EE5027 Adaptive Signal Processing
Final Project: Adaptive Beamforming

Notice

- **Due at 9:00pm, December 29, 2022 (Thursday)** $= T_d$ for the electronic copy of your final report.
- Please submit **your report (.pdf file)**, **all your MATLAB files (.m files)**, **all your MATLAB figures (.fig files)**, and **your results (.mat file)** to NTU COOL (`https://cool.ntu.edu.tw/courses/15994`)
- Please **type your final report**. This report can be written either in English or in Chinese.
- No extensions, unless granted by the instructor one day before T_d.

Introduction

In this course, we went through several adaptive filters, such as LMS adaptive filters, RLS adaptive filters, and Kalman filters. We also introduced the array data model and adaptive beamforming. In this final project, we will explore a numerical example that combines all these topics together.

Recently, communications with *moving sources* have become a significant research topic. This scenarios find applications in self-driving vehicles, satellite communications, and unmanned aerial vehicles (UAV). In this applications, it is common that the signals are emitted from a moving source. If we design adaptive algorithms and adaptive beamformers properly, it might be possible to recover the source signals emitted from moving objects.

Problem Formulation

We consider a uniform linear array with N elements and the inter-element spacing $d = \lambda/2$. The output $\mathbf{x}(t)$ of this array is modeled as

$$\mathbf{x}(t) = \mathbf{a}(\theta_{\mathrm{s}}(t))\mathfrak{s}(t) + \mathbf{a}(\theta_{\mathrm{i}}(t))\mathfrak{i}(t) + \mathbf{n}(t), \tag{1}$$

where the time index $t = 1, 2, \ldots, L$. The steering vector is defined as

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 & e^{j\pi\sin\theta} & e^{j2\pi\sin\theta} & \ldots & e^{j(N-1)\pi\sin\theta} \end{bmatrix}^T. \tag{2}$$

The source signal is $\mathfrak{s}(t)$, the interference signal is $\mathfrak{i}(t)$, and the noise vector is $\mathbf{n}(t)$. We have some prior knowledge about (1):

- The source DOA $\theta_{\mathrm{s}}(t)$ and the interference DOA $\theta_{\mathrm{i}}(t)$ vary with the time index t. However, the exact values of $\theta_{\mathrm{s}}(t)$ and $\theta_{\mathrm{i}}(t)$ are unknown. Instead, we have *noisy measurements* of these DOAs. These noisy measurements are contaminated by additive white real-valued Gaussian noise with zero mean.

- The noise term $\mathbf{n}(t)$ is a complex-valued circularly-symmetric white Gaussian vector with zero mean.
- The source signal $\mathfrak{s}(t)$, the interference signal $\mathfrak{i}(t)$, and the noise $\mathbf{n}(t)$ are uncorrelated.
- For simplicity, the Doppler effect is omitted in (1).

According to (1) and the assumptions, three variables are given in `ASP_Final_Data.mat`:

$$\texttt{theta\_s\_noisy} = \begin{bmatrix} \widetilde{\theta}_{\mathrm{s}}(1) & \widetilde{\theta}_{\mathrm{s}}(2) & \ldots & \widetilde{\theta}_{\mathrm{s}}(L) \end{bmatrix}, \tag{3}$$

$$\texttt{theta\_i\_noisy} = \begin{bmatrix} \widetilde{\theta}_{\mathrm{i}}(1) & \widetilde{\theta}_{\mathrm{i}}(2) & \ldots & \widetilde{\theta}_{\mathrm{i}}(L) \end{bmatrix}, \tag{4}$$

$$\texttt{matX} = \begin{bmatrix} \widetilde{\mathbf{x}}(1) & \widetilde{\mathbf{x}}(2) & \widetilde{\mathbf{x}}(3) & \ldots & \widetilde{\mathbf{x}}(L) \end{bmatrix}, \tag{5}$$

where $\widetilde{\theta}_{\mathrm{s}}(t)$ represents the *noisy source DOA*, $\widetilde{\theta}_{\mathrm{i}}(t)$ stands for the *noisy interference DOA*, and $\widetilde{\mathbf{x}}(t)$ denotes the *noisy array measurement* at time t.

Objective

Given the data in (3), (4), and (5), estimate the *the source DOA*, *the interference DOA*, and *the source signal*.

