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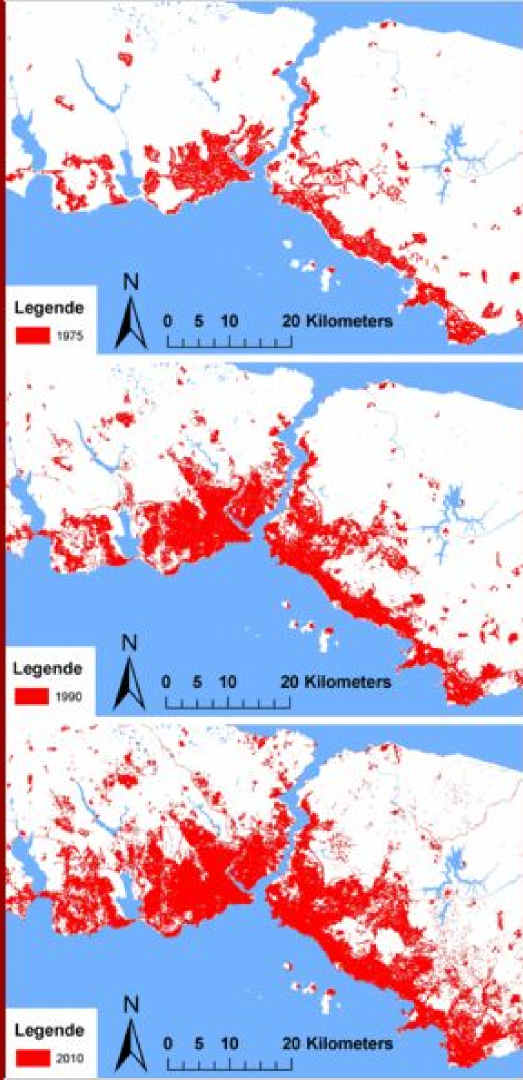
Overview

Istanbul Metropolitan Municipality

- The Istanbul Metropolitan Municipality (IMM) needs to determine where to build new fire stations
- Istanbul is the largest city in Turkey
- Population: 13.5 million
- Population Density: 2,523 people per square kilometer
- Bosphorus Bridge, located at the center of Istanbul, connects the European side of the city to the Asian side
- Home to many historical and cultural sights



Purpose

- Make residences and historic sites reachable by emergency vehicles within five minutes
- Increase safety

Problem

Problem

- There are 790 districts in Istanbul
- Fire stations come in 4 forms:
  A, B, C, D
- Some districts are weighted more as they hold cultural & historic sights
- Set budget
- A district is considered covered if a fire station is within 5 minutes

