle In Haystack High Memory= 100

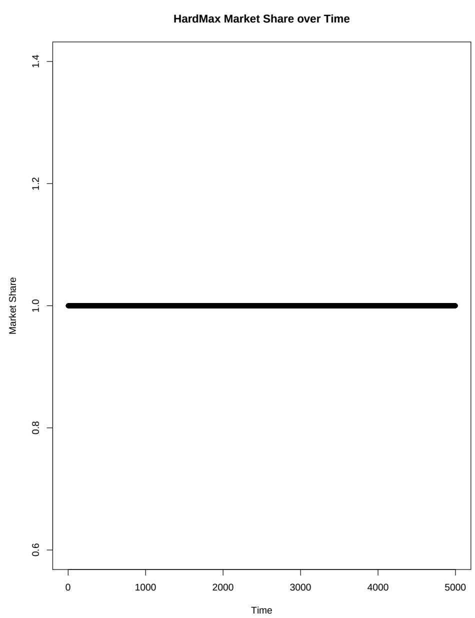
	TS vs BEG	TS vs BG	BG vs BEG	TS vs TS	BEG vs BEG	BG vs BG
EU	**0.52** +/- 0.06 Var: 0.24 Share: 96 %	**0.49** +/- 0.06 Var: 0.24 Share: 94 %	**0.43** +/- 0.06 Var: 0.23 Share: 92 %			
Noisy EU	**0.48** +/- 0.05 Var: 0.16 Share: 62 %	**0.59** +/- 0.05 Var: 0.14 Share: 56 %	**0.48** +/- 0.05 Var: 0.15 Share: 58 %			
Logit	**0.55** +/- 0.03 Var: 0.057 Share: 3.2 %	**0.53** +/- 0.03 Var: 0.063 Share: 2 %	**0.45** +/- 0.03 Var: 0.075 Share: 6.4 %			

Results for t= 15000 Needle In Haystack High Memory= 100

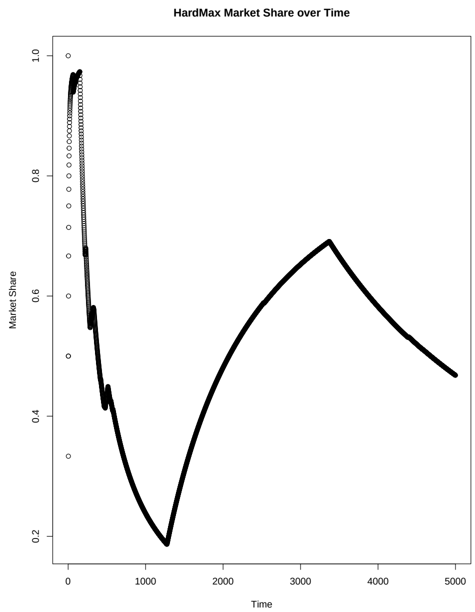
	TS vs BEG	TS vs BG	BG vs BEG	TS vs TS	BEG vs BEG	BG vs BG
EU	**0.54** +/- 0.06 Var: 0.24 Share: 94 %	**0.5** +/- 0.06 Var: 0.23 Share: 92 %	**0.42** +/- 0.06 Var: 0.23 Share: 92 %			
Noisy EU	**0.57** +/- 0.02 Var: 0.04 Share: 11 %	**0.64** +/- 0.03 Var: 0.067 Share: 33 %	**0.44** +/- 0.04 Var: 0.09 Share: 34 %			
Logit	**0.54** +/- 0.01 Var: 0.006 Share: 0 %	**0.59** +/- 0.02 Var: 0.034 Share: 0.8 %	**0.4** +/- 0.03 Var: 0.046 Share: 2 %			

The conjecture for HMR and SM seems to be consistent with the data generated from the simulation. However, what is going on with HardMax appears confusing. To get some more evidence for the original conjecture, we re-run simulations but track market share over time. In most cases, we get evidence of this and the vast majority of market shares over time look as follows:



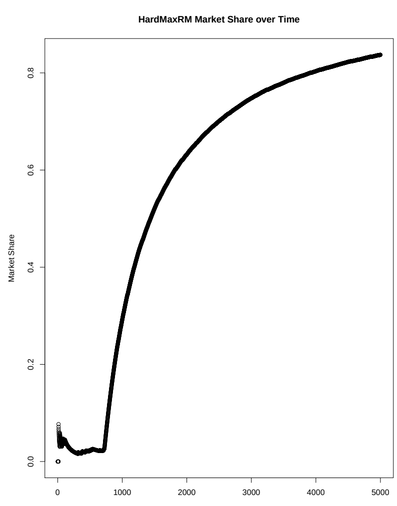


However, there are some simulations where we do not observe this. For instance,



I am not sure what happens in these cases, but one conjecture is that these arise from random noise from a low memory size (memory here is 100, as we have been using in our experiment). I've re-run this experiment for a memory size of 1000 and there are fewer cases which have odd switches as shown above. However, would need to look into this more if we think this is worth digging into more.

