Recommenders' Originals:

Integrated Recommender Systems and Vertical Foreclosure*

Guy Aridor† Duarte Gonçalves‡

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Abstract

We study a model of strategic interaction between producers and a monopolist platform that employs a recommendation system. We characterize the consumer welfare implications of the platform's entry into the production market. The platform's entry induces the platform to bias recommendations to steer consumers towards its own goods, which leads to equilibrium investment adjustments by the producers and lower consumer welfare. Further, we find that a policy separating recommendation and production is not always welfare improving. Our results highlight the ability of integrated recommender systems to foreclose competition on online platforms.

Keywords: Recommender System; Biased Intermediation; Vertical Integration; Bayesian Persuasion.

JEL Classifications: L11; L42; D83; L81.

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†Department of Economics, Columbia University; g.aridor@columbia.edu.

‡Department of Economics, Columbia University and BRU-IUL; duarte.goncalves@columbia.edu.

1. Introduction

An increasing number of online platforms deploy recommender systems to assist consumers with purchase decisions by providing information on available goods, which determinedly impacts consumer choice. These systems facilitate information acquisition on product quality by consumers in environments where there are thousands or even millions of alternatives available. Existing experimental literature provides causal evidence of these systems' immense power in steering demand, with market shares even being significantly affected by recommendation systems that supply simple information to consumers (Salganik, Dodds and Watts 2006). Moreover, anedoctal evidence reflects the huge influence the information provided by these systems have on consumption choices individuals make: recommendations are said to account for 75% of consumed content on Netflix and 35% of page views on Amazon.[1]

Online platforms increasingly not only deploy recommender systems but also develop and make available their own goods alongside other firms', with unclear implications to consumer welfare. Major platforms and technological leaders in the development and deployment of recommender systems such as Amazon, Netflix and Spotify, all now develop their own goods that are then made available on their platforms: Amazon produces more than 23,000 goods that are available on the firm's platform,[2] Netflix hosts more than 1,300 "Netflix originals" titles,[3] and Spotify is now investing in developing its own audio content.[4] There is a substantial amount of popular press coverage suggesting that these platforms systematically bias their search and recommendation systems towards their own goods.[5] Further, antitrust authorities have recognized this as a problem in online markets with the EU Competition Commission imposing a €2.42 billion fine on Google in June 2017 for giving an unfair advantage to its Google Shopping service at the expense of other comparison shopping services.[6]

In this paper we study the welfare consequences of a platform acting as both a producer and a recommender and consider the specific role played by the deployment of a recommender system. We set up a stylized model of a pay-for-access platform where producers make investment decisions about the quality of their goods and revenue is split according to each goods' market share. Our main focus is in contrasting resulting consumer welfare, investments and market shares across three different scenarios: **no platform production** case, where only a good by an independent

[1] How retailers can keep up with consumers (2013). `https://www.mckinsey.com/industries/retail/our-insights/how-retailers-can-keep-up-with-consumers`. Retrieved December 29th, 2019.

[2] Amazon Private Label brands. `https://www.marketplacepulse.com/amazon-private-label-brands`. Retrieved on December 29th, 2019.

[3] Complete List of Netflix Originals. `https://www.whats-on-netflix.com/originals/`. Retrieved on December 20th, 2019.

[4] Spotify bets big on podcasts with acquisition of Gimlet and Anchor. `https://mashable.com/article/spotify-acquires-podcast-gimlet-anchor/`. Retrieved on December 29th, 2019.

[5] For instance, Amazon The Brand Buster (`https://www.nytimes.com/2018/06/23/business/amazon-the-brand-buster.html`) discusses Amazon's bias towards its own goods via recommendation and search. How Google Interferes With Its Search Algorithms and Changes Your Results (`https://www.wsj.com/articles/how-google-interferes-with-its-search-algorithms-and-changes-your-results-11573823753`) discusses how Google manipulates its search results to steer consumers. According to Giving Viewers What They Want (`https://www.nytimes.com/2013/02/25/business/media/for-house-of-cards-using-big-data-to-guarantee-its-popularity.html`) upon release of *House of Cards*, Netflix recommended that consumers watch it regardless of their past behavior. Pandora has stated in court that it manipulates its recommendations based on the ownership of the sound recordings.

[6] Antitrust: Commission fines Google €2.42 billion for abusing dominance as search engine by giving illegal advantage to own comparison shopping service. `https://ec.europa.eu/commission/presscorner/detail/en/IP_17_1784`. Retrieved December 27th, 2019.

firm is available, the **platform dual role** case where the platform can both produce a good and design recommendations and both the platform's good and the firm's are available, and the **unbiased recommendations** case where we modify the dual role case by imposing an exogenous policy that requires that recommendation be unbiased, or truthful and neutral. An important component of our analysis is that we allow the independent firm to have access to alternative sources of revenue and characterize how the strength of these alternative sources impacts the resulting investment decisions and welfare of consumers on the platform. This is a reduced-form proxy for the extent to which the independent firm is reliant on the platform for revenues. For instance, movie producers can garner revenues from movie theaters or sales of DVDs and independent producers on e-commerce platforms can earn revenues from brick-and-mortar stores.

Unlike other papers that study the consequences of platform steering (e.g. Hagiu and Jullien (2011); de Corniere and Taylor (2019); Teh and Wright (2019)), we model the platform's recommendation as providing information on product quality to consumers as opposed to directly influencing the search order or choosing the consumed product for a fraction of consumers. Producers have access to stochastic investment technology that affects the likelihood that goods turn out to be of high versus low quality and consumers take as their prior on product quality the observed investments and update their beliefs based on the recommendation policy of the platform. This allows us to build credibility of recommendation directly into our model, where the platform's ability to steer the behavior of rational and Bayesian consumers is limited and depends on the design of its recommendations. In our model, the primary mechanism for the platform to bias recommendation towards its own goods is that the revenue generated by the platform is split between the platform and the independent firm according to their goods' consumption share. The optimal recommendation policy is simple, leading to market shares that are linear in the difference in the expected good quality.

We first consider the extent to which the platform's entry into the upstream production market influences investment levels and overall consumer welfare. We show that, despite increasing competition, the platform's entry results in lower quality investments and lower welfare when the independent firm is heavily dependent on the platform. However, we find that if the alternative sources of revenue are sufficiently large, the platform's entry increases consumer welfare.

The resulting differences in consumer welfare are driven by two opposing forces on changes in investment levels that result from the platform's entry. On the one hand, compared to when the independent firm is the sole producer, the dual role of the platform leads the independent firm to get a smaller share of the total platform revenue and, therefore, the independent firm only receives a fraction of its marginal impact on total platform revenue – opposite to the case where it is the sole producer. On the other hand, additional investment by the independent firm increases not only the total platform revenue but also the independent firm's market share, which expands the marginal return to investment relative to the sole producer case. These two opposing forces generate a threshold effect: for investment levels below a given threshold, the independent firm's marginal return to investment is weaker in the dual role case than in the no platform production setting, but above that threshold it is stronger. This directly leads to a threshold result for the investment levels of the independent firm in terms of the relevance of these alternative sources of revenue: if these are not large enough, the platform's dual role effectively depresses its incentives to invest in quality, resulting in lower consumer welfare; if instead these are large enough, the firm to invests more strongly than when it is the sole producer, driving up consumer welfare.

Another important consequence of the platform's dual role is the possibility of vertical foreclosure. This manifests itself in several ways. When the alternative revenue sources are small, the platform may find it profitable to become the product quality leader. Then, due to to the ability to bias recommendations towards its own products, it completely drives demand away from its competition, capturing the entire market on the platform. Although this is a sharp prediction, it echoes recent trends in video streaming markets, where platforms' original content quality – as indicated by awards received – has notably risen while their own content has simultaneously dominated platform viewership. Even when the independent firm's revenue sources are large enough compared to the platform's revenue potential that it becomes a product quality follower, the platform can still partially foreclose the independent firm by biasing recommendations in favor of its own goods. However, in this case the fact that consumers are rational and Bayesian prevents the platform from steering all demand towards its own good. Nevertheless, the ability to bias recommendations still enables the platform to steer some portion of the demand, enabling it to achieve a higher market share and profit than otherwise.

We then explore a natural policy remedy: ensuring that the platform cannot simultaneously provide recommendation services and produce goods or, equivalently, a policy that prevents the platform from biasing recommendations towards its own goods. Although it would be reasonable to expect an unambiguous improvement in consumer welfare, we find that, this policy can actually harm consumers under certain conditions. There are two observations that lead to this result. The first is that, under unbiased recommendation, the marginal gain in market share from increased investment is lower under unbiased recommendations for both the independent firm and the platform. The second is that total platform revenue is now more responsive to platform investment, but less responsive to firm investment. Combined, these two observations again lead to a threshold result in terms of the relevance of these alternative sources of revenue for the firm: if these are not large enough, unbiased recommendations are consumer welfare improving; if these are above a certain level, imposing unbiased recommendations lowers welfare when compared to the platform-optimal biased recommendations.

Our results illustrate that platform's entering the upstream production market leads to incentives to bias recommendation and has equilibrium effects on product quality. However, the resulting distortion from this entry does not necessarily harm consumers. An important element to consider is how dependent are independent firms on platform revenue: only when this dependence is high will policies targeting bias in recommender systems or separating recommendation and production altogether have a positive effect. This suggests caution when considering policy interventions, the bias in recommender system may be inducing independent firms to produce higher quality goods than what they otherwise would.

Related Work

Our paper lies in the intersection of three different literatures: biased intermediation, recommender systems and, more broadly, vertical integration and foreclosure.

The most relevant literature that our paper fits into is the nascent biased intermediation literature and, more generally, biased information provision. This literature emerged from the traditional intermediation and two-sided market (Rubinstein and Wolinsky 1987; Rochet and Tirole 2003) and focuses on the incentives of an intermediary to bias consumers' consumption decisions. However, while the majority of the literature focuses on the consequences of an intermediary manipulating the search process of a user, we instead model the intermediary as providing information to consumers. The intermediary in our model, the recommender, is thus an information designer as in the Bayesian persuasion literature (Kamenica and Gentzkow 2011; Bergemann and Morris 2019). This has two main advantages. The first is that it provides a more accurate model of how recommender systems function, where recommendations are used to provide imperfectly informed consumers with information about goods on the platform whose true consumption values are only learned from experience. The second is that it builds in restrictions on the credibility of the recommendations allowing for us to study the possibilities and investment consequences of bias even with sophisticated consumers.

Within this literature, the papers closest to ours are Bourreau and Gaudin (2018), de Corniere and Taylor (2019) and Hagiu and Jullien (2011). Bourreau and Gaudin (2018) study the incentives of a pay-for-access platform to bias their recommendation in order to reduce the market power of the upstream content providers. They also consider that the payouts between the producers and the platforms are split via royalty fees that depend on consumer's consumption choices. However, they focus on perfectly horizontal preferences with fixed good location, whereas our model considers the effect that platform recommendation bias has on quality investments. de Corniere and Taylor (2019) study a model of biased intermediation where the recommendation is sold through an auction while Hagiu and Jullien (2011) examine a setting where consumers perform costly and sequential search and the intermediary directs the consumers to a seller; in both cases, uniformed consumers naively follow the recommendation, without any credibility constraint. In contrast, our model imposes discipline on recommendation so that the recommender has to be credible as consumers are assumed to be rational and Bayesian. Furthermore, in our model recommendation depends on investment levels and both are endogenous elements of the model. Other related papers – where investment decisions are absent – focus on price competition among sellers on the platform (Armstrong, Vickers and

Zhou 2009), advertising and search (Hagiu and Jullien 2014; Burguet, Caminal and Ellman 2015; de Cornière 2016) and commission and price setting (Inderst and Ottaviani 2012*a*; 2012*b*, Teh and Wright 2019).