Methods

Min Cost

Minimize the cost of building fire station such that all districts are covered

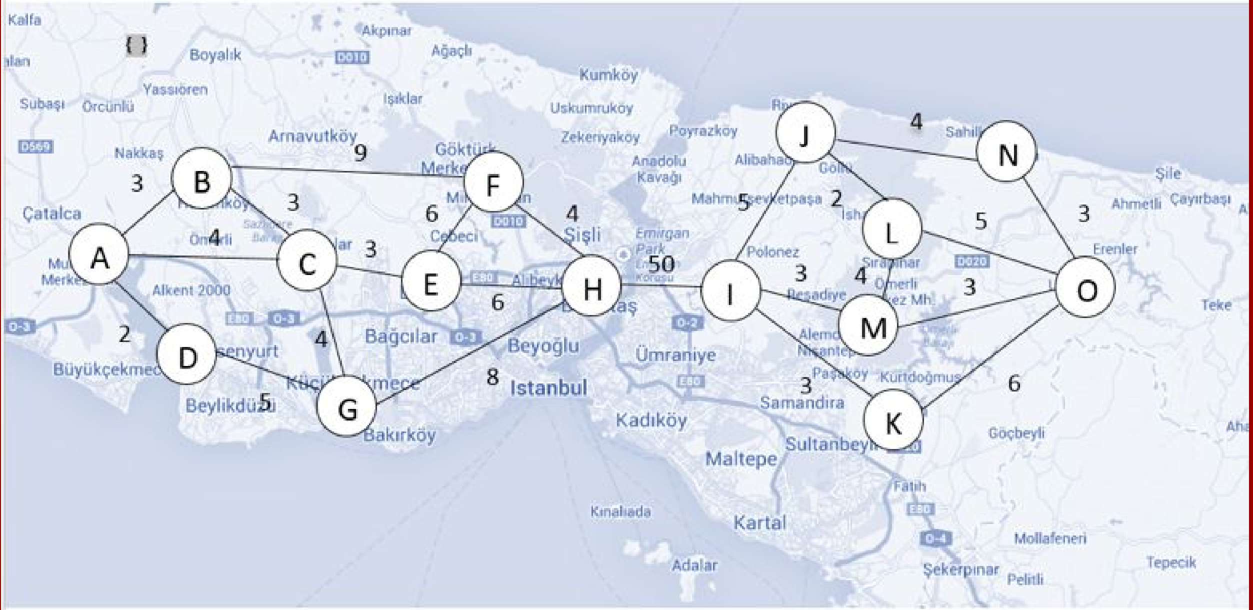
Max Coverage

Maximize the number of districts covered, given a fixed budget

Assumptions

- 15 Districts
- Covered if reached within 10 minutes
- Only one type of fire station that has unlimited capacity
- No weights on historical sites

Network Map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Şekerpınar
Pelitli
Tepecik

Methodology

Data

- We have a fixed cost of building each fire station

Cost i = Fixed cost for building one fire station

- From our network, we establish the shortest paths between any two nodes i and j (Total 210 data points)

Tij = Shortest Path (in units of Time) from node i to j

- We then “clean up” (process) the data by removing all data points greater than 10 minutes, since we will never use these paths

FTij = Feasible Shortest Path (Time) from node i to j given by Tij<=10

Decision Variables

- One decision variable is whether or not we build or don’t build a fire station at any given node i

  X_i = *whether or not to build at Node i*

- And an indicator variable to reflect whether or not any given node is covered after having built a fire station

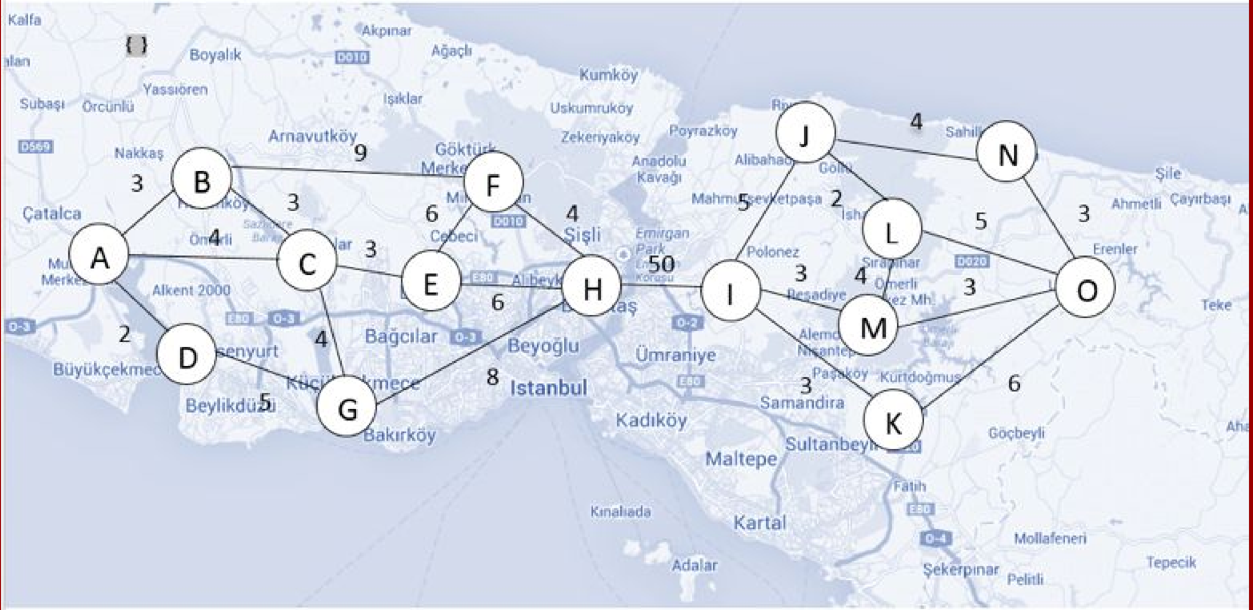
  Y_i = *whether or not Node i is covered by* X_i

Intermediate Variables

- We also have an intermediate variable, to reflect the time of the active shortest paths from Node i to j, for any given Node i
- An ACTIVE shortest path is a shortest path that is both feasible and in use after the fire stations are built
- Note that some paths are feasible, but are not used - this will be important in our constraints