As a comparison, what do the trajectories for the HardMaxWithRandom agent response functions look like? Haven't dug too deeply into the data for these simulations, but here is a sample one (ThompsonSampling vs DEG - plotted market share is for ThompsonSampling) that is similar to many in those that I have looked at:



Incumbent Experiment

Now, we change $X = 200$. In words, we give principal 2 X free observations (the "incumbent") and this updates her information set and reputation amongst the agents. In this setting what should the incumbent do? What should the entrant do? Does this depend on the agent model? I would conjecture that when we have sufficient randomness in the agent model the same conclusions as before should hold - namely that the entrant, even if getting beat upon entry, should eventually get enough "free" observations that she can catch up and take more of the market. However, the time horizon, in contrast to the simultaneous entry case, seems like it may be way longer! So far, most of the simulations have simply looked at $t = 1000$, but it may make sense to look at longer time horizons due to the conjecture from above.

What about with HardMax? The only intuition I have is for the incumbent. For the entrant, it is not clear to me what the entrant should do. For the incumbent, in our simulations we fix a X when the entrant will come in and the incumbent should want to maximize both the information that she accrues before then and the reputation that she amongst the agents. However, this seems exactly like the standard exploration exploitation dilemma where reputation is simply the cumulative reward and "smart" algorithms like Thompson Sampling ought to be better than the "dumb" algorithms at maximizing information gain while at the same time optimizing cumulative reward. Thus, I expect that Thompson Sampling should be in a better starting position at the start of the competing bandits game (we can view the incumbent experiment as being the simultaneous entry game where the incumbent starts with a head start in terms of information and reputation) and thus control more of the market. This seems to be the case in the reported simulations, both with large $X = 200$ and $X = 100$. Below are the results, the first for $X = 200$ and the second for $X = 100$.

To read the tables: the columns represent the algorithm played by the incumbent and the row represents the algorithm played by the incumbent. For instance, looking at the second cell in the first row we have that the Incumbent played Dynamic (Bayesian) Epsilon Greedy and the entrant played Thompson Sampling and the reported mean is the average market share for the entrant.

Results for HardMax t = 1000 Needle In Haystack High

	Thompson Sampling	Bayesian Epsilon Greedy	Bayesian Greedy
Thompson Sampling	**0.1** +/- 0.022 0.063 (0.056, 0.072) Extreme Shares: 87 %	**0.13** +/- 0.026 0.086 (0.076, 0.097) Extreme Shares: 88 %	**0.28** +/- 0.036 0.17 (0.15, 0.19) Extreme Shares: 84 %
Bayesian Epsilon Greedy	**0.093** +/- 0.022 0.064 (0.056, 0.072) Extreme Shares: 91 %	**0.18** +/- 0.031 0.12 (0.11, 0.14) Extreme Shares: 89 %	**0.26** +/- 0.035 0.16 (0.14, 0.18) Extreme Shares: 86 %
Bayesian Greedy	**0.11** +/- 0.024 0.072 (0.064, 0.081) Extreme Shares: 87 %	**0.18** +/- 0.03 0.12 (0.1, 0.13) Extreme Shares: 88 %	**0.23** +/- 0.033 0.14 (0.13, 0.16) Extreme Shares: 85 %

Results for HardMaxWithRandom t = 1000 Needle In Haystack High

	Thompson Sampling	Bayesian Epsilon Greedy	Bayesian Greedy
Thompson Sampling	**0.17** +/- 0.023 0.067 (0.059, 0.076) Extreme Shares: 73 %	**0.3** +/- 0.031 0.13 (0.11, 0.14) Extreme Shares: 66 %	**0.34** +/- 0.034 0.15 (0.13, 0.17) Extreme Shares: 71 %
Bayesian Epsilon Greedy	**0.22** +/- 0.026 0.088 (0.078, 0.1) Extreme Shares: 67 %	**0.26** +/- 0.029 0.11 (0.094, 0.12) Extreme Shares: 68 %	**0.37** +/- 0.034 0.15 (0.13, 0.17) Extreme Shares: 66 %
Bayesian Greedy	**0.23** +/- 0.026 0.086 (0.076, 0.097) Extreme Shares: 65 %	**0.29** +/- 0.03 0.12 (0.1, 0.13) Extreme Shares: 65 %	**0.37** +/- 0.033 0.14 (0.12, 0.16) Extreme Shares: 63 %