Naturally, this paper contributes to the literature on recommender systems which analyzes the economic consequences of recommendation on consumer choice, pricing and sales. The most relevant paper to ours is Bergemann and Ozmen (2006) which analyzes model with horizontally differentiated products where a platform with a recommender system competes with a fringe of distribution channels without such system. In this paper, the recommender's information advantage is modeled as deriving observing past users' experience in a two-period model and consumers are able to obtain a recommendation at no cost. Opposite to our model, the focus in this paper is on the optimal pricing by the recommender and investment decisions are absent as all firms are intermediaries selling the same products. Other, less related works are Che and Hörner (2017), Fleder and Hosanagar (2009) and Hosanagar, Krishnan and Ma (2008). Che and Hörner (2017) characterizes the optimal information provision by a welfare maximizing recommender that learns a product's quality through consumer feedback. Fleder and Hosanagar (2009) discuss the role that recommender systems can play in diversifying sales due to their personalized nature, but do not endogenize production or consider incentives to bias recommendation. Hosanagar, Krishnan and Ma (2008) study how a recommender would trade-off between optimizing for profit and maintaining reputation amongst consumers, but do not consider good investment or platform production.

Finally, our paper broadly contributes to a classic literature in industrial organization that focuses on vertical integration and investment and investment (Grossman and Hart 1986; Williamson 1971; Perry 1989) and vertical foreclosure (Hart et al. 1990; Ordover, Saloner and Salop 1990). We are most interested in the dual role of platforms as producers – vertical integration – and information providers, which has not received as much attention in the literature. Two papers that look at related problems are Asker and Bar-Isaac (2016) – studying the role of vertical information restraints in a retail market that involves search frictions with a focus on understanding minimum advertising price restrictions – and Janssen and Shelegia (2015), who introduces vertical industry structure into a consumer search model where consumers are uninformed about wholesale prices. These papers are complementary to ours, but, to our knowledge, we are the first to study the integration of recommendation and production. In particular, we are the first to study how this integration can lead to a novel form of vertical foreclosure – shifting market share from the non-integrated producers to the integrated producers by biasing the information that is revealed to rational, Bayesian consumers.

The remainder of the paper is structured as follows: Section 2. provides the setup for the model. Section 3. characterizes the equilibrium consequences and impact of the platform's dual role. Section 4. explores the value of recommendation and characterizes the equilibrium welfare consequences of a policy that imposes unbiased recommendations. Finally, section 5. concludes.

2. Model Setup

Production. There are two firms, the independent firm F and the platform P. We will consider two cases: one in which only F, the independent firm, makes production decisions, and another where both F and P make production decisions. In the case where both F and P make production decisions, we suppose that the independent firm's investment q_F is observable by the platform before deciding its investment. We argue that this timing assumption is realistic as platforms are usually second-movers in production decisions. However, our conclusions do not rely on this: a setting where production decisions are simultaneous yields the same qualitative results.[7]

We will denote by J the set of firms making production decisions. Each firm $j \in J$ produces a single good x_j which can be of two types: high quality, $x_j = 1$, and low quality, $x_j = 0$. However, the realized quality of the goods ultimately produced – x_j – is stochastic and depends on the firms' investments. Each firm initially makes investment decisions $q_j \in [0,1]$ which determines the probability that the realized quality of the good is high, i.e. $q_j = \Pr(x_j = 1)$. This investment in stochastic quality has quadratic cost $C(q_j) = q_j^2$.

Good Distribution. Firm P serves as a platform for consumers in the market to access the goods produced. P sets a single price, τ, that consumers need to pay in order to access the platform. Once consumers are on the platform they can consume a single item on the platform without additional cost.[8]

Consumers. There is measure $M_P > 0$ of risk-neutral expected-utility-maximizing consumers in the market that can only consume the produced goods through the platform. Moreover, we assume that all consumers prefer high-quality goods to low-quality goods so that consumer i's indirect utility from consuming good x and paying access fee τ is given by $u(x,\theta_i,\tau) = \theta_i x - \tau$, where θ_i denotes consumer i's willingness-to-pay for high quality goods.[9] The utility of consuming the outside good – that is, the utility from not joining the platform – is normalized to 0 and θ is distributed according to distribution G, denoting the uniform distribution on $[0,\bar{\theta}]$. We model consumers as being well-informed and able to observe the investment probabilities by each of the firms on the platform so that they use this information to form their beliefs about whether the resulting good is high quality or low quality.

Recommendation Policy. We model P as an information designer who commits to a recommendation policy before the true quality of x_P and x_F is realized. The recommendation policy is then a mapping from the states, the realized good qualities $\{0,1\}^J$ to distributions over an arbitrary message space $\mathcal{M}$. The commitment assumption is common

[7]The only difference, aside from minor changes in the actual equilibrium investment values, is that in such setting for some range of parameters there are two equilibria: one where the platform acts as a monopolist, invests more than the firm and the firm gets no market share and another where the firm invests higher amounts than the platform and consequently gets a positive market share. Consistently selecting one or the other over the ranges of multiplicity results in the same patterns.

[8]We do not allow the producing firms to set prices for their individual goods. This is the case in many markets where recommender systems are widely deployed such as media streaming or news platforms where consumers pay for access to the bundle of content on the platform. Further, it allows us to restrict focus on the interplay between production quality and recommendation without introducing the additional complexity of pricing.

[9]Some of the markets, such as movies and music, described in the introduction where recommendation systems play a large role have horizontally differentiated preferences. However, vertical differentiation case has a cleaner interpretation in the context of stochastic investment technology and additionally gives us a cleaner interpretation of the consequences of the integration between recommendation and production.

in the literature in economics and computer science that studies issues related to strategic interactions between recommender systems and Bayesian consumers (Che and Hörner 2017; Kremer, Mansour and Perry 2014; Mansour, Slivkins and Syrgkanis 2015). This assumption is often justified by both the need for credibility in the context of repeated interactions but also by the fact that existing recommender systems are stable deployed algorithms which imply commitment to a predefined recommendation strategy.

Revenue Sources Outside the Platform. We suppose that the integrated producer, P, only gets revenue from the platform, as is common for private-label goods such as ("recommender's originals" such as Netflix's, Amazon's, Hulu's) where their good is only available on their own platform. Contrastingly, we allow the independent producer, F, to obtain additional revenue streams off the platform, $R_F(q_F)$. We interpret these additional revenue as the reliance of the independent firm on the platform revenues. This can be seen as a reduced-form proxy of revenues from offline alternatives to the online platform. For instance, in the case of Amazon this could be the revenue achieved from sales in brick and mortar stores and for Netflix it could be the revenue achieved from movie theaters or other streaming platforms. In order to study how equilibrium and consumer welfare change as the strength of the outside option for the producing firms increases we parameterize $R_F(q_F) = r_F \cdot q_F$ with $r_F \geq 0$.

Revenue on the Platform. The platform revenue R_P will depend on the recommendation policy, pricing and quality investments of both firms. We suppose that is split between the platform and the firm according to their expected consumption share on the platform, which we denote as α_P and α_F.[10] The overall expected payoffs for each firm is therefore given by:

$$\pi_P = \alpha_P \cdot R_P - C(q_P)$$

$$\pi_F = \alpha_F \cdot R_P + R_F - C(q_F)$$

Timing. The timing of events in the model is summarized as follows:

1. Production decisions are made
2. The platform determines access fee τ and consumers decide whether to join the platform or not, resulting in revenue R_P
3. The platform commits to a recommendation policy
4. True good qualities realize
5. The platform makes recommendations

[10] This split rule can be interpreted as the reduced form of a linear contract agreed to by the platform and the independent producer where the independent producer gets a royalty fee for each consumer that consumes her good. This type of contractual relationship is common on pay for access platforms. For instance, Spotify pays musicians a royalty in accordance with the number of times their song is played on the platform and on YouTube Premium the membership fees are distributed based on how many members watch a producer's content. We take this split rule as exogenous and study its consequences.

6. Consumers decide which good to consume conditional on investment probabilities and platform recommendation
7. The independent firm and the platform split the platform revenue according to their consumption share and the independent firm accrues outside revenue R_F

One aspect of the timing which requires further justification is the timing of P's commitment to a recommendation policy which occurs after consumers decide to join the platform. There are two justifications for this. The first is that it is arguably the most realistic timing as it is common that products are constantly added and removed from the set of available products while pricing and consumer's decision to join the platform change rather infrequently. Furthermore, platforms are continually iterating on their recommendation system design even after consumers join the platform. Thus, this timing is more in line with what occurs in many online markets. The second justification is that it allows us to isolate the role of recommendation as being utilized purely for steering subject to the constraint of rational, Bayesian consumers. As a result, this setup allows us to isolate the equilibrium implications of steering of rational consumers on production decisions.

3. Consequences of the Platform's Dual Role

In this section we study the consequences of the platform's dual role. In order to do so, we contrast the cases where only the firm's good is available on the platform to the equilibrium in the case where the platform can itself choose to produce a good and steer consumers via its recommender system.

3. 1. No Platform Production

We first consider the case where the independent firm is the sole producer in the market. The timing is the same, but the platform does not make production decisions. As there is only one good available on the platform and the platform does not observe product quality before consumers make the decision to join the platform, there is no scope for recommendation policy to impact consumers' valuation of paying to access the platform.

We assume that pricing is still profit maximizing.[11] The expected value of subscribing to the platform for a consumer of type θ is then $\mathbb{E}u(x,\theta,\tau) = \theta\mathbb{E}[x] - \tau$, where in the case we are analyzing we have the expected quality of the consumed good is $\mathbb{E}[x] = \mathbb{E}[x_F] = q_F$. A consumer of type θ then subscribes to the platform whenever $\mathbb{E}u(x_F,\theta,\tau) \geq 0$, where, though the tie-breaking rule favors subscribing, it is not consequential for investment decisions as types are continuously distributed.

[11] Although the platform has no incentive to set profit maximizing prices in the current setup, as it effectively accrues no revenue, it can be seen – and indeed it is – the limit case of a related setup where the platform gets a fixed share s of the overall revenue, where this share is arbitrarily small. This analysis of this exact case yields the same qualitative results.

The platform's pricing problem is then

$$\tau \in \arg\max_{t\in\mathbb{R}} M_p \cdot t \cdot \int_0^{\bar{\theta}} \mathbf{1}_{\mathbb{E}u(x_F,\theta,t)\geq 0} dG(\theta)$$
$$= \arg\max_{t\in\mathbb{R}} t \cdot \left(1 - G\left(\frac{t}{q_F}\right)\right)$$

Given the uniform distribution assumption, the pricing problem is equivalent to

$$\tau \in \arg\max_{t\in\mathbb{R}} t \cdot \left(1 - \frac{t}{\bar{\theta} q_F}\right)$$

with solution $\tau = \frac{1}{2}\bar{\theta} q_F$. This implies that $R_P = \frac{M_P \cdot \bar{\theta}}{4} q_F$.

We define $r_P := \frac{M_P \cdot \bar{\theta}}{4}$, so that $R_P = r_P q_F$. r_P represents the total potential revenue that can be accrued on the platform. Both r_P and r_F can be taken as reduced form measures of market size and relative comparison between the two provides us with a notion of the relative dependence of the independent firm on the platform. The comparison between r_P and r_F will then provide insight on how consumer welfare and other considerations depend on how relevant platform market size is relative to alternative markets available to the independent firm.

The firm's production decision problem is then given by

$$q_F \in \arg\max_{q\in[0,1]} \alpha_F \cdot R_P + R_F - C(q)$$
$$= \arg\max_{q\in[0,1]} r_P \cdot q + r_F \cdot q - q^2$$

and therefore the firm's optimal production decision in the "no platform production" (NP) case is

$$q_F^{NP} = \min\left\{\frac{r_P + r_F}{2}, 1\right\}$$

Consumer welfare is then given by

$$W^{NP} = \mathbb{E}_{\theta,x_F} \max\{u(x_F,\theta,\tau), 0\}$$
$$= \int_{\frac{\bar{\theta}}{2}}^{\bar{\theta}} \theta q_F - \tau dG(\theta) = \frac{3}{8}\bar{\theta}^2 q_F^{NP}$$

which turns out to be linear in the expected good quality of the good consumed in the platform due to the assumption of types being uniformly distributed.

3.2. Platform's Dual Role

We now characterize the equilibrium investment levels of the case where the platform can also make production decisions. Given that now more than one good is available on the platform, recommendations play a role in determining how much consumers value having a platform subscription.

The solution concept we adopt is sub-game perfect equilibrium and thus we solve for the equilibrium investment levels via backwards induction. An equilibrium is a tuple $\{\rho, \tau, q_P, q_F\}$ where ρ is the recommendation policy of the platform, τ is the access fee set by the platform, q_P is the investment level of the platform, and q_F is the investment level of the independent firm.