*Active ij = Time of Active Shortest Path from Node i to j, given by FTij * Xi*

Network Map



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Objective Function

- We want to <u>maximize</u> coverage of the nodes, so our objective function is simply given by

$$\sum_{i=A}^{O} Y_i$$

Constraints

- The straightforward constraints

$$X_i, Y_i \text{ are binary}$$

$$X_i, Y_i >= 0$$

- Fix the maximum number of stations we can built, as determined by the budget

$$\sum_{i=A}^{O} X_i = \frac{\text{Fire Station Budget}}{Cost_i}$$

Constraints

- And the most important constraint

$$Y_i \leq \sum_j Active_{ij} + \sum_j Active_{ji}$$

- The RHS is the sum of the lengths of the active shortest paths flowing into AND out of node i
- If there is even one active shortest path connected to node i (RHS >= 1), we consider that node to be covered. If not (RHS = 0), the node is not covered
- Since we are maximizing, Solver will first try to put 1 if RHS >=1, otherwise it has to put 0 if RHS = 0

Model

Variables, Objective Function, Constraints

	I	J	K	L	M	N
1	Nodes	Build/ don't build		Is the node covered?		Sum of Shortest Paths for each Node after Building
2	A	0		1	<=	7
3	B	0		1	<=	6
4	C	0		1	<=	3
5	D	0		1	<=	9
6	E	1		1	<=	44
7	F	0		1	<=	6
8	G	0		1	<=	7
9	H	0		1	<=	6
10	I	1		1	<=	36
11	J	0		1	<=	5
12	K	0		1	<=	3
13	L	0		1	<=	7
14	M	0		1	<=	3
15	N	0		1	<=	9
16	O	0		1	<=	9
17		2		15		
18		=				
19	Max # allowable stations	2				
20						
21	Budget	$ 2,000,000				
22	Cost per fire station	$ 1,000,000				

'Build/Don't Build' - Binary decision variable

'Is the node covered?' - Auxiliary binary decision variable

Orange cell - Objective function, sum of all 'Is the node covered?'

'Sum of Shortest Paths for each Node after Building' - Sum of all the flows coming through the node =SUMIF(To, I2, SPafterB) +SUMIF(From, I2, SPafterB)

Max # of Allowable stations - determined by the budget and cost

Data Inputs

	A	B	C	D	E	F	G
1	From	To	Shortest Path (Time)	Is Data >10?	Keep only the data that is <10	Build/Don't Build?	SP after Building
44	D	A	2	1	2	0	0
45	D	B	5	1	5	0	0
46	D	C	6	1	6	0	0
47	D	E	9	1	9	0	0
48	D	F	15	0	0	0	0
49	D	G	5	1	5	0	0
50	D	H	13	0	0	0	0
51	D	I	63	0	0	0	0
52	D	J	68	0	0	0	0
53	D	K	66	0	0	0	0
54	D	L	70	0	0	0	0
55	D	M	66	0	0	0	0
56	D	N	72	0	0	0	0
57	D	O	72	0	0	0	0
58	E	A	7	1	7	1	7
59	E	B	6	1	6	1	6
60	E	C	3	1	3	1	3
61	E	D	9	1	9	1	9
62	E	F	6	1	6	1	6
63	E	G	7	1	7	1	7
64	E	H	6	1	6	1	6
65	E	I	56	0	0	1	0
66	E	J	61	0	0	1	0
67	E	K	59	0	0	1	0
68	E	L	63	0	0	1	0
69	E	M	59	0	0	1	0
70	E	N	65	0	0	1	0
71	E	O	65	0	0	1	0