Results for SoftMax t = 1000 Needle In Haystack High

	Thompson Sampling	Bayesian Epsilon Greedy	Bayesian Greedy
Thompson Sampling	**0.43** +/- 0.0082 0.0088 (0.0078, 0.01) Extreme Shares: 0 %	**0.46** +/- 0.012 0.018 (0.016, 0.02) Extreme Shares: 0 %	**0.52** +/- 0.016 0.033 (0.029, 0.037) Extreme Shares: 0 %
Bayesian Epsilon Greedy	**0.42** +/- 0.011 0.016 (0.014, 0.018) Extreme Shares: 0 %	**0.46** +/- 0.013 0.023 (0.021, 0.027) Extreme Shares: 0 %	**0.5** +/- 0.017 0.036 (0.032, 0.041) Extreme Shares: 0 %
Bayesian Greedy	**0.39** +/- 0.013 0.022 (0.02, 0.025) Extreme Shares: 0 %	**0.44** +/- 0.015 0.03 (0.026, 0.034) Extreme Shares: 0 %	**0.49** +/- 0.017 0.039 (0.034, 0.044) Extreme Shares: 0 %

Results for HardMax t = 1000 Heavy Tail

	Thompson Sampling	Bayesian Epsilon Greedy	Bayesian Greedy
Thompson Sampling	**0.042** +/- 0.014 0.026 (0.023, 0.03) Extreme Shares: 93 %	**0.081** +/- 0.021 0.06 (0.053, 0.068) Extreme Shares: 93 %	**0.12** +/- 0.027 0.091 (0.081, 0.1) Extreme Shares: 93 %
Bayesian Epsilon Greedy	**0.11** +/- 0.022 0.062 (0.055, 0.071) Extreme Shares: 84 %	**0.15** +/- 0.026 0.087 (0.077, 0.099) Extreme Shares: 81 %	**0.16** +/- 0.028 0.099 (0.088, 0.11) Extreme Shares: 83 %
Bayesian Greedy	**0.14** +/- 0.024 0.077 (0.068, 0.088) Extreme Shares: 82 %	**0.2** +/- 0.031 0.13 (0.11, 0.14) Extreme Shares: 83 %	**0.18** +/- 0.027 0.097 (0.086, 0.11) Extreme Shares: 76 %

Results for HardMaxWithRandom t = 1000 Heavy Tail

	Thompson Sampling	Bayesian Epsilon Greedy	Bayesian Greedy
Thompson Sampling	**0.12** +/- 0.018 0.042 (0.037, 0.048) Extreme Shares: 80 %	**0.17** +/- 0.024 0.073 (0.065, 0.083) Extreme Shares: 75 %	**0.21** +/- 0.028 0.1 (0.092, 0.12) Extreme Shares: 76 %
Bayesian Epsilon Greedy	**0.17** +/- 0.021 0.055 (0.049, 0.063) Extreme Shares: 67 %	**0.27** +/- 0.026 0.09 (0.08, 0.1) Extreme Shares: 54 %	**0.25** +/- 0.027 0.095 (0.084, 0.11) Extreme Shares: 62 %
Bayesian Greedy	**0.21** +/- 0.024 0.077 (0.068, 0.087) Extreme Shares: 63 %	**0.31** +/- 0.029 0.11 (0.096, 0.12) Extreme Shares: 55 %	**0.3** +/- 0.028 0.1 (0.091, 0.12) Extreme Shares: 54 %

Results for SoftMax t = 1000 Heavy Tail

	Thompson Sampling	Bayesian Epsilon Greedy	Bayesian Greedy
Thompson Sampling	**0.44** +/- 0.0059 0.0045 (0.004, 0.0052) Extreme Shares: 0 %	**0.48** +/- 0.0083 0.009 (0.008, 0.01) Extreme Shares: 0 %	**0.48** +/- 0.011 0.015 (0.013, 0.017) Extreme Shares: 0 %
Bayesian Epsilon Greedy	**0.43** +/- 0.0084 0.0092 (0.0081, 0.01) Extreme Shares: 0 %	**0.46** +/- 0.0087 0.0098 (0.0087, 0.011) Extreme Shares: 0 %	**0.47** +/- 0.012 0.018 (0.016, 0.021) Extreme Shares: 0 %
Bayesian Greedy	**0.44** +/- 0.0099 0.013 (0.011, 0.014) Extreme Shares: 0.2 %	**0.47** +/- 0.011 0.016 (0.014, 0.018) Extreme Shares: 0.2 %	**0.49** +/- 0.011 0.015 (0.014, 0.017) Extreme Shares: 0 %