The (inverse) timing of events is the following:

8. The independent firm and the platform split the platform revenue according to their consumption share and the independent firm accrues outside revenue R_F.
7. Consumers decide which good to consume by selecting a good x_j with the highest expected utility conditional on recommendation and investment probabilities:
   $x_j \in X_i^*(\rho, m, \theta, \tau) := \arg\max_{x_j \in \{x_P, x_F\}} \mathbb{E}[u(x_j, \theta, \tau) \mid \rho(x_P, x_F) = m]$ given ρ,
   $\forall m : \mathbb{P}(\rho(x_P, x_F) = m) > 0$ and $\theta : \mathbb{E}\left[u(x_j, \theta, \tau)\right] = \mathbb{E}\left[\mathbb{E}\left[u(x_j, \theta, \tau) \mid \rho(x_P, x_F) = m\right]\right] \geq 0$.
6. The platform makes recommendations:
   $\rho(x_P, x_F) = m$, where $\rho : \{0,1\}^2 \to \Delta\mathscr{M}$ and $m \in \mathscr{M}$.
5. True good qualities realize:
   $x_j \sim \mathbb{P}(x_j = 1) = q_j$ and $\mathbb{P}(x_j = 0) = 1 - q_j$, with $j = P, F$.
4. The platform commits to a recommendation policy ρ:
   $\rho \in \arg\max_{r:\{0,1\}^2 \to \Delta\mathscr{M}} \alpha_P \cdot R_P - C(q_P)$ given R_P, q_P
   and where the platform's market share is given by $\alpha_P = \mathbb{E}\left[\mathbf{1}_{x_P \in X_i^*(\rho,m,\theta,\tau)} \mid \theta : \max_{x_j \in \{x_P, x_F\}} \mathbb{E}[u(x_j, \theta, \tau) \mid \rho(x_P, x_F) = m] \geq 0\right]$
3. The platform determines access fee τ and consumers decide whether to join platform, resulting in revenue R_P:
   $\tau \in \arg\max_{t \in \mathbb{R}} M_P \cdot t \cdot \int_0^{\bar{\theta}} \mathbf{1}_{\mathbb{E}[\mathbb{E}[u(x_j,\theta,t) \mid \rho(x_P,x_F)=m]] \geq 0} dG(\theta)$
2. The platform decides what level of quality to produce, q_P:
   $q_P \in \arg\max_{q_P \in [0,1]} \alpha_P \cdot R_P - C(q_P)$
1. The independent firm determines what level of quality to produce, q_F:
   $q_F \in \arg\max_{q_F \in [0,1]} \alpha_F \cdot R_P + R_F - C(q_F)$

Optimal Recommendation Policy

The first step is to solve for the optimal recommendation policy, as from period 5 onwards the sequence of events is mechanical. We assume, as is customary in the information design literature (Bergemann and Morris 2019), that the recommender is able to select its preferred equilibrium. Noting that $\arg\max_{x_j \in \{x_P, x_F\}} \mathbb{E}[u(x_j, \theta, \tau) \mid \rho(x_P, x_F) = m] = \arg\max_{x_j \in \{x_P, x_F\}} \theta \mathbb{E}[x_j \mid \rho(x_P, x_F) = m] - \tau = \arg\max_{x_j \in \{x_P, x_F\}} \mathbb{E}[x_j \mid \rho(x_P, x_F) = m]$, we have that the event $\left\{x_P \in \arg\max_{x_j \in \{x_P, x_F\}} \mathbb{E}[u(x_j, \theta, \tau) \mid \rho(x_P, x_F) = m]\right\}$ is independent from θ for any recommendation policy ρ and so α_P reduces to $\mathbb{P}\left(x_P \in \arg\max_{x_j \in \{x_P, x_F\}} \mathbb{E}[x_j \mid \rho(x_P, x_F) = m]\right)$. As the recommendation policy is determined after revenue R_P is collected and q_P is chosen, the problem of designing a recommendation policy that maximizes the platform's payoffs collapses to maximizing the platform's market share subject to consumer credibility constraints:

$$\begin{aligned} & \arg \max_{r:\{0,1\}^2 \to \Delta \mathcal{M}} \alpha_P \cdot R_P - C(q_P) \\ \text{subject to} \quad & \alpha_P = \mathbb{E}\left[\mathbf{1}_{x_P \in X_i^*(\rho, m, \theta, \tau)} \mid \max_{x_j \in \{x_P, x_F\}} \mathbb{E}[u(x_j, \theta, \tau) \mid \theta; \rho(x_P, x_F) = m] \geq 0\right] \\ & = \arg \max_{r:\{0,1\}^2 \to \Delta \mathcal{M}} \mathbb{P}\left(x_P \in \arg \max_{x_j \in \{x_P, x_F\}} \mathbb{E}[x_j \mid \rho(x_P, x_F) = m]\right) \end{aligned} \tag{RP}$$

where $X_i^* = \arg\max_{x_j \in \{x_P, x_F\}} \mathbb{E}[u(x_j, \theta, \tau) \mid \rho(x_P, x_F) = m]$.

Proposition 1. *The optimal recommendation policy by the platform is as follows:*

- *if $q_P \geq q_F$, the platform always recommends its good*
- *if $0 = q_P < q_F$ or $q_P < q_F = 1$ the platform sends unbiased recommendation and, whenever possible, breaks indifference in favor of its good*
- *if $0 < q_P < q_F < 1$, then P commits to recommend its own good whenever it has weakly higher quality than the non-integrated firm's, $\mathbb{P}\left\{\rho(x_P, x_F) = P \mid x_P \geq x_F\right\} = 1$, but to spam consumers only a fraction of the time when this is not the case, that is $\mathbb{P}\left\{\rho(x_P, x_F) = P \mid x_P < x_F\right\} = \frac{q_P}{1-q_P} \frac{1-q_F}{q_F} < 1$.*

The proof for proposition 1 is deferred to the appendix.

Proposition 1 specifies the optimal recommendation policy for the platform,. The intuition for the recommendation policy is as follows. When $q_P \geq q_F$, that is, when the platform invests at least as much as the firm in good quality, then the platform has no incentive to provide any information since, without additional information, consumers believe that the platform's good is more likely to be of high quality than the firm's and so will always choose it. When $q_P < q_F$, that is, when the independent firm invests more than the platform, then the platform commits to truthfully recommending the best option (breaking ties in its favor) when the best option is hers. However, when the best option for consumers is the independent firm's good then the platform can sometimes bias its recommendations.

However, this bias is constrained by the need for the policy to be credible in the consumers' eyes and so the platform can only recommend its product $\frac{q_P}{1-q_P}\frac{1-q_F}{q_F}$ fraction of the time in this case.

It immediately follows from proposition 1 that

Corollary 1.

- *The expected market share of the platform* α_P *is given by* $\min\{1, 1-(q_F - q_P)\}$ *and that of the firm is* $\alpha_F = 1-\alpha_P = \max\{0, q_F - q_P\}$.
- *The expected good quality is given by* $\mathbb{E}[x(m)] = \max\{q_P, q_F\}$*, for* $x(m) \in X_i^*(\rho, m, \theta, \tau)$.

Corollary 1 highlights how the optimal recommendation policy heavily favors the platform's own goods and how, whenever the firm invests more than the platform, the recommendation policy results in consumer welfare only depending on the investment level of the independent firm. In order to be recommended and have its good subsequently consumed on the platform, the firm not only needs to initially invest more than the platform but also needs its realized quality to be higher than that of the platform's good. Even then, the platform can still induce Bayesian consumers to consume its good sometimes. The credibility constraints that the platform needs to satisfy in order to persuade consumers to pick its own good leads to a lower bound on the expected quality of the chosen good: the consumers cannot be left worse off when compared to just picking the independent firm's good regardless of the recommendation. Importantly, this lower bound is attained whenever the firm invests more than the platform.

Optimal Access Fee

The pricing problem faced by the platform is then

$$\tau \in \arg\max_{t\in\mathbb{R}} M_P \cdot t \cdot \int_0^{\bar{\theta}} \mathbf{1}_{\mathbb{E}[\mathbb{E}[u(x_j,\theta,\tau)|\rho(x_P,x_F)=m]]\geq 0} dG(\theta)$$

As the optimal recommendation policy is independent of the access fee, the problem simplifies to

$$\tau \in \arg\max_{t\in\mathbb{R}} M_P \cdot t \cdot \int_0^{\bar{\theta}} \mathbf{1}_{\theta\cdot\max\{q_P,q_F\}\geq t} dG(\theta)$$

which then has a similar solution and implications to the "no platform production" case:

Proposition 2.

- *The optimal access fee is given by* $\tau = \frac{1}{2}\bar{\theta}\max\{q_P, q_F\}$.
- *The total revenue collected by the platform from subscriptions is* $R_P = r_P \max\{q_P, q_F\}$.
- *The resulting consumer welfare, given investment in quality* q_P *and* q_F*, is* $\frac{3}{8}\bar{\theta}^2 \max\{q_P, q_F\}$.

Proposition 2 gives the optimal access fees given production decisions. When $q_P \geq q_F$, being forward-looking, consumers know that the platform will always recommend its own good and consumers will take this into account when determining the expected value of the platform. Since consumers form their expectations only based on the likelihood that the platform's good will be high quality, the optimal access fee is the same as if the firm was not on the platform.

When $q_P < q_F$, however, the optimal access fee *only* depends on q_F. Interestingly, the platform increasing q_P has no effect on the optimal access fee, exactly because it has no effect on the expected good quality that consumers expect to be recommended. The intuition for this is that sometimes the platform will recommend its own good even when its good is low quality. However, when the independent firm is recommended, the consumer knows that it is because it is a high-quality good. An increase in q_F leads to higher probability that the consumer will be recommend a high-quality good but an increase in q_P may only lead to an increase in the probability that the consumer gets a biased recommendation.

Production Decisions

Finally, we characterize the optimal investment decisions of the firms.

We assume that the firm chooses quality investment first and the platform second. The motivation for this assumption is that platforms are often seen as second-movers in terms of production decisions, being able to benefit from the information obtained from having other firms' goods available.

The platform's production problem is given by

$$\max_{q_P \in [0,1]} \alpha_P \cdot R_P - C(q_P)$$

As $\alpha_P = \min\{1, 1-(q_F - q_P)\}$ and $R_P = r_P \max\{q_P, q_F\}$, we have that the objective function is continuous and piecewise strictly concave in q_P, with

$$\pi_P(q_P, q_F) = \alpha_P \cdot R_P - C(q_P) = \begin{cases} r_P \cdot (1-(q_F - q_P)) \cdot q_F - {q_P}^2 & \text{if } q_P < q_F \text{ and } r_P < 2 \quad (1) \\ r_P \cdot q_P - {q_P}^2 & \text{if } q_P \geq q_F \text{ and } r_P < 2 \text{ or } r_P \geq 2 \quad (2) \end{cases}$$

Proposition 3. *The equilibrium investment levels for the platform are given by:*

$$q_P(q_F) = \begin{cases} \frac{r_P}{2} q_F & \text{if } q_F \geq \tilde{q}_F \text{ and } r_P < 2 \\ \min\left\{1, \frac{r_P}{2}\right\} & \text{if } q_F < \tilde{q}_F \text{ and } r_P < 2, \text{ or } r_P \geq 2 \end{cases}$$

where $\tilde{q}_F \equiv \frac{r_P}{4 - r_P}$.

Proposition 3 immediately follows from two observations. The first is that if $r_P \geq 2$, then the maximizer of both (1) and (2) is $q_P = 1$. The second is that if $r_P < 2$, then the maximizer of (2), $\frac{r_P}{2} < 1$, is always weakly larger than the maximizer of (1), $\frac{r_P}{2} q_F$, and therefore $\frac{r_P}{2}$ will be the maximizer in the case when $\pi_P(\frac{r_P}{2} q_F, q_F) < \pi_P(\frac{r_P}{2}, q_F)$ and $\frac{r_P}{2} \geq q_F$, which reduces to the case when $q_F < \tilde{q}_F$. Further, we break indifference in favor of the second-arm, when $q_F = \tilde{q}_F$. The tie-breaking will be immaterial in characterizing the equilibrium investment levels, and so is without loss for the overall conclusions.