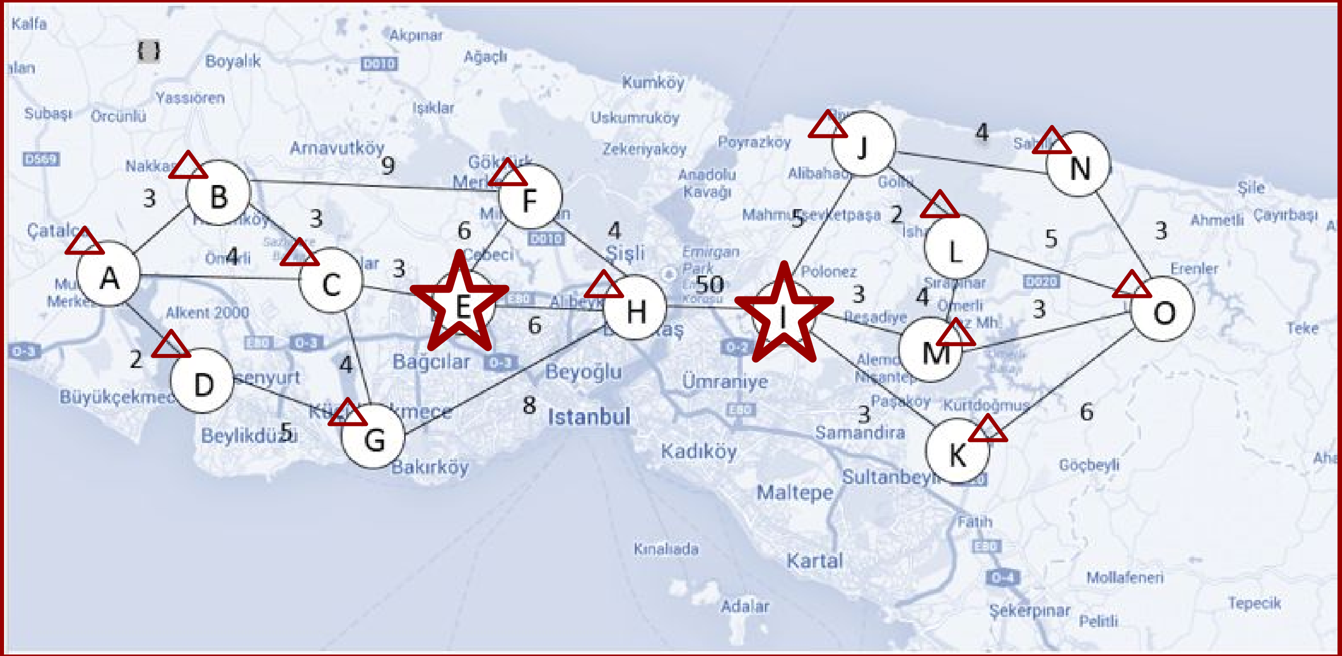
'From' Column - origin node
'To' Column - supply node
'Shortest Path (Time)' - Shortest Path to node i to node j
'Is Data > 10?' - =IF(C2<=10,1,0)
'Keep only data that is <10' - =C2*D2
'Build/Don't Build?' - based on decision variable
'SP after Building' - =E2*F2

Complete Model

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	From	To	Shortest Path (Time)	Is Data >10?	Keep only the data that is <10	Build/Don't Build? (	SP after Building		Nodes	Build/ don't build		Is the node covered?		Sum of Shortest Paths for each Node after Building
2	A	B	3	1	3	0	0		A	0		1	<=	7
3	A	C	4	1	4	0	0		B	0		1	<=	6
4	A	D	2	1	2	0	0		C	0		1	<=	3
5	A	E	7	1	7	0	0		D	0		1	<=	9
6	A	F	12	0	0	0	0		E	1		1	<=	44
7	A	G	7	1	7	0	0		F	0		1	<=	6
8	A	H	15	0	0	0	0		G	0		1	<=	7
9	A	I	65	0	0	0	0		H	0		1	<=	6
10	A	J	70	0	0	0	0		I	1		1	<=	36
11	A	K	66	0	0	0	0		J	0		1	<=	5
12	A	L	67	0	0	0	0		K	0		1	<=	3
13	A	M	68	0	0	0	0		L	0		1	<=	7
14	A	N	74	0	0	0	0		M	0		1	<=	3
15	A	O	74	0	0	0	0		N	0		1	<=	9
16	B	A	3	1	3	0	0		O	0		1	<=	9
17	B	C	3	1	3	0	0			2		15		
18	B	D	5	1	5	0	0			=				
19	B	E	6	1	6	0	0		Max # allowable stations	2				
20	B	F	9	1	9	0	0							
21	B	G	7	1	7	0	0		Budget	$ 2,000,000				
22	B	H	12	0	0	0	0		Cost per fire station	$ 1,000,000				

Recommendations

Solution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Analysis

- Reasonable to have one fire station on each side, because in reality it takes awhile to cross from the European side to the Asian side of Istanbul
- Multiple optimal solutions due to the network design
  - there are only 15 nodes
  - the time between each node is relatively short
  - there is a high level of interconnectedness between nodes
- Improvements:
  - more accurate data on time it takes to travel between districts
  - reduce the simplifications
    - account for different types of fire stations and capacity
    - add weights to districts based on their importance
    - add more nodes

Conclusion

- It only takes two fire stations to cover all nodes in our network
- Theoretically, this model can determine where to build fire stations in order to maximize coverage for all networks, given uniform capacity and uniform importance of districts
- Realistically, the model is limited by the amount of data (or nodes) it can handle