HW2 Part I

Show

$$\exp[-\frac{1}{2}((\sum_{i=1}^{n}\phi(x_i-\theta)^2)+\tau(\theta-\theta_0)^2)]$$
$$\propto \exp[-\frac{1}{2}(\tau+n\phi)(\theta-\frac{1}{\tau+n\phi}(\tau\theta_0+\phi\sum_{i=1}^{n}x_i))^2]$$

$$\exp[-\frac{1}{2}((\sum_{i=1}^{n}\phi(x_i-\theta)^2)+\tau(\theta-\theta_0)^2)]$$
$$=\exp[-\frac{1}{2}(\sum_{i=1}^{n}(\phi x_i^2-2\phi x_i\theta+\phi\theta^2)+(\tau\theta^2-2\tau\theta\theta_0+\tau\theta_0^2))]$$
$$=\exp[-\frac{1}{2}(\phi\sum_{i=1}^{n}x_i^2-2\phi\theta\sum_{i=1}^{n}x_i+n\phi\theta^2+\tau\theta^2-2\tau\theta\theta_0+\tau\theta_0^2)]$$
$$=\exp[-\frac{1}{2}(\theta^2(\tau+n\phi)-2\theta(\tau\theta_0+\phi\sum_{i=1}^{n}x_i)+(\tau\theta_0^2+\phi\sum_{i=1}^{n}x_i^2))]$$
$$=\exp[-\frac{1}{2}(\theta^2(\tau+n\phi)-2\theta(\tau\theta_0+\phi\sum_{i=1}^{n}x_i))]\exp[-\frac{1}{2}(\tau\theta_0^2+\phi\sum_{i=1}^{n}x_i^2)]$$
$$\propto \exp[-\frac{1}{2}(\theta^2(\tau+n\phi)-2\theta(\tau\theta_0+\phi\sum_{i=1}^{n}x_i))] \qquad (\exp[-\tfrac{1}{2}(\tau\theta_0^2+\phi\textstyle\sum_{i=1}^{n}x_i^2)] \text{ is constant})$$
$$=\exp[-\frac{1}{2}((\tau+n\phi)(\theta-\frac{(\tau\theta_0+\phi\sum_{i=1}^{n}x_i)}{2(\tau+n\phi)})^2)]$$
$$=\exp\left[-\frac{1}{2}(\tau+n\phi)\left(\theta-\frac{1}{\tau+n\phi}\left(\tau\theta_0+\phi\sum_{i=1}^{n}x_i\right)\right)^2\right]$$