MATH 4329
HOMEWORK #1

(ALL FIVE PROBLEMS ARE DUE WEDNESDAY, JANUARY 29 BY THE END OF OUR CLASS)

SHOW ALL WORK

1. Prove that if $z_1, z_2 \in \mathbb{C}$ and $z_1 z_2 = 0$ then either $z_1 = 0$ or $z_2 = 0$.

2. Do each of the following:

   (a) Write the complex number $\dfrac{1+2i}{3+4i}$ in the form $a + bi$.

   (b) Find $\text{Re}\left(\dfrac{2-i}{2+i}\right)$ and $\text{Im}\left(\dfrac{2-i}{2+i}\right)$.

3. Prove that if $|z| = 2$, then

$$\frac{1}{|z^4 - 4z^2 + 3|} \leq \frac{1}{3}.$$

   (Hint: factor $z^4 - 4z^2 + 3$ and then use the reverse triangle inequality.)

4. Prove:

   (a) z is real if and only if $z = \overline{z}$.

   (b) z is either real or purely imaginary if and only if $(\overline{z})^2 = z^2$.

5. With $z = x + yi$, write the equation $|2\overline{z} + i| = 4$ in terms of x and y. Sketch the graph of this equation and identify what kind of equation it is.