2 Quantum Mechanics

2.1 Photon Polarization

2.1.1 Linear Polarization

Problem 1. *Consider the following state vector $|s\rangle$ and measurement vectors $|m_1\rangle$ and $|m_2\rangle$ for photon polarization:*

$$|s\rangle = \begin{pmatrix} 4/5 \\ 3/5 \end{pmatrix}$$

$$M = (|m_1\rangle, |m_2\rangle) = \left(\begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}, \begin{pmatrix} -4/5 \\ 3/5 \end{pmatrix} \right).$$

(a) Show that $|s\rangle$ is indeed a legitimate state vector and that M is indeed a legitimate pair of measurement vectors.

(b) When the measurement M is performed on a photon in the state $|s\rangle$, what are the probabilities of the two outcomes?

For part (a), we need to check whether $||s\rangle| = 1$, as all valid polarization state vectors are *normalizable*[1]. We know that

$$\begin{aligned} ||s\rangle| &= \sqrt{(4/5)^2 + (3/5)^2} \\ &= a = \qquad\qquad 1 \end{aligned}$$

2.1.2 Review of Complex Numbers

Problem 1. *Let z_1 and z_2 be complex numbers.*

(a) Show that the complex conjugate of $z_1 z_2$ is $\bar{z_1}\bar{z_2}$.

(b) Show that if $z_2 \neq 0$, the complex conjugate of z_1/z_2 is $\bar{z_1}/\bar{z_2}$.

(c) Show that $|z_1 z_2| = |z_1||z_2|$.

(d) Show that if $z_2 \neq 0$, $|z_1/z_2| = |z_1|/|z_2|$.

Answer here...

Problem 2. *Starting with $e^{3i\theta} = (e^{i\theta})^3$, derive a formula for $\cos 3\theta$ in terms of $\cos\theta$ and $\sin\theta$.*

Answer here...

Problem 3. *Evaluate each of the following quantities.*

(a) $e^{i\pi}$

(b) $|1 + i|$

[1] I believe this can be decided in a more general case if the state vector is square-integrable.

(c) $\frac{2+3i}{3+2i}$

(d) $(1+i)^{16}$. *(Hint: There is more than one way to approach this problem.)*