Tutorial 3

Exercise 4: The Lotka-Volterra equations

The Lotka-Volterra equations (also known as the predator-prey equations) model the dynamics of an ecosystem in which a species of predators and a species of prey coexist. The abundance of the predators depends on the population of the prey and vice versa. The equations are given as:

$$\frac{dx}{dt} = \alpha x - \beta xy, \tag{1a}$$

$$\frac{dy}{dt} = \delta xy - \gamma y, \tag{1b}$$

where x represents the population of the prey, y the population of the predators, and t the time. α, β, δ, and γ are constants that determine the interaction between predators and prey.

Tasks:

1. Find out what are the physical meaning behind the constants α, β, δ, and γ in the model. Can you develop an intuition of what the equations model? (Hint: the reference helps.)
2. Discretise equation (1) with the Störmer-Verlet method. (Hint: It may be easier to discretise the equation with the explicit Euler method as a first step.)
3. Implement the code required to model equation (1).
4. With the initial conditions given in the section "A simple example" of the reference, are you able to reproduce the figure on the population dynamics of the cheetahs and the baboons?

   For ease of reference, the initial conditions and the figure are reproduced below. Letting x be the baboons and y the cheetahs,

$$\alpha = 1.1, \qquad \beta = 0.4, \qquad \delta = 0.1, \qquad \gamma = 0.4,$$
$$x^0 = 10, \qquad y^0 = 10. \tag{2}$$

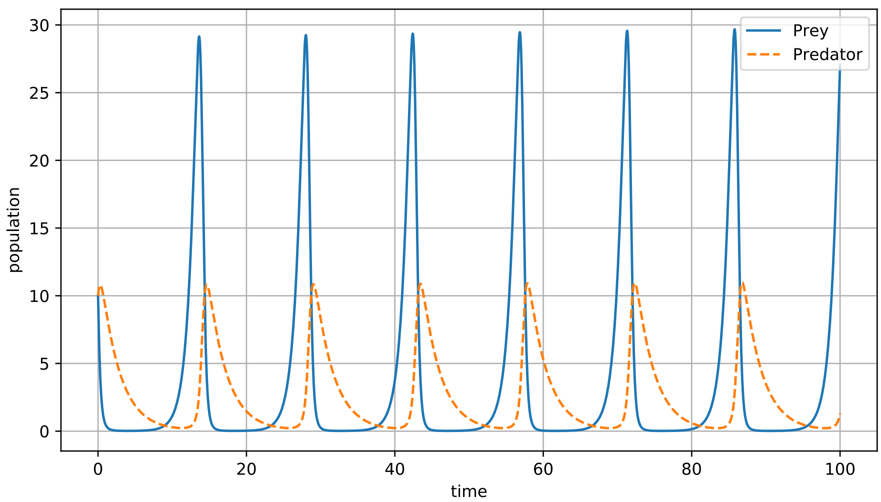

Figure 1: Dynamics of the cheetah and baboon populations following the Lotka-Volterra model and the initial conditions given in (2). Can you reproduce this plot? [source]

5. Does the result fit your intuition of the physical model?
6. Play around with different experimental parameters to get an intuition of the dynamics modelled by the Lotka-Volterra model, as well as the limits of the Störmer-Verlet numerical method.

Reference

https://en.wikipedia.org/wiki/Lotka-Volterra_equations