The solution to the platform's production decision has a very natural interpretation: If the independent firm's investment in quality is too low, then the platform is better off by investing as if it were the only good available and, indeed, it is going to recommend only its own goods. If instead the independent firm's investment is high enough, then the platform invests below what it would if its good was the only one on the platform – as a single good monopoly case – saving in investment costs at the expense of the independent firm. It can still enjoy some positive market share and get a part of the revenue R_P as it will bias recommendations towards its own goods when its realized quality is weakly higher than the firm and, sometimes, even when it isn't.

The independent firm's production problem is then to choose q_F in order to maximize $\pi_F(q_F) = \alpha_F \cdot R_P + R_F - C(q_F)$, given $q_P(q_F)$. By backward induction, the independent firm's payoffs can be written as follows:

$$\pi_F(q_F) = \begin{cases} (q_F - q_P(q_F)) \cdot r_P \cdot q_F + r_F \cdot q_F - {q_F}^2 & \text{if } r_P < 2 \text{ and } q_F \geq \tilde{q}_F = \frac{r_P}{4 - r_P} \qquad (3) \\ r_F \cdot q_F - {q_F}^2 & \text{otherwise} \qquad (4) \end{cases}$$

Proposition 4. *The equilibrium investment level for the firm is given by*

$$q_F^{DR} = \begin{cases} \frac{r_F}{2} & \text{if } r_F \leq \underline{r_F} \text{ and } r_P < 2, \text{ or } r_P \geq 2 \\ \tilde{q}_F & \text{if } \underline{r_F} \leq r_F < \overline{r_F} \text{ and } r_P < 2 \\ \min\left\{1, \frac{r_F}{2(1-r_P)+{r_P}^2}\right\} & \text{if } \overline{r_F} \leq r_F \text{ and } r_P < 2 \end{cases}$$

where $\tilde{q}_F \equiv \frac{r_P}{4-r_P}$, $\underline{r_F} \equiv \frac{r_P}{4-r_P}\left(2 - \sqrt{2r_P(2-r_P)}\right)$, $\overline{r_F} \equiv \frac{r_P}{4-r_P}\left(2(1-r_P)+{r_P}^2\right)$.
Except when $\underline{r_F} = r_F$, *the investment levels are uniquely determined.*

The proof for proposition 4 is deferred to the appendix.

The solution to the independent firm's production decision has a clear interpretation. If $r_P \geq 2$, the platform will always set $q_P = 1$ and recommends only its good, regardless of the firm's investment level, thereby excluding the firm from considering platform revenue. When instead $r_P < 2$, then there are three cases to consider, which depend on $\underline{r_F}$ and $\overline{r_F}$, both strictly increasing in r_P and implicitly defining conditions on r_F relative to r_P. In the first case, where r_F is small enough relative to r_P, we have that the independent firm's investment decisions only depend on the relative strength of the outside markets since the platform's ability to bias recommendations is sufficiently strong to foreclose the independent firm entirely from the platform. In the second case, where r_F attains intermediate values relative to r_P, the independent firm invests in quality just enough to leave the platform indifferent between foreclosing the independent firm and investing in lower quality and allowing the firm some consumption share. In this case, the investment that the firm would make in the absence of platform revenue would be considerably lower and fighting for higher consumption share on the platform is still not worthwhile. However, the firm strictly benefits from getting enough of the platform demand and investing more than it would when considering only the outside revenue. Finally, in the last case, where r_F is large relative to r_P, it is now worthwhile for the firm to invest in even higher quality, simultaneously allowing it to compete for a larger share of the platform demand as well as leading to higher revenue from platform subscriptions.

3.3. Welfare Consequences of the Dual Role

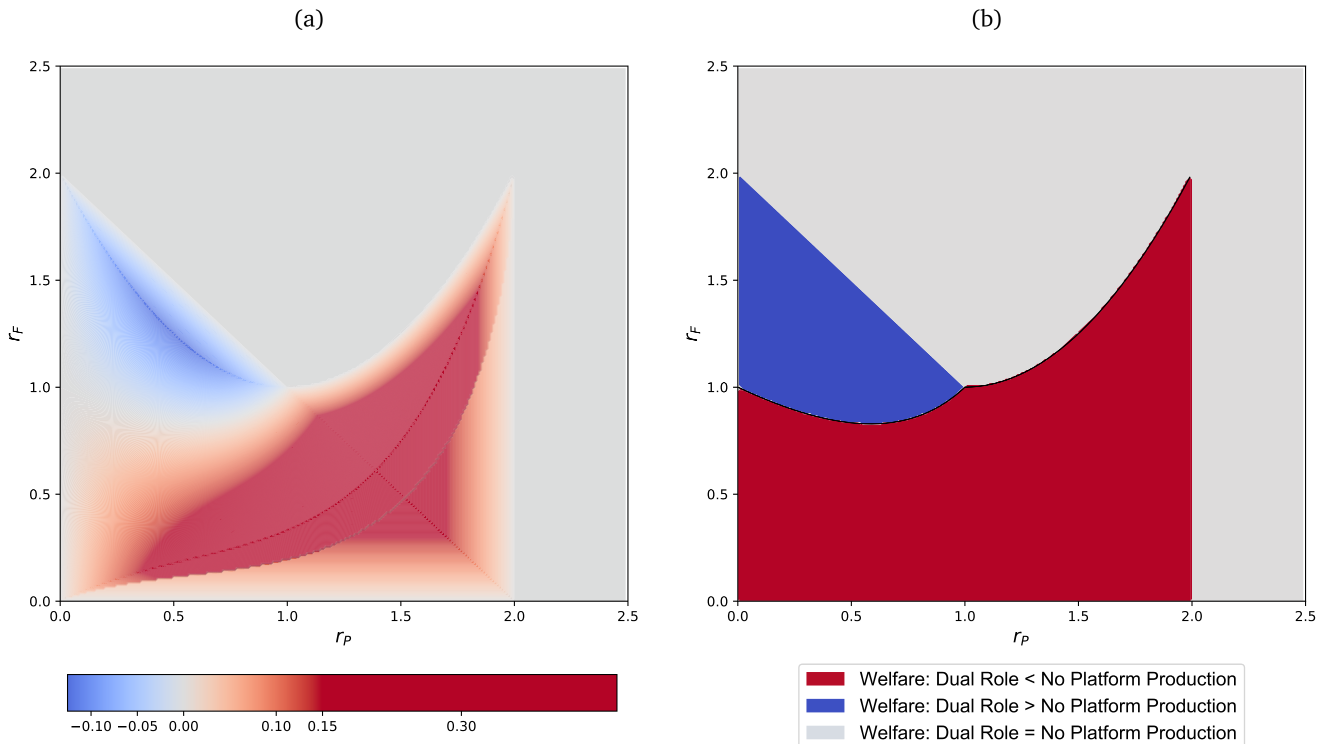
We now study the implications of the dual role on consumer welfare. Our primary question of interest is whether the increased competition in the production market through the entry of platform increases or decreases consumer welfare. We now state the main result comparing welfare between the two cases, whose proof is deferred to the appendix:

Proposition 5. *Consumer welfare is higher under the dual role than under the no platform production case if and only if* $r_F \geq \frac{2(1-r_P)+r_P^2}{\max\{1,2-r_P\}}$ *or* $r_P \geq 2$.
It is strictly higher if and only if $r_F > \frac{2(1-r_P)+r_P^2}{2-r_P}$, $\frac{r_F+r_P}{2} < 1$ *and* $r_P < 2$.

Proposition 5 shows that, despite the increased competition in the upstream market, the platform's dual role results in lower quality investments and lower welfare when the independent firm's alternative market relevance is small compared to the platform's – cf. figure 1 and figure 2. This shows that when the platform steers a substantial fraction of the demand utilizing recommendation, platform upstream competition decreases consumer welfare. It not only adversely impacts consumer welfare but also strictly depresses the industry's total profits since, if the firm is the sole producer, its profits attain the industry's maximum. Therefore the increased competition leads to an overall, unambiguous, decrease in total surplus. However, as the size of the alternative market grows relative to the size of the platform's potential revenues, eventually the platform's dual role is welfare improving.

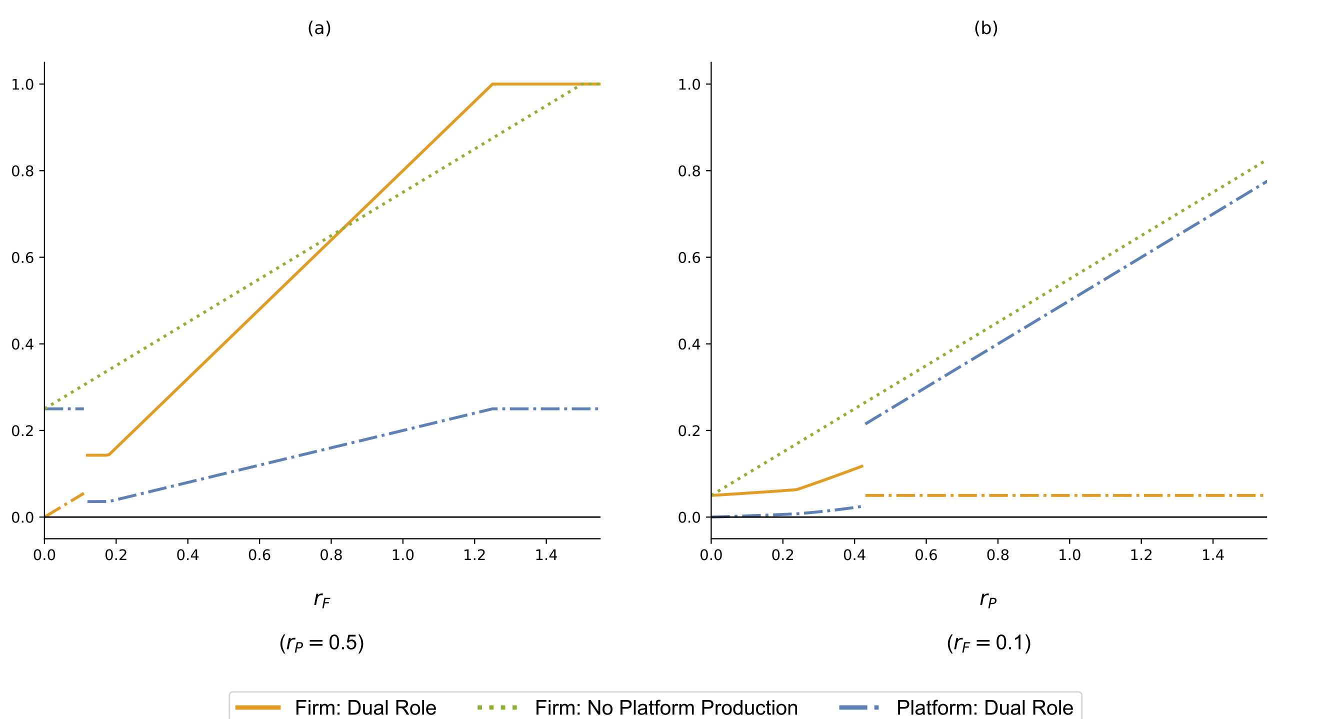
The model holds another significant implication for streaming platforms: once the platform relevance becomes significant relative to other revenue sources, the platform becomes the product quality leader and uses recommendations to perform vertical foreclosure. As illustrated in figure 2b, in equilibrium the platform may benefit from completely disregarding the independent firm, producing as if it were a monopolist. This occurs when the firm's alternative revenue sources are not strong enough relative to platform potential revenue to induce it to compete for market share with the platform as a producer. The outcome of this interaction is then a form of vertical foreclosure by

Figure 1. **Difference in Welfare: Dual Role – No Platform Production**



Notes: This figure displays the difference in expected product quality, which corresponds to average consumer welfare on the platform, between the no platform production and dual role cases. Subfigure (a) shows a heatmap of the differences between welfare for varying r_F and r_P and subfigure (b) shows for which values of r_F and r_P the different cases differ in resulting welfare.

Figure 2. **Equilibrium Investment Levels**



Notes: This figure displays the investment levels across the no platform production and dual role cases. Subfigure (a) shows how investment levels change as we increase the alternative revenue sources for a fixed level of platform potential revenue. Subfigure (b) shows how investment levels change as we increase the platform potential revenue for a fixed level of alternative revenue sources.

means of the platform's use of biased recommendations, which establishes the platform as the quality leader by precluding the independent firm's access to the platform's demand. This necessarily has a negative effect on consumer welfare as the expected good quality on the platform is lower than it would be when compared to a case where only the independent firm's good is available.

