Practice Paper

Design and Analysis of Algorithms

Q1 Solve the following recurrence equations. Make your asymptotic bounds as tight as possible

(i) $T(n) = T(9n/10) + n$

(ii) $T(n) = 2T(n/5) + n^{\log_5 2}$

(iii) $T(n) = 2T(\sqrt{n}) + \lg n$

(iv) $T(n) = 2T(n/4) + \sqrt{n}$

Q2 Let $n = 2^k - 1$. An array $A[1 \ldots n]$ contains all integers from 0 to 1 except one. The elements of A are stored as k bit vectors. Assume that only operation we can use to examine the integers is $Bit_Lookup(i, j)$ which returns j^{th} bit of $A[i]$. Each $Bit_Lookup(i, j)$ operation takes constant time. Design a $O(n)$ time algorithm to find the missing integer.

Q3 Show how quick sort can be made to run in $O(n \lg(n))$ time in the worst case.

Q4 Let $X[1 \ldots n]$ and $Y[1 \ldots n]$ be two arrays each containing n numbers already in sorted order. Give an $O(\lg(n)$ time algorithm to find the median of $2n$ elements in arrays X and Y.

Q5 For a given pattern *aabaabaaa* construct a shift table that cab be used to apply string matching from any given text using Knuth-Morris-Prat algorithm.

Q6 Design a $\theta(n^{\lg 3})$ algorithm that computes the multiplication of two polynomials of degree $n - 1$,

$P(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1x^1 + a_0$
$P(x) = b_{n-1}x^{n-1} + b_{n-2}x^{n-2} + \ldots + b_1x^1 + b_0$

Q7 Given a weighted directed graph $G = (V, E, w)$ and a shortest path p from s to t, if we doubled the weight of every edge to produce $G' = (V, E, w')$, then explain if p is also a shortest path in G'? What if weights on edges are increased by 1 ?

Q8 Give an algorithm that determines whether or not a given undirected graph $G = (V, E)$ contains a cycle. Your algorithm should run in $O(V)$ time independent of $|E|$.

Q9 Explain how a vertex u of a directed graph can end up in a depth first tree containing only u even though u may have both incoming and outgo ing edges in G.

Q10 Suppose all the weights in a graph G are in the range of 1 to $|V|$. How can you make Prim's algorithm run fast for finding MST. What is the weights are in the range 1 to W for some positive integer W.

Q11 Let $G = (V, E)$ be a weighted directed graph with non-negative weight function $w : E \longrightarrow \{0, 1, \ldots, W\}$ for some non-negative integer W. Modify Dijkstra's algorithm to compute the shortest path from a given source s in $O(WV + E)$ time.

Q12 Can you describe an algorithm for finding maximum spanning tree ? What about finding the longest shortest path ? Justify your answer.

Q13 Show that problem of finding clique of an undirected graph is NP complete.

Q14 How can you find transitive closure of a weighted directed graph ?