1 Division Algorithm

Problem 5. *Use old-fashioned long division to implement the Division Algorithm and write* $a = bq + r$, $0 \le r < b$, *for each* a *and* b *listed below:*

(a) $a = 20$, $b = 3$.

(b) $a = 54$, $b = 7$.

Problem 8. *Show that when the square of an odd integer is divided by 8, the remainder is 1. (Hint: remember* $2|n(n+1)$*.)*

Problem 10. *Let* $n, m \in \mathbb{N}$ *with* $m \neq 1$. *Show* n *can be uniquely written in the form* $n = \sum_{k=0}^{N} a_k m^k$ *for some* $N \in \mathbb{Z}_{\geq 0}$ *and* $a_k \in \{0, 1, \ldots, (m-1)\}$ *with* $a_N \neq 0$. *Hint: Use induction on* n *and begin by choosing the largest* $N \in \mathbb{Z}_{\geq 0}$ *so that* $m^N \leq n$. *Use the Division Algorithm to write* $n = a_N m^N + r$ *and then apply the inductive hypothesis to* r.

2 Divisors

Problem 12. *List all of the divisors of the following:*

(a) *52,*

(b) ...

Problem 14. *Evaluate the following:*

(a) *(42,56),*

(b) ...

Problem 17. *Let* $b, q, r \in \mathbb{Z}$ *and let* $a = bq + r$ *with* a *and* b *not both* 0.

(a) *Show a common divisor of* a *and* b *is a divisor of* r *and that a common divisor of* b *and* r *is a divisor of* a.

(b) *Conclude that* $(a, b) = (r, b)$

Problem 19. *Let* $a, b \in \mathbb{Z}$, *not both 0.*

(a) *If* $(a, b) = d$, *then* $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$. *Hint: Write* $ax + by = d$ *so that* $\frac{a}{d}x + \frac{b}{d}y = 1$.

(b) ...