This prediction echoes anecdotal evidence on streaming platforms. First, the already mentioned concerns that platforms bias their recommendation systems towards their own goods to the detriment of goods produced by other firms manifests itself in our model through the biased recommendations the platform uses to steer consumers towards its own goods. Second, data on viewer and subscriber patterns on streaming platforms indicates that there is a positive correlation between number of subscribers, platform original content's share of the total platform viewership and ranking of platform's original content.[12] Even though the model's prediction is sharp and reality is necessarily more complex and nuanced, the evidence is consistent with the main intuition from the model.

Nevertheless, the need to compete for market share on the platform can still drive the firm to stronger quality investments as the platform's audience loses its overwhelming relevance within the industry, possibly due to the emergence of alternative platforms. When the independent firm has a base incentive to invest in quality that is sufficiently high – i.e. when the alternative revenue sources are significant enough relative to the platform's – it is worthwhile for the firm to compete for market share on the platform. In this case, the platform becomes a quality follower relative to the independent firm, making use of recommendations to appropriate substantial market share that the independent firm's production decisions attract.

4. Unbiased Recommendations

In this section we consider a natural policy remedy that imposes that the platform needs to provide unbiased - truthful and neutral - recommendations. While it would be natural to expect that such a policy is unambiguously beneficial for consumers, we find that when the platform potential revenue is large relative to the firm's alternative revenue sources such a policy may in fact harm consumers. The intuition for this is that when the firm's alternative revenue is low then unbiased recommendation levels competition conditions between the platform and the independent firm. However, when the firm's alternative revenue is high, biased recommendations may spur the firm to invest more strongly in quality than it would otherwise under unbiased recommendations to the point that it overcomes the negative effect of biased recommendations on consumer welfare.

Another interpretation of the unbiased recommendation case is as a separation (or divestiture) between the platform's production and recommendation activities. Therefore, this also corresponds to a benchmark of two producers,

[12] See, for the case of Netflix, e.g.
Netflix Original Series Viewing Climbs, but Licensed Content Remains Majority of Total U.S. Streams. `https://variety.com/2018/digital/news/netflix-original-series-licensed-viewing-friends-the-office-1203085230/` Retrieved on November 10th, 2019. and Netflix Subscriber Numbers Soar, Showing the Sky Is Not Falling. `https://www.fool.com/investing/2019/10/17/netflix-subscriber-numbers-soar-showing-the-sky-is.aspx`. Retrieved on December 1st, 2019.

independent from the platform.[13] This provides a different angle on the consequences of the integration between recommendation and production than in section 3. In section 3. we explored the effect on consumer welfare of moving from a monopolist producer to a duopoly where one of the producers is vertically integrated with the platform and therefore benefits from biased recommendation. In this section we maintain the duopoly in the upstream market, but we eliminate the ability for the platform to bias recommendation.

The model remains the same, up to imposing unbiased recommendations: The recommender system now recommends whichever good realizes the highest quality with uniform tie-breaking. This immediately implies that the expected good quality that consumers end up getting is

$$q^U := q_P \cdot q_F + q_P \cdot (1 - q_F) + (1 - q_P) \cdot q_F = q_P + (1 - q_P) \cdot q_F = q_F + (1 - q_F) \cdot q_P$$

The solution to the optimal access fee is analogous to the one from before, leading also to a similar expression for platform revenue R_P. In fact, the revenue maximizing access fee is the same up to replacing $\max\{q_P, q_F\}$ with the new expression for expected good quality q^U. Thus, the optimal access fee and resulting revenue are given by $\tau = \frac{1}{2}\bar{\theta} \cdot q^U$ and $R_P = r_P \cdot q^U$.

Expected market shares are given by $\alpha_P = 1 - \alpha_F = \frac{1}{2}(1 - (q_F - q_P))$, resembling the form of market share under biased recommendations with two differences. On the one hand, market shares are always strictly positive for any strictly positive quality investment. This immediately implies that the independent firm now is able to capture a share of the market even it invests less than the platform, as imposing unbiased recommendations precludes the use of this policy instrument by the platform to induce vertical foreclosure. On the other hand, the incentives for the firm to compete for market share are dampened: the marginal change in the firm's market share from increasing its investment is halved when compared to the biased recommendation case with positive market share.

The platform's production problem is given by the same expression up to the changes in R_P and α_P:

$$\pi_P(q_P, q_F) = \alpha_P R_P - C(q_P) = \frac{1}{2}(1 - (q_F - q_P)) \cdot r_P \cdot (q_F + (1 - q_F) q_P) - {q_P}^2$$

Lemma 1. *The platform's optimal investment,* $q_P(q_F) := \arg\max_{q_P \in [0,1]} \pi_P(q_P, q_F)$ *is unique and continuous in* q_F.

The firm's production problem is now:

$$\pi_F(q_P, q_F) = \alpha_F R_P + R_F - C(q_F) = \frac{1}{2}(1 - (q_P - q_F)) \cdot r_P \cdot (q_F + (1 - q_F) q_P) + r_F \cdot q_F - {q_F}^2$$

[13] Although in this case the sequentiality of investments is less appealing, the main patterns also hold in the simultaneous investment case, fixing unbiased recommendations.

with $q_P = q_P(q_F)$.

It results that this problem has a unique maximizer and therefore:

Proposition 6. *Equilibrium investment levels with unbiased recommendations are uniquely defined.*

The proofs for both Lemma 1 and Proposition 6 are deferred to the appendix. Unfortunately, the solution for the independent firm investment levels has no closed form; hence, we evaluate the effect of such policy by numerical computation.

We now directly compare the equilibrium investment levels in the different cases. Figure 3 plots the equilibrium investment levels as we vary the significance of the firm's alternative revenue, r_F, for different representative values of platform market sizes, r_P. The figure shows that imposing unbiased recommendations has an ambiguous effect on firm equilibrium investment levels.

It is immediate that similarly to before, the independent firm invests less under unbiased than under biased recommendations when r_F is high and r_P is low (see b 3). There are two channels that depress the incentives for the independent firm to invest. The first channel arises since the marginal gain on market share is halved under unbiased recommendations compared to biased recommendations. The second channel arises since the marginal increase in overall platform revenue as a result of increasing independent firm investment is r_P, but, under unbiased recommendation, the marginal effect on total revenue is only $r_P(1-q_P)$.

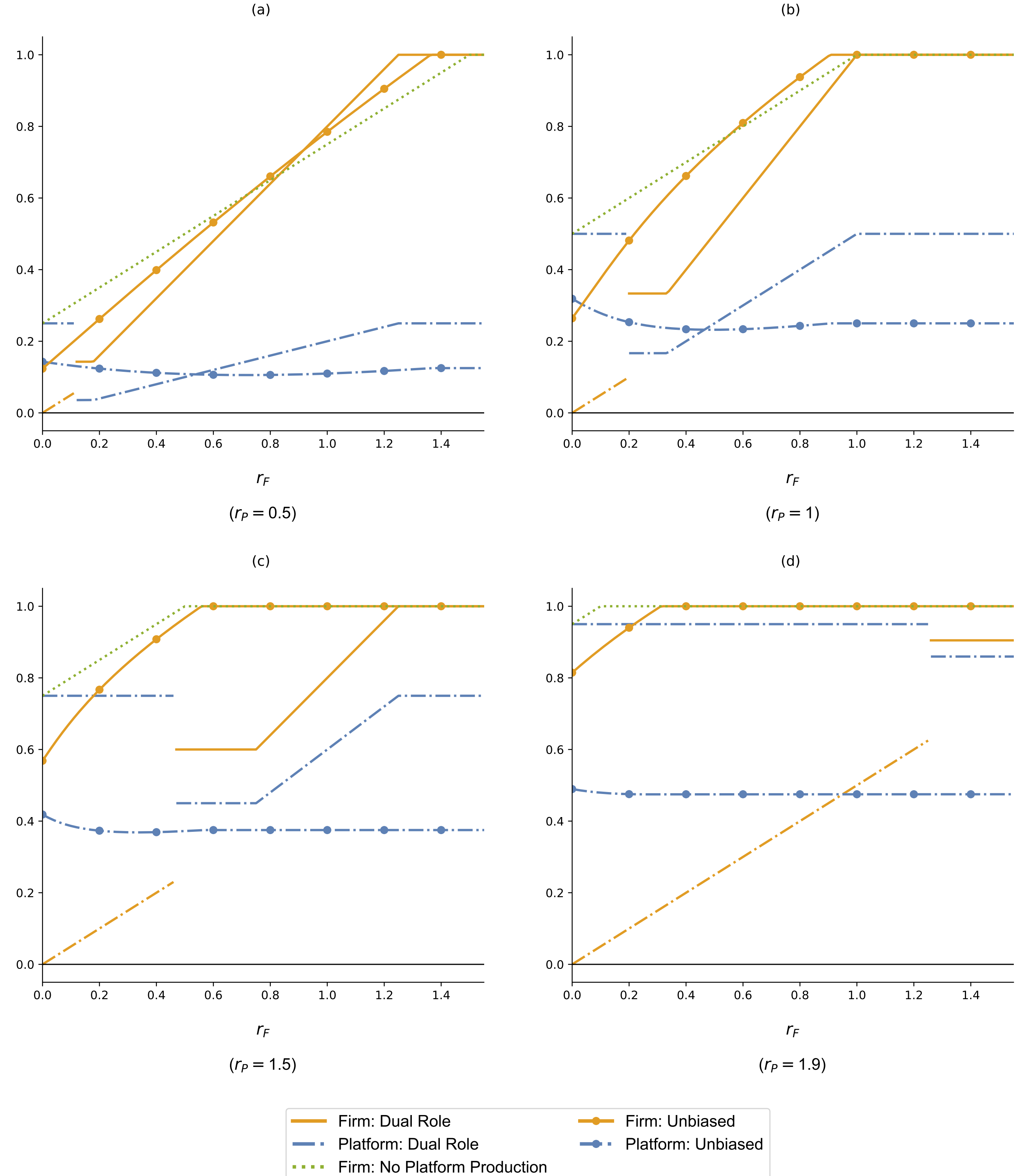
However, there are two channels that act in the opposite direction and encourage the independent firm to increase investment. The first is that the platform's reaction to the independent firm's quality investment is softer as it can no longer make use of biasing recommendations in its favor. The second is that even though the marginal effect of investment on market share is lower, unbiased recommendation still leads to an overall higher level of market share for the independent firm.

When r_F is high enough and r_P is low enough, the channels that depress incentives are stronger and drive the independent firm to invest less under unbiased recommendations compared to biased recommendations. However, as r_P increases, the channels that drive the firm to invest more become stronger than the channels that depress investment. Thus, when r_P is sufficiently high, the channels that lead to higher investment dominate the investment-depressing channels leading to overall high investment levels in the unbiased case.

We now investigate the effect that unbiased recommendation has on the platform's production decision, which can similarly be seen in figure 3. We observe three main results.

First, when the platform potential revenue is high relative to the firm's alternative markets, we previously noted that the platform is able to use recommendation to effectively become a monopolist. However, now that recommendation

Figure 3. **Equilibrium Investment Levels including Unbiased Recommendation**



Notes: This figure displays the investment levels across the unbiased, sole producer, and dual role cases. Each figure plots the changes in investment levels as we vary the strength of the alternative revenue sources for representative values of platform potential revenue.

has to be unbiased, the resulting competition leads to a decrease in the platform's equilibrium investments. Although it is reasonable to expect that the fact that the platform is now unable to capture the whole market would lead to more aggressive investment by the platform, this is not the case. As the independent firm has higher returns to investments than the platform due to its alternative revenue sources, it will always obtain at least half of the market share under unbiased recommendations. This leads to competition resulting in lower investment by the platform.

Second, unbiased recommendations depress platform investment when the firm's alternative revenue is large enough. When the platform can bias recommendations, it has a stronger incentive to keep up with the firm's investments as r_F increases. However, this effect turns negative as the alternative revenue sources grow more and more significant. In this case, as happens for the independent firm, unbiased recommendations actually dampen competition incentives, making it less worthwhile for the platform to produce higher quality goods given the halved effect on the market share and virtually no marginal change in total revenue when q_F is close to 1.

