

The method is more suitable for a gray image. The result is usually over-segmented.

It is based on the concept that:

- (1) The center of a region should have small gradient.
- (2) The boundary of regions should have large gradient.

(Step 1) Determine the gradient of an image.

Perform edge detection along x-axis and y-axis and obtain g_x , g_y , respectively.

Then calculate

$$g[m, n] = (g_x[m, n]^2 + g_y[m, n]^2)^{0.5}$$

(Step 2) Quantize $g[m, n]$ into several level

$$L[m, n] = \text{round}(g[m, n]/Q)$$

Q can be adjusted to obtain a better result.

(ex: $Q=3$)

(Step 3) For the case of $L == 0$, perform binary segmentation

Suppose that the output of the 2nd step is

$$L = \begin{bmatrix} 2 & 1 & 3 & 3 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 3 & 2 & 0 & 0 & 1 \\ 3 & 4 & 4 & 5 & 2 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 3 & 3 \\ 0 & 1 & 2 & 4 & 2 & 2 & 3 \\ 0 & 1 & 3 & 3 & 2 & 1 & 1 \end{bmatrix}$$

We first segment it into 3 regions

$$L = \begin{bmatrix} 2 & 1 & 3 & 3 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 3 & 2 & 0 & 0 & 1 \\ 3 & 4 & 4 & 5 & 2 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 3 & 3 \\ 0 & 1 & 2 & 4 & 2 & 2 & 3 \\ 0 & 1 & 3 & 3 & 2 & 1 & 1 \end{bmatrix}$$

region 1: orange; region 2: blue; region 3: red

Binary segmentation can be done by

for Matlab: `R = bwlabel(L==0)`

for Python: `from skimage import measure`

`R = measure.label(L==0)`

(Step 4) Increase the level by 1 (level=level + 1 or level +=1)

(Step 5) Assign the region number for the pixel that satisfies $L[m, n] = \text{level}$

(Step 5-1) If $L[m, n] = \text{level}$ and it is adjacent to some existing region, then we classify it into the region.

$$L = \begin{bmatrix} 2 & 1 & 3 & 3 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 3 & 2 & 0 & 0 & 1 \\ 3 & 4 & 4 & 5 & 2 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 3 & 3 \\ 0 & 1 & 2 & 4 & 2 & 2 & 3 \\ 0 & 1 & 3 & 3 & 2 & 1 & 1 \end{bmatrix}$$

(Step 5-2) If some pixel with $L[m, n] = \text{level}$ is adjacent to two or more regions, we can assign the priority (For example, the order of priority can be assigned according to the **difference of level**. If the differences of level are the same, we can assign **the prior according to the direction**. In this example, the prior according to the direction is up, down, left, right).

$$L = \begin{bmatrix} 2 & 1 & 3 & 3 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 3 & 2 & 0 & 0 & 1 \\ 3 & 4 & 4 & 5 & 2 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 3 & 3 \\ 0 & 1 & 2 & 4 & 2 & 2 & 3 \\ 0 & 1 & 3 & 3 & 2 & 1 & 1 \end{bmatrix}$$

(Step 5-3) If some pixel with $L[m, n] = \text{level}$ has not been assigned by Steps (5-1) and (5-2), repeat Steps (5-1) and (5-2) again and again until no more pixel with $L[m, n] = \text{level}$ can be assigned to some region.

$$L = \begin{bmatrix} 2 & 1 & 3 & 3 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 3 & 2 & 0 & 0 & 1 \\ 3 & 4 & 4 & 5 & 2 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 3 & 3 \\ 0 & 1 & 2 & 4 & 2 & 2 & 3 \\ 0 & 1 & 3 & 3 & 2 & 1 & 1 \end{bmatrix}$$

(Step 6) After performing step (Step 5), if some pixel with $L[m, n] = \text{level}$ can not be assigned to any region, we can treat them as new regions.

$$L = \begin{bmatrix} 2 & 1 & 3 & 3 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 3 & 2 & 0 & 0 & 1 \\ 3 & 4 & 4 & 5 & 2 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 3 & 3 \\ 0 & 1 & 2 & 4 & 2 & 2 & 3 \\ 0 & 1 & 3 & 3 & 2 & 1 & 1 \end{bmatrix} \quad \text{region 4: pink}$$

(Step 7) Repeat Steps 4, 5, and 6 again and again. Then all the pixels in the image can be assigned to some region.

Level = 2

$$L = \begin{bmatrix} 2 & 1 & 3 & 3 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 3 & 2 & 0 & 0 & 1 \\ 3 & 4 & 4 & 5 & 2 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 3 & 3 \\ 0 & 1 & 2 & 4 & 2 & 2 & 3 \\ 0 & 1 & 3 & 3 & 2 & 1 & 1 \end{bmatrix}$$

Level = 3

$$L = \begin{bmatrix} 2 & 1 & 3 & 3 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 3 & 2 & 0 & 0 & 1 \\ 3 & 4 & 4 & 5 & 2 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 3 & 3 \\ 0 & 1 & 2 & 4 & 2 & 2 & 3 \\ 0 & 1 & 3 & 3 & 2 & 1 & 1 \end{bmatrix}$$

Level = 4

$$L = \begin{bmatrix} 2 & 1 & 3 & 3 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 3 & 2 & 0 & 0 & 1 \\ 3 & 4 & 4 & 5 & 2 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 3 & 3 \\ 0 & 1 & 2 & 4 & 2 & 2 & 3 \\ 0 & 1 & 3 & 3 & 2 & 1 & 1 \end{bmatrix}$$

Level = 5

$$L = \begin{bmatrix} 2 & 1 & 3 & 3 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 3 & 2 & 0 & 0 & 1 \\ 3 & 4 & 4 & 5 & 2 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 3 & 3 \\ 0 & 1 & 2 & 4 & 2 & 2 & 3 \\ 0 & 1 & 3 & 3 & 2 & 1 & 1 \end{bmatrix}$$