Results for HardMax t = 1000 Uniform

	Thompson Sampling	Bayesian Epsilon Greedy	Bayesian Greedy
Thompson Sampling	**0.092** +/- 0.02 0.053 (0.047, 0.06) Extreme Shares: 86 %	**0.14** +/- 0.026 0.089 (0.079, 0.1) Extreme Shares: 84 %	**0.19** +/- 0.03 0.11 (0.1, 0.13) Extreme Shares: 81 %
Bayesian Epsilon Greedy	**0.16** +/- 0.026 0.089 (0.079, 0.1) Extreme Shares: 80 %	**0.18** +/- 0.028 0.1 (0.089, 0.11) Extreme Shares: 76 %	**0.19** +/- 0.03 0.12 (0.1, 0.13) Extreme Shares: 82 %
Bayesian Greedy	**0.18** +/- 0.029 0.11 (0.095, 0.12) Extreme Shares: 81 %	**0.19** +/- 0.029 0.11 (0.099, 0.13) Extreme Shares: 79 %	**0.23** +/- 0.031 0.12 (0.11, 0.14) Extreme Shares: 75 %

Results for HardMaxWithRandom t = 1000 Uniform

	Thompson Sampling	Bayesian Epsilon Greedy	Bayesian Greedy
Thompson Sampling	**0.22** +/- 0.024 0.076 (0.067, 0.086) Extreme Shares: 60 %	**0.27** +/- 0.028 0.1 (0.091, 0.12) Extreme Shares: 61 %	**0.31** +/- 0.031 0.12 (0.11, 0.14) Extreme Shares: 61 %
Bayesian Epsilon Greedy	**0.27** +/- 0.026 0.088 (0.078, 0.1) Extreme Shares: 54 %	**0.3** +/- 0.028 0.1 (0.091, 0.12) Extreme Shares: 54 %	**0.35** +/- 0.03 0.12 (0.11, 0.14) Extreme Shares: 53 %
Bayesian Greedy	**0.25** +/- 0.026 0.086 (0.076, 0.098) Extreme Shares: 59 %	**0.32** +/- 0.029 0.11 (0.099, 0.13) Extreme Shares: 55 %	**0.36** +/- 0.03 0.12 (0.11, 0.14) Extreme Shares: 52 %

Results for SoftMax t = 1000 Uniform

	Thompson Sampling	Bayesian Epsilon Greedy	Bayesian Greedy
Thompson Sampling	**0.45** +/- 0.0084 0.0091 (0.0081, 0.01) Extreme Shares: 0 %	**0.47** +/- 0.0099 0.013 (0.011, 0.014) Extreme Shares: 0 %	**0.51** +/- 0.013 0.021 (0.018, 0.024) Extreme Shares: 0 %
Bayesian Epsilon Greedy	**0.44** +/- 0.01 0.013 (0.012, 0.015) Extreme Shares: 0 %	**0.46** +/- 0.011 0.016 (0.014, 0.018) Extreme Shares: 0 %	**0.49** +/- 0.014 0.024 (0.021, 0.027) Extreme Shares: 0.2 %
Bayesian Greedy	**0.43** +/- 0.011 0.017 (0.015, 0.019) Extreme Shares: 0 %	**0.46** +/- 0.012 0.019 (0.017, 0.021) Extreme Shares: 0 %	**0.49** +/- 0.013 0.022 (0.02, 0.025) Extreme Shares: 0.2 %

What next? Have re-run this for lower values of X and higher T (3000) and seen qualitatively similar results. Some potential follow-up questions of interest are:

1. Does reputation or information play a bigger role in the dominance of the incumbent? Can test this by re-initializing either reputation or information of the incumbent when the entrant wins the market (and re-running simulations of that).

2. Even for HardMax it seems that the entrant gets a non-zero market share (the average seems to be between .05 and .25 usually). What do the market share trajectories look like? Are there a lot of simulations where the entrant simply gets zero and some where the entrant seems to get lucky upon entry and has an increasing market share over time?

3. Run for a larger time horizon for the models with agent randomness to see if better algorithms win in the long-run.