Finally, if the platform's potential revenue is high enough and the firm's alternative markets are not too large, the firm's investment under unbiased recommendations is lower than the platform's when it can bias recommendations. As it is possible to observe in figures 3c and 3d, when r_P is high enough and r_F is low, in order to capture full platform demand in the dual role case, the platform invests higher quality than it does in the unbiased recommendations case. Moreover, the platform's equilibrium investment levels are then consistently higher under biased recommendations than it is when imposing unbiased recommendations.

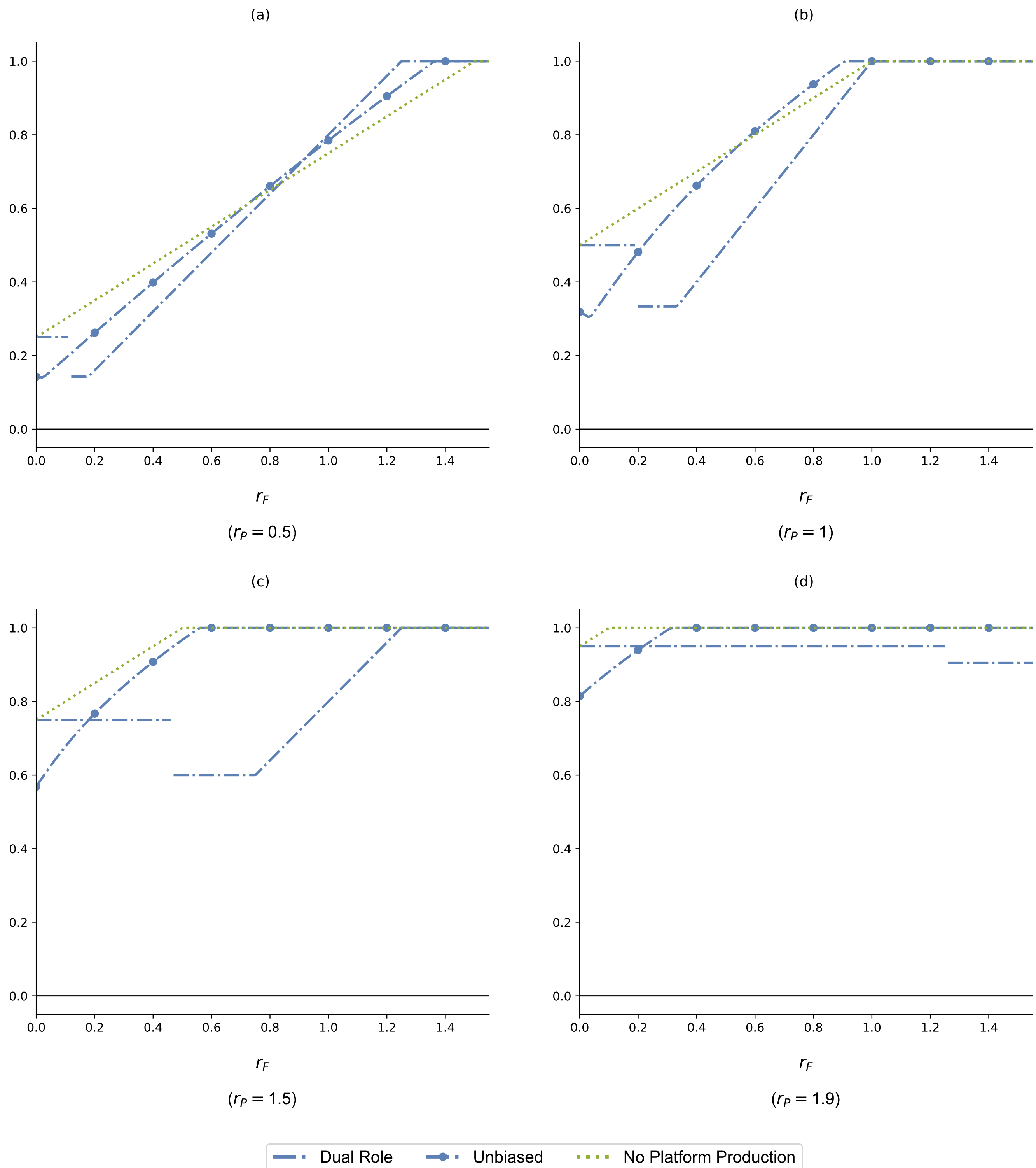
The two main findings with regards to investment levels are then the following:

Remark 1.

1. *When the platform's market size is low and the firm's alternative market size is large, then imposing unbiased recommendations leads to lower investments by the independent firm.*
2. *When the platform's market size is large relative to the firm's alternative market size or when the latter is large enough, then imposing unbiased recommendations leads to lower investments by the platform.*

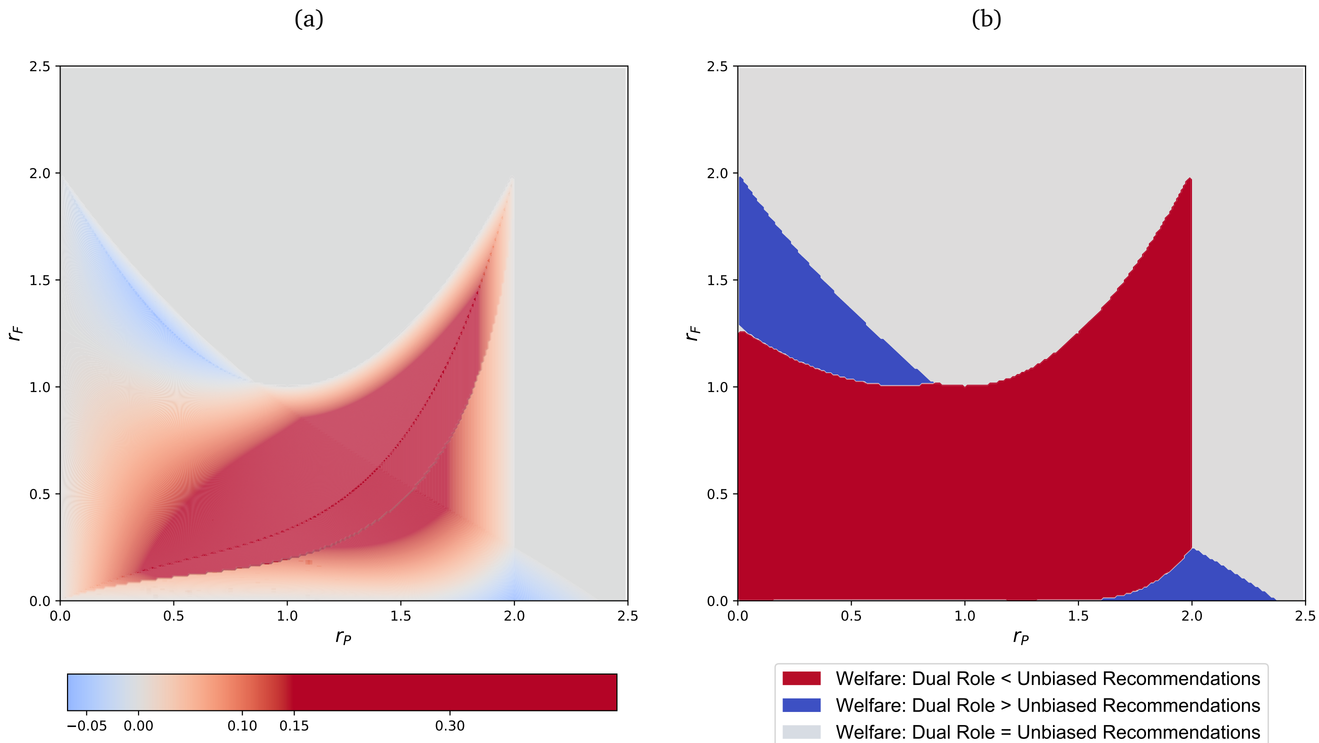
The analysis of the resulting differences in consumer welfare between the two regimes does not follow immediately from assessing the platform's and the firm's investment levels independently since the expected market shares further differ as a result of the change in recommendation policy, even if the investment levels would be the same in both the biased and unbiased regimes. However, recall that, in both the biased and unbiased regimes, consumer welfare is a linear function of expected good quality and so our results here have direct parallels to the previously discussed changes in investment levels. There are two channels that lead to welfare increases. The first is that, fixing investment levels and prices, unbiased recommendations leads to a clear increase in consumer welfare due to the improved

Figure 4. **Equilibrium Welfare Levels Across Unbiased, Dual Role, and Sole Producer Cases**



Notes: This figure displays the welfare values across the unbiased, sole producer, and dual role cases. Each figure plots the changes in welfare as we vary the strength of the alternative revenue sources for representative values of platform potential revenue.

Figure 5. **Difference in Welfare: Unbiased – Dual Role**



Notes: This figure displays the difference in expected product quality, which corresponds to average consumer welfare on the platform, between the dual role and unbiased recommendations cases. Subfigure (a) shows a heatmap of the differences between welfare for varying r_F and r_P and subfigure (b) shows for which values of r_F and r_P the different cases differ in resulting welfare.

information consumers have about good each good's quality. This leads to differences in expected market shares for the producers and, as a result, the second channel comes from responses in equilibrium investment levels.

Interestingly, imposing unbiased recommendations can lower consumer welfare both when the platform market size relative to the size of the firm's alternative market is large and when it is small – cf. figure 5. Immediately, remark 1 implies that when r_P is low and r_F is high, both the independent firm and the platform are investing less in quality under the new policy than when the platform is able to bias the recommendations. However, by itself, this need not imply that welfare is lower, as de-biasing recommendations leads to a higher informational value of recommendations for consumers which could potentially outweigh the lower investments. When looking at panel (a) of figure 4, we can observe this is not the case: when the platform's market size is low and the firm's alternative market size is large enough, imposing unbiased recommendation leads to a welfare loss.

There is also a second case where unbiased recommendations can be welfare depressing: when the platform's market size is large relative to the firm's alternative market. In this case, in order for the platform to completely foreclosure the independent firm utilizing biased recommendations would require it to undertake higher investments in quality than the both the platform and the independent firm do with unbiased recommendations. This results in the platform increasing its investment in order to shut out the independent firm, but this increased investment leads to higher welfare for consumers. Therefore, it is exactly the ability to engage in anti-competitive practices enabled by the ability to bias recommendations that leads to a higher consumer welfare with biased recommendations in this case.

These cases are clearly identifiable in figure 5, resulting in the main observation of this section:

Remark 2. *Imposing unbiased recommendations depresses consumer welfare when either the platform's market size is low and the firm's alternative market size is large or vice-versa.*

Although the remark 2 highlights the striking result that preventing platforms from biasing their recommender systems may be harmful for consumers, figure 5a shows that the welfare gains may be very significant when the platform's and the firm's alternative markets' potential revenue are comparable. Thus, it becomes crucial to understand not only the industry's structure but also its returns when considering the consequences of such a policy.

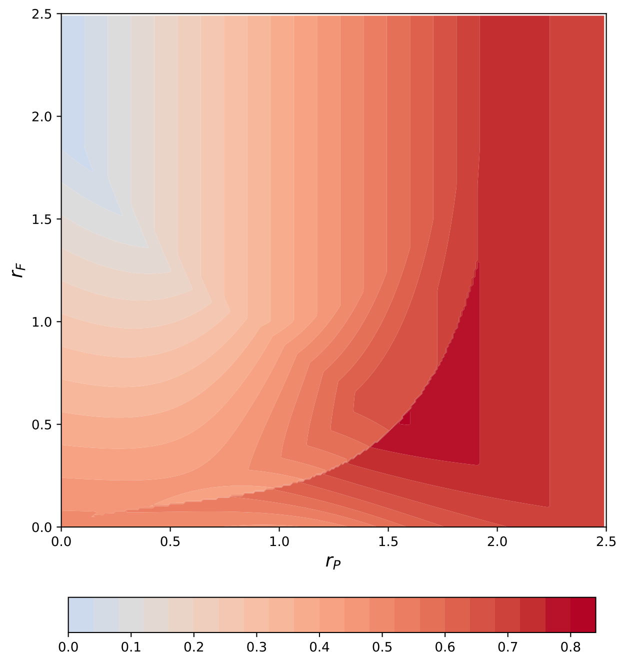
Finally, we characterize the extent to which market shares differ between the unbiased and biased recommendation regimes. Figure 6a displays the overall differences in market share between the two regimes. As expected, overall the unbiased recommendation regime leads to a larger market share for the independent firm. Figures 6b and 6c break down the extent to which these changes are a result of unbiased recommendation itself and changes in investment levels, respectively. These figures suggest that the influence of both is non-trivial, especially when r_F is small. On the one hand, the effect of unbiased recommendation itself, and not necessarily the resulting equilibrium adjustments, appears to play a bigger role when the firm was already obtaining positive market share under biased recommendations – with a clearly identifiable discontinuity – besides the case where it was already producing at maximum quality. On the other hand, equilibrium adjustments in quality lead to extremely significant changes in market share when the firm's alternative revenue sources are meager, as under unbiased recommendations it can do always at least as well as the platform, increasing market share from zero to over 50%. This underscores the extent to which the ability of platforms to bias recommendation can undeservedly shift substantial market share from independent firms' goods to the platform's goods.

