Note: unrequired parts of multi-part problems are listed obfuscated as ... to recognize they are multi-parted.

Problem 0.1.

Answer here...

1 Arithmetic

1.2 Modular Arithmetic

1.2.4 Structure

Problem 1.48. *List all units and (separately) all zero divisors of:*

(a) $\mathbb{Z}_3$,

(b) $\mathbb{Z}_4$,

(c) $\mathbb{Z}_7$,

(d) $\mathbb{Z}_8$,

(e) $\mathbb{Z}_{12}$,

(f) $\mathbb{Z}_{15}$.

Answer here...

Problem 1.49. *Determine how many solutions there are to the following equations.*

(a) $[2]x = [9]$ *in* $\mathbb{Z}_7$.

(b) $[15]x - [1] = [5]$ *in* $\mathbb{Z}_{25}$.

(c) $[2]x + [2] = [3]$ *in* $\mathbb{Z}_6$.

(d) $[35]x - [20] = [20]$ *in* $\mathbb{Z}_{100}$.

Answer here...

Problem 1.50. *Using the proof of Theorem 1.26, write down all solutions to the following equations.*

(a) $[3]x - [1] = [2]$ *in* $\mathbb{Z}_{12}$.

(b) ...

Answer here...

Problem 1.51. *Let* $n \in \mathbb{N}$, $n \geq 2$. *This exercise proves* Wilson's Theorem, *which says* $(n-1)! \equiv -1 \pmod{n}$ *if and only if* n *is prime.*

(a) If n *is not prime, show that a factor of* n *appears in* $(n-1)!$. *Conclude that* $[(n-1)!]$ *is a zero divisor and that* $(n-1)! \not\equiv -1$.

(b) If $n = p$ *is prime, show the only solutions to* $x^2 = [1]$ *in* $\mathbb{Z}_p$

Answer here...

1.2.5 Applications

1.2.6 Equivalence Relations