Guidelines for Your Final Report

Please address the following items:

1. (20 points) What are the details of these beamformers? Please describe the details using mathematical equations and proper explanations. Also define all the notations properly.
   - The beamformer with uniform weights
   - The beamformer with array steering
   - The MVDR beamformer
   - The LCMV beamformer
2. (15 points) Design an algorithm to denoise $\widetilde{\theta}_{\mathrm{s}}(t)$ and $\widetilde{\theta}_{\mathrm{i}}(t)$. The denoised results are denoted by $\widehat{\theta}_{\mathrm{s}}(t)$ and $\widehat{\theta}_{\mathrm{i}}(t)$.
   - Describe the details of your algorithm using mathematical equations.
   - Summarize your algorithm similar to the format of that in Page 27 of `10_LS_RLS.pdf`.
   - What are the advantages of your algorithm?
3. (10 points) Plot the estimated DOAs $\widehat{\theta}_{\mathrm{s}}(t)$ and $\widehat{\theta}_{\mathrm{i}}(t)$ over the time index t, using your estimator in Item 2.
   - Please create only one plot in this figure. There are two curves for $\widehat{\theta}_{\mathrm{s}}(t)$ and $\widehat{\theta}_{\mathrm{i}}(t)$ with proper legends.

- **Save your figure to `ASP_Final_DOA.fig`.**

4. (15 points) Design a beamformer that extracts the source signal in (1). Your beamformer should be *different* from those in Item 1. Two beamformers are different if the weight vectors are different.
   - Elaborate on the details of your beamformer using mathematical equations.
   - Summarize your beamformer similar to the format of that in Page 27 of `10_LS_RLS.pdf`.
   - What are the advantages of your beamformer?

5. (10 points) Plot the *real and imaginary parts* of estimated source signal $\widehat{s}(t)$ over the time index t, using your beamformer in Item 4.
   - Please create two subplots in one figure. One subplot for the real part and the other for the imaginary part.
   - **Save your figure to `ASP_Final_Source.fig`.**

Performance Evaluation

We will assess your estimation performance according to **the source DOA**, **the interference DOA**, and the **source signal**. Please submit your processed data in a mat file. This file contains three vectors `theta_s_hat`, `theta_i_hat`, and `s_t_hat`. These vectors are defined as

$$\texttt{theta\_s\_hat} \triangleq \begin{bmatrix} \widehat{\theta}_{\mathrm{s}}(1) & \widehat{\theta}_{\mathrm{s}}(2) & \dots & \widehat{\theta}_{\mathrm{s}}(L) \end{bmatrix}, \tag{6}$$

$$\texttt{theta\_i\_hat} \triangleq \begin{bmatrix} \widehat{\theta}_{\mathrm{i}}(1) & \widehat{\theta}_{\mathrm{i}}(2) & \dots & \widehat{\theta}_{\mathrm{i}}(L) \end{bmatrix}, \tag{7}$$

$$\texttt{s\_t\_hat} \triangleq \begin{bmatrix} \widehat{s}(1) & \widehat{s}(2) & \dots & \widehat{s}(L) \end{bmatrix}, \tag{8}$$

where $\widehat{\theta}_{\mathrm{s}}(t)$ is the estimated source DOA, $\widehat{\theta}_{\mathrm{i}}(t)$ is the estimated interference DOA, and $\widehat{s}(t)$ is the estimated source signal. We will compare your results with the *true* DOAs and signals. Let $\theta_{\mathrm{s}}(t)$, $\theta_{\mathrm{i}}(t)$, and $s(t)$ be the true parameters. We define three vectors

$$\texttt{theta\_s} \triangleq \begin{bmatrix} \theta_{\mathrm{s}}(1) & \theta_{\mathrm{s}}(2) & \dots & \theta_{\mathrm{s}}(L) \end{bmatrix}, \tag{9}$$

$$\texttt{theta\_i} \triangleq \begin{bmatrix} \theta_{\mathrm{i}}(1) & \theta_{\mathrm{i}}(2) & \dots & \theta_{\mathrm{i}}(L) \end{bmatrix}, \tag{10}$$

$$\texttt{s\_t} \triangleq \begin{bmatrix} s(1) & s(2) & \dots & s(L) \end{bmatrix}. \tag{11}$$

We will evaluate your grade in this part by the MATLAB codes

```
% theta_s_hat: 10 points
max([ round( 10 * ( 1 - norm(theta_s_hat(:) - theta_s(:)) / norm(theta_s) ) ), 0])
% theta_i_hat: 10 points
max([ round( 10 * ( 1 - norm(theta_i_hat(:) - theta_i(:)) / norm(theta_i) ) ), 0])
% s_t_hat: 10 points
max([ round( 10 * ( 1 - norm(s_t_hat(:) - s_t(:)) / norm(s_t) ) ), 0])
```

In other words, if your results are closer to the true parameters, then you get more grades.

Last updated December 7, 2022.