5. Conclusion

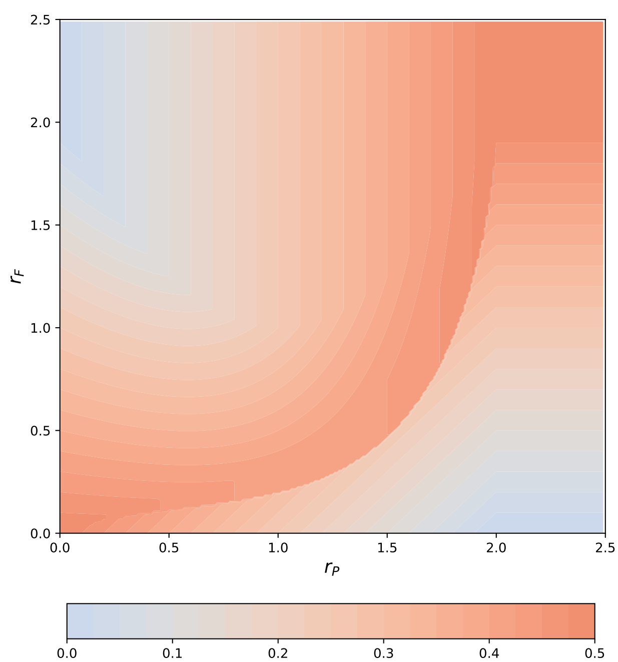
This paper studied a stylized model of strategic interaction between a platform that deploys a recommender system and producers of the goods distributed on this platform. Using this model we explored the welfare consequences of the entry of the platform into the production market. Opposite to the common intuition that increased competition in good production is welfare improving, we found that the ability by the platform to deploy a recommender system enables it to steer demand towards its own goods which leads to lower consumer welfare in equilibrium as a result of the platform's entry. When the primary revenue sources for the independent producers are from the platform, the

Figure 6. **Difference in Market Share for Indepenent Firm**

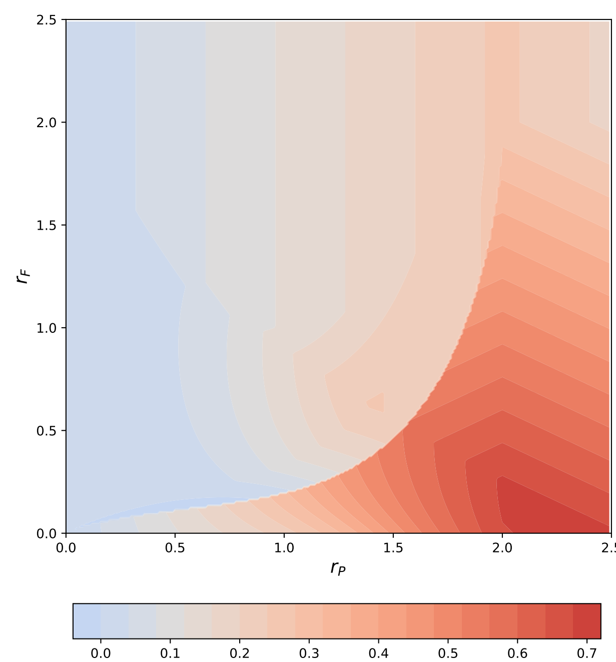
(a) **Difference in Market Share: Unbiased – Dual Role**



(b) **Difference in Market Share: Unbiased w/ Dual Role Investments – Dual Role**



(c) **Difference in Market Share: Unbiased - Unbiased w/ Dual Role Investments**



Notes: This figure displays the differences in market share between the unbiased and dual role cases for the independent firm. Subfigure (a) displays the overall difference in market share in equilibrium between the unbiased and dual role case. Subfigure (b) displays the difference in market share for the independent firm between the unbiased recommendation regime with investment levels fixed at the dual role levels and the equilibrium in the dual role. Subfigure (c) displays the difference in market share between the equilibrium in the unbiased recommendation regime and unbiased recommendation with investment levels fixed at the equilibrium values for the dual role.

bias in recommendation leads to depressed incentives to invest both for the independent producers and the platform itself.

The policy implications from our model are clear – the increasing trend of online platforms to produce their own goods should be viewed with caution by regulators. A unique element of these platforms is their deployment of recommender systems, which provide utility for consumers by providing them with information on which goods on the platform they should consume. However, as our model points out, the clear evidence of bias in both search and recommendation can lead to negative equilibrium effects on the quality of the goods that get produced and threatens the ability of independent producers to thrive when they are dependent on the platform as their primary revenue source.

A natural policy remedy is to require that platforms have unbiased recommendations or, equivalently, force a separation between recommendation and production. Surprisingly, we find that the equilibrium effects of biased recommendation lead to this policy not being unambiguously welfare-improving for consumers. In the case when the platform is the primary revenue source for the independent producers, we find that this policy does improve welfare for consumers. This happens not only from the welfare gains as a result of unbiased information disclosure, but also from the equilibrium investment response to unbiased recommendations. However, when the alternative revenue sources for the independent producers are large relative to the platform potential revenue, biased recommendations induce higher investment levels and lead to higher consumer welfare when compared to unbiased recommendations.

As a result, if independent producers are primarily dependent on the platform for revenue then policies targeting the integration of recommendation and production will be welfare-improving. However, if independent producers have access to large alternative revenue sources relative to the platform's potential revenue, then policies targeting the integration of recommendation and production will have adverse effects on consumer welfare.

Finally, there are several aspects of the integration between recommendation and production that warrant further study. The first is a better understanding of the interaction between the information accumulated about consumer preferences due to intermediation and dynamic production decisions. This is particularly amplified in the case of recommender systems since, in order to develop a good recommendation system, the intermediary needs to collect fine-grained information about consumer preferences. For instance, Netflix and Amazon are primarily relying on "data-driven" approaches to production decisions using the data they get to power their recommendation systems. Further, consumer choices in these markets may have a path-dependence as illustrated by Aridor, Goncalves and Sikdar (2020) and so understanding the dynamic consequences of the integration of production and recommendation seems a fruitful and important direction for future work. As the role of recommender systems in online platforms increases and platforms increasingly integrate production and recommendation understanding the effect on competition, investment, and consumer welfare of all these considerations becomes increasingly more important.

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Appendix

The appendix contains proofs that are omitted from the main text.

Proof of Proposition 1

Proof. Given that there are only two relevant actions that the recommendation policy induces x_P, x_F, the problem is equivalent to having (stochastic) direct recommendations, that is, to having $\rho : \{0,1\}^2 \to \Delta\{P, F\}$. We can then recast the optimal recommendation policy from the optimization problem given in (RP) to:

$$\max_{\rho:\{0,1\}^2 \to \Delta\{P,F\}} \sum_{a,b \in \{0,1\}} \mathbb{P}\left(\rho(x_P, x_F) = P \mid x_P = a, x_F = b\right) \mathbb{P}(x_P = a, x_F = b)$$

subject to credibility constraints

$$\mathbb{E}[x_P \mid \rho(x_P, x_F) = P] \geq \mathbb{E}[x_F \mid \rho(x_P, x_F) = P] \tag{1}$$

$$\mathbb{E}[x_F \mid \rho(x_P, x_F) = F] \geq \mathbb{E}[x_P \mid \rho(x_P, x_F) = F] \tag{2}$$

Given independence of x_P and x_F, the objective function becomes $q_P q_F \mathbb{P}\left(\rho(1,1) = P\right) + q_P(1-q_F)\mathbb{P}\left(\rho(1,0) = P\right) + (1-q_P)q_F\mathbb{P}\left(\rho(0,1) = P\right) + (1-q_P)(1-q_F)\mathbb{P}\left(\rho(0,0) = P\right)$, which is linear and increasing in $\mathbb{P}\left(\rho(a,b) = P\right)$, $a, b \in \{0,1\}$. Note that the unconstrained optimum is trivially setting $\mathbb{P}\left(\rho(x_P, x_F) = P\right) = 1$ regardless of the quality realizations, implying that the platform always recommends its own goods. This is indeed the solution to the optimal recommendation policy problem whenever $q_P \geq q_F$ as, in this case, the unconstrained optimum is feasible as, without further information, the consumers will always consume the platform's good.

The solution to the case where $0 = q_P < q_F$ is similarly trivial, as recommendations are ineffective and thus the only policy that complies with obedience is to send consumers unbiased recommendations to choose the independent firm's good. Similarly, when $q_P < q_F = 1$, it should be straightforward that the optimal recommendation policy is to send consumers unbiased recommendations, but breaking indifference in favor of the platform's good. For the case where $0 < q_P < q_F < 1$, as the constraints do not depend on $\mathbb{P}\left(\rho(j,j) = P\right)$, $j = 0,1$, we can set $\mathbb{P}\left(\rho(j,j) = P\right) = 1$ noting that the objective function is strictly increasing in $\mathbb{P}\left(\rho(j,j) = P\right)$ given that $q_P > 0$ and $q_F < 1$. Note that when $q_P < q_F$, the constraint (2), is redundant as $\mathbb{P}(x_P = 1, x_F = 0) - \mathbb{P}(x_P = 0, x_F = 1) = q_P(1-q_F) - (1-q_P)q_F < 0$. Rearranging the the terms in the constraint (1), we have $0 \leq \mathbb{P}\left(\rho(x_P = 0, x_F = 1) = P\right) \leq \frac{q_P(1-q_F)}{q_F(1-q_P)}\mathbb{P}\left(\rho(x_P = 1, x_F = 0) = P\right) \leq 1$. Again by strict monotonicity of the objective function in $\mathbb{P}\left(\rho(x_P, x_F) = P\right)$, the optimal policy is given by setting $\mathbb{P}\left(\rho(x_P = 1, x_F = 0) = P\right) = 1$ and $\mathbb{P}\left(\rho(x_P = 0, x_F = 1) = P\right) = \frac{q_P}{1-q_P}\frac{1-q_F}{q_F}$. □

Proof of Proposition 4

Proof. Note that $\pi_F(q_F)$ is also a piecewise strictly concave function, but it is not continuous. Immediately, if $r_P \geq 2$, we have that the independent firm sets quality at $\frac{r_F}{2}$. We now consider the case where $r_P < 2$. Let $\pi^{(1)}(q_F) := r_F \cdot q_F - {q_F}^2$ and $\pi^{(2)}(q_F) := (q_F - q_P(q_F)) \cdot r_P \cdot q_F + r_F \cdot q_F - {q_F}^2$. The maximizer of $\pi_F^{(1)}$ is $\min\{1, \frac{r_F}{2}\}$, while that of $\pi_F^{(2)}$ is $\min\left\{1, \frac{r_F}{2(1-r_P)+{r_P}^2}\right\}$. We have split the exogenous parameters into different cases and find the maximum under each of these cases.

