The following is a compiled review for the second exam according to the study guide.

2 Groups

A Definitions

Exercise 2.72 (Normalizer $N(S)$)**.** *Let G be a group and let $S \subseteq G$ be nonempty. The normalizer of S in G is $N(S) = N_G(S) = \{g \in G \mid gSg^{-1} = S\}$...*

Exercise 2.73 (Commutator)**.** *Suppose G is a group and let G' be the subgroup generated by $\{g_1 g_2 g_1^{-1} g_2^{-1} \mid g_i \in G\}$, called the commutator subgroup of G...*

Definition 2.1 (Torsion)**.** *Let G be an abelian group and let T be the set of all elements of G with finite order; T is a subgroup called the torsion subgroup.*

Definition 2.20 (Homomorphism, Isomorphism, Automorphism)**.** *Let G and H be groups and let $\varphi : G \to H$ be a function.*

1. *If $\varphi(g_1 g_2) = \varphi(g_1)\varphi(g_2)$ for all $g_1, g_2 \in G$, then φ is a homomorphism.*
2. *A bijective (one-to-one and onto) homomorphism is called an isomorphism. In that case, G amd H are said to be isomorphic and we write $G \cong H$.*
3. *If $G = H$, an isomorphism is also called an automorphism.*

Definition 2.22 (Conjugation Map)**.** *Let G be a group and let $g, h \in G$. The conjugation map $c_g : G \to G$ is defined by $c_g(h) = ghg^{-1}$.*

Definition 2.24 (Image and Kernel)**.** *Let $\varphi : G \to H$ be a homomorphism between groups.*

1. *The image of φ is $\operatorname{Im} \varphi = \{\varphi(g) \mid g \in G\}$.*
2. *The kernel of φ is $\ker \varphi = \{g \in G \mid \varphi(g) = e_H\}$.*

Definition 2.27 (Cosets)**.** *Let G be a group and let H be a subgroup of G. Let $g \in G$.*

1. *The left coset of g with respect to H is $gH = \{gh \mid h \in H\}$.*
2. *If C is a left coset with respect to H and $C = gH$, then g is called a representative of C.*
3. *The set of left cosets is denoted G/H and is called the quotient of G by H.*
4. *The index of H in G is $|G/H|$ and is denoted $[G : H]$.*

Definition 2.30 (Normality)**.** *Let G be a group, let H be a subgroup of G, and let $g \in G$.*

1. *The right coset of g with respect to H is $Hg = \{hg \mid h \in H\}$.*
2. *H is called normal if $gHg^{-1} \subseteq H$ for all $g \in G$ where $gHg^{-1} = \{ghg^{-1} \mid h \in H\}$.*

Theorem 2.31 (Normality T.F.A.E.s). *The following are equivalent:*

1. H *is normal.*
2. $gHg^{-1} = H$ *for all* $g \in G$.
3. *Left cosets are right cosets; i.e.,* $gH = Hg$ *for all* $g \in G$.
4. $gH \subseteq Hg$ *for each* $g \in G$.
5. $Hg \subseteq gH$ *for each* $g \in G$.
6. *The equation* $(g_1H)(g_2H) = (g_1g_2)H$ *gives a well-defined binary operation on* G/H *where* $g_1, g_2 \in G$.
7. $ghg^{-1} \in H$ *for all* $g \in G, h \in H$.

Definition 2.32. *Let* G *be a group and let* H *be a normal subgroup of* G. *Under the bilinear operation* $(g_1H)(g_2H) = (g_1g_2)H$ *for* $g_1, g_2 \in G$, G/H *is a group and is called the* quotient group *or* factor group.

Exercise 2.151. *Let* N *and* H *be groups along with a homomorphism* $\varphi : H \to \mathrm{Aut}(N)$. *For* $n \in N$ *and* $h \in H$, *write* $\varphi_h(n)$ *for* $(\varphi(h))(n)$. *Define the* semidirect product *of* N *and* H *as*

$$N \rtimes H = \{(n, h) \mid n \in N, h \in H\}$$

with a group law given by

$$(n_1, h_1)(n_2, h_2) = (n_1\varphi_{h_1}(n_2), h_1h_2) \ldots$$

B Book Proofs

C Homework Exercises

D Homework Proofs

E New Exercises