1. When $\tilde{q}_F \geq \frac{r_F}{2}$, then the maximizer can only be that of $\pi_F^{(2)}$. This follows by strict concavity of $\pi_F^{(1)}$ which then leads to the fact that $\frac{d}{dq_F}\pi_F^{(1)}(q_F)\mid_{q_F=\tilde{q}_F} > 0$. As such, $\max_{q_F\in[\tilde{q}_F,1]}\pi_F^{(2)}(q_F) \geq \pi_F^{(2)}(\tilde{q}_F) > \pi_F^{(1)}(\tilde{q}_F)$. Finally, when $\tilde{q}_F \geq \frac{r_F}{2}$ we also have that $\frac{d}{dq_F}\pi_F^{(2)}(q_F)\mid_{q_F=\tilde{q}_F} \geq 0$, which implies that $\arg\max_{q_F\in[0,1]}\pi_F^{(2)}(q_F) = \min\left\{1, \frac{r_F}{2(1-r_P)+{r_P}^2}\right\}$.

2. When $\frac{r_F}{2} \geq \tilde{q}_F \geq \frac{r_F}{2(1-r_P)+{r_P}^2}$, then a matter of simple computations show that $\max_{q_F\in[0,\tilde{q}_F]}\pi^{(1)}(q_F) < \max_{q_F\in[\tilde{q}_F,1]}\pi^{(2)}(q_F)$ whenever this is the case and therefore $\arg\max_{q_F\in[0,1]}\pi_F^{(2)}(q_F) = \min\left\{1, \frac{r_F}{2(1-r_P)+{r_P}^2}\right\}$.
   From the conditions in this and the above case we have that $\min\left\{1, \frac{r_F}{2(1-r_P)+{r_P}^2}\right\}$ is a maximizer whenever (i) $\tilde{q}_F \geq \frac{r_F}{2}$ or (ii) $\frac{r_F}{2} \geq \tilde{q}_F \geq \frac{r_F}{2(1-r_P)+{r_P}^2}$, which, given $r_P < 2$, leads to the sufficient condition that $\frac{r_F}{2} \geq \tilde{q}_F\frac{2(1-r_P)+{r_P}^2}{2}$.

3. Finally, when $\frac{r_F}{2(1-r_P)+{r_P}^2} < \tilde{q}_F$, there are two candidates for maximizers: the discontinuity point, $\tilde{q}_F$, which corresponds to the unique (corner) solution to $\arg\max_{q_F\in[\tilde{q}_F,1]}\pi^{(2)}(q_F)$, and $\frac{r_F}{2} = \arg\max_{q_F\in[0,\tilde{q}_F]}\pi^{(1)}(q_F)$. The discontinuity point is a maximizer whenever, together with the above inequalities,

$$
\begin{aligned}
&\pi_F(\tilde{q}_F) \geq \pi_F\left(\frac{r_F}{2}\right)\\
\Leftrightarrow &\left(r_P\frac{2-r_P}{2}-1\right)(\tilde{q}_F)^2 + r_F\cdot\tilde{q}_F \geq \frac{{r_F}^2}{4}\\
\Leftrightarrow &\frac{r_F}{2} \in \left[\tilde{q}_F\left(1-\sqrt{r_P\frac{2-r_P}{2}}\right), \tilde{q}_F\left(1+\sqrt{r_P\frac{2-r_P}{2}}\right)\right]
\end{aligned}
$$

   As $\tilde{q}_F\left(1+\sqrt{r_P\frac{2-r_P}{2}}\right) \geq \frac{r_F}{2(1-r_P)+{r_P}^2}$ whenever $r_P < 2$, we have that the discontinuity point is a maximizer whenever $\frac{r_F}{2} \in \left\{\tilde{q}_F\left(1-\sqrt{r_P\frac{2-r_P}{2}}\right), \tilde{q}_F\frac{2(1-r_P)+{r_P}^2}{2}\right)$ and $\frac{r_F}{2}$ is a maximizer when $\frac{r_F}{2} \leq \tilde{q}_F\left(1-\sqrt{r_P\frac{2-r_P}{2}}\right)$.

□

Proof of Proposition 5

Proof. As welfare in both cases is given by $K\cdot\max\{q_P^{DR}, q_F^{DR}\}$ and $K\cdot q_F^{NP}$ for the same positive constant K, it suffices to compare the resulting quality investments in both cases. If $r_P \geq 2$, then $q_P^{DR} = q_F^{NP} = 1$, attaining the same welfare.

We proceed by analyzing the case where $r_P < 2$. Note that

$$\max\{q_P^{DR}, q_F^{DR}\} = \begin{cases} \min\left\{1, \frac{r_F}{2(1-r_P)+r_P^2}\right\} & \text{if } \frac{2(1-r_P)+r_P^2}{2}\tilde{q}_F \leq \frac{r_F}{2} \\ \tilde{q}_F & \text{if } \tilde{q}_F\left(1-\sqrt{r_P\frac{2-r_P}{2}}\right) \leq \frac{r_F}{2} < \frac{2(1-r_P)+r_P^2}{2}\tilde{q}_F \\ \frac{r_P}{2} & \text{if } \frac{r_F}{2} \leq \tilde{q}_F\left(1-\sqrt{r_P\frac{2-r_P}{2}}\right) \end{cases}$$

As $q_P^{DR} \leq \frac{r_P}{2} < \frac{r_P+r_F}{2} \leq q_F^{NP}$, if $\max\{q_P^{DR}, q_F^{DR}\} = q_P^{DR}$, welfare is lower in the dual role case.

Suppose that $\max\{q_P^{DR}, q_F^{DR}\} = \tilde{q}_F \geq \frac{r_P+r_F}{2} = q_F^{NP} \Leftrightarrow \tilde{q}_F - \frac{r_P}{2} \geq \frac{r_F}{2}$. As $\max\{q_P^{DR}, q_F^{DR}\} = \tilde{q}_F$ implies that $\tilde{q}_F\frac{2\left(1-\sqrt{r_P\frac{2-r_P}{2}}\right)}{2} \leq \frac{r_F}{2}$. However, $\tilde{q}_F\frac{2\left(1-\sqrt{r_P\frac{2-r_P}{2}}\right)}{2} > \tilde{q}_F - \frac{r_P}{2}$ $\forall r_P < 2$, which then leads to a contradiction.

We then have the case where $\max\{q_P^{DR}, q_F^{DR}\} = \min\left\{1, \frac{r_F}{2(1-r_P)+r_P^2}\right\}$. Note that $r_F 2(1-r_P)+r_P^2 \geq \frac{r_P+r_F}{2} \Rightarrow r_F \geq \frac{2(1-r_P)+r_P^2}{2-r_P}$. Moreover, as $\max\{q_P^{DR}, q_F^{DR}\} = \frac{r_F}{2(1-r_P)+r_P^2} \Rightarrow r_F \geq \left(2(1-r_P)+r_P^2\right)\frac{r_P}{4-r_P}$ and $\frac{1}{2-r_P} \geq \frac{r_P}{4-r_P}$, we have that only if $r_F \geq \frac{2(1-r_P)+r_P^2}{2-r_P}$ do we have $q_F^{DR} \geq q_F^{NP}$ and that if $r_F \geq \frac{2(1-r_P)+r_P^2}{2-r_P}$ and if $q_F^{NP} = \min\{1, \frac{r_P+r_F}{2}\} = \frac{r_P+r_F}{2}$ then $q_F^{DR} \geq q_F^{NP}$.

Finally, note that $\frac{r_P+r_F}{2} \geq q_F^{NP} = 1 > \frac{r_F}{2(1-r_P)+r_P^2}$ implies that $\frac{2(1-r_P)+r_P^2}{2-r_P} > r_F$, which is necessary and sufficient for this case. To see this note that $\max\{q_P^{DR}, q_F^{DR}\} = \frac{r_F}{2(1-r_P)+r_P^2}$ implies $r_F \geq \left(2(1-r_P)+r_P^2\right)\frac{r_P}{4-r_P}$ and as $\frac{1}{2-r_P} \leq \frac{r_P}{4-r_P}$ when $r_P < 2$, this imposes no further constraint.

Consequently, $r_F \geq \frac{2(1-r_P)+r_P^2}{\max\{1,2-r_P\}}$ is a necessary and sufficient for $q_F^{NP} \leq \max\{q_P^{DR}, q_F^{DR}\}$. Moreover, $q_F^{NP} < \max\{q_P^{DR}, q_F^{DR}\}$ if and only if $r_F > \frac{2(1-r_P)+r_P^2}{2-r_P}$ and $1 \geq \frac{r_P+r_F}{2}$. □

Proof of Lemma 1

Proof. If $r_P \geq \frac{2}{1-q_F}$, then $\pi_P(q_P, q_F)$ is convex and strictly increasing in q_P, which immediately implies that the platform optimally sets $q_P = 1$. If $\frac{2}{1-q_F} > r_P \geq \frac{4}{3(1-q_F)+q_F^2}$, then $\pi_P(q_P, q_F)$ is strictly concave but $\frac{\partial}{\partial q_P}\pi_P(q_P, q_F)|_{q_P=1} \geq 0$ and still implies that the platform optimally sets $q_P = 1$. Finally, if $\frac{4}{3(1-q_F)+q_F^2} > r_P$, then $\pi_P(q_P, q_F)$ is strictly concave and the platform sets investments optimally at $q_P = \hat{q}_P(q_F) := \frac{r_P}{2}\frac{1-q_F(1-q_F)}{2-r_P(1-q_F)}$. As $\forall q_F \in [0,1]$ and $\forall r_P > 0$, π_P is either strictly increasing or strictly concave in q_P, it is strictly quasiconcave in q_P. Moreover, as π_P is continuous in (q_P, q_F), then we have that $q_P(q_F)$ is continuous, by Berge's theorem of the maximum. Hence, the platform's optimal investment, as a function of the firm's investment, is given by

$$q_P(q_F) = \begin{cases} 1 & \text{if } \frac{4}{3(1-q_F)+q_F^2} > r_P \\ \hat{q}_P(q_F) & \text{if otherwise} \end{cases}$$

and is a continuous function of q_F. □

Proof of Proposition 6

Proof. Recall that the condition under which $q_P(q_F) = 1$ is $\frac{4}{3(1-q_F)+{q_F}^2} > r_P$. Note that $\frac{4}{3(1-q_F)+{q_F}^2} > r_P \Leftrightarrow \frac{4}{r_P} - 3 + 3q_F - {q_F}^2 > 0 \Leftrightarrow q_F \geq \frac{1}{2}\left(3 - \sqrt{\frac{16-3r_P}{r_P}}\right) \equiv \hat{q}_F$. Clearly, $\hat{q}_F \geq 1 \Leftrightarrow r_P \geq 4$, which implies that if $r_P \geq 4$, then $q_P(q_F) = 1\ \forall q_F$, in which case $q_F^U := \arg\max_{q_F \in [0,1]} \pi_F(1, q_F) = \min\left\{1, \frac{r_F + r_P/2}{2}\right\}$.

If $r_P < 4$, then $\hat{q}_F < 1$. We define:

$$\pi_F^{(1)}(q_F) = \pi_F(1, q_F)$$

$$\pi_F^{(2)}(q_F) = \pi_F(\hat{q}_P(q_F), q_F)$$

where $\pi_F(q_P(q_F), q_F) = \pi_F^{(1)}(q_F)$ if $q_F \leq \hat{q}_F$ and $\pi_F(q_P(q_F), q_F) = \pi_F^{(2)}(q_F)$ if otherwise. Note that

1. $\pi^{(1)}$ is strictly concave
2. When $r_P < 4$, it is also the case that $\frac{d}{dq_F}\pi_F^{(1)}(q_F) > 0\ \forall q_F \in [0, \hat{q}_F]$.
3. Straightforward computations show that $\frac{d}{dq_F}\pi_F^{(2)}(q_F)\,|_{q_F=\hat{q}_F} > 0$
4. $\frac{d^2}{(dq_F)^2}\pi_F^{(2)}(q_F) < 0\ \forall q_F \in [0,1]$ when $r_P < 4$.

(1) - (3) directly imply that $\arg\max_{q_F \in [0,1]} \pi_F(q_P(q_F), q_F) = \arg\max_{q_F \in [\hat{q}_F, 1]} \pi_F^{(2)}(q_F)$.

(4) implies that $\arg\max_{q_F \in [\hat{q}_F, 1]} \pi_F^{(2)}(q_F)$ is a singleton.

The direct implication of these two results is that $\arg\max_{q_F \in [\hat{q}_F, 1]} \pi_F^{(2)}(q_F) = \arg\max_{q_F \in [0,1]} \pi_F^{(2)}(q_F) = \arg\max_{q_F \in [0,1]} \pi_F(q_P(q_F), q_F)$ is uniquely defined. □